

Structural Estimation of the Scoring Auction Model

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April 17, 2013

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Abstract

We establish a structural estimation method of the scoring auction model. We show a semi-parametric procedure to identify the joint distribution of bidders' multi-dimensional private signals from observed multi-dimensional bids in a first-score (FS) auction and demonstrate sufficient conditions for identification. A simulation experiment is also conducted to verify the consistency of our estimation procedure. The scoring auction model in our analysis allows not only the quasilinear scoring rule but also a broad class of non-quasilinear rules including price-over-quality ratio (PQR) scoring rules. An empirical analysis of multi-dimensional bidding in the Japanese public procurement auctions is provided. A series of counterfactual analyses quantify the impact of the change of auction formats and scoring rules on the utilities of both bidders and the auctioneer.

Key words: scoring auctions, structural estimation, procurement
JEL classification: D44, H57

1 Introduction

Public sectors purchase a variety of goods and services from the private sector, encompassing from snow removal services to weapons systems. OECD (2007) reports that the amount of expenditure incurred on public procurement accounts for 10 to 15 % of GDP in many countries. For public funds to be spent efficiently and effectively, value for money is the key principle in public procurement. More and more procurement buyers, thus, introduce awarding mechanisms with which relevant prices and qualities of proposals in the

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whole procurement cycle are assessed. Scoring auctions, or equivalently multi-parameter bidding, are one of the most prevailing mechanisms that meet their need.

In the scoring auction, bidders are asked to submit a set of multi-dimensional bids that include price and some non-price attributes, such as quality. The multiple bids are evaluated into a score by an *ex ante* publicly announced scoring rule, and the awardee is the bidder whose score is the highest or lowest. Thus, the procurement buyer may obtain a greater value-for-money contract without reducing the bidder's profit (Milgrom (2004)).

A variety of forms of scoring rules are used in real-world public procurement. In U.S. state departments of transportation, for instance, Delaware, Idaho, Oregon, Massachusetts, Utah, and Virginia, use quasilinear (QL) scoring rules, in which the score is price subtracted by other non-monetary attributes (quality), whereas Alaska, Colorado, Florida, Michigan, North Carolina, and South Dakota use price-over-quality ratio (PQR) rules, in which the score is computed based on the price divided by quality. The PQR scoring rule is also extensively used in Japan.¹ In addition, quite a few governments use the scoring auction in which the score is the sum of price and quality measurements but the score is non-linear in the price bid. Note that, any monotonic function cannot transform these non-quasilinear scoring rules into a QL form since a necessary condition for quasi-linearity requires that the price be linear in score.

The theory of scoring auctions was first introduced by Che (1993) who characterizes the equilibrium bidding function in a scoring auction, assuming that *ex ante* symmetric bidders have a single-dimensional, independent-private-value signal and that the scoring function is QL. The model has been extended to the correlated signal environment by Branco (1997), to the multi-dimensional signal case by Asker and Cantillon (2008), and to scoring auctions with non-quasilinear scoring rules or with QL rules with reservation prices by Hanazono, Nakabayashi and Tsuruoka (2013). As for empirical literature, a growing number of works have been developed (*e.g.* Bajari et al. (2006), Lewis and Bajari (2011), and Tunca et al. (2010)) but confined to non-structural approaches or scoring auctions with QL awarding rules and no reservation price, because the equilibrium characterization is more complex in the scoring auction with a non-quasilinear scoring rule; the bidder's effective type, as denoted by *productive potential* in Che (1993) or *pseudo-type* in Asker and Cantillon (2008), hinges on the equilibrium strategy and, thus, cannot be defined as an exogenous variable under a non-quasilinear scoring rule. In this sense, a general procedure for the structural estimation of the scoring auction model has been limited in the literature.

In this paper, we propose a structural estimation procedure of the scoring auction model established by Hanazono, Nakabayashi and Tsuruoka (2013), in which *ex ante* symmetric, risk-neutral bidders with l dimensional private information submit $l + 1$ dimensional bid. Based on a pre-announced scoring rule, multi-dimensional bids are mapped into a real

¹Ministry of Land, Infrastructure and Transportation in Japan allocates most of the public construction project contracts through scoring auctions based on PQR awarding rules.

value, *the score*, and a lowest scored bidder is selected as a winner. The model is an extension of Che (1993), allowing a broad class of independent scoring rules,² including non-quasilinear forms. In addition, with imposing a separability condition on the bidder’s cost function, the model accepts multi-dimensional signals. Therefore, quite a few scoring auction data can be analyzed in our framework.

Several assumptions are made in the scoring auction model. First, multi-dimensional signals are separable and monotone in conjunction to the bidder’s cost function and the score function (Assumption 1). More specifically, the marginal cost of an additional provision for a component of l -dimensional quality rises separately as the signal value in the corresponding dimension increases in the bidders’ cost schedules. The signal value of the remaining one dimension, denoted by dimension zero in our model, is strictly monotone in the total cost given a vector of the multi-component quality level. Although restrictive, this specification dramatically simplifies the mechanism design problem with multi-dimensional signals in a way that the bidder’s information rent in the scoring auction hinges solely on the dimension zero signal distribution. Furthermore, the model fits the typical scoring auction data in which price and quality are scattered in the price quality space. The bidder’s strategic interaction in the score choice is thus reduced into a single dimensional problem with a single dimensional private information. Assumption 1 is satisfied in a PQR scoring rule if a set of cost functions with an identical dimensional-zero signal are homothetic with each other. In addition, Assumption 1 constitutes a sufficient condition for the identification of the scoring auction model with multi-dimensional signals.

Second, the bidder’s expected payoff function satisfies the log-supermodularity condition (Assumption 2). In the scoring auction model, the bidder’s strategy space is reduced into a single dimensional value, the score, as the bidder’s choice in quality components (non-price attributes) is thoroughly endogenous in the score. Hence, the positiveness of the cross partial derivative of the bidder’s expected payoff function with respect to both the score and private signal is equivalent to the Spence-Mirrlees’s single-crossing condition in the scoring auction model. The single-crossing condition is sufficient for the log-supermodularity of the payoff function. The log-supermodularity of the bidder’s expected payoff guarantees the existence of the Bayesian Nash equilibrium in a first-score (FS) auction.

To identify the bidder’s multi-dimensional signals from observed multi-dimensional bid data, we choose a semi-parametric estimation methodology. Assuming that the bidder’s cost function is known except for the multi-dimensional parameters (signals), we proceed the identification of the bidder’s signals in the following three steps. First, we recover an indicator of the bidder’s equilibrium production cost, namely the *quality adjusted cost* $k(s(\theta), \theta)$ in Hanazono et al. (2013), applying the method proposed by Guerre et al. (2000).³

²A scoring rule is *interdependent* if the bidder’s score is determined not only by his/her p and q but also other bidder’s p and q such that $S^i(p^1, \dots, p^n, q^1, \dots, q^n)$. In this paper, we restrict attention to independent scoring rules. See Albano et al. (2009) for the classification of scoring rules.

³The quality adjusted cost is called the bidder’s *productive potential* in Che (1993) or *pseudo-type* in

Second, we obtain the pseudo values of bidders' multi-dimensional signals, using the pseudo value of the quality adjusted cost and a cost function that meets the sufficient conditions for identification. Finally, we identify the distribution of the bidder's multi-dimensional private signals. The structural estimation method of first-price auctions has been developed by Laffont et al. (1995), Guerre et al. (2000), Li et al. (2002), and a growing number of empirical analyses of first-price auction data have been provided in the literature. Our methodology is an extension of Guerre et al. (2000) to the scoring auction model in which *ex ante* symmetric bidders draw a multi-dimensional signal from a publicly known common joint distribution.

We conduct a numerical experiment to verify the consistency of our identification method. Simulated bid data samples are created assuming that θ^0 follows a uniform distribution, so that the symmetric Bayesian Nash equilibrium bidding strategy is explicitly obtain in a FS auction. Then, our structural estimation method is applied to the simulation data to recover the distribution of θ . The recovered cumulative distribution functions are presented in Section 3.

As an empirical application, we conduct a series of counterfactual analyses using the scoring auction data. The data is from public procurement auctions for construction projects in Japan, where a PQR scoring rule is used. The parametric cost functions used in our empirical application are quadratic, cubic, and quartic polynomials, which also satisfy the regularity conditions for the existence of a unique Bayesian Nash equilibrium in the observed non-quasilinear scoring rule. The counterfactual analyses measure the impact of the change of the scoring rule or the auction format on both the procurement buyer's and suppliers' utilities, assuming that the procurement buyer's true preference is represented by the observed PQR scoring rule. In addition, the extent to which the utility of the buyer using scoring auctions would change by the use of price-only auctions is quantified.

For ranking the expected scores of scoring auctions, the following two auction formats are examined, the first-score (FS) and second-score (SS) auctions. In a FS auction, the bidder with the lowest score wins and follows the contract as specified in his winning bid. In a SS auction, the bidder with the lowest score wins and is free to choose the contracted price and quality such that the score value based on the contracted price quality combination matches the second-lowest score. In the data, multi-dimensional bids in the (FS) auction is observed. From the estimated bidder's cost, we create the equilibrium multi-dimensional bids of the SS auction for each bidder and compare the expected winning bids as well as the payoffs of both the procurement buyer and bidders between the observed PQR FS auction and the hypothetical PQR SS auction. Hanazono et al. (2013) show that the expected score equivalence holds only in a ql scoring rule and that the procurement buyer has a lower gain from FS than SS auctions in many non-quasilinear scoring functions such as PQR. Assuming that the PQR score represents the buyer's true utility, we quantify the

Asker and Cantillon (2008) under the QL scoring rule.

inequivalence in the buyer's welfare between FS and SS auctions.

For the measurement of the impact of the change in the scoring rule, we suppose that the buyer employed a QL rule which would distort the contracted quality downward effectively. The PQR FS auctions in the data consists of two disadvantages from the viewpoint of the expected contracted score minimization. First, the PQR scoring rule induces a far excessive quality provision when it is used in a FS auction. This causes a higher expected score in a FS auction than in a SS auction, where quality bids of all losing bidders are their first-best levels. Second, even in a SS auction, the observed PQR scoring rule results in excessive quality provision relative to the optimal outcome. As Che (1993) indicated, an optimal outcome can be achieved in a multi-dimensional bidding game by distorting the bidder's quality provision downward to limit the bidder's informational rents. Therefore, introducing a QL rule immediately resolves the first inefficiency problem and will give some remedy for the second inefficiency problem if the QL rule limits the bidder's informational rents effectively. We designed a QL scoring rule so that the marginal rate of substitution (the slope of the iso-score curve) coincides the second-lowest bidder's break-even score and show that, even if the buyer's true utility is represented by the observed PQR scoring rule, this suboptimal QL scoring rule improves the buyer's utility more than the PQR SS auction does.

The results of our empirical application are given as follows; first, under the PQR scoring rule, the procurement buyer has approximately .03 percent lower utility by the use of FS than SS auctions while the winning bidder earns the payoff greater by approximately .03 through .12 percent in FS than in SS auctions. Theory suggests that non-equivalence stems from the overproduction in quality in a FS auction. Accordingly, we observe that the expectation of the winner's quality provision is 3 through 4 percent larger in FS than in SS auctions. Second, we explore a QL scoring rule with which a FS auction dominates the currently used PQR FS auction. With a well-designed QL scoring rule, the procurement buyer improves utility by approximately .27 to .28 percent while bidders earn lower payoffs by 3.6 to 4.5 percent. Finally, a first-price auction is always dominated by the currently used FS auction with the PQR scoring rule. In such a price-only auction, bidders earn lower payoffs by 1.8 to 4.0 percent, while the procurement buyer's utility can be 5 percent higher than in the FS auction. These results suggest that the currently used scoring auction may perform poorly especially for an experienced procurement buyer with a PQR utility function. A procurer who has enough information regarding the distribution of the bidder's cost structure will obtain a higher gain with the use of a price-only auction with a well-designed fixed quality standard or a FS auction with a well-designed QL scoring rule.

The remaining part of this paper is organized as follows. Section 2 describes the theoretical consideration on scoring auctions with general scoring rules. Section 3 discusses the identification of the distribution of bidders' cost schedule parameters. Section 4 conducts empirical examinations using the structural estimation method. The final section is the

conclusion.

