Winning back the unfaithful while exploiting the loyal

Retention offers and heterogeneous switching costs

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Very Preliminary

Abstract

We study a market with behavior-based price discrimination and heterogeneous switching costs. If consumers indicate an intention to switch suppliers, their current supplier can make a retention offer, discounting the price they would otherwise pay. In equilibrium, only consumers with low switching costs will make an effort to switch. Hence, retention offers effectively allow firms to price discriminate against consumers with high switching costs. The possibility of retention offers hurts consumers with high, but benefits those with low switching costs. Welfare deceases as consumers spend more on switching costs and are matched less efficiently.

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JEL classification: D11, D43, L13.

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1 Introduction

In recent years firms have increased the practice of "behavior-based" price discrimination in many markets as the information required for it has become increasingly available. With online shopping becoming more popular and the introduction of the smartphone, firms can optimize their (pricing) strategies even further as they are able to obtain more information than ever on consumer behavior. A particular interesting form of behavior-based price discrimination is the practice of retention offers. In markets for e.g. mobile phone subscriptions or insurance, firms actively try to win consumers back who have the intention to switch suppliers by providing them with a better deal than their more loyal customers are offered. Although these pricing strategies obviously affect profits, consumer welfare and efficiency of the market, there exists virtually no literature which investigates their precise impact. With this paper we fill this void.

Although the practice of retention offers provides the firm with an additional tool to undercut the deals of the competition, we find that it facilitates a price raise for certain groups of consumers. Consumers who signal their intention to switch and who are won back by the retention offer of the firm indeed pay a lower price. Only consumers with relatively low switching costs find such an action worthwhile and to keep them the firm has to charge a low price. On the other hand, consumers that even do not consider switching or consumers that end up at the competition will both pay a higher price than in the absence of retention offers. The latter group of consumers signals, with their intention to switch, that they have relatively strong preferences towards the brand of the competitor, which the competitor can exploit. Consumers that stay loyal to a firm are charged a higher price than in a setting without retention offers because they signal that they have high switching costs and strong preference for the brand of the firm and are therefore unlikely to go to the competition.

These price comparisons between situations with and without retention offers lead naturally to another finding that firms will only make these offers when the group with relatively high switching costs is sufficiently large or when their switching costs are relatively high. The explanation is that only in such a situation firms can increase profits by making retention offers. In addition we find that the introduction of retention offers reduces consumer and total welfare because consumers spend more on switching costs and it leads to less efficient matching between consumers and suppliers. A final, yet remarkable, result we obtain is that increases in switching costs of one group of consumers might be beneficial to the other consumer group, even in a framework without retention offers. The reason is that the incumbent firm wants to exploit this increase in switching costs, while the competitor will lower its price to generate sufficient demand. For the group that was unaffected by the change in switching costs it therefore becomes more attractive to switch, and consumers with relatively strong preferences for the competitor might be better off in the end.

Most closely related to our paper is probably the work by Gnutzmann (2012). In that paper the author extends Chen (1997) by introducing retention offers into a market with homogeneous products and finds results that oppose ours: retention offers reduce prices. The key difference with our paper is that we consider a market for heterogeneous products. This difference is an important driver for the discrepancies in results. Once consumers signal they intend to switch in the model of Gnutzmann (2012) they become more homogenous since they already sunk some switching costs, which reduces equilibrium prices. However, once a consumer indicates her intention (not) to switch in our model she reveals her preferences for a particular product as well. This is exploited by the firms and drives prices up. Another difference with Gnutzmann (2012) and our work is that in his paper there are no consumers who pretend to switch. All consumers that initiate the switching procedure in his model will decide to switch when they do not receive a better retention offer, while in our model some of these consumers will actually prefer not to switch in such a situation after all. An additional difference between our work and that of Gnutzmann (2012) is that switching costs are assumed to be uniformly distributed there while we focus on the discrete case with only two possible values for these costs.

Taylor (2003) is connected to our paper as well. That paper basically extends Chen (1997) by considering a market with homogenous goods and allowing for an arbitrary number of firms, an arbitrary number of periods and a more general distribution of switching costs. Taylor (2003) finds that consumers may change suppliers in order to establish a reputation as switcher and receive better future offers. This result is obviously related to our finding that consumers with low switching costs may pretend to switch to get a better deal. Taylor (2003) considers a case in which switching costs in each period are correlated and some consumers have on average higher switching costs than others (high and low types). There he finds that these low-types get better offers from their current supplier while high-types receive better offers from the competition. This is in contrast to our finding that only consumers who threaten to switch receive a better offer while all others are offered worse deals.

Our work connects to several other parts of the literature as well. First, it is closely related to Fudenberg and Tirole (2000). They consider a market with heterogeneous products on which firms practice pricing based upon purchase histories. Gehrig, Shy and Stenbacka (2007) evaluate this behavior-based price discrimination from an antitrust point-of-view. We generalize their models in two ways by allowing for both heterogeneous switching costs and retention offers. Second, the work on price-matching basically considers retention offers that are restricted to be better than that of the competitor, in that sense there is a connection with our paper. Arbatskaya, Hviid and Shaffer (2004) and Corts (1997) consider such price-matching practices. Finally, our work is obviously embedded in the vast literature of switching costs, of which excellent overviews can be found in Klemperer (1995) and Farrell and Klemperer (2007).

This paper is organized as follows. First, section 2 introduces the model. Section 3 considers a benchmark in which there are no retention offers. This model has been studied before by for instance Fudenberg and Tirole (2000) and Gehrig, Shy and Stenbacka (2007), but we now allow for heterogeneous switching costs. This leads some new and interesting insights which are summarized in section 3.1. We then turn to the main focus of our study in section 4 by solving Period 2 of the general model with retention offers and presenting some results. In section 5 we solve Period 1 of the model. Subsequently we present some results in Section 6 and conclude with Section 7.

2 The model

The world consists of consumers with preferences described by a Hotelling line of length one and two firms, labeled A and B. Consumers are uniformly distributed along the line and firms A and B are respectively located at the far left and far right of the line. Firms face marginal costs c. Travel costs for the consumers are equal to $\tau = 1$. This means that a consumer located at x obtains the following level of utility from buying from firm A or B respectively:

$$r - x - p_A$$

$$r - (1 - x) - p_B.$$

The game consists of 2 periods. Consumers have unit demand in each period. We assume r is sufficiently large so that every consumer will buy a product each period in the equilibrium. If a consumer buys in period 1 from firm A she can switch to firm B at a heterogeneous costs z (and vice versa). z can take on two possible values:

$$z = \begin{cases} z_L \text{ with probability } \lambda \\ z_H \text{ with probability } 1 - \lambda, \end{cases} \tag{1}$$

where $z^H \geq z^L$ and $\lambda \in [0, 1]$. Consumers with switching costs z_L will be referred to as low-types, consumers with switching costs z_H will referred to as high-types.[1] We make the following assumption on switching costs.

Assumption 1 $z_H \leq 1/3$.

The assumption makes sure that we arrive at an interior solution. The assumption is sufficient, but far from necessary in most situations.

[1]We restrict analysis to two possible values of z for traceability.

These switching costs and consumer locations are independent (hence, uncorrelated). Firms and consumers use the same discount factor $\delta \in [0, 1]$ for the second period. We assume consumer preferences remain the same in every period.[2]

Due to the presence of retention offers some consumers might pretend to intend to switch in order to get a better offer from the firm they bought from in the last period. Sending such a signal credibily comes at a cost. We split up the switching costs into two elements: $z_K = z_K^1 + z_K^2$, for type $K \in \{L, H\}$. We impose the following restriction:

Assumption 2 *$z_H^l \geq z_L^l$ for $l = 1, 2$.*

This means that high types constantly have switching costs at least as high as low types. If $z_K^1 > 0$ consumes incur some costs or invest some effort before the actual switch occurs, so, these costs are sunk even if a consumer threatens to switch. The timing of the game is as follows.

- Stage 1.a: Firms simultaneously set first period prices p_i^1, $i = A, B$.
- Stage 1.b: Consumers decide which firm to buy from in Period 1.
- Stage 2.a: Firms simultaneously set second period prices. Firm $i = A, B$ can identify previous customers thus it sets prices $p_{i,A}^2$ for consumers who previously bought from firm A and $p_{i,B}^2$ for consumers who previously bought from B. We will use the term *loyalty price* for prices p_{AA}^2 and p_{BB}^2. Prices p_{BA}^2 and p_{AB}^2 we be referred to as *switching prices*.
- Stage 2.b: Consumers have the opportunity signal that they will switch given prices $p_{i,A}^2$ and $p_{i,B}^2$. To make this so-called *signal decision* a consumer has to incur cost or invest effort z^1 in order to get an offer from the firm she did not buy from in period 1.[3]

[2] Although switching costs figure in our model, consumers are perfectly informed about the location and prices firms set at the start of each periods, thus, search costs do not figure in our model.

[3] For instance, a consumer might want to switch to another insurer or phone subscription. Although the consumer already knows the exact offer of the new insurer or provider (price and package) she might have to fill in several forms in order to switch. Note that

- Stage 3.a: The firm has the option to make an offer to retain its consumers if the consumer made the signal decision, this offer is denoted by p_i^3. We will label p_A^3 and p_B^3 as *retention prices.* [4]

- Stage 3.b: Consumers decide which firm to buy from in stage 3, i.e. Period 2, if they decide to switch they incur z^2.

