Transactions in Two-Sided Markets*

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Abstract

We show that equilibrium in a two-sided market can be characterized as a one-sided market in which transaction demand depends on the markup of the intermediary (or the platform). We start by outlining the intuition in the case of a monopolist one-to-one intermediary. We then show that the characterization also holds for multiplicative matching, for settings with network effects, if the intermediary can practice two-part pricing, or with differentiated Bertrand competition between intermediaries. We extend the setup to show the equivalences to matching with Nash bargaining and the marriage market. As a result, intuition from the standard one-sided markets can be applied to many two-sided problems.

1 Introduction

Settings involving intermediaries, two-sided markets, and matching are at the forefront of economic research for at least the past fifteen years. Countless articles were written contrasting these markets to the standard one-sided markets. We show that for many purposes

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these markets can be thought of as the standard one-sided markets after a proper transformation. The key is to view these markets as one-sided markets, where demand and supply functions are demand and supply of transactions between the two sides. We outline how far this transformation carries us, and we also discuss what this transformation cannot achieve, namely identifying effects on either of the sides in particular.

In the rest of the paper we show that this equivalence works in many other contexts: much more general retailer intermediary setup, two-part tariffs, multiplicative two-sided market setup (where the number of transactions is demand times supply), and matching (both one-to-one with bargaining and the multiplicative marriage model).

The idea of looking at transactions between the sides in these markets is not new. One of the first U.S. Supreme Court cases clarifying the Sherman Antitrust Act, the Chicago Board of Trade v. United States (1918), considered what we would now call a platform (the Chicago Board of Trade, where buyers and sellers would get together to trade commodities). The court ruled that the Board disallowing after hour trading was not anticompetitive. The reasoning was that the Board's action effectively expanded the market, increased the number of transactions, and thus benefited the market welfare.

Coming back to more modern time, in the academic two-sided market literature, Rochet and Tirole (2006) discuss 'interactions,' that are effectively our transactions. To illustrate what we mean by transactions, below is part of a paragraph from their article:

"The interaction can be pretty much anything, but must be identified clearly. In the case of videogames, an interaction occurs when a buyer (gamer) buys a game developed by a seller (game publisher), and plays it using the console designed by the platform. Similarly, for an operating system (OS), an interaction occurs when the buyer (user) buys an application built by the seller (developer) on the platform. In the case of payment cards, an interaction occurs when a buyer (cardholder) uses his card to settle a transaction with a seller (merchant). The interaction between a viewer and an advertiser mediated by a newspaper or a TV channel occurs when the viewer reads the ad. The interaction between a caller and a receiver in a telecom network is a phone conversation and that between a website and a web user on the Internet is a data transfer."

In this paper, we propose examining the standard one-sided market for transactions that arises out of any intermediary/two-sided market/matching setting. We show several settings where it is possible to transform the two-sided setting into a market for transactions, and we outline how this transformation occurs. From that point on, if one is interested in the overall number of transactions as a welfare measure, one can examine a standard one-sided market, and apply any intuition developed for one-sided markets.

We continue to express everything in terms of transactions; however, one could similarly express everything as a function of per-transaction revenue – the sum of prices that the firm receives from both sides (of course, one of these prices could be negative). Thus, the volume of transactions is the analog of quantity, and the per-transaction revenue is the analog of price.

For a given transaction volume, a firm operating in a two-sided market has an optimal combination of prices to charge (to each side) to achieve the maximum profit conditional on transacting only that volume. These two prices combine to determine their sum, the per-transaction revenue of the firm. Thus, as long as we have a bijection, or at least know which solution to pick, from a given transaction volume to the optimal prices, and thus their sum, the transaction volume provides a sufficient statistic of the two optimal prices.

The applications range from the simple Microeconomics 101 price ceilings and price floors to tax incidence to any pass-through argument (see Weyl and Fabinger, 2013). A particularly relevant concern lately is antitrust. We provide an example of how to use our framework for antitrust purposes. The process of applying our framework is similar for any of the applications listed above.