2 A theoretical consideration

2.1 The model

A procurement buyer auctions a project contract to n risk-neutral bidders.⁴ The scoring function $S(p, \mathbf{q}) : \mathbb{R}_+^{L+1} \rightarrow \mathbb{R}$ is common knowledge, mapping the bidder's price-bid $p \in \mathbb{R}_+$ and a quality level $\mathbf{q} = (q^1, \dots, q^L) \in [\underline{q}^1, \bar{q}^1] \times \dots \times [\underline{q}^L, \bar{q}^L]$ into a single dimensional value, the score, denoted by $s \in \mathbb{R}$. The scoring function is monotonic; in particular, $S_p(p, \mathbf{q}) > 0$ and $S_{q^\ell}(p, \mathbf{q}) < 0$ for all $\ell = 1, \dots, L$. For instance, the PQR scoring rule with an unbinding reservation price is $S(p, \mathbf{q}) = p/V(\mathbf{q})$ where $V_{q^\ell} > 0$ for all $\ell = 1, \dots, L$ while the QL scoring rule with an unbinding reserve is $S(p, \mathbf{q}) = p - V(\mathbf{q})$. The procurement buyer's utility function is represented by the scoring function, namely $U(p, \mathbf{q}) = S(p, \mathbf{q})$.⁵

At the bid preparation stage, each bidder obtains a two-dimensional signal $\boldsymbol{\theta} \in [\underline{\theta}^0, \bar{\theta}^0] \times \dots \times [\underline{\theta}^L, \bar{\theta}^L]$ distributed following the publicly known cumulative joint distribution $F(\boldsymbol{\theta})$. We allow θ^ℓ and $\theta^{\tilde{\ell}}$ with $\ell \neq \tilde{\ell}$ to be correlated with each other; however, we assume that θ^0 is identically and independently distributed for all n bidders following the marginal distribution function $F_0(\theta^0)$. Finally, we denote by $F_\ell(\theta^\ell)$ the the marginal distribution of θ^ℓ with $\ell = 1, \dots, L$.

The bidder's cost function $C(\mathbf{q}|\boldsymbol{\theta})$ is increasing and strictly convex in q^ℓ for all $\ell = 1, \dots, L$. For simplicity, we assume that $C(\mathbf{q}|\boldsymbol{\theta})$ is differentiable and $C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta})$ is strictly increasing in q^ℓ for any $\ell = 1, \dots, L$. Furthermore, we normalize the cost function such that i) $C(\mathbf{q}|\boldsymbol{\theta})$ is strictly increasing in θ^0 and ii) C_{q^ℓ} is strictly decreasing in θ^ℓ for all $\ell = 1, \dots, L$. The interpretation of this specification is that the dimension-zero signal θ^0 represents the bidder's productivity. The signals in the rest of dimensions θ^ℓ with $\ell = 1, \dots, L$ are scale parameters in technology. The bidder with a larger θ^ℓ has a lower marginal cost to make an additional provision of quality in the ℓ th-dimension.

Two auction formats are considered. In a FS auction, the successful bidder receives a payment p . In a SS auction, the successful bidder can freely choose the contracted p and \mathbf{q} as long as the score resulted by the contracted price and the quality level equals the second lowest score in the auction.

The scoring auction game can be equivalently considered as follows. The bidders are asked to submit a scoring-bid $s \in \mathbb{R}$. The lowest-score bidder wins the contract. Only the winner chooses a quality level \mathbf{q} with which the contractor performs the project. The monotonicity of the scoring function implies that, for any score value s , the payment

⁴The argument in this section follows Hanazono et al. (2013).

⁵We relax this assumption in Section 4.

function $P(s, \mathbf{q})$ is defined such that

$$S(P(s, \mathbf{q}), \mathbf{q}) \equiv s.$$

Let s^e be the exercised score. In a FS auction, the exercised score is the winning bidder's score, *i.e.*, $s^e = s$. In a SS auction, it is equal to the second-lowest score. Then, the bidder's problem in a scoring auction is given by

$$\max_{s, q^1, \dots, q^L} [P(s^e, \mathbf{q}) - C(\mathbf{q}|\boldsymbol{\theta})] \Pr\{\text{win}|s\}.$$

We assume that, for any s^e , there exists a unique internal solution $(q^1, \dots, q^L) \in [\underline{q}^1, \bar{q}^1] \times \dots \times [\underline{q}^L, \bar{q}^L]$ that maximizes the bidder's payoff upon winning, $P(s^e, \mathbf{q}) - C(\mathbf{q}|\boldsymbol{\theta})$. Let $q^\ell(s^e, \boldsymbol{\theta})$ denote the maximizer of the bidder's payoff upon winning for each $\ell = 1, \dots, L$ dimension such that

$$q^\ell(s^e, \boldsymbol{\theta}) = \arg \max_{q^\ell} P(s^e, \mathbf{q}) - C(\mathbf{q}|\boldsymbol{\theta}). \quad (1)$$

A sufficient condition for the uniqueness of quality choice is that, for all $\ell = 1, \dots, L$, $P_{q^\ell}(s^e, \mathbf{q}(s^e, \boldsymbol{\theta})) - C_{q^\ell}(\mathbf{q}(s^e, \boldsymbol{\theta})|\boldsymbol{\theta}) = 0$ with $P_{q^\ell q^\ell}(s, \mathbf{q}(s, \boldsymbol{\theta})) - C_{q^\ell q^\ell}(\mathbf{q}(s, \boldsymbol{\theta})|\boldsymbol{\theta}) < 0$. For notational convenience, we define $u(s^e, \boldsymbol{\theta}) = P(s^e, \mathbf{q}(s^e, \boldsymbol{\theta})) - C(\mathbf{q}(s^e, \boldsymbol{\theta})|\boldsymbol{\theta})$. Then, the bidder's maximization problem in a scoring auction is reduced into the following one-dimensional optimization problem:

$$\max_s u(s^e, \boldsymbol{\theta}) \Pr\{\text{win}|s\}. \quad (2)$$

Recall that we have assumed that $P_s(\cdot) = 1/S_p(\cdot) > 0$ and $C_{\theta^0} > 0$. Therefore, the derivatives of $u(\cdot)$ with respect to s^e and θ^0 are given by

$$\begin{aligned} u_s(s^e, \boldsymbol{\theta}) &= P_s(s^e, q(s^e, \boldsymbol{\theta})) > 0, \\ u_{\theta^0}(s^e, \boldsymbol{\theta}) &= -C_{\theta^0}(q(s^e, \boldsymbol{\theta})|\boldsymbol{\theta}) < 0. \end{aligned}$$

It suggests that the scoring auction game is a single-dimensional auction game in which bidders with non-linear utility functions submit scores.

2.2 Equilibrium in a FS auction

A symmetric Bayesian Nash equilibrium in a FS auction with the multi-dimensional type space is analyzed based on the following two additional conditions on the bidder's utility function. The first assumption (Assumption 1) simplifies the analysis of the scoring auction with the multi-dimensional type space, whereas the second assumption (Assumption 2) is required for the existence of an equilibrium in a FS auction.

Let $\underline{u}(s, \theta^0)$ be the payoff of the smallest-scale bidder whose efficiency level is θ^0 so that $\underline{u}(s, \theta^0) = u(s, \theta^0, \underline{\theta}^1, \dots, \underline{\theta}^L)$. Then, the assumption imposed here is summarized as follows.

Assumption 1 (Separability). *There exists a monotonic function $h(\boldsymbol{\theta}) = h^1(\theta^1)h^2(\theta^2) \dots h^L(\theta^L)$ with $h^\ell(\theta^\ell) \geq 1$ and $h(\theta^0, \underline{\theta}^1, \dots, \underline{\theta}^L) = 1$ such that, for any θ^0 , $dh^\ell(\theta^\ell)/d\theta^\ell > 0$ for all $\ell = 1, \dots, L$ and, for all s and θ^0 ,*

$$u(s, \boldsymbol{\theta}) = h(\boldsymbol{\theta})\underline{u}(s, \theta^0). \quad (3)$$

Assumption 1 ensures that the equilibrium bidding strategy is a sole function of θ^0 , *i.e.*, $s(\theta^0)$. Together with the specification of the cost function such that C_{q^ℓ} is decreasing in θ^ℓ for all $\ell = 1, \dots, L$, Assumption 1 implies that bidders with an identical θ^0 but different θ^ℓ never choose the same quality set in equilibrium. The monotonicity of the marginal cost is needed for the identification of the bidder's type from observable s and \mathbf{q} . A detailed discussion is delivered in Section 3.1.

To see that Assumption 1 is sufficient for the bidding strategy $s(\cdot)$ to be independent of θ^ℓ with $\ell = 1, \dots, L$, suppose that two bidders have an identical θ^0 but different θ^ℓ for some or all $\ell = 1, \dots, L$. Let $\boldsymbol{\theta}$ and $\tilde{\boldsymbol{\theta}}$ be their $L + 1$ dimensional signals. The equilibrium bid strategy $s(\cdot)$ maximizes the bidder's expected payoff. The bidders' objective functions are given by

$$\begin{aligned} \max_s h(\boldsymbol{\theta})\underline{u}(s, \theta^0) \Pr\{win|s\}, \\ \max_s h(\tilde{\boldsymbol{\theta}})\underline{u}(s, \theta^0) \Pr\{win|s\}. \end{aligned}$$

Since the two maximization problems are monotonic transforms of each other, the two objective functions are maximized at the same s . This implies that the equilibrium bid strategy $s(\cdot)$ depends solely on θ^0 .

Assumption 1 is interpreted as a generalization of the homothetic cost function. If the scoring function is PQR such that $S(p, \mathbf{q})$, the cost function $C(\mathbf{q}|\boldsymbol{\theta})$ is a homothetic function of $C(\mathbf{q}|\theta^0, \underline{\theta}^1, \dots, \underline{\theta}^L)$, where $h(\boldsymbol{\theta})$ is a multiplier. In other words, $C(\mathbf{q}|\boldsymbol{\theta})$ is homogeneous of degree zero such that

$$C(q^1(s, \boldsymbol{\theta})h^1(\theta^1), \dots, q^L(s, \boldsymbol{\theta})h^L(\theta^L)|\boldsymbol{\theta}) = h(\boldsymbol{\theta})C(\mathbf{q}|\theta^0, \underline{\theta}^1, \dots, \underline{\theta}^L),$$

if the scoring rule is PQR.

Given Assumption 1, only one of all dimensions of the bidder's multi-dimensional signal, θ^0 , associates the strategic interaction in the score choice game. Therefore, the existence of a Bayesian Nash equilibrium in a FS auction only requires that the cross-partial derivative of the log of $u(s^e, \boldsymbol{\theta})$ with respect to s^e and θ^0 is strictly positive.

Assumption 2 (Supermodularity). *The smallest-scale bidder's expected payoff*

$$\underline{u}(s, \theta^0) \Pr\{\text{win}|s\}$$

is log-supermodular.

Note that Assumption 2 is required only in the analysis of a FS auction since a truth-telling dominant strategy equilibrium exists in a SS auction, as will be seen in the next subsection. Note also that all bidder's payoff functions are log-supermodular since Assumption 1 ensures that the cross partial derivative of the log of $u(s^e, \theta)$ is independent of θ^ℓ with $\ell = 1, \dots, L$:

$$\frac{\partial^2 u(s^e, \theta)}{\partial s \partial \theta^0} = \frac{\partial}{\partial \theta^0} \left(\frac{u(s^e, \theta)}{u_s(s^e, \theta)} \right) = \frac{\partial}{\partial \theta^0} \left(\frac{h(\theta) \underline{u}(s^e, \theta^0)}{h(\theta) \underline{u}_s(s^e, \theta^0)} \right) = \frac{\partial^2 h(\theta) \underline{u}(s^e, \theta^0)}{\partial s \partial \theta^0}. \quad (4)$$

Given these assumptions, a symmetric, increasing equilibrium strategy in a FS auction is characterized as follows. Let $s(\theta^0)$ be a symmetric increasing equilibrium in a FS auction. The log-supermodularity of the bidder's utility function is sufficient to guarantee the existence of a strictly increasing Bayesian Nash equilibrium shown by Athey (2001). Then, the bidder's problem (2) is given by

$$\max_s u(s, \theta) [1 - F_0(s^{-1}(s))]$$

in equilibrium. By imposing the symmetric condition, the first-order condition is given by

$$u_s(s(\theta^0), \theta) s'(\theta^0) [1 - F_0(\theta^0)]^{n-1} = u(s(\theta^0), \theta) (n-1) f_0(\theta^0) [1 - F_0(\theta^0)]^{n-2}. \quad (5)$$

Solving the differential equation for $u(s(\theta^0), \theta)$ yields

$$P(s(\theta^0), q(s(\theta^0), \theta)) = C(q(s(\theta^0), \theta) | \theta) + \int_{\theta^0}^{\bar{\theta}^0} C_{\theta^0}(q(s(\tau), \tau) | \tau) \left[\frac{1 - F_0(\tau)}{1 - F_0(\theta^0)} \right]^{n-1} d\tau, \quad (6)$$

which characterizes the equilibrium strategy $s(\theta^0)$ in a FS auction.