Stage 1.a and 1.b form together Period 1, stage 2.a, 2.b, 3.a and 3.b form Period 2.

Switching costs in stage 2 may for instance the action of filling in forms. The stage 3 switching costs are incurred after the purchase. These might include (temporary) usage of another cellphone number. Number portability for instance will reduce these costs. Hence, markets for mobile phone subscription are characterized by high second stage switching costs, but low third stage. On the other hand, if one buys a new type of phone (or start using a new operating system) stage 2 switching costs tend to be low, but stage 3 costs relatively high because one has to learn how to use the new phone (or system). Note that phone companies sometimes bundle subscription and phone deals, which yields a hybrid combination of these two types of switching costs.

For clarity we present the segmentation of the market when firms make no retention offers in Figure 1. Here, $\hat{x}_K^1$ is the point where consumers are indifferent between firm A and B in Period 1, and consumers of type $K \in \{L, H\}$ with $x < \hat{x}_K^1$ ($x > \hat{x}_K^1$) buy from firm A (B) in Period 1. In Period 2 there are two disjoint market segments A and B that are separated at $\hat{x}_K^1$ for each type of consumer and firms fight for consumers on these segments with different prices. Consumers that are located closer to $\hat{x}_K^1$ than $\hat{x}_{JK}^2$,

this *signal decision* is not the same as a purchase decision: the consumer might still change her mind and stay with the current insurer or provider. For instance, in the Netherlands consumers only switch from insurer at the first of January, so a consumer can register at several insurers, but the one she registered last with will be her new insurer. In the case of a mobile phone subscription the current provider is notified when the customer intends to leave and has the opportunity to make a retention offer before the new subscription starts.

[4] We assume $p_{B,A}^2$ is known by firm A at this stage, or its equilibrium value can be derived. A similar assumption holds for $p_{A,B}^2$ and firm B.

for $J \in \{A, B\}$ and $K \in \{L, H\}$, will make the *signal decision*. When firms make no retention offers $\hat{x}^2_{JK}$ coincides with the consumers that are indifferent between offers p^2_{AA} and p^2_{BA} on segment A and offers p^2_{BB} and p^2_{AB} on segment A. Note that $|\hat{x}^2_{JL} - \hat{x}^1_L| > |\hat{x}^2_{JH} - \hat{x}^1_H|$ because high and low types face the same prices on a segment and high types have to pay higher switching costs.

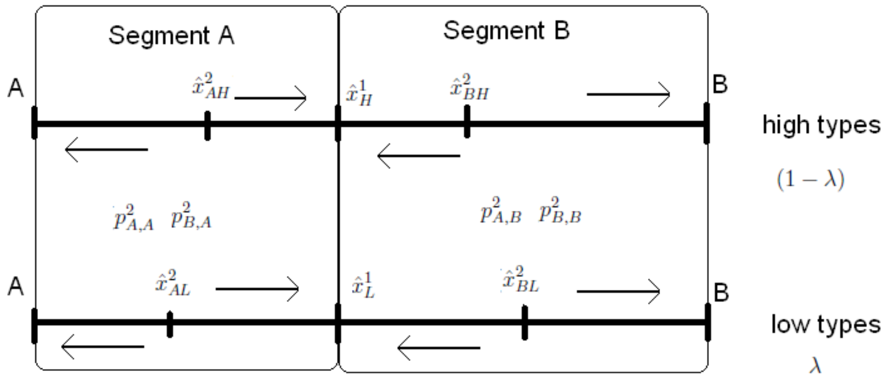

Figure 1: The segmentation of the market in Period 2 when firms make no retention offers.

This model is basically that of Fudenberg and Tirole (2000) with switching costs explicitly modeled, which is described in Gehrig et all (2007) as well. When firms are allowed to make retention offers two changes take place. First, consumers in the intervals $[\hat{x}^2_{AK}, \hat{x}^1_K]$ and $[\hat{x}^1_K, \hat{x}^2_{BK}]$ get a retention offer, and a fraction of them closest to the firm they bought from in the first period might be retained. Second, some consumers might realize that they will receive a retention offer and might adjust their behavior accordingly.[5] In particular, more consumers might want to make to make the signal decision in order to receive a better offer of their current supplier. We will denote the consumer who made the signal decision and is indifferent between the switching and retention offer by $\hat{x}^3_{JK}$. $\hat{x}^2_{JK}$ with $J \in \{A, B\}$ and $K \in \{L, H\}$ still denotes the consumer who is indifferent between making the signal decision and staying loyal to her current supplier. Hence, $\hat{x}^2_{JK}$ will be at least as far from $\hat{x}^1_K$ located as $\hat{x}^3_{JK}$.

[5]Throughout the paper we assume consumers are forward-looking.

The model collapses to a benchmark model in which firms make no retention offers, as treated in Fudenberg and Tirole (2000) and Gehrig et all (2007), in two situations. First, when $z_K^1 = 0$ for $K \in \{L, H\}$ since in such a situation every consumer will make the signal decision and all will buy at the retention price when staying loyal to the current supplier. Second, even when firms are allowed to make retention offers, they might refrain from it because it is unprofitable. Below we show that this is an equilibrium strategy when the group with high switching costs is relatively small or their switching costs in the third stage are small.

As mentioned, our model connects to Gnutzmann (2012) as well. Contrary to that paper, we do not impose $z_K^2 = \alpha z_K = \alpha(z_K^1 + z_K^2)$. The other differences between the papers were already mentioned: heterogeneous products instead of homogeneous ones and binary instead of uniformly distributed switching costs.

Before we continue it is convenient to define the following:

$$\tilde{z} = (1 - \lambda)z_H + \lambda z_L, \tag{2}$$

and

$$\hat{x}^1 = \lambda(1 - \lambda)\hat{x}_H^1 + \hat{x}_L^1. \tag{3}$$

In a similar fashion we define:

$$\tilde{z}^2 = \lambda z_L^2 + (1 - \lambda)z_H^2, \tag{4}$$

$$\tilde{z}^1 = \lambda z_L^1 + (1 - \lambda)z_H^1. \tag{5}$$

This implies $\tilde{z} = \tilde{z}^1 + \tilde{z}^2$. $\tilde{z}$ is increasing in both z_H and z_L. In Period 2 not only switching costs prevent consumers to switch, but the firm has in addition the possibility to make a retention offer. Exactly because of this we expect, and show below, prices $p_{i,i}^2$ for $i = A, B$ will be higher when firms make retention offers. In addition we derive the novel result that $p_{i,j}^2$ is higher when retention offers are possible, for $i \neq j$.

In section 4 we turn to the situation of interest in which firms get to make retention offers. In the upcoming section we establish the prevailing equilibrium prices for the benchmark case without such offers to allow for a comparison and in addition derive some results. These findings are not the main focus of this paper but we present them since these, to the best of our knowledge, have not been established in the literature and are quite intriguing. This benchmark, in which firms refrain from making retention offers, provides actually an equilibrium strategy in a certain parameter space of the game above in which firms are allowed to make these retention offers.

3 Benchmark Model

Equilibrium prices can be derived by following steps similar to those in Gehrig, Shy and Stenbacka (2007). They consider this benchmark model but with homogenous switching costs. However, they allow the switching costs to differ in another respect: switching from A to B might be more costly than switching from B to A (or vice versa). Let us first consider the case in which no single firm is too dominant: $\hat{x}^1 \in [\frac{\tilde{z}+1}{4}, \frac{3-\tilde{z}}{4}]$. We find that in that case period 2 equilibrium prices on segment A are given by:

$$p^2_{A,A} = c + \frac{1}{3}(2\hat{x}^1 + 1 + \tilde{z}) \quad \text{and} \quad p^2_{B,A} = c + \frac{1}{3}(4\hat{x}^1 - 1 - \tilde{z}), \tag{6}$$

and for market segment B these are given by:

$$p^2_{A,B} = c + \frac{1}{3}(4[1-\hat{x}^1] - 1 - \tilde{z}) \quad \text{and} \quad p^2_{B,B} = c + \frac{1}{3}(2[1-\hat{x}^1] + 1 + \tilde{z}). \tag{7}$$

Note that switch costs increase the equilibrium price for the firm that tries to retain consumers and decreases the equilibrium price of firms that try to steal consumers on the segment.

When $\hat{x}^1 \notin [\frac{\tilde{z}+1}{4}, \frac{3-\tilde{z}}{4}]$ a single firm is rather dominant after Period 1, we call this the case of Strong Dominance. Actually, a firm is in such a case is so dominant that on the rival's segment it fails to steal customers since they are located too far away. This firm will set a switching-price equal to marginal costs and serve no new customers in Period 2. More in particular, if $\hat{x}^1 < \frac{\tilde{z}+1}{4}$ prices on segment A will be

$$p^2_{A,A} = c + \frac{\tilde{z}+1}{2} \quad \text{and} \quad p^2_{B,A} = c, \tag{8}$$

while prices are as in the non-dominant case on segment B. Similarly, if $\hat{x}^1 > \frac{3-\tilde{z}}{4}$ prices on segment B will be

$$p^2_{B,B} = c + \frac{\tilde{z}+1}{2} \quad \text{and} \quad p^2_{A,B} = c, \tag{9}$$

while prices are as in the non-dominant case on segment A.

