Do higher search costs make markets less competitive?*

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Abstract

We study price formation in models with search cost heterogeneity. In doing so, we dispense with the usual assumption that all consumers search at least once in equilibrium. This allows for an important role of the price mechanism often neglected by the literature, namely, that the price ought to affect the share of consumers who choose to search for a product rather than not searching for it at all. Recognising this role turns out to be critical for our understanding of the effect of higher search costs on prices and profits. We show that higher search costs may result in more elastic individual demand functions, and therefore lead to lower prices. This happens because an increase in search costs affects two margins, the *intensive search margin*, or search intensity, and the *extensive search margin*, or participation. Higher search costs result in less search intensity, making demand more inelastic; however, higher search costs also result in a change in the composition of demand because a larger share of high search cost consumers chooses not to search, which makes demand more elastic. We identify conditions on the search cost density/distribution for higher search costs to result in higher or lower prices. The main insights of the paper hold no matter whether products are differentiated or homogenous and irrespective of whether consumers search sequentially or non-sequentially.

Keywords: non-sequential search, sequential search, oligopoly, search cost heterogeneity, homogeneous products, differentiated products

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1 Introduction

Ever since Stigler (1961), search theory has become an important tool for understanding the functioning of real-world markets. Seminal contributions include the homogeneous product market models of Burdett and Judd (1983), Reinganum (1979) and Stahl (1989), where the important phenomenon of price dispersion is given a microfoundation based on search theory. When products are differentiated, the market models of Wolinsky (1986) and Anderson and Renault (1999) show that prices remain above costs even if firm entry is costless and correspondingly infinitely many firms enter the market. Another well-known and important result in this literature that is common to homogeneous and differentiated product market models is that higher search costs lead to higher prices, thus benefiting firms at the expense of consumers.

This paper is devoted to the study price formation under consumer search cost heterogeneity. When studying search markets with search cost heterogeneity, it is natural to dispense with an assumption that the consumer search literature has almost invariably made: most models assume that the cost consumers have to incur in order to conduct a search is "sufficiently low," *de facto* implying that all consumers search at least once in equilibrium (e.g. Stahl, 1989; Burdett and Judd, 1983; Wolinsky, 1986). As pointed out by Stiglitz (1989), assuming otherwise that search costs are large can potentially cause the market to collapse. For example, in the setting of Diamond (1971), the only price that can be part of a market equilibrium is the monopoly price –this result is referred to as the "Diamond paradox". If the search cost is relatively high, the surplus consumers derive at the monopoly price may be insufficient to cover the cost of the first search, in which case consumers rather do not search at all and the market fails to exist.

Apart from being simply less general, the standard "low-search-cost" assumption is also restrictive. To understand why, note that in search markets the price mechanism ought to affect the share of consumers who choose to search for a good deal in the first place (which we refer to as the *extensive search margin*), as well as the intensity with which consumers search (which we call the *intensive search margin*). The literature, by assuming that all consumers search at least once, has focused on the effects of the intensive search margin on price determination and has neglected the role of the extensive search margin.

Failing to recognise this role of the price is critical for our complete understanding of the functioning of consumer search markets. For example, Janssen et al. (2005) show that in the standard consumer search model of Stahl (1989) (the distribution of) equilibrium prices will surely decrease as the search cost goes up provided that the search cost is sufficiently high. This point is also the central tenet in Anderson and Renault (2006), who, by allowing for arbitrary search costs, are able to reconcile the empirical observation that much of the advertising we observe in arguably search environments does impart only match information and not price information. Our paper shows that the behavior of the equilibrium price (distribution) depends on the *range* of search costs as well as on the *properties* of the search cost density/distribution. As we will see later in more detail, the equilibrium price can increase, remain constant or decrease as search costs increase. This implies that whether firms benefit from higher information frictions depends on the context.

We start the analysis by considering models in which products are differentiated and the price equilibrium is in pure-strategies, as in the model of Wolinsky (1986). In these models, consumers do not search for lower prices but for satisfactory products and have correct expectations about the equilibrium price. We study both the case in which consumers search sequentially and the case in which they search non-sequentially. The chief difference between these two search protocols is in terms of the ability of an individual firm to affect the different aforementioned margins. While under sequential search an individual firm can affect the intensive search margin, this is not the case under non-sequential search. Nonetheless the main insights we derive are common to the two modes of search.

In the standard sequential search model of Wolinsky (1986), consumers visit a first store and conditional on the utility they obtain at the current store they decide whether to walk away and visit a second store or to acquire the product at hand, and so on and so forth. When the search cost of every consumer is sufficiently low, then they all choose to search in the market until they find a satisfactory product. However, when the range of search costs is sufficiently large, some consumers choose not to search at all just because what they expect to get in the market falls short of their costs of search. An individual firm, facing the various consumer types, chooses its price to maximise the profits obtained from the average consumer that chooses to search. We show that the model has a unique equilibrium in pure strategies when the density of match values is increasing and log-concave and the search cost density is also log-concave.

We then move to study the relation between search costs and prices. We demonstrate that the effect of an increase in search costs on the equilibrium price in a situation where all consumers have relatively low search costs and correspondingly they all search at least one time differs fundamentally from the case in which search costs are more dispersed and some consumers opt out of the search market altogether. The reason is that, everything else constant, in the first case an increase in search costs has a bearing only on the intensive search margin while in the second case both the intensive and the extensive search margins are affected by the cost shock.

In the first case, we show that higher search costs unambiguously lead to higher prices, in line with the literature results. Intuitively, what happens is that when search costs are low and all consumers participate, an increase in search costs results in a clockwise rotation of the demand, making it more inelastic. This occurs because all consumers choose to search less after search costs go up. Firms, anticipating a more inelastic demand, raise their prices to maximise profits.

However, when the range of search costs is sufficiently large, a shock that makes search costs higher has the additional implication that it decreases consumer participation. If participation were not affected by the increase in search costs, only the intensive search margin would play a role and demand would again become more inelastic. However, since consumer participation falls, an increase in search costs changes the composition of demand. In particular, because a larger share of high search cost consumers choose to abandon the market, the average consumer, whom determines firm pricing, changes. If the average consumer becomes more elastic than before, then the equilibrium price falls; otherwise, it increases. We derive a necessary and sufficient condition under which *higher search costs result in lower, constant or higher prices.*

The condition has to do with how the different percentiles of the search cost density are affected by a shock that raises search costs. When an increase in search costs is felt more at the higher percentiles of the search cost distribution than at the lower, then the impact on the extensive search margin has a dominating influence over the effect on the intensive search margin; in such a case, prices decrease as search costs increase. Otherwise prices stay constant or increase.

The insight that higher search frictions can result in lower prices and profits depends neither on our assumption of consumers searching for differentiated products nor on the search protocol. We extend the analysis by considering a non-sequential search version of the model. Under non-sequential search, consumers choose how many products they wish to inspect, including none, in order to maximise the expected gains from search minus the search costs.[1] When the search cost of every consumer is sufficiently low, then they all search the entire market in equilibrium. In this case, the equilibrium price is the same as in Perloff and Salop (1985) and therefore small changes in the search cost distribution have no bearing on the equilibrium price.

As search costs become higher (in a first-order stochastic dominance sense), search intensities

[1] As far as we know, the only study of a non-sequential search model with differentiated products is in Anderson *et al.* (1992); in their model, however, all consumers have the same cost of search and this makes their analysis very different from ours.

become more heterogeneous. Some consumers continue to choose to inspect all the products in the market while other consumers with higher search costs choose to search fewer products. From the point of view of an individual firm, consumers who sample many firms are more elastic than consumers who visit just a few. Optimal pricing trades-off the incentives to extract profits from less elastic consumers and the incentives to compete for the more elastic ones. Eventually as search costs continue to rise, some consumers choose to opt out of the market altogether. Under duopoly we show that a symmetric equilibrium in pure-strategies exists provided that the density of match values is uniform. For this model we derive similar results in regard to the relation between search costs and prices. When all consumers search at least once, higher search costs unambiguously raise prices. However, when the extensive search margin is also affected by an increase in search costs, then prices can increase, decrease or stay constant. Though the conditions we derive in this case have to do with the distribution of search costs, instead of with the density, the economic intuition is exactly the same.

We finally move to examine the case of search in homogeneous product markets. The case of sequential search was studied by Stahl (1996) and, unfortunately, such a model proves non-tractable. We instead proceed by examining a non-sequential consumer search model in the spirit of Burdett and Judd (1983) and allow for search cost heterogeneity. [2] In such a model, the equilibrium is in mixed pricing strategies. Nevertheless, in line with the results for the differentiated product models, we show that the entire equilibrium price distribution can shift to the right, to the left or stay fixed when consumer search costs increase. Interestingly, the necessary and sufficient condition we derive for this to occur is exactly the same as that in the case of the model of non-sequential search for differentiated products. This is somewhat surprising because in the model with homogeneous products the distribution of prices is endogenous while in the model with differentiated products the distribution of match values is exogenous.

An important result of this paper is to derive conditions on search cost heterogeneity under which higher search costs may result in lower prices and lower profits for the firms. This result is relevant for the recent literature on obfuscation, which points out that firms have incentives to obfuscate their products (Ellison and Wolitzky, 2012; Wilson, 2010) by raising the costs consumers have to incur to inspect their offers. Our result tells that under some conditions firms may benefit from doing exactly the opposite, that is, to lower search costs instead.

[2] Though this model has seen various empirical applications (see e.g. Hong and Shum (2006), Moraga-González and Wildenbeest (2008) and Moraga-González *et al.* (2013), as far as we know, the questions of existence of equilibrium and the comparative statics effects of an increase in the number of firms have not been studied.

This is not the only paper showing that higher search costs need not result in higher prices. As mentioned above, Janssen et al. (2005) study the effects of higher search costs in the Stahl's (1989) homogeneous products sequential search setting and show that prices will surely fall provided the search cost is sufficiently high. As shown in this paper, this outcome is the result of a special assumption about consumer heterogeneity. Chen and Zhang (2011), who enrich Stahl's (1989) setting by adding loyal consumers, show that a reduction in the search cost sometimes leads to higher equilibrium prices. In a different framework where search is price-directed, Armstrong and Zhou (2011) show that higher search costs lead to lower prices.

The structure of the remainder of the paper is as follows. In Section 2 we explore the case of sequential search for differentiated products. Section 3 is also devoted to differentiated products but with non-sequential search. In Section 4 we study the case of non-sequential search for homogeneous goods. The paper closes with a Conclusions section. Some of the proofs have been placed in the appendix to ease the reading of the paper.

2 Sequential search for differentiated products

We adopt the framework proposed in the seminal contribution of Wolinsky (1986), where consumers search sequentially for differentiated products and firms compete in prices. Our consumers have, however, heterogeneous search costs.[3]

Consider a market with infinitely many consumers and firms and, without loss of generality, normalise the number of consumers per firm to 1. Firms produce horizontally differentiated products using the same constant returns to scale technology of production; let r be the marginal cost of the firms. Aiming at maximizing their expected profits, firms choose their prices simultaneously. We focus on symmetric Nash equilibrium (SNE); let p^* denote the SNE price.

A consumer m has tastes for a product i described by the following indirect utility function: $u_{im} = \varepsilon_{im} - p_i$, if she buys product i at price p_i; and zero if she does not buy it. The parameter ε_{im} is a match value between consumer m and product i. We assume that the match value ε_{im} is the realization of a random variable distributed on the interval $[0, \overline{\varepsilon}]$ according to a differentiable cumulative distribution function denoted by F. Match values ε_{im} are independently distributed across consumers and products. Moreover, they are private information of consumers so personalized pricing is not possible. Let f be the density function of F. We assume that f is log-concave. For later use,

[3]Later in Sections 3 and 4 we study models where consumers search non-sequentially for differentiated and homogeneous products, respectively. There we show that the main insights from this section do not qualitatively depend on the fact that consumers search sequentially.

we define the monopoly price as $p^m = \arg\max_p (p-r)(1-F(p))$.

Consumers search sequentially in order to maximize expected utility. While searching, they have correct beliefs about the equilibrium price and can recall previously inspected products costlessly. Consumers differ in their costs of search. A buyer's search cost is drawn independently from a differentiable cumulative distribution function G with support $(0, \bar{c})$ and positive density g everywhere.

We now characterise the SNE price. In order to do so, we derive the payoff of a firm, say i, that deviates from the SNE price p^* by charging a price $p_i \neq p^*$. Then, we compute the first order condition (FOC), apply the symmetry condition $p_i = p^*$ and study the existence and uniqueness of SNE.

Consider the (expected) payoff to a firm i that deviates from equilibrium by charging a price p_i. In order to compute firm i's demand, we first need to characterize consumer search behavior. Since consumers do not observe deviations before searching, we can rely on Kohn and Shavell (1974), who study the search problem of a consumer who faces a set of independently and identically distributed options with known distribution. Kohn and Shavell show that the optimal search rule is static in nature and has the stationary reservation utility property. Accordingly, consider a consumer with search cost c and denote the solution to

$$h(x) \equiv \int_x^{\bar{\varepsilon}} (\varepsilon - x) f(\varepsilon) d\varepsilon = c \tag{1}$$

in x by $\hat{x}(c)$. The left-hand-side (LHS) of (1) is the expected benefit in symmetric equilibrium from searching one more time for a consumer whose best option so far is x. Its right-hand-side (RHS) is her cost of search. Hence $\hat{x}(c)$ represents the threshold match value above which a consumer with search cost c will optimally decide not to continue searching for another product. The function h is monotonically decreasing. Moreover, $h(0) = E[\varepsilon]$ and $h(\bar{\varepsilon}) = 0$. It is readily seen that for any $c \in [0, \min\{\bar{c}, E[\varepsilon]\}]$, there exists a unique $\hat{x}(c)$ that solves (1). Differentiating (1) successively, we obtain

$$\hat{x}'(c) = -\frac{1}{1 - F(\hat{x}(c))} < 0.$$
$$\hat{x}''(c) = \frac{f(\hat{x}(c)) \, [\hat{x}'(c)]^2}{1 - F(\hat{x}(c))} > 0$$

which implies that $\hat{x}(c)$ is a decreasing and convex function of c on $[0, \min\{\bar{c}, E[\varepsilon]\}]$, with $\hat{x}(E[\varepsilon]) = 0$ and $\hat{x}(0) = \bar{\varepsilon}$.[4]

[4]Consumers with search cost $c > E[\varepsilon]$, if any, will automatically drop from the market and can therefore be ignored right away. Therefore $\hat{x}(c)$ is well-defined for every consumer that matters for pricing.

In order to compute firm i's demand, consider a consumer with search cost c who shows up at firm i to inspect its product after possibly having inspected other products. Let $\varepsilon_i - p_i$ denote the utility the consumer derives from the product of firm i. Obviously, if alternative i is not the best one so far, the consumer will discard it and search again. Therefore, only when the deal offered by firm i happens to be the best so far will the consumer consider stopping searching and buying it right away. For this decision, the consumer compares the gains from an additional search with the costs of such a search. In this comparison, the consumer holds correct expectations about the equilibrium price so she expects the other firms to charge the equilibrium price p^*. The expected gains from searching one more firm, say j, are equal to $\int_{\varepsilon_i - p_i + p^*}^{\overline{\varepsilon}} [\varepsilon_j - (\varepsilon_i - p_i + p^*)] f(\varepsilon_j) d\varepsilon_j$. Comparing this to (1), it follows that, conditional on having arrived at firm i, the probability that buyer c stops searching at firm i is equal to $\Pr[\varepsilon_i - p_i > \hat{x}(c) - p^*] = 1 - F(\hat{x}(c) + p_i - p^*)$. With the remaining probability, the consumer finds the product of firm i not good enough and continues searching; with infinitely many firms, such a consumer will surely buy at another firm. Because consumer c may visit firm i after having visited no, one, two, three, etc. other firms, the unconditional probability she stops searching and buys at firm i is

$$\frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))} \tag{2}$$

To obtain the payoff of firm i we need to integrate (2) over the consumers who decide to search the market for a satisfactory product; in other words, we need to integrate over those consumers who derive expected positive surplus from participation. To compute the surplus a consumer with search cost c obtains from participation, we note that the consumer will stop and buy after the first search when $\varepsilon > \hat{x}(c)$; otherwise she will drop the first option and continue searching, in which case she will encounter herself exactly in the same situation as before because, conditional on participating, the consumer will continue searching until she finds a match value for which it is worth to stop searching. Recursively, denoting by $CS(c)$ her consumer surplus, we must have:

$$CS(c) = -c + (1 - F(\hat{x}(c))) \frac{\int_{\hat{x}(c)}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon}{1 - F(\hat{x}(c))} + F(\hat{x}(c)) CS(c).$$

Solving for $CS(c)$ we obtain the surplus consumer c derives from participation:

$$CS(c) = \frac{\int_{\hat{x}(c)}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon - c}{1 - F(\hat{x}(c))}.$$

Setting this surplus equal to zero we get that the critical search cost value above which consumers will refrain from participating in the market is given by the solution to

$$\int_{\hat{x}(c)}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon - c = 0.$$

Let $\tilde{c}(p^*)$ denote such a solution. Note that $\tilde{c}(\cdot)$ is the inverse function of $\hat{x}(\cdot)$ on $[0, E[\varepsilon]]$. Therefore,

$$\tilde{c}(p^*) = \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon \tag{3}$$

Depending on how large the range of search costs is, more or less consumers will choose not to participate in the market. Correspondingly we define

$$c_0(p^*) \equiv \min\{\overline{c}, \tilde{c}(p^*)\}.$$

We refer to the effect of the equilibrium price on the decision to search as the *extensive search margin*. If consumers expect a higher equilibrium price, then fewer consumers will search the market for a good deal. The standard assumption in the search cost literature has been that $c_0(p^*) \equiv \overline{c}$, which implies that all consumers search at least once. Because $c_0(p^*)$ might depend on the equilibrium price, this assumption is clearly restrictive: it basically boils down to assuming little consumer heterogeneity, or relatively low search costs for all consumers, which implies that the extensive search margin will be unresponsive to the equilibrium price.[5]

The payoff to the deviant firm i is then:[6]

$$\pi(p_i; p^*) = (p_i - r) \int_0^{c_0(p^*)} \frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c)} g(c) dc. \tag{4}$$

The FOC is given by:

$$\int_0^{c_0(p^*)} \frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c)} g(c) dc - (p_i - r) \int_0^{c_0(p^*)} \frac{f(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))} g(c) dc = 0.$$

Applying symmetry, $p_i = p^*$, we can rewrite the FOC as:

$$p^* = r + \frac{G(c_0(p^*))}{\int_0^{c_0(p^*)} \frac{f(\hat{x}(c))}{1 - F(\hat{x}(c))} g(c) dc}. \tag{5}$$

We now show that a candidate symmetric equilibrium price exists and is unique. First consider the case in which search costs are low, i.e. $\overline{c} < \tilde{c}(p^*)$. Under this parameter constraint, $c_0(p^*) = \overline{c}$

[5] The assumption that all consumers search is akin to a fully-covered market assumption where all consumers are required to buy. While such an assumption is often adopted for convenience, it is known to be restrictive.

