THE COMPETITIVE EFFECT OF MULTIMARKET CONTACT

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Abstract Changes in the extent of multi-market contact (MMC) between firms often affect market outcomes – quantities and prices. This paper challenges the standard economic interpretation of this phenomena as an indication of tacit collusion. We show that a strategic but purely competitive effect of changes in MMC can change the quantity provided in a market by a firm by as much as 50%. Moreover, changes in demand for a firm one market may affect equilibrium quantities in markets where the firm is not active.

1. INTRODUCTION

Large firms are often active in more than one market and commonly compete with each other in many, but not necessarily all, markets. For example, United Airlines and American Airlines compete on some but not all of their routes. There is a large empirical literature on the effect of changes in the extent of frims' multi-market contact (MMC) on market outcomes.[1] While inconclusive, most empirical work finds that increases in MMC tend to soften competition. The standard interpretation for this finding is that of MMC as facilitating tacit collusion (a.k.a. mutual forebearance). This paper studies a different mechanism from which similar results may be obtained. Specificaly, we show how sunk transferable investments can cause firms to compete less aggresively in industries with significant MMC compared to industries with lesser MMC.

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[1]See e.g. Evans and Kessides (1994); Gimeno and Woo (1999); Parker and Roller (1997); Jans and Rosenbaum (1997); Fernandez and Marin (1998); Pilloff (1999)

Sunk transferable investments are common in many industries: WalMart may buy a container of dolls from China and decide upon its arrival into LA what stores to send the dolls to; GE may invest in a centralized HR department and allocate resources from this department across its different divisions; American Airlines may purchase 60 gates at O'Hare and then allocate the gates to flights serving distinct markets. Common examples of such investments are production facilities, overhead, and specialized inputs.

This paper shows that the allocation of these investments into different markets and the initial investment decision itself are affected by the extent of MMC between firms.

The non-commitment of investments to a specific market allows firms the *flexibility* to reallocate a sunk investment. If market conditions deteriorate for Walmart in north LA, it can sell less there and shift the stock to it's other stores in the region. This flexibility can be abused by a rival. If a rival can be aggresive in a joint market, the firm will use its flexibility to allocate the investment into other markets in which the rival does not exist. This strategy requires the rival to *commit* to an aggressive behavior: increase its sunk investments that will be used in the joint market. We show that the MMC equilibrium outcome can be defined in terms of the firms' *flexibility* and *commitment power* whether firms compete in prices or quantities.

In industries with significant overlap, firms would compete softly as rivals would lack the ability to reallocate sunk investments into private markets due to their scarcity. In industries with little overlap firms would compete softly as no firm would be able to commit the allocation of additional investments into the overlapping market. In industries with intermediate overlap firms would compete aggresively as rivals have the ability to reallocate investments and firms have the ability to commit the allocation of additional investments into the overlapping markets. This non-monotonic relationship between competition and MMC may explain some of the empirical evidence on MMC.

The results have important impacts on merger discussions and trade policy. Changing the degree of multimarket contact may change how aggressive firms behave even if the number of firms in each market remains constant. In some recent US airline mergers firms argued heavily that their own overlap was minimal (Delta and Northwest) and thus consumers would not be harmed by the merger. This paper shows how changes in rival carriers investment decisions as a result of the merger could in fact hurt consumers. In a different example, the US has a non-reciprocal trade agreement with Japan in which Japanesse steel manufacturers may sell in the US but US steel manufacturers may not sell in Japan. This treaty might in fact benefit American steel manufacturers' position in the US, as they can credibly commit their capacity to the American market while Japannesse companies cannot.

This paper bridges together, and builds on, three strands of the literature: mutual forebearance driven my multimarket contact, pre-emption through investments, and competition in strategic substitutes vs strategic complements. The possibility that MMC affects tacit cooperation was formalized and studied in detail by Bernheim and Whinston (1990).[2] The concern has been the subject of many empirical studies. For example, considering the US domestic airlines industry, Evans and Kessides (1994), Gimeno and Woo (1999) and more recently Ciliberto and Williams (2012) find that an increase in MMC between carriers reduces prices. This is considered by the authors as evidence of the mutual forbearance (anti-competitive) effect of MMC. Similar studies have been conducted

[2] Bernheim and Whinston (1990) credit Edwards (1955) with the introduction of this idea to the academic economic discourse.

in different industries. See e.g. Parker and Roller (1997), Jans and Rosenbaum (1997), Fernandez and Marin (1998) and Pilloff (1999).

Spence (1977) and Dixit (1980) formalized the economic use of committed investments in capacity or in cost reduction to deter entry and help incumbents achieve a Stackelberg type leadership position in the market. By making committed investments in an early stage, incumbents can "push out" their reaction functions. Potential entrants that have not made such committed investments are thus deterred to enter or to obtain significant market share. The key difference between the potential entrant and the incumbent relied on the committed investments. In our framework, all firms make non-reversible investment decisions, but these are not fully committed to the markets in which firms overlap. The size of private markets limit how much of the initial investments are truly committed to the joint markets. As such, all firms' reaction curves are pushed out but the amount by which they are pushed out depends on the size of the private markets.

The terms strategic complements and strategic substitutes were coined in the seminal work of Bulow et al. (1985), which showed how the type of competition (complements or substitutes) and the cost structure of firms (economies of scale or diseconomies of scale) affected how firms competed when one firm had access to private markets. The basic model presented here extends such analysis in two important ways. First, firms *choose* investment decisions, which determine the extent of the diseconomies of scale. In this sense we endogenize the cost structure of the firm. Second, we allow for both firms to have access to private markets. By adding these extensions, some of Bulow et al. (1985)'s results regarding MMC no longer hold and the two forms of competition (substitutes and complements) no longer mirror each other.

Section 2 presents and solves the model based on quantity competition. Section 3 assumes price competition. Section 4 discusses the implications for both mergers and trade policy.

2. COURNOT MODEL

2.1. *Setup*

Consider an industry with two firms, identified by $i \in \{A, B\}$ and three markets: an *overlapping* market in which both firms are active, and a *private* market for each firm – one market in which firm A operates but not firm B, and one in which firm B operates but not firm A.[3] The three markets may vary in size. In particular, we use m_o, m_A, and m_B for the measure, or size, of the overlapping market, A's private market, and B's private market.