3.1 Benchmark Results

Gehrig, Shy and Stenbacka (2007) have studied a version of our benchmark model and they explore conditions under which a dominant firm remains dominant in the market or the entrant becomes dominant. However, to the best of our knowledge, the current literature has not studied the impact of switching costs in a setting where there is pricing based on purchasing history, or the impact of the introduction of pricing based on purchasing history in a situation where there are switching costs. We present here some results on these issues, and in addition investigate the effects of heterogeneous switching costs. Proofs of these findings are available from the author upon request.

Proposition 1 *Increases of switching costs z_K for type $K \in \{L, H\}$ lead in Period 2 to, ceteris paribus:*

1. *a lower amount of switchers of type K,*
2. *more switchers of the other type (K^-),*
3. *a lower total amount of switchers,*
4. *higher firm profits,*
5. *lower welfare of consumers of type K,*

6. *lower consumer welfare of type K^- when they were plan to remain loyal before the change,*

7. *higher consumer welfare of type K^- when they were plan to switch before the change.*

From point 3 immediately follows that there are less switchers in the current model than in a model without switching costs. Point 2 can be explained by the fact that if switching costs of a group increase, the difference in equilibrium prices between firm A and B on a particular segment increases. Therefore it becomes more attractive to switch for the group of consumers of which the switching costs did not change, because the cost of doing so remained the same for that group.

We can derive first period equilibrium prices as well. We found the following equilibrium prices:

$$p_A^1 = p_B^1 = c + 1 - \frac{2\delta}{3}\tilde{z}, \tag{10}$$

when consumers are myopic, and

$$p_A^1 = p_B^1 = c + 1 + \frac{\delta}{3} - \frac{2}{3}\tilde{z}\delta, \tag{11}$$

when consumers are forward-looking. Being forward-looking is now harmful to consumers, a result that was found by Fudenberg & Tirole (2000) as well.

We now compare prices in the current benchmark model with three other models listed below to evaluate the impact of the introduction of switching costs, behavior-based pricing or a combination of the two.

- A model without switching costs and without pricing based on purchasing history.

- A model with switching costs and without pricing based on purchasing history.

- A model without switching costs and with pricing based on purchasing history.

We find that second period prices in the current model, in which there are both switching costs and behavior based pricing, are lower than in all these models except for loyal customers in the model with discriminative pricing but no switching costs. First period prices on the other hand are higher than in these benchmarks, except for the one without switching costs but with discriminative pricing. The findings hold for both forward-looking and myopic consumers. These observations lead to results that are in line with the literature and are summarized in Proposition 2.

Proposition 2 *First, switching costs tend to reduce prices in Period 1 and firms reap the benefits from it by charging a higher price for consumers that are locked in in Period 2. Second, pricing based upon purchasing history leads to lower prices in Period 2 but higher prices in Period 1. In addition it is true that the possibility of pricing based upon purchasing history leads to lower firm profits, higher consumer welfare and lower total welfare. Increases in switching costs affect consumer and total welfare negatively.*

4 Retention offers: solving period 2

Since we have two periods in the model we will use backward induction for our analysis. We analyze segment A of the market, by symmetry a similar analysis can be conducted for segment B. Since we assume that making a retention offer is costless, a consumer that makes the signal decision will always receive a retention offer p_A^3.

We first solve for $\hat{x}_{AK}^3$: the consumer who made the *signal decision* and is indifferent between firms A and B. Hence, z_K^1 are sunk costs for for this consumer (of type K) and we have:

$$r - \hat{x}_{AK}^3 - p_A^3 = r - [1 - \hat{x}_{AK}^3] - p_{B,A}^2 - z_K^2 \tag{12}$$

$$\Leftrightarrow$$

$$\hat{x}_{AK}^3 = \frac{1 + p_{B,A}^2 - p_A^3 + z_K^2}{2}. \tag{13}$$

Similarly we find the consumer who is indifferent between making the signal decision or not ($\hat{x}^2_{AK}$):

$$r - \hat{x}^2_{AK} - p^2_{A,A} = r - [1 - \hat{x}^2_{AK}] - p^2_{B,A} - z_K \tag{14}$$

$$\Leftrightarrow$$

$$\hat{x}^2_{AK} = \frac{1 + p^2_{B,A} - p^2_{A,A} + z_K}{2}. \tag{15}$$

Note that some consumers might pretend to switch in order to get a better offer from A. This happens whenever $p^2_{A,A} > p^3_A + z^1_K$ and in that case $\hat{x}^2_{AK}$ is not given by the expression above but equals zero. We will therefore consider three cases in this section, each considered separately in a separate subsection.

1. **Everybody pretends to switch** which happens when $p^2_{A,A} > p^3_A + z^1_H$. A sufficient condition for this to happen is $z^1_H = 0$.

2. **Nobody pretends to switch** which happens when $p^2_{A,A} < p^3_A + z^1_L$.

3. **Only low types pretend to switch** which happens when $z^1_L < p^2_{A,A} - p^3_A < z^1_H$.

Note that case 2 collapses to the benchmark model: the signal and purchase-decision coincide and consumers only make the signal decision when they indeed will buy from the other firm. No retention offers will be made then. Below we will show that this can be an equilibrium when λ is sufficiently large. The reason is that making retention offers in that case is unprofitable since there are too many low types. In addition ws show below that case 1, in which everybody pretends to switch, can be no equilibrium.

4.1 Everybody pretends to switch

When everybody makes the signal decision and either buys from firm B or from firm A at price p^3_A, not at the loyalty price, we require the following to hold:

$$p^2_{A,A} > p^3_A + z^1_H. \tag{16}$$

Proposition 3 *There is no equilibrium in which*

$$p^2_{A,A} > p^3_A + z^1_H \tag{17}$$

and where everybody makes the signal decision and nobody buys at price $p^2_{A,A}$.

Proof. Suppose (17) holds and therefore every consumer makes the signal decision. Then consumers never will buy at firm A against price $p^2_{A,A}$ because p^3_A is smaller; that is why they made the signal decision in the first place. Now suppose firm A sets a new loyalty price $p^2_{A,A} = p^3_A + z^1_H - \varepsilon$ with $\varepsilon \in (0, z^1_H)$. In that case some consumers will not make the signaling decision and they will buy against this new price $p^2_{A,A}$ from firm A instead of the smaller retention price p^3_A. Moreover, firm A obviously does not lose any consumers to firm B by this price reduction. It follows that such an action is profitable and therefore (17) cannot hold in equilibrium. ■

4.2 Nobody pretends to switch

Suppose that nobody pretends to switch, which happens when

$$p^2_{A,A} < p^3_A + z^1_L. \tag{18}$$

In section 4.3 we will see that this condition translates to:

$$z^1_L > \frac{z_H - z^2_L}{2}. \tag{19}$$

Note that given that no consumers will pretend to switch and return to firm A, prices p^2_{AA} and p^2_{BA} should be as in the benchmark without the possibility of retention offers, see the equations in (6):

$$p^2_{A,A} = c + \frac{1}{3}(2\hat{x}^1 + 1 + \tilde{z}) \quad \text{and} \quad p^2_{B,A} = c + \frac{1}{3}(4\hat{x}^1 - 1 - \tilde{z}). \tag{20}$$

Consider the case in which firm A deviates from the above to a price $p^3_A = p^2_{A,A} - z^1_L - \varepsilon$, so that $z^1_L < p^2_{A,A} - p^3_A < z^1_H$. In that case all low types that initially bought against p^2_{AA} now pretend to switch and will buy at a lower price p^3_A. Such a change will thus impact the profits on the low types negatively. However, firm A might at the same time adjust $p^2_{A,A}$ as well. Since now only high types buy against this price, firm A can increase it without losing any low types, i.e. the price-elasticity is reduced because low types buy against another price. This strategy of separating the low and high types will be considered in the next subsection. It turns out that setting $p^2_{A,A} - p^3_A \in (z^1_L, z^1_H)$ is more profitable whenever λ is sufficiently small, that is, when there are enough high types.

To figure out under which conditions there is an equilibrium in which (18) holds and nobody pretends to switch, we have to calculate the profits under this strategy and compare these to the profits found in the next subsection where $z^1_L < p^2_{A,A} - p^3_A < z^1_H$ and low types pretend to switch.

Profits (of firm A on her own segment in period 2) for the case (18) can be found by first calculating demand under this strategy by substituting (20) into (15), which yields:

$$\hat{x}^2_{AK} = \frac{1}{2} + \frac{1}{3}(\hat{x}^1 - 1 - \tilde{z}) + \frac{1}{2}z_K. \tag{21}$$

Secondly, we substitute this result along with the prices into the profit function. In this way we find profits in period 2 for firm A on her own segment under strategy (18) to be:

$$\begin{aligned} \Pi^2_A &= \left(\lambda\hat{x}^2_{AL} + (1-\lambda)\hat{x}^2_{AH}\right)\left(p^2_{A,A} - c\right) && (22)\\ &= \left(\frac{1}{2} + \frac{1}{3}(\hat{x}^1 - 1 - \tilde{z}) + \frac{1}{2}\tilde{z}\right)\frac{1}{3}(2\hat{x}^1 + 1 + \tilde{z}) && (23)\\ &= \frac{1}{18}\left(2\hat{x}^1 + 1 + \tilde{z}\right)^2. && (24) \end{aligned}$$

We will compare these to the profits found for the case considered in the next subsection.

