Open Source Software:

Competition with A Public Good

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Abstract

This paper looks at price and quality competition in software markets under two different forms of competition - one where two proprietary firms first choose quality and then engage in price competition, and second where a proprietary firm faces competition from an open source software firm that allows its users to determine quality level and provides the software at zero price. We find that OSS competition never improves quality for consumers who value quality highly. However, it may provide greater quality to users with a low valuation for quality. In addition, we find that although OSS has a zero market price, the public good nature of OSS competition can lessen price competition, making the proprietary firm better-off with increased profit but leaving consumers worse-off with lower surplus.

Keywords: open source software, duopoly, price competition, vertical differentiation

JEL Classification Codes: L17, D4, L11

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1 Introduction

The emergence of open source software (OSS) has widely been regarded as changing the competitive landscape of software markets and challenging the proprietary model of production. The OSS model of software development is characterized by voluntary contributions to software development where developers do not own the copyright for their contributions and as a result cannot appropriate the value of their contributions as a proprietary developer does.

While there has been varied academic opinion about which model is more profitable and which generates the best outcome in terms of quality and innovation, the reality is that software markets today comprise of both types of firms, in some cases competing in a duopoly fashion (as with Apple’s iPhone OS and Google’s open source Android OS) and in others dominated by one form or another. One question that arises in this context is how the nature of competition differs in each case. In other words, how does competition between OSS and proprietary software (PS) differ from competition between two proprietary firms; and how does this difference in competition then matter for quality provision, prices and consumer welfare? Broadly speaking, there are two important features that distinguish OSS from PS. First, in most cases, OSS is available free or at a much lower price than PS. Second, the quality of OSS is determined by the joint effort of many different user-developers and hence there may be an effort cost from users for quality provision. OSS has public good characteristics and thus is vulnerable to free-riding and this can potentially be exploited by the proprietary seller in her pricing decisions and quality choice. In this paper, we examine that.

We consider a model of price and quality competition between two firms that have different costs of providing quality and where consumers differ in their value for quality. We assign one firm to be a proprietary firm and then consider two organizational forms for the second firm - one where it is also proprietary and the other where it is organized as an OSS.

We find that price competition with an OSS firm may be stronger or weaker than with another proprietary firm. The price set by the proprietary firm depends on the value of the competing product to consumers. In case of proprietary competition, this is the actual quality provided by the other firm. However, in the case of OSS competition, this is the potential quality that would result if current consumers switched to the OSS firm and then contributed towards its quality. If the proprietary firm’s consumers have very high value for quality and hence could potentially make large contributions to the OSS, then price competition to retain these consumers is very intense.
On the other hand, OSS under-provides quality due to free-riding by its users. Thus proprietary prices may be higher or lower under OSS competition depending on how much consumers value quality and how large the market is.

Another interesting finding in our analysis is that, in the presence of OSS competition, it is possible for the market to be served entirely by a single firm though not necessarily by the lower cost firm. By comparison, proprietary competition always leads to a vertically segmented market with the high (low) cost firm selling a low (high) quality product. This result is particularly interesting when the proprietary firm has a higher cost of providing quality. In this case, it can never profitably serve only to the low value consumers. If high value consumers join the OSS, low value consumers can free-ride on their contributions without incurring a cost. This leaves no room for the proprietary firm to profitably get their demand. At the same time, it cannot provide a low enough price to attract high value consumers without also attracting low value consumers into its market. As a result, it either stays out of the market if its cost is prohibitive or it serves all consumers foreclosing the OSS firm.

In terms of quality provision, we find that proprietary competition always provides greater quality to high value users, but not necessarily so for low value consumers. Even with lower quality, consumers’ surplus may be higher with OSS competition if price competition is more intense.

Our paper contributes to the growing discussion in the economics literature on the impact of OSS on traditional proprietary firms. Many economists have examined the economic incentives governing OSS provision and have recognized the public good under-provision problem surrounding the open source (Lerner and Tirole, 2002; Johnson, 2002; Atal and Shankar, 2013.) A number of recent papers have also analyzed various aspects of the interaction between proprietary software and OSS software as we do. One stream of research explores the complementary interaction between OSS and proprietary firms. Bonaccorsi et al. (2006) explore hybrid business models adopted by large technology firms. Others such as Kumar et al. (2011), Economides and Katsamakas (2006), Casadesus-Masanell and Ghemawat (2006) and Mustonen (2003) study the effect of competition between proprietary and OSS firms on prices, quality, profitability and welfare. In contrast to them, we explore the difference in market outcomes in prices and quality when software competition occurs between two proprietary firms and when a single proprietary firm faces competition from an OSS firm.
## 2 A Brief Historical Overview

The software market provides an interesting yet complicated subject for economic analysis because of its highly dynamic nature. In this section, we provide a brief historical background by looking at three markets within computer software - (1) the desktop operating system market where competition has primarily revolved around two proprietary firms - Apple and Microsoft; (2) the web server operating system market where open source software (in particular, Apache) has been the dominant player, and (3) mobile operating system, currently a duopoly between Apple’s proprietary iOS and Google’s open source Android platform.

At the beginning of the PC era, Apple and Microsoft were the two dominant companies. Until the 1990s, Apple commanded more than 10 percent market share and dominated much of graphical desktop computing. However, Apple started to lose market share as desktop manufacturers, most notably, IBM adopted Microsoft’s MSDOS in the 1990s. Instead of aggressively competing, Apple chose to keep prices high to attract consumers who valued quality. Apple’s dominance from the market however began to wane through the 1990s and Microsoft Windows became a virtual monopoly with over 90 percent market share by the end of that decade. The Apple-Microsoft duopoly surfaced again in the 2000s. Microsoft released Windows XP, which became the company’s primary operating system (OS) while Apple debuted its Mac OS X 10.0. Microsoft released a series of updates to XP, few had any major updates. By contrast, Apple released several new versions of Mac OS X containing significant feature updates and performance enhancements that made its OS run faster. Apple also developed exclusive retail stores that enabled it to vertically differentiate its more expensive products from cheaper Windows machines more effectively.

While Apple’s OS X has been making steady gains, open source software such as Linux has not seen the same success, making up less than 1 percent of the market. On the other hand, OSS has had a major influence on web technology and the web server market. The leading server, Apache’s HTTP, controls more than 50 percent of the market and is far ahead of Microsoft which has a slim 20 percent share. As cloud computing and internet usage becomes an integral part of the future, the role of OSS in the coming years cannot be discounted.

In recent years, mobile computing has become important. At present, this market is a duopoly with Apple’s iOS and Google’s Android OS making up more than 90 percent of the market. As of August 2013, the market shares for Android and Apple were 51.6 percent and 40.7 percent respectively. The Android OS is released under the Apache license which is an open source license.
Although there is no conclusive evidence on whether open source Android produces a higher quality experience, the general consensus seems to be that the market is vertically segmented with Apple producing the higher priced better quality product. While the open nature of the Android OS has enabled it to support many more apps relative to what the iOS provides, there is a growing worry that individual decisions from application developers do not factor in the greater good of a unified Android platform. This concern highlights the public good problem faced by OSS.