An equivalent interpretation of the model is that there are many identical markets where m_o, m_A, and m_B are the number of markets that are respectively overlapping and private for firm A and B. In this model, m_o is the measure of the markets in which both firms operate (i.e. markets that generate MMC) while m_A and m_B are the measures of the markets in which one of the firms does not operate (i.e. markets that do not generate MMC).

The model has two stages. In the first stage firms simulteanously make an investment in capacity, denoted k_A, k_B. Firms pay a constant marginal cost c per unit of capacity.[4] To fix ideas, we refer to this investment as production capacity. However, it may be interpreted as any investment that is sunk, transferable across markets and can be utilized to increase production.

After both capacities are fixed and known, in a second stage, firms *compete*

[3]The qualitative results remain if each firm faces competition from other firms in their 'private' markets.

[4]The results are qualitatively unaffected when allowing for weakly convex costs and cost asymmetries between firms.

in quantities subject to their installed capacity.[5] Each firm chooses how many units to provide to the overlapping market and how many units to provide in its private market. For simplicity we assume there is no direct cost when choosing quantities. Nevertheless, a firm's total output across both markets cannot be larger than its installed capacity. Markets clear accordingly.

Denote by q_A and q_B the quantity sold by each firm in the overlapping market per market size m_o. That is, the total quantity offered in the overlapping market is $m_o\left(q_A+q_B\right)$. Similarly, denote by $\hat{q}_A$ and $\hat{q}_B$ the quantity sold in the private market per market size (respectively m_A and m_B) so that the total quantity offered in firm i's private market is $m_i \cdot q_i$. The degree of multi-market contact (MMC) is determined by the relative sizes of m_o, m_A and m_B. When m_o is large (resp. small) relative to both m_i, the degree of MMC is large (resp. small). MMC can also be assymetric. If $m_A \gg m_o \gg m_B$ then MMC is small for firm A and large for firm B.

The inverse demand curve for the overlapping and private markets per consumer measure are $P\left(Q\right)$ so that the market price in the overlapping market is $P\left(q_A+q_B\right)$ and the market price in each private market is $P\left(q_i\right)$. The results below will be provided for a general inverse demand function $P\left(\cdot\right)$ and for the standard linear curve:

$$P\left(Q\right)=a-b\cdot Q\ .$$

An *equilibrium* $\left(\mathbf{k}^{\star},\mathbf{q}_A^{\star},\mathbf{q}_B^{\star}\right)$ is a pair of scalars $\mathbf{k}=\left(k_A,k_B\right)$ indicating the capacity set by each firm, and two pairs $\mathbf{q}_A^{\star}=\left(q_A^{\star},\hat{q}_A^{\star}\right)$ and $\mathbf{q}_B^{\star}=\left(q_B^{\star},\hat{q}_B^{\star}\right)$ indicating the quantity allocation chosen by each firm in each market.

The game is solved through backward induction. In the second stage, given

[5] Competition in prices is solved in the next section.

capacities k_A and k_B, firms choose quantities to be set in each market, so as to maximize revenue:

$$(2.1)\qquad R_i(k_A,k_B) = \max_{q_i,\hat{q}_i\geq 0} \quad m_o q_i \cdot P\left(q_i+q_{-i}\right) + m_i\cdot\hat{q}_i\cdot P\left(\hat{q}_i\right) \qquad \text{s.t.} \quad m_o q_i + m_i\hat{q}_i \leq k_i$$

A sub game equilibrium $(\mathbf{q}_A, \mathbf{q}_B; \mathbf{k})$ is the set of quantity allocations that form an equlibrium given the first stage capacities $\mathbf{k}$. Throughout, we use ρ_i and $\hat{\rho}_i$ to denote firm i's marginal revenue curve per consumer measure in the overlapping and private markets, respectively.[6]

A natural regularity assumption for the analysis is that the inverse demand curves are such that the sub-game equilibrium for any first stage capacities is unique and differentiable in $\mathbf{k}$:

Assumption All inverse demand functions $P(\cdot)$ are strictly decreasing and log concave in all their arguments.

This assumption allows us to describe the equilibrium strategies of the subgame as a function of the first stage investments, which we define with a slight abuse of notation as: $q_i(k_A,k_B)$ and $\hat{q}_i(k_A,k_B)$.

The first stage problem for each firm is given by:

$$(2.2)\qquad \max_{k_i\geq 0} R_i(k_A,k_B) - ck_i \quad .$$

The first stage profit functions determine the equilibrium capacities $(k_A^\star, k_B^\star)$, which in turn map into the equilibrium second stage quantities: $q_i^\star = q_i(k_A^\star, k_B^\star)$.

2.2. *Equilibrium*

The next lemma identifies a basic characteristic of the equilibrium that will be used throughout.

[6]That is $\rho_i = q_i \cdot P'\left(q_i+q_{-i}\right) + P\left(q_i+q_{-i}\right)$ and $\hat{\rho}_i = \hat{q}_i\cdot P'\left(\hat{q}_i\right) + P\left(\hat{q}_i\right)$.

LEMMA 1 *In equilibrium, in the second stage:*

1. *The capacity constraint binds:*$k_i = m_o q_i + m_i \hat{q}_i$.
2. *Firm i's marginal revenue is identical in both of it's markets:*$\rho_i = \hat{\rho}_i$.

The first claim is identical to those made in Spence (1977) and Dixit (1980). Firm B's best response in the second stage depends only on A's quantity in the overlapping market (q_A). Therefore, if for any first stage decisions k_A,k_B, A expects to have excess capacity ($k_A > q_A + \hat{q}_A$), it should reduce k_A and set the same second stage quantities. Firm B's best response is unaffected and so firm A's revenues are unaffected, but A's costs are strictly lower.

The reasoning behind the second claim is standard. If the marginal revenue in one market is larger than the other (e.g. $\rho_i > \hat{\rho}_i$), optimality requires diverting quantity from the market with lower marginal revenue to the market with the larger marginal revenue.

Before characterizing the equilibrium we formalize two concepts: *flexibility* and *commitment power*.