[6] In writing this payoff we have assumed that $p_i < p^*$. For $p_i > p^*$ the payoff is slightly different because the expression $\hat{x}(c) + p_i - p^*$ involved in $D(p_i, p^*)$ can exceed $\overline{\varepsilon}$. Denote by $\hat{c}(p_i)$ the solution in c of the equation $\hat{x}(c) + p_i - p^* = \overline{\varepsilon}$, and since $\hat{x}(c)$ is strictly decreasing in c, $\hat{x}(c) + p_i - p^* < \overline{\varepsilon}$ for all $c > \hat{c}(p_i)$. Therefore the payoff would be

$$\pi(p_i; p^*) = (p_i - r) \int_{\hat{c}(p_i)}^{c_0(p^*)} \frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))} g(c) dc.$$

The first order condition at the symmetric equilibrium price is the same though.

and therefore expression (5) gives the candidate equilibrium price explicitly. In this case, obviously, there exists a unique candidate equilibrium price.[7]

When search costs are not restricted to be small then $\overline{c} > \tilde{c}(p^*)$ and correspondingly $c_0(p^*) = \tilde{c}(p^*)$. In this case the equilibrium price is given implicitly by the solution to (5). We now show that (5) has a unique solution also in this case. For this we define the function

$$L(p) \equiv G(\tilde{c}(p)) - (p-r)\int_0^{\tilde{c}(p)} \frac{f(\widehat{x}(c))}{1-F(\widehat{x}(c))} g\left(c\right) dc$$

for $p \in [r, p^m]$, where p^m, recall, denotes the monopoly price. Note that

$$L\left(r\right) = G(\tilde{c}(r)) > 0.$$

Observe also that $L(p^m)$ can be written as

$$L\left(p^m\right) = \int_0^{\tilde{c}(p^m)} \frac{1-F(\hat{x}(c)) - (p^m - r) f(\hat{x}(c))}{1-F(\hat{x}(c))} g\left(c\right) dc. \tag{6}$$

The sign of this expression depends on the sign of the numerator of the fraction in the integrand. We now argue that $L(p^m) < 0$ because $1 - F(\hat{x}(c)) - (p^m - r)f(\hat{x}(c)) \leq 0$ for all $c[0, \tilde{c}(p^m)]$. In fact, note that by logconcavity of f, because $\hat{x}(c)$ decreases in c, it follows that $f(\hat{x}(c))/\left[1 - F(\hat{x}(c))\right]$ decreases in c, which implies that $1 - F(\hat{x}(c)) - (p^m - r)f(\hat{x}(c))$ increases in c. Because $\hat{x}(c(p)) = p$, if we set $c = \tilde{c}(p^m)$ in the expression $1 - F(\hat{x}(c)) - (p^m - r)f(\hat{x}(c))$ we get the monopoly pricing rule $(1 - F(p^m) - (p^m - r)f(p^m) = 0)$. We can now conclude that $L\left(p^m\right) < 0$ because the expression $1 - F(\hat{x}(c)) - (p^m - r)f(\hat{x}(c))$ is increasing in c and takes on value zero when we compute it at the upper bound of the integral.

These two facts together, $L\left(r\right) > 0$ and $L\left(p^m\right) < 0$, imply that a candidate equilibrium price $p^* \in [r, p^m]$ exists. We finally note that

$$\begin{aligned}\frac{dL\left(p\right)}{dp} &= g\left(\tilde{c}\left(p\right)\right)\frac{d\tilde{c}\left(p\right)}{dp} - \left(p-r\right)f\left(p\right)g\left(\tilde{c}\left(p\right)\right)\frac{d\tilde{c}\left(p\right)}{dp} - \int_0^{\tilde{c}(p)} f(\hat{x}(c))g\left(c\right)dc \\ &= \frac{1-F\left(p\right)-\left(p-r\right)f\left(p\right)}{1-F(p)} g\left(\tilde{c}\left(p\right)\right)\frac{d\tilde{c}\left(p\right)}{dp} - \int_0^{\tilde{c}(p)} f(\hat{x}(c))g\left(c\right)dc \end{aligned} \tag{7}$$

is negative for any $p \in [r, p^m]$, which implies that there exists a unique candidate equilibrium price. This follows from the fact that $1 - F\left(p\right) - \left(p-r\right)f\left(p\right) \geq 0$ (because it is the first order derivative of the monopoly payoff $\left(p-r\right)\left(1-F\left(p\right)\right)$, which is log-concave) and $d\tilde{c}\left(p\right)/dp < 0$ (because, from (3), $\tilde{c}$ is decreasing in p). In particular, at the candidate equilibrium price p^* we must have $dL(p^*)/dp < 0$. Figure 1 illustrates these observations.

Our next result provides conditions for existence of a symmetric equilibrium.

[7] To be more precise, the condition we need for this is $\overline{c} < \int_{\hat{x}(\tilde{c}(p^*))}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon$, where $p^* = r + \frac{\int_0^{\overline{c}}[1-F(\widehat{x}(c))]g(c)dc}{\int_0^{\overline{c}} f(\widehat{x}(c))g(c)dc}$.

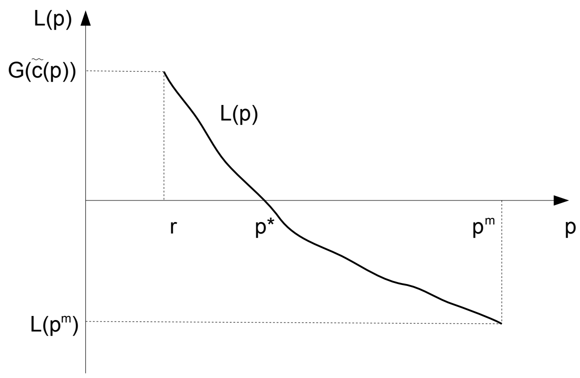


Figure 1: Existence and uniqueness of a candidate equilibrium price

Proposition 1 *In the model of sequential search for differentiated products there may be two types of symmetric equilibria:*

(A) A SNE where all consumers conduct at least a first search, in which case $\bar{c} < \tilde{c}(p^*)$, *where the equilibrium price is given by the expression (5) after setting* $c_0(p^*) = \bar{c}$, *and where* $\hat{x}(c)$ *solves (1).*

(B) A SNE where only a fraction of the consumers search for a satisfactory product, in which case $\bar{c} > \tilde{c}(p^*)$ *where the equilibrium price is given by the expression (5) after setting* $c_0(p^*) = \int_{p^*}^{\bar{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon$. *In this type of equilibrium the fraction of consumers* $G\left(\int_{p^*}^{\bar{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon\right)$ *conducts at least a first search while the rest of the consumers do not search at all.*

When the density of match values f *is increasing and the search cost density* g *is log-concave, then the SNE exists and is unique.*

Proof. It remains to prove that the equilibrium exists when f is increasing and g is log-concave. For this, we prove that the demand function of a firm i in (4), i.e.,

$$D(p_i, p^*) \equiv \int_0^{c_0(p^*)} \frac{1 - F(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c))} g(c) dc \tag{8}$$

is a log-concave function of its own price p_i. Given this, the inverse of the demand function of a firm i is convex and by Proposition 3 in Caplin and Nalebuff (1991), the firm profit function (4) is quasi-concave in own price so that the unique candidate equilibrium price given by (5) is indeed an equilibrium.

We start by showing that the integrand in (8) is log-concave in c and in p_i under the conditions that f is increasing and g is log-concave. For this we first note that the product of log-concave functions is log-concave. The term $(1 - F(\hat{x}(c))^{-1}$ in the integrand of (8) is readily seen to be log-concave in c provided that f is increasing. Under log-concavity of g, the term $(1 - F(\hat{x}(c) + p_i))g(c)$ is log-concave in c and in p_i provided that $1 - F(\hat{x}(c) + p_i - p^*)$ is log-concave in c and in p_i. Let us now show that this is indeed the case. For this we need to prove that the function $m(c, p_i) \equiv \ln[1 - F(\hat{x}(c) + p_i - p^*)]$ is concave in c and in p_i, where the symbol ln denotes the natural logarithm. Taking derivatives we have:

$$\frac{\partial m}{\partial c} = -\frac{f(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c) + p_i - p^*)}\hat{x}'(c)$$
$$\frac{\partial m}{\partial p_i} = \frac{f(\hat{x}(c) + p_i - p^*)}{1 - F(\hat{x}(c) + p_i - p^*)}$$

To construct the Hessian matrix, we now compute the necessary second order derivatives:

$$\begin{aligned}\frac{\partial^2 m}{\partial c^2} &= -\frac{1}{[1 - F(\hat{x}(c) + p_i - p^*)]^2}\Big\{\Big[f'(\hat{x}(c) + p_i - p^*)\left[\hat{x}'(c)\right]^2 + f(\hat{x}(c) + p_i - p^*)\hat{x}''(c)\Big]\left[1 - F(\hat{x}(c) + p_i - p^*)\right] \\ &\quad + \left[f(\hat{x}(c) + p_i - p^*)\hat{x}'(c)\right]^2\Big\} \\ &= -\frac{1}{[1 - F(\hat{x}(c) + p_i - p^*)]^2}\Big\{\left[\hat{x}'(c)\right]^2\Big[f'(\hat{x}(c) + p_i - p^*)\left[1 - F(\hat{x}(c) + p_i - p^*)\right] + \left[f(\hat{x}(c) + p_i - p^*)\right]^2\Big] \\ &\quad + f(\hat{x}(c) + p_i - p^*)\hat{x}''(c)\left[1 - F(\hat{x}(c) + p_i - p^*)\right]\Big\}\end{aligned}$$

The sign of this expression depends on the sign of the part in curly brackets. We note that, because f is log-concave, the first summand of the expression in curly brackets is positive. To determine the sign of the second summand, we need to study the sign of $\hat{x}''(c)$. Differentiating successively (1), we obtain

$$\hat{x}'(c) = -\frac{1}{1 - F(\hat{x}(c))} < 0.$$
$$\hat{x}''(c) = \frac{f(\hat{x}(c))\left[\hat{x}'(c)\right]^2}{1 - F(\hat{x}(c))} > 0$$

Because $\hat{x}''(c) > 0$, we conclude that $\partial^2 m/\partial c^2 < 0$.

We now observe that

$$\frac{\partial^2 m}{\partial p_i^2} = -\frac{1}{[1 - F(\hat{x}(c) + p_i - p^*)]^2}\Big\{f'(\hat{x}(c) + p_i - p^*)\left[1 - F(\hat{x}(c) + p_i - p^*)\right] + \left[f(\hat{x}(c) + p_i - p^*)\right]^2\Big\}$$

which is negative again by the log-concavity of f.

Finally we derive

$$\frac{\partial^2 m}{\partial p_i \partial c} = -\frac{\hat{x}'(c)}{[1 - F(\hat{x}(c) + p_i - p^*)]^2}\Big\{f'(\hat{x}(c) + p_i - p^*)\left[1 - F(\hat{x}(c) + p_i - p^*)\right] + \left[f(\hat{x}(c) + p_i - p^*)\right]^2\Big\}$$

Defining

$$\psi(c,p_i) \equiv \frac{f'(\hat{x}(c)+p_i-p^*)\left[1-F(\hat{x}(c)+p_i-p^*)\right]+\left[f(\hat{x}(c)+p_i-p^*)\right]^2}{\left[1-F(\hat{x}(c)+p_i-p^*)\right]^2}$$

the Hessian matrix is

$$H = \begin{pmatrix} -\left[\hat{x}'(c)\right]^2\psi(c,p_i) - \frac{f(\hat{x}(c)+p_i-p^*)\hat{x}''(c)}{1-F(\hat{x}(c)+p_i-p^*)} & -\hat{x}'(c)\psi(c,p_i) \\ -\hat{x}'(c)\psi(c,p_i) & -\psi(c,p_i) \end{pmatrix}$$

It is straightforward to check that the determinant of H is equal to $\frac{f(\hat{x}(c)+p_i-p^*)\hat{x}''(c)}{1-F(\hat{x}(c)+p_i-p^*)}\psi(c,p_i)$, which is strictly positive. As a result the function $m(c,p_i)$ is strictly concave and by implication the integrand of (8) is log-concave in c and in p_i.

We now invoke Theorem 6 in Prékopa (1973) showing that the integral taken over a convex subset of the real line of a log-concave function is also log-concave, which implies that the demand function (8) is log-concave in p_i. Therefore, an equilibrium exists and is unique. ■

2.1 The effect of higher search costs on the SNE price

We now study the impact of higher search costs on the equilibrium prices of Proposition 1. In order to address this question, we parametrize the search cost distribution G by a positive parameter β and use the notation $G(c;\beta)$. Specifically, we assume that an increase in β implies an increase in search costs in the sense of first-order stochastic dominance (FOSD), i.e. $\partial G(c;\beta)/\partial\beta < 0$ for all c. Let $p^*(\beta)$ be the corresponding SNE price for a given β. We are interested in the behaviour of $p^*(\beta)$ with respect to β.

Consider first the case of Proposition (1)A. When the upper bound of the search cost distribution is sufficiently low, the SNE price is

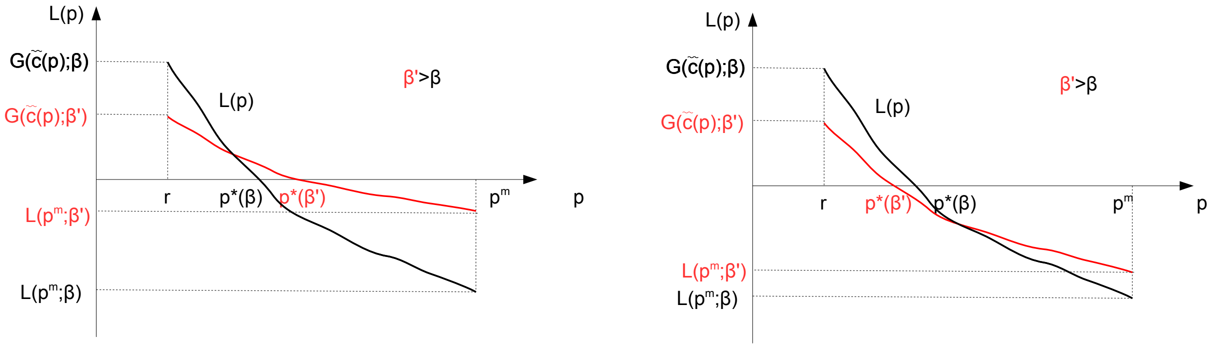
$$p^*(\beta) = r + \frac{1}{\int_0^{\bar{c}(\beta)} \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} g(c,\beta)dc}. \tag{9}$$

(Notice that we allow the upper bound of the search cost distribution to increase in β.) The effect of an increase in search costs on the equilibrium price follows from taking the derivative of (9) with respect to β. Because f is log-concave, the hazard rate $f(\hat{x}(c))/(1-F(\hat{x}(c)))$ increases in $\hat{x}(c)$ and decreases in c. As a result, the expectation in the denominator of (9) falls as β goes up. This implies that the equilibrium price (9) *unambiguously* increases as search costs rise.

We now move to the the case of Proposition (1)B. In this case the equilibrium price $p^*(\beta)$ is given by the unique solution to equation

$$L(p;\beta) \equiv G(\tilde{c}(p);\beta) - (p-r)\int_0^{\tilde{c}(p)} \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} g(c,\beta)dc = 0. \tag{10}$$

Upon observing (10) we see that an increase in search costs affects two terms. The term $G(\tilde{c}(p);\beta)$ goes down because of FOSD, while the integral term goes up by the log-concavity of f. As a result, an increase in β has, potentially, an ambiguous effect on the SNE price. This is illustrated in Figure 2. In the graph of Figure 2(a), we depict a case where higher search costs result in a higher SNE price. The black decreasing function shows the function $L(p;\beta)$. When β increases to β', the function $L(p;\beta)$ rotates from the black to the red curve. In this case, the new equilibrium price increases in search costs. By contrast, in the graph of Figure 2(b) we observe the opposite case where higher β results in a lower SNE price.



(a) Higher $\beta \to$ higher price

(b) Higher $\beta \to$ lower price

Figure 2: The effect of higher search costs on the SNE price ($\bar{c}$ sufficiently large)

The economic intuition for why the effect of higher search costs on the SNE price is ambiguous when search costs are sufficiently high is as follows. An increase in search costs has two effects on demand. On the one hand, search becomes more costly and as a result of that consumer reservation values become lower. This decreases search activity and demand becomes less elastic. By this effect, firms have an incentive to raise their prices. On the other hand, when search costs increase a larger fraction of consumers choose not to search at all. This alters the composition of demand in particular making the average consumer more elastic because the consumers who choose to leave the market are the ones with higher search costs. By this effect, firms have an incentive to lower their prices. Interestingly, as the following example shows, these two effects exactly offset one another when the search cost distribution is uniform.

The uniform-uniform example. *Assume match values are uniformly distributed on* $[0,1]$ *while search costs are uniformly distributed on* $[0,\beta]$*. Then:*

(A) If $\beta \leq \frac{2}{9}(1-r)^2$*, an equilibrium exists and is unique where all consumers search at least one*

time. The equilibrium price and profits are given by

$$p^* = r + \sqrt{\frac{\beta}{2}} \text{ and } \pi^* = \sqrt{\frac{\beta}{2}},$$

while consumer surplus (CS) and social welfare (W) are equal to

$$CS = 1 - r - \frac{5}{3}\sqrt{\frac{\beta}{2}} \text{ and } W = 1 - r - \frac{2}{3}\sqrt{\frac{\beta}{2}}.$$

In this case, price and profits increase as search costs go up, while consumer surplus and social welfare decrease.