It is tempting to draw similarities between the Microsoft-Apple competition in the desktop OS market with today's Apple-Google competition in the mobile market. However, as we argue in this paper, the public good nature of Android substantively changes the playing field and has very different implications for how consumers are affected and how the proprietary firm (or Apple) responds in terms of prices and quality. Competition between two proprietary firms is essentially dominated by competition for software consumers. An open source firm by contrast allows users of the software to voluntarily contribute and add value to the software. It thus integrates the roles of consumers as both users and developers. As a result, by competing for users, the proprietary firm simultaneously competes for developers who influence the quality of open source software. At the same time, the public good under-provision problem of open source software gives an inherent advantage to the proprietary firm. By attracting the best user-developers, it not only enhances its market share but also lowers the quality of the competing open source product.

3 Model and Analysis

There are two consumer groups with value for quality, \( v_i \in \{v^H, v^L\} \), \( v^H > v^L \). Let us call the high value consumers group \( H \) and the low value consumers group \( L \). If quality is \( Q \) and price is \( p \), consumer \( i \)'s utility is \( [v_i f(Q) - p] \), where \( f(Q) = Q^{1/2} \). We assume that there are \( N > 1 \) consumers of each type. On the production side, there is a proprietary seller, firm 1, who chooses quality \( Q_1 \) in the first stage and then a single price \( p_1 \). There are no production costs, but the seller's cost of quality is \( c_1 Q_1 \), \( c_1 > 0 \). Firm 1 may face competition from another seller, firm 2, who provides software of quality \( Q_2 \) at a cost of \( c_2 Q_2 \), where \( c_1 \neq c_2 \). We consider two organizational forms for firm 2; one where it is also proprietary like firm 1 and second where it is organized as an OSS with users making individual voluntary contributions towards the quality of the software. In the context
of an OSS, we consider the consumer population to be comprised of user-developers.¹

A few points about the difference between the proprietary and open source models of production are worth elucidating. First, a proprietary firm charges a positive price for the software that it sells to consumers. On the other hand, OSS is available freely to all users. However, users who contribute towards its quality incur a marginal cost of $c_2$. Second, OSS users determine their optimal contribution level by maximizing their own utility and the quality of the OSS is the aggregate contributions from all users. So every consumer who joins and uses OSS contributes so that her marginal benefit from contribution, $v_i f'(Q)$ is just equal to marginal cost $c_2$. As a result, individual users ignore the positive externality that their contribution provides to other users in the network leading to the classic under-provision problem faced by a public good. By contrast, a proprietary firm appropriates revenue from all its users by charging a positive price. Hence it chooses its profit maximizing quality level where its marginal cost equals the total additional benefit over all its users.

In order to ensure the existence of a price equilibrium for every quality choice, we make the following tie-breaking assumptions. If a consumer is indifferent between buying the good and not buying at all, she buys the good. If the consumer is indifferent between the two firms, she chooses the higher quality firm. And if the firm is indifferent between selling to only one group and selling to both groups, it chooses to cover the market and sells to both groups. We also restrict our attention to symmetric equilibria where consumers of the same type make the same choices. Finally, in order to ensure that the market is vertically segmented when two proprietary firms compete, we assume that there is sufficient variation in the value for quality by the two consumer groups, i.e., $v^H > 3v^L$. The equilibrium concept used is Subgame Perfect Nash Equilibrium.

Below we describe two market situations for software production; the first where both firms are proprietary, and the second where firm 1 is proprietary but firm 2 is organized as an OSS.

### 3.1 Duopoly Competition between Proprietary Firms

Let us start by looking at the case where there are two proprietary firms in the market. Firms first choose quality simultaneously and then compete in prices.

We solve the equilibrium in prices and quality by backward induction. In the second stage, firms choose prices given quality choice $Q_i$ and $Q_j$ by firms $i$ and $j$ respectively in stage 1. Except for the difference in quality, the two firms are otherwise identical in this stage since there are no

¹This is a common modeling approach used when considering open source software development. See Atal and Shankar (2013), Economides and Katsamakas (2006).
production costs. The lemma below summarizes the price equilibrium in the second stage, given $Q_i$ and $Q_j$. Without loss of generality, let us assume that $Q_i \geq Q_j$.

**Lemma 1** If $Q_i = Q_j$, then $p_i^* = p_j^* = 0$. If the difference in qualities is small enough, then in every price equilibrium firm $i$ covers the entire market at a price $p_i^* = \left[ v^L \left( Q_i^{3/2} - Q_j^{3/2} \right) + p_j^* \right]$. If the difference in qualities is large enough, then the unique price equilibrium is: $p_i^* = \left[ v^H Q_i^{3/2} - (v^H - v^L) Q_j^{3/2} \right]$ and $p_j^* = v^L Q_j^{3/2}$. All high value consumers buy from firm $i$ and all low value consumers buy from firm $j$.

If $Q_i = Q_j$, then Bertrand price competition drives prices and profits to zero. If the difference in quality between the two firms is small but positive, then the higher quality firm $i$ sells to both consumer groups driving the other firm out of the market. Firm $i$ then simply charges a price equal to group $L$’s reservation value that makes them indifferent between firms $i$ and $j$. Firm $j$ can choose a range of prices in this equilibrium, but it does not get any demand. This is because when the difference in quality between the two firms is small, the price at which $i$ can get group $H$’s demand is not much greater than the price at which it can get $L$’s demand. However, as we show below in Proposition 1, firms, in the first stage, never choose quality in this range. Finally, if $Q_i$ is substantially higher than $Q_j$, then the unique price equilibrium is a vertically differentiated market where $H$ consumers buy from $i$ and $L$ consumers buy from $j$. Firm $i$ cannot extract the entire valuation from $H$ consumers. In order to ensure their demand, $i$ has to make them indifferent between buying $Q_j$ at price $p_j$ and buying $Q_i$. Firm $j$, however, does not face a binding incentive constraint with the low value consumers, so it can charge the highest possible price from them.

Now let us look at what happens during quality competition in the first stage. Without loss of generality, let us assume that $i$ is the low cost firm, i.e., $c_i < c_j$. The two firms choose different quality levels in order to separate the market and limit price competition in the second stage. However, it is not necessarily the case that the low cost firm produces the higher quality product. If the difference in cost between the two firms is low enough, then an equilibrium exists where $j$ produces the higher quality product. But if the difference in the cost of quality provision is sufficiently high, then a unique equilibrium emerges where $Q_i > Q_j$. This is summarized in the proposition below.

**Proposition 1** In a duopoly where both firms are proprietary with $c_i < c_j$, the equilibrium is as follows: $Q_i^* = \left( \frac{N_i v^H}{2c_i} \right)^2 > \left( \frac{N_i v^L}{2c_j} \right)^2 = Q_j^*$, where firm $i$ sells to high value consumers and firm $j$ sells
to low value consumers. In addition, if the difference in marginal cost of quality is small enough, then there exists another equilibrium given by: 

$$Q_i^* = \left( \frac{Nv_i}{2c_i} \right)^2 < \left( \frac{Nv_j}{2c_j} \right)^2 = Q_j^*,$$

where the low-cost firm $i$ sells to low value consumers and high-cost firm $j$ sells to high value consumers.

In cases where there are multiple equilibria, we restrict the equilibrium to the one where the lower cost firm provides the higher quality. Note that the quality provided by both firms increase as $N$ increases. This is because the proprietary firm’s revenue from providing a unit of quality increases as the number of consumers increase, while the cost of providing that quality does not.

