Unplanned Purchases and Retail Competition

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Abstract. I propose a framework in which consumers have biased beliefs about their future purchase probabilities, leading them to make unplanned purchases. Using this framework, I examine the phenomenon of loss leading and the welfare effects of banning it. When the market is covered, a ban on below-cost pricing raises total surplus, although consumers need not gain—but they are guaranteed to benefit if the market is perfectly competitive. When the market is not covered, a ban always hurts at least some consumers, but may help others. Also, there is a pro-competitive justification for the observation that larger firms sometimes charge below cost on the core product lines of smaller firms—consumer bias is crucial for this justification.

Multi-product retailers sometimes sell certain products beneath cost, with the hope of attracting additional customers who will also buy other, higher-margin items. This practice, often referred to as “loss leading,” is controversial, with below-cost pricing having been banned in a number of countries, including Belgium, Germany, France, and Ireland, and in approximately half of all US states.† Many US cities also have such prohibitions.

In this article I examine the welfare consequences of prohibitions on below-cost pricing, in a market where multi-product retailers compete for boundedly rational consumers who practice one-stop shopping (that is, visit at most one store). My results indicate that it is simplistic to argue that, by limiting the scope of competition, below-cost pricing bans must hurt consumers and society. Indeed, when rivals are symmetric and the market is covered (so that all consumers shop), a ban on loss leading raises total surplus. When the market is not covered, a ban presents a social welfare tradeoff: fewer consumers choose to shop, although those who do generate higher levels of social surplus. Perhaps surprisingly, for those consumers who shop both before and after a ban, a ban is guaranteed to make them better off if the market is perfectly competitive, but otherwise may harm them.

But I also assess below-cost pricing laws in the presence of asymmetric competition, as when a “large” firm with a full product line competes against a “small” firm with a limited product line. In a number of markets including that for groceries, a serious concern of antitrust authorities is that larger firms often price below cost on the core product lines of smaller

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†See the prepared testimony, “The Impact of Sales-Below-Cost Laws on the U.S. Retail Gasoline Market,” by Ronald N. Johnson, from February 1999. As of 1992, a total of twenty-two US States had general below-cost pricing laws. Some states without such general laws nonetheless had laws that specifically banned the practice for gasoline.
rivals. I provide an explanation for this phenomenon, showing that it need not be anti-competitive. Indeed, in my analysis, banning loss leading among asymmetric competitors can lower total surplus and harm all consumers, even when the market is covered.

Before further discussing my results, I explain what drives loss leading in my model. I assume that consumers make “unplanned purchases,” systematically purchasing more types of goods than anticipated; consumers have ex-ante biased beliefs about their own propensity for future purchases. For example, a consumer on his way to a grocery store may intend to buy only select items, but walk out with additional ones. He might buy these extra items because being in the store reminds him of pre-existing needs about which he had forgotten, or because of new needs created while at the store. Many people have probably experienced some type of unplanned purchasing personally, and so the basic idea is easy to understand. Additionally, this phenomenon is widely documented and seemingly sizable in magnitude, with some studies suggesting that more than 60% of purchases are unplanned (see Kollat and Willett (1967), Park, Iyer, and Smith (1989), and Heilman, Nakamoto, and Rao (2002)).

A crucial preliminary result is that loss leading never arises unless consumers make unplanned purchases, when demand is independent across goods. Indeed, for below-cost pricing to exist, consumers need not only have biased beliefs about their purchase probabilities, but must have asymmetrically biased beliefs. Thus, for example, it must be that consumers are (correctly) confident that they need some particular product, such as milk or bread (which will be priced below cost), but underestimate their tendency to purchase other goods.

Although it is intuitive that consumer bias influences which products are priced below cost as just suggested, I show a much stronger and surprising result, which is that such bias is the only thing that matters. That is, factors such as (arbitrary) differences in the shapes of demand and marginal costs of the different products, or how important goods are in an absolute sense for a consumer’s overall utility, are effectively irrelevant in determining the set of loss leaders—only a simple measure of consumer bias matters.

I now provide a bit more detail about the effects of a loss-leading ban, beginning with the case of symmetric firms. As mentioned above, when the market is covered, a ban raises total surplus. The reason is that below-cost pricing is a socially inefficient means of attracting consumers, which firms resort to only because consumers have biased beliefs. Limiting the ability of firms to so distort their prices—by imposing a ban on loss leading—necessarily raises the prices of goods that were priced below cost, but also leads to lower prices on goods that were priced above cost. This price compression increases the overall efficiency of any customer’s visit to a store. However, because a ban rules out a pricing practice that

\[2\text{Keep in mind that consumers who are one-stop shoppers choose where to shop based on the overall utility they believe a visit will give them, and similarly pricing below cost to attract multi-stop shoppers cannot be profitable, so the explanation for why larger firms charge less on products also carried by smaller firms is not as simple as there being more competitors supplying that product.}\]
is effective at attracting consumers with biased beliefs, consumers contemplating a trip to a store believe they are worse off under the post-ban prices than under the pre-ban prices, meaning that fewer consumers actually visit a store if the market is not covered. This is bad for overall welfare. Interestingly, consumers who continue shopping after a ban can be either better or worse off, with respect to their true (that is, unbiased) demands. That is, they may or may not be beneficiaries of the increase in total surplus that their visit to a store generates. In the case of perfect competition, however, such consumers are indeed better off.

Now consider the case in which competition is among firms with asymmetric product lines. Chen and Rey (2012) identify the importance of this matter, noting that the prevalence of loss leading among asymmetric rivals poses important questions for antitrust authorities. For example, many large supermarkets compete primarily against smaller grocers, and utilize loss-leader strategies. The United Kingdom Office of Fair Trading reports that such behavior is extremely common among all leading UK grocers, and likely growing in prevalence due to continuing consolidation in the sector. However, significant controversy remains as to whether such a practice is a healthy manifestation of competition or instead a harmful act.

If loss leading is an innocuous manifestation of competition, then why are the products chosen as loss leaders by large firms often products that are also carried by smaller firms? Are larger firms trying to induce the exit of smaller rivals? Although a tempting conclusion, predation is inconsistent with the observed persistence of loss leading—a rational predator must expect eventually to recoup its losses.

My explanation for why larger firms charge below cost on the same goods that smaller firms carry builds on two key ideas. First, that consumers make unplanned purchases, and second, that product lines are endogenous. I show that smaller firms, in selecting which products to carry, optimally eschew higher priced goods exhibiting high levels of unplanned purchases, and instead specialize in lower priced goods for which consumers have unbiased beliefs. They do so even though they know these are the same goods that larger firms are pricing below cost, and even though they could instead carry products for which the larger firm charges higher prices. Similarly, large firms decide which products to price below cost based on the extent of consumer bias associated with the products, not based on which products the smaller firm carries. In other words, it is not the fact that small firms carry particular products that itself causes large firms to charge below cost on those same products. Instead, the same consumer bias that causes small firms optimally to carry certain products leads the larger firm to charge below cost on those same products.

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4For example, in its 2000 report, the United Kingdom Competition Commission reports that the same products regularly priced below cost by larger retailers are disproportionately relied upon by smaller firms to generate revenue.
Thus, it is underlying heterogeneity of the consumer bias associated with the products (not heterogeneity of the products themselves), along with the fact that product lines are endogenous, that explains the phenomenon. Building on this result, I show that banning loss leading among asymmetric rivals harms all consumers, rather than helping them as argued by Chen and Rey (2012), and lowers overall surplus.

Despite the policy importance and controversy surrounding below-cost pricing, there is little formal analysis of the effects of bans on such behavior. Instead, it is popularly suggested that either a ban restricts competition between firms and so hurts welfare, or is harmful predatory conduct. As Chen and Rey (2012) and I emphasize, the situation is more complex. Lal and Matutes (1994) examine competition when consumers cannot perfectly observe prices, and Ellison (2005) and Gabaix and Laibson (2006) consider the ideas of add-on pricing and product shrouding. Below-cost pricing can arise in these articles, but they do not focus on antitrust aspects such as the effects of banning loss leading. Grubb (2009, 2012) considers consumers with behavioral biases in the mobile-phone-service market, with Grubb (2009), for example, showing that such bias can lead firms to price below marginal cost on some units within a mobile-service plan. Armstrong and Chen (2012) discuss a behavioral motive for the practice of marketing a price at a discount to an earlier price.

The remainder of my paper is organized as follows. Section 1 presents the basic model of competition between firms with symmetric product lines. Below-cost pricing is considered in Section 2 (with symmetric firms) and Section 3 (with asymmetric firms and endogenous product lines). Section 4 presents some comparative statics on consumer bias.

### 1. Model

Consider the following single-period model of competition among multi-product retailers. There are $N \geq 2$ independent products (neither substitutes nor complements), each of which is sold by each of $M \geq 2$ differentiated retailers. Each retailer has access to product $n$ at constant marginal cost $c_n > 0$. There is a unit mass of consumers with ex-ante homogeneous preferences for the $N$ products, each of whom will shop at exactly one retailer.

The timing is as follows. First, each retailer $m$ simultaneously sets prices $\{p_n(m)\}_{n=1}^N$ for each of the $N$ goods. All prices are perfectly observed by consumers. Second, consumers decide which single retailer to visit. Third, each consumer learns about his demand and makes final purchasing decisions at the retailer he has chosen to visit.

A consumer who visits retailer $m$ purchases quantities $\{x_n\}_{n=1}^N$ to maximize

$$
\sum_n \xi_n [u_n(x_n) - p_n(m)x_n],
$$
where $\xi_n \in \{0, 1\}$ is a binary random variable that is realized after the consumer chooses a retailer but before final in-store purchasing decisions are made. Hence, for any good $n$, a consumer either has zero demand for it (so that $\xi_n = 0$) and so buys zero units, or instead has positive demand for it (so that $\xi_n = 1$) and so buys quantity $x_n$ to maximize $u_n(x_n) - p_n(m)x_n$, where $u_n$ is an increasing, continuous, and concave function.