(B) If $\beta > \frac{2}{9}(1-r)^2$, then an equilibrium exists and is unique where a fraction of consumers $\beta - \frac{2}{9}(1-r)^2$ do not search at all. The equilibrium price and profits are given by

$$p^* = \frac{1}{3}(1+2r) \text{ and } \pi^* = \frac{2(1-r)^3}{27\beta}$$

while consumer surplus and social welfare are

$$CS = \frac{4(1-r)^3}{81\beta} \text{ and } W = \frac{10(1-r)^3}{81\beta}$$

In this case, the equilibrium price is independent of the level of search costs, while profits, consumer surplus and social welfare also decrease in search costs.

In order to determine conditions under which the demand composition effect dominates the effect on search intensity we invoke the implicit function theorem. The effect of an increase in β on the equilibrium price given by (10) is given by the sign of

$$\frac{dp^*(\beta)}{d\beta} = -\frac{\frac{\partial L(p^*;\beta)}{\partial \beta}}{\frac{\partial L(p^*;\beta)}{\partial p^*}}. \tag{11}$$

We have already argued above that the function $L(p;\beta)$ is monotone decreasing in p so the denominator of (11) is negative. In regard to the numerator of (11), using the notation $g_\beta(c;\beta) = \partial g(c;\beta)/\partial\beta$, we note that

$$\begin{aligned}
\frac{\partial L(\cdot)}{\partial \beta} &= \int_0^{\tilde{c}(p^*)} g_\beta(c;\beta)dc - (p^*-r)\int_0^{\tilde{c}(p^*)} \frac{f(\widehat{x}(c))}{1-F(\widehat{x}(c))} g_\beta(c;\beta)\,dc \\
&= \int_0^{\tilde{c}(p^*)} g_\beta(c;\beta)dc - \frac{\int_0^{\tilde{c}(p^*)} g(c;\beta)dc}{\int_0^{\tilde{c}(p^*)} \frac{f(\widehat{x}(c))}{1-F(\widehat{x}(c))} g(c;\beta)\,dc} \int_0^{\tilde{c}(p^*)} \frac{f(\widehat{x}(c))}{1-F(\widehat{x}(c))} g_\beta(c;\beta)\,dc \\
&= \int_0^{\tilde{c}(p^*)} g(c;\beta)dc \left[\frac{\int_0^{\tilde{c}(p^*)} g_\beta(c;\beta)dc}{\int_0^{\tilde{c}(p^*)} g(c;\beta)dc} - \frac{\int_0^{\tilde{c}(p^*)} \frac{f(\widehat{x}(c))}{1-F(\widehat{x}(c))} g_\beta(c;\beta)\,dc}{\int_0^{\tilde{c}(p^*)} \frac{f(\widehat{x}(c))}{1-F(\widehat{x}(c))} g(c;\beta)\,dc} \right]
\end{aligned} \tag{12}$$

where the second equality follows form using the equilibrium condition (5). Therefore:

Proposition 2 *Let $G(c;\beta)$ be a parametrized search cost cdf with positive density on $[0, \overline{c}(\beta)]$ and with derivative $\partial G(\cdot)/\partial\beta < 0$. Then the comparative statics of the SNE price in Proposition 1 is as follows:*

(A) The equilibrium price given by Proposition 1A unambiguously increases in β.

(B) The equilibrium price given by Proposition 1B increases (decreases) in β if and only if

$$\frac{\int_0^{\tilde{c}(p^*)} g_\beta(c;\beta)dc}{\int_0^{\tilde{c}(p^*)} g(c;\beta)dc} - \frac{\int_0^{\tilde{c}(p^*)} \frac{f(\widehat{x}(c))}{1-F(\widehat{x}(c))} g_\beta\,(c;\beta)\,dc}{\int_0^{\tilde{c}(p^*)} \frac{f(\widehat{x}(c))}{1-F(\widehat{x}(c))} g\,(c;\beta)\,dc} > 0(<0). \tag{13}$$

Moreover, if g is from the family of power distributions on $[0, \beta]$ (which includes the uniform), then the equilibrium price does not depend on β.

Proof. It remains to prove that when g is from the family of power distributions on $[0, \beta]$, then the equilibrium price does not depend on β. Let $g(c) = \alpha c^{\alpha-1}/\beta^\alpha$, on $[0, \beta]$, $\alpha > 0$. In this case, it is straightforward to check that the SNE price is given by the solution to

$$p^* = r + \frac{(\tilde{c}(p^*))^\alpha}{\int_0^{\tilde{c}(p^*)} \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} \alpha c^{\alpha-1} dc}, \tag{14}$$

which is clearly independent of β. ■

Condition (13) is a necessary and sufficient condition under which the SNE price will go down (or up) after search costs increase for all consumers. Intuitively, for prices to increase in search costs, we need that the effect on search intensity is stronger than the effect on consumer participation. This occurs when, in relative terms, the search cost shock is not felt very strongly at the higher percentiles of the search cost density. Our next result provides a sufficient condition under which this is indeed the case.

Proposition 3 *Assume that $g(c;\beta)$ has the monotone increasing likelihood ratio property (MILRP). Then, no matter whether the range of search costs is small or large, the SNE price given by Proposition 1 unambiguously increases in β.*

Proof. It remains to prove that the SNE price given by Proposition 1B will also increase under MILRP. In order to prove this statement, we make use of the following version of Theorem 9 in Menezes and Monteiro (2009).[8]

[8] We are indebted to Paulo Monteiro for alerting us about this theorem and showing us how to use it for this proof. For the proof of the theorem, we refer the reader to the original source Menezes and Monteiro (2009). (We have nevertheless developed a more detailed proof, which is available from us upon request.)

Theorem (Menezes and Monteiro (2009). Let $f_1,\ f_2,\ f_3,\ f_4$ be non-negative functions on $[a,b]$ such that $f_1(x) f_2(y) \leq f_3(x \vee y) f_4(x \wedge y)$ for all $x,\ y \in [a,b]$, where $x \vee y \equiv \max\{x,y\}$, $x \wedge y \equiv \min\{x,y\}$. Then

$$\int_a^b f_1(x)\,dx \int_a^b f_2(x)\,dx \leq \int_a^b f_3(x)\,dx \int_a^b f_4(x)\,dx.$$

Let $\gamma > \beta$. We prove that

$$\frac{\int_0^{c_0(p^*)} g(c;\beta)dc}{\int_0^{c_0(p^*)} \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} g(c;\beta)dc} \leq \frac{\int_0^{c_0(p^*)} g(c;\gamma)dc}{\int_0^{c_0(p^*)} \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} g(c;\gamma)dc}$$

$$\frac{\int_0^{\widetilde{c}(p)} [1-F(\widehat{x}(c))]\, g(c,\beta)\, dc}{\int_0^{\widetilde{c}(p)} f(\widehat{x}(c))\, g(c,\beta)\, dc} \leq \frac{\int_0^{\widetilde{c}(p)} [1-F(\widehat{x}(c))]\, g(c,\gamma)\, dc}{\int_0^{\widetilde{c}(p)} f(\widehat{x}(c))\, g(c,\gamma)\, dc}$$

by using the Theorem. Let

$$f_1(c) = g(c,\beta), \quad f_2(c) = \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} g(c,\gamma),$$
$$f_3(c) = g(c,\gamma), \quad f_4(c) = \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} g(c,\beta).$$

We show that for all $c,\ d \in [0, \widetilde{c}(p)]$

$$f_1(c) f_2(d) \leq f_3(c \vee d) f_4(c \wedge d), \tag{15}$$

i.e.,

$$g(c,\beta) \frac{f(\hat{x}(d))}{1-F(\hat{x}(d))} g(d,\gamma) \leq g(c \vee d,\gamma) \frac{f(\hat{x}(c \wedge d))}{1-F(\hat{x}(c \wedge d))} g(c \wedge d,\beta).$$

Take first $c < d$; we have $c \vee d = d,\ c \wedge d = c$. So we need to prove that

$$g(c,\beta) \frac{f(\hat{x}(d))}{1-F(\hat{x}(d))} g(d,\gamma) \leq g(d,\gamma) \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} g(c,\beta).$$

This is equivalent to

$$g(d,\gamma) g(c,\beta) \left(\frac{f(\hat{x}(d))}{1-F(\hat{x}(d))} - \frac{f(\hat{x}(c))}{1-F(\hat{x}(c))} \right) \leq 0,$$

which is true because f is log-concave ($f(\hat{x}(c))/[1-F(\hat{x}(c))]$ is decreasing in c and $c < d$).

Now take $c \geq d$; we have $c \vee d = c,\ c \wedge d = d$. So we need to prove that

$$g(c,\beta) \frac{f(\hat{x}(d))}{1-F(\hat{x}(d))} g(d,\gamma) \leq g(c,\gamma) \frac{f(\hat{x}(d))}{1-F(\hat{x}(d))} g(d,\beta). \tag{16}$$

If $g(c,\beta) = 0$ then the inequality holds clearly for any d. If $g(d,\beta) = 0$ then $d = 0$. Then the fact $g(d,\beta) = 0$ means that the derivative of $G(c,\beta)$ is 0 at 0. Since from FOSD (implied by MLRP) $G(c,\gamma) \leq G(c,\beta)$, the derivative of $G(c,\gamma)$ must also be 0 at 0. Therefore, $g(d,\gamma) = 0$, so the inequality holds.

Next, by dividing (16) with $g(c,\beta)$ and $g(d,\beta)$, this will be equivalent to

$$\frac{f(\hat{x}(d))}{1-F(\hat{x}(c))}\left(\frac{g(d,\gamma)}{g(d,\beta)}-\frac{g(c,\gamma)}{g(c,\beta)}\right)\leq 0.$$

This holds for all $c\geq d$ if and only if $g\left(c,\beta\right)$ has the MILRP. ■

We have shown in Proposition 2 that for the case when the density of search costs is from the family of power densities the SNE price given in Proposition 1B is constant in parameter β. Note that the family of power densities has *constant* likelihood ratio with respect to β. Moreover, Proposition 3 has demonstrated that when the search cost density has the MILRP, then higher search costs result in higher prices. These observations lead us to conjecture that when the density has the monotone *decreasing* likelihood ratio property, the SNE price falls in search costs. Though a general proof of this result has so far escaped us, we now provide two different examples, one analytical and one numerical where this is indeed the case.

The family of MDLR search cost densities $g=(1+t)\left[(c/\beta)^{t}+1\right]/(\beta(2+t))$, $0\leq t\leq 1$**.**

Consider the case in which match values are distributed uniformly on $[0,1]$ and search costs are distributed on $[0,\beta]$ according to the following distribution function:

$$G(c;\beta)=\frac{c}{\beta(2+t)}\left[1+t+\left(\frac{c}{\beta}\right)^{t}\right],\text{ with }0\leq t\leq 1,$$

Notice that an increase in β shifts the search cost distribution to the right, so higher β implies a FOSD shift of the search cost distribution. The corresponding density is:

$$g(c;\beta)=\frac{1+t}{\beta(2+t)}\left[1+\left(\frac{c}{\beta}\right)^{t}\right].$$

We now make two observations about this density function. First, the family of densities g is log-concave, which together with the uniform density for the match values ensures that a SNE exists and is unique (Proposition 1). To see this, we note that

$$\frac{\partial^{2}\ln[g(c;\beta)]}{\partial c^{2}}=-\frac{t\left[1-t+\left(\frac{c}{\beta}\right)^{t}\right]\left(\frac{c}{\beta}\right)^{t}}{c^{2}\left[1+\left(\frac{c}{\beta}\right)^{t}\right]^{2}}<0.$$

In addition, we notice that g has the MDLRP. To see this, we note that

$$\frac{g_{\beta}'}{g}=\frac{-\frac{1+t+(1+t)^{2}\left(\frac{c}{\beta}\right)^{t}}{\beta^{2}(2+t)}}{\frac{1+t}{\beta(2+t)}\left[1+\left(\frac{c}{\beta}\right)^{t}\right]}=-\frac{1+(1+t)\left(\frac{c}{\beta}\right)^{t}}{\beta\left[1+\left(\frac{c}{\beta}\right)^{t}\right]}$$

Taking the derivative with respect to c gives

$$\frac{\partial(g'_\beta/g)}{\partial c} = -\frac{t^2\left(\frac{c}{\beta}\right)^{t-1}}{\beta^2\left[1+\left(\frac{c}{\beta}\right)^t\right]^2} < 0.$$

We now compute the equilibrium prices by following Proposition 1). We have that:

(A) When $\beta \leq \frac{8(1-r)^2(1+t)^4}{(6t^2+13t+6)^2}$ the equilibrium price is

$$p^*(\beta) = r + \frac{\sqrt{\beta}(2+t)(1+2t)}{2\sqrt{2}(1+t)^2},$$

which clearly increases in β.

(B) When $\beta > \frac{8(1-r)^2(1+t)^4}{(6t^2+13t+6)^2}$, the equilibrium price is given by the solution to (10). Using the formulas above for the distribution and the density this equation is:

$$L(p;\beta) \equiv \frac{(1-p)^2}{2\beta(2+t)}\left[1+t+\left(\frac{(1-p)^2}{2\beta}\right)^t\right] - (p-r)\int_0^{(1-p)^2/2}\frac{1+t}{\sqrt{2c}\beta(2+t)}\left[1+\left(\frac{c}{\beta}\right)^t\right]dc = 0$$

Integrating and simplifying we obtain that the price is given by the solution to

$$\widetilde{L}(p;\beta) \equiv (1-p)\left[1+t+\left(\frac{(1-p)^2}{2\beta}\right)^t\right] - 2(t+1)(p-r)\left[1+\frac{(1-p)^{2t}}{2^t\beta^t(1+2t)}\right] = 0$$

From the analysis above, we know that $\widetilde{L}(p;\beta)=0$ has a unique solution; let $p(\beta)$ be such a solution. Applying the implicit function theorem we have that

$$\frac{dp}{d\beta} = -\frac{\partial\widetilde{L}/\partial\beta}{\partial\widetilde{L}/\partial p} = -\frac{-\frac{(1-p)^3}{4\beta^2} + (p-r)\frac{(1-p)^2}{3\beta^2}}{-1-\frac{3(1-p)^2}{4\beta} - \frac{(1-p)^2+6\beta}{3\beta} + (p-r)\frac{2(1-p)}{3\beta}}$$

where

$$\frac{\partial\widetilde{L}}{\partial\beta} = \frac{2^{1-t}t(1+t)(p-r)(1-p)^{2t}}{\beta^{t+1}(1+2t)} - \frac{t(1-p)^{2t+1}}{2^t\beta^{t+1}} \tag{17}$$

$$\frac{\partial\widetilde{L}}{\partial p} = \frac{2^{2-t}t(t+1)(p-r)(1-p)^{2t-1}}{\beta^t(1+2t)} - 2(t+1)\left(\frac{(1-p)^{2t}}{2^t\beta^t(1+2t)}+1\right) - \frac{2^{1-t}t(1-p)^{2t}}{\beta^t} - \left(\frac{(1-p)^2}{2\beta}\right)^t - t - 1 \tag{18}$$

From the equilibrium condition equation $\widetilde{L}(p;\beta)=0$ we obtain that

$$p-r = \frac{(1-p)\left(1+t+\left(\frac{(1-p)^2}{2\beta}\right)^t\right)}{2(t+1)\left(1+\frac{(1-p)^{2t}}{2^t\beta^t(1+2t)}\right)}.$$

Substituting $p - r$ in (17) and (18) by this expression and simplifying we obtain

$$\frac{\partial \widetilde{L}}{\partial \beta} = -\frac{t^2(1-p)^{2t+1}}{\beta\left((1-p)^{2t} + 2^t\beta^t(1+2t)\right)} < 0$$

$$\frac{\partial \widetilde{L}}{\partial p} = -\frac{2t^2(1-p)^{2t}}{(1-p)^{2t} + 2^t\beta^t(1+2t)} - \frac{(3+4t)(1-p)^{2t}}{2^t\beta^t(1+2t)} - 3(t+1) < 0.$$

From this, we conclude that $dp/d\beta < 0$. As a result, the equilibrium price unambiguously decreases in search costs when β is sufficiently large. ■

The Kumaraswamy's (1980) density

Definition: *The Kumaraswamy distribution has cdf G and pdf g given by*

$$G(c) = 1 - \left[1 - \left(\frac{c}{\beta}\right)^a\right]^b, \; c \in [0, \beta], \; a, b > 0$$

$$g(c) = \frac{ab}{\beta}\left(\frac{c}{\beta}\right)^{a-1}\left[1 - \left(\frac{c}{\beta}\right)^a\right]^{b-1}. \tag{19}$$

The Kumaraswamy's (1980) distribution is often used in lieu of the beta-distribution (see e.g. Ding and Wolfstetter (2011)). This distribution turns out to be quite useful in our setting because it has increasing (for $b > 1$), decreasing (for $0 < b < 1$) and constant (for $b = 1$) likelihood ratio with respect to the shifter parameter β.[9] Note that parameter β multiplies the search cost c and scales the support of the distribution. An increase in β therefore shifts the search cost distribution rightward, which signifies that search costs are higher for all consumers.

Table 1 reports some of the numerical results we obtain using the Kumaraswamy distribution. While computing the equilibrium we set $r = 0$ and $a = 1$. The nature of the results does not depend on these parameter choices. When $b = 1.5$ the density function satisfies the MILRP; in this case, as shown in Proposition 2, the equilibrium price increases in search costs. We have also computed profits and consumer surplus, which go down in search costs. We also report consumer surplus conditional on searching, which also decreases in this case. When $b = 1$ the price is constant and so is consumer surplus conditional on searching; however, profits, consumer surplus and welfare fall in search costs. Finally, when $b = 0.5$ the density function satisfies the MDLRP; in this case, price decreases in search costs and consumer surplus conditional on searching therefore goes up; profits, consumer surplus and welfare go down anyway.