### 3.2 Duopoly with an Open Source Competitor

Now suppose firm 2 operates as an open source firm where consumers themselves can determine quality jointly through contributions. Let $q^H$ and $q^L$ represent the contribution from an $H$ and $L$ consumer respectively. Then the expected payoff to consumer $k$ belonging to group $H$ from contributing $q^H_k$ is 

$$v^H \left( (N - 1) q^H + q^H_k + Nq^L \right)^2 - c_2 q^H_k.$$ 

Let us define,

$$Q^H_o := \left( \frac{v^H}{2c_2} \right)^2.$$ 

$Q^H_o$ represents the ideal quality level that a contributing $H$ user would like to achieve in the network. So if aggregate contributions from other users exceeds this ideal quality level, then $q^H_k = 0$. In other words, user $k$ then free-rides on other users’ contributions. If, however, total contributions from all other users is lower than $Q^H_o$, then the $H$ user simply contributes the residual amount to achieve her ideal quality, i.e.,

$$q^H_k = \begin{cases} 
0, & \text{if } [(N - 1) q^H + Nq^L] > Q^H_o, \\
Q^H_o - \left( (N - 1) q^H + Nq^L \right), & \text{otherwise}. 
\end{cases}$$

Similarly, for consumer $l$ from group $L$, let $Q^L_o$ be her ideal quality:

$$Q^L_o := \left( \frac{v^L}{2c_2} \right)^2.$$ 

Note that $Q^L_o < Q^H_o$. Then,

$$q^L_l = \begin{cases} 
0, & \text{if } [(N - 1) q^L + Nq^H] > Q^L_o, \\
Q^L_o - \left( (N - 1) q^L + Nq^H \right), & \text{otherwise}. 
\end{cases}$$

We assume that all users of the same type contribute equal amounts in equilibrium. Then it is straightforward to see that in any equilibrium, where users from group $H$ join the OSS, the quality
is $Q^H$. If $L$ users join, they free-ride since $Q^H < Q^L$. Given that $L$ users do not contribute even if they join, $H$ users find it optimal to contribute $Q^H \over N$ in aggregate. Under a symmetric equilibrium, this means that $q^H_k = Q^H \over N$. If, in equilibrium, $H$ users do not join the OSS while $L$ users do, then each $L$ user contributes $q^L_k = Q^L \over N$, to achieve their ideal quality $Q^L$.

In every case, the quality of the software does not depend on the number of users. This is because each user only cares about her own value and not about the value that her contribution generates to the other users in the network. This is the classic public good under-provision problem. Further, in this market, the quality of OSS is determined during the price competition phase itself. Since the price chosen by firm 1 determines who joins the OSS firm, it affects contributions made to OSS and hence the quality of the competing good.

In the following lemma, we describe the equilibrium price given $Q_1$ when the proprietary seller faces competition from an OSS firm.

Lemma 2 There exist $Q^h_1$, $\tilde{Q}_1$, $p^l_1$, and $p^h_1$ (defined in the proof in the appendix) such that if $Q_1 < Q^h_1$, then no one buys from the proprietary seller; if $Q^h_1 \leq Q_1 \leq \tilde{Q}_1$, then the proprietary seller sells to both consumer groups at a price $p_1 = \min \{p^h_1, p^l_1\}$; if $Q_1 > \tilde{Q}_1$, then the proprietary seller sells to group $H$ consumers only at a price $p_1 = p^h_1$.

Similar to what we found in Lemma 1 with proprietary competition, it continues to be the case that when firm 1’s quality is high enough, a vertically differentiated market emerges; while it is also possible for the market to be served by a single firm that provides the software to both consumer groups. However, it is worth noting the following difference in the price equilibrium that emerges with OSS competition compared to what we see with proprietary competition when the proprietary firm covers the market. With proprietary competition, group $L$’s reservation price to buy firm 1’s good is always lower than group $H$’s reservation price. However, with OSS competition, group $L$’s reservation price, $p^l_1$, could be higher or lower than group $H$’s reservation price, $p^h_1$. This is because the quality of the competing good is different depending on whether it is comprised of $H$ consumers or $L$ consumers. Although $L$ consumers value quality less than $H$ consumers, they may still get higher utility from joining the OSS firm since their contribution levels and hence costs in the OSS firm are lower. It is easy to see that when firm 1’s quality is high enough, $p^h_1 > p^l_1$.

Further, this also means that whereas with two proprietary firms quality competition ensures that firms differentiate their quality levels enough to enjoy a positive share of the market, with OSS competition this need not happen. In particular, depending on costs, the market may be served by
a single OSS firm, a single proprietary firm or by both firms vertically segmenting the market. The proposition below describes this equilibrium. In order to restrict the number of cases to consider here, we assume that when the proprietary firm covers the market, it chooses the lowest quality required to induce both $H$ and $L$ consumers, by setting $p^1_L = p^h_1$. Let us denote this quality level by $\tilde{Q}_1$.

**Proposition 2** When the proprietary firm (firm 1) faces competition from an OSS competitor, there exist $c^H_1$ and $c^L_1$ (defined in the proof in the appendix) with $c^H_1 \geq c^L_1 > c_2$ such that firm 1 does not produce and all consumers join the OSS firm if $c_1 > c^H_1$, it sells only to high value consumers if $c_1 \leq c^L_1$ with quality $Q_1 = \left( \frac{Nv_H}{2c_1} \right)^2$ and it covers the market if $c^L_1 < c_1 \leq c^H_1$ with quality $\tilde{Q}_1 < \left( \frac{Nv_H}{2c_1} \right)^2$; the intermediate range is empty if $v_H > (3 + 2\sqrt{2}) v_L$.

As explained above, it is possible for the proprietary seller to stay out of the market if her costs are high relative to OSS costs. For intermediate costs, she may cover the market if the difference in valuation between the two consumer groups is not too high. Finally, if the proprietary seller’s cost is low enough, she provides the monopoly quality level to the $H$ consumers, while pushing the $L$ consumers into the OSS firm. Note that a proprietary firm competing with an OSS never serves just $L$ consumers. If $H$ consumers join the OSS firm, the quality of OSS is $Q_o^H > Q_o^L$. Then $L$ consumers can free-ride on the resulting high quality OSS so that the seller does not find it profitable to serve the $L$ consumers by matching their OSS payoff. When this happens, firm 1 prefers to stay out of the market altogether.

## 4 Proprietary Competitor vs. Open Source Competitor

In this section, we compare the market outcomes under both forms of competition. We first look at firm 1’s profits and then analyze quality outcomes and consumers’ surplus.

It is straightforward to see that when firm 1 is the higher cost firm, i.e., $c_1 > c_2$, it always has a positive market share when firm 2 is proprietary. However, if firm 2 is organized as OSS, then for $c_1 > c^H_1$, firm 1 does not produce. So if firm 1’s costs are very high relative to its competitor, its profits are always higher when the competitor is proprietary. When firm 1 is the lower cost firm, i.e., $c_1 < c_2$, then firm 1’s performance with an OSS competitor relative to a proprietary competitor

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2 Details regarding this assumption can be found in the proof of Proposition 2 in the appendix. The results presented in the paper are not affected by this restriction; however the analysis is more tractable.
depends on the size of the market and the difference in valuation of the two consumer groups. This is explained in the next proposition.