Let $v_n(p_n(m))$ denote the indirect utility associated with product $n$ at retailer $m$, conditional on this consumer having positive demand for it ($\xi_n = 1$). That is,

$$v_n(p_n(m)) = \max_{x_n} [u_n(x_n) - p_n(m)x_n].$$

The values $\{\xi_n\}$ are realized independently of each other, and independently and identically across consumers (I will not introduce consumer-specific notation). The true probability that a consumer has positive demand for $n$ is given by $\theta_n$. That is, for any given consumer,

$$\Pr[\xi_n = 1] = \theta_n > 0.$$

The bulk of the analysis and results in this article hinge on the prospect that consumers may have ex-ante biased beliefs about their future propensity to consume. To this end I suppose that (prior to selecting a retailer) each consumer believes that he will have positive demand for product $n$ with some value $\hat{\theta}_n > 0$. I will say that consumers make unplanned purchases of product $n$ if $\theta_n \neq \hat{\theta}_n$. For each $n$, define the “accuracy ratio” $\alpha_n$ as

$$\alpha_n = \frac{\hat{\theta}_n}{\theta_n}.$$

These accuracy ratios are crucial in the analysis of this model. I assume that $\alpha_n \leq 1$ for each $n$, so that for a given good $n$ consumers either have unbiased beliefs ($\alpha_n = 1$) or underestimate their purchase probability ($\alpha_n < 1$) and so make unplanned purchases. This corresponds to a situation in which, for example, being in the store refreshes a consumer’s memory of his true needs, or in which being in the store creates entirely new needs.

Because consumers believe they will have positive demand for $n$ with probability $\hat{\theta}_n$, each consumer forecasts his expected “in-store utility” of shopping at retailer $m$ to be

$$\hat{U}_m = \sum_n \hat{\theta}_nv_n(p_n(m)).$$

Each consumer has idiosyncratic preferences for the $M$ retailers, and evaluates the attractiveness of any option $m$ by considering the sum of $\hat{U}_m$ and some random shock that is specific to the match of that consumer and firm.\footnote{Note that there is no signalling or other form of information transmission in this model—consumers are naive and act as if the values $\{\hat{\theta}_n\}$ accurately describe their future purchase probabilities. In some work on hyperbolic discounting, such as in O’Donoghue and Rabin (1999), consumers may be sophisticated and understand that they will not optimize in the future, leading them to different decisions in the present. The function $\hat{U}_m$ is similar to the decision function adopted by consumers in the study of “economic focusing” by Köszegi and Szieki (2013).} Because I will only focus on equilibria
that are symmetric or in which there are a total of two firms, the following specification is sufficient. The number of consumers shopping at firm \( m \) is given by \( y(\hat{U}_m, \hat{U}) \), if all firms \( k \neq m \) are offering forecast utility values \( \hat{U}_k = \hat{U} \). This function satisfies several properties. First, it is strictly decreasing in \( \hat{U} \). Second, it is increasing and log-concave in \( \hat{U}_m \). Third, letting \( y_1 \) denote the derivative with respect to the first argument, the function

\[
h(x) = \frac{y_1(x, x)}{y(x, x)}
\]

is weakly decreasing in \( x \).

These properties are consistent with both a covered Hotelling-line specification and with consumers having logit preferences among the retailers. In cases where consumers have an outside option, I suppose that they have unbiased forecasts of its value, given by \( \hat{U}_0 \). Thus, for example in the logit model with parameter \( \sigma > 0 \) and \( \hat{U}_0 = 0 \),

\[
y(\hat{U}_m, \hat{U}) = \frac{\exp(\hat{U}_m / \sigma)}{\exp(\hat{U}_m / \sigma) + (M - 1) \exp(\hat{U} / \sigma) + 1}.
\]

Note that in cases with no outside option (that is, with a covered market), \( h(x) \) is constant.

Firms know the true probabilities \( \{\theta_n\} \), but also know that consumers forecast their utility values \( \{\hat{U}_m\} \) based on the values \( \{\hat{\theta}_n\} \). As such, in a symmetric equilibrium in which firms \( k \neq m \) are choosing prices such that \( \hat{U}_k = \hat{U} \), and suppressing retailer-specific notation on prices where so doing causes no confusion, firm \( m \) chooses prices \( \{p_n\} \) to maximize

\[
\Pi_m = y(\hat{U}_m, \hat{U}) \sum_n \theta_n (p_n - c_n) x_n(p_n).
\]

(1)

To ensure that the maximization program of firms is well behaved, a minimal requirement is that the within-product maximization program is well behaved. To this end, define

\[
L_n(p_n) = \frac{p_n - c_n}{p_n} \epsilon_n(p_n), \quad \text{where} \quad \epsilon_n(p_n) = \frac{p_n x_n'(p_n)}{x_n(p_n)}.
\]

\( L_n \) is the Lerner index of good \( n \) multiplied by its elasticity, so that if the firm were simply maximizing \((p_n - c_n)x_n(p_n)\), it would set \( L_n = -1 \). Attention is restricted to positive prices, for otherwise demand would be infinite. Hence, \( \epsilon_n < 0 \). I assume that \( L_n(p_n) \) is decreasing in \( p_n \), which, along with the log-concavity of \( y \), ensures a unique solution in prices to each firm’s maximization program.\(^6\) \( L_n \) is decreasing, for example, if the underlying demand \( x_n \) exhibits constant elasticity or is linear.\(^7\) I assume a symmetric equilibrium exists.\(^8\)

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\(^6\)This is stronger than assuming concave demand. The reason is that concave demand only ensures that \( L_n \) is decreasing in the region where \( p_n > c_n \), but—as I will discuss at length later—the possibility that \( p_n < c_n \) for some \( n \) exists in this model. In the region \( p_n < c_n \), this requires that \( \epsilon_n' \) is not too large compared to \( \epsilon_n \).

\(^7\)Linear demand can be incorporated in this model for most values of \( p_n > 0 \), but near zero demand must become infinite. That is, demand can be linear outside of some tiny neighborhood around zero.

\(^8\)As shown in the beginning of the Appendix, if there is a symmetric equilibrium, then it is unique.
Suppressing retailer-specific notation on prices, define
\[ \pi_m = \sum_n \theta_n(p_n - c_n)x_n(p_n) \]
as the per-customer profits of firm \( m \). Hence the overall profits of \( m \) are
\[ \Pi_m = y(\hat{U}_m, \hat{U})\pi_m. \]
An increase in \( p_n \) has the following effect.
\[
\frac{d\Pi_m}{dp_n} = y(\hat{U}_m, \hat{U})\theta_n[x_n(p_n) + (p_n - c_n)x'_n(p_n)] + y_1(\hat{U}_m, \hat{U})\hat{\theta}_n \frac{dv_n(p_n)}{dp_n} \pi_m \\
= y\theta_n[x_n(p_n) + (p_n - c_n)x'_n(p_n)] - y_1\hat{\theta}_nx_n(p_n)\pi_m, \tag{2}
\]
using the fact that \( dv_n/dp_n = -x_n(p_n) \) and suppressing the arguments of \( y \) and \( y_1 \).

Recalling the definition of the accuracy ratio \( \alpha_n = \hat{\theta}_n/\theta_n \) for product \( n \), and building on Equation (2), if \( m \) is maximizing its profits then for each \( n \) it is the case that
\[
\frac{1}{\alpha_n}[1 + L_n(p_n)] = \frac{y_1\pi_m}{y}. \tag{3}
\]
Because the term on the right-hand side is identical for all \( n \), a necessary condition for firm maximization is that, for each \( n \) and \( k \),
\[
\alpha_k(1 + L_n(p_n)) = \alpha_n(1 + L_k(p_k)). \tag{4}
\]

2. Loss Leading among Symmetric Rivals

Laws banning below-cost pricing are common—do such laws help or harm society? Here I answer this question, assuming that firms are symmetric. Along the way, I assess when loss leading does or does not arise, and identify which products (if any) are loss leaders.

Although consumers choose where to shop based on their forecasts \( \{\hat{U}_m\} \), I take the perspective that the correct measure of in-store utility is that associated with the unbiased assessments \( \{U_m\} \), where \( U_m = \sum_n \theta_n v_n(p_n(m)) \). This is as if consumers are forgetful of their needs prior to visiting a store, or possibly have needs created at the store, and would not wish to avoid any ex-post purchases from an ex-ante perspective. Thus, putting aside a consumer’s idiosyncratic preferences amongst retailers, consumers would prefer to shop at the store with the highest value of \( U_m \).

2.1. When does loss leading occur? Loss leading never arises without consumer bias.

**Proposition 1.** Suppose that there is no consumer bias, so that \( \alpha_n = 1 \) for each \( n \). Then in equilibrium, no loss leading occurs: the equilibrium price \( p^*_n \) of good \( n \) exceeds its marginal
cost \( p^*_n \geq c_n \) for each \( n \). The same result holds if there is symmetric bias across all products, so that \( \alpha_n = \alpha \) for all \( n \) for some \( \alpha \).

Proposition 1 is especially interesting given that the model allows for significant asymmetry across products. Thus, neither differing shapes of demand nor differing marginal costs by themselves lead to loss leading. Likewise, the fact that demand is stochastic and that there is rivalry among firms is not enough. Rather, if loss leading is to exist—which it can—it must be that consumers make unplanned purchases.\(^9\)

To see why Proposition 1 is true, suppose there is no bias but that a firm is charging \( p_n < c_n \) for some \( n \). By instead setting \( p_n = c_n \), the firm clearly increases its profit per customer and raises total surplus per customer, but may lose some customers. However, if it also lowers the price of some non-loss leader \( k \), it will further raise overall surplus but increase utility \( U \), where \( U = \hat{U} \) because of the lack of bias. Thus, it need only lower \( p_k \) to the point that the consumer is as well off as originally, so that the same number of customers visit. At these prices, this firm’s profit must be higher because overall surplus is higher. (Because the lack of bias is used for this line of reasoning, loss leading cannot be universally ruled out.)