[9] A proof can be obtained form the authors upon request. Observe that the uniform density case obtains by setting $a = b = 1$ in the Kumaraswamy distribution above.

	$b = 1.5$			$b = 1.00$			$b = 0.5$		
	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$
p^*	0.3287	0.3311	0.3319	0.3333	0.3333	0.3333	0.3378	0.3354	0.3347
π	0.1045	0.0539	0.0363	0.0740	0.0370	0.0246	0.0393	0.0190	0.0125
CS	0.0729	0.0367	0.0245	0.0493	0.0246	0.0164	0.0250	0.0124	0.0082
$CS/\int_0^{\frac{(1-p)^2}{2}} gdc$	0.2294	0.2255	0.2244	0.2222	0.2222	0.2222	0.2153	0.2189	0.2200
$Welfare$	0.1775	0.0907	0.0609	0.1234	0.0617	0.0411	0.0643	0.0314	0.0208

Table 1: Sequential search for differentiated products (Uniform-Kumaraswamy with $a = 1$)

The results reported in the Table do not change when we modify the parameters r, a and b. On the basis of these numerical results, we state the following:

Result: *Assume that match values are uniformly distributed on [0,1] and that search costs are distributed on the interval* $[0, \beta]$ *according to the Kumaraswamy distribution. Then:*

(A) The equilibrium price in Proposition 1A increases in β; *as a result, higher search costs unambiguously result in higher prices.*

(B) The equilibrium price in Proposition 1B decreases in β *if* $0 < b < 1$, *is constant in* β *if* $b = 1$, *and increases in* β *if* $b > 1$; *as a result, higher search costs result in lower (higher) prices if* $0 < b < 1$ $(b > 1)$.

The results obtained in this section are rather intuitive and we expect them to be robust across model specifications. In the next sections we study models of non-sequential search, both for differentiated and homogeneous products. As far as we know, the model of non-sequential search for differentiated products with heterogeneous search costs has not been studied before and the one with homogeneous products has only been used in empirical work. In this sense, the theoretical analysis we perform has separate interest for its own sake.

3 Non-sequential consumer search for differentiated products

We next present a *duopoly* model of firms selling horizontally differentiated products to consumers who search the market for satisfactory goods non-sequentially.[10] Otherwise, the model is identical to the one in the previous section.[11] The critical distinction between sequential and non-sequential

[10]The N-firm model is examined in section 3.2.

[11]A classic paper on non-sequential search is Burdett and Judd (1983), where firms sell homogeneous products. Later in Section 4 we introduce search cost heterogeneity into a setting similar to Burdett and Judd (1983). To the best of our knowledge, only Anderson *et al.* (1992, ch. 7) has considered non-sequential search with differentiated products. In their model, search costs are assumed to be small and all consumers search the same number of firms in

search is that with non-sequential search consumers commit ex-ante to a number of searches. In terms of price formation, since an individual firm can influence the intensity of search in the sequential search model and cannot do it in the non-sequential search model, prices are typically lower in the first type of model.

There are two firms producing horizontally differentiated products with marginal cost equal to r. The two firms choose their prices simultaneously to maximize profits. There is a unit mass of consumers with utility $u_{im} = \varepsilon_{im} - p_i$ from buying product i at price p_i. The match value ε_{im} has again distribution F and density f, defined over the support $[0, \overline{\varepsilon}]$. As before, f is assumed to be log-concave. The monopoly price is denoted p^m.

Consumers have heterogeneous search costs. The distribution of search cost is G and the density g, with support $(0, \overline{c})$. A consumer with search cost c sampling k firms incurs a total search cost kc, $k = 0, 1, 2$. As mentioned above, we assume that consumers search non-sequentially; this means that they choose the number of firms to visit, including none, in order to maximize expected utility. Once they have visited the desired number of firms, they buy from the store offering them the best deal. While making such a decision, they have correct beliefs about the equilibrium price.[12]

In what follows we characterize a symmetric pure-strategy Nash equilibrium. Let us start examining the problem of the consumers. Assume both firms charge a price $p^* \in [r, p^m]$. Because consumers have correct expectations about the equilibrium price, a consumer with search cost c that chooses to sample one firm only expects to obtain a utility equal to

$$E[\varepsilon - p^* | \varepsilon \geq p^*] = \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon - c. \tag{20}$$

For a consumer to conduct at least one search, such an expected utility has to be positive. If an individual with search cost $c \in [0, \overline{c}]$ exists such that (20) is equal to zero, then this means that for some consumers it is not worthwhile to conduct a first search. Correspondingly, we define the critical search cost value:

$$c_0(p^*) \equiv \min \left\{ \overline{c}, \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon \right\}. \tag{21}$$

Like in the previous model, if $c_0(p^*)$ is strictly lower than the upper bound of the search cost distribution $\overline{c}$, a fraction of the consumer population will abstain from searching.

equilibrium. The equilibrium price (weakly) increases in the search cost. For an empirical application of our model, see Moraga-González *et al.* (2012).

[12] The choice of search protocol is based on theoretical tractability. Later in Section 2 we examine a model where consumers search sequentially, as in Wolinsky (1986), and show that the main result of this paper does not qualitatively depend on whether consumers search non-sequentially or sequentially.

Consider now a consumer with search cost c for whom it is worth to conduct at least one search. This consumer has to choose between searching one firm only or searching the two firms. Let $z_2 \equiv \max\{\varepsilon_1, \varepsilon_2\}$ and note that the distribution of z_2 is $F(\varepsilon)^2$. Then, the utility a consumer expects to get when sampling the two firms is equal to

$$E[z_2 - p^*|z_2 \geq p^*] = \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) 2F(\varepsilon) f(\varepsilon) d\varepsilon - 2c.$$

Comparing this utility with that derived from searching only one firm, she will prefer to visit the two firms provided that

$$\int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) 2F(\varepsilon) f(\varepsilon) d\varepsilon - 2c \geq \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon - c.$$

Correspondingly, we define the critical search cost value $c_1(p^*)$ above which and below $c_0(p^*)$ consumers prefer to search one time only:

$$c_1(p^*) \equiv \min\left\{\overline{c}, \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*)[2F(\varepsilon) - 1] f(\varepsilon) d\varepsilon\right\}. \tag{22}$$

It is straightforward to check that $c_1(p^*) \leq c_0(p^*)$. Individuals with search cost below $c_1(p^*)$ prefer to search twice. Hence, the population of consumers can be split into three groups of consumers. These three groups comprise consumers not searching at all, searching one time and searching two times. Denoting the group of consumers searching k times by $\mu_k(p^*)$, we have:

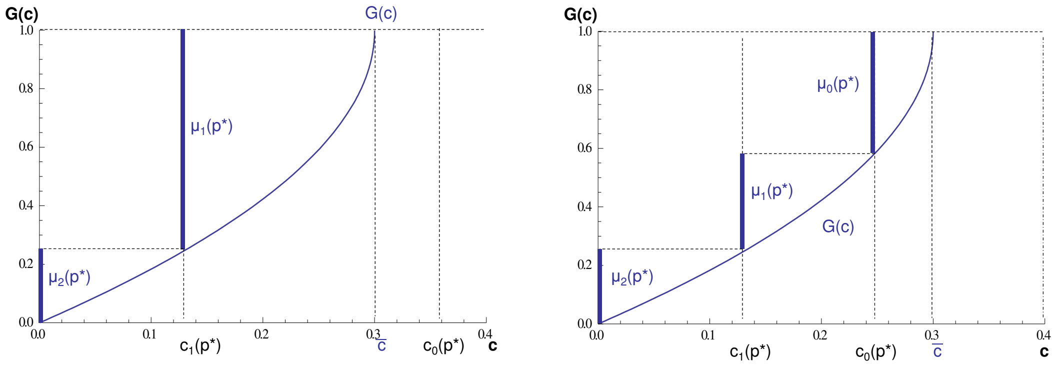
$$\mu_0(p^*) = 1 - G(c_0(p^*));\ \mu_1(p^*) = G(c_0(p^*)) - G(c_1(p^*)),\ \text{and}\ \mu_2(p^*) = G(c_1(p^*)) \tag{23}$$

Figure 3 illustrates. In the graph of Figure 3(a) we represent a case where all consumers search; in particular the vertical (blue) line denoted $\mu_1(p^*)$ depicts the share of consumers who search once, while the vertical (blue) line denoted $\mu_2(p^*)$ shows the fraction of consumers who search twice. Note that when $\overline{c}$ is very low, all consumers will search twice. By contrast, in the graph of Figure 3(b) the fraction of consumers $\mu_0(p^*)$ chooses not to search at all.

We now move to the problem of the firms. To characterize the symmetric pure-strategy equilibrium we start by deriving the payoff of a firm i that deviates from equilibrium pricing by charging a price $p_i \neq p^*$, given that the rival firm charges p^* and given consumer search behaviour. The expected payoff to the deviant firm i is:

$$\pi_i(p_i; p^*) = (p_i - r)\left(\frac{\mu_1(p^*)}{2}\Pr[\varepsilon_i \geq p_i] + \mu_2(p^*)\Pr\left[\varepsilon_i - p_i \geq \max\{\varepsilon_j - p^*, 0\}\right]\right), \tag{24}$$

where the symbol Pr stands for probability. This payoff formula is easily understood. The per-consumer profit is $p_i - r$. Consumers who search only once happen to visit firm i with probability



(a) c_0 sufficiently large

(b) c_0 sufficiently small

Figure 3: Equilibrium search intensities and search costs

1/2; these consumers buy firm i's product when the match values they obtain there are higher than the price p_i. Consumers who search twice only buy from firm i when firm i's deal is better than the rival's and the outside option of 0.

When firm i deviates by charging a higher price than the rival, $p_i > p^*$, the payoff in (24) can be written as follows:[13]

$$\pi_i(p_i > p^*; p^*) = (p_i - r)\left(\frac{\mu_1(p^*)}{2}(1 - F(p_i)) + \mu_2(p^*)\int_{p_i}^{\overline{\varepsilon}} F(\varepsilon - (p_i - p^*))f(\varepsilon)d\varepsilon\right). \quad (25)$$

The first order condition (FOC) in this case is:

$$\begin{aligned}\frac{d\pi_i\left(p_i\right)}{dp_i} &= \frac{\mu_1(p^*)}{2}(1 - F(p_i)) + \mu_2(p^*)\int_{p_i}^{\overline{\varepsilon}} F\left(\varepsilon - p_i + p^*\right) f\left(\varepsilon\right) d\varepsilon \\ &\quad - (p_i - r)\left\{\frac{\mu_1(p^*)}{2} f(p_i) + \mu_2(p^*)\left[\int_{p_i}^{\overline{\varepsilon}} f\left(\varepsilon - p_i + p^*\right) f\left(\varepsilon\right) d\varepsilon + F(p^*)f(p_i)\right]\right\} = 0 \quad (26)\end{aligned}$$

Setting $p_i = p^*$ in (26), replacing $\mu_1(p^*)$ and $\mu_2(p^*)$ by their corresponding values in terms of the search cost distribution and rearranging, we obtain the necessary condition for a symmetric equilibrium price p^*. Let us define the function

$$H\left(p\right) \equiv N(p)G\left(c_1(p)\right) - D(p)G\left(c_0(p)\right). \quad (27)$$

[13]When firm i deviates by charging a lower price, the payoff formula is different:

$$\pi_i(p_i < p^*; p^*) = (p_i - r)\left(\frac{\mu_1(p^*)}{2}(1 - F(p_i)) + \mu_2(p^*)\left[1 - F(\overline{\varepsilon} + p_i - p^*) + \int_{p_i}^{\overline{\varepsilon}+p_i-p^*} F(\varepsilon - (p_i - p^*))f(\varepsilon)d\varepsilon\right]\right).$$

However, the condition that a symmetric price equilibrium must satisfy is the same as the one we derive below in equation (28).

where the functions $D(p)$ and $N(p)$ are given by

$$D(p) \equiv -\left[1 - F(p) - (p-r)\, f(p)\right]$$
$$N(p) \equiv F(p)(1-F(p)) - 2\,(p-r)\left(\int_p^{\overline{\varepsilon}} f(\varepsilon)^2\, d\varepsilon + F(p)f(p) - \frac{1}{2}f(p)\right).$$

The necessary condition for a symmetric equilibrium price p^* is

$$H(p^*) = 0 \tag{28}$$

Equation (28) cannot be solved for an explicit solution in p^*. However, we now note that a candidate equilibrium price $p^* \in [r, p^m]$ exists. We observe first that when we set $p = r$ we obtain

$$H(r) = (1 - F(r))\left[F(r)G(c_1(r)) + G(c_0(r))\right] > 0.$$

Second, if we set $p = p^m$ then we get that

$$H(p^m) = N(p^m)G(c_1(p^m)) \tag{29}$$

just because the price p^m satisfies the first order condition for the monopoly problem: $1 - F(p^m) - (p^m - r)\, f(p^m) = 0$. The sign of $H(p^m)$ depends on the sign of $N(p^m)$, for which we can write:

$$\begin{aligned} N(p^m) &= F(p^m)(1-F(p^m)) - 2\,(p^m - r)\left(\int_{p^m}^{\overline{\varepsilon}} f(\varepsilon)^2\, d\varepsilon + F(p^m)f(p^m) - \frac{1}{2}f(p^m)\right) \\ &= \left[1 + F(p^m)\right]\left[1 - F(p^m)\right] - 2\,(p^m - r)\left(\int_{p^m}^{\overline{\varepsilon}} f(\varepsilon)^2\, d\varepsilon + F(p^m)f(p^m)\right) \\ &= (p^m - r)\left[f(p^m)\left[1 - F(p^m)\right] - 2\int_{p^m}^{\overline{\varepsilon}} f(\varepsilon)^2\, d\varepsilon\right], \end{aligned} \tag{30}$$

where we have used again the relation $1 - F(p^m) - (p^m - r)\, f(p^m) = 0$. Upon observing (30) it follows that the sign of $H(p^m)$ depends on the sign of the expression inside the squared brackets. Let us define

$$M(p) \equiv f(p)\left[1 - F(p)\right] - 2\int_p^{\overline{\varepsilon}} f(\varepsilon)^2\, d\varepsilon.$$

Taking the derivative of M with respect to p gives $f'(p)(1 - F(p)) + f(p)^2$, which is greater than zero by logconcavity of f (see Corollary 2 in Bagnoli and Bergstrom (2005)). Since M is increasing in p and is equal to zero when we set $p = \overline{\varepsilon}$, we conclude that $M(p^m) < 0$. Hence $H(p^m) < 0$.

Since H is a continuous function with $H(r) > 0$ and $H(p^m) < 0$, we conclude that for any log-concave density f, there exists a candidate price equilibrium $p^* \in [r, p^m]$. Note also that at the candidate equilibrium price p^* we must have $dH(p^*)/dp < 0$.[14] Further, we can prove that:

[14] In case there are multiple equilibria, because the number of equilibria will generically be odd, this will also be true for the highest price equilibrium.

Proposition 4 *Depending on the magnitude of the upper bound of the search cost distribution $\overline{c}$, in the non-sequential search duopoly model there may exist three types of SNE.*

(A) A SNE where all consumers search twice and firms charge a price given by the solution to

$$\frac{1}{2}(1-F^2(p^*))-(p^*-r)\left[\int_{p^*}^{\overline{\varepsilon}} f(\varepsilon)^2\,d\varepsilon+F(p^*)f(p^*)\right]=0. \tag{31}$$

This equilibrium is unique and exists provided that

$$\overline{c}\leq\int_{p^*}^{\overline{\varepsilon}}(\varepsilon-p^*)[2F(\varepsilon)-1]f(\varepsilon)d\varepsilon. \tag{32}$$

(B) A SNE where a fraction $G\left(\int_{p^*}^{\overline{\varepsilon}}(\varepsilon-p^*)[2F(\varepsilon)-1]f(\varepsilon)d\varepsilon\right)$ of consumers searches the two firms and the rest just one, in which case the equilibrium price p^* is given by the solution to (28). For this equilibrium to exist $\overline{c}$ must satisfy the inequality

$$\int_{p^*}^{\overline{\varepsilon}}(\varepsilon-p^*)f(\varepsilon)d\varepsilon\geq\overline{c}>\int_{p^*}^{\overline{\varepsilon}}(\varepsilon-p^*)[2F(\varepsilon)-1]f(\varepsilon)d\varepsilon, \tag{33}$$

and when F is the uniform distribution, an equilibrium exists.

(C) Finally, a SNE where a fraction $G\left(\int_{p^*}^{\overline{\varepsilon}}(\varepsilon-p^*)[2F(\varepsilon)-1]f(\varepsilon)d\varepsilon\right)$ of consumers searches the two firms, a fraction $G\left(\int_{p^*}^{\overline{\varepsilon}}(\varepsilon-p^*)f(\varepsilon)d\varepsilon\right)-G\left(\int_{p^*}^{\overline{\varepsilon}}(\varepsilon-p^*)[2F(\varepsilon)-1]f(\varepsilon)d\varepsilon\right)$ of consumers searches one firm only and the rest do not search at all, in which case the equilibrium price p^* is given by the solution to (28). For this equilibrium to exist $\overline{c}$ must satisfy the inequality

$$\overline{c}>\int_{p^*}^{\overline{\varepsilon}}(\varepsilon-p^*)f(\varepsilon)d\varepsilon, \tag{34}$$

and when F is the uniform distribution, an equilibrium exists.

Proof. (A) If all consumers search both firms, the payoff function in (25) coincides with that in Perloff and Salop (1985):

$$\pi_i(p_i>p^*,p^*)=(p_i-r)\int_{p_i}^{\overline{\varepsilon}}F(\varepsilon-p_i+p^*)f(\varepsilon)d\varepsilon$$

From Caplin and Nalebuff (1991) we know that under log-concavity of f, this payoff function is quasi-concave in p_i and therefore p^* is the unique symmetric equilibrium price. In order for all consumers to search twice, we need that $c_1(p^*)=\overline{c}$, which is guaranteed under condition (32).