4.3 Low types pretend to switch

The final and most interesting case we consider is is the one in which only the low types pretend to switch. This happens when:

$$z_L^1 < p_{A,A}^2 - p_A^3 < z_H^1. \tag{25}$$

In this case $\hat{x}_{AH}^2 = \frac{1+p_{B,A}^2-p_{A,A}^2+z_H}{2}$ gives the high type consumer who is indifferent between offers of A and B while all the low types pretend to switch and they contribute $\hat{x}_{AL}^3 = \frac{1+p_{B,A}^2-p_A^3+z_L^2}{2}$ to the demand of firm A. Note that no high type buys at a price p_A^3. Of a type either all pretend to switch or no one. If no one decides to pretend to switch of a type, as is now true for the high types, consumers that make the *signal decision* will actually buy from the competitor.

It follows that the profit functions that firms A and B are maximizing in stages 2 and 3 on segment A are given by, respectively:

$$\begin{aligned}\Pi_A^{2A} &= \lambda(p_A^3 - c)\hat{x}_{AL}^3 + (1-\lambda)(p_{AA}^2 - c)\hat{x}_{AH}^2 \\ &= \lambda(p_A^3 - c)\frac{1+p_{B,A}^2 - p_A^3 + z_L^2}{2} \\ &\quad +(1-\lambda)(p_{AA}^2 - c)\frac{1+p_{B,A}^2 - p_{A,A}^2 + z_H}{2}\end{aligned} \tag{26}$$

and

$$\begin{aligned}\Pi_B^{2A} &= \lambda(p_{BA}^2 - c)[\hat{x}_L^1 - \hat{x}_{AL}^3] + (1-\lambda)(p_{BA}^2 - c)[\hat{x}_H^1 - \hat{x}_{AH}^2] \\ &= \frac{1}{2}(p_{BA}^2 - c)[2\hat{x}^1 - 1 - p_{BA}^2 - \lambda z_L^2 - (1-\lambda)z_H + \lambda p_A^3 + (1-\lambda)p_{AA}^2] \\ &= \frac{1}{2}(p_{BA}^2 - c)[2\hat{x}^1 - 1 - p_{BA}^2 - \tilde{z}^2 - (1-\lambda)z_H^1 + \lambda p_A^3 + (1-\lambda)p_{AA}^2]\end{aligned} \tag{27}$$

where the superscript on Π denotes period and market segment. Note that maximization is done under restriction (25).

Firm B her maximization problem has obviously the following solution:

$$p_{BA}^2 = \frac{2\hat{x}^1 - 1 - \tilde{z}^2 - (1-\lambda)z_H^1 + \lambda p_A^3 + (1-\lambda)p_{AA}^2 + c}{2}. \qquad (28)$$

The solutions to the maximization problems of A (plural, since she sets two prices) can straightforwardly be found to be:

$$p_A^3 = \frac{1 + p_{BA}^2 + z_L^2 + c}{2} \qquad (29)$$

$$p_{AA}^2 = \frac{1 + p_{BA}^2 + z_H + c}{2}. \qquad (30)$$

Note that

$$p_{AA}^2 - p_A^3 = \frac{z_H - z_L^2}{2}, \qquad (31)$$

which we require to lay in the interval $[z_L^1, z_H^1]$. Therefore it is assumed that

$$z_L^1 < \frac{z_H - z_L^2}{2} \qquad (32)$$

for this subsection, otherwise we end up in the case in which no consumer will pretend to switch, see subsection (4.2). A sufficient condition for (32) to hold is $z_L^1 = 0$.

We consider 2 subcases depending on the values of switching costs.

1. $\frac{z_H - z_L^2}{2} < z_H^1 \Leftrightarrow z_H^2 - z_H^1 - z_L^2 < 0$.
   In this case firm A sets $p_A^3 = \frac{1+p_{BA}^2+z_L^2+c}{2}$ and $p_{AA}^2 = \frac{1+p_{BA}^2+z_H+c}{2}$.

2. $\frac{z_H - z_L^2}{2} > z_H^1$.
   In this case firm A will set $p_{A,A}^2 = p_A^3 + z_H^1$ because we showed in Proposition (3) that $p_{AA}^2 > p_A^3 + z_H^1$ cannot hold in equilibrium.

Before we continue we define the following.

$$Z_L = 3z_L^2 - \tilde{z}^2 - (1-\lambda)z_H^1 \qquad (33)$$

$$Z_H = 3z_H - \tilde{z}^2 - (1-\lambda)z_H^1. \qquad (34)$$

Since $z_H > z_L > z_L^2$ we obviously have $Z_H > Z_L$.

Case 1: $\frac{z_H - z_L^2}{2} < z_H^1$.

Using equations (29), (30) and (28) we find:

$$p_{BA}^2 = c + \frac{4\hat{x}^1 - 1 - \tilde{z}^2 - (1-\lambda)z_H^1}{3}. \tag{35}$$

Note that the expression is similar to (6). In that expression $\tilde{z}$ gives the total switching costs (including both those that are incurred before and after the *signal decision*). Hence, firm B will try to steal customers against a higher price when retention offers are possible because:

$$\tilde{z}^2 + (1-\lambda)z_H^1 = \tilde{z} - \lambda z_L^1 < \tilde{z}. \tag{36}$$

The reason is that for firm B it is easier to attract low types in the current setting because they incur switching costs in stage 2 (z_L^1) anyhow, even if it is only to get a better deal from firm A.

To find the optimal prices for firm A we substitute (35) into (29) and (30), which yields:

$$p_{AA}^2 = c + \frac{2 + 4\hat{x}^1 + Z_H}{6} \tag{37}$$

$$p_A^3 = c + \frac{2 + 4\hat{x}^1 + Z_L}{6}. \tag{38}$$

Market shares under this pricing regime are the following.

$$\hat{x}_{AH}^2 = \frac{4\hat{x}^1 + 2 + Z_H}{12} \tag{39}$$

and

$$\hat{x}_{AL}^3 = \frac{2 + 4\hat{x}^1 + Z_L}{12}. \tag{40}$$

The market share of firm A on high types is higher than that of low types since $Z_H > Z_L$.

Note that the tem $\tilde{z}^2 + (1-\lambda)z_H^1$ is common in all expression. This is present because it is part of the optimal strategy of firm B: she reduces prices with the weighted average switching costs $\tilde{z}$ as in the benchmark, except for the term λz_L^1 because low types incur stage 2 search costs anyhow (possibly to get a better deal from firm A).

Recall that in the benchmark $p_{AA}^2 = c + \frac{1}{3}(2\hat{x}^1 + 1 + \tilde{z})$ was the prevailing price for consumers on segment A that bought in Period 2 from firm A. In the current model with retention offers low types only buy at price p_A^3 from firm A, and high types only at p_{AA}^2. We now proof that the offering of retention offers leads to a higher price for firm A her retained consumers of the high type. This claim is true whenever:

$$\tilde{z} \leq \frac{Z_H}{2} = \frac{3z_H - \tilde{z}^2 - (1-\lambda)z_H^1}{2}. \tag{41}$$

This condition is equivalent to:

$$2\tilde{z} + \tilde{z}^2 + (1-\lambda)z_H^1 \leq 3z_H. \tag{42}$$

For the left-hand side we have:

$$2\tilde{z} + \tilde{z}^2 + (1-\lambda)z_H^1 \leq 2\tilde{z} + z_H^2 + (1-\lambda)z_H^1 \leq 2z_H + z_H^2 + (1-\lambda)z_H^1 \leq 3z_H,$$

which follows from the definitions and $\lambda \in [0,1]$. This concludes the proof.

On the other hand, consumers that are conquered back by firm A and pay p_A^3 (the *retention price*) will pay a lower price than in the benchmark. To see this note that:

$$\tilde{z} \geq \frac{Z_L}{2} = \frac{3z_L^2 - \tilde{z}^2 - (1-\lambda)z_H^1}{2}. \tag{43}$$

$$\Leftrightarrow$$

$$2\tilde{z} + \tilde{z}^2 \geq 3z_L^2 - (1-\lambda)z_H^1. \tag{44}$$

and (44) holds since $\tilde{z} = \tilde{z}^1 + \tilde{z}^2 \geq \tilde{z}^2 = \lambda z_L^2 + (1-\lambda)z_H^2 \geq z_L^2$. Hence, consumers that are retained pay a lower price than in a model without the possibility of retention offers. Consumers that would buy from firm A in the benchmark are actually better off if this price difference is larger than z_L^1, since that is the cost they incur to get the new price quote from firm A. This is true whenever:

$$2\tilde{z} + \tilde{z}^2 > 3z_L^2 - (1-\lambda)z_H^1 + 6z_L^1. \tag{45}$$

This condition will hold when z_L^1 is sufficiently small. A sufficient condition is $z_L^1 = 0$. Hence, low types are under the current pricing regime better off when this condition holds than in the benchmark when they decide to stay at firm A after pretending to switch.