2.3 Equilibrium in a SS auction

Let $s_{(2)}$ be the second-lowest score in a SS auction. Then, the bidder's payoff upon winning in a SS auction:

$$u(s_{(2)}, \theta),$$

is independent of his own scoring bid. Since the winning bidder has a non-negative payoff, bidding the break-even score (the minimum score the bidder with type θ makes with a non-negative utility) is a dominant strategy in a SS auction. Therefore, a dominant strategy

equilibrium $s_{\Pi}(\cdot)$ in a SS auction satisfies

$$u(s_{\Pi}(\theta^0), \boldsymbol{\theta}) = 0. \quad (7)$$

As in the case of a FS auction, the equilibrium strategy in a SS auction is independent of θ^ℓ with $\ell = 1, \dots, L$ since $u(s_{\Pi}(\theta^0), \boldsymbol{\theta}) = \underline{u}(s_{\Pi}(\theta^0), \theta^0) = 0$ for any $\boldsymbol{\theta}$.

In the scoring auction, the profit maximizing quality is first-best if the exercised score is equal to the bidder's break-even score.⁶ Therefore, the bidder's quality choice at bidding is always equal to first-best in a SS auction. Let $\mathbf{q}^{FB}(\boldsymbol{\theta})$ be the first-best quality. Under the PQR scoring rule, for instance, $\mathbf{q}^{FB}(\boldsymbol{\theta})$ satisfies

$$C_{q^\ell}(\mathbf{q}^{FB}(\boldsymbol{\theta})|\boldsymbol{\theta})q^{FB,\ell}(\boldsymbol{\theta}) = C(\mathbf{q}^{FB}(\boldsymbol{\theta})|\boldsymbol{\theta}). \quad (8)$$

2.4 Revenue ranking

Revenue ranking is possible in scoring auctions. The exercised score s^e represents the auctioneer's utility from the scoring auction. Hanazono et al. (2013) show that the equivalence regarding expected exercised scores (revenue) does not generally hold in the scoring auction. If we restrict attention to a class of scoring rules that are linear in price *i.e.*, PQR and QL are both included, then the expected exercised score is weakly greater in FS than in SS auctions. In particular, if the scoring rule is PQR, a FS procurement auction creates a higher expected score than a SS procurement auction. Thus, the auctioneer prefers a SS auction if his true preference is PQR and the score represents his/her true preference.

Using the characterization of the equilibrium strategies as well as the equilibrium properties in FS and SS auctions, a series of empirical examinations are taken place in Section 4. As in the theoretical model, risk-neutral bidders and independent private value are assumed.

3 Structural estimation of the scoring auction model

3.1 Identification of the bidder's cost function in a FS auction

If bidders have non-linear utility functions which are unknown to econometricians, the auction model may not be identified from the observed bid data. Guerre et al. (2009), for instance, show that the model of auctions with risk-averse bidders is generally unidentified. A similar problem occurs in the identification of the scoring auction model with a non-quasilinear scoring rule. The induced utility of a risk-neutral bidder, $u(s, \boldsymbol{\theta})$, is generally an unknown non-linear function if the bidder's cost function is unknown. Clearly, the observed $L + 1$ -dimension scoring auction data do not allow us to identify the cost function

⁶The profit maximizing quality is always first-best under the QL scoring rule even if the bidder's score is strictly greater than the break-even score (Che (1993)).

with $L + 2$ or more dimensional parameters. Therefore, we choose a semi-parametric procedure, assuming that the bidder's cost function is parametric and that it is known to econometricians except for $L + 1$ dimensional parameters. Then, with Assumption 1, a non-linear function of the bidder's induced utility can be characterized so that we can estimate the latent parameters from the $L + 1$ dimensional bid data (p, \mathbf{q}) .

We first show that, given the assumptions discussed in the previous section, an identification of the cost function parameters, $\boldsymbol{\theta} = (\theta^0, \dots, \theta^L)$, is possible as follows. Since the equilibrium strategy depends only on θ^0 in a FS auction and since $s(\theta^0)$ is a strictly increasing function of θ^0 , the inverse function of the equilibrium strategy, $s^{-1}(\cdot)$, exists. Therefore, the distribution of parameter θ^0 is identified from observed score s . Next, from (1), ignoring boundary solution (i.e., $q^\ell = \underline{q}^\ell$ or \bar{q}^ℓ for some $\ell = 1, \dots, L$), the optimal quality \mathbf{q} satisfies $P_{q^\ell}(s, \mathbf{q}) = C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta})$ for all $\ell = 1, \dots, L$. Note that the value of $P_{q^\ell}(s, \mathbf{q})$ is observable from observed score s and observed quality \mathbf{q} . Furthermore, since θ^0 can be recovered from observed score s , $C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta})$ is known up to $(\theta^1, \dots, \theta^L)$. If $C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta})$ is a strictly decreasing function of θ^ℓ , the function $y(\theta^\ell) = C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta})$ has its inverse. Therefore, if $C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta})$ is a strictly decreasing function of θ^ℓ for all $\ell = 1, \dots, L$, parameter θ^ℓ is also identified from observed score s and quality \mathbf{q} . The following proposition summarizes this point.

Proposition 1. *We define that a distribution $G(\cdot)$ of observed scores (s_1, \dots, s_n) is rationalized by the distribution of the bidder's multi-dimensional private signal $F(\cdot)$ in the scoring auction if $G(\cdot)$ is the distribution of the equilibrium score bid. Then, the model of scoring auctions with symmetric risk-neutral bidders is identified if i) the scoring rule is independent, ii) the bidder's utility function, $u(s, \boldsymbol{\theta})$ is log-supermodular, and iii) the marginal cost of the bidder for each quality dimension ℓ is monotone in θ^ℓ for all $\ell = 1, \dots, L$.*

We have two remarks on the monotonicity condition of $C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta})$. First, in the PQR scoring rule, Assumption 1 implies that $C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta})$ is a strictly decreasing function of θ^ℓ for all $\ell = 1, \dots, L$. Furthermore, in the PQR scoring rule, we have

$$mC(\mathbf{q}|\boldsymbol{\theta}) = C(m\mathbf{q}|\theta^0, m\theta^1, \dots, m\theta^L)$$

for all $m > 0$. Therefore, we gain

$$C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta}) = C_{q^\ell}(m\mathbf{q}|\theta^0, m\theta^1, \dots, m\theta^L). \quad (9)$$

If $m > 1$, since $C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta})$ is a strictly convex function of \mathbf{q} , $C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta})$ is a strictly increasing function of \mathbf{q} . That is,

$$C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta}) < C_{q^\ell}(m\mathbf{q}|\boldsymbol{\theta}). \quad (10)$$

From (9) and (10),

$$C_{q^\ell}(m\mathbf{q}|\boldsymbol{\theta}) > C_{\mathbf{q}}(m\mathbf{q}|\theta^0, m\theta^1, \dots, m\theta^L) = C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta}).$$

Therefore, $C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta})$ is strictly decreasing with respect to θ^ℓ . Similarly, if $m \in (0, 1]$, $C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta})$ is a strictly decreasing function of \mathbf{q} . Thus, we do not have to impose any additional assumptions to identify parameter vector $\boldsymbol{\theta}$ under Assumption 1.

Second, in general, Assumption 1 does not imply the monotonicity condition of $C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta})$. In other words, in general, the monotonicity condition can not be excluded. A typical example is QL awarding rule.

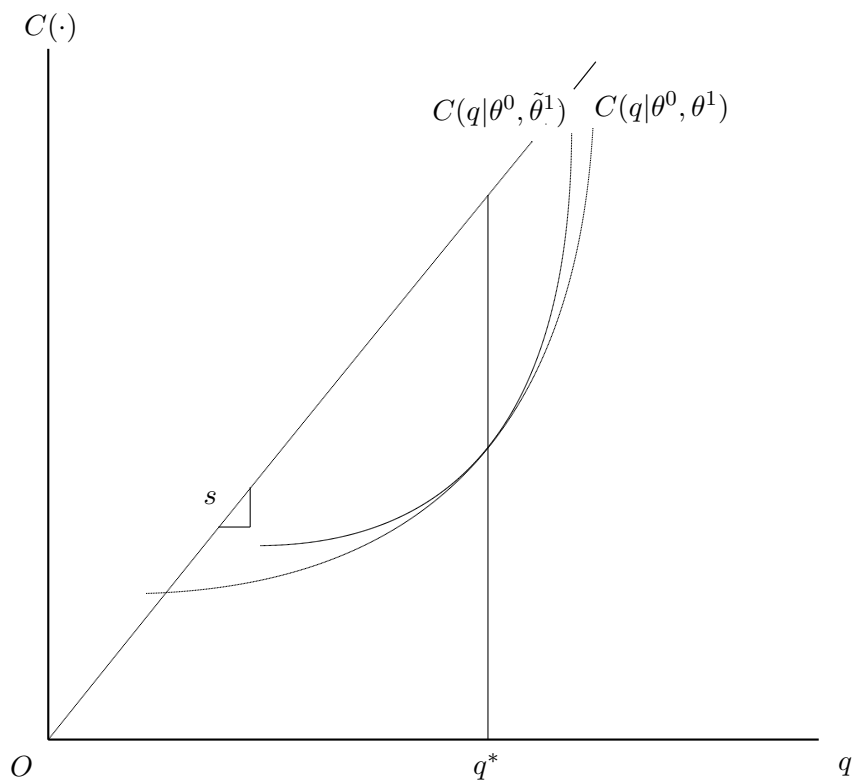


Figure 1: Example of non-identifiable parameter (PQR scoring rule)

Figure 1 shows an example that parameter $\boldsymbol{\theta} = (\theta^0, \theta^1)$ is not identifiable from the observed score s and quality $q \in \mathbb{R}_+$. In this example, a bidder with cost function $C_q(q|\theta^0, \theta^1)$ submits quality q^* and score s . Similarly, a bidder with cost function $C_q(q|\theta^0, \tilde{\theta}^1)$ with $\theta^1 \neq \tilde{\theta}^1$ also submits quality q^* and score s . Therefore, from observed quality q^* and score s , parameter θ^1 is not identified.

3.2 Estimation for the distribution of cost function parameter vector θ

The estimation of θ^0 and θ^1 from the observed data, $s_{i,t}$, $q_{i,t}$ proceeds as follows. By the equilibrium bidding function, we have

$$s(\theta^0) = s_{i,t}. \quad (11)$$

In addition, from (1), ignoring boundary solution (i.e., $q = \underline{q}$ or \bar{q}), optimal quality q satisfies $P_{q^\ell}(s, q) = C_{q^\ell}(\mathbf{q}|\boldsymbol{\theta})$ for all $\ell = 1, \dots, L$. Therefore, we obtain the following equation:

$$P_{q^\ell}(s_{i,t}, \mathbf{q}_{i,t}) = C_{q^\ell}(\mathbf{q}_{i,t}|\boldsymbol{\theta}_{i,t}) \text{ with } \ell = 1, \dots, L. \quad (12)$$

Let $\hat{\boldsymbol{\theta}}_{i,t} = (\hat{\theta}_{i,t}^0, \dots, \hat{\theta}_{i,t}^L)$ be the solution of the simultaneous equations (11) and (12). Since $s(\theta^0)$ is strictly increasing, parameter θ^0 could be obtained, using the inverse function, $s^{-1}(\cdot)$. Furthermore, the assumption that $C_q(\mathbf{q}|\boldsymbol{\theta})$ is monotone in θ^ℓ for all $\ell = 1, \dots, L$ implies that, for given $s_{i,t}$, $q_{i,t}$, and $\theta_{i,t}^0$, parameter $\theta_{i,t}^\ell$ is obtained for all $\ell = 1, \dots, L$. Therefore, $\hat{\boldsymbol{\theta}}$ would be estimated.

Unfortunately, the inverse function $s^{-1}(\cdot)$ can not be obtained analytically in general. It could be possible to obtain the inverse function $s^{-1}(\cdot)$ directly from (6) with a numerical computation. However, given the fact that the distribution of $\boldsymbol{\theta}$ is unknown, it is a computational burden. Therefore, we estimate θ from the first-order condition instead of solving the equilibrium strategy explicitly.