One should identify the transaction occurring and examine the effects on the number of these transactions using standard techniques. For example, the issue might be Google favoring sites that they own in search results for a particular keyword (like Google Shopping appearing while searching for products available there). Then, the antitrust authorities should examine the market for transactions: the number of these products bought by consumers. It would be hard to examine this market using standard tools otherwise, in particular because the search is free to consumers, and Google derives no markup from that side of the market. However, using our metric of transactions, one can apply the standard Upward Pricing Pressure and the Critical Loss Analysis tools, and define the market in a familiar way.[1] Similarly when one is examining credit card rules such as network not allowing surcharge on their credit cards, one can see this setting as a market for the number of times consumers swipe a card or make the purchase, depending on how one defines the market. With either of these two metrics, it is relatively easy to quantify the metric, apply the standard tools, and decide which market definition is the proper one if one wants to. Moreover, the model is flexible enough to account for cases like some consumers receiving additional benefits from swiping a card by getting rewards.

Of course, there are several limitations to our model, some conceptual and some technical. The conceptual limitation is whether it is enough to examine the number of transactions and the corresponding overall markup that the firm derives. If the public policy, or whatever the researcher's interest is, only concerns social welfare, then the answer is likely affirmative, at least with a fixed number of firms in the market. But if one is interested in one side of the market in particular, then looking only at transactions is not enough. Our model does not provide much guidance over and above the existing literature for the purposes of examining one side of the market at a time, and thus we recommend using already existing models for these purposes.

A more technical issue is ensuring the existence of our solution. We assume throughout the paper that there is either a unique Nash equilibrium or all firms and researchers know which equilibrium is being played. It is not a new assumption in either theoretical or empirical literatures.[2] At the heart of our solution, we invert a matrix of functions. Of course, with

[1] See, for example, Farrel and Shapiro (2010) and Werden (1998).

[2] See Konovalov and Sandor (2010) and Ciliberto and Tamer (2009) respectively on theoretical and em-

many solutions, and no ways of knowing which one is which, this inversion is not defined. And unfortunately, as the literature still does not have an answer to this general problem (when is the matrix invertible or when Nash is unique), we have to simply assume that the matrix is invertible. We leave it to future researchers to deal with this issue.

Another issue is dealing with multiple coordination equilibria that can arise with network effects. Along with the majority of the literature, we do not explicitly model expectations and tipping. Alexandrov (2013) and Hagiu and Spulber (2013) show possible ways to overcome tipping issues with equilibria (via product differentiation and content).

With both of these technical issues the problem is arguably not with our technique, but rather with the fact that these are not solely intermediary/two-sided market/matching issues. Multiple equilibria, tipping, and not being able to identify tight sufficient or necessary assumptions for the existence of a unique Nash are phenomena that occur in one-sided markets as well. Thus, our technique of converting an intermediary/two-sided market/matching problem into a standard one-sided market is naturally not able to correct the existing issues with standard one-sided markets.

Several papers have explicitly tried to uncover when two-sided markets effectively collapse into a one-sided analog. Alexandrov et. al. (2011), building on Stahl (1988) and Spulber (1999), examine intermediaries a two-sided Salop model, and lay out a similar intuition for symmetric oligopolistic competition in that particular model, applying it to market definition and critical elasticity concepts from the antitrust literature.[3] Rozanski and Thompson (2011) examine agricultural markets, and use the farm-to-retail spreads to analyze buyer power.[4] Gans and King (2003) outline the conditions for the interchange fees to be neutral in a payment cards market.[5] Emch and Thompson (2006) argue that in the context of credit cards, the SSNIP test should be applied to the sum of prices on both sides.

pirical ways of at least partially dealing with this problem.

[3]Alexandrov et. al. (2011) also show how a similar framework can be applied to yield results akin to competitive bottlenecks in Armstrong (2006). Also see Loertscher (2007) and Reisinger and Schnitzer (2012) for two-sided spatial models.

[4]Also see Marx and Shaffer (2008) and Esö et. al. (2010) for buyer power arguments in similar contexts.

[5]Also see Prager et. al. (2009) for a discussion of interchange fee neutrality in payment card markets.

Many articles summing up two-sided literature were written, and there are even more original articles regarding two-sided markets and other possible settings of our work, like matching and intermediaries. Thus, we are not even attempting to compare our paper to each one of the existing papers, and have to make a call about which papers are natural and informative counterparts. We compare and contrast our findings with Wright (2004) and Rochet and Tirole (2003).