**Proposition 3** If the proprietary firm is the lower cost seller, i.e., \( c_1 < c_2 \), then the proprietary firm makes higher profit with a proprietary competitor than with an OSS competitor if and only if either \( N \leq 4 \) or \( \nu_H > \beta \nu_L \) where \( \beta \) is defined in the proof in appendix. If the proprietary firm is the higher cost seller, i.e., \( c_1 > c_2 \), then there exists \( \tilde{c}_1 > c_2 \) such that the proprietary firm makes higher profit with a proprietary competitor than with an OSS competitor if and only if \( c_1 > \tilde{c}_1 \).

The above proposition highlights the role of price competition and free-riding in an OSS. In order to understand the dynamics of competing with an OSS firm, consider the case where \( c_1 < c_2 \). Here firm 1 provides quality \( Q^*_1 = \left( \frac{N\nu^H}{4c_1} \right)^2 \) to group \( H \) consumers irrespective of whether it competes with another proprietary firm or an OSS firm. In this case, the difference in profits arises from the outside option for group \( H \) users. Under proprietary duopoly competition, the price that firm 1 can extract depends on the quality provided by the other firm, i.e., \( Q_2 \). When firm 2 is an OSS, then firm 1 has to compete against the potential quality that would result if \( H \) users joined the OSS, i.e., \( Q_o^H \). Comparing \( Q_2 \) and \( Q_o^H \), we see that there are two factors that affect the difference in the two quality levels. First, \( Q_o^H \) depends on the valuation of group \( H \) users, while \( Q_2 \) depends on \( \nu_L \). This tends to make \( Q_o^H \) relatively bigger. Second, however, because of potential free-riding by group \( H \) users, the quality provision in the OSS is not affected by the number of users. By contrast, for a proprietary firm, the marginal revenue from quality increases as the number of users increase. Thus \( Q_2 \) increases with \( N \) making it harder for firm 1 to charge a higher price for her good in the presence of a proprietary competitor.

Comparing quality, we find that group \( H \) users get an equal or higher quality product when firm 1 competes with a proprietary seller. If firm 1 is the lower cost seller, then they enjoy the same quality under both markets. If \( c_2 < c_1 \leq c_1^H \), then they are served by the lower cost firm in the proprietary duopoly market and hence quality is higher. If \( c_1 > c_1^H \), then free-riding by group \( H \) users in the OSS firm drives down the quality of the software below that provided by the proprietary firm in a proprietary duopoly. For group \( L \) consumers, on the other hand, quality may be higher or lower under OSS. The quality enjoyed by \( L \) users is higher under proprietary duopoly only when \( c_1 \leq c_1^L \). This is the case where group \( H \) users are served by the proprietary seller and group \( L \) users contribute to the OSS firm.

**Proposition 4** The equilibrium quality of the product for group \( H \) consumers is (weakly) better...
when duopoly competition is among two proprietary firms than with an OSS firm. Existence of an OSS competitor can improve quality to group L consumers if $c_1 > c_1^L$.

In the sense of consumers’ surplus, group L consumers are always better-off with OSS competition since they receive zero surplus in a proprietary duopoly. However, group H consumers may see a lower surplus if potential free-riding mitigates price competition with OSS. As a result, overall consumers’ surplus may be higher or lower depending on the market size and the relative valuation of group H and group L consumers.

**Proposition 5** There exists $\bar{c}_1 < c_1^H$ such that consumers’ surplus under duopoly competition between two proprietary firms is higher than that under duopoly with an OSS firm if and only if $\max\{c_1, c_2\} < \bar{c}_1$. As the number of consumers $N$ increases or the relative valuation $v_H$ decreases, $\bar{c}_1$ increases.

The price charged by the proprietary firm under an OSS depends on how high the potential quality under OSS is if group H joined the OSS firm. This is increasing in $v_H$. As a result, when $v_H$ is high, group H consumers are better off under OSS competition as price competition is more intense. Similarly, a low $N$ implies that the loss in quality from free-riding in an OSS is smaller.

**5 Conclusion**

The nature of competition in software markets has evolved over the decades as open source firms are playing an increasingly important role in influencing the competitive landscape. By integrating the role of users and developers, OSS provides leverage to users in the price that proprietary firms can charge. But at the same time, it is well recognized that public good aspects of OSS make quality provision difficult. This paper looked at the incentives of a seller of proprietary software to provide quality when she faces competition from an open source software product that has public good characteristics. Quality provision in the OSS product occurs through contributions by developers. Since the OSS product is available to all consumers, it functions as a public good and hence is prone to under-provision from free-riding among developers. The paper looked at how the public good nature of the competing product’s quality can affect the incentives for investing in quality by proprietary software firm. We compared the price and quality outcomes to the case where two proprietary firms competed in a traditional duopoly market. We found several interesting results. First, while a proprietary duopoly always lead to a vertically differentiated market, OSS
competition may lead to a single firm serving all consumers. Second, profits to the proprietary firm can be higher or lower with an OSS competitor as compared to another proprietary competitor depending on the intensity of price competition under the two market situations, which in turn depended on the severity of the free-riding problem faced by OSS. Conversely, consumers were better-off with OSS competition when it increased the intensity of price competition.

Appendix

Proof of Lemma 1. If \( Q_i = Q_j \), then Bertrand competition drives prices and profits to zero for both firms. Without loss of generality, let \( Q_i > Q_j \).

When \( P_j \geq v^HQ_j^\frac{1}{2} \), no one buys from firm \( j \) because firm \( i \) maximizes profits by selling to \( H \) at \( P_i = v^HQ_i^\frac{1}{2} \). But at this price, \( j \) can improve its profits by dropping price below \( v^HQ_j^\frac{1}{2} \). Let us now look at \( v^LQ_j^\frac{1}{2} < P_j < v^HQ_j^\frac{1}{2} \). In this case, \( L \) never buys from \( j \). Firm \( i \) can get \( L \)'s demand as long as \( P_i \leq v^LQ_i^\frac{1}{2} \), but to get \( H \)'s demand, it has to compete with \( j \)'s prices and set \( P_i \leq v^H \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) + P_j \). Firm \( i \) can then cover the market by setting price low enough or only sell to \( H \). In either case, \( j \) does not get any demand and hence earns zero profits. Thus \( v^LQ_j^\frac{1}{2} < P_j < v^HQ_j^\frac{1}{2} \) cannot be an equilibrium since \( j \) can improve its profits by lowering price at least to \( L \) consumers. This means that \( P_j^* \leq v^LQ_j^\frac{1}{2} \). Then demand for firm \( i \) is:

\[
D_i = \begin{cases} 
0, & \text{if } P_i > v^H \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) + P_j, \\
N, & \text{if } v^H \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) + P_j \geq P_i > v^L \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) + P_j, \\
2N, & \text{if } P_i \leq v^L \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) + P_j.
\end{cases}
\]

So the seller covers the market if and only if \( P_j \geq (v^H - 2v^L) \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) \). This is possible under this case only if \( Q_i \leq \left( \frac{v^H-v^L}{v^H-2v^L} \right)^2 Q_j \).