Period 2 (stage 2 and 3) profits of firm A on segment A are given by:

$$\begin{aligned}\Pi_A^{2A} &= (1-\lambda)\hat{x}_{AH}^2\left(p_{A,A}^2 - c\right) + \lambda\hat{x}_{AL}^2\left(p_A^3 - c\right) \\ &= (1-\lambda)\frac{1}{18}\left(2\hat{x}^1 + 1 + \frac{Z_H}{2}\right)^2 + \lambda\frac{1}{18}\left(2\hat{x}^1 + 1 + \frac{Z_L}{2}\right)^2 \end{aligned} \tag{46}$$

Note that above we found:

$$\frac{Z_L}{2} \leq \tilde{z} \leq \frac{Z_H}{2}. \tag{47}$$

Period 2 profits for firm A on segment A when it decides not to make retention offers are given in (24). It follows that when λ is sufficiently small firm A will make offers because she then can make higher profits. More precise, A will make retention offers whenever:

$$\lambda < \frac{\left(2\hat{x}^1 + 1 + \frac{Z_H}{2}\right)^2 - \left(2\hat{x}^1 + 1 + \tilde{z}\right)^2}{\left(2\hat{x}^1 + 1 + \frac{Z_H}{2}\right)^2 - \left(2\hat{x}^1 + 1 + \frac{Z_L}{2}\right)^2}. \tag{48}$$

In this way the additional profits she gets from the high types outweighs the losses she makes on selling to the low types at a lower price. This result holds under the assumption that when firm A makes a retention offer all low types consumers immediately make the signal decision. In reality not all consumers might be so rational or forward-looking and the it might be profitable for firm A to make retention offers for even larger values of λ .

Case 2: $\frac{z_H - z_L^2}{2} > z_H^1$.

Now we consider the other case where $\frac{z_H - z_L^2}{2} > z_H^1$ and firm A sets $p_{AA}^2 = p_A^3 + z_H^1$. In this case the optimal strategy of firm B is still that as given in (28). By imposing $p_{AA}^2 = p_A^3 + z_H^1$ in equation (26) one can straightforwardly derive the reaction function of firm A to be:

$$p_A^3 = \frac{1 + p_{BA}^2 + \tilde{z}^2 - (1-\lambda)z_H^1 + c}{2}. \tag{49}$$

Solving the resulting system of equations gives:

$$p_{BA}^2 = c + \frac{4\hat{x}^1 - 1 - \tilde{z}^2 - (1-\lambda)z_H^1}{3} \tag{50}$$

$$p_A^3 = c + \frac{1}{3}\left(2\hat{x}^1 + 1 + \tilde{z}^2 - 2(1-\lambda)z_H^1\right) \tag{51}$$

$$p_{AA}^2 = c + \frac{1}{3}\left(2\hat{x}^1 + 1 + \tilde{z}^2 - 2(1-\lambda)z_H^1\right) + z_H^1. \tag{52}$$

Note that p_{BA}^2 is the same as in the case of $\frac{z_H - z_L^2}{2} < z_H^1$. Hence, this price is higher than in the benchmark without the possibility of retention offers. The price charged by firm A in the benchmark without retention offers was $c + \frac{1}{3}\left(2\hat{x}^1 + 1 + \tilde{z}\right)$. Under the current pricing regime consumers who are loyal will pay a higher price whenever:

$$\tilde{z}^2 - 2(1-\lambda)z_H^1 + 3z_H^1 > \tilde{z}. \tag{53}$$

Since the left-hand side of the equation is larger than $\tilde{z}^2 + z_H^1$, and since this is in turn larger than $\tilde{z}$, it follows that these consumers pay a higher price than in the benchmark.

Consumers who pretend to switch and pay the *retention price* pay a lower price than in the case without retention offers since:

$$\tilde{z}^2 - 2(1-\lambda)z_H^1 < \tilde{z}. \tag{54}$$

Hence, price changes with respect to the benchmark without retention offers are of the same sign as before.

Consumers who would buy from firm A in the benchmark and now pretend to switch, are not necessarily better off than in the benchmark case. For this to be the case z_L^1 should be small enough such that the following will hold:

$$\tilde{z}^2 - 2(1-\lambda)z_H^1 + 3z_L^1 < \tilde{z}. \tag{55}$$

Now the market shares can be found using equations (13) and (15). Substitution of the found prices gives:

$$\hat{x}_{AH}^2 = \frac{1 + 2\hat{x}^1 - 2\tilde{z}^2 + (1-\lambda)z_H^1 + 3z_H^2}{6} \tag{56}$$

and

$$\hat{x}_{AL}^3 = \frac{1 + 2\hat{x}^1 - 2\tilde{z}^2 + (1-\lambda)z_H^1 + 3z_L^2}{6}. \tag{57}$$

Hence, in this case total Period 2 profits for firm A on segment A will be:

$$\begin{aligned}
\Pi_A^{2A} &= (1-\lambda)\hat{x}_{AH}^2\left(p_{A,A}^2 - c\right) + \lambda\hat{x}_{AL}^2\left(p_A^3 - c\right) \\
&= (1-\lambda)\frac{1 + 2\hat{x}^1 - 2\tilde{z}^2 + (1-\lambda)z_H^1 + 3z_H^2}{6}\frac{1}{3}\left(2\hat{x}^1 + 1 + \tilde{z}^2 - 2(1-\lambda)z_H^1 + 3z_H^1\right) \\
&\quad +\lambda\frac{1 + 2\hat{x}^1 - 2\tilde{z}^2 + (1-\lambda)z_H^1 + 3z_L^2}{6}\frac{1}{3}\left(2\hat{x}^1 + 1 + \tilde{z}^2 - 2(1-\lambda)z_H^1\right) \\
&= \frac{1 + 2\hat{x}^1 + (1-\lambda)z_H^1 + \tilde{z}^2}{18}\left(2\hat{x}^1 + 1 + \tilde{z}^2\right) \\
&+ \frac{(1-\lambda)z_H^1}{18}\left[1 + 2\hat{x}^1 - 8\tilde{z}^2 + (1-\lambda)z_H^1 + 9z_H^2\right].
\end{aligned} \tag{58}$$

The term on the last line is clearly positive since $z_H^2 > \tilde{z}^2$. Moreover, the term on the one but last line is greater than

$$\frac{1}{18}\left(2\hat{x}^1 + 1 + \tilde{z}\right)^2, \tag{59}$$

which is the equilibrium profit in case of no retention offers. Hence, when $\frac{z_H - z_L^2}{2} > z_H^1$, that is when $z_H - 2z_H^1 - z_L^2 > 0$, firm A will always make a retention offer. The reason is that stage 3 switching costs for the high type (z_H^2) are relatively high compared to those of the low type (z_L^2). This difference can be exploited by charging these groups a different price.

4.4 Summarized results on retention offers

We now summarize our findings on retention offer in this section in several propositions.

Proposition 4 *Firms will decide to make retention offers for consumers who are on the verge of switching when the group with high switching costs is relatively large or their switching costs in the third stage are relatively high. Introducing retention offers then raises firm profits. Formally, firm A will make retention offers whenever:*

$$\lambda < \frac{\left(2\hat{x}^1 + 1 + \frac{Z_H}{2}\right)^2 - \left(2\hat{x}^1 + 1 + \tilde{z}\right)^2}{\left(2\hat{x}^1 + 1 + \frac{Z_H}{2}\right)^2 - \left(2\hat{x}^1 + 1 + \frac{Z_L}{2}\right)^2}, \tag{60}$$

or when

$$z_H - 2z_H^1 - z_L^2 > 0. \tag{61}$$

Proposition 5 *High type consumers will never pretend to switch once retention offers are introduced. The fraction of them that are loyal to the firm after Period 1 will pay a higher price in Period 2 after the introduction. The* *fraction of consumers that planned to buy from the closest firm in the benchmark case now obtains a lower welfare. Low type consumers will always make the signal decision and a positive fraction will return once retention offers are made. The fraction that returns will pay a lower price than in the benchmark situation. Their welfare might increase as compared to the case where no retention offers where made, for this to be true one requires:*

$$2\tilde{z} + \tilde{z}^2 > 3z_L^2 - (1-\lambda)z_H^1 + 6z_L^1 \quad \textit{when } \frac{z_H - z_L^2}{2} < z_H^1 \tag{62}$$
$$\tilde{z}^2 - 2(1-\lambda)z_H^1 + 3z_L^1 < \tilde{z} \quad \textit{when } \frac{z_H - z_L^2}{2} > z_H^1. \tag{63}$$

If z_L^1 is sufficiently small both conditions will hold. Consumers that decide to switch from supplier in Period 2 will pay a higher price than in a setting without retention offers.

4.5 Welfare and efficiency

We now turn to the question whether allowing firms to make retention offers is welfare improving and efficient. We compare the number of consumers who buy from firm A in the scenario with retention offers to the scenario without. Again cases $z_H^1 < \frac{z_H - z_L^2}{2}$ and $z_H^1 > \frac{z_H - z_L^2}{2}$ are considered separately. Without loss of generality we focus on segment A.

Note that the number of buyers from A in the benchmark model is, by (21), equal to:

$$\lambda \hat{x}_{AL}^2 + (1-\lambda)\hat{x}_{AH}^2 = \frac{4\hat{x}^1 + 2 - 4\tilde{z} + 6(\lambda z_L + (1-\lambda)z_H)}{12}. \tag{64}$$

Now we focus on market shares when there are retention offers.