Wright (2004) suggests that "using conventional wisdom from one-sided markets in two-sided settings" leads to various "fallacies." Our results suggest in contrast that careful application of conventional wisdom in one-sided settings can generate further insights into how two-sided markets work. Additionally, rejecting conventional wisdom from one-sided markets can itself lead to logical fallacies. Consider for example, Wright's Fallacy 1, which states that "An efficient price structure should be set to reflect relative costs (user-pays)." Although it is true that bid and ask prices can depart from transaction costs on either side of the market, our analysis suggests that the user-pays principle certainly continues to hold. The bid-ask spread is the price of a buy-sell transaction and thus must cover the sum of direct transaction costs. Moreover, the dealer's buy and sell transactions are necessarily connected by resale as well as by possible network effects. Even though an efficient pricing structure could yield prices that are greater than or lower than either side's direct transaction cost, that does not negate the user-pays principle. Rather the price charged to each side of the market reflects an allocation that includes both direct and indirect transaction costs.

Similarly, consider Wright's Fallacy 2 "A high price-cost margin indicates market power." This is not a fallacy in a two-sided market; a high margin based on the bid-ask spread and the sum of direct transaction costs continues to indicate market power. Although a profit-maximizing price structure may involve a higher margin on one side of the market than another relative to attributable costs, the margin reflects an allocation of monopoly rents charged to the two sides of the market. Just as in standard one-sided markets, an increase in competition generally results in lower overall markups and a larger volume of transactions.[6]

Wright's (2004) Fallacy 8 states that "Regulating prices set by a platform in a two-sided market is competitively neutral." However, regulating the bid-ask spread is competitively neutral since it does not provide any advantage to the rivals. The regulated firm can adjust its prices on both sides given the constraint, and the optimal adjustment to a lower markup results in more customers for the regulated firm and lower profits than without regulation. Schiff (2008) explores the "waterbed effect": regulation of on one of the prices of a multi-product firm affects the firm's other prices. Schiff (2008) shows that a regulatory price ceiling in a two-sided market can result in a higher price on the other side of the market.[7] This result is closely related to the LeChatelier principle for constrained optimization (Samuleson, 1972, Milgrom and Roberts, 1996).

In summary, while we agree that each of these eight statements above is a fallacy, we offer a simple and an intuitive way of reconstructing each of those statements into a true statement, based on the well-developed understanding of one-sided markets.

We start from the same premise as Rochet and Tirole (2003): "Economic value is created by "interactions" or "transactions" between pairs of end users, buyers (superscript B) and sellers (superscript S)." We agree, and as our discussion above shows, many people before us have viewed the market in this way.

Since we agree with the premise, we better agree with the main result (their Proposition 1): "A monopoly platform's total price, $p = p^B + p^S$, is given by the standard Lerner formula for elasticity equal to the sum of the two elasticities, $\eta = \eta^B + \eta^S$." One way to interpret our paper is as taking Rochet and Tirole's Proposition 1 much further: beyond simply the total price, and onto a full-fledged standard one-sided market of interactions, that in many cases can answer the questions that one wants to answer in the two-sided model.

[6]However, see Chen and Riordan (2008) for how an increase in competition in a standard one-sided market can lead to higher prices.

[7]Genakos and Valletti (2011) confirm the waterbed effect empirically, using data from mobile phone market.

2 The Monopoly Dealer

Consider a two-sided market in which a monopoly dealer buys and resells a product. The dealer's customers and suppliers are price takers. Let Q_d be the quantity demanded by customers and let Q_s be the quantity offered by suppliers. Let T_d be the dealer's sales to customers and T_s be the dealer's purchases from suppliers. The dealer cannot sell more than the quantity demanded by customers, $T_d \leq Q_d$. In addition, the dealer cannot purchase more than the quantity offered by suppliers, $T_s \leq Q_s$. Also, the dealer cannot sell more that is purchased, $T_d \leq T_s$. The dealer offers customers an ask price $p > 0$ and offers suppliers a bid price $w > 0$.

The monopoly dealer provides a two-sided transaction platform for buyers and sellers so that there may be network effects in the market. Customer demand depends on the dealer's ask price p and on the volume of purchases from suppliers T_s,

$$Q_d = D(p, T_s). \tag{1}$$

Supply depends on the dealer's bid price w and the volume of sales to customers T_d,

$$Q_s = S(w, T_d). \tag{2}$$

Demand and supply are continuously differentiable and have the following properties.