DEFINITION 1 Firm i's *flexibility*, ϕ_i, is the firm's second stage reaction to a change in its competitor's quantity allocation:

$$\phi_i \equiv \frac{\partial q_i}{\partial q_{-i}} \tag{2.3}$$

Flexibility measures A's best response to B's second stage deviation in the equilibrium neighbourhood. In the standard Cournot with fixed marginal cost, this is the firm's reaction curve, which we denote $\overline{\phi}_i$.[7] Flexibility differs from

[7] Using marginal revenues, the standard result is

$$\overline{\phi}_i = -\left(\frac{\partial \rho_i}{\partial q_{-i}} / \frac{\partial \rho_i}{\partial q_i}\right) \quad .$$

the standard best response because the economic cost of increasing q_i is not constant (i.e. c). In particular, as all production costs have been paid for in the previous stage, the cost of an increase in quantity in the overlapping market is the required decrease in the private market quantity. Therefore, in the second stage, the marginal cost for firm i in the overlapping market is it's marginal revenue in the private market, and vice versa. Accordingly, firm i's flexibility will depend on the extent of A's MMC $\left(\frac{m_0}{m_A}\right)$, amd the relative sensitivity of its marginal revenues to quantity changes:

LEMMA 2 *In equilibrium,*

$$\phi_i = -\frac{\frac{\partial \rho_i}{\partial q_{-i}}}{\frac{\partial \rho_i}{\partial q_i} + \frac{m_o}{m_i}\frac{\partial \hat{\rho}_i}{\partial \hat{q}_i}} \in \left[\overline{\phi}_i, 0\right] \ .$$

If the demand curve is linear,

$$\phi_i = -\frac{1}{2} \cdot \frac{m_i}{m_i + m_o}$$

As expected, flexibility is negative as competition is in strategic substitutes. When A's extent of MMC, $\frac{m_o}{m_A}$, is large, $\phi_A \to 0$. In these settings, A's first stage investment was made primarily to serve the joint market and A cannot profitably divert the investment to it's private market. In contrast, when A's extent of MMC is small, it's private market can absorb any excess capacity without effect and as a result $\phi_A \to \overline{\phi}_A$, the standard Cournot reaction curve.

While flexibility may be profitable when dealing with uncertainty, in our setting, flexibility in the second stage makes the firm vulnerable to its rival's leadership. If firm A is flexible (ϕ_A large in absolute terms) while firm B is not ($\phi_B \to 0$), B can make it's second stage decision in the first stage. The resulting dynamics are as if the flexible firm is a Stackelberg follower and it's rival a Stackelberg leader. However, for this to work, firm B needs to be able to make a credible first stage commitment:

DEFINITION 2 Firm i's *commitment power*, σ_i, is the change in the firm's second stage overlapping market quantity following an additional unit of investment:

$$\sigma_i \equiv m_o \frac{\partial q_i}{\partial k_i}$$

Commitment power is simply the fraction of the additional unit of invested capacity that would be allocated to the overlapping market. Ideally, the firm would like to set it's investment level to the sum of the optimal investment in it's private market, and the optimal investment in the overlapping market. However, this is generally not possible. As lemma 1 indicates, the second stage allocation equalizes marginal revenues across both markets for any first stage investment. For example, suppose k_i is set so that i can set the monopoly quantity in it's private market and the duopoly quantity in the overlapping market. With those allocations in the second stage, i's MR is the same in both markets. If i now tries to take advantage of it's rival's flexibility and increase quantity in the overlapping market, this will reduce i's MR in the overlapping market: $\rho_i < \hat{\rho}_i$. As a result, it is strictly optimal for i to also increase quantity a bit in it's private market.

A firm's commitment power therefore depends on the shape of the MR curves and the extent of MMC. However, holding the rival's first stage decision fixed, the change in the overlapping market MR will be accomodated by the rival, depending on its flexbility. Thus:

LEMMA 3 *In equilibrium,*

$$\sigma_i = \frac{\frac{\partial \hat{\rho}_i}{\partial \hat{q}_i}}{\frac{m_i}{m_o}\left(\frac{\partial \rho_i}{\partial q_i} + \frac{\partial \rho_i}{\partial q_{-i}}\phi_{-i}\right) + \frac{\partial \hat{\rho}_i}{\partial \hat{q}_i}} \in [0,1]$$

If the demand curve is linear

$$\sigma_i = \frac{1 + \frac{m_{-i}}{m_o}}{1 + \frac{m_i}{m_o} + \frac{m_{-i}}{m_o} + \frac{3}{4}\frac{m_i}{m_o}\frac{m_{-i}}{m_o}}$$

Both flexibilty and commitment power depend only on the relative sizes of the private markets compared to the overlapping market. In the competitive analysis, this is the relevant measure of MMC. As an increase in any quantity reduces the marginal revenue, σ_i is positive.[8] Without a private market $\left(\frac{m_i}{m_o} \to 0\right)$, any first stage investment will be used in the overlapping market and $\sigma_i \to 1$. If the private market is large relative to the overlapping market $\left(\frac{m_i}{m_o} \to \infty\right)$, the bulk of i's capacity increase will be used in it's private market, and $\sigma_i \to 0$.

We now characterize the equilibrium.

PROPOSITION 1 *The equilibrium of the game is characterized by the following equation*

$$c - \rho_i = c - \hat{\rho}_i = q_i P^{'}(q_i + q_{-i}) \cdot \phi_{-i} \cdot \sigma_i > 0 \qquad i \in \{A, B\}\ .$$

Firms behave more aggresively under MMC than they would have under standard oligopoly.

If $\frac{m_i}{m_o} \to \infty$ *and* $\frac{m_{-i}}{m_o} \to 0$ *, then the equilibrium in the overlapping market is the Stackelberg equilibrium with firm i the Stackelberg follower.*

[8]This also relies on the fact that the rival effect on MR is weaker than the own effect, and that $\phi_i \geq -1$. Both are intuitive and proved in the appendix.

Partial MMC provides an extra incentive for firms to be aggressive: the rival has the flexibility to accomodate aggressive behavior by hurting other markets, in which the aggressive firm is not active. However, as MMC (m_o) increases, the rival "runs out" of flexiblity ($\phi_{-i} \to 0$), and the firms converge to the standard Cournot result.

Partial MMC also affects the private - non-overlapping markets. To commit to being more aggressive in the overlapping market, the firm must increase it's private market quantity.

