Oligopolistic Competition and Search Without Priors *

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Abstract

I study a model of oligopolistic competition in which consumers search for prices, but have no idea about the underlying price distribution. Consumers' behaviour satisfies four consistency requirements such that beliefs about the underlying distribution maximize Shannon entropy. I derive the optimal stopping rule and equilibrium price distribution of the model. Unlike in Stahl (1989), the expected price is decreasing in the number of firms. Moreover, consumers can benefit from being uninformed, if the number of firms is sufficiently large.

Keywords: consumer search, search without priors, bounded rationality, entropy.

JEL-Codes: D83, D43, L11

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1 Introduction

The purpose of this paper is to study the effect of bounded rationality of consumers on market outcomes in a search model. In standard search models with homogeneous goods (e.g. Stahl (1989), Janssen et al. (2005)), in order to pick the reservation price correctly consumers have to know the price distribution. This assumption is rather restrictive, since consumers have to know all the parameters of the model and be able to derive firms' equilibrium strategies. I construct a model of oligopolistic competition with *truly uninformed* customers, i.e. when there is a group of customers which have no idea about the shape of the equilibrium price distribution. Consumers' behaviour is only restricted by four consistency assumptions. These assumptions result in the maximum entropy estimation procedure, which gives a very simple decision rule.

The idea of looking together at search and learning is pioneered by Rothschild (1974). However, incorporating this idea into an equilibrium framework, when firms act strategically is not an easy task. The first attempts to look at search with learning in an equilibrium framework are due to Benabou and Gertner (1993) and Dana (1994). The latter paper has a setup similar to my paper, but with two important differences: search protocol is not sequential ("newspaper search" – uninformed consumers ask informed for the information), and consumers are Bayesian. Tappata (2009) uses search model with (Bayesian) learning to study asymmetric pricing. The closest paper to mine is Janssen et al. (2011). They study a case when consumers are not aware of production costs of the firms, but have certain prior distribution over them. After learning the price, consumers re-estimate the price distribution in a Bayesian way and solve the optimal stopping problem. However, this paper requires even more sophisticated consumers that those by Stahl. Moreover, the choice of prior is still an open question. This two drawbacks are applicable for all aforementioned papers. My approach follows Chou and Talmain (1993) and does not require any priors (say, about production costs, or the fraction of informed consumers), except for knowing the boundaries of the support of the distribution.

Modeling boundedly rational consumers should have two desired properties. Firstly, the choice of a behavioural heuristic should not be ad hoc, but rather be based on certain plausible axioms. Suppose consumers observed several prices and need to make an inference about the underlying distribution. Following Shore and Johnson (1980) I impose four consistency requirements: uniqueness, invariance, system and subset independence. Shore and Johnson (1980) showed that any statistical procedure that satisfies these requirements should give the same result as maximization of entropy. This is the approach I am going to use in the paper. The second property is that the procedure should not be too complicated, so one can believe that consumers can follow the prescribed heuristics. The entropy maximization leads to a piece-wise uniform belief about the underlying distribution, probably the simplest belief one can hold. Say, if a consumer knows that the prices should be between 20 and 100 dollars, and observes a price of 50, she assigns 50% of the probability mass to the interval $[20, 50]$ and the rest to the interval $[50, 100]$ and assumes that it is uniformly distributed on both intervals. Thus, the modeling of consumer behaviour satisfies both properties.

An alternative approach to search without priors is presented in Bergemann and Schlag (2011). They look for the search rule which allows to a searcher without priors to be as close to the Bayesian searcher as possible. Though surprisingly an uninformed searcher can be rather close to the Bayesian searcher, their approach is hard to apply to a framework of oligopolistic competition. Firstly, the search rule is not necessarily represented by a unique reservation price. Secondly, the model is built for a no-recall search protocol, which is not appropriate for the analysis of consumer markets. Finally, the optimal stopping rule can be derived only for the case of two alternatives.