Let $G(s)$ be the cumulative distribution function of $s_1(\theta^0)$ and $g(s)$ be its density. Then, letting $s^{-1}(\cdot)$ be the inverse function of $s(\cdot)$ such that $s^{-1}(s(\theta^0)) = \theta^0$, we have $G(s) = 1 - F_0(s^{-1}(s))$. By the inverse function theorem, $g(s) = f_0(s^{-1}(s))/s'(\theta^0)$ holds. From the bidder's first-order condition in a FS auction, (5), and given an optimal quality $\mathbf{q}(s, \boldsymbol{\theta})$ and parameter vector $\boldsymbol{\theta}$, we obtain

$$H(s, \theta^0) \equiv \frac{u(s, \boldsymbol{\theta})}{u_s(s, \boldsymbol{\theta})} - \frac{1}{n-1} \frac{1-G(s)}{g(s)} = 0, \quad (13)$$

where Assumption 1 (separability) ensures that $H(\cdot)$ depends only on dimension zero signal, θ^0 . In equilibrium, (13) satisfies $H(s(\theta^0), \theta^0) \equiv 0$ for all θ^0 . In other words, the first-order condition constitutes an implicit function that uniquely defines the inverse of a strictly increasing equilibrium strategy $s(\theta^0)$. Therefore, with the use of (13), we will estimate θ^0 from the observed score s . The following proposition summarizes this result.

Proposition 2. *Let $\Delta = \frac{d}{ds} \frac{1}{n-1} \frac{1-G(s)}{g(s)}$. In addition, let $G(s_1, \dots, s_n)$ be the joint distribution of (s_1, \dots, s_n) with support $[s, \bar{s}]$. Then, there exists a distribution of bidders' private signal $F(\cdot)$ such that $G(s_1, \dots, s_n)$ is the distribution of the equilibrium scores in a FS auction with symmetric, risk-neutral bidders if*

1. $G(s_1, \dots, s_n) = \prod_{i=1}^n G(s_i)$.

2. Assumption 1 and 2 are met.

3. $\underline{u}_s^2 \cdot \Delta - \underline{u} \cdot \underline{u}_{ss} > 0$.

Moreover, the following implicit function,

$$H(s_{i,t}, \theta_{i,t}^0) \equiv \frac{\underline{u}(s_{i,t}, \theta_{i,t}^0)}{\underline{u}_s(s_{i,t}, \theta_{i,t}^0)} - \frac{1}{n-1} \frac{1 - G(s_{i,t})}{g(s_{i,t})} = 0,$$

uniquely defines a strictly increasing and differentiable function which coincides the inverse bidding strategy $s^{-1}(s_{i,t}) = \theta_{i,t}^0$

Proof. By the implicit function theorem, there are a neighborhood U of θ^0 and a unique C^1 function φ such that $\theta^0 = \varphi(s)$ and $H(s, \varphi(s)) = 0$ for all $\theta^0 \in U$. Furthermore, the derivative of φ at θ^0 is

$$\varphi'(s) = -\frac{H_s(s, \theta^0)}{H_{\theta^0}(s, \theta^0)}.$$

Since $H_{\theta^0} < 0$ by log-supermodularity, the implicit function theorem suggests that $\varphi(s)$ is unique. Therefore, $\varphi(s)$ must be equivalent with the inverse bidding strategy s^{-1} if the observed score is the outcome of a symmetric increasing bidding strategy $s(\theta^0)$. \square

Several observations are made here. First, $H_s(s(\theta^0), \theta) > 0$, or equivalently,

$$\underline{u}_s^2 \cdot \Delta - \underline{u} \cdot \underline{u}_{ss} > 0, \tag{14}$$

holds if $G = F(s^{-1}(\theta))$.⁷ Therefore, (14) is necessary for a distribution $G(\cdot)$ to be rationalized by the scoring auction model. Note that $\underline{u}_{ss} = 0$ if the scoring rule is QL whereas $\underline{u}_{ss} > 0$ if the scoring rule is PQR as shown in Hanazono et al. (2013). Therefore, learning the restriction by the scoring auction model with a non-quasilinear scoring rule requires the estimate of θ^0 . Regardless of whether the observables are rationalized, we can uniquely

⁷For an arbitrary $\tilde{\theta}^0 \in [\theta^0, \bar{\theta}^0]$ and a strictly increasing strategy $s(\theta)$, we have

$$H(s(\tilde{\theta}^0), \theta^0) \equiv \frac{\underline{u}(s(\tilde{\theta}^0), \theta^0)}{\underline{u}_s(s(\tilde{\theta}^0), \theta^0)} - \frac{1}{n-1} \frac{1 - G(s(\tilde{\theta}^0))}{g(s(\tilde{\theta}^0))}.$$

Suppose that $G(s) = F(s^{-1}(s))$. Then, the log-supermodularity implies that $H_{\theta^0} < 0$ for all θ^0 . Therefore, for all θ^0 , we have

$$H(s(\tilde{\theta}^0), \theta^0) \leq 0 \text{ if } \tilde{\theta}^0 \geq \theta^0.$$

Suppose that $s(\cdot)$ is strictly increasing. Then, for all θ^0 , we have

$$H(s, \theta^0) \leq 0 \text{ if } s \geq s(\theta^0),$$

in a neighborhood of $(\theta^0, s(\theta^0))$. Hence, $H_s(s, \theta_{i,t}^0) < 0$ at $s = s(\theta_{i,t}^0)$.

estimate θ^0 from the observable s if Assumption 2 (log-supermodularity) is met since the implicit function theorem ensures that φ is continuously differentiable.

Second, since we have that $\underline{u}/\underline{u}_s = s - k(s, \theta^0)$, the first-order condition (13) also gives $k(s, \theta)$ explicitly. However, estimating k is not, in general, sufficient to obtain the parameter θ^0 ; if $u_{ss} < 0$, then the equilibrium quality-adjusted cost $k(s(\theta^0), \theta^0)$ may not be strictly increasing in θ^0 .⁸ In other words, a Bayesian Nash equilibrium is characterized in a FS auction in general even if the equilibrium s is not strictly increasing in the equilibrium k (Hanazono et al. (2013) Theorem 2). Therefore, no one-to-one mapping is guaranteed from the estimated k to the private signal θ^0 in the scoring auction model.

Finally, Assumption 1 makes it simple for H to be obtained explicitly from the score distribution. Without Assumption 1, the first-order condition depends on an unknown parameter θ^ℓ with $\ell = 1, \dots, L$. Because of Assumption 1, we can set $\theta^\ell = \underline{\theta}^\ell$ for all $\ell = 1, \dots, L$ and obtain a pseudo function $H(s, \theta^0)$, which uniquely defines the inverse strategy s^{-1} , regardless of the true value of $(\theta^1, \dots, \theta^L)$. Thus, given Assumption 1 and Assumption 2, using $H(\cdot)$ is a general procedure to estimate θ^0 from the data of a scoring auction with an independent scoring rule.

Since score s is observable, the cumulative distribution function of s , $G(\cdot)$, and its density, $g(\cdot)$, can be estimated by the standard kernel estimator. Let there be T scoring auction samples, each indexed by $t = 1, \dots, T$. The observations involve heterogeneity in the number of bidders and auction heterogeneity including location, time, and the maximum quality level. To control these auction specific effect, let n_t and x_t denote the number of bidders and the covariates of auction t , respectively. Then, the kernel estimator for the distribution function of s given the number of bidder n and the covariate vector $x = (x_1, \dots, x_d)$ is provided by

$$1 - \hat{G}(s|n, x) = \frac{1}{Th_{G_n}h_{G_x}^d} \sum_{t=1}^T \frac{1}{n} \sum_{i=1}^n \mathbf{1}(s \leq s_{i,t}) K_G\left(\frac{n-n_t}{h_{G_n}}, \frac{x_1-x_{1,t}}{h_{G_x}}, \dots, \frac{x_d-x_{d,t}}{h_{G_x}}\right), \quad (15)$$

where $\mathbf{1}(\cdot)$ is the indicator function, K_G is a kernel with bounded support, h_{G_n} and h_{G_x} are some bandwidths, and d is the number of dimensions of covariate vector x . Similarly, the kernel density estimator for the density function of score s given the number of bidder n and the covariate vector x is given by

$$\hat{g}(s|n, x) = \frac{1}{Th_s h_{g_n} h_{g_x}^d} \sum_{t=1}^T \frac{1}{n} \sum_{i=1}^n K_g\left(\frac{s-s_t}{h_s}, \frac{n-n_t}{h_{g_n}}, \frac{x_1-x_{1,t}}{h_{g_x}}, \dots, \frac{x_d-x_{d,t}}{h_{g_x}}\right), \quad (16)$$

where K_g is a kernel with a bounded support, and h_s , h_{g_n} and h_{g_x} are some bandwidths. In practice, the discrete variables such as the number of bidders and the maximum quality level are smoothed out in the way discussed in Li and Racine (2006). Using the estimated

⁸See Hanazono et al. (2013) for an example of the non-monotonic $k(\cdot)$.

distribution function $\hat{G}(s|n, x)$ and the estimated density function $\hat{g}(s|n, x)$, we have the implicit function (13) as

$$\hat{H}(s_{i,t}, \hat{\theta}_{i,t}) \equiv \frac{u(s_{i,t}, \hat{\theta}_{i,t})}{\underline{u}_s(s_{i,t}, \hat{\theta}_{i,t})} - \frac{1}{n-1} \frac{1 - \hat{G}(s_{i,t}|n_t, x_t)}{\hat{g}(s_{i,t}|n_t, x_t)} = 0. \quad (17)$$

Therefore, we can gain the plausible estimator of θ^0 .

Then the distribution of θ can be estimated by the standard kernel method. The conditional distribution function of θ given the covariate vector x is estimated by

$$\hat{F}(\theta|x) = \frac{\frac{1}{Th_{F_x}^d} \sum_{t=1}^T \mathbf{1}(\theta^0 \leq \theta_{i,t}^0, \dots, \theta^L \leq \theta_{i,t}^L) K_F\left(\frac{x_1 - x_{1,t}}{h_{F_x}}, \dots, \frac{x_d - x_{d,t}}{h_{F_x}}\right)}{\frac{1}{Th_{F_x}^d} \sum_{t=1}^T K_F\left(\frac{x_1 - x_{1,t}}{h_{F_x}}, \dots, \frac{x_d - x_{d,t}}{h_{F_x}}\right)},$$

where K_F is a kernel with bounded support and h_{F_x} is some bandwidth. Similarly, the kernel density estimator for the conditional density function of θ given the covariate vector x is given by

$$\hat{f}(\theta|x) = \frac{\frac{1}{Th_{f_0} \dots h_{f_L} h_{f_x}^d} \sum_{t=1}^T K_f\left(\frac{\theta^0 - \theta_{i,t}^0}{h_{f_0}}, \dots, \frac{\theta^L - \theta_{i,t}^L}{h_{f_L}}, \frac{x_1 - x_{1,t}}{h_{f_x}}, \dots, \frac{x_d - x_{d,t}}{h_{f_x}}\right)}{\frac{1}{Th_{f_x}^d} \sum_{t=1}^T K_f\left(\frac{x_1 - x_{1,t}}{h_{f_x}}, \dots, \frac{x_d - x_{d,t}}{h_{f_x}}\right)}$$

where K_f is a kernel with bounded support, and h_{f_0} , h_{f_1} and h_{f_x} are some bandwidths. The property of the estimator $\hat{f}(\theta|x)$ is examined in Guerre et al. (2000).

3.3 Simulation experiments

To illustrate the our identification procedure, a numerical simulation is conducted. We have $T = 500$ auctions, two bidders in each. The cost function is assumed to be

$$C(q|\boldsymbol{\theta}) = (1 + \theta^1) \left[\left(\frac{q}{1 + \theta^1} - 1 \right)^2 + \theta^0 \right].$$

Signal $\boldsymbol{\theta}$ is identically and independently distributed with the marginal distributions of θ^0 and θ^1 being Uniform ($U(0, 1)$) and Beta ($B(3, 2)$), respectively. Then, we generate a thousand of sample bids $s(\theta^0)$ based on the random samples $\boldsymbol{\theta}$, where the equilibrium bidding function is given by ⁹

$$s(\theta^0) = -2 + \sqrt{2\theta^0 + 6},$$

$$q(s(\theta^0), \boldsymbol{\theta}) = (1 + \theta^1) \left[\frac{s(\theta^0)}{2} + 1 \right].$$

⁹See Appendix A for the derivation of the equilibrium strategy.