Whenever quantities in duopoly and monopoly markets are well below the surplus maximizing level, the welfare effect is unambiguous. Total market quantity, consumer welfare and surplus are always higher in all markets as a result of MMC, while total industry profit in equilibrium is always lower. As the result depends on the equilibrium without MMC, we show this is the case for linear demand:

PROPOSITION 2 *If demand is linear, consumer and total surplus is always at least as high in industries with MMC as without. Total industry profit is always at most as high in industries with MMC as without.*

We call the difference between cost and marginal revenue in proposition 1 firm *i's strategic effect of MMC*. As σ_i and ϕ_i depend only on the ratios $\frac{m_i}{m_o}$, so does the strategic effect.

Underlying the strategic effect of MMC is the observation that the existence of the private markets affects the firm's reaction in the second stage, and that the firm's rival takes advantage of this. Fix firm A's first stage capacity, k_A and consider an increase in B's capacity. This will imply an ε increase in it's quantity in the overlapping market. If A does not have a private market, it will still sell the same number of units in the overlapping market. If A's private market is sufficiently 'large', however, A can shift some of the losses from B's additional quantity to it's private market, accomodating B. By increasing it's quantity in the overlapping market, B profits from A's accomodation, in addition to the standard marginal revenue effect. The extent of A's accomodation is captured by ϕ_A. The extent of B's additional profit is B's overlapping market quantity, q_B, multiplied by the price implication of A's accomodation, $P^{'}()$.

The final piece in the equilibrium condition is B's ability to influence it's overlapping market quantity through it's first stage solution, captured by σ_B. B can only commit a fraction of any first stage additional capacity to the overlapping market since it will always be optimal for B to equate marginal revenue across markets in the second stage. When B's private market is 'small' relative to the overlapping market, σ_B is large. The majority of the additional capacity will be allocated to the overlapping market. B's first stage decision influences the market quantity and so B's strategic effect of MMC increases. If B's private market is 'large' relative to the ovelapping market. The majority of the additional capacity will be allocated to the private market and B's first stage capacity choice does not credibly affect the second stage overlapping quantity. As a result, B's strategic effect of MMC will be small.

The importance of the overlapping market for each firm relative to it's private market determines the firm's strategy and the resulting equilibrium. This is demonstrated in the equilibrium outcome with linear demand:

PROPOSITION 3 *If demand is linear, let* $\lambda_i = \frac{m_o}{m_o+m_i}$, *then the equilibrium* *quantities are:*

$$q_i = \frac{a-c}{3b} \cdot \frac{9+3\lambda_i+3\lambda_{-i}-3\lambda_i\lambda_{-i}}{\chi_i}$$

$$\hat{q}_i = \frac{a-c}{2b} \cdot \frac{9+\lambda_i-\lambda_{-i}+3\lambda_i\lambda_{-i}}{\chi_i}$$

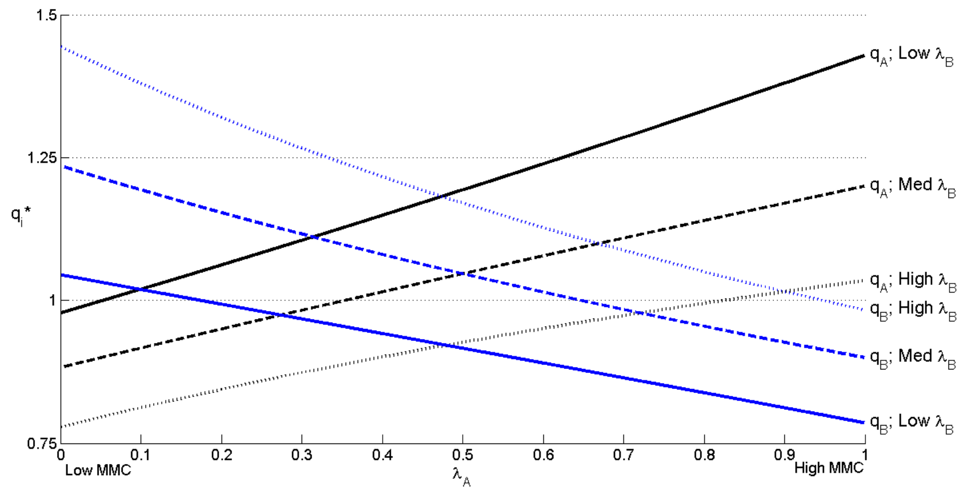
Where

$$\chi_i \equiv \frac{(9-\lambda_{-i})(3+\lambda_{-i})-(\lambda_i(1-\lambda_{-i}))^2+2\lambda_i(1+\lambda_{-i})(3+\lambda_{-i})}{3-\lambda_i+\lambda_{-i}-\lambda_i\lambda_{-i}}$$

The fractions identify the deviation from the Cournot standard, illustrated in figure 2.1. The use of the relative size of the overlapping and private market for the firm ,λ_i, formalizes the intuition that only the relative sizes matter (and simplifies the notation). Holding m_o fixed, if $m_A = m_B \to 0$ (and so $\lambda_i \to 1$) then neither firm has flexibility to accomodate in the second stage ($\phi_{-i} \to 0$). Market quantities exactly equal the standard Cournot. Similarly, if $m_A = m_B \to \infty$ (and so $\lambda_i \to 0$), neither firm can commit to allocate additional investment in the overlapping market ($\sigma_i \to 0$) and again market quantities are the standard Cournot quantities. In all other cases, however the market quantities are larger than Cournot.