I model oligopolistic competition using the seminal model by Stahl (1989). The only conceptual difference is that consumers follow the entropy maximization rule. Introducing bounded rationality in the model changes its game-theoretical structure. In Stahl (1989) consumers are active players and play a best response to the equilibrium pricing strategy of the firms. In my model, the reservation price is no longer a best response in a strict game-theoretical sense. The question *how* consumers get the "correct" reservation price is not a relevant question in game theory, but it is the main focus of this paper. The stopping rule is optimal based on the information consumers have, but it is not the best response strategy. Thus, my paper is closer to Varian (1980), where only firms are strategic players.

My analysis shows two notable differences with the existing literature. Firstly, the original model by Stahl has a disturbing property that the expected price is increasing with the number of firms in the industry. This prediction is reversed with boundedly rational consumers: consumers base their decision on the lower bound of the support of the price distribution, which is decreasing in the number of firms (while the length of the support of price distribution is constant), and put too little weight on the probability mass at the upper part of the support. Secondly, one of the results of Janssen et al. (2011) is that the reservation price (if it exists) is higher when consumers are incompletely informed. This is not true in my model. Though consumers have minimum information about the price distribution, the reservation price can be either lower or higher than in Stahl's model, depending on parameters. For a large number of firms, prices tend to be lower than in the Stahl model.

The rest of the paper organised as follows. Section 2 presents the model and axiomatic approach. Analysis of the model can be found in Section 3. Section 4 gives a brief discussion of incorrect beliefs about the support. Section 5 concludes. All the proofs (including intermediate steps lemmas) are provided in the appendix.

2 Model

There are N firms in the industry, where N is fixed. Firms produce a homogeneous product and compete in prices. Since both in my model and a reference model of Stahl (1989) (against which results will be compared) marginal costs are just a shift parameter, I assume that marginal costs equal to zero. There is a unit mass of consumers. Consumers have unit demand and valuation of the good v. A fraction λ of the consumers is informed about all the prices in the market. Those are so called "shoppers", who enjoy shopping and have zero search costs. The rest $(1-\lambda)$ of the consumers are uninformed. I assume like in Stahl (1989) that they get the first price quotation for free[1]. After observing the first price then they can either buy the product, or engage in sequential search. Search costs are equal to c and it is constant across consumers and search rounds. I also assume perfect recall: consumers can go to previously visited firms for free.

The important difference from the model by Stahl (1989) is that the consumers are not aware of the equilibrium price distribution. I assume that they have correct beliefs about the boundaries of the price distribution, which I denote $\underline{p}$ and $\overline{p}$, with $\overline{p} \leq v$. This assumption is common in the literature on search without priors, regardless the underlying search model (see, for example Chou and Talmain (1993) or Bergemann and Schlag (2011)).

Suppose a customer has observed some price sample: $(p_1, \ldots, p_n)$. In order to make a decision whether to continue searching or to stop, the consumer has to make some inference about the price distribution. Shore and Johnson (1980) propose four axioms for this statistical procedure:

1. *Uniqueness*: For each price sample the result should be unique.

2. *Invariance*: The result should not depend on the choice of measurement units.

3. *System Independence*: It should not matter whether one accounts for independent information about independent systems separately in terms of different densities or together in terms of joint density.

4. *Subset Independence*: It should not matter whether one treats an independent subset of system states in terms of separate conditional densities or in terms of one full density.

[1] Janssen et al. (2005) relax this assumption.

They show, that any statistical procedure which satisfies these axioms must give the same result as maximization of Shannon entropy (and minimization of cross-entropy):

$$h[f] = -\int_{-\infty}^{\infty} f(x) \log f(x) dx$$

where $f(\cdot)$ is the density function of the price distribution.