Assumption 1: $D_p(p,T) < 0$ and $S_w(w,T) > 0$.

Assumption 2: $0 < D_T(p,T) < 1$ and $0 < S_T(w,T) < 1$.

The dealer has a unit cost per transaction with customers c_d and with suppliers c_s so that the dealer's profit equals

$$\Pi = pT_d - wT_s - c_dT_d - c_sT_s. \tag{3}$$

The dealer maximizes chooses the ask price p, the bid price w, sales T_d, and purchases T_s to maximize profit Π subject to $T_d \leq Q_d$, $T_s \leq Q_s$, and $T_d \leq T_s$.

The dealer gains nothing from generating supplier offers that exceed purchases from suppliers and incurs the bid price w for each unit purchased. Because the supply is increasing in the bid price w, the dealer will lower the bid price until the amount offered by suppliers equals the amount purchased from suppliers, $T_s = Q_s = S(w, T_d)$. For convenience, drop the subscript d and let T indicate dealer sales. So, the dealer chooses p, w, and T to maximize profit

$$\Pi = pT - wS(w,T) - c_d T - c_s S(w,T) \tag{4}$$

subject to

$$T \leq \min\{D(p, S(w,T)), S(w,T)\}. \tag{5}$$

The dealer does not gain from generating customer demand in excess of supplier offers, so that if this were the case for any given w and T the dealer would lower the ask price p until $D(p, S(w,T)) \leq S(w,T)$. The dealer also does not gain from generating customer demand in excess of sales so that if this were the case for any given w and T he dealer would lower the ask price p until $T = D(p, S(w,T))$. So, the dealer chooses p, w, and T to maximize profit $\Pi = pT - wS(w,T) - c_d T - c_s S(w,T)$ subject to $T = D(p, S(w,T))$ and $T \leq S(w,T)$.

It remains to be determined whether or not sales can be strictly less than supplier offers, $D(p, S(w,T)) < S(w,T)$. Define the total cost of a two-sided transaction by $c = c_d + c_s$. Then, $D(p, S(w,T)) < S(w,T)$ implies that

$$\Pi = pT - wS(w,T) - c_d T - c_s S(w,T) < (p - w - c)S(w,T),$$

which contradicts profit maximization. It follows that the dealer has an incentive to clear the market, $T = D(p,T) = S(w,T)$, for any $T > 0$. Also, profit equals $\Pi = (p - w - c)T$.

By Assumption 1 and the implicit function theorem, $T = D(p,T) = S(w,T)$ implies that there exist unique continuously differentiable inverse demand and inverse supply that are functions of the volume of transactions $p = P(T)$ and $w = W(T)$. Let ρ represent the bid-ask spread. It follows that the bid-ask spread depends on the volume of transactions,

$$\rho(T) \equiv P(T) - W(T). \tag{6}$$

The dealer's profit function can be written as a function of the volume of transactions,

$$\Pi(T) = (\rho(T) - c)T. \tag{7}$$

This reduces the monopolist's problem to the standard-one-dimensional output choice problem.

Lemma 1 shows that we can define a market demand for transactions as a function of the bid-ask spread, $T = T(\rho)$.

Lemma 1. The volume of transactions $T(\rho)$ is a differentiable function and $T'(\rho) < 0$.

Proof. By Assumptions 1 and 2, the inverse demand function is strictly decreasing in the volume of transactions and the inverse supply function is strictly increasing in the volume of transactions,

$$P'(T) = \frac{1 - D_T(p, T)}{D_p(p, T)} < 0. \tag{8}$$

$$W'(T) = \frac{1 - S_T(w, T)}{S_w(w, T)} > 0. \tag{9}$$

This implies that $\rho(T)$ is differentiable and $\rho'(T) = P'(T) - W'(T) < 0$. Therefore, $\rho(T)$ is invertible so that transaction demand can be expressed as $T = T(\rho) = \rho^{-1}(\rho)$ where $T(\rho)$ is a differentiable function and $T'(\rho) < 0$. $\square$

Lemma 1 implies that the dealer's profit function can be written as a function of the bid-ask spread,

$$\pi(\rho) = (\rho - c)T(\rho). \tag{10}$$

The two-sided market with a monopoly dealer and network effects therefore reduces to a one-sided market problem in which intermediary's output is a two-sided transaction and dealer's bid-ask spread is the price of a transaction. The one-sided market problem and the two-sided market problem are equivalent.