(B and C) When $\overline{c}$ is relatively large some consumers search once and some search twice. In such a case, the candidate equilibrium price is given by the solution to equation (28). We now note that the payoff (25) involves the sum of two log-concave functions. Unfortunately, such a sum need not be quasi-concave, which implies that we need to impose additional restrictions on the primitives of the model in order to guarantee the existence of a pure-strategy equilibrium.[15] We now show that when F is the uniform distribution, the payoff function in (25) is strictly concave. The second order derivative of (25) is:

$$\begin{aligned}\frac{d^2\pi_i\left(p_i>p^*\right)}{dp_i^2} &= -2\frac{\mu_1(p^*)}{2}f(p_i)-2\mu_2(p^*)\left(\int_{p_i}^{\overline{\varepsilon}} f\left(\varepsilon-p_i+p^*\right)f\left(\varepsilon\right)d\varepsilon+F(p^*)f(p_i)\right)\\ &\quad -\left(p_i-r\right)\left[\frac{\mu_1(p^*)}{2}f'(p_i)-\mu_2(p^*)\left(\int_{p_i}^{\overline{\varepsilon}} f'\left(\varepsilon-p_i+p^*\right)f\left(\varepsilon\right)d\varepsilon+f(p^*)f(p_i)-F(p^*)f'(p_i)\right)\right]\end{aligned} \tag{35}$$

where f' denotes the derivative of f.

For the uniform distribution, we have $F(\varepsilon)=\varepsilon/\overline{\varepsilon}$, $f(\varepsilon)=1/\overline{\varepsilon}$ and $f'(\varepsilon)=0$. Plugging these values in (35) and simplifying gives:

$$\begin{aligned}\frac{d^2\pi_i\left(p_i>p^*\right)}{dp_i^2} &= -\frac{\mu_1(p^*)}{\overline{\varepsilon}}-2\mu_2(p^*)\left(\frac{\overline{\varepsilon}-p_i}{\overline{\varepsilon}^2}+\frac{p^*}{\overline{\varepsilon}^2}\right)+\left(p_i-r\right)\mu_2(p^*)\frac{1}{\overline{\varepsilon}^2}\\ &= -\frac{\overline{\varepsilon}\mu_1(p^*)+\mu_2(p^*)(2\overline{\varepsilon}-3p_i+2p^*+r)}{\overline{\varepsilon}^2}\end{aligned}$$

which is clearly negative because p_i cannot be greater than the monopoly price, which in this case of the uniform distribution is given by $p^m=(\overline{\varepsilon}+r)/2$. In a similar way, we can compute the second order condition for prices $p_i<p^*$, which gives

$$\frac{d^2\pi_i\left(p_i<p^*\right)}{dp_i^2}=-\frac{1}{\overline{\varepsilon}}\left(\mu_1(p^*)+2\mu_2(p^*)\right)<0.$$

Because of strict concavity of the payoff, the equilibrium exists.

In order for consumers to search as prescribed in part B, we need that $c_1(p^*)<\overline{c}<c_0(p^*)$, which is guaranteed under condition (33). Finally, for consumers to search as prescribed in part C, we need that $c_0(p^*)<\overline{c}$, which gives condition (34). ■

3.1 The effect of higher search costs on the equilibrium price

We now study how the equilibrium price derived in Proposition 4 depends on the magnitude of search costs. We proceed as before: we parametrize the search cost density by a scalar β and study how the equilibrium price $p^*(\beta)$ responds to a change in β.

[15] This problem is quite common in search models where demand stems from various consumer types. For example, in the sequential search model of Anderson and Renault (1999) demand stems from consumers who happen to visit a firm for the first time, and from consumers who happen to walk away from a firm and later return to it to conduct a purchase.

The first observation we make is that the price in part (A) of Proposition 4, given by the solution to the FOC (31), is completely independent of a small change in the search cost distribution. As mentioned above, this is because search costs are so low in this case that they do not restrict consumers' search behaviour at all and, as a result, all consumers search the two firms in equilibrium.

The other cases, namely (B) and (C), are the most interesting ones. In the cases of (B) and (C), if an equilibrium exists, the price is given by the solution to the FOC (28). Because we have parametrized G by β, let us denote by $H(p^*;\beta)$ the corresponding parametrized function defined by the FOC (28). By the implicit function theorem, the comparative statics effect of an increase in search costs is then given by

$$\frac{dp^*(\beta)}{d\beta} = -\frac{\frac{\partial H}{\partial \beta}}{\frac{\partial H}{\partial p^*}}. \tag{36}$$

We have already noted above that the denominator of (36), $\partial H/\partial p^*$, is negative. We now study the sign of the numerator of (36). For this we now distinguish between cases (B) and (C) in Proposition 4. Consider first the situation in (B). In this case, the upper bound of the search cost distribution is neither too high nor too low, which implies that all consumers search at least once, i.e., $G\left(c_0(p^*),\beta\right) = 1$, and some consumers do search twice, i.e. $\mu_2(p^*) = 1 - \mu_1(p^*) = G(c_1(p^*),\beta)$. In such a case, the numerator of (36) is

$$\frac{\partial H}{\partial \beta} = N(p^*)\frac{\partial G\left(c_1(p^*),\beta\right)}{\partial \beta} > 0,$$

because $D(p^*) < 0$ and existence of a candidate equilibrium implies that $N(p^*) < 0$. As a result, since $\partial H/\partial p^* < 0$ and $\partial H/\partial \beta > 0$, we have demonstrated that $dp^*(\beta)/d\beta > 0$. That is, an increase in search costs results in higher prices, which is the standard result in the search cost literature. In the present case where all consumers search, an increase of search costs has only a bearing on consumers' search intensity, the intensive search margin, and not at all on consumers' participation, the extensive search margin. When search costs increase, consumers search less and prices go up. That consumers search less is reflected here in $G(c_1(p^*),\beta)$ falling in β, which, by definition, means that the fraction of consumers searching twice decreases and, by implication, the fraction of consumers searching once increases. Facing fewer consumers who compare the products of the two firms after search costs increase, the producers safely increase their prices.

Consider now the situation in case (C). In this situation $G\left(c_0(p^*),\beta\right) < 1$ and therefore for the numerator of (36) we have

$$\frac{\partial H}{\partial \beta} = N(p^*)\frac{\partial G\left(c_1(p^*),\beta\right)}{\partial \beta} - D(p^*)\frac{\partial G\left(c_0(p^*),\beta\right)}{\partial \beta}$$

Using the equilibrium condition (28), we can rewrite this as follows

$$
\begin{aligned}
\frac{\partial H}{\partial \beta} &= D(p^*) \frac{G\left(c_0(p^*), \beta\right)}{G\left(c_1(p^*), \beta\right)} \frac{\partial G\left(c_1(p^*), \beta\right)}{\partial \beta} - D(p^*) \frac{\partial G\left(c_0(p^*), \beta\right)}{\partial \beta} \\
&= D(p^*) \left[\frac{G\left(c_0(p^*), \beta\right)}{G\left(c_1(p^*), \beta\right)} \frac{\partial G\left(c_1(p^*), \beta\right)}{\partial \beta} - \frac{\partial G\left(c_0(p^*), \beta\right)}{\partial \beta} \right] \\
&= D(p^*) G\left(c_0(p^*), \beta\right) \left[\frac{1}{G\left(c_1(p^*), \beta\right)} \frac{\partial G\left(c_1(p^*), \beta\right)}{\partial \beta} - \frac{1}{G\left(c_0(p^*), \beta\right)} \frac{\partial G\left(c_0(p^*), \beta\right)}{\partial \beta} \right]. \qquad (37)
\end{aligned}
$$

The sign of $\partial H/\partial \beta$ is therefore ambiguous; it depends on the values that the hazard rate G'_β/G takes at the cutoff points $c_0(p^*)$ and $c_1(p^*)$, where G'_β is short-hand notation for $\partial G/\partial \beta$. The interesting issue is that, as in the previous model, this derivative can be negative, in which case the equilibrium price will *decrease* when search costs increase. The next proposition summarizes our findings and provides a sufficient condition for the equilibrium price to decrease in search costs. We explain the intuition behind this result after stating it precisely.

Proposition 5 *Let $G(c;\beta)$ be a search cost cdf with positive density on $[0,\overline{c}]$ and with derivative $\partial G(\cdot)/\partial \beta < 0$. Then the comparative statics with respect to β of the SNE price of the non-sequential search duopoly model described in Proposition 4 is as follows:*

(A) The equilibrium price given by Proposition 4A is independent of β. Therefore, higher search costs do not have a bearing on the equilibrium price.

(B) The equilibrium price given by Proposition 4B unambiguously increases in β. Therefore, higher search costs always result in higher prices.

(C) The equilibrium price given by Proposition 4C decreases in β if and only if

$$
\frac{1}{G\left(c_1(p^*), \beta\right)} \frac{\partial G\left(c_1(p^*), \beta\right)}{\partial \beta} - \frac{1}{G\left(c_0(p^*), \beta\right)} \frac{\partial G\left(c_0(p^*), \beta\right)}{\partial \beta} > 0
$$

Moreover, if G'_β/G increases (decreases) in c, then the equilibrium price increases (decreases) in β. The price is independent of β if G'_β/G is constant in c.

The contrast between the results in parts B and C of the Proposition is important in that it again demonstrates that the standard result about the relationship between search costs and prices is based on a restriction on the magnitude of search costs. When search costs are initially low, an increase in search costs has effects only on the intensive search margin. Confronting more difficulties to try products, consumers engage in less product-comparison. Buyers who stop comparing products enlarge the group of buyers who do not, and this results in a lower elasticity of the demand of an individual firm. Correspondingly, firms adjust their prices upwards.

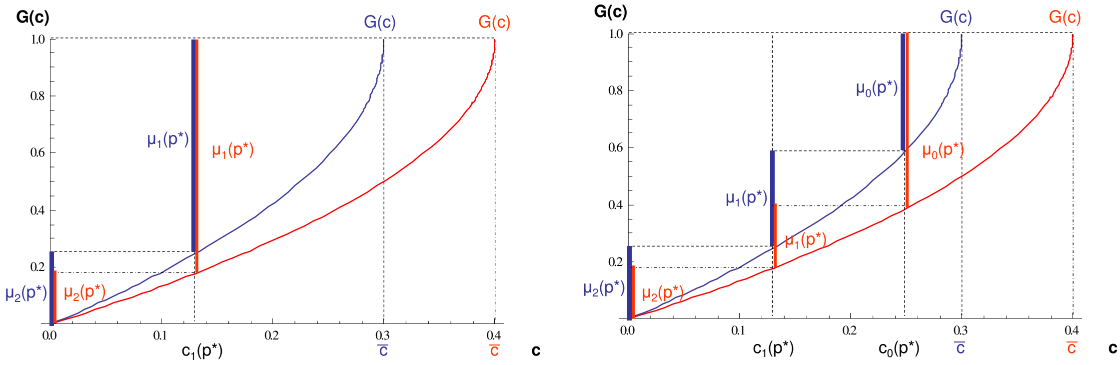
However, when search costs are not restricted to be initially low, increases in search costs have a bearing on both the intensive and the extensive search margins. At the intensive search margin, the same effect happens. The share of consumers who used to inspect the two products goes down and this tends to decrease the elasticity of demand of an individual firm. However, at the extensive search margin, more consumers drop from the market altogether when search costs go up, which changes demand composition and the nature of the average consumer. This results in an increase rather than in a decrease of the elasticity of demand. This tradeoff is resolved in favour of lowering prices when the hazard rate G'_β/G is increasing in search costs. The reason for this is that in such a case, an increase in search costs is more noticeable at higher percentiles of the search cost distribution than at lower, which implies that the effect on the extensive search margin is stronger than the effect on the intensive search margin.

In order to illustrate these arguments, we refer to Figure 4. In this figure we represent the effect of an increase in search costs on the intensive and extensive search margins. Initially consumer search costs are given by the blue search cost distribution. This search cost distribution has the property that G'_β/G is increasing in c.[16] The increase in search costs is represented by the shift from the blue distribution to the red one. As the graph shows, the increase in search costs is much more felt at the higher percentiles of the search cost distribution.

In the graph of Figure 4(a) we represent the case discussed in Proposition 5B. Before the increase in search costs, the blue fractions of consumers $\mu_1(p^*)$ and $\mu_2(p^*)$ represent the equilibrium fractions of consumers searching once and twice, respectively. Because here search costs are small for all consumers ($c_0(p^*) = \bar{c}$), they all search at least once. Keeping prices constant, an increase in search costs results in a fall in the number of consumers who search twice and, correspondingly, in an increase in the number of consumers who search once. This lowers demand elasticity and firms raise their prices.

The graph of Figure 4(b) shows the case discussed in Proposition 5C. In this case search costs are sufficiently large ($c_0(p^*) < \bar{c}$) and the fraction of consumers $\mu_0(p^*)$ does not find it worth to search thereby opting out of the market altogether. When search costs increase, keeping prices fixed, the share of consumers who do not even start searching increases a lot. This causes the share of inelastic consumers to fall more than the share of elastic consumers; this demand composition effect increases overall elasticity of demand and firms lower their prices as a result.

[16]To be sure, we plot here the Kumaraswamy's (1980) distribution with parameters $a = 1, b = 1/2$ and upper bound $\beta = 0.3$; in section 3.1.1 we introduce such a distribution and demonstrate that it has an increasing elasticity for those parameter values.



(a) c_0 sufficiently large (b) c_0 sufficiently small

Figure 4: The effect of an increase in search costs

3.1.1 The Kumaraswamy's distribution

Note that for the Kumaraswamy distribution we have

$$\frac{\partial G(c;\beta)}{\partial\beta} = -\frac{ab}{\beta}\left(\frac{c}{\beta}\right)^a\left(1-\left(\frac{c}{\beta}\right)^a\right)^{b-1} < 0;$$

correspondingly, the hazard ratio G'_β/G is then

$$\frac{G'_\beta(c;\beta)}{G(c;\beta)} = -\frac{\frac{ab}{\beta}\left(\frac{c}{\beta}\right)^a\left(1-\left(\frac{c}{\beta}\right)^a\right)^{b-1}}{1-\left[1-\left(\frac{c}{\beta}\right)^a\right]^b}. \tag{38}$$

We now let

$$t \equiv 1-\left(\frac{c}{\beta}\right)^a.$$

Note that $t \in (0,1)$ and that t is monotonically decreasing in c. We can rewrite (38) as

$$\frac{G'_\beta}{G} = -\frac{ab(1-t)t^{b-1}}{\beta(1-t^b)},$$

and then take the derivative of G'_β/G with respect to t. This gives

$$\frac{d[G'_\beta/G]}{dt} = -\frac{abt^{b-2}(b-1-bt+t^b)}{\beta(1-t^b)^2}.$$

We now argue that this derivative is negative for all $b > 1$ and positive for all $0 < b < 1$. Consider first the $b > 1$ case. Let $h(t) \equiv b-1-bt+t^b$. Then $h(0) = b-1 > 0$, $h(1) = 0$, and $h'(t) = -b\left(1-t^{b-1}\right) < 0$. So h is monotonically decreasing and hence $h(t) > 0$ for any $t \in (0,1)$. As a result, G'_β/G decreases in t (and thus increases in c). By Proposition 5, this implies that when condition (34) holds, for the Kumaraswamy family of search cost distributions with parameter $b > 1$, the equilibrium price increases as search costs rise.

Second, assume $0 < b < 1$. In this case we have $h(0) = b - 1 < 0$, $h(1) = 0$ and $h'(t) = -b\left(1 - t^{b-1}\right) > 0$. Hence $h(t) < 0$ for any $t \in (0,1)$. As a result, G'_β/G increases in t (and therefore decreases in c). By Proposition 5, this implies that when condition (34) holds, for the Kumaraswamy family of search cost distributions with parameter $0 < b < 1$, the equilibrium price decreases as search costs go up.

For completeness, let $b = 1$. Plugging $b = 1$ in (38) gives $G'_\beta/G = -a/\beta$, which is constant in c and therefore the equilibrium price when condition (34) holds does not vary with β.

The following result summarizes these findings.

Corollary to Proposition 5 *Assume that search costs are distributed on the interval* $[0, \beta]$ *according to the Kumaraswamy distribution. Then:*

(A) The equilibrium price in Proposition 4A is independent of β*.*

(B) The equilibrium price in Proposition 4B unambiguously increases in β*.*

(C) For all a*, the equilibrium price in Proposition 4C decreases in* β *if* $0 < b < 1$*, is constant in* β *if* $b = 1$*, and increases in* β *if* $b > 1$*.*

3.2 The N-firm model

The previous non-sequential search model with differentiated products can be generalized to the case of $N > 2$ firms. The problem of a consumer with search cost c is to choose a number k of firms to be sampled in order to maximize her expected utilitity:

$$\max_k \left\{ \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) k F(\varepsilon)^{k-1} f(\varepsilon) d\varepsilon - kc \right\}.$$

It can easily be checked that this problem is well-behaved and that a unique solution exists. Such a solution defines a partition of the consumer population into groups of buyers $\mu_k(p^*)$ that search $k = 0, 1, 2, ..., N$ firms, with $\sum_{k=0}^{N} \mu_k(p^*) = 1$; as above, some of these groups may have zero mass as the upper bound of the search cost distribution decreases.

In order to determine the size of these groups, let us define the critical search cost parameters

$$c_0(p^*) = \min\left\{ \overline{c}, \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon \right\}$$
$$c_k(p^*) = \min\left\{ \overline{c}, \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) \left[(k+1)F(\varepsilon) - k\right] F(\varepsilon)^{k-1} f(\varepsilon) d\varepsilon \right\}, \ k = 1, 2, ..., N-1.$$

The fractions of consumers searching k times are then given by the expressions:

$$\begin{aligned}
\mu_0 &= 1 - G(c_0(p^*)) \\
\mu_k &= G(c_{k-1}(p^*)) - G(c_k(p^*)),\ \ k = 1, 2, ..., N-1 \\
\mu_N &= G(c_{N-1}(p^*)) - G(c_N(p^*)) = G(c_{N-1}(p^*)) \text{ since } c_N = 0.
\end{aligned} \tag{39}$$

If $c_{N-1}(p^*) = \bar{c}$ for example, then all consumers will search the N firms in equilibrium and the situation will again resemble the perfect information model of Perloff and Salop (1985). When $c_{N-1}(p^*) < \bar{c} < c_{N-2}(p^*)$, a fraction $\mu_N = G(c_{N-1}(p^*))$ of consumers will visit the N firms and the remaining consumers will each visit $N-1$ randomly selected firms; and so on and so forth.