Case 1: $\frac{z_H - z_L^2}{2} < z_H^1$.

By using (39) and (40) that the number of consumers buying from firm A in this case is given by:

$$\lambda \hat{x}_{AL}^3 + (1-\lambda)\hat{x}_{AH}^2 = \frac{4\hat{x}^1 + 2 - \tilde{z}^2 - (1-\lambda)z_H^1 + 3(\lambda z_L^2 + (1-\lambda)z_H)}{12}.$$

It follows that there are less consumers that buy from firm A than in the benchmark whenever:

$$3(\lambda z_L^2 + (1-\lambda)z_H) - \tilde{z}^2 - (1-\lambda)z_H^1 < 6(\lambda z_L + (1-\lambda)z_H) - 4\tilde{z} \quad (65)$$

$$\Leftrightarrow$$

$$6\lambda z_L^1 + 3(\lambda z_L^2 + (1-\lambda)z_H) + (1-\lambda)z_H^1 > 3\tilde{z} + \tilde{z}^1. \quad (66)$$

The left-hand side of this equation is greater than the right-hand side when $z_L^1 = 0$. Since the derivative of the former with respect to z_L^1 is 6λ while that of the latter is 3λ, it follows that the inequality holds for any $z_L^1 > 0$. Hence, firm A serves less consumers on its own segment when it makes retention offers under the condition that $\frac{z_H - z_L^2}{2} < z_H^1$.

Case 2: $\frac{z_H - z_L^2}{2} > z_H^1$.
By equations (56) and (57) we find that the number of consumers buying from firm A is in this case given by:

$$\lambda \hat{x}_{AL}^3 + (1-\lambda)\hat{x}_{AH}^2 = \frac{4\hat{x}^1 + 2 + 2\tilde{z}^2 + 2(1-\lambda)z_H^1}{12}. \quad (67)$$

Note that (64) can be rewritten as:

$$\frac{4\hat{x}^1 + 2 + 2\tilde{z}}{12}, \quad (68)$$

which is obvious larger than in the market share in (67). Hence, in case 2 firm A on its turf sells to less consumers than in the benchmark model.

From these findings we conclude that the introduction of retention offers leads to more consumers selling from firm B when they bought in Period 1 from firm A. Since the consumers in market segment A are closer located to firm A this is inefficient.

Proposition 6 *The introduction of retention offers leads to a less efficient matching between consumers and firms (products). On segment A more consumers will buy from firm B.*

Besides the matching inefficiency, retention offers lead to more switching costs to be paid (i.e. lost) in equilibrium.

Proposition 7 *The introduction of retention offers give rise to an equilibrium in which consumers spend in total more on switching costs.*

Proof. By proposition 6 more consumers on segment A buy from firm B. These incur $z_K^1 + z_K^2$ more switching costs when they are of type $K \in \{L, H\}$ than in the benchmark. Moreover, all low types pay z_L^1 in order to get the price quote p_A^3 from firm A even if they do not switch, while in the model without retention offers they did not have to pay these costs. Hence, in a model with retention offers more switching costs are incurred. ■

Combining the fact that prices are only transfers between firms and consumers with Propositions 6 and 7 gives rise to the following Proposition:

Proposition 8 *Total welfare is lower when firms practice retention offers as compared to a situation where they do not.*

On firm profits we found the following to be true.

Proposition 9 *Making retention offers is profit increasing for firm A when:*

$$\lambda < \frac{\left(2\hat{x}^1 + 1 + \frac{Z_H}{2}\right)^2 - \left(2\hat{x}^1 + 1 + \tilde{z}\right)^2}{\left(2\hat{x}^1 + 1 + \frac{Z_H}{2}\right)^2 - \left(2\hat{x}^1 + 1 + \frac{Z_L}{2}\right)^2}, \tag{69}$$

or when

$$z_H - 2z_H^1 - z_L^2 > 0. \tag{70}$$

When neither condition holds firms will not make retention offers and profits are the same as in the benchmark model.

4.6 Segment B

All the results above carry over to segment B of the market. We will need equilibrium prices, market shares and profits later on, so we state those here.

First, when firms refrain from making retention offers, the equilibrium prices that prevail are:

$$p^2_{A,B} = c + \frac{1}{3}(4[1-\hat{x}^1] - 1 - \tilde{z}) \quad \text{and} \quad p^2_{B,B} = c + \frac{1}{3}(2[1-\hat{x}^1] + 1 + \tilde{z}). \tag{71}$$

The condition in (48) is slightly different. Formally, firm B will make retention offers when

$$\lambda < \frac{\left(3 - 2\hat{x}^1 + \frac{Z_H}{2}\right)^2 - (3 - 2\hat{x}^1 + \tilde{z})^2}{\left(3 - 2\hat{x}^1 + \frac{Z_H}{2}\right)^2 - \left(3 - 2\hat{x}^1 + \frac{Z_L}{2}\right)^2}. \tag{72}$$

or when

$$z_H - 2z^1_H - z^2_L > 0. \tag{73}$$

Note that the conditions for retention offers of firm A and B coincide when $\hat{x}^1 = 1/2$.

Now consider the case that $\frac{z_H - z^2_L}{2} < z^1_H$, then on segment B the equilibrium prices that will prevail are:

$$p^2_{A,B} = c + \frac{3 - 4\hat{x}^1 - \tilde{z}^2 - (1-\lambda)z^1_H}{3} \tag{74}$$

$$p^2_{BB} = c + \frac{6 - 4\hat{x}^1 + Z_H}{6} \tag{75}$$

$$p^3_B = c + \frac{6 - 4\hat{x}^1 + Z_L}{6}. \tag{76}$$

Under this pricing regime the indifferent consumers are located at:

$$\hat{x}^2_{BH} = \frac{4\hat{x}^1 + 6 - Z_H}{12} \tag{77}$$

$$\hat{x}^3_{BL} = \frac{4\hat{x}^1 + 6 - Z_L}{12}. \tag{78}$$

And equilibrium profits of B on segment B are in period 2:

$$\Pi_B^{2B} = (1-\lambda)\frac{1}{18}\left(3-2\hat{x}^1+\frac{Z_H}{2}\right)^2+\lambda\frac{1}{18}\left(3-2\hat{x}^1+\frac{Z_L}{2}\right)^2. \quad (79)$$

It is straightforward to derive the equilibrium profits of firm B on segment A and these are given by:

$$\Pi_B^{2A} = \frac{1}{18}\left(4\hat{x}^1-1-\tilde{z}^2-(1-\lambda)z_H^1\right)^2. \quad (80)$$

Similarly one can find that firm A her equilibrium profits on segment B are given by:

$$\Pi_A^{2B} = \frac{1}{18}\left(3-4\hat{x}^1-\tilde{z}^2-(1-\lambda)z_H^1\right)^2. \quad (81)$$

Next consider the case that $\frac{z_H-z_L^2}{2} > z_H^1$. On segment B we have:

$$p_{AB}^2 = c+\frac{3-4\hat{x}^1-\tilde{z}^2-(1-\lambda)z_H^1}{3} \quad (82)$$

$$p_B^3 = c+\frac{1}{3}\left(3-2\hat{x}^1+\tilde{z}^2-2(1-\lambda)z_H^1\right) \quad (83)$$

$$p_{BB}^2 = c+\frac{1}{3}\left(3-2\hat{x}^1+\tilde{z}^2-2(1-\lambda)z_H^1\right)+z_H^1. \quad (84)$$

Indifferent consumers are located at:

$$\hat{x}_{BH}^2 = \frac{3+2\hat{x}^1+2\tilde{z}^2-(1-\lambda)z_H^1-3z_H^2}{6} \quad (85)$$

$$\hat{x}_{BL}^3 = \frac{3+2\hat{x}^1+2\tilde{z}^2-(1-\lambda)z_H^1-3z_L^2}{6}. \quad (86)$$

And in this case total Period 2 profits for firm B on segment B will be:

$$\begin{aligned}\Pi_B^{2B} &= \frac{3-2\hat{x}^1+(1-\lambda)z_H^1+\tilde{z}^2}{18}\left(3-2\hat{x}^1+\tilde{z}^2\right)\\ &+ \frac{(1-\lambda)z_H^1}{18}\left[3-2\hat{x}^1-8\tilde{z}^2+(1-\lambda)z_H^1+9z_H^2\right]. \quad (87)\end{aligned}$$

Firm A her profits (in this period and on this segment) can be shown to equal:

$$\Pi_A^{2B} = \frac{1}{18}\left(3 - 4\hat{x}^1 - \tilde{z}^2 - (1-\lambda)z_H^1\right)^2. \tag{88}$$

Frim B her profits in the same period are on segment A:

$$\Pi_B^{2A} = \frac{1}{18}\left(4\hat{x}^1 - 1 - \tilde{z}^2 - (1-\lambda)z_H^1\right)^2. \tag{89}$$

5 Period 1

We now turn to Period 1. Above we already gave the prevailing prices when no retention offers will be made. We thus focus on the case with retention offers. This puts conditions on z and/or λ (which are different for segment A and B).

5.1 Period 1 analysis

We assume consumers are forward-looking. A rational consumer of type $K \in \{L, H\}$ who is indifferent between firms A and B in period 1 foresees that if she chooses product A in period 1, she will switch to product B in period 2, whereas if she chooses product B in period 1 she will switch to product A in period 2.