The ability to commit in the first stage is closely related to the Stacklberg analysis. Suppose firm A mainly serves the overlapping market ($\lambda_A \to 1$) while firm B mainly serves it's private market ($\lambda_B \to 0$). This would be the case, for example, if firm A is a local (mom and pop) firm while firm B is a 'big-box' firm that serves many localities. Assume that firms set quantities ignoring the MMC effects, so that in all markets marginal revenue equals marginal cost. Note that the marginal revenue in all markets for all firms is identical ($= c$). Now suppose that, between the first and the second stage, the demand in the overlapping market m_o decreases. Both firms are "stuck" with over-capacity destined to the overlapping market. Firm A does not have much alternative options for it's excess capacity. However, firm B can divert the quantity destined for the overlapping

FIGURE 2.1.— Firm's quantity in the overlapping market
– Strategic Substitutes –



* Plot shows quantities normalized by the Cournot benchmark: $q_A^\star = q_A/\tilde{q}$ where $\tilde{q} = \frac{a-c}{3b}$. λ_A measures A's MMC: $\lambda = \frac{m_o}{m_A+m_o}$. Three lines are plotted for each firm: when $\lambda_B = 0.1$ (Low), $\lambda_B = 0.5$ (Med), and $\lambda_B = 0.9$ (High).

market to any of it's other markets. As the marginal revenue in those other markets equals marginal cost, if there are enough of these other local markets, firm B's profit is almost uneffected. This is exactly the implication of B's low MMC: firm B is flexible and can absorb any excess capacity from the overlapping market in it's private markets.

A low MMC implies that the firm has an alternative use to it's overlapping quantity and as a result, the overlapping market quantity decision for the firm is done in the second stage. In contrast, a high MMC implies that the firm chooses its overlapping quantity in the first stage. As m_i decreases, it is as if firm i moves from being a second mover to a first mover.

At the extreme ($\lambda_A \to 1$ and $\lambda_B \to 0$), firm A moves first and firm B moves second – the Stackelberg result is obtained. As m_B decreases, or m_A increases,

A's first mover advantage decreases: A's quantity decreases and B's quantity increases.

PROPOSITION 4 *A firm's equilibrium quantity in both markets* ***increases*** *with the firm's MMC and* ***decreases*** *with the rival's MMC*[9]

$$\frac{dq_i}{dm_i} < 0 \quad \frac{d\hat{q}_i}{dm_i} \leq 0 \qquad\qquad \frac{dq_i}{dm_{-i}} > 0 \quad \frac{d\hat{q}_i}{dm_{-i}} \geq 0$$

Firm's total profits are increasing in it's own MMC and decreasing in rival's MMC.

Propositon 4 identifies three implications of the game. First, a change in MMC has the same qualitative effect on the firm's private and overlapping market. Second, an increase in firm i's MMC, makes firm i more aggressive in both the overlapping market and it's private market. Finally, an increase in firm i's MMC makes it's rival less aggressive in both the overlapping market and it's own private market. This last result $\left(\frac{d\hat{q}_i}{dm_{-i}} \geq 0\right)$ implies that a change in one firm's private market conditions may affect the outcome in other firms' private market conditions. Firm A may decrease it's quantities in markets it is serving as a monopolist as a result of a decrease in demand in markets monopolized by firm B.[10]

The above proposition allows for a straightforward assessment of MMC on the welfare of the private markets:

COROLLARY 1 *Total output and consumer surplus increase in the firm's private market and decrease in the rival's private market in response to an increase in the firm's MMC.*

[9] The inequalities are weak only if λ_i or λ_{-i} are exactly zero.

[10] A similar result is suggested in Bulow et al. (1985). However, there, only one firm has a private market and so the same result cannot be obtained.

While the effects of MMC on total output and consumer surplus are straightforward in the private markets, they are not so in the overlapping market. Proposition 4 states that firms' equilibrium strategies move in opposite direction in the overlapping markets. This leaves open the question on what is the net effect. Does consumer welfare increase or decrease? The next proposition answers this question:

PROPOSITION 5 *When competition is in strategic substitutes there is a non-monotonic relationship between total output and the extent of MMC. In particular, an increase in* m_o *increases (resp. decreases) the overlapping market quantity if the extent of MMC is small (resp. large) for both firms. For any* m_o*, there are* $\underline{m} > 0$ *and* $\overline{m} < \infty$ *such that*

- *If* $\max_i \{m_i\} < \underline{m}$*, then* $\frac{dq_i}{dm_o} \leq 0$*.*
- *If* $\min_i \{m_i\} > \overline{m}$*, then* $\frac{dq_i}{dm_o} \geq 0$*.*

The non-monotonocity can be understood under similar lines as those used above. When MMC is very small or very large, it has no effect. When MMC is partial, quantities are higher. Thus, an increase in MMC should initially increase quantities and eventually decrease quantities.

It may be surprising that both firms are more agressive as a result of MMC. To see this, suppose that $m_A = m_B = m_o$ and that firm A sets its first stage capacities so that it can exactly provide the monopoly quantity in its private market and the Cournot quantity in the private market. Firm B's best response in the first stage is to commit to a slightly larger capacity, knowing well that firm A will be able to accomodate B's additional quantity in the overlapping market by shifting some of it's quantity intended to the overlapping market into the private market. As the Cournot quantities are optimal holding the rival quantities fixed, the additional profit for firm B from the devation is exactly the effect of A's accomodation on B's profit – $q_B P^{'}()$. Thus, both firms deviate from the Cournot equilbrium by increasing their capacities whenever the firms are symmetric and there is some overlap. Proposition 5 implies that if firms are symmetric $(m_A = m_B)$, quantities initially increase and then decrease in MMC. Propostion 6 provides the explicit solution to the symmetric case for linear demand:

PROPOSITION 6 *If demand is linear and firm are symmetric ($m_A = m_B = m_i$), let $\lambda = \frac{m_o}{m_i+m_o}$. Then in equilibrium:*

$$q_i \quad = \tfrac{2}{3}\hat{q}_i \quad = \frac{a-c}{3b} \cdot \frac{9+6\lambda-3\lambda^2}{9+4\lambda-\lambda^2} \quad .$$

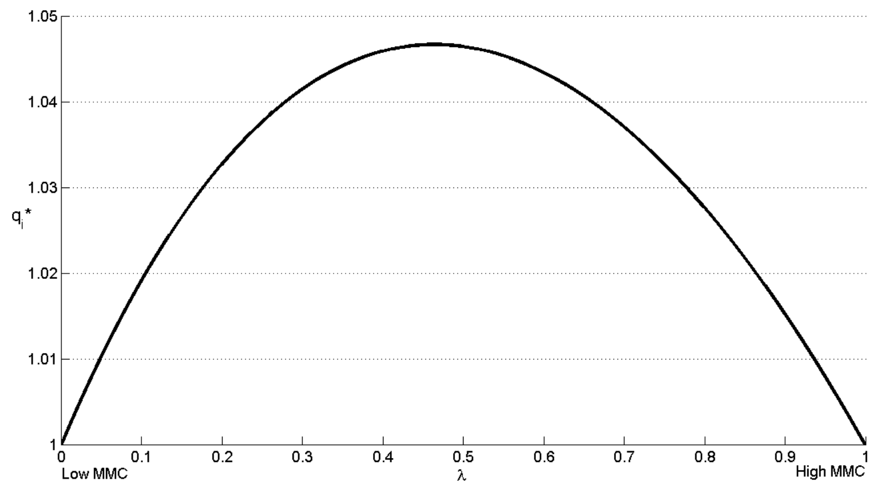
If demand is linear, the relative effect of MMC is identical in overlapping and private markets. That is, the firm increases quantity in all its markets by the same percentage. The effect is monotonic and peaks to about 4% increase when firms' private markets are slightly smaller than the overlapping market. Figure 2.2 illustrates the change in market quantities for symmetric firms as MMC increases.