Let $z_k \equiv \max\{\varepsilon_1, \varepsilon_2, ..., \varepsilon_k\}$. In general, the expected payoff of a firm i that deviates from the symmetric equilibrium price by charging a price $p_i \neq p^*$ is

$$\pi_i(p_i; p^*) = (p_i - r)\left(\frac{\mu_1(p^*)}{2}\Pr[\varepsilon_i \geq p_i] + \sum_{k=2}^{N}\frac{k\mu_k(p^*)}{N}\Pr\left[\varepsilon_i - p_i \geq \max\{z_{k-1} - p^*, 0\}\right]\right) \tag{40}$$

As before, the demand of the deviant firm i stems from the various consumer groups and a consumer who searches k times compares the offer of firm i with the offers of $k-1$ other firms.

For the case where the deviant firm charges a higher price than the rest of the firms, the expression in (40) becomes

$$\pi_i(p_i > p^*; p^*) = (p_i - r)\left[\frac{\mu_1(p^*)}{N}(1 - F(p_i)) + \sum_{k=2}^{N}\frac{k\mu_k(p^*)}{N}\int_{p_i}^{\bar{\varepsilon}} F(\varepsilon - (p_i - p^*))^{k-1} f(\varepsilon)d\varepsilon\right]. \tag{41}$$

Taking the FOC gives:

$$\begin{aligned}
&\mu_1(p^*)(1 - F(p_i)) + \sum_{k=2}^{N} k\mu_k(p^*)\int_{p_i}^{\bar{\varepsilon}} F(\varepsilon - p_i + p^*))^{k-1} f(\varepsilon)d\varepsilon - (p_i - r)\mu_1(p^*)f(p_i) \\
&- (p_i - r)\sum_{k=2}^{N} k\mu_k(p^*)\left(\int_{p_i}^{\bar{\varepsilon}}(k-1)F(\varepsilon - p_i + p^*)^{k-2} f(\varepsilon - p_i + p^*)f(\varepsilon)d\varepsilon + F(p^*)^{k-1}f(p_i)\right) = 0.
\end{aligned} \tag{42}$$

After imposing symmetry, simplifying and rearranging we obtain:

$$\begin{aligned}
&\mu_1(p^*)\left[1 - F(p^*) - (p^* - r)f(p^*)\right] + \sum_{k=2}^{N} k\mu_k(p^*)\int_{p^*}^{\bar{\varepsilon}} F(\varepsilon)^{k-1} f(\varepsilon)d\varepsilon \\
&- (p^* - r)\sum_{k=2}^{N} k\mu_k(p^*)\left(\int_{p^*}^{\bar{\varepsilon}}(k-1)F(\varepsilon)^{k-2} f(\varepsilon)^2 d\varepsilon + F(p^*)^{k-1}f(p^*)\right) = 0.
\end{aligned} \tag{43}$$

In the appendix we show that a candidate equilibrium $p^* \in [r, p^m]$ exists.

Depending on the magnitude of the upper bound of the search cost distribution $\overline{c}$, there may exist $N+1$ types of equilibria:

1. When

$$\overline{c} \leq \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) \left[(N+1)F(\varepsilon) - N\right] F(\varepsilon)^{N-1} f(\varepsilon) d\varepsilon$$

then all consumers search the N-firms in the market and the equilibrium price is given by the solution to the FOC (43) after setting $\mu_i(p^*) = 0$ for all $i = 1, 2, ..., N-1$ and $\mu_N(p^*) = 1$. This equilibrium exists an is unique, as it is the same as that in Perloff and Salop (1985).

2. When

$$\int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) \left[(N+1)F(\varepsilon) - N\right] F(\varepsilon)^{N-1} f(\varepsilon) d\varepsilon < \overline{c} \leq \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) \left[NF(\varepsilon) - (N-1)\right] F(\varepsilon)^{N-2} f(\varepsilon) d\varepsilon$$

then a fraction of consumers $\mu_N(p^*) = G(c_{N-1}(p^*))$ searches N firms and the rest of the consumers search $N-1$ firms, and the equilibrium price is given by the solution to the FOC (43) after setting $\mu_i(p^*) = 0$ for all $i = 1, 2, ..., N-2$ and replacing $\mu_{N-1}(p^*)$ and $\mu_N(p^*)$ by their corresponding values in (39).

3. When

$$\int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) \left[NF(\varepsilon) - (N-1)\right] F(\varepsilon)^{N-2} f(\varepsilon) d\varepsilon < \overline{c} \leq \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) \left[(N-1)F(\varepsilon) - (N-2)\right] F(\varepsilon)^{N-3} f(\varepsilon) d\varepsilon$$

then a fraction of consumers $\mu_N(p^*) = G(c_{N-1}(p^*))$ searches N firms, a fraction of consumers $\mu_{N-1}(p^*) = G(c_{N-2}(p^*)) - G(c_{N-1}(p^*))$ searches $N-1$ firms and the rest of the consumers search $N-2$ firms, and the equilibrium price is given by the solution to the FOC (43) after setting $\mu_i(p^*) = 0$ for all $i = 1, 2, ..., N-3$ and replacing $\mu_{N-2}(p^*), \mu_{N-1}(p^*)$ and $\mu_N(p^*)$ by their corresponding values in (39).

4,5,...,N-1. So and so forth.

N. When

$$\int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) \left[2F(\varepsilon) - 1\right] F(\varepsilon) f(\varepsilon) d\varepsilon < \overline{c} \leq \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon$$

then a fraction of consumers $\mu_N(p^*) = G(c_{N-1}(p^*))$ searches N firms, a fraction of consumers $\mu_k(p^*) = G(c_{k-1}(p^*)) - G(c_k(p^*))$ searches $k = 2, 3, ..., N-1$ firms and the rest of the consumers search just one firm. In this case the equilibrium price is given by the solution to the FOC (43) after replacing $\mu_1(p^*), \mu_2(p^*), ..., \mu_N(p^*)$ by their corresponding values in (39).

N+1. Finally, when

$$\overline{c} > \int_{p^*}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon$$

then a fraction of consumers $\mu_N(p^*) = G(c_{N-1}(p^*))$ searches N firms, a fraction of consumers $\mu_k(p^*) = G(c_{k-1}(p^*)) - G(c_k(p^*))$ searches $k = 1, 2, 3, ..., N-1$ firms and the rest of the consumers do not search at all. In this case the equilibrium price is given by the solution to the FOC (43) after replacing $\mu_1(p^*), \mu_2(p^*), ..., \mu_N(p^*)$ by their corresponding values in (39).

Except in case 1 above, the payoff functions involve sums of quasiconcave functions and because of this feature the payoff function need not be quasiconcave. This makes it very hard to provide existence results. Nevertheless, we can prove the following existence result:

Proposition 6 *Let $N = 3$ and assume that F is the uniform distribution. Then, depending on the magnitude of the upper bound of the price distribution $\bar{c}$ there exist 4 types of equilibria. These equilibria can be obtained from equation (43) after replacing $\mu_k(p^*)$, $k = 1, 2, 3$ by their respective values given in (39).*

Proof. See the appendix.

To study how an increase in search costs affects the equilibrium price we proceed by solving the model numerically. We focus on the most novel case, i.e. where search costs are sufficiently large so that not all consumers search in equilibrium. Assuming that search costs follow the Kumaraswamy distribution with upper bound β, we set $a = 1$, pick β sufficiently high so that all fractions of consumers defined above in (39) are strictly positive and compute the price equilibrium and search intensities for various levels of the parameter b. For the case $N = 2$, our Proposition 5 shows mathematically that, after an increase in search costs, prices go down when the parameter b of the Kumaraswamy search cost distribution is less than 1; for $b = 1$, prices do not change; while for $b > 1$, prices increase. Table 2 shows that the same results obtain in a market where $N = 5$, $r = 0$ and match values are uniformly distributed on the set $[0, 1]$.

The table also illustrates the impact of higher search costs on profits, consumer surplus and welfare. What we see is that, even if higher search costs result in lower prices, consumer surplus goes down in search costs. This is clearly due to the impact higher search costs have on the extensive search margin, which is of first order. In fact notice that conditional on searching, consumers benefit from higher search costs just because prices fall.

Another interesting result is that firm profits always decrease when search costs increase, even if prices increase. Once again, this is due to the impact of higher search costs on the extensive search margin.

	$b = 1.5$			$b = 1.00$			$b = 0.5$		
	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$
μ_0	0.7008	0.8467	0.8970	0.7910	0.8955	0.9303	0.8905	0.9465	0.9646
μ_1	0.1251	0.0651	0.0439	0.0900	0.0450	0.0300	0.0484	0.0233	0.0153
μ_2	0.0668	0.0341	0.0228	0.0463	0.0231	0.0154	0.0241	0.0118	0.0078
μ_3	0.0370	0.0187	0.0125	0.0253	0.0126	0.0084	0.0129	0.0064	0.0042
μ_4	0.0216	0.0108	0.0072	0.0146	0.0073	0.0048	0.0074	0.0036	0.0024
μ_5	0.0484	0.0243	0.0162	0.0325	0.0162	0.0108	0.0163	0.0081	0.0054
p^*	0.3504	0.3521	0.3526	0.3536	0.3536	0.3536	0.3567	0.3551	0.3545
π	0.0171	0.0087	0.0059	0.0120	0.0060	0.0040	0.0062	0.0030	0.0020
CS	0.0792	0.0402	0.0269	0.0544	0.0272	0.0181	0.0280	0.0138	0.0091
$CS/(1-\mu_0)$	0.2647	0.2626	0.2619	0.2606	0.2606	0.2606	0.2566	0.2587	0.2593
$Welfare$	0.1650	0.0842	0.0565	0.1144	0.0572	0.0381	0.0595	0.0291	0.0193

Table 2: Non-sequential search for differentiated products: price equilibrium and search intensities (Kumaraswamy distribution, $a = 1$)

4 Homogeneous products

In this Section we study the effects of higher search costs in a non-sequential consumer search models for homogeneous products. The main difference with the case of differentiated products is that in homogeneous product models consumers search for low prices and the symmetric equilibrium is characterized by mixed-strategies. We will show that our result above in Proposition 5 that higher search costs can result in lower prices also arises with homogeneous products. This is interesting because the main insight of this paper has nothing to do with the mixed- or pure-strategy nature of equilibria.

We start by examining a *duopoly* model similar to Burdett and Judd (1983).[17] Except in that products are homogeneous, the model is similar to the model in Section **??**. Firms produce a homogeneous good at constant unit costs r. There is a unit mass of buyers. Each buyer inelastically demands one unit of the good and is willing to pay for it a maximum of $v > r$. Let $\theta \equiv v - r$. Consumers search for prices non-sequentially and buy from the cheapest store they know. Search costs are distributed on $(0, \bar{c})$,with distribution G and density g.[18] Searching k times costs the consumer kc, $k = 0, 1, 2$.

Firms and buyers play a simultaneous moves game. An individual firm chooses its price taking rivals' prices as well as consumers' search behavior as given. A firm i's strategy is denoted by a distribution

[17] For a dynamic version, see Fershtman and Fishman (1992) and for an oligopoly version see Janssen and Moraga-González (2004). These models do not allow for genuine search cost heterogeneity.

[18] As before, the critical issue will be the relationship between $\bar{c}$ and v. We will see that when $\bar{c} > v$, some consumers will opt out of the market and will not search at all. By contrast, when $\bar{c} < v$ every consumer will make at least one search.

of prices $F_i(p)$. Let $F_{-i}(p)$ denote the vector of prices charged by firms other than i. The (expected) profit to firm i from charging price p_i given rivals' strategies is denoted by $\Pi(p_i, F_{-i}(p))$. Likewise, an individual buyer takes as given firm pricing and decides on his/her optimal search strategy to maximize his/her expected utility. The strategy of a consumer with search cost c is then a number k of prices to sample, $k = 0, 1, 2$. Let the fraction of consumers sampling k firms be denoted by μ_k. We shall concentrate on symmetric Nash equilibria. A symmetric equilibrium is a distribution of prices $F(p)$ and a collection $\{\mu_0, \mu_1, \mu_2\}$ such that (a) $\Pi_i(p, F_{-i}(p))$ is equal to a constant $\overline{\Pi}$ for all p in the support of $F(p)$, $\forall i$; (b) $\Pi_i(p, F_{-i}(p)) \leq \overline{\Pi}$ for all p, $\forall i$; (c) a consumer with search cost c chooses to sample $k(c)$ firms such that $k(c) = \arg\min_{k\in\{0,1,2\}} \left[kc + \int_{\underline{p}}^{v} pk(1 - F(p))^{k-1} f(p)dp\right]$; and (d) $\sum_{k=0}^{2} \mu_k = 1$. Let us denote the equilibrium density of prices by $f(p)$, with maximum price $\overline{p}$ and minimum price $\underline{p}$.

The following 2 lemmas follow directly from Burdett and Judd (1983). The first indicates that, for an equilibrium to exist, there must be some consumers who search just once and others who search twice. The second shows that prices must be dispersed in equilibrium.

Lemma 1 *If a symmetric equilibrium exists, then* $1 > \mu_k > 0$, $k = 1, 2$, *and* $\mu_0 \geq 0$.

The intuition behind this result is simple. Suppose all searching consumers did search twice ($\mu_0 + \mu_2 = 1$); then pricing would be competitive. This however is contradictory because then consumers would not be willing to search that much in the first place. Suppose now that no consumer did compare prices ($\mu_0 + \mu_1 = 1$); then firms would charge the monopoly price. This is also contradictory because in that case consumers would not be willing to search at all.[19]

Lemma 2 *If a symmetric equilibrium exists,* $F(p)$ *must be atomless with upper bound equal to* v.

This is easily understood. If a particular price is chosen with strictly positive probability then a deviant can gain by undercutting such a price and attracting all price-comparing consumers. This competition for the price-comparing consumers cannot drive the price down to zero since then a deviant would prefer to raise its price and sell to the consumers who do not compare prices.

[19]In the original model of Burdett and Judd (1983), it is assumed that the search cost is lower than the surplus consumers get at the monopoly price. As a result, all consumers buy no matter the equilibrium price distribution and therefore there always exists an equilibrium where all firms charge the monopoly price (cf. Diamond, 1971). Since we have arbitrary search cost heterogeneity, this assumption is relaxed. A by-product is that a Diamond-type of result cannot be an equilibrium any longer.

We now turn to consumers' search behavior. Expenditure minimization requires a consumer with search cost c to continue to draw prices from the price distribution $F(p)$ till the expected gains of drawing one more price fall below her search cost. The expected net gains from searching once rather than not searching at all are given by $v - E[p] - c$, while the expected net gains from searching twice rather than once are given by $E[p] - E[\min\{p_1, p_2\}] - c$, where E denotes the expectation operator. Since the search cost distribution has support on $[0, \bar{c}]$, we can define the critical consumers c_0 and c_1 satisfying the following equalities:

$$c_0 = \min\{\bar{c}, v - E[p]\}, \tag{44}$$

$$c_1 = E[\min\{p_1, p_2\}] - E[p]. \tag{45}$$

From Lemma 1, it must be the case that $c_1 > 0$ and $c_0 > c_1$. c_0 is the minimum of the search cost of the consumer who is indifferent between searching and not searching at all and of the upper bound of the search cost distribution. When the upper bound of the search cost distribution $\bar{c}$ is sufficiently high $c_0 = v - E[p]$ and all consumers with search cost above c_0 will not search at all. When $\bar{c}$ is small enough, all consumers will search at least once. In particular, consumers for whom $c_1 < c \leq c_0$ will indeed search once and consumers for whom $c \leq c_1$ will search twice.

Lemma 3 *Given any atomless price distribution $F(p)$, optimal consumer search behavior is uniquely characterized as follows: the fractions of consumers searching once and twice are given by*

$$\mu_1 = \int_{c_1}^{c_0} dG(c) > 0; \; \mu_2 = \int_0^{c_1} dG(c) > 0 \tag{46}$$

while the fraction of consumers not searching at all is

$$\mu_0 = \int_{c_0}^{\bar{c}} dG(c) \geq 0, \tag{47}$$

where c_0 and c_1 are given by (44)-(45)

We now examine firm pricing behavior taking consumer search strategies as given. Following Burdett and Judd (1983), a firm i charging a price p_i sells to a consumer who searches one time provided the consumer samples firm i, which happens with probability $1/2$, and sells to a consumer who searches twice provided the rival firm charges a price higher than p_i, which happens with probability $1 - F(p_i)$. Therefore the expected profit to firm i from charging price p_i when its rivals draw a price from the cdf $F(p)$ is

$$\Pi_i(p_i; F(p)) = (p_i - r)\left\{\frac{1}{2}\mu_1 + \mu_2\left[1 - F(p_i)\right]\right\}.$$

In equilibrium, a firm must be indifferent between charging any price in the support of $F(p)$ and charging the upper bound $\overline{p}$. Thus, any price in the support of $F(p)$ must satisfy $\Pi_i(p_i; F(p)) = \Pi_i(\overline{p}; F(p))$. Since $\Pi_i(\overline{p}; F(p))$ is monotonically increasing in $\overline{p}$, it must be the case that $\overline{p} = v$. As a result, equilibrium pricing requires

$$(p_i - r)\left\{\mu_1 + 2\mu_2\left[1 - F(p_i)\right]\right\} = \mu_1(v - r). \tag{48}$$

Solving this equation for $F(p_i)$ leads to the following result, also in Burdett and Judd (1983):

Lemma 4 *Given μ_1 and μ_2, there exists a unique symmetric equilibrium price distribution $F(p)$. In equilibrium firms charge prices randomly chosen from the set $\left[\frac{(v-r)\mu_1}{\mu_1+2\mu_2} + r, v\right]$ according to the price distribution*

$$F(p) = 1 - \frac{\mu_1}{2\mu_2}\frac{v-p}{p-r}. \tag{49}$$

Notice that $F(p)$ depends on the search cost distribution via its effect on μ_1 and μ_2; moreover, notice that $F(p)$ is increasing in μ_2 and decreasing in μ_1. Hence, if an increase in search costs results in a higher (lower) ratio of "price-comparing to non-price-comparing" consumers, then the price distribution shifts up (down) and prices decrease (increase).