a) When \( Q_j < Q_i \leq \left( \frac{v^H-v^L}{v^H-2v^L} \right)^2 Q_j \), then \( (v^H - 2v^L) \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) \leq v^LQ_j^\frac{1}{2} \). (i) If \( P_j \leq (v^H - 2v^L) \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) \), we have \( P_i = v^H \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) + P_j \). Firm \( j \) can never get \( H \) consumers. But since she can get \( L \) consumers, she will maximize profits by setting \( P_j = v^LQ_j^\frac{1}{2} > (v^H - 2v^L) \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) \). So this cannot be an equilibrium. (ii) If \( (v^H - 2v^L) \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) \leq P_j \leq v^LQ_j^\frac{1}{2} \), then \( P_i = v^L \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) + P_j \). Every price within this range can qualify as an equilibrium. This is because given that \( P_j^* = v^L \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) + P_j^* \), firm \( j \) can never get any demand. To ensure continuity
in i’s profit function in stage 1, let us assume that:

\[ p_j^* = \frac{1}{2} \left[ (v^H - 2v^L) \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) + v^L Q_j^\frac{1}{2} \right], \]

so that:

\[ p_i^* = \frac{1}{2} \left[ v^H Q_i^\frac{1}{2} - (v^H - v^L) Q_j^\frac{1}{2} \right]. \]

b) When \( Q_i > \left( \frac{v^H - 2v^L}{v^H - v^L} \right)^2 Q_j \), then \( v^H - 2v^L \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) > v^L Q_j^\frac{1}{2} \). This means that for \( p_j \leq v^L Q_j^\frac{1}{2} \), we have \( p_j < \left( v^H - 2v^L \right) \left( Q_i^\frac{1}{2} - Q_j^\frac{1}{2} \right) \). Hence i never finds it profitable to cover the market. So j can serve L by setting \( p_j^* = v^L Q_j^\frac{1}{2} \), and i sells to H at \( p_i^* = \left[ v^H Q_i^\frac{1}{2} - (v^H - v^L) Q_j^\frac{1}{2} \right] \).

**Proof of Proposition 1.** First, note that \( Q_i^* > 0 \) and \( Q_j^* > 0 \). If firm i chooses \( Q_i^* \geq 0 \), then by Lemma 1, j can make positive profits by choosing \( Q_j > Q_i^* \). So \( Q_i^* > 0 \) and \( Q_j^* > 0 \). Also, given \( Q_i^* > 0 \), j never chooses \( Q_j \in \left( \left( \frac{v^H - 2v^L}{v^H - v^L} \right)^2 Q_i^*, Q_i^* \right) \) because by Lemma 1, j makes no revenue. Similarly, \( Q_i \notin \left( \left( \frac{v^H - 2v^L}{v^H - v^L} \right)^2 Q_i^*, Q_j^* \right) \).

If \( Q_i < \left( \frac{v^H - 2v^L}{v^H - v^L} \right)^2 Q_j \), then i sells to L and earns a profit of \( \left( N v^L Q_i^\frac{1}{2} - c_i Q_i \right) \), which is concave in \( Q_i \) and increasing at \( Q_i = 0 \). If \( Q_j \leq \left( \frac{N v^L}{2c_i} \left( \frac{v^H - v^L}{v^H - 2v^L} \right) \right)^2 \), then profit of i is always increasing and hence i does not choose \( Q_i < \left( \frac{v^H - 2v^L}{v^H - v^L} \right)^2 Q_j^* \). If \( Q_j > \left( \frac{N v^L}{2c_i} \left( \frac{v^H - v^L}{v^H - 2v^L} \right) \right)^2 \), then i may choose \( Q_i = \left( \frac{N v^L}{2c_i} \right)^2 \). The maximized profits are \( \left( \frac{N v^L}{4c_i} \right)^2 \).

If \( Q_i > Q_j^* \), then i covers the market and earns a profit of \( \left[ N \left( v^H Q_i^\frac{1}{2} - (v^H - v^L) \left( Q_j^* \right)^{\frac{1}{2}} \right) - c_i Q_i \right] \), which is concave in \( Q_i \). If \( Q_j^* \geq \left( \frac{N v^H}{2c_i} \right)^2 \), then profit of i is always decreasing and hence i does not choose \( Q_i > Q_j^* \). If \( Q_j^* < \left( \frac{N v^H}{2c_i} \right)^2 \), then i may choose \( Q_i = \left( \frac{N v^H}{2c_i} \right)^2 \) if the maximized profit is positive, i.e., if \( Q_j^* < \left( \frac{N \left( v^H \right)^2}{4c_i \left( v^H - v^L \right)} \right)^2 \). Combining these, we can say that in this case i may choose \( Q_i = \left( \frac{N v^H}{2c_i} \right)^2 \) if \( Q_j^* < \left( \frac{N \left( v^H \right)^2}{4c_i \left( v^H - v^L \right)} \right)^2 \) and the maximized profits are \( \left( \frac{N \left( v^H \right)^2}{4c_i} - N \left( v^H - v^L \right) \left( Q_j^* \right)^{\frac{1}{2}} \right) \).

Comparing the two threshold values for \( Q_j \), we get \( \left( \frac{N v^L}{2c_i} \left( \frac{v^H - v^L}{v^H - 2v^L} \right) \right)^2 < \left( \frac{N \left( v^H \right)^2}{4c_i \left( v^H - v^L \right)} \right)^2 \) because \( v^H > 3v^L \). Also, comparing the two maximized profits for firm i, we get: \( \left( \frac{N \left( v^H \right)^2}{4c_i} - N \left( v^H - v^L \right) \left( Q_j^* \right)^{\frac{1}{2}} \right) > \)
\frac{(Nv^L)^2}{4c_i} \text{ if and only if } \left( \frac{N(v^H+v^L)}{4c_i} \right)^2 > Q^*_j. \text{ Hence the best response of firm } i \text{ is given as follows:}

\begin{align*}
Q^*_i &= \begin{cases} 
\left( \frac{Nv^H}{2c_i} \right)^2, & \text{if } 0 < Q^*_j \leq \left( \frac{N(v^H+v^L)}{4c_i} \right)^2, \\
\left( \frac{Nv^L}{2c_i} \right)^2, & \text{if } Q^*_j > \left( \frac{N(v^H+v^L)}{4c_i} \right)^2.
\end{cases}
\end{align*}

Similarly, we get the best response of firm } j. \text{ Without loss of generality, let } c_i \leq c_j. \text{ Hence, possible equilibria are:}

(i) \quad Q^*_i = \left( \frac{Nv^H}{2c_i} \right)^2 \text{ and } Q^*_j = \left( \frac{Nv^L}{2c_j} \right)^2;
(ii) \quad Q^*_i = \left( \frac{Nv^L}{2c_i} \right)^2 \text{ and } Q^*_j = \left( \frac{Nv^H}{2c_j} \right)^2 \text{ only if } \frac{2v^L}{(v^H+v^L)} c_j \leq c_i.