Maximization of Shannon entropy leads to a quantile preserving estimation procedure, described, for example in Chou and Talmain (1993). Suppose prices in the sample $(p_1, \ldots, p_n)$ are ordered, so $\underline{p} \leq p_i \leq p_j \leq \overline{p}, i < j$, which divides an interval $[\underline{p}, \overline{p}]$ on $n+1$ subintervals. The estimation procedure works as follows:

- assign equal probability mass $(\frac{1}{n+1})$ to each of subintervals $\{[\underline{p}, p_1], \ldots, [p_i, p_{i+1}], \ldots [p_n, \overline{p}]\}$ in case all the prices are different;
- assign $\frac{k-1}{n+1}$ to the price, which appeared k times in the sample;
- distribute probability mass uniformly within each interval.

After the next price is observed the probability distribution is updated. This estimation procedure is consistent and asymptotically efficient (Chou and Talmain (1993)).

Once behavioural assumptions for uninformed customers are fully specified I move towards the analysis of the optimal search strategy and the equilibrium model.

3 Analysis

I start my analysis with a derivation of the optimal stopping rule. In fact, I adapt the analysis of Chou and Talmain (1993) to my setup. As it is typical for search models of this type, I assume in my model that firms in equilibrium play a mixed strategy distribution which is defined by cumulative distribution function $F(p)$ with the support $[\underline{p}, \overline{p}]$. In order to get solutions in a simple form I assume here that $F(p)$ is atomless. These assumptions will be verified later, when I derive firms' strategies.

Using backward induction I derive the optimal stopping rule, which is summarized in the following theorem.

Theorem 1. *A reservation price on step k is given by $r_k = \underline{p} + 2(k+1)c$.*

Note, that the optimal stopping rule is no longer stationary like in Stahl (1989). This is a common feature of models with learning. For example, reservation prices in Janssen et al. (2011), if they exist, are also non-stationary and increasing in the number of search rounds. In my model, reservation prices between the subsequent rounds always differ by $2c$. It might look surprising that the reservation prices do not depend on the search history itself. However, this fact has very simple intuition behind. For the definition of the reservation price only the lowest price in the sample matters. This price fully characterize the lowest part of distribution, upon which consumer computes the expected offer and (together with search costs) compares it with current offer. Thus, both options only depend on the lowest price in the sample – the reservation price.

Now consider the pricing behaviour of the firms. It is known that firms can not play pure strategies and therefore have to mix (see Varian (1980), Stahl (1989)). I am going to look for symmetric mixed strategy equilibria in the model. There are two well-known properties of mixed strategies of the firms in similar models. If $F(p)$ is a symmetric equilibrium pricing strategy, it has to be atomless, otherwise there is a positive probability of a tie, and firms would like to undercut at this point. Another property is that the upper bound of the equilibrium distribution function has to be equal to the first round reservation price. If it is higher, then consumers prefer to search further and find the lower price with probability one, so firms make zero profit at the upper bound. If it is lower, than firms only sell to uninformed consumers, so they have incentives to increase the price, since uninformed consumers would buy at any price below the reservation one. Equipped with these two properties I can proceed with the characterization of the equilibrium pricing strategies.

Proposition 2. *The equilibrium pricing strategy is given by*

$$F(p) = \begin{cases} 0, & p \leq 4c\frac{1-\lambda}{\lambda N} \\ 1 - \left(\frac{1-\lambda}{\lambda N} \cdot \frac{4(1-\lambda)c+\lambda N(4c-p)}{\lambda N p}\right)^{\frac{1}{N-1}}, & 4c\frac{1-\lambda}{\lambda N} < p \leq 4c\frac{1-\lambda+\lambda N}{\lambda N p} \\ 1, & p > 4c\frac{1-\lambda+\lambda N}{\lambda N} \end{cases}$$

and the equilibrium level of profits equals to $\pi = 4c\frac{(1-\lambda)(1-\lambda+N\lambda)}{\lambda N^2}$.