Proposition 1 further indicates that there must be asymmetric consumer bias for loss leading to exist. That is, there must be asymmetry of bias across products, so that \( \alpha_n \neq \alpha_k \) for some \( n \) and \( k \). Thus, if consumers are just generally bad at planning ahead, where this forgetfulness is not more severe for some products than others, no product is priced below cost. Instead, loss leading requires that consumers are, for example, ex-ante prone to forget their need for certain products but likely to remember it for others. This may be if the needs for certain staples are easier to remember than other needs.

If there is loss leading, which products are loss leaders? To investigate, reconsider the necessary conditions for optimality given in Equation (4),

\[ \alpha_k (1 + L_n(p_n)) = \alpha_n (1 + L_k(p_k)). \]

An optimizing firm chooses prices such that \( 1 + L_n(p_n) \geq 0 \) for each \( n \).\(^{10}\) This implies that

\[ L_n(p_n) \geq L_k(p_k) \iff \alpha_n \geq \alpha_k. \]

Recall that

\[ L_k(p_k) = \frac{p_k - c_k}{p_k} \epsilon_k(p_k). \]

\(^9\)Proposition 1 is consistent with Ambrus and Weinstein (2008), who find, with perfectly rational consumers with unit demands, that loss leading does not arise with independent demands. Bliss (1988) argues that a complementary demand structure can drive loss leading. DeGraba (2006) shows that loss leading can be a way of offering targeted discounts to more-profitable customers. These articles do not focus on the effects of banning below-cost pricing.

\(^{10}\)The reason is that \( L_n(p_n) < -1 \) corresponds to pricing above the level that maximizes per-customer profits for product \( n \). As such, the firm would do better by lowering \( p_n \), thereby increasing per-customer profit and its share of the overall customer base.
Because $\epsilon_k < 0$, below-cost pricing for $k$ is identical to $L_k > 0$. But this means that if $k$ is a loss leader, and if $\alpha_n \geq \alpha_k$, then $L_n(p_n) \geq L_k(p_k) > 0$, so that $n$ is also a loss leader. Thus, the following has been shown.\textsuperscript{11}

**Proposition 2.** If $k$ is a loss leader and $\alpha_n \geq \alpha_k$, then $n$ is a loss leader. Put differently, there is a value $\alpha^*$ such that $n$ is a loss leader if and only if $\alpha_n \geq \alpha^*$.

To emphasize the implications, suppose there are a total of two products, and that product 2 is priced below cost. Notably, then, the fact that product 2 is chosen as the loss leader is not driven by whether product 2 is more or less significant for the consumer’s overall utility, or whether the consumer’s demand function for 2 is more or less elastic than his demand for product 1. Similarly, whether product 2 would be more or less profitable to sell as a single-product firm than product 1, given whatever the demand functions and marginal costs are, is also irrelevant. Indeed, the levels of demand $\theta_1$ and $\theta_2$ are irrelevant, conditional on the accuracy ratios: product 2 instead of product 1 is the loss leader because $\alpha_2 > \alpha_1$.

Note further that the ranking of the accuracy ratios does not typically imply a ranking of prices, again emphasizing the strength of this result. That is, $\alpha_n > \alpha_k$ does not in general suggest whether $p_n > p_k$ or $p_n < p_k$. Instead, as demonstrated above, $\alpha_n > \alpha_k$ merely implies that the elasticity-weighted Lerner index of $n$ exceeds that of $k$, $L_n(p_n) > L_n(p_k)$, at the optimal prices. However, because $L_n$ crosses zero only at $c_n$ and $L_k$ crosses zero only at $c_k$, the characterization of loss leaders given in Proposition 2 emerges.

That said, a loose intuition behind Proposition 2 is as follows. A price cut is more effective at generating in-store traffic when customers expect that they will buy it, that is, when $\hat{\theta}_n$ is high. And such a price cut is less damaging to per-customer profits when it is less likely that consumers will actually buy it, that is, when $\theta_n$ is low. Each of these circumstances coincides with a high accuracy ratio $\alpha_n$, and so such products are natural loss leaders.

Proposition 2 provides an explanation of loss leaders that resonates well with the actual world. For example, among grocers actual loss leaders tend to be products that consumers purchase regularly, such as bread or milk, so that it is reasonable to expect that consumers have relatively accurate assessments of their needs for such goods (indeed, the need for such staples may well be the impetus for a visit to the store). Similarly, some gasoline retailers have attached convenience stores and use low prices on gas to drive sales in the convenience store. This makes sense because it seems likely that consumers accurately know when they need gas, but may not fully consider their need for other items.

\textsuperscript{11}Note that Proposition 1 is in fact a corollary of Proposition 2. I present them in the order I do because I feel it improves the overall intuition.
Proposition 2 is reminiscent of a key result that emerges in the study of nonlinear pricing with overconfident consumers by Grubb (2009). Grubb shows that if consumers are overconfident in the precision of their demand forecasts (such as for mobile phone usage), then firms price below marginal cost on units beneath some quantity and above marginal cost on units above that quantity. The logic builds on the fact that such consumers overestimate the probability that they will purchase at least some low level of units but underestimate the probability that they will purchase a high level, so that initial units are similar to goods that have high accuracy ratios in my model. DellaVigna and Malmendier (2004) and Eliaz and Spiegler (2006) also consider contracting with boundedly rational consumers, with DellaVigna and Malmendier (2004) showing that firms price below cost on “investment goods” that consumers pay for now but enjoy later, and price above cost on “leisure goods” that consumers enjoy now but pay for later. Thus, leisure goods correspond to goods with a low accuracy ratio in my model, whereas investment goods correspond to higher accuracy ratios.

I now confirm that below-cost pricing can exist in equilibrium. For emphasis, consider the benchmark case of perfect retailer competition, the equilibrium of which I define to be the set of prices that maximize $\hat{U}$, subject to firms earning zero profits: no retailer can do more to attract consumers without earning negative profits.

Remark 1. Suppose that there is perfect competition and that there is asymmetry in the accuracy ratios: $\alpha_n \neq \alpha_k$ for some $n$ and $k$. Then there is loss leading in equilibrium: there is some $\alpha^*$ such that $p_l < c_l$ if and only if $\alpha_l > \alpha^*$, where there is at least one such product.

When beliefs are (asymmetrically) biased, perfect competition does not lead to marginal-cost pricing. Rather, because perfectly competitive firms work to maximize the surplus that consumers believe they will receive, rather than the surplus that they will actually receive, there are incentives to diverge from marginal-cost pricing, and loss leading always exists.

2.2. Banning loss leading. Here I consider the effect of a ban on loss leading. I assume in this section that some below-cost pricing occurs in the absence of such a ban, which, given Proposition 1, requires asymmetry of the accuracy ratios (that is, $\alpha_n \neq \alpha_k$ for some $n$ and $k$). Denote the set of loss leaders by $\mathcal{L}$, so that

$$\mathcal{L} = \{n : p^*_n < c_n \text{ in the absence of a ban on loss leading}\}.$$ 

Let $U^*$ denote the unbiased forecast of equilibrium in-store utility, and $\hat{U}^*$ be a consumer’s biased equilibrium forecast of in-store utility. Also, let $\pi^*$ denote the equilibrium per-customer profit of firms (that is, the profit associated with any customer who actually shops).
Proposition 3. A ban on below-cost pricing has the following effects.

(1) (Price compression) Prices increase to marginal cost for products that were loss leaders, whereas prices decrease for products that were not loss leaders.

(2) (In-store efficiency) The social surplus created by each consumer who goes shopping increases. That is, \( \pi^* + U^* \) increases.

(3) (Softened competition)
   (a) The utility of shopping that consumers forecast, \( \hat{U}^* \), decreases. If the market is not covered, the number of consumers who shop decreases.
   (b) If the market is covered, per-customer profit \( \pi^* \) increases.

Proposition 3 indicates that there are competing welfare effects of a ban on below-cost pricing. The in-store-efficiency effect says that the surplus generated by each consumer who shops increases following a ban. However, the softened-competition effect says that \( \hat{U}^* \) decreases so that, unless the market is covered, fewer consumers actually shop following a ban. Moreover, if the market is covered, per-customer profits increase, suggesting that equilibrium in-store utility \( U^* \) might fall even though surplus rises.

To understand Proposition 3 and the resulting welfare implications in more detail, observe that the first part of the proposition indicates that a ban causes the price of each good to change, not just those of loss leaders: prices are compressed. The intuition is as follows. Because products in \( L \) were generating losses on a per-customer basis in the absence of a ban, firms had weak incentives to lower the prices of goods not in \( L \). Under a ban, products in \( L \) are priced at marginal cost and so generate zero profits on a per-customer basis, rather than a loss, so that firms have stronger incentives to lower the prices of other goods.

Naturally, price compression leads to an increase in in-store efficiency. That is, because both above-cost and below-cost pricing are inefficient from the standpoint of in-store surplus, price compression must raise the surplus generated by consumers who actually visit a store. It follows that, if the market is covered, then social surplus unambiguously increases following a ban on loss leading. As I discuss below, this conclusion is exactly the opposite of the central message from the literature on competitive price discrimination, which is that adding constraints to firms’ pricing policies generally harms welfare (Armstrong and Vickers (2001)).

Unfortunately for consumers, the fact that in-store efficiency increases need not imply that equilibrium in-store utility \( U^* \) increases. The reason is that a ban constrains firms to compete on products for which doing so is fundamentally less effective. That is, a ban limits the ability of firms to compete on products in \( L \), for which they manifestly had intense incentives to cut prices, and forces them to compete on products for which such incentives are weaker. To see this, note that the first-order condition from Equation (3) indicates that, absent a ban

\[
y[1 + L_k(p_k)] - \alpha_k y_1 \pi = 0, \text{ for each } k.
\]
Everything else equal, firms have the strongest incentives to cut prices for products with larger accuracy ratios. But because it is such products that are loss leaders, an effect of the ban is to transfer competition to products for which competition is softer.