For the price distribution (49) to be an equilibrium of the game, the conjectured groupings of consumers has to be the outcome of optimal consumer search. This requires that

$$c_0 = \min\left\{\overline{c}, \int_0^v F(p)dp\right\} \text{ and } c_1 = \int_0^v F(p)(1-F(p))dp \tag{50}$$

Since the price distribution $F(p)$ in (49) is strictly increasing in p, we can find its inverse:

$$p(z) = \frac{v-r}{1+2\frac{\mu_2}{\mu_1}(1-z)} + r. \tag{51}$$

Using this inverse function, integration by parts and the change of variables $z = F(p)$, we can state that:

Proposition 7 *If a symmetric equilibrium exists in the non-sequential search duopoly model with homogeneous products then consumers search according to Lemma 3, firms set prices according to Lemma 4, and c_0 and c_1 are given by the solution to the following system of equations:*

$$c_0 = \min\left\{\overline{c}, (v-r)\left[1 - \int_0^1 \frac{G(c_0) - G(c_1)}{G(c_0) - G(c_1)(1-2u)}du\right]\right\}, \tag{52}$$

$$c_1 = (v-r)\int_0^1 \frac{\left[G(c_0) - G(c_1)\right](1-2u)}{G(c_0) - G(c_1)(1-2u)}du \tag{53}$$

Relative to Burdett and Judd (1983), this Proposition is our first contribution. It is useful because of two reasons. First, it provides a straightforward way to compute the market equilibrium. For fixed v, r, $\overline{c}$ and $G(c)$, the system of equations (52)–(53) can be solved numerically. If a solution exists, then the consumer equilibrium is given by equations (46)–(47) and the price distribution follows readily from equation (49). Secondly, this result enables us to address the issues of existence and uniqueness of equilibrium, which are the subject of our second contribution.

Proposition 8 *In the non-sequential search duopoly model with homogeneous products: (A) For any consumer valuation v and firm marginal cost r such that $v > r \geq 0$ and for any search cost distribution function $G(c)$ with support $(0, \overline{c})$ such that either $g(0) > 0$ or $g(0) = 0$ and $g'(0) > 0$, a symmetric Nash equilibrium exists. (B) For the family of polynomial distribution functions $G(c) = (c/\overline{c})^a$, $a > 0$, the equilibrium is unique.*

The proof of this result is in the Appendix.[20] Proposition 8 establishes uniqueness of equilibrium when the search cost distribution has the described polynomial form. General results on uniqueness prove to be very difficult because we cannot compute the equilibrium explicitly and the system of equations (52)–(53) is non-linear.

4.1 The effect of higher search costs on prices

The next step in the analysis is to study how an increase in search costs affects prices. As mentioned above, for this it suffices to study how the ratio of "price-comparing to non-price comparing" consumers λ is affected by an increase in search costs. To do so, as before, let $G(c; \beta)$ be a parametrized search cost CDF with $\partial G(c; \beta)/\partial \beta < 0$ and denote the equilibrium price distribution corresponding to a given β by $F(p; \beta)$. We now examine how F changes with β.

To understand the effect of an increase in β on the equilibrium price distribution, we study how the solution to the system of equations that determines c_0, c_1 and c_2 depends on β; this, in turn, determines how μ_1 and μ_2, and therefore λ, depend on β. We start with the (most interesting) case where the upper bound of the search cost distribution is sufficiently high so that $c_0 < \overline{c}$ and some consumers opt out of the market altogether. Using the change of variables $x_k \equiv G(c_k; \beta)$ in (52)–(53)

[20]Elsewhere, we have extended this existence result to the case of an arbitrary number of firms N (see Moraga-González et al., 2010).

gives

$$x_0 = G\left(\theta - \theta \int_0^1 \frac{x_0 - x_1}{x_0 - x_1 + 2x_1 u} du; \beta\right),$$
$$x_1 = G\left(\theta \int_0^1 \frac{x_0 - x_1}{x_0 - x_1 + 2x_1 u}(1-2u)\, du; \beta\right).$$

Let $y \equiv x_1/x_0 \in [0,1]$. Then the previous system of equations is equivalent to

$$yG(c_0(y);\beta) - G(c_1(y);\beta) = 0. \tag{54}$$

where

$$c_0(y) = \theta\left[1 - \int_0^1 \frac{1-y}{1-y(1-2u)} du\right] = \theta\left[1 + \frac{(1-y)}{2y}\ln\left(\frac{1-y}{1+y}\right)\right] \text{ and} \tag{55}$$

$$c_1(y) = \theta \int_0^1 \frac{(1-y)(1-2u)}{1-y(1-2u)} du = -\frac{\theta(1-y)}{2y^2}\left[2y + \ln\left(\frac{1-y}{1+y}\right)\right]. \tag{56}$$

Note that $0 \le c_1(y) \le c_0(y) \le \theta$ for any $y \in [0,1]$. For later use, notice that $0 = c_1(0) = c_0(0)$ and that $c_1'(0) > 0$.

Rewriting (54) gives:

$$H(y;\beta) \equiv yG\left(\theta\left\{1 + \frac{1-y}{2y^2}\left[2y + \ln\left(\frac{1-y}{1+y}\right)\right]\right\};\beta\right) - G\left(\theta\left[1 + \frac{1-y}{2y}\ln\left(\frac{1-y}{1+y}\right)\right];\beta\right) = 0. \tag{57}$$

An equilibrium of the model is given as a solution to equation $H(y;\beta) = 0$. Let $y(\beta)$ denote such a solution. If we obtain $y(\beta)$, then using (55) and (56) we can immediately derive the corresponding $c_0(\beta)$ and $c_1(\beta)$ and hence $\mu_1(\beta)$, $\mu_2(\beta)$, $\lambda(\beta)$ and the equilibrium price distribution $F(p;\beta)$. To be sure, for a given β, we notice again the relationship between the variables we have introduced

$$y = x_1/x_0,\ x_0 = G(c_0),\ x_1 = G(c_1),\ \mu_2 = x_1,\ \mu_1 = x_0 - x_1. \tag{58}$$

Since

$$\frac{\mu_2}{\mu_1} = \frac{1}{\frac{1}{y} - 1},$$

a decrease in y results in an decrease in μ_2/μ_1 and, correspondingly, in an increase in prices. We now study how $y(\beta)$ depends on the shifter β of the search cost distribution $G(c;\beta)$

Let $y(\beta)$ be the solution to equation (57). The Implicit Function Theorem implies

$$\frac{dy(\beta)}{d\beta} = -\frac{\frac{\partial H(y;\beta)}{\partial \beta}}{\frac{\partial H(y;\beta)}{\partial y}}. \tag{59}$$

In order to sign this derivative, we consider first its numerator.

$$\begin{aligned}\frac{\partial H(y;\beta)}{\partial\beta} &= yG'_\beta\left(c_0\left(y\right);\beta\right) - G'_\beta\left(c_1\left(y\right);\beta\right)\\ &= \frac{G\left(c_1\left(y\right);\beta\right)}{G\left(c_0\left(y\right);\beta\right)}G'_\beta\left(c_0\left(y\right);\beta\right) - G'_\beta\left(c_1\left(y\right);\beta\right)\\ &= G\left(c_1\left(y\right);\beta\right)\left[\frac{G'_\beta\left(c_0\left(y\right);\beta\right)}{G\left(c_0\left(y\right);\beta\right)} - \frac{G'_\beta\left(c_1\left(y\right);\beta\right)}{G\left(c_1\left(y\right);\beta\right)}\right]\end{aligned}$$

where the second equality follows from the equilibrium condition (57). Therefore, we conclude that

$$\frac{\partial H(y;\beta)}{\partial\beta} > 0 \text{ if and only if } \frac{G'_\beta\left(c_0\left(y\right);\beta\right)}{G\left(c_0\left(y\right);\beta\right)} - \frac{G'_\beta\left(c_1\left(y\right);\beta\right)}{G\left(c_1\left(y\right);\beta\right)} > 0. \tag{60}$$

This condition is exactly the same as in the previous section.

Consider now the denominator of (59). For a given β, $\partial H(y;\beta)/\partial y$ is the derivative of H at the solution y. We note that for $y = 0$ and $y = 1$ we have

$$\begin{aligned}&H(0;\beta) = 0\cdot G\left(c_0\left(0\right);\beta\right) - G\left(c_1\left(0\right);\beta\right) = -G\left(0;\beta\right) = 0,\\ &H(1;\beta) = G\left(c_0\left(1\right);\beta\right) - G\left(c_1\left(1\right);\beta\right) = G\left(1;\beta\right) - G\left(0;\beta\right) = G\left(1;\beta\right) > 0.\end{aligned} \tag{61}$$

Consider now the value of $\partial H\left(y;\beta\right)/\partial y$ at $y = 0$. Since $0 = c_1\left(0\right) = c_0\left(0\right)$ and $c'_1\left(0\right) > 0$ we have

$$\frac{\partial H(0;\beta)}{\partial y} = G\left(0;\beta\right) - G'\left(0;\beta\right)c'_1\left(0\right) = -G'\left(0;\beta\right)c'_1\left(0\right) < 0.$$

Given these three observations (i.e. $H\left(0,\beta\right) = 0, H\left(1,\beta\right) > 0$ and $\partial H\left(0,\beta\right)/\partial y < 0$), we conclude that there exists at least one equilibrium at which H is increasing in y.[21] We then obtain the following result:

Proposition 9 *Consider the non-sequential search duopoly model with homogeneous products and let $G\left(c;\beta\right)$ be a parametrized search cost cdf with positive density on $[0,\overline{c}]$ and with derivative $\partial G(c;\beta)/\partial\beta < 0$ for all c. Assume that $\overline{c}$ is sufficiently large so that c_0 defined in (52) satisfies $c_0 < \overline{c}$. Then, if there exists a unique equilibrium $F(p;\beta)$ increases in β for all p (so prices decrease in search costs) if and only if*

$$\frac{G'_\beta\left(c_1\left(y\right);\beta\right)}{G\left(c_1\left(y\right);\beta\right)} - \frac{G'_\beta\left(c_0\left(y\right);\beta\right)}{G\left(c_0\left(y\right);\beta\right)} > 0,$$

[21] We ignore ill-behaved situations where at the solutions of (57) $H\left(\cdot;\beta\right)$ is tangent to the horizontal axes, that is, we assume that $\partial H\left(y,\beta\right)/\partial y \neq 0$ at any solution y. Moreover, if there are multiple equilibria, the number of equilibria is odd. In such situation each odd-numbered solution $y\left(\beta\right)$ satisfies $\partial H\left(y,\beta\right)/\partial y > 0$, while each even-numbered solution $y\left(\beta\right)$ satisfies $\partial H\left(y,\beta\right)/\partial y < 0$.

where $c_0(y)$ and $c_1(y)$ are given by (55) and (56), respectively.[22] *Moreover, when the hazard ratio G'_β/G decreases (increases) in c, prices fall (rise) in search costs.*

This result is similar to that in the previous section on differentaited products. Prices can increase or decrease after search costs go up for all consumers provided that not all consumers search. What is needed for prices to decrease is that the impact of higher search costs on the *intensive search margin* is weaker than that on the *extensive search margin*, and this is guaranteed when the hazard ratio G'_β/G decreases in c.

We now continue with the case where the upper bound of the search cost distribution is sufficiently low so that $c_0 = \bar{c}$. This implies that $\mu_0 = 0$. In this case higher search costs only have an effect at the intensive search margin and thereby we should obtain the standard result that higher search costs lead to higher prices. For this case,

$$\frac{\mu_2}{\mu_1} = \frac{1}{\mu_1} - 1.$$

As a result, the equilibrium price distribution is uniquely determined by μ_1, which in turn depends on

$$c_1 = \theta \int_0^1 \frac{[1 - G(c_1;\beta)]\,(1-2u)}{1 - G(c_1;\beta)(1-2u)} du \tag{62}$$

Using the change of variables $y \equiv G(c_1)$ we can write (62) as $y - G(c_1(y)) = 0$ where $c_1(y)$ is given in (56). Rewriting gives:

$$H(y;\beta) \equiv y - G\left(-\frac{\theta\,(1-y)}{2y^2}\left[2y + \ln\left(\frac{1-y}{1+y}\right)\right];\beta\right) = 0 \tag{63}$$

As above, an equilibrium of the model is given as a solution to equation (63). Note that since G is monotone and $0 = c_1(0) = c_1(1)$, the equilibrium is unique in this case.

If we consider the parametrized search cost distribution above $G(c;\beta)$ and compute the derivative of H with respect to β we get $\partial H(y;\beta)/\partial\beta = -G'_\beta(c_1(y);\beta) > 0$. This implies that the sign of (59) is negative. As a result:

Proposition 10 *Consider the non-sequential search duopoly model with homogeneous products and let $G(c;\beta)$ be a parametrized search cost cdf with positive density on $[0,\bar{c}]$ and with derivative $\partial G(c;\beta)/\partial\beta < 0$ for all c. Assume that $\bar{c}$ is sufficiently low so that c_0 defined in (52) is equal to $\bar{c}$. Then, $F(p;\beta)$ decreases in β for all p (so prices unambiguously increase in search costs).*

[22] If there exist multiple equilibria, this result also holds for the odd-numbered equilibria. For the even-numbered equilibria, we have the opposite.

4.1.1 The general N-firms case.

The non-sequential search model we have presented above can easily be generalized to the case of N firms.[23] In such a case, the payoff to a firm i charging price p_i given the $N-1$ rivals use the equilibrium price distribution $F(p)$ is

$$\Pi_i(p_i; F(p)) = (p_i - r)\left[\sum_{k=1}^{N} \frac{k\mu_k}{N}(1-F(p_i))^{k-1}\right].$$

The fractions of consumers searching k times are given by

$$\mu_0 = \int_{c_0}^{\overline{c}} dG(c) \tag{64}$$

$$\mu_k = \int_{c_k}^{c_{k-1}} dG(c), \text{ for all } k = 1, 2, \ldots, N; \tag{65}$$

where

$$c_0 = \min\left\{\overline{c}, \int_{\underline{p}}^{v} F(p)dp\right\}; \tag{66}$$

$$c_k = \int_{\underline{p}}^{v} F(p)(1-F(p))^k dp, \; k = 1, 2, \ldots, N-1; \; c_N = 0 \tag{67}$$

The equilibrium distribution function follows from the constancy-of-profits condition

$$\sum_{k=1}^{N} k\mu_k(1-F(p_i))^{k-1} = \frac{\mu_1\theta}{(p_i - r)}, \tag{68}$$

from which we can calculate the inverse of the equilibrium price distribution

$$p(z) = \frac{\mu_1\theta}{\sum_{k=1}^{N} k\mu_k(1-z)^{k-1}} + r. \tag{69}$$

Using (69, we can rewrite the critical points $\{c_k\}_{k=0}^{N}$ as:

$$c_0 = \min\left\{\overline{c}, \theta\left(1 - \int_0^1 \frac{G(c_0) - G(c_1)}{\sum_{k=1}^{N} k[G(c_{k-1}) - G(c_k)]u^{k-1}} du\right)\right\}; \tag{70}$$

$$c_k = \theta \int_0^1 \frac{[G(c_0) - G(c_1)]\left[ku^{k-1} - (k+1)\, u^k\right]}{\sum_{k=1}^{N} k[G(c_{k-1}) - G(c_k)]u^{k-1}} du, \; k = 1, 2, ..., N-1; \; c_N = 0. \tag{71}$$

As mentioned above, we have proven elsewhere that an equilibrium always exists (see Moraga-González et al., 2010).

[23] For empirical applications of the N-firm model see Moraga-González and Wildenbeest (2008) and Moraga-González, Sándor and Wildenbeest (forthcoming). Hong and Shum (2006) estimates search costs using a model with infinitely many firms.

The impact of higher search costs on the equilibrium price distribution (69) is however very difficult to analyze in the general N-firms case because the system of equations (70)–(71) is non-linear and therefore it is hard to say something about how its solution depends on β. Nevertheless, it is straightforward to check numerically that the spirit of the result in Theorem 8 remains. For this we take a market with $N = 10$ firms, set $v = 10$ and $r = 0$ and use the family of Kumaraswamy search cost distributions presented above. We choose β high enough so we are sure $c_0 < \bar{c}$. Table 3 shows how market equilibria evolve as we increase the parameter β from 8 to 9 and to 10. We do this for $a = 1$ and let b take on values that cover the regions in Proposition 3.1.1A, in particular $b = \{0.5, 1, 1.25\}$.

Table 3 clearly shows that the results in Proposition 9 hold true more in general. In particular, when $b = 0.5$ an increase in search costs leads to lower prices. We can see that higher search costs lead to overall less search, i.e., as search costs increase a given consumer searches (weakly) less (all μ's decrease except μ_0). The effect is more noticed at the higher quantiles of the search cost distribution. This is due to the fact that for $b = 0.5$, the search cost density is increasing and thereby there is more mass of consumers at higher search costs. Relative to the non-price-comparing consumers, the number of price-comparing consumers increases (all fractions mu_k/μ_1 increase), which makes the market more competitive. As a result prices decrease. Though aggregate social welfare falls as search costs increase, some consumers benefit. This can be seen in the row $CS/(1 - \mu_0)$, which is the consumer surplus conditional on searching at least one time.

When search costs follow the uniform distribution ($b = 1$), prices are constant. What happens is that the numbers of price-comparing and non-price comparing consumers fall exactly in the same proportion. Consumer surplus conditional on searching also goes up in this case.

Finally, when the search cost density is decreasing ($b = 1.25$), an increase in search costs results in higher prices, lower consumers surplus (conditional and unconditional) and lower welfare.