We start with the low types. The location of the indifferent consumer is implicitly given by:

$$r - \hat{x}_L^1 - p_A^1 + \delta(r - (1-\hat{x}_L^1) - p_{BA}^2 - z_L) = r - (1-\hat{x}_L^1) - p_B^1 + \delta(r - \hat{x}_L^1 - p_{AB}^2 - z_L). \tag{90}$$

Note that the switching costs figure on both sides and therefore do not impact the location of the indifferent consumer. Rewriting and substitution of p_{AB}^2

and p^2_{BA} gives:

$$\hat{x}^1_L = \frac{1 + p^1_B - p^1_A - \delta\left(1 + \frac{8\hat{x}^1 - 4}{3}\right)}{2 - 2\delta}, \tag{91}$$

which holds independent of $\frac{z_H - z^2_L}{2} > z^1_H$ or $\frac{z_H - z^2_L}{2} < z^1_H$.

For the high types we can do a similar exercise to arrive at the following:

$$\hat{x}^1_H = \frac{1 + p^1_B - p^1_A - \delta\left(1 + \frac{8\hat{x}^1 - 4}{3}\right)}{2 - 2\delta}, \tag{92}$$

which holds independent of $\frac{z_H - z^2_L}{2} > z^1_H$ or $\frac{z_H - z^2_L}{2} < z^1_H$ and is at the same location as before.

We now thus have two equations and the restriction that $\lambda\hat{x}^1_L + (1-\lambda)\hat{x}^1_H = \hat{x}^1$. Substitution and rewriting gives:

$$\hat{x}^1 = \frac{3(1 + p^1_B - p^1_A) + \delta}{6 + 2\delta}, \tag{93}$$

which is the same as in the benchmark model. This result is caused by the fact that switching costs on both sides cancel out as well as their impact on prices p^2_{AB} and p^2_{BA}. We now derive equilibrium strategies for the cases $\frac{z_H - z^2_L}{2} < z^1_H$ and $\frac{z_H - z^2_L}{2} > z^1_H$ separately.

Case 1: $\frac{z_H - z^2_L}{2} < z^1_H$.

We now derive equilibrium strategies when $\frac{z_H - z^2_L}{2} < z^1_H$.

Firm A maximizes, by equations (46) and (81), the following function:

$$\begin{aligned}
\Pi_A &= (p^1_A - c)\hat{x}^1 + \delta\Pi^{2A}_A + \delta\Pi^{2B}_A \\
&= (p^1_A - c)\hat{x}^1 + (1-\lambda)\frac{\delta}{18}\left(2\hat{x}^1 + 1 + \frac{Z_H}{2}\right)^2 + \lambda\frac{\delta}{18}\left(2\hat{x}^1 + 1 + \frac{Z_L}{2}\right)^2 \\
&\quad + \frac{\delta}{18}\left(3 - 4\hat{x}^1 - \tilde{z}^2 - (1-\lambda)z^1_H\right)^2.
\end{aligned} \tag{94}$$

By (93) $\frac{\partial \hat{x}^1}{\partial p_A^1} = \frac{-3}{6+2\delta}$ and therefore:

$$\begin{aligned}\frac{\partial \Pi_A}{\partial p_A^1} &= \hat{x}^1 + (p_A^1 - c)\frac{-3}{6+2\delta} - \frac{\delta}{3}\frac{1}{3+\delta}\left(2\hat{x}^1 + 1 + \frac{(1-\lambda)Z_H + \lambda Z_L}{2}\right) \\ &\quad + \frac{4\delta}{3}\frac{1}{6+2\delta}\left(3 - 4\hat{x}^1 - \tilde{z}^2 - (1-\lambda)z_H^1\right). \qquad (95)\end{aligned}$$

The second order conditions of firm A her problem are obviously satisfied.

Now for firm B we do something similar. Firm B maximizes, by equations (79) and (80), the following function:

$$\begin{aligned}\Pi_B &= (p_B^1 - c)[1 - \hat{x}^1] + \delta\Pi_B^{2B} + \delta\Pi_B^{2A} \\ &= (p_B^1 - c)[1 - \hat{x}^1] + (1-\lambda)\frac{\delta}{18}\left(3 - 2\hat{x}^1 + \frac{Z_H}{2}\right)^2 + \lambda\frac{\delta}{18}\left(3 - 2\hat{x}^1 + \frac{Z_L}{2}\right)^2 \\ &\quad + \frac{\delta}{18}\left(4\hat{x}^1 - 1 - \tilde{z}^2 - (1-\lambda)z_H^1\right)^2. \qquad (96)\end{aligned}$$

By (93) $\frac{\partial \hat{x}^1}{\partial p_B^1} = \frac{3}{6+2\delta}$ and therefore:

$$\begin{aligned}\frac{\partial \Pi_B}{\partial p_B^1} &= 1 - \hat{x}^1 + (p_B^1 - c)\frac{-3}{6+2\delta} - \frac{\delta}{3}\frac{1}{3+\delta}\left(2[1 - \hat{x}^1] + 1 + \frac{(1-\lambda)Z_H + \lambda Z_L}{2}\right) \\ &\quad + \frac{4\delta}{3}\frac{1}{6+2\delta}\left(3 - 4[1 - \hat{x}^1] - \tilde{z}^2 - (1-\lambda)z_H^1\right). \qquad (97)\end{aligned}$$

The second order conditions of firm B her problem are obviously satisfied.

Notice that

$$1 - \hat{x}^1 = \frac{3(1 + p_A^1 - p_B^1) + \delta}{6+2\delta}, \qquad (98)$$

which, up to the interchange of p_B^1 and p_A^1, equals $\hat{x}^1$. Hence, the first order conditions are the same for firm A and B and therefore we can impose symmetry $p_A^1 = p_B^1$. Then $\hat{x}^1$ reduces to:

$$\hat{x}^1 = \frac{3+\delta}{6+2\delta} = \frac{1}{2}. \tag{99}$$

We equate $\frac{\partial \Pi_A}{\partial p_A^1}$ to zero to find first period equilibrium prices. By using (99) we find:

$$\begin{aligned} \frac{\partial \Pi_A}{\partial p_A^1} &= 0 \qquad (100) \\ &\Leftrightarrow \\ p_A^1 &= c + 1 + \frac{\delta}{3}\left(1 - 2\tilde{z}^2 - 2(1-\lambda)z_H^1\right). \qquad (101) \end{aligned}$$

Hence, first period equilibrium prices are:

$$p_A^1 = p_B^1 = c + 1 + \frac{\delta}{3}\left(1 - 2\tilde{z}^2 - 2(1-\lambda)z_H^1\right). \tag{102}$$

Note that these prices are increasing in δ (when switching costs are not too large) and decreasing in switching costs.

Case 2: $\frac{z_H - z_L^2}{2} > z_H^1$.

Next we consider the case in which $\frac{z_H - z_L^2}{2} > z_H^1$. Firm A maximizes, by equations (58) and (88), the following function:

$$\begin{aligned} \Pi_A &= (p_A^1 - c)\hat{x}^1 + \delta\Pi_A^{2A} + \delta\Pi_A^{2B} \\ &= (p_A^1 - c)\hat{x}^1 + \delta\frac{1 + 2\hat{x}^1 + (1-\lambda)z_H^1 + \tilde{z}^2}{18}\left(2\hat{x}^1 + 1 + \tilde{z}^2\right) \\ &+ \frac{\delta(1-\lambda)z_H^1}{18}\left[1 + 2\hat{x}^1 - 8\tilde{z}^2 + (1-\lambda)z_H^1 + 9z_H^2\right] \\ &+ \frac{\delta}{18}\left(3 - 4\hat{x}^1 - \tilde{z}^2 - (1-\lambda)z_H^1\right)^2. \end{aligned} \tag{103}$$

Since $\frac{\partial \hat{x}^1}{\partial p_A^1} = \frac{-3}{6+2\delta}$ we find:

$$\begin{aligned}
\frac{\partial \Pi_A}{\partial p_A^1} &= \hat{x}^1 - (p_A^1 - c)\frac{3}{6+2\delta} - \frac{2\delta}{18}\frac{3}{6+2\delta}\left(2\hat{x}^1 + 1 + \tilde{z}^2\right) \\
&- \frac{1 + 2\hat{x}^1 + (1-\lambda)z_H^1 + \tilde{z}^2}{18}\frac{6\delta}{6+2\delta} - \frac{(1-\lambda)\delta z_H^1}{18}\frac{6}{6+2\delta} \\
&+ \frac{\delta 2}{18}\left(3 - 4\hat{x}^1 - \tilde{z}^2 - (1-\lambda)z_H^1\right)\frac{12}{6+2\delta}. \qquad (104)
\end{aligned}$$

The second order conditions of firm A her problem are obviously satisfied.