We then recover the private signal θ . We follow Guerre et al. (2000) for the non-parametric estimation: the use of the triweight kernel and the selection of the bandwidth. The estimated density and distribution are, respectively, given as follows. The results imply that our non-parametric estimation method identifies the private signals from bid data.

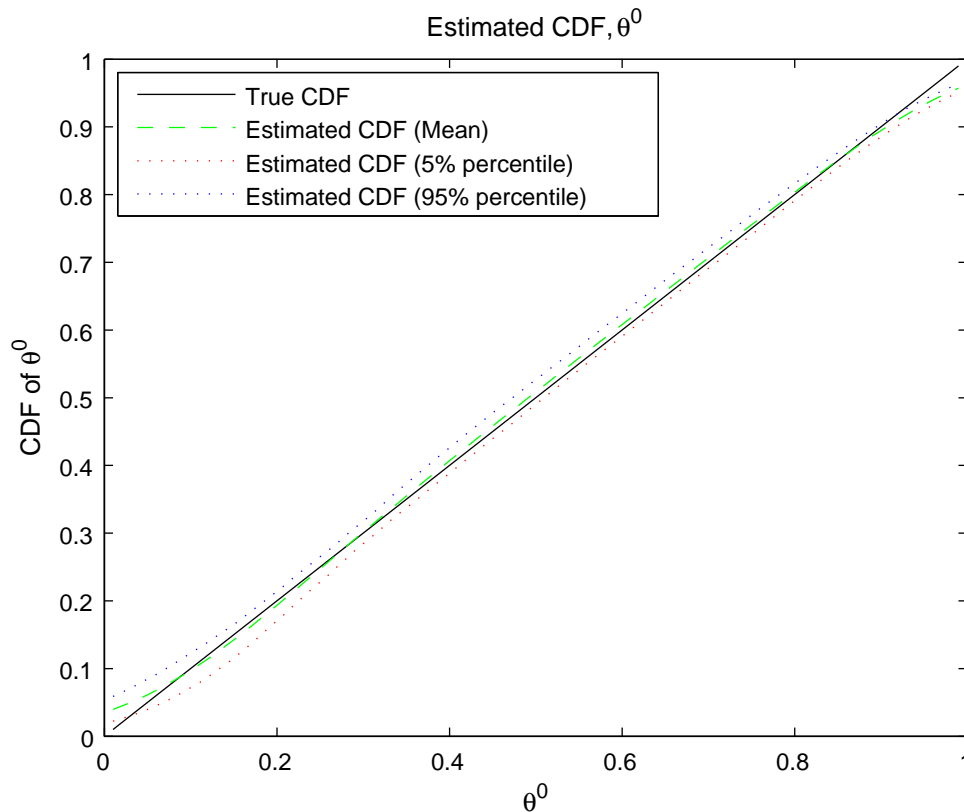


Figure 2: Estimated CDF of θ^0 [Uniform(0,1)]

4 Empirical experiments

4.1 Data

The data used in our analysis contain the bid results of the procurement auctions for civil engineering projects by Ministry of Land, Infrastructure, and Transportation (MLIT) in Japan from April 2010 through January 2013. The number of contracts awarded was 7,538 during this period.

MLIT posts the bid results on the Public Works Procurement Information Service (PPI) website.¹⁰ The information available in PPI includes the names of procurement

¹⁰The address is “<http://www.ppi.go.jp>.”

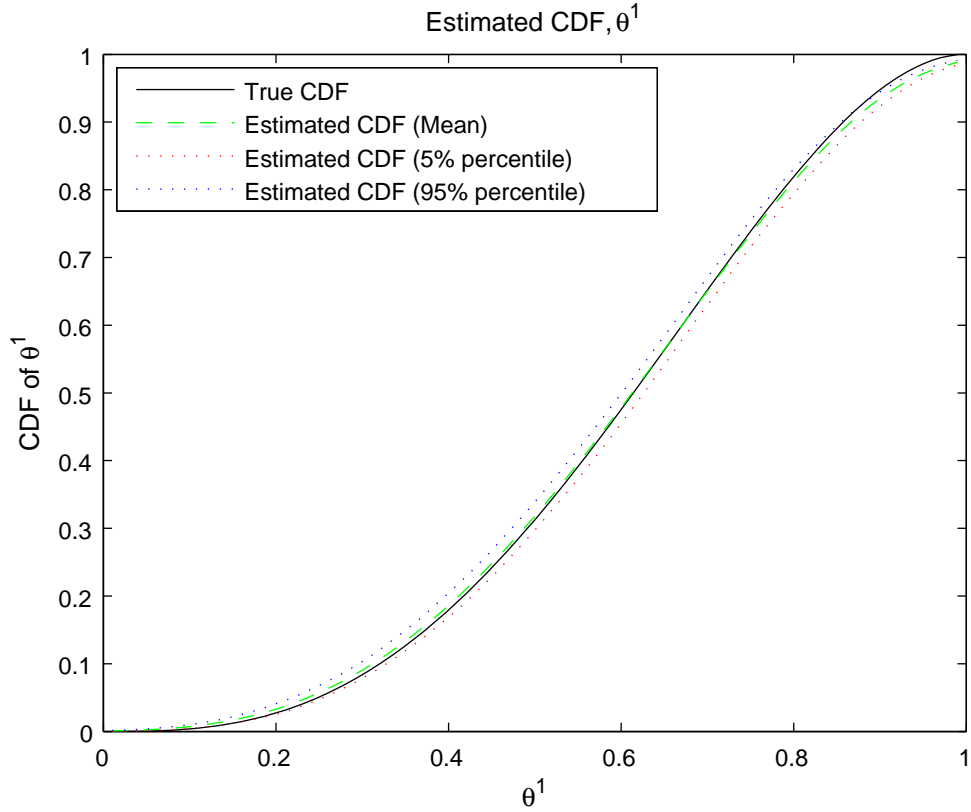


Figure 3: Estimated CDF of θ^1 [Beta(3, 2)]

buyers (local branch names), project names, project types, dates of auctions, engineer's estimates, auction formats (open competitive bidding, invited bidders, scoring or price-only auctions), and submitted bids with the bidder's name.¹¹

MLIT procures 21 types of construction works including civil engineering (or heavy and general construction works), buildings, bridges, paving, dredging, and painting. The amount of civil engineering projects is approximately 750 billion yen a year, which accounts for approximately 54 percent of the entire expenditure of the ministry as well as for approximately 7 percent of the public construction investment in the country.

4.1.1 Percentage bids

Most of the procurement contracts of the civil engineering projects (7,489 out of 7,538) are allocated through scoring auctions. Therefore, we remove the data on price-only auctions. In MLIT scoring auctions, the bidder with the highest-scoring bid wins the project in which the scoring bid is calculated as the factor bid divided by the price bid. The factor

¹¹The information concerning work location is not generally available.

bid consists of multiple components, such as noise level, completion time, and experience.

The dataset records each bidder’s quality bid Q as a number. The lower bound of the factor bids is 100 for all auctions, and the upper bound is 110 through 200 depending on the auction. In practice, each bidder submits a technical proposal that is converted into the factor bid according to the publicly announced tender notice for the auction. The bidder proposing nothing has a factor bid equal to 100. The method of converting a technical proposal into a factor bid differs for each project. For instance, every one decibel reduction in noise accounts for five additional factor bid points.

To incorporate the data of scoring auctions into the model, let B_i and Q_i be the values of the price and factor bids. Let S_i be bidder i ’s score. Under the price–factor ratio scoring rule, $S_i = B_i/Q_i$. To obtain a percentage score bid, let \bar{B} and \underline{Q} be the engineer’s estimated cost and the factor bid evaluating nothing (the lowest possible factor bid), respectively. Then, a base score \bar{S} is defined such that $\bar{S} \equiv \bar{B}/\underline{Q}$. Then, the percentage scoring bid of bidder i is defined as

$$s_i = \frac{S_i}{\bar{S}}. \quad (18)$$

The winner is the bidder with the lowest percentage scoring bid. Let T denote the number of procurement contracts to be auctioned off by the buyer. Furthermore, let $S_{(1),t}$, \bar{S}_t , and \bar{B}_t be the winning bidder’s score, the base score, and the engineer’s estimated price in auction $t = 1, \dots, T$, respectively. Our model assumes that the scoring rule represents the procurement buyer’s utility. Thus, a higher value-for-money contract (Q/B is higher) implies a contract with a lower *quality-adjusted* cost (B/Q is lower), where the winning score is the quality-adjusted procurement cost. The effective procurement cost of purchasing T contracts is thus given by $\sum_{t=1}^T S_{(1),t}$. In our data, \underline{Q} is normalized to be 100 for all T projects. Hence, the average of the winning percentage scoring bid as

$$\frac{1}{T} \sum_{t=1}^T s_{(1),t}, \quad (19)$$

where $s_{(1),t} = S_{(1),t}/\bar{S}_t$. In what follows, this amount is considered to be the effective procurement cost.

4.1.2 Covariates

The sample auction data involve significant heterogeneity such as the number of bidders, the project size, the maximum quality level. Using the percentage bid somehow mitigates these auction specific heterogeneity but not perfectly. Therefore, we introduce a covariate vector x to capture the residual auction specific effects. In our analysis, the covariates include the maximum quality level, auction date, and the log of engineer’s estimated costs

(as a proxy of project sizes).

4.2 Specifications under the PQR scoring rule with a parametric cost function

4.2.1 Estimation of θ

Let us assume that the cost function we estimate is parameterized with the following two-dimensional signal $\theta = (\theta^0, \theta^1)$ as

$$C(q|\theta) = \theta^1 \left[\left(\frac{q}{\theta^1} - \alpha \right)^\beta + \theta^0 \right], \quad (20)$$

with $q = \alpha\theta^1$. We assume that $\alpha = 1$ and that the cost function is quadratic, cubic, and quartic polynomials so that β is either 2, 3, or 4.

As described before, θ^0 represents the efficiency parameter; the lower is θ^0 , the lower is the bidder's cost given all other things are constant. The second dimension θ^1 represents the scale parameter; the higher is θ^1 , the greater is the bidder's quality provision level at the break-even score even if the value of the break-even score is constant. Cost functions with an identical θ^0 are homothetic with each other. Consequently, the marginal cost is monotonic in θ_1 .

A pseudo implicit function that provides the inverse bidding function is given by

$$H(s, \theta^0) = s - \frac{\theta^1}{q(s, \theta^0, \theta^1)} \left[\left(\frac{q(s, \theta^0, \theta^1)}{\theta^1} - \alpha \right)^\beta + \theta^0 \right] - \frac{1 - G(s)}{(n-1)g(s)}.$$

Here, $q(s, \theta^0, \theta^1)$ is the solution of $\arg \max_q P(s_{i,t}, q) - C(q|\theta^0, \theta^1)$, which is explicitly obtained by

$$q(s, \theta^1) = \theta^1 \left[\left(\frac{s}{\beta} \right)^{\frac{1}{\beta-1}} + \alpha \right].$$

This equation is also used for the estimation of θ^1 , with the fact that the observation, $q_{i,t}$, must satisfy $q(s_{i,t}, \theta_{i,t}) = q_{i,t}$. Therefore, with the observations, $q_{i,t}$ and $s_{i,t}$, and the estimated distribution and density \hat{G} and \hat{g} , we have

$$\hat{\theta}_{i,t}^0 = \left[s_{i,t} - \frac{1 - \hat{G}(s_{i,t})}{(n-1)\hat{g}(s_{i,t})} \right] \cdot \left[\left(\frac{s_{i,t}}{\beta} \right)^{\frac{1}{\beta-1}} + \alpha \right] - \left(\frac{s_{i,t}}{\beta} \right)^{\frac{\beta}{\beta-1}},$$

$$\hat{\theta}_{i,t}^1 = \frac{q_{i,t}}{\left(\frac{s_{i,t}}{\beta} \right)^{\frac{1}{\beta-1}} + \alpha}.$$

For the estimation of \hat{G} and \hat{g} , the following tri-weight kernel is used:

$$K(u) = \frac{35}{32}(1 - u^2)^3 \mathbf{1}(|u| < 1).$$

with the bandwidth As usual, the bandwidths, h_s and h_x , are given by the so-called rule of thumb; $h_s = \eta_s(\sum_{k=1}^m n_k)^{-1/6}$ and $h_x = \eta_x(\sum_{k=1}^m n_k)^{-1/6}$, where $\eta_s = 2.978 \times 1.06\hat{\sigma}_s$ and $\eta_x = 2.978 \times 1.06\hat{\sigma}_x$. Both $\hat{\sigma}_s$ and $\hat{\sigma}_x$ are sample variances of the normalized scoring bids and the observed covariate.