3. PRICE COMPETITION

We now turn to the case of price competition in the second stage. Using the same three markets as before, the only difference is that in the second stage, instead of choosing its quantity per market, each firm chooses it's price in each market. The second stage problem for firm i is:

$$\begin{aligned} (3.1) \qquad R_i(k_A, k_B) = \max_{p_i, \hat{p}_i \geq 0} \quad & m_o q_i\left(p_i, p_{-i}\right) \cdot p_i + m_i \cdot \hat{q}_i\left(\hat{p}_i\right) \cdot \hat{p}_i \\ \text{s.t.} \quad & m_o q_i\left(p_i, p_{-i}\right) + m_i \hat{q}_i\left(\hat{p}_i\right) \leq k_i \end{aligned}$$

FIGURE 2.2.— Firm's quantity in the overlapping market
– Symmetric MMC, Strategic Substititutes –



* Plot shows Firm A's quantity normalized by the Cournot benchmark: $q_A^{\star} = q_A/\tilde{q}$ where $\tilde{q} = \frac{a-c}{3b}$. Graph is for symmetric MMC: $\lambda_A = \lambda_B = \lambda$.

The second stage game is now in strategic complements rather than strategic substitutes. The economics of capacity followed by pricing requires additional structure. Kreps and Scheinkman (1983) and Davidson and Deneckere (1986) show that if the second stage is a standard homogenous goods pricing game the equilibrium depends on how the residual demand is split between firms. The overall game may in fact be in strategic complements or substitutes. The complication arises mainly from the discontinuity of the payoff functions (see **??**). In other settings (e.g. discrete choice with no outside good), there are multiple equilibria are unavoidable and a specific equilibrium selection rule is required.

We wish to avoid these complications and focus on the case in which price competition qualitatively changes the game compared to quantity competition.

Assumption Regularity assumptions for the pricing game:

1. For any first stage choices k_i, the second stage equilibrium is unique

and differentiable in k_i

2. The own price effect on demand is at least as strong as the rival's price effect

$$\left|\frac{\partial q_i(\cdot)}{\partial p_i}\right| \geq \left|\frac{\partial q_i(\cdot)}{\partial p_{-i}}\right|$$

3. In the overall game, the firms equilibrium prices in the overlapping market are strategic complements

The first two assumptions are intuitive. Common demand functions that satisfy these are the Differentiated Bertrand / Hotelling / circular city models and the discrete choice (logit) models with an outside good.

The last assumption is used only for propostion 8. To understand the need for the last assumption, suppose firm A reduces it's equilibrium price. This will be associated with an increase in A's first stage capacity. The price change generates a complementing response from the B (price decrease), while the second generates a substituting effect (capacity decrease). Now, B's capacity decrease drives it to increase prices. What the assumption states is that the first order effect dominates the second order effect.

LEMMA 4 *If the demand function is linear in all prices and satisfies the first two parts of the regularity assumption, the third part is also satisfied.*

Lemma 4 implies that the Differentiated Bertrand / Hotelling / circular city models with linear price effects satisfy the third part of the assumption as well. Although we cannot prove a more general result, the result of the lemma held for any specific demand model we checked.

We will use the following Differentiated Bertrand model to illustrate the results.

$$q_i\left(p_i, p_{-i}\right) = a - bp_i + \frac{b}{2}p_{-i} \tag{3.2}$$

As in the previous section, the results do not rely on the linear structure unless this is mentioned explicitly.

3.1. *Benchmark*

The combination of a pricing game following a capacity setting game already distorts outcomes. In the quantity setting game, if markets are disconnected, the two stage game has the same outcome as the standard one stage Cournot game in which firms pay the production costs only when selling. However, the equilibrium for our two stage game differs from the standard one shot differentiated Betrand game even if firms only compete in the overlapping market. In the standard game, firms pay the production costs only after the quantity is determined. The equilibrium is defined by the solution to

$$q_i + \left(p_i - c\right)\frac{\partial q_i}{\partial p_i} = 0$$

Letting ρ_i denote the effective marginal revenue

$$\rho_i \equiv p_i + q_i / \frac{\partial q_i}{\partial p_i} \ ,$$

the firm's equilibrium price solves

$$\rho_i = c \ .$$

Accordingly, the price reaction curves maintain the marginal revenue fixed and are given by:

$$\overline{\phi}_i^1 = -\left(\frac{\partial \rho_i}{\partial p_{-i}} / \frac{\partial \rho_i}{\partial p_i}\right) \in (0,1)$$

In our setting, however, firms pay the production costs and then compete on prices. When firms only compete in one, overlapping market, the firm's second stage reactions will keep the quantity sold fixed rather than the marginal revenue, and are therefore given by

$$\overline{\phi}_i^2 = -\left(\frac{\partial q_i}{\partial p_{-i}} / \frac{\partial q_i}{\partial p_i}\right) \in (0,1)$$

The resulting equilibria for the two models differ. For example, in the linear case (see eq. 3.2), market quantities are smaller by a factor of $\frac{9}{10}$ in the two-stage benchmark compared to the one-shot benchmark (see appendix). Intuitively, the reaction when capacity is fixed $\left(\overline{\phi}^2\right)$ is stronger than when capacity can be adjusted to the new demand conditions. As prices are strategic complements, the steeper reactions curves push the equilibrium up.[11]

3.2. *Equilibrium*

The first stage problem for each firm is formally the same as in the Cournot case, with the revenue function R_i referring to the pricing game revenue 3.1 instead:

$$\max_{k_i \geq 0} R_i(k_A, k_B) - ck_i \quad . \tag{3.3}$$

[11] If demand is linear, we have that

$$\overline{\phi}_i^1 = -\frac{-\frac{1}{2}}{2} = \frac{1}{4} \quad ; \quad \overline{\phi}_i^2 = -\frac{\frac{b}{2}}{-b} = \frac{1}{2} \quad .$$

The next lemma confirms that the basic characteristic of the equilibrium are unaffected:

LEMMA 5 *In the equilibrium of the pricing game:*

1. *The capacity constraint binds:*$k_i = m_o q_i + m_i \hat{q}_i$.
2. *Firm i's marginal revenue is identical in both of it's markets:*$\rho_i = \hat{\rho}_i$.