It can be checked that for } v^H > 3v^L, \text{ when } Q^*_i > Q^*_j, \text{ we also have } Q^*_i > \left( \frac{v^H-v^L}{v^H-2v^L} \right)^2 Q^*_j, \text{ so that at the price competition stage, we get a vertically differentiated market.} \quad \square

Proof of Lemma 2. \text{ Given the price of proprietary good } p_1, \text{ let us derive the OSS network equilibrium. We consider only symmetric equilibrium, where all consumers from the same group contribute the same amount. Let us first derive } L \text{ consumer’s best response, given that } H \text{ consumers contribute } Nq^H. \text{ In a symmetric equilibrium, it means that:}

\begin{align*}
q^{L*} = \begin{cases} 
0, & \text{if } Nq^H > Q^L_o, \\
\left( \frac{1}{N}Q^L_o - q^H \right), & \text{otherwise}.
\end{cases}
\end{align*}

Similarly, let us derive } H \text{ consumer’s best response. Note that } Nq^{L*} \leq Q^L_o < Q^H_o. \text{ So } H \text{ consumers always contribute a positive amount to the network if they decide to use the OSS. They contribute:}

\begin{align*}
q^{H*} = \left( \frac{1}{N}Q^H_o - q^L \right).
\end{align*}

If } H \text{ joins the network, then quality of the OSS is at least } Q^H_o. \text{ This means that, given } H \text{ consumers are in the OSS, if } L \text{ consumers join, they do not contribute anything. So if } H \text{ joins the OSS, the quality is } Q^H_o \text{ and their utility from the OSS is } \left[ v^H \left( Q^H_o \right)^\frac{1}{2} - \frac{c_2}{N} Q^H_o \right]. \text{ If } L \text{ consumers join when } H \text{ consumers are in the OSS, they free ride so their utility is } v^L \left( Q^H_o \right)^\frac{\frac{1}{2}}{2}. \text{ Finally, if } L \text{ consumers join by themselves without } H \text{ contributing in OSS, their utility is } \left[ v^L \left( Q^L_o \right)^\frac{1}{2} - \frac{c_2}{N} Q^L_o \right].

So, given } p_1, \text{ } H \text{ joins the OSS if and only if } p_1 > v^H \left[ Q^\frac{1}{2} \frac{1}{2} - \left( Q^H_o \right)^\frac{1}{2} \right] + \frac{c_2}{N} Q^H_o. \text{ Define,}

\begin{align*}
p_1^h := v^H \left[ Q^\frac{1}{2} \frac{1}{2} - \left( Q^H_o \right)^\frac{1}{2} \right] + \frac{c_2}{N} Q^H_o.
\end{align*}
Hence, \( p_1^h \geq 0 \) if and only if \( Q_1^h \geq (1 - \frac{1}{2N}) (Q_o^H)^{\frac{1}{2}} \). Define,

\[
    Q_1^h := Q_o^H \left(1 - \frac{1}{2N}\right)^2 < Q_o^H.
\]

Therefore, to win \( H \)'s demand, the proprietary firm has to set \( Q_1 \geq Q_1^h \) and \( p_1 \leq p_1^h \).

Suppose \( H \) joins the OSS, then \( L \) joins the OSS if and only if \( p_1 > v^L \left[ Q_1^h \right] - (Q_o^H)^{\frac{1}{2}} \). In this case, to win \( L \)'s demand, the firm has to set \( Q_1 \geq Q_o^H \) and \( p_1 \leq p_1^h \) where,

\[
    \rho_1 := v^L \left[ Q_1^h \right] - (Q_o^H)^{\frac{1}{2}} < p_1^h.
\]

If \( H \) does not join the OSS, then \( L \) joins the OSS if and only if \( p_1 > v^L \left[ Q_1^h \right] + \frac{c_2}{N} Q_o^L \). Define,

\[
    p_1^h := v^L \left[ Q_1^h \right] + \frac{c_2}{N} Q_o^L.
\]

Hence, \( p_1^h \geq 0 \) if and only if \( Q_1^h \geq (1 - \frac{1}{2N}) (Q_o^L)^{\frac{1}{2}} \). Define,

\[
    Q_1^l := Q_o^L \left(1 - \frac{1}{2N}\right)^2 < Q_1^h.
\]

Therefore, to win \( L \)'s demand, the proprietary firm has to set \( Q_1 \geq Q_1^l \) and \( p_1 \leq p_1^l \).

Let us compare the price cutoffs. Note that \( \rho_1 < \min \{ p_1^h, p_1^l \} \). \( p_1 < p_1^h \) if and only if \( Q_1^h > \frac{1}{2c_2} (1 - \frac{1}{2N}) (v^H + v^L) \). Let us define,

\[
    \tilde{Q}_1 := \left[ \frac{1}{2c_2} \left(1 - \frac{1}{2N}\right) (v^H + v^L) \right]^2 > Q_1^h.
\]

Now we can consider different cases to explain who buys from where. If \( Q_1 < Q_1^l \), then no one buys from the proprietary seller. If \( Q_1 \leq Q_1^l < Q_1^h \), \( H \) never buys the proprietary good and since \( H \) is in the OSS and \( Q_1 < Q_o^H \), \( L \) also joins the OSS. Now suppose \( Q_1^h \leq Q_1 < Q_o^H \). (i) If \( Q_1^h \leq Q_1 < \min \{ \tilde{Q}_1, Q_o^H \} \), then \( p_1 > p_1^h \). Hence, if \( p_1 \leq p_1^h \), then \( H \) and \( L \) both buy from the proprietary seller. If \( p > p_1^h \), then \( H \) joins the OSS and since \( Q_1 < Q_o^H \), \( L \) also joins the OSS. (ii) If \( \min \{ \tilde{Q}_1, Q_o^H \} \leq Q_1 < Q_o^H \), then either this range is empty or we have \( p_1 ^h > p_1 ^h \). Hence, if \( p_1 \leq p_1^h \), then \( H \) and \( L \) both buy from the proprietary seller. If \( p_1 > p_1^h \), then \( L \) joins the OSS while \( H \) buys from the proprietary seller. If \( p > p_1^h \), then no one buys from the proprietary seller. Finally consider the case when \( Q_1 \geq Q_o^H \). (i) If \( Q_o^H \leq Q_1 \leq \max \{ \tilde{Q}_1, Q_o^H \} \), then either this range is empty or we have \( p_1 > p_1^h \). Hence, if \( p_1 \leq p_1^h \), then \( H \) and \( L \) both buy from the proprietary seller. If \( p > p_1^h \), then \( H \) joins the OSS and \( L \) also joins the OSS since \( p > p_1^h \). (ii) If \( Q_1 \geq \max \{ \tilde{Q}_1, Q_o^H \} \), then \( p_1 \leq p_1^h \). If \( p_1 \leq p_1^h \), then \( H \) and \( L \) both buy from the proprietary seller. If \( p_1 > p_1 \), then
L joins the OSS while H buys from the proprietary seller. If \( p > p^h_1 \), then no one buys from the proprietary seller.

As shown above, if \( Q_1 > \tilde{Q}_1 \) so that \( 0 < p_1^l < p_1^h \), the proprietor can sell her goods to everyone by setting \( p_1 = p_1^l \) or to \( H \) only by setting \( p_1 = p_1^h \). Hence, she covers the market if and only if \( 2p_1 > p_1^h \), or,

\[
Q_1^2 < \frac{1}{2c_2} \left( 1 - \frac{1}{2N} \right) \frac{(v^H)^2 - 2(v^L)^2}{(v^H - 2v^L)} =: Q_1^\ast, \text{ say.}
\]

Note that \( \tilde{Q}_1 > Q_1 \).

Hence, combining all the cases above, we can summarize that if \( Q_1 < Q_1^h \), then no one buys from the proprietary seller. If \( Q_1^h < Q_1 \leq \tilde{Q}_1 \), then the proprietor charges \( p_1 = \min \{ p_1^l, p_1^h \} \) and covers the entire market. Finally, if \( Q_1 > \tilde{Q}_1 \), then the proprietor sells to \( H \) only at the price \( p_1 = p_1^h \).