From the pseudo-values of θ^0 , we compute the quality-adjusted costs, $\hat{k}_{i,t} = s_{i,t} - u(s_{i,t}, \hat{\theta})/u_s(s_{i,t}, \hat{\theta})$. Corollary 1 in Hanazono et al. (2013) suggests that, under an IPV environment, the expectation of the lowest scoring bid coincides with the expectation of the second-lowest bidder's k . The average of the obtained 6,088 pseudo values of the second-lowest bidders' $\hat{k}_{i,t}$ is 0.583358. The average of the winning bidder's scores is 0.583358. Therefore, our estimation result is in line with the theoretical prediction.

The following figures are the estimated joint density functions assuming that the cost function is the quadratic polynomial ($\beta = 2$). Axis x (horizontal) and y (depth) represent θ^0 and θ^1 , respectively. Recall that we parameterize the cost function so that the cost function shifts up vertically as the efficiency parameter θ^0 rises. Given this specification, a strong negative correlation is observed between θ^0 and θ^1 ($R^2 = -0.8657$), suggesting that more (less) efficient supplies tend to be larger (smaller).

4.2.2 Rationalizability

The scoring auction model imposes an additional restriction on the observations such that $H_s(s_{i,t}, \theta_{i,t}) > 0$. In this subsection, we focus on the restriction so that the scoring auction data is rationalizable. Since H_s contains the latent variable θ^0 , the restriction is not directly obtained from the observations and their distributions. Furthermore, our observations include covariates, which also prevents us to obtain the restriction explicitly from the data. Therefore, we choose to check whether the estimated $\theta_{i,t}$ s are indeed strictly increasing in $s_{i,t}$ in each auction.

We have 6,115 auction samples from which θ s are effectively obtained. Among these, 22 auctions, accounting just for 0.36 % of all auction samples, exhibit non-monotonic $\hat{\theta}^0$ with respect to s . Except for one auction, the non-monotonicity is observed in a pair of two bidders such that one bids a lower score but the bidder's θ^0 is estimated to be higher than that of the other. The observed scores resulting in the non-monotonic θ^0 are relatively close to the lower bound of the observed scores. It is well-known that the non-parametric estimation suffers from biases close to the boundary. Therefore, it is hard to conclude that the non-monotonicity occurs simply because the auction samples are not rationalizable.

On the other hand, the rest of the auction samples exhibit a strict monotonicity between the observed scores and the estimated θ^0 . Hence, we conclude that our scoring auction

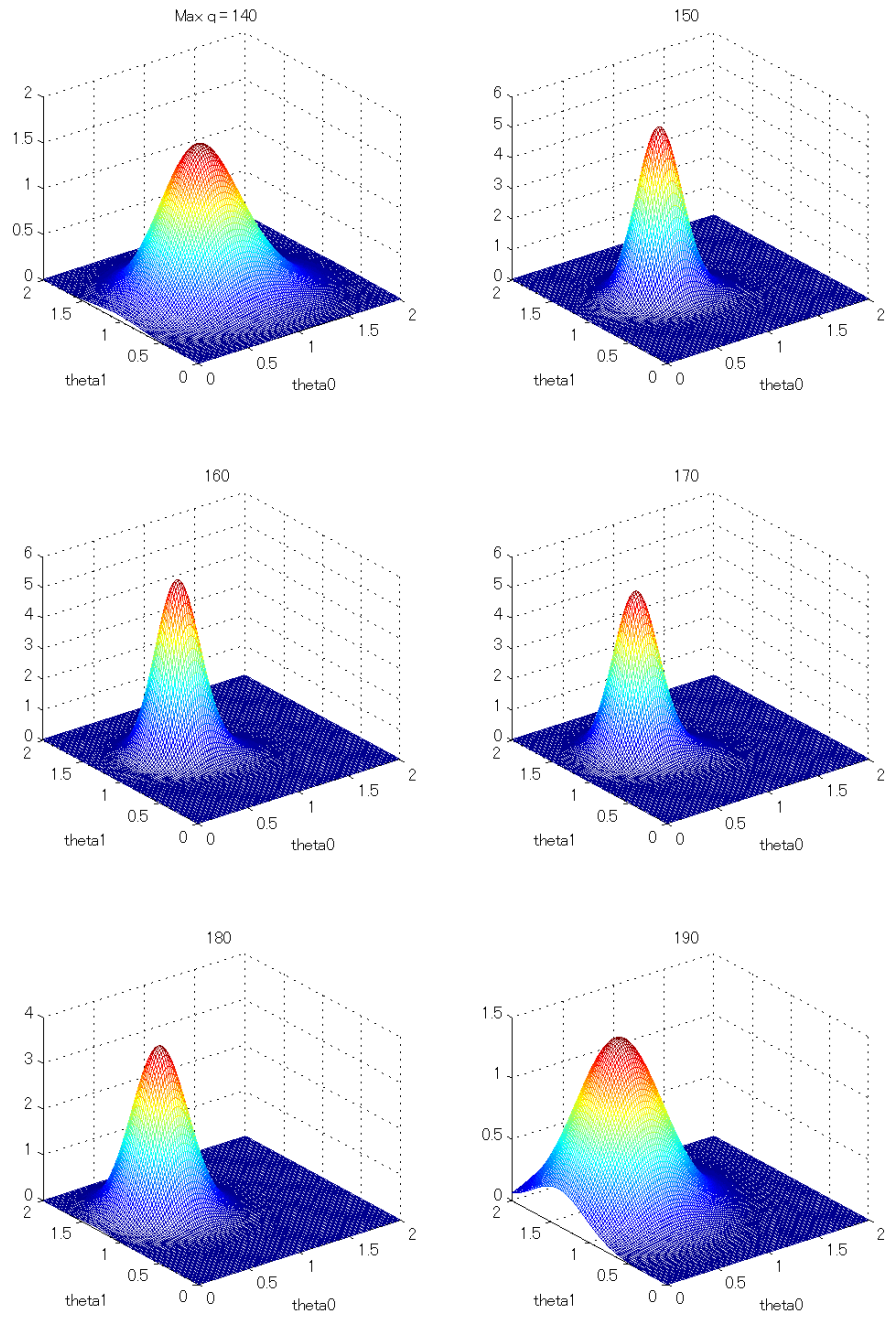


Figure 4: Estimated PDF (3D)

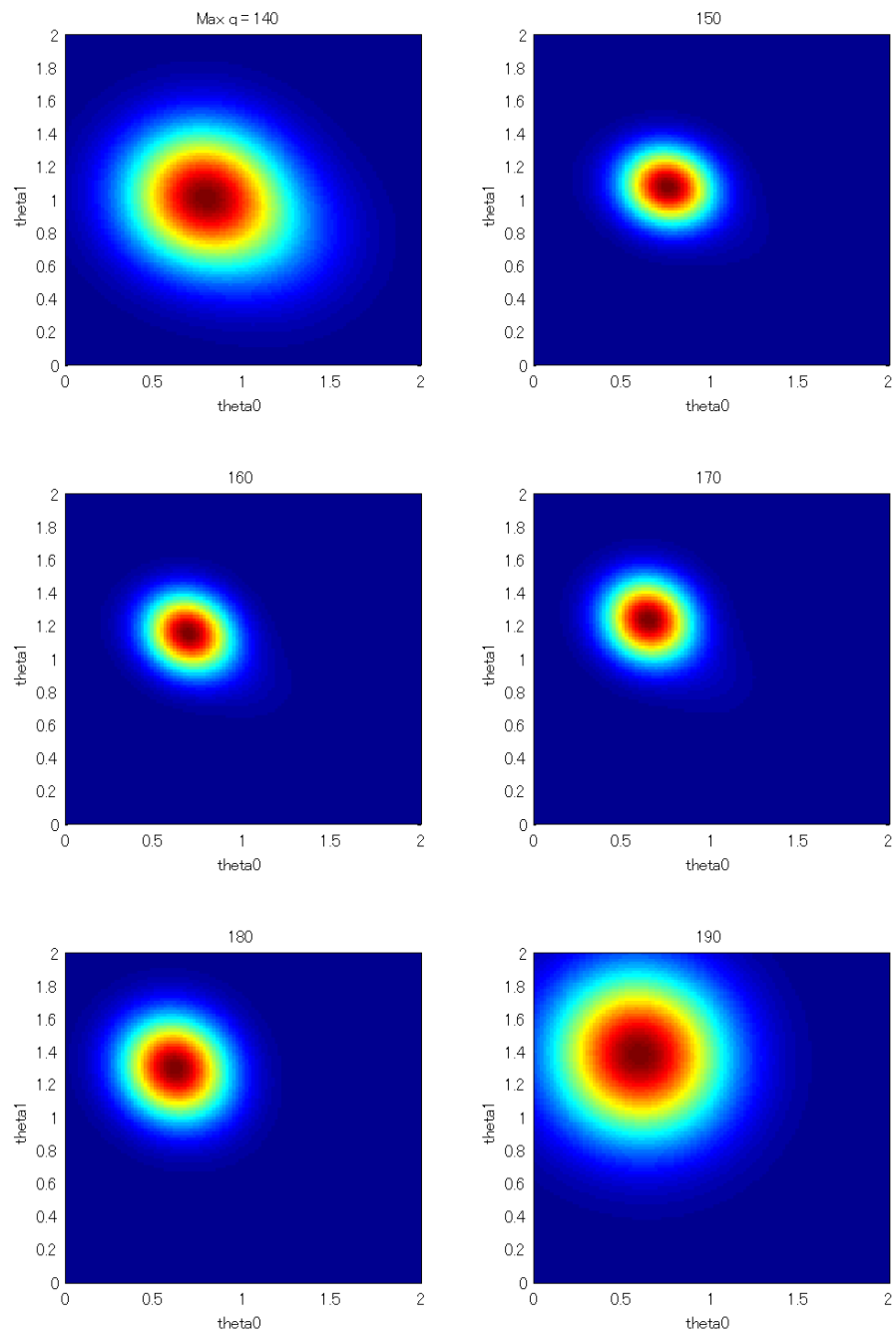


Figure 5: Estimated PDF (Pseudo Color)

data is rationalizable from the scoring auction model with symmetric risk-neutral bidders.

4.3 Counterfactual analyses

4.3.1 First-price vs. FS auctions

We first explore the effect of the use of scoring auctions. As Milgrom (2004) addressed, one of the appeals of multi-parameter auctions is that bidders increase profits without reducing the auctioneer’s utility. Our first empirical examination thus focuses on measuring how much the use of scoring auctions raises the procurement buyer’s utility.

Given the estimated bidder’s private information, bidders’ costs are computed for $q = 1.3, 1.4,$ and 1.5 . Then, to predict the contract prices when second-price (SP) auctions are used, the second-lowest costs are selected from all auction samples. Then, the buyer’s utility of the contract is measured by the selected second-lowest cost divided by q , where $q = 1.3, 1.4,$ or 1.5 . Since cost functions are differentiated by $\beta = 2, 3,$ and 4 , nine types of price-only auctions are simulated.

Table 1 compares the procurement buyer’s quality-adjusted procurement costs (price quality ratios) in the observed FS auction versus those in cases that price-only auctions take place instead. The procurement costs drop by approximately 2 percent if a first-price auction with $q = 1.3$ were to be used. Table 2 reports the winning bidder’s payoffs. Bidders earn significantly lower payoffs upon winning in a first-price auction by 3 through 16 percent.