5 Conclusions

This paper has studied models of price competition under search cost heterogeneity and has revisited the question how an increase in search costs affects the level of prices. Traditional consumer search models have typically assumed that all consumers search at least once in equilibrium. By doing so, the standard literature has neglected an important role of the price mechanism, namely, that the price ought to affect the number of consumers who choose to search for a product in the first place. This assumption cannot easily be reconciled with the idea that search costs, to the extent

	$b = 0.50$			$b = 1.00$			$b = 1.25$		
	$\beta = 8$	$\beta = 9$	$\beta = 10$	$\beta = 8$	$\beta = 9$	$\beta = 10$	$\beta = 8$	$\beta = 9$	$\beta = 10$
μ_0	0.800	0.823	0.842	0.622	0.664	0.697	0.541	0.591	0.630
μ_1	0.131	0.116	0.103	0.241	0.214	0.193	0.287	0.257	0.232
μ_2	0.032	0.029	0.026	0.064	0.057	0.051	0.080	0.071	0.064
μ_3	0.013	0.012	0.011	0.027	0.024	0.022	0.034	0.030	0.027
μ_4	0.007	0.006	0.006	0.014	0.013	0.011	0.018	0.016	0.014
μ_5	0.004	0.004	0.003	0.008	0.007	0.007	0.011	0.009	0.008
μ_6	0.003	0.002	0.002	0.005	0.005	0.004	0.007	0.006	0.005
μ_7	0.002	0.002	0.001	0.004	0.003	0.003	0.005	0.004	0.004
μ_8	0.001	0.001	0.001	0.003	0.002	0.002	0.003	0.003	0.003
μ_9	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002
μ_{10}	0.005	0.004	0.004	0.010	0.009	0.008	0.012	0.011	0.010
μ_2/μ_1	0.247	0.249	0.251	0.267	0.267	0.267	0.278	0.277	0.275
μ_3/μ_1	0.103	0.104	0.105	0.113	0.113	0.113	0.118	0.118	0.117
μ_4/μ_1	0.054	0.054	0.055	0.059	0.059	0.059	0.062	0.062	0.061
μ_5/μ_1	0.032	0.032	0.032	0.035	0.035	0.035	0.037	0.037	0.036
μ_6/μ_1	0.021	0.021	0.021	0.023	0.023	0.023	0.024	0.024	0.023
μ_7/μ_1	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016	0.016
μ_8/μ_1	0.010	0.010	0.010	0.011	0.011	0.011	0.012	0.012	0.012
μ_9/μ_1	0.008	0.008	0.008	0.008	0.008	0.008	0.009	0.009	0.009
μ_{10}/μ_1	0.037	0.037	0.037	0.040	0.040	0.040	0.042	0.041	0.041
$E[p]$	7.118	7.100	7.086	6.973	6.973	6.973	6.894	6.905	6.913
$\underline{p}$	3.433	3.409	3.390	3.240	3.240	3.240	3.139	3.152	3.163
PS	1.313	1.155	1.032	2.409	2.141	1.927	2.873	2.569	2.323
CS	0.683	0.608	0.548	1.355	1.207	1.087	1.687	1.502	1.354
$CS/(1-\mu_0)$	3.4132	3.4401	3.4612	3.583	3.588	3.593	3.676	3.669	3.664
Total Welfare	1.996	1.763	1.580	3.764	3.348	3.015	4.560	4.072	3.677

Table 3: Equilibrium search intensities for Kumaraswamy distribution ($a = 1$)

that they are related to consumer demographics such as income, age, marital status etc., might be very heterogeneous. In this paper we have shown that recognising this role of the price turns out to be critical for our understanding of the effect of higher search costs on prices and profits. The main results of the paper have been on characterising conditions of search cost distributions under which higher search costs result in lower prices.

Allowing for search cost heterogeneity, besides being more realistic, allows for prices to increase or decrease when search costs go up. We have identified a critical property of search cost distributions that play a decisive role, namely, the elasticity of the search cost distribution with respect to the parameter that shifts it. When this elasticity is increasing in search costs, an increase in search frictions affects consumers with higher search costs more strongly than it affects consumers with low search costs. This makes demand more elastic and correspondingly prices decrease. We have shown that these insights are quite robust since they hold no matter whether products are differentiated or homogenous and irrespective of whether consumers search sequentially or non-sequentially.

Appendix

Proof that a candidate equilibrium price $p^* \in [r, p^m]$ exists in the non-sequential search N-firm model with differentiated products.

Recall that the FOC (43) is given by

$$\mu_1(p^*)\left[1 - F(p^*) - (p^* - r)f(p^*)\right] + \sum_{k=2}^{N} k\mu_k(p^*) \int_{p^*}^{\overline{\varepsilon}} F(\varepsilon)^{k-1} f(\varepsilon) d\varepsilon$$
$$- (p^* - r)\sum_{k=2}^{N} k\mu_k(p^*) \left(\int_{p^*}^{\overline{\varepsilon}} (k-1)F(\varepsilon)^{k-2} f(\varepsilon)^2 d\varepsilon + F(p^*)^{k-1} f(p^*)\right) = 0.$$

Note that when we set $p^* = r$, the LHS of this equation is strictly positive. We now show that when we set $p^* = p^m$, then it is strictly negative, which implies that there exists a candidate equilibrium price $p^* \in [r, p^m]$.

Since the monopoly price p^m satisfies $1 - F(p^m) - (p^m - r)f(p^m) = 0$, when we evaluate the LHS of the FOC at p^m we obtain:

$$\sum_{k=2}^{N} \mu_k(p^*)\left[1 - F(p^m)^k\right] - (p^m - r)\sum_{k=2}^{N} k\mu_k(p^*) \left(\int_{p^m}^{\overline{\varepsilon}} (k-1)F(\varepsilon)^{k-2} f(\varepsilon)^2 d\varepsilon + F(p^m)^{k-1} f(p^m)\right), \tag{72}$$

where we have used the fact that $\int_{p^m}^{\overline{\varepsilon}} F(\varepsilon)^{k-1} f(\varepsilon) d\varepsilon = 1 - F(p^m)^k$.

We now claim (72) is negative. To show it, we first observe that

$$1 - F(p^m)^k = (1 - F(p^m))\sum_{j=0}^{k-1} F(p^m)^j = (p^m - r)f(p^m)\sum_{j=0}^{k-1} F(p^m)^j,$$

where we have used again the monopoly pricing rule, and write (72) as follows:

$$(p^m - r)\left\{f(p^m)\sum_{k=2}^{N} \mu_k(p^*)\frac{1 - F(p^m)^k}{1 - F(p^m)} - \sum_{k=2}^{N} k\mu_k(p^*)\left(\int_{p^m}^{\overline{\varepsilon}} (k-1)F(\varepsilon)^{k-2} f(\varepsilon)^2 d\varepsilon + F(p^m)^{k-1} f(p^m)\right)\right\}.$$

Putting terms together, this simplifies to

$$(p^m - r)\sum_{k=2}^{N} \mu_k(p^*)\left\{f(p^m)\left[\frac{1 - F(p^m)^k}{1 - F(p^m)} - kF(p^m)^{k-1}\right] - k\int_{p^m}^{\overline{\varepsilon}} (k-1)F(\varepsilon)^{k-2} f(\varepsilon)^2 d\varepsilon\right\}.$$

We now note that the expression within curly brackets is increasing in p^m. In fact, its derivative is equal to

$$f'(p^m)\left[\frac{1 - F(p^m)^k}{1 - F(p^m)} - kF(p^m)^{k-1}\right] + f^2(p^m)\left[\frac{1 - F(p^m)^k - kF(p^m)^{k-1}}{(1 - F(p^m))^2}\right]$$
$$= \frac{1 - F(p^m)^k - kF(p^m)^{k-1}(1 - F(p^m))}{(1 - F(p^m))^2}\left[f'(p^m)(1 - F(p^m) + f^2(p^m)\right] > 0$$

where the sign follows by log-concavity of f. Since it is increasing in p^m and it is equal to zero when we set $p^m = \overline{\varepsilon}$, we conclude it is always negative. This shows that a candidate equilibrium price $p^* \in [r, p^m]$ exists in the non-sequential search N-firm model with differentiated products.

Proof of Proposition 6. In order to prove the result we compute the second order derivative of the payoff function and show that for the case $N = 3$ it is strictly negative so the payoff is strictly concave. The proof also serves to illustrate the difficulties to prove strict concavity when $N \geq 4$.

Let us denote the demand of a firm i charging price p_i as

$$d_i(p_i, p^*) = \frac{\mu_1(p^*)}{N}(1 - F(p_i)) + \sum_{k=2}^{N} \frac{k\mu_k(p^*)}{N} \int_{p_i}^{\overline{\varepsilon}} F(\varepsilon - (p_i - p^*))^{k-1} f(\varepsilon) d\varepsilon$$

The first order derivative of demand with respect to own price is:

$$\frac{\partial d_i(p_i, p^*)}{\partial p_i} = -\frac{\mu_1(p^*)}{N} f(p_i) - \sum_{k=2}^{N} \frac{k\mu_k(p^*)}{N} \left(\int_{p_i}^{\overline{\varepsilon}} (k-1) F(\varepsilon - p_i + p^*)^{k-2} f(\varepsilon - p_i + p^*) f(\varepsilon) d\varepsilon + F(p^*)^{k-1} f(p_i) \right)$$

and the second order derivative is:

$$\frac{\partial^2 d_i(p_i, p^*)}{\partial p_i^2} = -\frac{\mu_1(p^*)}{N} f'(p_i) - \sum_{k=2}^{N} \frac{k\mu_k(p^*)}{N} \left[F(p^*)^{k-1} f'(p_i) - (k-1) F(p^*)^{k-2} f(p^*) f(p_i) \right]$$
$$+ \sum_{k=2}^{N} \frac{k\mu_k(p^*)}{N} \left(\int_{p_i}^{\overline{\varepsilon}} (k-1) \left[(k-2) F(\varepsilon - p_i + p^*)^{k-3} f(\varepsilon - p_i + p^*)^2 + F(\varepsilon - p_i + p^*)^{k-2} f'(\varepsilon - p_i + \right.\right.$$

For the case of the uniform distribution, these derivatives simplify to:

$$\begin{aligned}
\frac{\partial d_i(p_i, p^*)}{\partial p_i} &= -\frac{\mu_1(p^*)}{N} \frac{1}{\overline{\varepsilon}} - \sum_{k=2}^{N} \frac{k\mu_k(p^*)}{N} \left(\int_{p_i}^{\overline{\varepsilon}} (k-1) \frac{(\varepsilon - p_i + p^*)^{k-2}}{\overline{\varepsilon}^k} d\varepsilon + \frac{p^{*k-1}}{\overline{\varepsilon}^k} \right) \\
&= -\frac{\mu_1(p^*)}{N} \frac{1}{\overline{\varepsilon}} - \sum_{k=2}^{N} \frac{k\mu_k(p^*)}{N} \frac{(\overline{\varepsilon} - p_i + p^*)^{k-1}}{\overline{\varepsilon}^k}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial^2 d_i(p_i, p^*)}{\partial p_i^2} &= \sum_{k=2}^{N} \frac{k\mu_k(p^*)}{N} \frac{(k-1)p^{*k-2}}{\overline{\varepsilon}^k} + \sum_{k=2}^{N} \frac{k\mu_k(p^*)}{N} \left(\int_{p_i}^{\overline{\varepsilon}} (k-1)(k-2) \frac{(\varepsilon - p_i + p^*)^{k-3}}{\overline{\varepsilon}^k} d\varepsilon \right) \\
&= \sum_{k=2}^{N} \frac{k(k-1)\mu_k(p^*)}{N} \frac{(\overline{\varepsilon} - p_i + p^*)^{k-2}}{\overline{\varepsilon}^k}
\end{aligned}$$

The second order condition of the maximization problem is then:

$$\begin{aligned}
\frac{\partial^2 \pi_i(p_i, p^*)}{\partial p_i^2} &= 2\frac{\partial d_i(p_i, p^*)}{\partial p_i} + (p_i - r) \frac{\partial^2 d_i(p_i, p^*)}{\partial p_i^2} \\
&= -\frac{2\mu_1(p^*)}{N\overline{\varepsilon}} - \sum_{k=2}^{N} \frac{2k\mu_k(p^*)}{N} \frac{(\overline{\varepsilon} - p_i + p^*)^{k-1}}{\overline{\varepsilon}^k} + (p_i - r) \sum_{k=2}^{N} \frac{k(k-1)\mu_k(p^*)}{N} \frac{(\overline{\varepsilon} - p_i + p^*)^{k-2}}{\overline{\varepsilon}^k} \\
&= -\frac{2\mu_1(p^*)}{N\overline{\varepsilon}} + \frac{1}{N} \sum_{k=2}^{N} \frac{k\mu_k(p^*)}{N} \frac{(\overline{\varepsilon} - p_i + p^*)^{k-2}}{\overline{\varepsilon}^k} \left[(p_i - r)(k-1) - 2(\overline{\varepsilon} - p_i + p^*) \right] \qquad (73)
\end{aligned}$$

For the $N = 3$ case we obtain

$$\frac{\partial^2 \pi_i(p_i, p^*)}{\partial p_i^2} = -\frac{2\mu_1}{3\bar{\varepsilon}} + \frac{2\mu_2}{9\bar{\varepsilon}^2}\left[(p_i - r) - 2(\bar{\varepsilon} - p_i + p^*)\right] + \frac{\mu_3}{3}\frac{(\bar{\varepsilon} - p_i + p^*)}{\bar{\varepsilon}^3}\left[2(p_i - r) - 2(\bar{\varepsilon} - p_i + p^*)\right]$$

The sign of this second order derivative depends on the sign of the expressions in squared brackets. Take first the expression $(p_i - r) - 2(\bar{\varepsilon} - p_i + p^*)$ and notice that it increases in p_i. If we set p_i equal to the monopoly price $(\bar{\varepsilon} + r)/2$, which is the maximum price, we get $-(\bar{\varepsilon} - r) - 2p^*) < 0$. Since it is increasing in p_i and at teh maximum price it is negative, we conclude $(p_i - r) - 2(\bar{\varepsilon} - p_i + p^*)$ is negative for all $p_i > p^*$. Take now the second expression in squared brackets $2(p_i - r) - 2(\bar{\varepsilon} - p_i + p^*)$ and notice that it is also increasing in p_i. Setting p_i equal to the monopoly price we get $-2p^* < 0$ so, by the same argument, $(p_i - r) - 2(\bar{\varepsilon} - p_i + p^*)$ is negative for all $p_i > p^*$. We conclude that for $N = 3$, the payoff function is strictly concave so there exists a unique equilibrium that is symmetric.

For $N \geq 4$, the summation in (73) involves additional terms and some of these terms are not always negative; because of this, we can no longer prove that the payoff function is stricly concave. However, using numerical methods we have been able to check that the payoff function is quasi-concave so we are quite confident that the equilibrium exists and is unique. ■

Proof of Proposition 8. Since $x_0 = G(c_0)$ and $x_1 = G(c_1)$, we have

$$x_0 = G\left(\theta - \theta \int_0^1 \frac{x_0 - x_1}{x_0 - x_1 + 2x_1 u} du\right);$$
$$x_1 = G\left(\theta \int_0^1 \frac{(x_0 - x_1)(1 - 2u)}{x_0 - x_1 + 2x_1 u} du\right).$$

An equilibrium of the model is given by a solution to

$$H(y) \equiv yG(\theta - \theta(1 - y) I(y)) - G(\theta(1 - y) J(y)) = 0,$$

where

$$I(y) = \int_0^1 \frac{1}{1 - y + 2yu} du = \frac{\log(1 + y) - \log(1 - y)}{2y};$$
$$J(y) = \int_0^1 \frac{1 - 2u}{1 - y + 2yu} du = \frac{\log(1 + y) - \log(1 - y) - 2y}{2y^2}.$$

We note that for $y = 0$ and $y = 1$ we have

$$H(0) = 0 \cdot G(c_0(0)) - G(c_1(0)) = -G(0) = 0,$$
$$H(1) = G(c_0(1)) - G(c_1(1)) = G(1) - G(0) = G(1) > 0.$$

Consider now the value of $\partial H(y)/\partial y$ at $y=0$. Since $0=c_1(0)=c_0(0)$ and $c_1'(0)>0$ we have

$$\frac{\partial H(0)}{\partial y}=G(0)-G'(0)\,c_1'(0)=-G'(0)\,c_1'(0)<0.$$

Given these three observations (i.e. $H(0)=0, H(1)>0$ and $\partial H(0)/\partial y<0$), we conclude that there exists at least one equilibrium.

We now prove the part on uniqueness of equilibrium. Let $G(c)=(c/\beta)^a$ for some $a>0$ with support $[0,\beta]$. From equation (54), since the case $y=0$ is not interesting and $G(c_0(y))>0$ for $y>0$, it is sufficient to prove that the equation

$$y=\frac{G(c_1(y))}{G(c_0(y))} \tag{74}$$

has a unique solution. Since the LHS of (74) is increasing in y, it suffices to show that the RHS decreases in y. Let $h(y)$ denote the RHS of (74):

$$h(y)=\frac{\left(\frac{c_1(y)}{\beta}\right)^a}{\left(\frac{c_0(y)}{\beta}\right)^a}=\frac{c_1(y)^a}{c_0(y)^a}$$

The derivative of $h(y)$ is

$$\begin{aligned}\frac{dh(y)}{dy}&=\frac{a\frac{dc_1(y)}{dy}c_1^{a-1}(y)\,c_0^a(y)-ac_1^a(y)\,\frac{dc_0(y)}{dy}c_0^{a-1}(y)}{c_0^{2a}(y)}\\&=\frac{ac_1^{a-1}(y)\,c_0^{a-1}(y)}{c_0^{2a}(y)}\left(\frac{dc_1(y)}{dy}c_0(y)-c_1(y)\,\frac{dc_0(y)}{dy}\right).\end{aligned}$$

Since

$$\begin{aligned}\frac{dc_1(y)}{dy}&=\frac{2y(2+y)-(1+y)(2-y)\ln\frac{1+y}{1-y}}{2y^3(1+y)},\\\frac{dc_0(y)}{dy}&=\frac{-2y+(1+y)\ln\frac{1+y}{1-y}}{2y^2(1+y)},\end{aligned}$$

we obtain that

$$\frac{dc_1(y)}{dy}c_0(y)-c_1(y)\,\frac{dc_0(y)}{dy}=$$

$$=4y^2(1+2y)+2y(1+y)(2-y)\ln\frac{1-y}{1+y}+\left(1-y^2\right)(1-y)\ln^2\frac{1-y}{1+y}.$$

This expression is negative for $0<y<1$, so $dh(y)/dy<0$, and therefore, the equilibrium is unique.

■

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