**Proof of Proposition 2.** The proprietary seller’s profits are:

\[
\pi_1(Q_1) = \begin{cases} 
-c_1Q_1, & \text{if } Q_1 \leq Q_1^h, \\
2N \left[ v^H \left( Q_1^\ast - \left( \frac{Q_1^h}{N} \right)^{1/2} \right) + \frac{c_2}{N}Q_1^h \right] - c_1Q_1, & \text{if } Q_1^h < Q_1 \leq \tilde{Q}_1, \\
2N \left[ v^L \left( Q_1^\ast - \left( \frac{Q_1^h}{N} \right)^{1/2} \right) + \frac{c_2}{N}Q_1^h \right] - c_1Q_1, & \text{if } \tilde{Q}_1 \leq Q_1 \leq \tilde{Q}_1, \\
N \left[ v^H \left( Q_1^\ast - \left( \frac{Q_1^h}{N} \right)^{1/2} \right) + \frac{c_2}{N}Q_1^h \right] - c_1Q_1, & \text{if } Q_1 > \tilde{Q}_1.
\end{cases}
\]

First, note that if the seller provides the proprietary good at all, \( Q_1 > Q_1^h \). Let us look for an equilibrium in the range \( Q_1^h \leq Q_1 < \tilde{Q}_1 \). The maxima for profit in this range is \( Q_1^\ast = \left( \frac{Nv^H}{c_1} \right)^2 \). For this to be an equilibrium, it must be the case that \( \left( \frac{Nv^H}{c_1} \right)^2 < \tilde{Q}_1 \) and \( \pi_1(Q_1^\ast) > 0 \). Now \( \left( \frac{Nv^H}{c_1} \right)^2 < \tilde{Q}_1 \) if and only if \( c_1 > \frac{4v^H}{(v^H + v^L)(2N - 1)c_2} \), and \( \pi_1(Q_1^\ast) > 0 \) if and only if \( c_1 < \frac{2N^2}{(2N - 1)c_2} \), which cannot happen together and hence \( Q_1^\ast > \tilde{Q}_1 \).

Next, let us look for an equilibrium in the range \( \tilde{Q}_1 \leq Q_1 \leq \tilde{Q}_1 \). The maxima for profit in this range could be interior or at either of the two corners. If \( \pi_1' \left( \tilde{Q}_1 \right) \geq 0 \), then the maxima could be at \( \tilde{Q}_1 \), but it cannot be an equilibrium because \( \pi_1(Q_1) \) is continuous and \( \lim_{Q_1 \to \tilde{Q}_1^-} \pi_1(Q_1) > \lim_{Q_1 \to \tilde{Q}_1^+} \pi_1(Q_1) \geq 0 \). The maximum possible profit in the range \( \tilde{Q}_1 \leq Q_1 < \tilde{Q}_1 \) is \( \pi_1 \left( \left( \frac{Nv^L}{c_1} \right)^2 \right) = \frac{(Nv^L)^2}{c_1} - (2N - 1) \left( \frac{v^L}{2c_2} \right)^2 \), and in the range \( Q_1 > \tilde{Q}_1 \) is \( \pi_1 \left( \left( \frac{Nv^H}{2c_1} \right)^2 \right) = \frac{(Nv^H)^2}{4c_2} - (2N - 1) \left( \frac{v^H}{4c_2} \right)^2 \).

Hence, if \( c_1 \geq \frac{2N^2}{(2N - 1)c_2} \), then the profits become negative and the proprietary seller does not sell at all.

If \( \frac{N^2}{2N - 1}c_2 \leq c_1 \leq \frac{2N^2}{2N - 1}c_2 \), then if the proprietor sells, then he chooses a quality such that
\( \tilde{Q}_1 \leq Q_1 < \tilde{Q}_1 \) because \( \pi_1 \left( \left( \frac{Nv^H}{2c_1} \right)^2 \right) < 0 \). In this case, it is easy to check that \( \pi_1 \left( \tilde{Q}_1 \right) < 0 \) and hence the maxima is \( \tilde{Q}_1 \). For this to be an equilibrium, \( \pi_1 \left( \tilde{Q}_1 \right) \geq 0 \) which happens if and only if \( c_1 \leq \frac{8v^Hv^L}{(v^H+v^L)^2} \frac{N^2}{2N-1} c_2 \). If \( v^H > (3 + 2\sqrt{2}) v^L \), then this cannot be true.

Now suppose \( c_1 < \frac{N^2}{2N-1} c_2 \). In this case, note that \( \left( \frac{Nv^H}{2c_1} \right)^2 > \tilde{Q}_1 \) because \( c_1 < \frac{2v^H(v^H-2v^L)}{(v^H)^2-2(v^L)^2} \frac{N^2}{2N-1} c_2 \). Also, \( \tilde{Q}_1 \leq \left( \frac{Nv^L}{c_1} \right)^2 < \tilde{Q}_1 \) if and only if:

\[
\begin{align*}
\frac{4v^L}{(v^H+v^L)} \frac{N^2}{2N-1} c_2 &\geq c_1 > \frac{4v^L (v^H-2v^L)}{(v^H)^2-2(v^L)^2} \frac{N^2}{2N-1} c_2,
\end{align*}
\]

and \( \pi_1 \left( \left( \frac{Nv^H}{2c_1} \right)^2 \right) \leq \pi_1 \left( \left( \frac{Nv^L}{c_1} \right)^2 \right) \) if and only if \( c_1 \geq \frac{(v^H)^2-4(v^L)^2}{(v^H)^2-2(v^L)^2} \frac{N^2}{2N-1} c_2 \), but they cannot happen together as long as \( v^H > av^L \), where \( a \) solves \( a^3 - 3a^2 - 4a + 4 = 0 \). Hence, for the remainder of the analysis, we assume that \( v^H \geq 3.8v^L \). This ensures that \( Q_1 = \left( \frac{Nv^L}{c_1} \right)^2 \) is never an equilibrium and if firm 1 covers the market, she will choose \( \tilde{Q}_1 \).

Now \( \pi_1 \left( \left( \frac{Nv^H}{2c_1} \right)^2 \right) \leq \pi_1 \left( \tilde{Q}_1 \right) \) if and only if:

\[
\left[ c_1 \left( 1 - \frac{1}{2N} \right) (v^H + v^L) \right]^2 - c_1 c_2 (2N-1) \left( v^H \right)^2 + (v^H c_2)^2 \leq 0,
\]

which happens if and only if \( c_1 \geq \frac{N^2}{2N-1} \alpha c_2 \), where

\[
\alpha := \frac{2v^H}{(v^H+v^L)^2} \left( (v^H + v^L) - \sqrt{v^L (2v^H + 3v^L)} \right).
\]

Note that \( \frac{N^2}{2N-1} \alpha c_2 > \frac{N^2}{2N-1} \left( \frac{4v^L}{v^H+v^L} \right) c_2 \).