The reasoning is exactly the same as for the quantity game. For the firm's first stage problem, ρ_i reflects the marginal revenue from a unit of investment in the expected second stage equilibrium.

Before characterizing the equilibrium we apply the definition of *flexibility* and *commitment power* for pricing games

DEFINITION 3 In the pricing game, firm i's *flexibility*, ϕ_i, is the firm's second stage reaction to a change in its competitor's overlapping market price:

$$\phi_i \equiv \frac{\partial p_i}{\partial p_{-i}} \tag{3.4}$$

Flexibility measures A's best response to B's second stage deviation in the equilibrium neighbourhood. If A does not have a private market, it's optimal reaction to B's deviation must be to maintain the same quantity sold, and thus A's flexiblity is the same as in the two-stage benchmark. As A's MMC decreases (A's private market increases), A can divert units to/from it's private market to react more profitably to B's deviation. At the extreme, if $\frac{m_A}{m_o} \to \infty$, A's reaction aproaches the one-stage benchmark:

LEMMA 6 *In equilibrium of the price setting game,*

$$\phi_i = -\frac{\frac{m_i}{m_o}\frac{\partial \hat{q}_i}{\partial \hat{p}_i} \cdot \frac{\partial \rho_i}{\partial p_{-i}} + \frac{\partial q_i}{\partial p_{-i}} \cdot \frac{\partial \hat{\rho}_i}{\partial \hat{p}_i}}{\frac{m_i}{m_o}\frac{\partial \hat{q}_i}{\partial \hat{p}_i} \cdot \frac{\partial \rho_i}{\partial p_i} + \frac{\partial q_i}{\partial p_i} \cdot \frac{\partial \hat{\rho}_i}{\partial \hat{p}_i}} \in \left[\overline{\phi}_i^1, \overline{\phi}_i^2\right] .$$

If the demand curve is linear,

$$\phi_i = \frac{1}{4}\frac{\frac{m_i}{m_o}+2}{\frac{m_i}{m_o}+1}$$

As expected, flexibility is positive as competition is in strategic complements. Moreover, when MMC is large ($m_A \to 0$), flexibilty is as in the two stage benchmark ($\phi_A \to \overline{\phi}^2$), when MMC is small ($m_o \to 0$), flexibility is as in the single stage benchmark $\phi_A \to \overline{\phi}^1$. The intuition is the same as in the Cournot game. When MMC is small the firm can divert its first stage investment that was planned to be used in the overlapping market to the private market at close to marginal cost. Thus, the firm's reaction is the same as if costs are only paid in the second stage, imitating the one stage game. Thus, we should again expect that as one firm moves from very low MMC to very high MMC with it's rival, the equilibrium will move from the "Stackelberg equivalent" to the "Cournot equivalent". However, while in quantity competition, the move from Stackelberg to Cournot implies that one firm produces less and one firm produces more, we shall see that in price competition the effect is different.

The definition for commitment power also translates directly to the pricing game:

DEFINITION 4 In the pricing game, firm i's *commitment power*, σ_i, is the change in the firm's second stage overlapping market price following an additional unit of investment:

$$\sigma_i \equiv m_o \frac{\partial p_i}{\partial k_i}$$

The commitment power measures the firm's ability to commit to allocating its additional capacity in the overlapping market. In terms of the second stage game, this is the firm's price change in the overlapping market following a change in it's first stage capacity. A firm's commitment power depends on the same forces as in the Cournot game – it's marginal revenues and the rival's flexibility – as well as the demand's reaction to price changes:

LEMMA 7 *In the equilibrium of the pricing game:*

$$\sigma_i = \frac{1}{\frac{m_i}{m_o}\frac{\partial \hat{q}_i}{\partial \hat{p}_i}/\frac{\partial \hat{\rho}_i}{\partial \hat{p}_i}\left(\frac{\partial \rho_i}{\partial p_i}+\frac{\partial \rho_i}{\partial p_{-i}}\phi_{-i}\right)+\left(\frac{\partial q_i}{\partial p_i}+\frac{\partial q_i}{\partial p_{-i}}\phi_{-i}\right)} < 0$$

As an increase in any quantity requires reducing prices, σ_i is negative. If the firm has no private market $\left(\frac{m_i}{m_o} \to 0\right)$, the price reduction is simply the reduction required to sell the additional quantity. If the overlapping market is small $\left(\frac{m_o}{m_i} \to 0\right)$, the private market will absorb the capacity increase.

We now characterize the equilibrium.

PROPOSITION 7 *The equilibrium of the game is characterized by the following equations*

$$c - \rho_i = c - \hat{\rho}_i = p_i \frac{\partial q_i}{\partial p_{-i}} \cdot \phi_{-i} \cdot \sigma_i < 0 \qquad i \in \{A, B\}\ .$$

Firms are less aggresive than in the one-shot pricing game.

Proposition 7 shows that the economics underlying the strategic effect are the same whether firms compete in prices or quantities. Given in terms of marginal effects, flexibility and commitment, the strategic effect is exactly the same in propositions 7 and 1. However, the economic forces themsleves change as a result of the move from quantity to price competition. In particular, with strategic complements, the marginal effect $\left(p_i \frac{\partial q_i}{\partial p_{-i}}\right)$ is positive, and thus MMC makes firms less aggressive.