Now for firm B we do something similar. Firm B maximizes, by equations (89) and (87), the following function:

$$\begin{aligned}
\Pi_B &= (p_B^1 - c)[1 - \hat{x}^1] + \delta\Pi_B^{2B} + \delta\Pi_B^{2A} \\
&= (p_B^1 - c)[1 - \hat{x}^1] + \frac{\delta}{18}\left(4\hat{x}^1 - 1 - \tilde{z}^2 - (1-\lambda)z_H^1\right)^2 \\
&+ \delta\frac{3 - 2\hat{x}^1 + (1-\lambda)z_H^1 + \tilde{z}^2}{18}\left(3 - 2\hat{x}^1 + \tilde{z}^2\right) \\
&+ \frac{\delta(1-\lambda)z_H^1}{18}\left[3 - 2\hat{x}^1 - 8\tilde{z}^2 + (1-\lambda)z_H^1 + 9z_H^2\right]. \qquad (105)
\end{aligned}$$

By (93) $\frac{\partial \hat{x}^1}{\partial p_B^1} = \frac{3}{6+2\delta}$ and therefore:

$$\begin{aligned}
\frac{\partial \Pi_B}{\partial p_B^1} &= -(p_B^1 - c)\frac{3}{6+2\delta} + [1 - \hat{x}^1] + \frac{2\delta}{18}\left(4\hat{x}^1 - 1 - \tilde{z}^2 - (1-\lambda)z_H^1\right)\frac{12}{6+2\delta} \\
&- \frac{2\delta}{18}\frac{3}{6+2\delta}\left(3 - 2\hat{x}^1 + \tilde{z}^2\right) - \delta\frac{3 - 2\hat{x}^1 + (1-\lambda)z_H^1 + \tilde{z}^2}{18}\frac{6}{6+2\delta} \\
&- \frac{\delta(1-\lambda)z_H^1}{18}\frac{6}{6+2\delta}. \qquad (106)
\end{aligned}$$

The second order conditions of firm B her problem are obviously satisfied.

As before $1-\hat{x}^1$ equals, up to the interchange of p_B^1 and p_A^1, $\hat{x}^1$. Hence, the first order conditions are the same for firm A and B and therefore we again can impose symmetry $p_A^1 = p_B^1$ and $\hat{x}^1 = 1/2$.

We equate $\frac{\partial \Pi_A}{\partial p_A^1}$ to zero to find first period equilibrium prices. By using

$\hat{x}^1 = 1/2$ we find:

$$\frac{\partial \Pi_A}{\partial p_A^1} = 0 \tag{107}$$

$$\Leftrightarrow$$

$$p_A^1 = c + 1 + \frac{\delta}{3}(1 - 2\tilde{z}^2 - 2(1-\lambda)z_H^1). \tag{108}$$

Hence, first period equilibrium prices are:

$$p_A^1 = p_B^1 = c + 1 + \frac{\delta}{3}(1 - 2\tilde{z}^2 - 2(1-\lambda)z_H^1). \tag{109}$$

Note that these prices are equal to those in the case of $\frac{z_H - z_L^2}{2} < z_H^1$.

6 Comparative Statics

6.1 Price comparison

We take as a benchmark a model in which there are both switching costs and behavior-based pricing. However, in that benchmark there are no retention offers. Proposition 5 compares second period prices of the benchmark to a model in which retention offers are made. In this subsection we will assume λ is sufficiently small or that $\frac{z_H - z_L^2}{2} > z_H^1$ so that firms will make retention offers.[6] We are thus left to compare first period findings and judge on (consumer) welfare over the entire game, as well as efficiency and firm profits.

Comparing equations (102) and (11) we find that first period equilibrium prices are a factor $\frac{\delta}{3}2\lambda z_L^1$ higher in the model with retention offers as compared to the benchmark. The reason is that in the model with retention offers low type consumers will always make the signaling decision: even if they do not switch they at least pretend to. These type consumers are therefore easier stolen from the rival's segment in the second period because they have already incurred stage 1 switching costs. High types, on the contrary,

[6] In all other cases the model collapses to the benchmark and there is nothing to compare.

are equally hard to steal from the competition as in the benchmark. Note that this reasoning is the intuition behind higher prices p_{BA}^2 and p_{AB}^2 in the model with retention offers as well.

Proposition 10 *When conditions are such that firms make retention offers in Period 2, equilibrium prices in Period 1 will be higher as compared to when firms will not make retention offers in Period 2.*

6.2 Equilibrium profits, (consumer) welfare and efficiency

In this subsection we combine all the above results for Period 1 and 2 and make some statements on equilibrium profits, (consumer) welfare and efficiency. We compare situations in which there are retention offers to the benchmark results, that is, either $\frac{z_H - z_L^2}{2} > z_H^1$ or λ is sufficiently small.

6.2.1 Equilibrium profits

Proposition 11 *Firms can obtain higher equilibrium profits by making retention offers as compared when they do not when*

$$\lambda < \frac{\left(2 + \frac{Z_H}{2}\right)^2 - (2 + \tilde{z})^2}{\left(2 + \frac{Z_H}{2}\right)^2 - \left(2 + \frac{Z_L}{2}\right)^2} \tag{110}$$

or

$$\frac{z_H - z_L^2}{2} > z_H^1. \tag{111}$$

Proof. By proposition 9 firm A profits in period 2 are higher when retention offers are made whenever (110) or $\frac{z_H - z_L^2}{2} > z_H^1$ hold. This proposition shows that in any other situation firms will not make retention offers and strategies in period 2, and thereby profits, will be as in the benchmark model. This straightforwardly implies that first period strategies and profits will be as in the benchmark. Hence when (110) does not hold and $\frac{z_H - z_L^2}{2} < z_H^1$ firms will not make retention offers and profits are as in the benchmark. We in addition showed that the same conditions are valid for firm B.

When (110) or $\frac{z_H - z_L^2}{2} > z_H^1$ hold firms will make retention offers. Above we showed that period 2 profits are higher than in the benchmark, see proposition (9). Furthermore, prices in period 1 are in such a situation higher than in the benchmark model. Since the market is covered (demand is unaffected by the retention offers and split equally in period 1), and because costs are the same as in the benchmark, it follows that period 1 profits are higher when retention offers are made.

We conclude that total profits are larger when retention offers are made because in each period they are higher under such a strategy. ■

6.2.2 Efficiency and total welfare

Above we showed that first period prices are higher when firms make retention offers in the second period. This means that several propositions above for period 2 straightforwardly generalize to the entire game.

Proposition 6 still holds:

Proposition 12 *The introduction of retention offers leads to a less efficient matching between consumers and firms (products). On segment A more consumers will buy from firm B.*

The result on more switching costs to be paid (i.e. lost) in equilibrium in Proposition 7 is not affected by first period results either:

Proposition 13 *The introduction of retention offers give rise to an equilibrium in which consumers spend in total more on switching costs.*

Combining the facts that prices are only transfers between firms, consumers are in the first period in the same way matched as in the benchmark, and propositions 6 and 7 for period 2 yields:

Proposition 14 *Total welfare is lower when firms make retention offers as compared to a situation where they do not.*

When retention offers are introduced by firms it increases their profit. By the above proposition, total welfare in such a situation is reduced. The following now straightforwardly follows.

Proposition 15 *Consumer welfare is lower when firms make retention offers as compared to a situation where they do not.*

7 Conclusions

This paper considered competition on a market with horizontally differentiated products and heterogeneous switching costs. In this market we assumed that firms are able to price based upon consumer purchase history. This is common practice in for instance the market for mobile phone subscriptions or health insurance. We are the first, to the best of our knowledge, to allow switching costs to be heterogeneous in such a setting. Besides this novelty, this paper contributes to the literature by introducing a new decision variable for firms: retention offers. When a firm learns that a consumer is about to switch to a competitor she has the possibility to make a retention offer. This offer can be better than the deal that the loyal customers get so that the firm is able to keep some of the customers that threaten to go to the competitor.

One particularly interesting result we obtain in this framework is that some consumers might pretend to go to the competition in order to get a better deal from the current supplier. We showed that this threat to switch will even be made in a situation where the deal of the competitor is worse than the one obtained from staying loyal to the current firm. We in addition show that the introduction of retention offers might actually benefit these "pretenders" who have relatively low switching costs, because they will end up paying a lower price.

On the other hand, consumers that are loyal to the firm will pay higher prices once firms start to make retention offers. The reason is that the firm receives no signal from the former type of consumers that they intend to switch. The lack of such signal indicates that these consumers have both high switching costs and strong preferences for the product of the firm over that of the competitor. This can be exploited by the firm by setting a high price.

Furthermore, we find that consumers who decide to switch pay a higher price as compared to the benchmark case without retention offers. The intuition is that they signal their intention to switch and the competitor thereby learns that these consumers have relatively strong preferences towards its products.

Another result we obtain is that firms only will make retention offers when the group with relatively high switching costs is sufficiently large or when their switching costs are relatively high. The explanation is that only in such a situation firms can increase profits by making retention offers.

In a situation where firms make retention offers because it is optimal for them, total profits are higher. Consumer, however, are worse off because they pay on average higher prices, are less efficient matched to products and spend in total more on switching costs. The aggregate effect is that total consumer welfare is lower. Society as a whole is worse off once firms are allowed to make retention offers since it reduces total welfare once firms decide to do so.

A final, yet remarkable, result we obtain is that increases in switching costs of one group might be beneficial to the other group, even in a framework without retention offers. The reasoning is that the incumbent firm wants to exploit this increase in switching costs, while the competitor will lower its price to generate sufficient demand. For the group that was unaffected by the change in switching costs it becomes now more attractive to switch, and consumers with relatively strong preferences for the competitor might be better off in the end.

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