Note, that the equilibrium distribution has exactly the same structure as in Stahl (1989) (or Varian (1980)) with the only difference in the reservation price, which is now defined by

$$r_1 = 4c\frac{1-\lambda+\lambda N}{\lambda N}. \tag{1}$$

However, this difference has very important consequences for welfare, which is summarized in the following proposition.

Proposition 3. *The expected price payed by consumers is:*

1. *increasing in search costs;*

2. *decreasing in the number of informed customers;*

3. *decreasing in the number of firms.*

Though the first two parts of this proposition coincide with standard theory, the third result is in contrast with the famous finding by Stahl (1989) – in his model expected price is increasing in N. The reason for Stahl's result is that firms are less likely to get shoppers when N is large, so they concentrate on non-shoppers. Though this logic is applicable here it is outweighed by two effects. Firstly, consumers do not take into account the fact that the probability mass is concentrated around the upper bound of the distribution. Thus, for large N they tend to use lower reservation prices in comparison with Stahl. Secondly, consumers base their decisions on the lower bound of the support of the distribution, which is decreasing in N and goes to zero as N goes to infinity. Moreover, in Stahl (1989) the length of the support $(r - \underline{p})$ is increasing in N, while in my case it is constant and equals to $4c$. These two effects together lower the reservation prices, which in turn has a stronger effect on prices than Stahl's mechanism. Thus, standard intuition that the increase in the number of firms should lead to more competition and lower prices is recovered. It is easy to see, that the total industry profit approaches $4c(1 - \lambda)$ as N increases, thus firms earn on average $4c$ (limit value of the reservation price r_1) on uninformed customers, without earning anything on informed:

$$\lim_{N\to\infty} \mathbb{E}p = \lim_{N\to\infty} \pi N = (1 - \lambda) \lim_{N\to\infty} r_1 = 4(1 - \lambda)c$$

Now I am going to compare prices in two models: model with boundedly rational and fully rational consumers. Janssen et al. (2011) showed, that if consumers are uninformed about the underlying distribution (level of production costs), they tend to pay higher prices. Same result is obtained by Dana (1994) under different search protocol. Though in my case consumers have minimum information – just the support of the distribution, this result is not necessary true. For industries with low number of firms consumers indeed tend to pay higher prices, but if the number of firms is sufficiently high, consumers pay lower prices if λ is large enough.

Proposition 4. *For any $\lambda \in (0, 1)$ there is N^*, such that for all $N > N^*$ $r_1(\lambda, N) < \rho(\lambda, N)$, where ρ is a reservation price of Stahl's model.*

Since firms' profits, consumer welfare and expected price are directly linked to the reservation price, this result is applicable to those variables as well. In particular, if N is large enough, firms' profits and expected price are lower and consumer welfare is higher under search without priors, than in the case of fully informed consumers.

4 Information About Support: Discussion

Following most of the literature on decision-making without priors, I assumed that customers know the support of the random variable which influences their decision (price). In this section I discuss possible consequences of relaxing this assumption. Suppose, consumers hold the beliefs about support $[\underline{p}', \overline{p}']$, while the true support of prices is $[\underline{p}, \overline{p}]$. In order to make the analysis more comprehensive, I assume, that it is not possible that some price "surprises" customer, i.e. that support of the true distribution falls into the interior of the beliefs: $\underline{p}' \leq \underline{p} \leq \overline{p} \leq \overline{p}'$. The best way to think about this restriction is that $\underline{p}' = \min(\underline{p}'', \underline{p})$, where $\underline{p}''$ is belief consumer holds when $\underline{p}$ is large enough. If $\underline{p}$ is small (which happens when c is small, or λ or N are large), the true lower bound coincides with the belief. This construction allows to analyze comparative statics in case of incorrect beliefs.