Softened competition ensures that firms offer lower forecast utility $\hat{U}^*$ to consumers. Of course, once arriving at a store, consumers care about $U^*$, not $\hat{U}^*$. Given that in-store surplus $\pi^* + U^*$ increases, whether $U^*$ increases or not is connected to how $\pi^*$ changes. It is possible for $\pi^*$ to increase sufficiently much that in-store utility $U^*$ falls.

The following two remarks exhibit the results in particular special cases. The first case of interest is that the market is perfectly competitive. Recall that, as shown in Remark 1, with perfect competition there are always some goods priced below cost, and others above cost.\(^\text{12}\)

**Remark 2.** If the market is perfectly competitive, then a ban on loss leading leads to marginal-cost pricing ($p_n = c_n$ for each $n$). Consumer in-store utility $U^*$ increases, but because forecast utility $\hat{U}^*$ falls, some consumers are worse off (if the market is not covered).

In a perfectly competitive market, a ban on below-cost pricing restores the marginal-cost pricing equilibrium that would obtain if consumers had unbiased beliefs. Because in-store surplus goes up, and because firms earn zero profits, consumers who shop must be the beneficiaries of these gains: $U^*$ is higher. Although intuitive, this is also somewhat unexpected: only when the market is perfectly competitive and covered can all consumers expect to benefit from restrictions on firms' pricing policies.

A different benchmark is that the market is covered and the demand functions for the goods are degenerate, so that that consumers have unit demand for each of the $n$ goods, being willing to pay no more than $v_n > 0$ for each. Note that some care must be taken when using unit demands, because of corner solutions.

**Remark 3.** If the market is covered and consumers have unit demands, then a ban on loss leading makes all consumers worse off (that is, lowers $U^*$ and $\hat{U}^*$).

In the degenerate case of unit demands, there is no consumption distortion either before or after a ban—the in-store efficiency effect is zero. Because $\pi^*$ must increase when the market is covered, in-store utility must fall, so that a ban hurts all consumers. Consider as an illustration a simple two-product covered Hotelling model in which consumers pay $\tau$ per unit in transportation costs, and in which $\alpha_1 > \alpha_2$. Before a ban, firms set $p_2 = v_2$ and compete on product 1, and the (unbiased) expected total outlay of consumers is

$$\theta_1p_1 + \theta_2p_2 = \frac{\tau}{\alpha_1} + \theta_1c_1 + \theta_2c_2.$$\(^{12}\)

\(^{12}\)Of course, this assumes there is asymmetry in the accuracy ratios, but throughout this section I am assuming that loss leading indeed exists in the absence of a ban, so such asymmetry is a given.
After a ban, competition shifts to product 2, and the expected outlay is 

$$\theta_1 p_1 + \theta_2 p_2 = \frac{\tau}{\alpha_2} + \theta_1 c_1 + \theta_2 c_2.$$ 

Effectively, the level of differentiation between firms is not simply the transportation cost $\tau$, but $\tau/\alpha_n$, where $\alpha_n = \alpha_1$ prior to the ban and $\alpha_n = \alpha_2$ after the ban. Because $\alpha_2 < \alpha_1$, price cuts are less effective after a ban, and so a ban raises the expected outlay.

Based on the two examples just given, an intuition for when consumers benefit from a ban on loss leading is as follows. Such a ban is more likely to benefit them when the in-store efficiency effect is greater (so that loss-leading is more inefficient), and when competition is more intense (so that consumers receive more of the gains from increased in-store surplus). Of course, this only applies to consumers who continue shopping after a ban.

My results overturn those of Armstrong and Vickers (2001), who consider the welfare effects of constraints on firms’ pricing policies, assuming consumers are fully rational. They show that, when firms are symmetric, the retail market is sufficiently competitive, and the market is covered, restricting pricing tools lowers social surplus. Similar implications emerge from the studies of non-linear pricing by Rochet and Stole (2002) and Yin (2004).

To explain why consumer bias leads to different results, it is helpful to first explain the intuition behind the results from the work just mentioned. Those articles show that sufficient competition disciplines firms to induce outcomes that are as efficient as possible given the pricing tools they have; doing otherwise is an ineffective way of attracting consumers. This logic breaks down when consumers are boundedly rational. That is, offering prices that induce inefficient in-store purchases is an effective way of attracting consumers, precisely because consumers do not correctly anticipate the value of in-store purchases. Consequently, pricing restrictions in principle might raise social surplus. In the case of a ban on loss leading, because of the resulting price compression, this indeed occurs.

It is straightforward to explore this insight further by assessing the role two-part tariffs play when consumers have biased beliefs. To this end, consider augmenting the model above so that each firm $m$ can also charge consumers an entry fee $F_m$. Consumers then choose to shop based on the quantities $\{\hat{U}_m - F_m\}$, where $\hat{U}_m$ is as defined earlier (that is, does not include the fixed fee). Similarly, let $\pi_m$ be as defined earlier, so that firm $m$’s per-customer profits are $\pi_m + F_m$.

Without bias, firm $m$ would set prices to maximize $\pi_m + U_m$, leading to marginal-cost pricing. However, with bias, it sets prices to maximize $\pi_m + \hat{U}_m$. Because this quantity can

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13This analysis is related to that in DellaVigna and Malmendier (2004), who also observe that competition for boundedly rational consumers need not result in efficient contracts being adopted.
be maximized product-by-product, it is straightforward to see that $p_n$ is chosen to maximize
\[
\hat{\theta}_n(u_n[x_n(p_n)] - c_n x_n) + (\theta_n - \hat{\theta}_n)(p_n - c_n)x_n(p_n).
\]
Because $\theta_n - \hat{\theta}_n \geq 0$, the optimal price satisfies $p_n^* \geq c_n$, where this inequality is strict if $\alpha_n < 1$. In other words, with unplanned purchases, two-part tariffs lead to prices being set strictly above marginal cost.\(^{14}\) Firms then compete in the fixed fees.

I now compare this to the outcome without two-part tariffs, in the special case where there is symmetric bias across products ($\alpha_n = \alpha < 1$ for each $n$). Equilibrium prices satisfy Equation (3), and in particular depend on $y_1/y$, which is a measure of the competitiveness of the market. When the market is very competitive, so that $y_1/y$ is large, prices tend towards marginal cost (and are never below marginal cost, because there is symmetric bias by assumption). In contrast, the price under two-part tariffs doesn’t depend on the competitiveness of the market (only the fixed fees do). Therefore, whether introducing two-part tariffs increases social surplus $\pi^* + U^*$ depends on whether it raises or lowers prices, and this depends solely on how competitive the market is.

**Remark 4.** In the case of symmetric bias ($\alpha_n = \alpha < 1$ for each $n$) with a covered market, introducing the ability to use two-part tariffs strictly lowers social surplus if and only if the market is sufficiently competitive.

With intense competition, the typical result from the competitive non-linear pricing literature is once again reversed. But, when competition is weak, two-part tariffs raise surplus.

### 3. Loss Leading among Asymmetric Rivals

In the previous section I identified the circumstances under which loss leading emerges, characterized the set of loss leaders, and determined the consequences of a ban on loss leading, all for the case of symmetric rivals. However, a significant cause of concern is loss leading among asymmetric rivals. The reason is that, in practice, larger firms often price below cost on the core product lines of smaller firms. As noted by Chen and Rey (2012), such behavior raises doubts that loss leading is an innocuous manifestation of competition. Two particular markets where this is a concern are the grocery market, in which full-service grocers charge below cost on the staple goods that largely comprise the product lines of smaller grocers, and the retail gasoline market, in which gas stations with large attached convenience stores charge below cost on gas in their competition with stations that primarily sell gas. Are large firms preying upon the smaller firms, seeking to induce their exit? Although a tempting explanation, predation is not consistent with the observed persistence of loss leading—a rational predator must expect eventually to recoup its losses.

\(^{14}\)If $\alpha > 1$, then prices would be below marginal cost. In essence, this would be the analysis of “investment goods” from DellaVigna and Malmendier (2004).
To investigate, in this section I examine competition and loss leading among asymmetric rivals, answering two questions. Why would a firm with a full product portfolio practice loss leading on products carried by a smaller rival? Is welfare in general and consumer welfare in particular harmed by this practice? My answers to these questions build on two ideas. First, that consumers make unplanned purchases, and second, that product lines are endogenous.

The model is extended as follows. First, there are two firms, a “small” firm $S$ and a “large” firm $L$. Firm $L$ sells each of the $N$ goods, but $S$ can only sell $|S|$ of them, where $|S| < N$. However, $S$ is able to choose which $|S|$ of the $N$ products it carries. In particular, at the same time that $L$ chooses the prices of its products, $S$ simultaneously chooses both which $|S|$ products it carries and the prices of those products.$^{15}$

Consumers observe product lines and prices, and know that if they visit $S$ and have a positive demand for some good $n$ ($\xi_n = 1$) that $S$ does not carry, then they will purchase zero units of that good (I assume $u_n(0) = 0$ for each $n$). Consumers then choose where to shop based on their utility forecasts $\hat{U}_S$ and $\hat{U}_L$, as in earlier analysis. Let $y_L(\hat{U}_L, \hat{U}_S)$ be the number of shoppers at $L$ and $y_S(\hat{U}_S, \hat{U}_L)$ be the number of shoppers at $S$. I assume that in equilibrium the small firm gets some market share, $y_S > 0$.

Second, $u_n(x_n) = u(x_n)$ for each $n$, so that the within-product demand curve $x_n(p_n) = x(p_n)$ for some demand function $x$, and $c_n = c$ for some $c$. The functions $u$ and $x$ satisfy all the properties assumed earlier. Third, there is at least one “staple good” $n$ such that $\theta_n = \hat{\theta}_n = 1$, so that $\alpha_n = 1$. Staple goods are products that consumers definitely need and know they need. As mentioned earlier, goods that consumers regularly purchase are often referred to as staples, and tend to be loss leaders.