Hence, summarizing, the proprietary seller does not produce and all consumers join the OSS if \( c_1 > \max \left\{ 1, \frac{8v^Hv^L}{(v^H+v^L)^2} \right\} \frac{N^2}{(2N-1) c_2} =: c^H \), say; she sells only to \( H \) at price \( p^h = \left( \frac{v^H}{2c_1} \right)^2 \left( \frac{N^2}{c_1} - \left( 1 - \frac{1}{2N} \right) \frac{1}{c_2} \right) \)

and quality \( Q^*_1 = \left( \frac{Nv^H}{2c_1} \right)^2 \) if \( c_1 \leq \min \{ \alpha, 1 \} \frac{N^2}{(2N-1) c_2} =: c^L \), say; she covers the market at price \( p^l = \frac{v^H v^L}{2c_1} (1 - \frac{1}{2N}) \) and quality \( \tilde{Q}_1 \) if \( c^L < c_1 \leq c^H \), in other words, if \( v^H \leq (3 + 2\sqrt{2}) v^L \) and \( \frac{N^2}{2N-1} \alpha c_2 < c_1 \leq \frac{N^2}{2N-1} \frac{8v^H v^L}{(v^H+v^L)^2} c_2 \). Note that the intermediate range is empty if \( v^H > (3 + 2\sqrt{2}) v^L \).

**Proof of Proposition 3.** Suppose \( c_1 < c_2 \). With a duopoly of two proprietary firms, we have \( Q^d_1 = \left( \frac{Nv^H}{2c_1} \right)^2 \) and

\[
\pi_1^d = \frac{(Nv^H)^2}{4c_1} - \frac{N^2}{2c_2} (v^H - v^L) v^L.
\]

\(^{3}a \approx 3.78.\)
Note that $c_1 < c_2 < c^f_1$, hence if faced by an OSS competitor, the proprietary firm sells only to $H$ at price $p_1 = \frac{(v^H)^2}{2} \left[ \frac{N}{c_1} - \left(1 - \frac{1}{2N} \right) \frac{1}{c_2} \right]$ and quality $Q_1 = Q^d_1$. Thus,

$$\pi^d_1 = \frac{(Nv^H)^2}{4c_1} - \frac{(2N - 1)(v^H)^2}{4c_2}.$$ 

Hence, when $c_1 < c_2$, then $\pi^d_1 > \pi^d_1^\text{oss}$ if and only if $(2N - 1)(v^H)^2 > 2N^2(v^H - v^L)v^L$ which holds true if and only if either $N < 4$ or $v^H > \beta v^L$, where

$$\beta := \frac{N}{(2N - 1)} \left[ N + \sqrt{(N - 2)^2 - 2} \right].$$

Now suppose $c_1 > c_2$. Hence we have $Q^d_1 = \left( \frac{Nv^H}{2c_1} \right)^2$ and $\pi^d_1 = \frac{(Nv^L)^2}{4c_1}$. When $c_2 < c_1 \leq c^f_1$, then if faced by an OSS competitor, the proprietary firm sells only to $H$. When $c^f_1 < c_1 \leq c^H$, then it covers the market at price $p_1 = \frac{v^Hv^L}{2c_2} \left( 1 - \frac{1}{2N} \right)$ and quality $\hat{Q}_1$. Thus, when $c_2 < c_1 \leq c^H$,

$$\pi^d_1^\text{oss} = \max \left\{ \frac{(Nv^H)^2}{4c_1} - \frac{(2N - 1)(v^H)^2}{4c_2} , \frac{(2N - 1)v^Hv^L - c_1\hat{Q}_1}{2} \right\}. $$

Thus, when $c_1 > c_2$, then $\pi^d_1 > \pi^d_1^\text{oss}$ if $c_1 > \hat{c}_1$, where

$$\hat{c}_1 = \max \left\{ \frac{(v^H)^2 - (v^L)^2}{(v^H)^2} , \frac{2v^L}{(v^H + v^L)^2} \left[ 2v^H + \sqrt{(2v^H)^2 - (v^H + v^L)^2} \right] \right\} \frac{N^2}{(2N - 1)c_2}.$$ 

Note that when $v^H > (3 + 2\sqrt{2})v^L$, we have $c^f_1 = c^H > \hat{c}_1$. When $v^H \leq (3 + 2\sqrt{2})v^L$, we have $c^L < c^f_1$, $\tilde{c}_1 < c^H$ and $(\pi^d_1 - \pi^d_1^\text{oss})$ is increasing in $\frac{c_1}{c_2}$. When $c_1 > c^H$, then if faced by an OSS competitor, the proprietary firm does not produce and sell. Thus $\pi^d_1 > \pi^d_1^\text{oss}$. Hence, combining, we can say that when $c_1 < c_2$, then $\pi^d_1 > \pi^d_1^\text{oss}$ if and only if $c_1 > \hat{c}_1$. ■

**Proof of Proposition 4.** Note that the equilibrium quality that $H$ receives in case the proprietor faces an OSS competitor is:

$$Q^H_\text{oss} = \begin{cases} \frac{(Nv^H)^2}{2c_1} , & \text{if } c_1 \leq c^L_1 \\ \frac{1}{2c_2} \left( 1 - \frac{1}{2N} \right)(v^H + v^L)^2 , & \text{if } c^L_1 < c_1 \leq c^H_1 \\ \frac{(v^H)^2}{2c_2} , & \text{if } c_1 > c^H_1. \end{cases}$$

Whereas if the proprietor competes with another proprietor, then the equilibrium quality that $H$ receives is:

$$Q_d^H = \left( \frac{Nv^H}{2 \min \{c_1, c_2\}} \right)^2.$$ 

Clearly, $Q_d^H \geq Q^H_\text{oss}$ with a strict inequality whenever $c_1 > c_2$. 


Similarly, the equilibrium quality that $L$ receives in case the proprietor faces an OSS competitor is:

$$Q_{oss}^L = \begin{cases} \left( \frac{v^L}{2c_2} \right)^2, & \text{if } c_1 \leq c_1^L \\ \left[ \frac{1}{2c_2} \left( 1 - \frac{1}{2N} \right) \left( v^H + v^L \right) \right]^2, & \text{if } c_1^L < c_1 \leq c_1^H \\ \left( \frac{v^H}{2c_2} \right)^2, & \text{if } c_1 > c_1^H. \end{cases}$$

Whereas if the proprietor competes with another proprietor, then the equilibrium quality that $L$ receives is:

$$Q_d^L = \left( \frac{N v^L}{2 \max \{c_1, c_2\}} \right)^2.$$

It is easy to check that $Q_d^L > Q_{oss}^L$ if and only if $c_1 \leq c_1^L$, where $L$ users can join the OSS and $H$ users are served by the proprietor.

**Proof of Proposition 5.** With a duopoly of two proprietary firms, the consumers’ surplus in duopoly between two proprietary firms is:

$$CS^d = \frac{N^2}{2 \max \{c_1, c_2\}} \left( v^H - v^L \right) v^L.$$

Similarly, the consumers’ surplus in duopoly between a proprietary firm and an OSS firm is:

$$CS^{oss} = \begin{cases} \left( \frac{2N-1}{4c_2} \right) \left( (v^H)^2 + (v^L)^2 \right), & \text{if } c_1 \leq c_1^H \\ \left( \frac{2N-1}{4c_2} \right) \left( v^H \right)^2 + \frac{N}{2c_2} v^H v^L, & \text{if } c_1 > c_1^H. \end{cases}$$

Suppose $c_1 \leq c_1^H$. We find that $CS^{oss} > CS^d$ if and only if:

$$\max \{c_1, c_2\} > 2 \left( \frac{v^H - v^L}{v^H + v^L} \right) v^L \frac{N^2}{(v^H)^2 + (v^L)^2 (2N-1)} c_2 =: \tilde{c}_1, \text{ say.}$$

If $c_1 > c_1^H$, we find that $CS^{oss} > CS^d$ always. ■

**References**