Note, that we if we define beliefs in this way, then the Theorem 1 still holds. The equilibrium price distribution is defined in the same way as in Proposition 2 with the only difference that now there is no link between lower bound of the support ($\underline{p}$) and the upper bound ($\overline{p} = \underline{p}' + 4c$), which now depends on incorrect belief. Thus, the equilibrium price distribution is

$$F(p) = \begin{cases} 0, & p \leq \frac{1-\lambda}{1+\lambda(N-1)}(p' - 4c) \\ 1 - \left(\frac{1-\lambda}{N\lambda}\frac{p'-p+4c}{p}\right)^{\frac{1}{N-1}}, & \frac{1-\lambda}{1+\lambda(N-1)}(p' - 4c) < p \leq p' - 4c \\ 1, & p > p' + 4c \end{cases}$$

Equilibrium profits equal to $\pi = \frac{1-\lambda}{N}(p' + 4c)$.

Comparative statics with respect to parameters c and λ is similar to the original model. Comparative statics with respect to N is a more delicate issue. Note, that $\underline{p}$ converges to zero, as N goes to infinity. In this case the expected price is constant in the number of firms if this number is small enough ($\underline{p} > \underline{p}''$), and decreases afterwards ($\underline{p} \geq \underline{p}''$).

5 Conclusions

I build a model of oligopolistic competition in which consumers search sequentially. Consumers have minimum information about the price distribution, they know only its support. Consumers' behaviour is restricted by four consistency requirements, and given those requirements is uniquely defined. Moreover, though the reservation prices are non-stationary in time, the optimal stopping rule can be represented by a simple heuristic. I derive the symmetric equilibrium and show that it exhibits several properties, which are in contradiction with the existing literature. Firstly, the unpleasant property of Stahl's model, that prices are increasing in the number of firms is reversed. The expected price is decreasing in the number of firms and approaches some positive number as the number of firms goes to infinity. I also show that unlike Dana (1994) and Janssen et al. (2011) limited information may both harm and benefit consumers, depending on the parameters of the model: if the number of firms is sufficiently large, then the expected price in the model with boundedly rational consumers is lower than in the case of full rationality.

Appendix: Proofs

In order to prove Theorem 1 I need a few preliminary statements.

Lemma A.1. *Let $g_k(x) : \mathbb{N} \times \mathbb{R} \to \mathbb{R}$ be defined by*

$$g_k(x) = \frac{k}{k+1}x + \frac{1}{k+1}\int_{\underline{p}}^{x} \frac{z}{x-\underline{p}}dz \tag{2}$$

Then, for any $k > 0$, $x > \underline{p}$, the function $g_k(x)$ is increasing both in k and x.

Proof. First, since both summands in 2 are increasing in x, $g_k(x)$ is increasing in x. Second, since $\frac{k}{k+1}$ is increasing in k, $\frac{1}{k+1}$ is decreasing in k (and they both sum up to 1) and $x > \int_{\underline{p}}^{x} \frac{z}{x-\underline{p}}dz$ for all $x > \underline{p}$, the function $g_k(x)$ in increasing in k. □

Corollary A.2. *If x_k is a solution for $x_k = g_k(x_k) + c$, $c > 0$, then $x_k > x_{k-1}$ for any k.*

Lemma A.3 provides the base for induction.

Lemma A.3. *Suppose a consumer has searched $N-1$ firms. Then the optimal stopping rule is characterized by reservation price $r_{N-1} = 2Nc$ and perceived continuation costs are equal to $g_{N-1}(p) + c$.*

Proof. Suppose if the consumer buys immediately she has to pay price p. If she decides to continue then she pays c and with perceived probability $\frac{N-1}{N}$ the price will be in one of the N higher quantiles, and thus the consumer will have to return back. With probability $\frac{1}{N}$ the price will fall to the lowest quantile and then the expected price will be $\int_{\underline{p}}^{p} \frac{z}{p-\underline{p}}dz$. Summing up these terms gives that continuation costs equal to $g_{N-1}(p) + c$. Now, for the reservation price it has to be that

$$r_{N-1} = c + \frac{N-1}{N}r_{N-1} + \frac{1}{N}\int_{\underline{p}}^{r_{N-1}} \frac{z}{r_{N-1}-\underline{p}}dz$$

which gives

$$r_{N-1} = 2Nc + \underline{p}$$

□

Now, Theorem 1 can be proved.