Before proceeding, I note that the characterization of loss leaders from Propositions 1 and 2 is robust to asymmetric competition.$^{16}$ In particular, below-cost pricing cannot emerge unless consumers have asymmetrically biased beliefs over the $N$ products. Also, firms use (firm-specific) cutoff rules for which products are priced below cost. That is, if firm $m$ sells both product $n$ and $k$, with $n$ priced below cost by $m$, and if $\alpha_k \geq \alpha_n$, then $k$ is also priced below cost by $m$.

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$^{15}$Endogenous product selection (without consumer bias) has been explored in a variety of settings, although typically not ones with one-stop shopping (see Klemperer (1992) for an exception). Several papers explore how firms strategically select their product lines, knowing that their selection influences consequent price or quantity choices of rivals, as in Brander and Eaton (1984), Champsaur and Rochet (1989), and Johnson and Myatt (2006). Other approaches instead focus more on the price discrimination rather than the strategic aspects of optimal product line selection, as in Mussa and Rosen (1978), Deneckere and McAfee (1996), and Johnson and Myatt (2003). There are many other contributions to this literature.

$^{16}$The existing proofs of these results lean entirely on matters of optimization by individual firms, and so work fine with asymmetric firms.
I begin with a useful characterization of the (endogenous) product portfolio that \( S \) selects. Recall that good \( n \) is a staple good if \( \theta_n = \hat{\theta}_n = 1 \), and that \( S \) chooses which \( |S| < N \) products to carry.

**Proposition 4.** Firm \( S \) carries as many staple goods as possible. That is, if \( S \) carries any product that is not a staple, then it also carries all available staples.

Proposition 4 holds in spite of the fact that staple goods are optimally priced lower than other goods, by either firm (and indeed are the products priced below cost if any are).\(^{17}\) In particular, it is not optimal for \( S \) to include a product exhibiting a high level of unplanned purchases in its portfolio unless all staple goods are already included, even though the decision of consumers about where to shop is not sensitive to price increases for such a good. For instance, consider some non-staple good \( k \) with \( \theta_k = 1 \) but \( \hat{\theta}_k \approx 0 \). This good, if carried by \( S \), would be priced near the monopoly level and always be purchased by consumers, but consumers would completely ignore its high price when choosing where to shop. Yet, \( S \) optimally eschews such a product for a staple good with a lower price.

The reason is that instead selling the staple good could always generate as much per-customer profit and be more effective at bringing customers in. That is, although it would not be optimal to do so, the staple good could be priced near the monopoly level just as a good with \( \hat{\theta}_k \approx 0 \) would be. However, even with monopoly pricing, some surplus is generated for consumers, and so selling the staple good at this (suboptimal) price raises \( \hat{U}_S \) compared to selling the other good (because consumers forecast this surplus with higher probability for a staple good compared to one with \( \hat{\theta}_k \approx 0 \)). This implies that selling the staple good at this price increases the number of customers who shop at \( S \), while generating the same profits per customer. Thus, it must be optimal to replace the non-staple good with a staple.

Similarly, the fact that \( L \) also charges lower prices for staples doesn’t entice \( S \) to instead carry goods for which \( L \) charges more. The reason is that consumers are one-stop shoppers, so that from the perspective of \( S \) it is irrelevant as such what prices \( L \) is charging for different products, conditional on the forecast utility \( \hat{U}_L \).

Proposition 4 seems to accord with reality. For example, in the grocery business, the core product lines of smaller stores are indeed goods that consumers frequently purchase. It seems likely that consumers correctly anticipate their purchases of such goods, and so don’t have biased beliefs. Indeed, it is likely that the need for such goods is what drives customers to go shopping in the first place.

\(^{17}\)It can be shown that the optimal prices for a firm are monotonically decreasing in the accuracy ratios. Because staples have the highest possible accuracy ratio, within either store these goods must be priced lower than all other goods within the same store. This conclusion relies on the assumption of this section that \( u_n = u \) for each \( n \), so that the shape of demand is identical across products.
It is now possible to answer the first question posed at the beginning of this section: why would a firm with a full product portfolio practice loss leading on products carried by a smaller rival? Let \( \mathcal{P}_m \) denote the product portfolio of \( m \in \{S, L\} \), and let \( \mathcal{L}_m \subset \mathcal{P}_m \) denote the (possibly empty) set of products that \( m \) prices beneath cost.

**Proposition 5.** If \( L \) practices loss leading, then at least one of its loss leaders is carried by \( S \). That is, if \( \mathcal{L}_L \) is nonempty, then \( \mathcal{L}_L \cap \mathcal{P}_S \) is also nonempty.

Proposition 5 is a central result of this article. It provides an explanation for why the products chosen as loss leaders by large firms are the same as the products carried by smaller rivals. There are several simple steps in reasoning underlying this result. First, it is optimal for \( S \) to carry staple goods. Second, such goods by definition exhibit the highest possible accuracy ratio, with \( \alpha_n = 1 \) for any such good \( n \). Third, if \( L \) prices any goods below cost, it must be pricing goods with high accuracy ratios below cost. Therefore, if \( L \) prices any goods below cost, it is pricing all staple goods below cost, at least some of which must be carried by \( S \).

It is imperative to note that this result does not imply that \( L \) prices below cost on particular goods simply because they are sold by \( S \). Rather, which goods are priced below cost by \( L \) is driven entirely by the nature of consumer bias, not as such by the product line of its rival \( S \). Similarly, \( S \) willingly chooses to carry goods that it knows \( L \) is pricing below cost, where its decision is also driven by the extent of consumer bias across the different products.

I now turn to the second question posed at the start of this section: is welfare in general and consumer welfare in particular harmed by loss leading by a large firm?

As a first step towards answering this question, I begin by noting a particular sense in which too few consumers shop at \( L \) in equilibrium. Agree that “removing consumer bias” refers to the following exercise. For each \( n, \check{\theta}_n \) is increased to \( \theta_n \); all other parameters are held fixed.

**Proposition 6.** Suppose that in equilibrium there is some good that \( S \) carries for which \( L \) charges a lower price: \( p_k(L) < p_k(S) \) for some \( k \in \mathcal{P}_S \). Then the following results hold.

1. \( L \) charges a lower price for each product that is carried by \( S \). That is, for each \( r \in \mathcal{P}_S \), \( p_r(L) < p_r(S) \).
2. Suppose also that there is at least one good \( n \) for which consumers have biased beliefs \((\alpha_n < 1)\). Then, fixing the prices of all goods at their equilibrium levels, removing consumer bias leads to an increase in the number of shoppers at \( L \).

The first result says that \( L \) either charges lower prices than \( S \) on all products in \( S \)’s portfolio, or charges higher prices for all such goods. This implies the second result. So long as \( L \) is not uniformly the high-price firm, then at the equilibrium prices too few consumers shop at
—there is an “under-patronage problem” in that more consumers would shop at \( L \) if their beliefs were unbiased. The logic relies both on the fact that consumers have biased beliefs and also on the result that \( L \) charges less for each product that \( S \) carries. In particular, for any product for which consumers have biased beliefs, they underestimate the value of the lower prices charged by \( L \) (and similarly underestimate the value of goods in \( L \)’s portfolio that \( S \) does not carry).

The fact that bias causes consumers to underestimate the value of shopping at \( L \) is important for assessing the effect of a ban on loss leading. Intuitively, such a ban makes it more difficult for the large firm to attract customers, and so exacerbates the under-patronage problem. This suggests that a ban on loss leading is bad for consumers.

To demonstrate that this can be so, I specialize to the case of two products \( (N = 2) \), each with unit demand (so that each consumer has a willingness to pay of \( v \) for a single unit of each good), and where good 2 is a staple.\(^{18}\) To rule out cases where there is no below-cost pricing or where \( S \) receives no sales, I suppose \( \theta_1 v \in (1.5, 3) \), and to ensure the market is covered I suppose \( v \) is not too small.\(^{19}\) Also, I explicitly model consumers as having unit linear transportation costs between the two firms arrayed on a segment of unit length, so that given utility forecasts \( \hat{U}_S \) and \( \hat{U}_L \), the number of consumers \( y_L \) shopping at \( L \) is such that \( \hat{U}_L - y_L = \hat{U}_S - (1 - y_L) \). It follows that

\[
y_L = \frac{\hat{U}_L - \hat{U}_S + 1}{2}.
\]

**Proposition 7.** Consider a covered market with two goods, each with unit demand, in which good 2 is a staple, in which consumers have biased beliefs over good 1 (so that \( \alpha_1 < 1 \)), and in which \( S \) can carry only one good. Then the following statements hold.

1. *In the absence of a ban on loss leading, \( L \) charges below cost on product 2, which is also the product that \( S \) chooses to carry.*
2. *A ban on loss leading reduces the number of consumers who shop at \( L \), and strictly raises the expected outlay associated with a trip to either store.*
3. *A ban on loss leading lowers the welfare of each consumer and also lowers social surplus.*

A loss-leading ban raises the expected outlay of visiting either store.\(^{20}\) Perhaps surprisingly, despite these price increases more must be shown to conclude that each consumer is worse off under a ban. The reason is that the bias associated with good 1 implies that some consumers

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\(^{18}\) Additionally, to avoid having to check for uninteresting corner solutions, I suppose that costs are zero and that firms may set negative prices.

\(^{19}\) If \( \theta_1 v < 1.5 \), then product 1 is not sufficiently attractive to spur below-cost pricing on good 2 by \( L \). If \( \theta_1 v > 3 \), then product 1 is so attractive that \( L \) is willing to price low enough on 2 that \( S \) receives no equilibrium sales. Additionally, in this range of parameters the market is covered.

\(^{20}\) That is, \( p_2(S) \) increases, as do both \( \theta_1 p_1(L) + p_2(L) \) and \( \theta_1 p_1(L) + p_2(L) \).
do not shop at the store that maximizes their true payoff. In principle, price increases might benefit such consumers by impelling them to switch from making the incorrect decision about where to shop (at the lower prices) to making the correct decision (at the higher prices).