This equivalence can be illustrated using the standard supply-and-demand graph (see Figure 1). A given transaction volume T uniquely defines the intermediary's spread ρ. Thus, putting aside transaction costs, one can transform the intermediary's problem of maximizing the area of the rectangle between supply and demand into the standard problem of maximizing the area of the rectangle under the demand curve simply by defining new demand as the difference between demand and supply.

The standard characterization of the monopoly problem then yields insights about the two-sided market dealer's problem. If the monopoly problem has an interior solution, the profit-maximizing bid-ask spread solves the standard one-sided first-order condition,

$$\frac{\rho(T) - c}{\rho(T)} = -\frac{\rho'(T)T}{\rho(T)}. \tag{11}$$

This implies that in a two-sided market, the relative markup of the bid-ask spread over marginal cost is a weighted average of the reciprocals of the elasticities of demand and supply,

$$\frac{P(T) - W(T) - c}{P(T) - W(T)} = \frac{1}{\eta(T)} \frac{P(T)}{P(T) - W(T)} + \frac{1}{\xi(T)} \frac{W(T)}{P(T) - W(T)}, \tag{12}$$

where the elasticities are given by $\eta(T) = -\frac{P(T)}{P'(T)T}$ and $\xi(T) = \frac{W(T)}{W'(T)T}$.

2.1 Extension: Differentiated Bertrand

Suppose that there are N firms selling differentiated products. Similarly to the base case, firm's demand and supply can be expressed as

$$Q_i^d = D_i(p_i, Q_i^s, p_{-i}, Q_{-i}^s), \tag{13a}$$

$$Q_i^s = S_i(w_i, Q_i^d, w_{-i}, Q_{-i}^d). \tag{13b}$$

where i denotes competitors' prices and quantities.

Konovalov and Sandor (2010) show existence and uniqueness of Nash equilibrium for multi-product firms for logit and CES consumer models with asymmetric cost firms. We assume the existence and uniqueness of the Nash equilibrium. While neither of these two papers account for network effects, both Alexandrov (2013) and Hagiu and Spulber (2013) show possible ways to overcome tipping issues with equilibria (via product differentiation and content). Given the unique Nash equilibrium, invertibility of the system of equations follows.

From here, the problem is effectively the same as in the base case, with the major difference being that instead of inverting a system of two equations (supply and demand of the monopolist) to get the wholesale and the retail prices as functions of all the other variables, one needs to invert a system of $2N$ equations (supply and demand of each of the N firms) to get the prices of each firm:

$$p_i = P_i(Q_i^d, Q_i^s, Q_{-i}^d, Q_{-i}^s), \tag{14a}$$

$$w_i = W_i(Q_i^d, Q_i^s, Q_{-i}^d, Q_{-i}^s). \tag{14b}$$

Then, firm i's maximization problem becomes (similarly to the base case) maximizing $\Pi_i(T_i)$:

$$\Pi_i(T_i) = T_i(P_i(T_i) - W_i(T_i) - c_i) - F_i = T_i(\rho_i(T_i) - c_i) - F_i, \tag{15}$$

where ρ_i is firm's markup (as a function of transactions), c_i is the marginal cost per transaction, and F_i is the fixed cost of the firm.

3 Monopolist Multiplicative Intermediary

Many of the intermediaries are in markets that are better defined by multiplicative transactions. For example, an internet search provider can potentially charge consumers and advertisers. Each consumer is going to see each advertisement, and thus for Q_1 consumers and Q_2 advertisers results in $Q_1 \times Q_2$ matches, instead of $min(Q_1, Q_2)$, as in the previous section. We assume that the demands are described by $D_1(p_1, D_2)$ and $D_2(p_2, D_1)$. The intermediary incurs a cost of c per transaction, and a fixed cost of F. The intermediary charges consumers i p_i per transaction. Thus, the intermediary's profit function is

$$\Pi(p_1, p_2) = (p_1 + p_2 - c)D_1(p_1)D_2(p_2). \tag{16}$$

From the first order conditions, suppressing some of the arguments:

$$p_1 + p_2 = c - \frac{D_1}{\frac{\partial D_1}{\partial p_1}}, \tag{17a}$$

$$p_1 + p_2 = c - \frac{D_2}{\frac{\partial D_2}{\partial p_2}}. \tag{17b}$$

Define the number of transactions to be

$$T \equiv D_1(p_1) \times D_2(p_2). \tag{18}$$

We can invert the two equations above:

$$p_1 = P_1(T), \tag{19a}$$

$$p_2 = P_2(T). \tag{19b}$$

Plugging these into the intermediary's profit function:

$$\Pi(T) = (P_1(T) + P_2(T) - c)T. \tag{20}$$

We can also define an overall price per transaction as

$$a \equiv p_1 + p_2, \tag{21}$$

resulting in

$$\Pi(T) = (a(T) - c)T. \tag{22}$$

Similarly to the retailer-intermediary section, the intermediary's problem can be expressed as a standard monopolist's problem, where both demand and supply are the functions of the number of transactions.

4 Matching

4.1 Trade with Bargaining

A matchmaker (eBay) gets buyers and sellers onto the same platform – either in the same building or on the same website. Buyers pay B and sellers pay R to enter the platform. The matchmaker cannot price discriminate. Once the buyers and the sellers have entered, they get matched assortatively: the highest valuation buyer gets matched with the lowest cost seller, and so on.[8] The buyer and the seller bargain, and the seller's bargaining power is α. The matchmaker incurs a transaction cost c per transaction. The buyer with the lowest valuation of the T highest buyers, has the valuation of $P(T)$ for the product. The seller with the highest cost of the T lowest sellers, has the opportunity cost of $W(T)$ of giving the product to the buyer.

Proposition 1 *The matchmaker's profit optimization problem can be expressed as a standard one-sided monopolist's problem, where the demand is the difference between the functions of buyers' valuations and sellers' opportunity costs. Moreover, the number of transactions is exactly the same as if the matchmaker were an intermediary from Section 2.*

Proof. As before, the matchmaker needs to match the number of buyers and the number of sellers, so assume that T buyers and T sellers enter the platform. The buyer-seller pair with the lowest utility from the trade is the pair where buyer's valuation of the product is $P(T)$ and seller's opportunity cost is $W(T)$. Thus, if the trade happens, the surplus is $P(T) - W(T)$. Given the bargaining power, the seller receives $\alpha(P(T) - W(T))$, net of the opportunity cost, and the buyer receives $P(T) - \alpha(P(T) - W(T))$. Thus, the matchmaker charges sellers $R = \alpha(P(T) - W(T)) - W(T)$ (to compensate sellers for their opportunity cost) and buyers $B = P(T) - \alpha(P(T) - W(T))$. The matchmaker's profit becomes

$$\Pi(T) = (B+R-c)T = (P(T)-\alpha(P(T)-W(T))+\alpha(P(T)-W(T))-W(T))T = (P(T)-W(T))T. \tag{23}$$

Defining $\rho(T) \equiv P(T) - W(T)$, we can see that the optimization problem again becomes the standard monopolist's optimization problem. ■

[8]The matching can occur either via some algorithm developed by the matchmaker or it could be driven by the buyers and the sellers themselves.

4.2 Marriage Market

A matchmaker matches men and women. Men pay M to enter the platform, women pay W. Once a man with value V_m and a woman with value V_w are matched, they both receive a surplus of $V_m V_w$, resulting in assortative matching, again either through the matchmaker's or through the consumers' efforts. The matchmaker incurs cost c per match. Men's values are distributed according to a p.d.f. f_m, and women's values are distributed according to a p.d.f. f_w.

Proposition 2 *The matchmaker's profit optimization problem can be expressed as a standard one-sided monopolist's problem, where the demand is the product of men's and women's valuations.*

Proof. As before, the matchmaker needs to match the number of men and the number of women, so assume that T men and T women enter the platform. The pair with the lowest utility from the match is the pair where men's value is $\overline{F_m}^{-1}(T)$ and women's is $\overline{F_w}^{-1}(T)$. Thus, if the trade happens, the surplus to each partner in the couple is $\overline{F_m}^{-1}(T) \times \overline{F_w}^{-1}(T)$. Thus, the profit function becomes

$$\Pi(T) = 2T\overline{F_m}^{-1}(T)\overline{F_w}^{-1}(T). \tag{24}$$

■

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