Form	β	q	Obs	Mean	Std. Dev.	Min	Max	Change	
FS* ¹	-		6,146	0.58335	0.07343	0.24046	1.18161	-	
		1.3	6,128	0.59543	0.07581	0.22578	1.34047	2.071%	
		2	1.4	6,128	0.59445	0.07674	0.22742	1.34780	1.903%
		1.5	6,128	0.59360	0.07768	0.22917	1.35525	1.758%	
SP* ²	3	1.3	6,128	0.60057	0.08655	0.22715	1.57453	2.953%	
		1.4	6,128	0.59952	0.08843	0.22847	1.59551	2.773%	
		1.5	6,128	0.59864	0.09037	0.22993	1.61703	2.621%	
		1.3	6,127	0.60674	0.10518	0.22849	1.92882	4.010%	
	4	1.4	6,128	0.60572	0.10887	0.22951	1.97449	3.835%	
		1.5	6,127	0.60468	0.11129	0.23070	1.71705	3.656%	

*¹ Observed FS auctions. *² Counterfactual second-price auctions. * Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 1: Buyer’s quality-adjusted procurement costs

Form	β	q	Obs	Mean	Std. Dev.	Min	Max	Change
FS* ¹	-		16,323	0.0914	0.0938	0.0038	0.7677	-
		1.3	16,318	0.0766	0.0964	0.0000	0.9222	-16.17%
	2	1.4	16,318	0.0810	0.1010	0.0000	0.9462	-11.35%
		1.5	16,318	0.0841	0.1042	0.0000	0.9628	-8.05%
		1.3	16,318	0.0785	0.0976	0.0000	0.9369	-14.15%
SP* ²	3	1.4	16,317	0.0828	0.1039	0.0000	0.9934	-9.42%
		1.5	16,316	0.0863	0.1087	0.0000	0.9997	-5.61%
		1.3	16,318	0.0790	0.0989	0.0000	0.9533	-13.60%
	4	1.4	16,314	0.0842	0.1072	0.0000	0.9988	-7.85%
		1.5	16,308	0.0889	0.1142	0.0000	0.9925	-2.78%

*¹ Observed FS auctions. *² Counterfactual second-price auctions. * Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 2: Bidders' payoffs

4.3.2 SS vs. FS auctions

Next, the extent to which the expected scores would be changed by introducing SS auctions is estimated. Given the cost function, the bidder's induced utility function $u(s, \theta)$ is convex in s for any $\beta > 1$ if the scoring rule is PQR, as the second derivative of u is given by

$$u_{ss}(s, \theta) = \theta^1 \frac{1}{\beta - 1} \left(\frac{s}{\beta} \right)^{-\frac{\beta-2}{\beta-1}} > 0 \text{ with } \beta \geq 2. \quad (21)$$

Therefore, the expected exercised score will be lower in SS than FS auctions as suggested by Theorem 3 in Hanazono et al. (2013). In the next subsection, we conduct a series of counterfactual analysis to empirically measure the difference between FS and SS auctions regarding expected exercised scores (the buyer's welfare), bidders' payoffs, and quality levels.

The counterfactual samples related to the SS auction is created from the estimated parameters, $\hat{\theta}_{i,t}$. First, the pseudo-samples of the first-best quality $q^{FB}(\theta)$ is created from (8), which is given by

$$\left(\frac{q^{FB}}{\theta_{i,t}^1} - \alpha \right)^\beta + \theta_{i,t}^0 = \frac{q^{FB}}{\theta_{i,t}^1} \beta \cdot \left(\frac{q^{FB}}{\theta_{i,t}^1} - \alpha \right)^{\beta-1},$$

under the specific cost function. Thus, the first-best quality of bidder i in auction t is created as

$$\hat{q}^{FB}(\hat{\theta}_{i,t}) \equiv \left\{ q : (1 - \beta)r_{i,t}^\beta(q) - \alpha\beta r_{i,t}^{\beta-1}(q) + \hat{\theta}_{i,t}^0 = 0 \right\}, \quad (22)$$

where $r_{i,t}(q) = q/\hat{\theta}_{i,t}^1 - \alpha$. Next, the counterfactual samples of the bidder's break-even score is created. From (7), the first-best quality, and the observed data, the break-even score of the bidder whose type is equal to $\theta_{i,t}$ is predicted as

$$k^-(\hat{\theta}_{i,t}^0) = \frac{\hat{\theta}_{i,t}^1}{\hat{q}_{i,t}^{FB}} \left[\left(\frac{\hat{q}_{i,t}^{FB}}{\hat{\theta}_{i,t}^1} - \alpha \right)^\beta + \hat{\theta}_{i,t}^0 \right], \quad (23)$$

under the PQR scoring rule.

The awarded bidder's quality choice in the SS auction is also estimated. Let $\hat{\theta}_{(i),t}$ be the signal of the bidder whose score is the i th lowest in auction t . In SS auctions, the exercised score is the second-lowest bidder's break-even score $k^-(\theta_{(2),t}^0)$. Thus, the winning bidder chooses the optimal quality level $q(k^-(\hat{\theta}_{(2),t}^0), \hat{\theta}_{(1),t})$. Let \hat{q}_t^π denote the quality level. The first-order condition of the bidder's quality choice given s suggests $C_q(\hat{q}_t^\pi | \hat{\theta}_{(1),t}) = \hat{k}^-(\theta_{(2),t}^0)$, which is expressed as

$$q_t^\pi = \hat{\theta}_{(1),t}^1 \cdot \left[\left(\frac{k^-(\hat{\theta}_{(2),t}^0)}{\beta} \right)^{\frac{1}{\beta-1}} + \alpha \right], \quad (24)$$

given our parametric cost functions. Thus, the awarded bidder's payoff, $u(\hat{k}^-(\theta_{(2),t}^0), \theta_{(1),t})$, is given by

$$u(k^-(\hat{\theta}_{(2),t}^0), \hat{\theta}_{(1),t}) = \hat{q}_t^\pi \left[k^-(\hat{\theta}_{(2),t}^0) - k(\hat{q}_t^\pi, \theta_{(1),t}) \right]. \quad (25)$$

In a SS auction, the score in the final contract equals the break-even score of the lowest losing bidder, denoted by $k^-(\theta_{(2),t}^0)$. The data on $k^-(\theta_{(2),t}^0)$ is shown in Table 3. The expected score declines approximately by .04 percent (when $\beta = 2$) and .02 percents (when $\beta = 4$) if the auction format alters from FS to SS mechanisms. The variances are greater than that in the FS auction similar to the difference in the variance of first- and second-price auctions.

	$C(q \theta)$	Obs	Mean	Std. Dev.	Min	Max	Change
FS* ¹	-	6,004	0.58030	0.04953	0.46244	0.91415	-
	Quadratic	6,004	0.58006	0.06796	0.23988	0.99445	-0.0417%
SS* ²	Cubic	6,004	0.58014	0.06788	0.24575	0.99449	-0.0288%
	Quartic	6,005	0.58015	0.06785	0.24967	0.99451	-0.0257%

*¹ Observed FS auctions (PQR). *² Hypothetical SS auctions with the PQR rule. * Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 3: Exercised scores in FS and SS auctions

Table 6 shows that the quality level finalized in the contract is, on average, declined approximately by 3 to 4 percents if SS auctions are used. The upward distortion of quality depends on the number of bidders. First, the cost function itself shifts to the right as the bidder has a more efficient type. Table 4 shows that the winning bidder’s scale parameter θ^1 is positively related with the number of competitors. [We will discuss more about this issue here.]

	Quadratic	Cubic	Quartic
nb	0.002191 (9.28)**	0.001489 (7.53)**	0.001188 (6.59)**
time	8.92E-06 (91.46)**	7.52E-06 (92.16)**	6.89E-06 (92.53)**
reserve	7E-11 (22.47)**	6E-11 (22.08)**	5E-11 (21.78)**
Auc Form	-0.03695 (7.23)**	-0.02964 (6.93)**	-0.02658 (6.81)**
Constant	-1.78E+02 (90.86)**	-1.50E+02 (91.51)**	-1.37E+02 (91.86)**
Observations	14342	14344	14346
R-squared	0.41	0.41	0.41

Absolute value of t statistics in parentheses
* significant at 5%; ** significant at 1%

Table 4: Winner’s θ^1

4.3.3 QL vs. PQR rules

Finally, we explore a QL scoring rule that dominates the current PQR scoring rule. Specifically, we suppose that the buyer uses a QL scoring rule that differs from the buyer’s true preference $U(p, q) = p/q$. To construct a well-performing QL rule, we relax the assumption that the quality price in the QL rule (the derivative of the score function with respect to q) is one such that, for some $\phi > 0$,

$$S(p, q) = p - \phi(\beta)q. \tag{26}$$

The lower utility caused by the use of a FS auction under PQR lies in over-provision in quality. In SS auctions, that upward distortion in quality provision is not observed. Therefore, a candidate of a QL rule that dominates the current PQR FS auction is such that the average of the winning bidders’ first-best quality is equivalent to the average of the quality level to be chosen in a SS auction. We thus choose the following three values of

	-	Quadratic	Cubic	Quartic
No. bidders	0.001561 (6.19)**	0.002843 (11.92)**	0.002051 (10.26)**	0.001707 (9.35)**
Time	9.79E-06 (94.94)**	9.09E-06 (93.25)**	7.66E-06 (93.81)**	7.02E-06 (94.14)**
Reserve price	6E-11 (19.66)**	6E-11 (21.50)**	5E-11 (21.13)**	5E-11 (20.83)**
Auction Forms	-0.03269 (6.12)**	-0.0351 (6.95)**	-0.02818 (6.66)**	-0.02523 (6.53)**
Constant	-1.95E+02 (94.20)**	-1.81E+02 (92.65)**	-1.53E+02 (93.18)**	-1.40E+02 (93.47)**
Observations	14844	14843	14843	14843
R-squared	0.41	0.41	0.41	0.41

Absolute value of t statistics in parentheses
* significant at 5%; ** significant at 1%

Table 5: Shift of cost functions

Form	$C(q \theta)$	Obs	Mean	Std. Dev.	Min	Max	Change
FS* ¹	-	6,004	1.53912	0.09679	1.31000	1.90000	-
	Quadratic	6,004	1.53894	0.10020	1.29678	1.90775	-0.0116%
SS* ²	Cubic	6,004	1.53869	0.09859	1.31695	1.89876	-0.0279%
	Quartic	6,005	1.53876	0.09795	1.31773	1.89850	-0.0235%

*¹ Observed FS auctions (PQR). *² Hypothetical SS auctions with the PQR rule. * Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 6: Contracted quality level in FS and SS auctions

Form	β	Obs	Mean	Std. Dev.	Min	Max	Change
FS* ¹	-	6,004	0.06368	0.07490	0.00178	0.71027	-
	Quadratic	6,004	0.06384	0.08204	0.00017	0.88633	0.2550%
SS* ²	Cubic	6,004	0.06380	0.08180	0.00017	0.88514	0.1936%
	Quartic	6,005	0.06377	0.08170	0.00017	0.88450	0.1509%

*¹ Observed FS auctions (PQR). *² Hypothetical SS auctions with the PQR rule. * Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 7: Bidder's payoffs in FS and SS auctions

the quality price: $\phi(2) = 0.6502278$, $\phi(3) = 0.6493106$, and $\phi(4) = 0.6477461$, each equal to the average of the exercised score in the counterfactual SS auction at $\beta = 2, 3$, and 4 , respectively. Given $\phi(\beta)$, we predict the expected value of the winning score in SS auctions with the QL rule.

Under the QL rule, the bidder's quality-adjusted cost is given by $k(q, \boldsymbol{\theta}) = C(q|\boldsymbol{\theta}) - \phi(\beta)q$, and the first-best quality $q^{FB}(\boldsymbol{\theta})$ satisfies $C_q(q^{FB}(\boldsymbol{\theta})|\boldsymbol{\theta}) = \phi(\beta)$. Given the parametric cost function, the marginal cost is given by $C_q(q|\boldsymbol{\theta}) = \beta \cdot (q/\theta^1 - \alpha)^{\beta-1}$. Therefore, $q^{FB}(\boldsymbol{\theta})$ is given by

$$q_{QL}^{FB}(\hat{\boldsymbol{\theta}}) = \hat{\theta}^1 \cdot \left(\alpha + \left(\frac{\phi(\beta)}{\beta} \right)^{\frac{1}{\beta-1}} \right). \quad (27)$$

Using $q_{QL}^{FB}(\boldsymbol{\theta})$ and the estimated $\boldsymbol{\theta}$, we compute the bidder's break-even score, $k^-(\hat{\theta}^0)$, under QL rules. With our parameterized cost function, this is expressed as

$$k^-(\hat{\theta}^0) \equiv \hat{\theta}^1 \cdot \left[\left(\frac{q_{QL}^{FB}(\hat{\boldsymbol{\theta}})}{\hat{\theta}^1} - \alpha \right)^\beta + \hat{\theta}^0 \right] - q_{QL}^{FB}(\hat{\boldsymbol{\theta}}). \quad (28)$$

Since bidders are symmetric, the bidder with the lowest $k(q_{QL}^{FB}(\hat{\boldsymbol{\theta}}), \hat{\boldsymbol{\theta}})$ is the awardee, receiving the payment $P_{QL} = C(q_{QL}^{FB}(\hat{\boldsymbol{\theta}}_{(1)})|\hat{\boldsymbol{\theta}}_{(2)})$ in a SS auction. Thus, both the contract price and the quality level are given by P_{QL} and $q_{QL}^{FB}(\hat{\boldsymbol{\theta}}_{(1)})$. The buyer's utility is thus computed by

$$s_{QL} = \hat{p}_{QL}/q_{QL}^{FB}(\hat{\boldsymbol{\theta}}_{(1)}).$$

Table 8 reports the buyer's utility s_{QL} in counterfactual SS auctions with QL rules. In all cases, s_{QL} s drop on average approximately by 5 to 15 percent. The greater variances in SS auctions due to the non-negative variance of the conditional second-order statistic can be remedied by the use of FS auctions. Table 9 shows the bidder's profit. The bidder's profit drops by 1 to 12 percent. Hence, the use of an appropriate QL rule extracts more rents from bidders.