Theorem 1. *A reservation price on step k is given by $r_k = \underline{p} + 2(k+1)c$.*

Proof. I construct a proof by induction. The base of induction is proved in Lemma A.3. Assume that the reservation price for steps $j \in \{k+1, \ldots, N-1\}$ is given by $r_j = \underline{p} + 2(j+1)c$.

Consider step k. Suppose that the consumer observes price p at this step.

If $p \leq r_k$ it is optimal to stop. Indeed, if the consumer continues, then she has to stop on the next step, since $p \leq r_k < r_{k+1}$. Then, the continuation cost on the next step is given by $g_{k+1}(p) + c$, which is higher than p for any $p < r_{k+1}$ due to A.2. Thus, it is optimal to stop.

If $p > r_k$ it is optimal to continue. Indeed, if we denote by $C_k(p)$ the costs of continuing to search on step k given price p, then

$$C_k(p) = c + \frac{k}{k+1}\min(p, C_{k+1}(p)) + \frac{1}{k}\mathbb{E}(\min(q, C_{k+1}(q)|q < p)) \leq c + g_k(p)$$

which is less than p since $p > r_k = \underline{p} + 2(k+1)c$. Thus, it is optimal to search further.

Therefore, r_k is the optimal stopping price. □

Proposition 2. *The equilibrium pricing strategy is given by*

$$F(p) = \begin{cases} 0, & p \leq 4c\frac{1-\lambda}{\lambda N} \\ 1 - \left(\frac{1-\lambda}{\lambda N} \cdot \frac{4(1-\lambda)c + \lambda N(4c-p)}{\lambda N p}\right)^{\frac{1}{N-1}}, & 4c\frac{1-\lambda}{\lambda N} < p \leq 4c\frac{1-\lambda+\lambda N}{\lambda N p} \\ 1, & p > 4c\frac{1-\lambda+\lambda N}{\lambda N} \end{cases}$$

and the equilibrium level of profits equals to $\pi = 4c\frac{(1-\lambda)(1-\lambda+N\lambda)}{\lambda N^2}$.

Proof. Since $F(p)$ is an equilibrium distribution profits should be constant over the support of $F(p)$. Then, since $F(p)$ is atomless I get

$$\pi(p) = \lambda(1 - F(p))^{N-1}p + \frac{1-\lambda}{N}p$$

where the first term comes from informed consumers and the second from the uninformed. Due to the fact that the upper bound of the distribution has to be equal to the first round reservation price I have $\pi(r_1) = \frac{1-\lambda}{N}(\underline{p} + 4c)$. On the other hand

$$\pi(\underline{p}) = \lambda\underline{p} + \frac{1-\lambda}{N}\underline{p}$$

which gives

$$\underline{p} = 4c\frac{1-\lambda}{\lambda N}$$

Therefore

$$r_1 = 4c\frac{1-\lambda+\lambda N}{\lambda N} \tag{3}$$

and the equilibrium distribution function over its support is

$$F(p) = 1 - \left(\frac{1-\lambda}{\lambda N} \cdot \frac{4(1-\lambda)c + \lambda N(4c-p)}{\lambda N p}\right)^{\frac{1}{N-1}}$$

□

Proposition 3. *The expected price payed by consumers is:*

1. *increasing in search costs;*

2. *decreasing in the number of informed customers;*

3. *decreasing in the number of firms.*

Proof. Note, that since consumers have unit demand the expected price is just equal to the total profit of the firms. Then,

$$\frac{\partial(\pi N)}{\partial c} == 4\frac{(1-\lambda)(1-\lambda+N\lambda)}{\lambda N} > 0$$

thus, the expected price is increasing in c.