However, because a ban reduces the number of shoppers at $L$, and because too few consumers shop at $L$ under the pre-ban prices, imposing a ban does not actually cause any consumer to switch from making an incorrect decision to making a correct decision about where to shop. Instead, all switchers were originally making the correct decision, and some of these end up making the incorrect decision. Thus, all consumers are indeed harmed by a ban.

The situation described above may be one of a small grocer competing with a full-service grocer, or of a gasoline retailer with an attached retail store competing with a station that only sells gas. Convenience leads some customers to shop at the small grocer or station that only sells gas. However, consumers underestimate their needs and tend to have unsatisfied demand—some would be better off if they instead shopped at the larger firm, even though it is less convenient. A ban on loss leading, by constraining the larger firm’s ability to attract consumers, leaves more needs ultimately unsatisfied and so reduces welfare.

I now discuss the insights of Chen and Rey (2012), who lead the way in providing meaningful analysis of loss leading among asymmetric competitors. They also consider one small and one large firm, with a total of two products for which consumers have unit demands. A crucial feature of their model is that some but not all consumers are multi-stop shoppers who visit both $S$ and $L$. They show that this allows the large firm to use loss leading as a discriminatory tool, effectively extracting more surplus from multi-stop shoppers. Such a screening device requires the presence of the small firm, and indeed the large firm’s profit are higher when it faces competition. They conclude that banning loss leading helps consumers.

The perspective I provide differs in several ways. First, I focus on the ideas of endogenous product line selection and consumer bias (indeed, there is no loss leading in my model unless such bias exists), which are absent from their model. Second, in their model, the large firm charges below cost on a good precisely because the small firm carries that good—exogenously switching the product carried by the small firm would change the loss leader selected by the large firm. In contrast, in my analysis such overlap is driven by the underlying heterogeneity in the bias across products. Third, and perhaps most significantly, because consumers with biased beliefs make suboptimal choices, and because a ban on loss leading raises average prices and exacerbates this problem, the final effect of a ban is to harm consumers.

At the broadest level, however, Chen and Rey’s model is meant to capture a different market structure than mine is. That is, we make differing assumptions about the relative strength of

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21More precisely, in their analysis the small firm carries a superior version of one of the products carried by the large firm, so it is more correct to say that switching the product category carried by the small firm would change the category in which the large firm practices loss leading.
the small firm, either of which could be appropriate depending on the situation. They assume that the small firm carries a product that is actually superior to the version carried by \( L \), and that no consumer has any locational preference for one store over the other. In contrast, in my model the strength of \( S \) is that some consumers prefer to shop there because of its convenience. To see why these differences matter, consider fixing prices at their equilibrium levels and removing shopping and transportation costs. If the strength of \( S \) is its superior product, then absent such costs all consumers would shop at both \( S \) and \( L \), whereas if the strength of \( S \) is its convenience for some customers, then absent such costs all customers would visit only \( L \).\(^{22}\)

Therefore, one main implication for proper antitrust assessment is that the source of any advantage of the small firm must be identified. If its main advantage lies in its location, so that absent transportation costs most consumers would choose to shop at the large firm, then that is closer to my model. On the other hand, if its main advantage is a superior although more limited product line, such that most consumers would choose to multi-stop shop in the absence of transportation costs, then that is closer to the model of Chen and Rey. In other words, conclusions about the social effects of loss leading depend on whether the small firms are boutiques providing niche or high-end products that offer a large quality advantage, or rather small firms are more convenient for some customers but carry products that tend to be no higher quality than those of the large firm.

A second main implication is that assessment of loss leading among asymmetric competitors should focus on why a large firm chooses particular products as loss leaders. If such selection is driven primarily by the product line of smaller rivals, then it may be more plausible to infer nefarious intent. On the other hand, if it is the intensity of competition itself that drives the large firm to offer loss leaders, and if such loss leaders are staple goods that would be chosen for reasons suggested by me, then it is less clear that consumer harm can be inferred.

4. The Effects of Unplanned Purchases

Although the main focus of this article is on below-cost pricing, it also makes sense to explore the welfare effects of changes in consumer bias. Thus, here I explore what happens when the level of unplanned purchases increases. Suppose that the accuracy ratios are initially \( \{\alpha_n\} \), and that \( \theta_n \) decreases to \( \theta'_n \), fixing \( \theta_n \), for each \( n \), and denote the new accuracy ratio for \( n \) by \( \alpha'_n < \alpha_n \). Further suppose that

\[
\alpha'_n = \mu \alpha_n, \text{ for some } \mu \in (0, 1), \text{ for each } n.
\]

\(^{22}\)This assumes that \( L \) is not the high-cost firm for all goods, so that by Proposition 6 it in fact charges lower prices on all goods.
Because this experiment fixes the probability $\theta_n$ that a consumer will demand product $n$ but decreases the anticipated probability of purchase, it can be interpreted as consumers becoming more forgetful of their needs prior to arriving at a store. I refer to this exercise as an “increase in unplanned purchases.”

Recall that the function

$$h(x) = \frac{y_1(x,x)}{y(x,x)}$$

is an important element of the model. Here I suppose that the elasticity of $y$ is increasing, that is, $xh(x)$ is increasing. This property is satisfied by both of the motivating demand structures mention earlier (a covered Hotelling-line model or a logit model).

**Proposition 8.** Suppose that there is an increase in unplanned purchases.

1. All prices strictly increase, so that per-customer profit $\pi^*$ strictly increases. However, the overall profits of firms need not go up.

2. All consumers who shop are strictly worse off, and fewer consumers choose to shop (if the market is not covered). That is, both $U^*$ and $\hat{U}^*$ decrease.

In the broadest sense, Proposition 8 contributes to the literature by emphasizing that bounded consumer rationality can have meaningful effect on the profits of competing firms. In contrast, the related work of Lal and Matutes (1994) and Gabaix and Laibson (2006) emphasizes that bounded consumer rationality transfers price competition to a certain subset of products upon which firms then compete intensely, fully dissipating potential gains.

More particularly, Proposition 8 identifies two distinct mechanisms that influence profits. The first is that an increase in consumer bias softens price competition between firms, thereby raising per-customer profits. There are two simple steps to this conclusion. First, if consumers are more biased about their propensity to purchase, price cuts are less effective at pulling consumers away from rival firms. Second, because the actual probability that consumers purchase any good has not changed, a price cut remains just as damaging to the per-customer profits of a given firm. Together, these effects imply that price cuts are less attractive when consumer bias is higher.\(^{23}\) It follows that prices are higher and moreover, if the market is covered (as it is in the analyses of Lal and Matutes (1994) and Gabaix and Laibson (2006)), then overall profits are higher.

The second mechanism is that increased bias negatively influences the decision of consumers to go shopping (by reducing $\hat{U}^*$), for two reasons. First, prices are higher, and second, consumers anticipate having a need for each good with lower probability. This second effect is particularly important because it implies that an increase in bias can lower the overall

\(^{23}\)Note that this reasoning also implies that, for prices to increase, it is crucial that consumer bias is increasing. If consumers always have unbiased beliefs but merely uncertain demand, then a proportional reduction in demand (that is, reducing each $\theta_n$ by the factor $\mu$) has no effect on prices whatsoever.
profits of a firm. With sufficiently sensitive aggregate demand, the drop in \( \{ \hat{\theta}_n \} \) causes enough consumers to stop shopping at this firm that its profits fall. Although the firm could lower prices to try and reclaim consumers, such price cuts have become less effective at drawing consumers in due to the increase in bias.

What would need to be true for there to be a neutral effect on profits following an increase in bias? First, the market must be covered. Second, there must be some good for which consumers have unit demand and unchanging bias. For example, suppose all consumers know that they will buy product one for sure, \( \theta_1 = \hat{\theta}_1 = 1 \), and that they have unit demand for it (a degenerate case of the model specified above). Then, an increase in unplanned purchases associated with goods \( n = 2, 3, \ldots, N \) causes their prices to increase, but \( p_1 \) decreases so that there is no equilibrium effect on firm profits—so long as the non-negativity constraint on \( p_1 \) is not violated.\(^{24} \)

5. Conclusion

I have presented a new, simple, and reasonably general model of consumer bias and consequent unplanned purchases that is easy to work with yet leads to rich results. Using this model, I investigated the welfare effects of banning below-cost pricing among symmetric and also asymmetric rivals. My results contribute to a body of literature exploring regulatory and antitrust issues in markets where consumers have bounded rationality, such as Grubb (2009, 2012), Heidhues and Kőszegi (2010), and Hoffman, Inderst, and Ottaviani (2013).

Consumer bias may be relevant to other areas of study within industrial organization in general and antitrust economics in particular. Such bias could take the form of unplanned purchases that I have suggested, or the equivalent formulation of consumers simply paying too little attention to certain aspects of a decision problem, perhaps because certain aspects are more important or salient in some sense, as in Kőszegi and Szeidl (2013). For example, manufacturers sometimes pledge to sell a product exclusively through a particular retailer. This behavior, or the related one of a manufacturer demanding exclusivity from a retailer, has been interpreted as a means of spurring efficient investment or avoiding opportunistic behavior (Marvel (1982) and McAfee and Schwartz (1994)), of softening competition among vertical chains (Bonanno and Vickers (1988)), or of deterring entry (Rasmusen, Ramseyer, and Wiley (1991)). Consumer bias and multi-product competition may provide another compelling explanation. And, the incentives of firms to merge or expand their product lines are sometimes predicated on the desire to become “one-stop shops” for customers (Klemperer

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\(^{24}\)Farrell and Klemperer (2007) (p. 1983) note that the presence of “worthless” customers may constrain firms’ ability to charge very low prices, and hence prevent them from competing away future profits in switching cost models. Even when the non-negativity constraint does not bind, so that firm profits do not increase as the bias over products \( n \geq 2 \) does, there is still a meaningful effect of increased bias. In particular, because consumers don’t have inelastic demands for all products, the price increases on goods \( n \geq 2 \) imply that overall surplus declines.
and Padilla (1997)), suggesting that a study of consumer bias may be applicable to questions of product line extension and merger policy. Also, the decisions of multi-product firms to offer components that are compatible or instead incompatible with those of their rivals (Einhorn (1992) and Matutes and Regibeau (1992)) may be influenced by consumer bias.