The additional rent extraction by the QL scoring rule stems from the downward distortion of the quality provision. Table 10 presents the contracted quality level in the observed FS auction and simulated QL scoring auctions. The quality levels would be sharply declined under the well-designed QL scoring rule. Although the well-designed QL scoring rule is not optimal, the lower contracted quality levels by the QL scoring rule limits the winner's informational rent, resulting in the greater welfare of the procurement buyer.

Form	β	Obs	Mean	Std. Dev.	Min	Max	Change
FS* ¹	-	6,004	0.58030	0.04953	0.46244	0.91415	-
	Quadratic	5,995	0.57859	0.06544	0.26503	0.94570	-0.2947%
SS* ²	Cubic	5,996	0.57864	0.06545	0.26532	0.94244	-0.2860%
	Quartic	5,997	0.57863	0.06544	0.26545	0.94474	-0.2884%

*¹ Observed FS auctions (PQR). *² Hypothetical SS auctions with the QL rule. * Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 8: Exercised scores under simulated QL rules

Form	β	Obs	Mean	Std. Dev.	Min	Max	Change
FS* ¹	-	6,004	0.06368	0.07490	0.00178	0.71027	-
	Quadratic	5,971	0.06098	0.07633	0.00002	0.86927	-4.2323%
SS* ²	Cubic	5,972	0.06135	0.07677	0.00000	0.87072	-3.6586%
	Quartic	5,972	0.06152	0.07696	0.00000	0.87143	-3.3857%

*¹ Observed FS auctions (PQR). *² Hypothetical SS auctions with the QL rule. * Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 9: Payoffs under simulated QL rules

Form	β	Obs	Mean	Std. Dev.	Min	Max	Change
FS* ¹	-	6,034	1.53883	0.09675	1.31000	1.90000	-
	Quadratic	6,009	1.51347	0.10507	0.84144	1.88924	-1.6484%
QL* ²	Cubic	6,010	1.52093	0.10345	0.76952	1.89201	-1.1632%
	Quartic	6,010	1.52488	0.10304	0.73337	1.89383	-0.9067%

*¹ Observed FS auctions (PQR). *² Hypothetical SS auctions with the QL rule. * Sample auctions with the number of bidders equal to or greater than 2; In FS auctions, profits are less than 1 and normalized bids are less than 150% of reservation prices; In simulated SP auctions, profits are less than 1 and price bids are less than 250% of reservation prices.

Table 10: Contracted quality levels in FS and QL scoring auctions

4.3.4 Summary of empirical experiments

Our counterfactual analyses suggest that FS auctions perform poor under PQR scoring rules. However, it does not mean that FS auctions never benefit procurement buyers whose preference is based on PQR. The performance of a price-only auction strongly depends on the choice of the fixed quality level. In many occasions, auctioneers have limited information regarding bidders' cost structures. Thus, only experienced buyers can choose the quality level that renders a higher expected utility to the buyer in a price-only auction than in a FS auction. For inexperienced buyers, the use of a FS auction is the best option even if their true preference is based on PQR. The same is true for QL scoring rules. We observed that, when a FS auction is used, a QL scoring rule may dominate the PQR rule in terms of the expected contracted score. However, for the procurement buyer with PQR preference, designing a well-performing QL scoring function, in particular, choosing the best quality price in a QL scoring function, requires accurate information on the bidders' cost structures. Less informed buyers with PQR preference will thus benefit from the use of his/her true preference as a scoring function since the quality price is determined in the market under a PQR scoring rule.

In scoring auctions, bidders' advantages in non-monetary attributes are evaluated. Therefore, the procurement buyer may obtain a better contract without reducing the bidder's profit. However, this is just an advantage of scoring auctions. Rather, the advantage of the use of scoring auction is in that even an unexperienced buyer can pursue the best value in procurement since he does not need to specify the quality level. If an unexperienced buyer is not familiar with bidders' advantages in non-monetary attributes rather than in costs, the scoring function selects the winner who provides the most valuable-for-money contract.

We found that bidders' bidding is greatest in the FS auction with the PQR scoring rule. This result suggests that a major advantage of the currently adopted FS auction format lies in the promotion of bidder participation. The intensified competition by a FS auction will lower the quality-adjusted procurement cost even if the procurement buyer has limited information on bidders' cost structures. An interesting extension will be to take into account potential bidders' endogenous participation in the structural model.

5 Conclusion

In this research, we provided a structural estimation method for a scoring auction with generalized scoring rule. From the scoring auction data that typically include scores and quality bids, latent parameters in the bidder's cost function was estimated. From observed quality levels, the bidder's marginal costs are estimated through the bidder's profit maximization behavior such that the marginal cost equals the quality price. Bidders' costs were

estimated through the first-order condition by the application of the semi-parametric estimation methodology for first-price auctions. It is obvious that the number of parameters capable to be identified is equal to or less than the number of dimensions of the observed data. Thus, for instance, the degree of concavity of the cost function is the one that is unable to be identified. We thus conducted a series of empirical experiments in which the parameters of the cost function vary to ensure the robustness of estimation results.

We also showed a simulation experiment to illustrate that our structural estimation method does identify the latent distribution of the bidder's signal. The recovered density and cumulative distribution functions were coincident with the true density and distribution functions except in the areas of boundaries. Therefore, our estimation method is effectively identify the bidder's multi-dimensional signal.

Furthermore, we applied our estimation technique to real world scoring auction data. Theory suggests that bidders the non-equivalence in the expected winning scores stems from the overproduction in quality in a FS auction with the PQR scoring rule. Accordingly, we observed that the expectation of the winner's quality provision is larger in FS than in SS auctions. Furthermore, with a well-designed QL scoring rule, we found that the procurement buyer improves utility while bidders earn lower payoffs. Generally, the optimal design problem is hard to be solved if the bid and signal are multi-dimensional. Therefore, our counterfactual analysis uses a standard FS or SS auction with a well-designed QL scoring rule as a suboptimal mechanism. Nevertheless, a flavor of the optimal design problem has been seen in our empirical result, the quality provision is distorted downward (allocative inefficiency) and the bidder's informational rents are limited.

In this paper, we restrict attention to the independent scoring rule, in which the bidder's score depends only on his or her price and quality bids. In the real-world procurement auctions, however, a wider-variety of scoring rules are used including the one in which the bidder's score depends also on the other bidders' price and quality bids (an interdependent scoring rule). Literature suggests that an interdependent scoring rule involves some inefficiency when bidders choose optimal quality levels since the realized exercised score generally differs from the score predicted by the bidder when choosing the quality bid. As a result, the expected exercised score (the procurement buyer's utility) is greater (smaller) than the scoring auction with an independent scoring rule. suggests that the welfare loss of the procurement buyer is approximately 11 %. Theoretical literature, on the other hand, has so far been silent on the equilibrium in the scoring auction with such an interdependent scoring rule. An interesting future research may lie in the structural analysis of the scoring auction with an interdependent scoring rule. A counterfactual analysis would quantify the expected score difference between the FS and SS auctions with an interdependent scoring rule.

Appendix A The equilibrium strategy in the simulation experiment

The optimal quality is given by $q(s, \theta) = (1 + \theta^1)(s/2 + 1)$. Therefore, we have

$$\frac{u(s, \boldsymbol{\theta})}{u_s(s, \boldsymbol{\theta})} = \frac{(s(\theta^0))^2 + 4s(\theta^0) - 4\theta^0}{2s(\theta^0) + 4}.$$

Given the uniform distribution of θ^0 , the first-order condition (5) is written as

$$s'(\theta^0) = \frac{1}{1 - \theta^0} \frac{(s(\theta^0))^2 + 4s(\theta^0) - 4\theta^0}{2s(\theta^0) + 4}.$$

Since $s(\theta^0)$ is strictly increasing in θ^0 and $u(s(\theta^0), \boldsymbol{\theta}) = 0$ at $\theta^0 = 1$, we obtain $s(1) = 2\sqrt{2} - 2$ as a boundary condition. Thus, the equilibrium bidding strategy is the solution of the differential equation

$$\begin{cases} s'(\theta^0) = \frac{1}{1 - \theta^0} \frac{(s(\theta^0))^2 + 4s(\theta^0) - 4\theta^0}{2s(\theta^0) + 4} \\ s(1) = 2\sqrt{2} - 2 \end{cases}$$

Solving the differential equation gives

$$(1 - \theta^0) [(s(\theta^0))^2 + 4s(\theta^0) - 2(1 + \theta^0)] = 0. \quad (29)$$

Applying the implicit function theorem ensures that (29) be the solution of the differential equation. Taking (29) as a quadratic equation, we obtain the equilibrium bidding function explicitly as

$$s(\theta^0) = -2 + \sqrt{2\theta^0 + 6}.$$

References

- Albano, Gian Luigi, Federico Dini, and Roberto Zampino**, “Bidding for Complex Projects: Evidence from the Acquisitions of IT Services,” *Lecture Notes in Computer Science*, 2009, 5693, 353–363.
- Asker, John and Estelle Cantillon**, “Properties of Scoring Auctions,” *RAND Journal of Economics*, 2008, 39 (1), 69–85.
- Athey, Susan**, “Single Crossing Properties and the Existence of Pure Strategy Equilibria in Games of Incomplete Information,” *Econometrica*, July 2001, 69 (4), 861–889.

- Bajari, Patrick, Stephanie Houghton, and Steve Tadelis**, “Bidding for Incomplete Contracts: An Empirical Analysis,” NBER Working Papers 12051, National Bureau of Economic Research, Inc February 2006.
- Branco, Fernando**, “The Design of Multidimensional Auctions,” *RAND Journal of Economics*, 1997, 28 (1), 63–81.
- Che, Yeon-Koo**, “Design Competition through Multidimensional Auctions,” *RAND Journal of Economics*, Winter 1993, 24 (4), 668–680.
- Guerre, Emmanuel, Isabelle Perrigne, and Quang Vuong**, “Optimal Nonparametric Estimation of First-Price Auctions,” *Econometrica*, 2000, 68 (3), 525–574.
- , – , and – , “Nonparametric Identification of Risk Aversion in First-Price Auctions Under Exclusion Restrictions,” *Econometrica*, 07 2009, 77 (4), 1193–1227.
- Hanazono, Makoto, Jun Nakabayashi, and Masanori Tsuruoka**, “Procurement Auctions with General Price-Quality Evaluation,” KIER Working Papers 845, Kyoto University, Institute of Economic Research January 2013.
- Laffont, Jean-Jacques, Herve Ossard, and Quang Vuong**, “Econometrics of First-Price Auctions,” *Econometrica*, July 1995, 63 (4), 953–80.
- Lewis, Gregory and Patrick Bajari**, “Procurement Contracting With Time Incentives: Theory and Evidence,” *The Quarterly Journal of Economics*, 2011, 126 (3), 1173–1211.
- Li, Qi and Jeffrey Scott Racine**, *Nonparametric Econometrics: Theory and Practice*, Vol. 1 of Economics Books, Princeton University Press, 2006.
- Li, Tong, Isabelle Perrigne, and Quang Vuong**, “Structural Estimation of the Affiliated Private Value Auction Model,” *RAND Journal of Economics*, Summer 2002, 33 (2), 171–193.
- Milgrom, Paul**, *Putting Auction Theory to Work*, Cambridge University Press, 2004.
- OECD**, “Bribery in Public Procurement: Methods, Actors and Counter-Measures,” 2007, p. 103.
- Tunca, Tunay I., D.J. Wu, and Fang (Vivian) Zhong**, “An Empirical Analysis of Price, Quality, and Incumbency in Service Procurement Auctions,” Umea Economic Studies 821, Umea University, Department of Economics August 2010.