$$\frac{\partial(\pi N)}{\partial \lambda} = 4c\left(-1+\frac{1-\frac{1}{\lambda^2}}{N}\right) < 0$$

thus, the expected price in decreasing in λ.

$$\frac{\partial(\pi N)}{\partial N} = \frac{\partial}{\partial N}\left(4c\frac{(1-\lambda)(1-\lambda+N\lambda)}{\lambda N^2}N\right) = -4c\frac{(1-\lambda)^2}{\lambda N^2} < 0$$

Thus, the expected price is decreasing in the number of firms. □

Proposition 4. *For any $\lambda \in (0,1)$ there is N^*, such that for all $N > N^*$ $r_1(\lambda, N) < \rho(\lambda, N)$, where ρ is a reservation price of Stahl's model.*

Proof. First note, that r_1 is decreasing in N and $\lim_{N\to\infty} r_1 = 4c$.

Then, consider an equation for ρ:

$$\int_{\frac{1-\lambda}{1+(N-1)\lambda}\rho}^{\rho} \left(1-\left(\frac{1-\lambda}{\lambda N}\frac{\rho-p}{p}\right)^{\frac{1}{N-1}}\right) dp =$$

$$= \frac{N\lambda}{1+(N-1)\lambda}\rho - \left(\frac{1-\lambda}{\lambda N}\right)^{\frac{1}{N-1}} \int_{\frac{1-\lambda}{1+(N-1)\lambda}\rho}^{\rho} \left(\frac{\rho-p}{p}\right)^{\frac{1}{N-1}} dp =$$

$$= \frac{N\lambda}{1+(N-1)\lambda}\rho - \rho\left(\frac{1-\lambda}{\lambda N}\right)^{\frac{1}{N-1}} \int_{\frac{1-\lambda}{1+(N-1)\lambda}}^{1} \left(\frac{1-t}{t}\right)^{\frac{1}{N-1}} dt =$$

$$= \rho\left(\frac{N\lambda}{1+(N-1)\lambda} - \left(\frac{1-\lambda}{\lambda N}\right)^{\frac{1}{N-1}}\left({}_2F_1\left(\frac{1}{1-N}, \frac{-2+N}{-1+N}, 2+\frac{1}{1-N}, 1\right) - \left(\frac{1-\lambda}{1+(-1+N)\lambda}\right)^{1-\frac{1}{N-1}} {}_2F_1\left(\frac{1}{1-N}, \frac{-2+N}{-1+N}, 2+\frac{1}{1-N}, \frac{1-\lambda}{1+(-1+N)\lambda}\right)\right)\right) = c$$

where ${}_2F_1$ is the hypergeometric function.

Now, note that

$$\lim_{N\to\infty} {}_2F_1\left(\frac{1}{1-N}, \frac{-2+N}{-1+N}, 2+\frac{1}{1-N}, \frac{1-\lambda}{1+(-1+N)\lambda}\right) = 1$$

$$\lim_{N\to\infty} {}_2F_1\left(\frac{1}{1-N}, \frac{-2+N}{-1+N}, 2+\frac{1}{1-N}, 1\right) = 1$$

$$\lim_{N\to\infty} \left(\frac{1-\lambda}{1+(-1+N)\lambda}\right)^{1-\frac{1}{N-1}} = 0$$

$$\lim_{N\to\infty} \left(\frac{1-\lambda}{\lambda N}\right)^{\frac{1}{N-1}} = 1$$

$$\lim_{N\to\infty} \frac{N\lambda}{1+(N-1)\lambda} = 1$$

Therefore for all λ $\lim_{N\to\infty} \rho = \infty$. Thus, there must exist such N^* that for all $N > N^*$

$\rho > r_1$.

$\square$

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