In any such antitrust application, biased consumer beliefs complicate the assessment of consumer harm. For example, a potential complexity that arises among asymmetric rivals is mentioned in the discussion following Proposition 7—it is possible that consumers might benefit from price increases. The reason is that in principle such increases could have a corrective effect on the incorrect (due to bias) decisions that consumers make about which firm to patronize. Note that this does not actually happen in my analysis; I merely point to the possibility in other settings.

APPENDIX

Here I present proofs of results that are not covered in the body of the article. The proofs of Proposition 2, Remark 4, and Proposition 5 are in the text.

Before proceeding, I show that there can not be more than one symmetric equilibrium. Such an equilibrium is the solution to the following system of equations, given by (for each $n = 1, 2, ..., N$)

$$1 + L_n(p_n) = \frac{y_1(\hat{U}, \hat{U})}{y(U, U)} \alpha \pi_n \alpha \pi_n.$$

Let $h(x) = y_1(x, x)/y(x, x)$. Suppose that there were multiple solutions to this system, and denote the values associated with one by $\{p_n\}, \hat{U}, h, \pi$, and the other by $\{\tilde{p}_n\}, \tilde{U}, \tilde{h}, \tilde{\pi}$. Because $L_n$ is strictly decreasing, if $\tilde{h} \tilde{\pi} = h \pi$, then the equilibria coincide, and so without loss suppose that $\tilde{h} \tilde{\pi} > h \pi$. It follows that $\tilde{p}_n < p_n$, for each $n$. Thus, $\tilde{U} > \hat{U}$ and so, because $h(x)$ is decreasing, $\tilde{h} \leq h$. Also, $\tilde{\pi} < \pi$. Hence, $\tilde{h} \tilde{\pi} \leq h \pi$, a contradiction.

**Proof of Proposition 1:** The argument presented in the body of the article suffices with slight modifications. Write $\hat{\theta}_n = \alpha \theta_n$ and note that, for any given prices,

$$\hat{U} = \sum_n \hat{\theta}_n v_n(p_n) = \alpha \sum_n \theta_n v_n(p_n) = \alpha U.$$

It follows that for any change in prices, the resulting changes in $\hat{U}$ and $U$, denoted $\Delta \hat{U}$ and $\Delta U$, satisfy $\Delta \hat{U} = \alpha \Delta U$.

Now consider any firm and suppose that $p_n < c_n$ and $p_k > c_k$ (if there is no $k$ such that $p_k > c_k$, then the firm is earning negative profits at the original prices, which cannot be optimal). Raise $p_n$ and lower $p_k$ slightly, so that $\hat{U}$ is unchanged (which ensures the same number of consumers visit). Thus, $0 = \Delta \hat{U} = \alpha \Delta U$, and so $\Delta U = 0$. Now, because overall
in-store surplus $\pi + U$ must be up, profits must be higher as well.

**Proof of Remark 1:** Without loss of generality assume that $\alpha_2 > \alpha_1$. If there is no loss leading, then because firms earn zero profits, they are pricing at marginal cost. Suppose this is the case, and a firm instead raises $p_1$ by one (infinitesimal) unit, and lowers $p_2$ by $\delta_2$, where $\delta_2$ is chosen so that $\hat{U}$ does not change:

$$-\hat{\theta}_1 x_1(c_1) + \delta_2 \hat{\theta}_2 x_2(c_2) = 0 \iff \delta_2 = \frac{\hat{\theta}_1 x_1(c_1)}{\hat{\theta}_2 x_2(c_2)}.$$

Now consider the change in the firm’s profits. Because this firm is offering consumers the same value $\hat{U}$ as before, the same number of consumers visit (or, alternatively, $\delta_2$ can be chosen slightly larger so that all consumers visit this firm). Because the original prices equal marginal cost, the first-order effect on profits per customer is

$$\theta_1 x_1(c_1) - \delta_2 \theta_2 x_2(c_2) = \theta_1 x_1(c_1) - \hat{\theta}_1 x_1(c_1) \frac{\theta_2}{\hat{\theta}_2} > 0 \iff \alpha_2 > \alpha_1,$$

which is true. Hence, it is clearly not optimal to set all prices at marginal cost. Because all prices weakly below marginal cost with some strictly below would result in negative profits, and because all prices weakly above cost with some strictly above would result in positive profits, in equilibrium some must be priced below and others above cost.

**Proof of Proposition 3:** Recall that $h(x) = y_1(x, x)/y(x, x)$. Consider any retailer and, suppressing retailer-specific notation, let $\pi$ and $\{p_n\}$ denote equilibrium values prior to the ban, and $\tilde{\pi}$ and $\{\tilde{p}_n\}$ equilibrium values after. Building on Equation (2) and the fact that in equilibrium $d\Pi/dp_n \leq 0$ with equality holding before the ban (and also after the ban for goods priced above cost), it follows that for each $n$

$$1 + L_n(p_n) = \alpha_n h \pi, \quad \text{and} \quad 1 + L_n(\tilde{p}_n) \leq \alpha_n \tilde{h} \tilde{\pi}.$$

Suppose for the sake of contradiction that $\tilde{h} \tilde{\pi} \leq h \pi$. Then

$$1 + L_n(\tilde{p}_n) \leq \alpha_n \tilde{h} \tilde{\pi} \leq \alpha_n h \pi = 1 + L_n(p_n).$$

Thus, $\tilde{p}_n \geq p_n$ for each $n$, with the gains being strict for some products because by assumption $\mathcal{L}$ is non-empty. Hence $\tilde{\pi} > \pi$ and $\hat{U} < \hat{U}$, so that, because $h$ is decreasing, $\tilde{h} \geq h$. All of this implies $\tilde{h} \tilde{\pi} > h \pi$, a contradiction (so that indeed it must be that $\tilde{h} \tilde{\pi} > h \pi$).

Using this fact, the price-compression result follows directly. To see this, suppose that $\tilde{p}_n > c_n$ for some $n$. Then

$$1 + L_n(\tilde{p}_n) = \alpha_n \tilde{h} \tilde{\pi} > \alpha_n h \pi = 1 + L_n(p_n),$$

so that $\tilde{p}_n < p_n$. That is, $\tilde{p}_n > c_n$ implies $\tilde{p}_n < p_n$. 

\[\square\]
One way for the price compression result to be false is if \( n \in \mathcal{L} \) and \( \bar{p}_n > c_n \). But using the implication just derived, this would imply that \( \bar{p}_n < c_n \), a contradiction. The only other way for the result to be false is if \( n \notin \mathcal{L} \) and \( \bar{p}_n > p_n \). But because \( p_n \geq c_n \) by assumption, this cannot happen by the implication just derived.

The second part of the proposition follows directly from the price compression result, and so all that is required is to prove the third part of the proposition. Suppose for the sake of contradiction that \( \tilde{U} \geq \hat{U} \), so that \( \tilde{h} \leq h \). Because it must be that \( \tilde{h}\tilde{\pi} > h\pi \), it follows that \( \tilde{\pi} > \pi \). But this means that at least as many consumers buy from each firm after the ban, and that each consumer generates more profit, so that the total profit of each firm is higher. But this cannot be, because each firm’s rivals are offering more forecast utility to consumers after the ban, and because each firm’s strategy set has been reduced. This contradiction establishes that \( \tilde{U} < \hat{U} \). To prove that \( \pi^* \) must increase if the market is covered, note that a covered market implies that \( \tilde{h} = h \). Because \( \tilde{h}\tilde{\pi} > h\pi \), it must be that \( \tilde{\pi} > \pi \).

**Proof of Remark 2:** A ban requires that \( p_n \geq c_n \) for each \( n \), and perfect competition requires that profits are zero. This means that \( p_n = c_n \) for each \( n \). It has already been shown that \( \pi^* + U^* \) has increased, which means that (because profits are zero) \( U^* \) has increased. To show that forecast utility has strictly decreased, suppose otherwise, letting \( \hat{U}_{MC} \) denote the forecast utility associated with marginal-cost pricing. The work in the proof of Remark 1 implies that a firm could earn strictly positive profits if its rivals were offering such values of forecast utility to consumers, in the absence of a ban. Thus, before a ban, it must be that \( \hat{U}^* > \hat{U}_{MC} \).

**Proof of Remark 3:** Label the goods so that \( \alpha_1 > \cdots > \alpha_N \). With unit demands, in the absence of a ban only good 1 (which has the highest accuracy ratio) is priced such that \( p_n < v_n \). After a ban, there is some good \( k \geq 2 \) that is priced strictly interior to \((c_k, v_k)\). (To avoid trivialities and needless extra cases, I am assuming not all goods are priced at corner solutions in this proof.) This means that Equation (3) holds (only) for good 1 prior to the ban, and (only) for some good \( k \) after the ban. Manipulating that equation, and noting that with unit demands \( L_n = 0 \) for each \( n \), it follows that, for \( n = 1 \) or \( n = k \) as is relevant,

\[
\sum_n \theta_n p_n = \frac{y}{y_1} \frac{1}{\alpha_n} + \sum_n \theta_n c_n.
\]

By hypothesis \( \alpha_k < \alpha_1 \), so that the expected outlay of each consumer has risen, and the result follows.
**Proof of Proposition 4:** Note that the assumption that $y_S > 0$ in equilibrium also ensures that the per-customer profits of the smaller firm are positive, $\pi_S > 0$, for otherwise the firm could slightly raise prices and still maintain positive market share. Suppose that the result is false, so that $S$ is carrying a non-staple and at least one staple is not carried. Then there must be some non-staple that is priced above cost. If instead all non-staples were priced weakly below cost, then all staples would also be priced below cost (because such goods have maximal accuracy ratios), ensuring non-positive profits.

Now, under the maintained assumption that the result is false, without loss of generality suppose that product 1 is a non-staple good carried by $S$ with $p_1(S) > c$ and that 2 is a staple good that is not carried by $S$. The profits $\Pi_S$ of $S$ are

$$\Pi_S = y_S(\hat{U}_S, \hat{U}_L)\pi_S,$$

where

$$\hat{U}_S = \sum_{n \in \mathcal{P}_S} \hat{\theta}_n v_n(p_n(S)), \text{ and } \pi_S = \sum_{n \in \mathcal{P}_S} \theta_n x(p_n(S))(p_n(S) - c).$$

Now consider replacing product 1 with product 2, but charging the same price currently charged for product 1, $p_1(S)$. By the definition of a staple, it must be that $\hat{\theta}_1 < \hat{\theta}_2$ (for otherwise the requirement that $\hat{\theta}_1 \leq \theta_1$ would imply that 1 is a staple as well), so that performing this switch strictly raises $\hat{U}_S$. Also by the definition of a staple, it must be that $\theta_2 \geq \theta_1$ so that (because $p_1(S) > c$) this switch raises $\pi_S$, so that $\Pi_S$ strictly increases. The result follows. \[\blacksquare\]

**Proof of Proposition 6:** Optimization by firm $m$ requires that, for any $n \in \mathcal{P}_m$,

$$\frac{1}{\alpha_n}[1 + L(p_n(m))] = \frac{y_1 \pi_m}{y},$$

where $y_1$ and $y$ are the relevant values for firm $m$ evaluated at the equilibrium levels. Note that the right-hand side of this equation does not vary with $n$. By hypothesis, $L$ charges less for some good $k \in \mathcal{P}_S$, from which it follows that the left-hand side of the above equation for good $k$ is larger when evaluated for firm $L$ than for firm $S$, so that for any $n \in \mathcal{P}_S$ the right-hand side of the above equation is larger for $L$ than for $S$. But this means that for any $n \in \mathcal{P}_S$, $L$ must be charging a lower price.

To prove the second part of the result, let $\Delta \hat{U}_m$ denote the increase in forecast utility for firm $m$ following the removal of bias. Because

$$\hat{U}_m = \sum_{n \in \mathcal{P}_m} \hat{\theta}_n v_n(p_n(m)),$$

it follows that

$$\Delta \hat{U}_m = \sum_{n \in \mathcal{P}_m} (\theta_n - \hat{\theta}_n)v_n(p_n(m)).$$
By hypothesis, \( L \) charges less for some good \( k \in \mathcal{P}_S \), and as was just shown this implies that \( L \) charges less for all goods \( n \in \mathcal{P}_S \). Thus, \( v_n(p_n(L)) > v_n(p_n(S)) \) for all such \( n \). Because \( L \) sells all products, it follows that \( \Delta \hat{U}_L \geq \Delta \hat{U}_S \). Furthermore, because by assumption there is some good \( r \) (possibly not in \( \mathcal{P}_S \)) for which consumers have biased beliefs, it must be that \( \theta_r - \hat{\theta}_r > 0 \) for this good, and this implies that \( \Delta \hat{U}_L > \Delta \hat{U}_S \). \( \blacksquare \)

Proof of Proposition 7: The proof is easy but there are many details and so I merely sketch certain elements that are easily verified. Because \( \hat{\theta}_1 < \theta_1 \), so that \( \alpha_1 < 1 \), while \( \alpha_2 = 1 \), \( S \) always chooses to carry good 2. It also ensures that, in the absence of a ban, \( L \) always sets \( p_1(L) = v \), and that under a ban \( L \) sets \( p_2(L) = 0 \) if \( p_2(L) < 0 \) in the absence of a ban.

Using these facts, absent a ban the profit function of \( S \) is

\[
\Pi_S = p_2(S)(1 - y_L) \equiv p_2(S)[1 - \hat{U}_L + \hat{U}_S] = p_2(S)[1 - (v - p_2(L)) + (v - p_2(S))]
\]

\[
= p_2(S)[1 + p_2(L) - p_2(S)],
\]

from which it follows that the best response of \( S \) is to set price

\[
p_2(S) = \frac{1 + p_2(L)}{2}.
\]

Similarly, absent a ban the profit function of \( L \) is

\[
\Pi_L = (\theta_1 v + p_2(L))y_L \equiv (\theta_1 v + p_2(L))[1 + p_2(S) - p_2(L)],
\]

from which it follows that, absent a ban, the best response of \( L \) is to set

\[
p_2(L) = \frac{1 + p_2(S) - \theta_1 v}{2}.
\]

Solving, it follows that in the absence of a ban, the equilibrium prices for good 2 are

\[
p_2^*(L) = 1 - \frac{2}{3} \theta_1 v, \quad \text{and} \quad p_2^*(S) = 1 - \frac{1}{3} \theta_1 v.
\]

Observe that \( p_2^*(L) < 0 \) whenever \( \theta_1 v > 1.5 \). Also, \( y_L < 1 \) whenever \( \theta_1 v < 3 \). By assumption, attention is restricted to these parameters.

Now, suppose a ban is in effect. There are two regimes. If \( \hat{\theta}_1 v \leq 1.5 \), then the new equilibrium is such that \( L \) does not lower \( p_1(L) \) from its pre-ban level of \( v \), and instead only raises \( p_2(L) \) to zero. \( S \) in turn raises its price to 0.5. Hence, both average prices increase, and it is readily verified that \( y_L \) decreases. The other regime is where \( \hat{\theta}_1 v > 1.5 \). In this case, \( L \) lowers \( p_1(L) \) beneath \( v \), and sets \( p_2(L) = 0 \). \( L \)'s profit function is proportional to

\[
\theta_1 p_1(L)[1 + p_2(S) + \hat{\theta}_1 v - \hat{\theta}_1 p_1(L)].
\]
Solving for best-responses of \( L \) and \( S \) yields equilibrium prices of

\[
p_1^*(L) = \frac{1}{\hat{\theta}_1} \left( 1 + \frac{\hat{\theta}_1 v}{3} \right), \quad \text{and} \quad p_2^*(S) = 1 - \frac{\hat{\theta}_1 v}{3}.
\]

Comparing these to the pre-ban prices, it can be shown that the assumption \( \hat{\theta}_1 < \theta_1 \) implies that average prices have increased, under either the biased or unbiased assessments. Furthermore, \( y_L \) has decreased.

To show that overall industry surplus falls under a ban, note that, in the absence of a ban, from a societal standpoint consumers under-weigh the value of shopping at \( L \) by \( \theta_1 v + p_2^*(L) \) and under-weigh the value of shopping at \( S \) by \( p_2^*(S) \). Plugging in the values given above, it can be seen that consumers under-weigh the value of shopping at \( L \) by more than they under-weigh the value of shopping at \( S \). Proposition 6 implies that strictly more consumers would optimally want to shop at \( L \), and so it follows that from a societal standpoint the optimal level of shoppers at \( L \) exceeds the equilibrium level.

All that remains is to show that all consumers are strictly worse off. Each firm has strictly raised the expected total price it charges consumers, and so \( \hat{U}_S \) and \( \hat{U}_L \) have both decreased. Consider any consumer who does not change where he shops as a consequence of a ban. His transportation costs don’t change, and because \( U_S \) and \( U_L \) have decreased due to the increase in prices, this consumer is worse off. Now consider any consumer who changes his decision about where to shop. Because \( y_L \) has decreased, these are consumers who originally shopped at \( L \) but now shop at \( S \). By the under-patronage effect identified in Proposition 6, prior to the ban each such consumer was correctly choosing the firm that maximized his true payoffs, where these payoffs are \( U_m - t_m \), where \( t_m \) is the transportation cost borne by this consumer to visit firm \( m \). Letting \( U'_m \) denote equilibrium values before the ban and \( U''_m \) equilibrium values after, and noting that \( U''_m < U'_m \), it must be that for any such consumer

\[
\max_r (U''_r - t_r) < \max_r (U'_r - t_r) = U'_L - t_L.
\]

The term on the right is this consumer’s true payoff before the ban (because he was correctly shopping at \( L \)), and the term on the left is an upper bound of his true payoff after the ban. ■

**Proof of Proposition 8:** Observe that, under the proposed comparative static, if prices move then they all move in the same direction. This follows if the right-hand side of

\[
1 + L_n(p_n) = \frac{y_1(\hat{U}, \hat{U})}{y(\hat{U}, \hat{U})} \alpha_n \pi.
\]
moves in the same direction for each $n$. To show this, let $\pi$ denote the original per-customer profit and $\pi'$ denote per-customer profit after the change. Then
\[ \alpha_n' \pi' - \alpha_n \pi = \mu \alpha_n \pi' - \alpha_n \pi = \alpha_n (\mu \pi' - \pi). \]
Hence, for each $n$, the sign is determined by that of $\mu \pi' - \pi$.

Now consider a decrease in unplanned purchases (note that the proposition is stated in terms of an increase, but it is slightly easier to prove this way). Let $h(x) = y_1(x,x)/y(x,x)$, and note that, keeping the starting values $\{\hat{\theta}_n\}$ fixed throughout, and starting from a value of $\mu = 1$, the direction of prices is determined by the direction of $h(\mu \hat{U}) \mu \hat{\theta}_n \pi$. All else fixed, differentiate the above with respect to $\mu$, giving a quantity proportional to
\[ h'(\mu \hat{U}) \mu \hat{U} + h(\mu \hat{U}) > 0, \]
because $xh(x)$ is increasing by assumption. If the proposition is false, then the equilibrium effect is that prices and hence $\pi$ have at least weakly increased, while $\hat{U}$ has decreased. But this means that the total effect of the change in $h(\mu \hat{U}) \mu \hat{\theta}_n \pi$ can be decomposed into the direct increase following the increase in $\mu$, and then the additional effect from an increase in $\pi$ (which obviously increases the term in question), and finally the effect of a decrease in $\hat{U}$ (which also increases the term in question because $h$ is decreasing). Hence, in fact, this term must have increased, so that prices fall.

References