Reducing capacity in the first stage is a way for the firm to commit to higher prices. As the rival reacts with higher prices as well ($\phi_{-i} > 0$), the firm gains some 'free' marginal consumers from it's capacity decrease. Firm *$i's$ strategic effect of MMC* is affected by exactly the same economic forces if competition in the second stage is in prices or quantities. However, the economic forces behave differently.

In contrast to the Cournot game, the effect is strongest when m_o is large. Here, both ϕ_{-i} and σ_i increase in absolute terms with m_o (see appendix). If MMC is high, the firm cannot divert quantity to it's private market in response to a rival price decrease and so it's price adjustment (ϕ) is larger. Commitment power increases in absolute terms with MMC because the firm must sell a larger share of it's additional capacity in the overlapping market. Thus, in constrast to the Cournot case, the results with $m_o \to 0$ and $m_o \to \infty$ are not identical.

Intuitively, while in quantity competition being a first mover is an advantage, in price competition the first mover takes a small loss to encourage it's rival to be less aggressive. Any increase in the overlap allows the firms to commit to compete softer and prices increase.

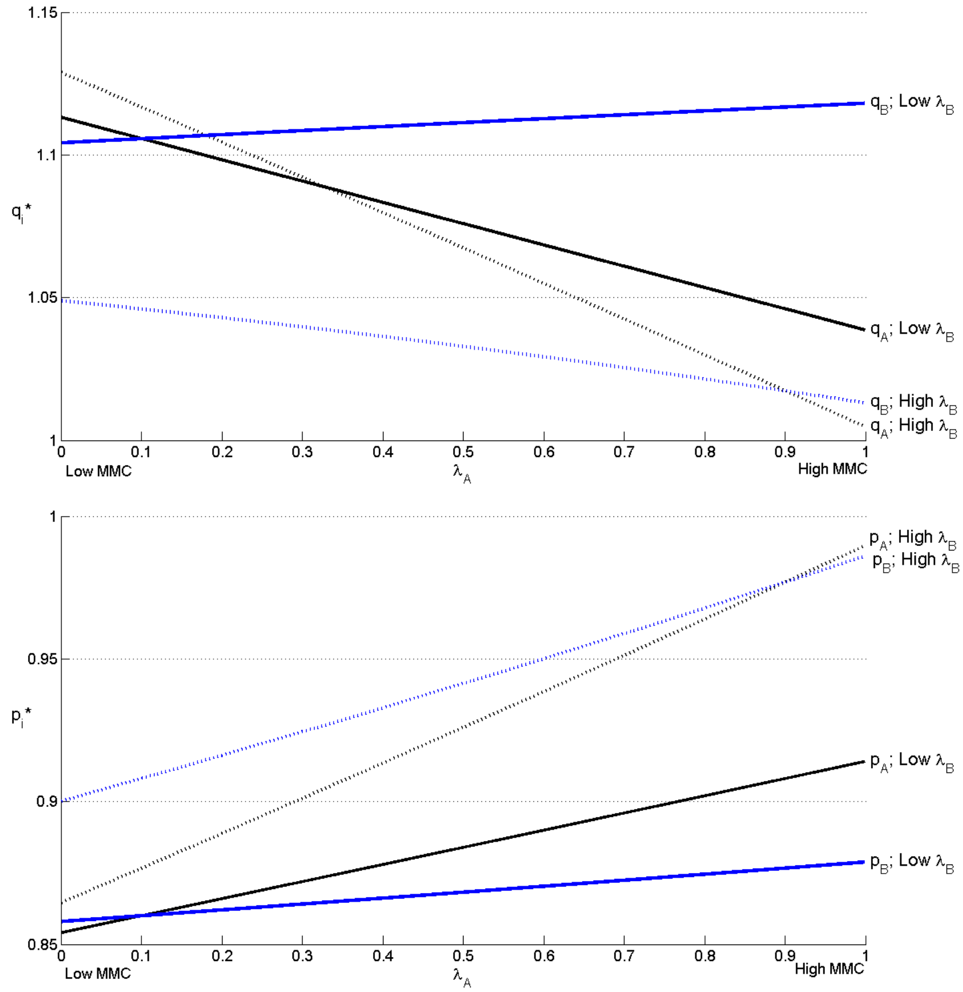
PROPOSITION 8 *If prices in the overall game are strategic complements, then an increase in MMC increases prices in the overlapping markets for both firms.*[12]

$$\frac{dp_i}{dm_i} < 0 \quad \frac{dp_i}{dm_{-i}} \leq 0 \quad \frac{dp_i}{dm_o} \geq 0$$

If firms compete in prices, increases in MMC can only reduce competition. It is therefore possible that the competitive effect of MMC would be confounded with a mutual forebearance effect. Closed form solutions for prices and quantities are

[12] The inequalities are weak only if λ_i or λ_{-i} are exactly zero.

FIGURE 3.1.— Firm's quantity and prices in the overlapping market
– Strategic Complements –



* Plots shows quantities and prices normalized by the by the appropriate benchmark: $q_A^\star = {}^{q_A}/_{\tilde{q}}$ where $\tilde{q} = \frac{3}{10}(2a - bc)$ and $p_A^\star = {}^{p_A}/_{\tilde{p}}$ where $\tilde{p} = (\tilde{q} - A)\,\frac{2}{3}b$. Quantity plot is invariant to A, b, and c. Price plot assumes $A = 10$, $b = 1$, and $c = 1$. λ_A measures A's MMC: $\lambda = \frac{m_o}{m_A + m_o}$. Two lines are plotted for each firm: when $\lambda_B = 0.1$ (Low) and $\lambda_B = 0.9$ (High).

provided in the appendix. Propostion 9 summarizes the symmetric case, which illustrates the results most clearly:

PROPOSITION 9 *Suppose* $m_A = m_B$ *then prices increase with MMC. When* $\frac{m_o}{m_o+m_i} \to 0$ *(no MMC), the equilibrium in the overlapping market is as in one-shot benchmark. When* $\frac{m_o}{m_o+m_i} \to 1$ *(full MMC), the equilibrium in the overlapping market is as in the two-stage benchmark. If demand is given by (3.2) then the overlapping market quantity per firm is*

$$q_i = q^* \frac{10}{3} \cdot \frac{10}{3} \frac{(3-\lambda)(5+\lambda)}{(5-\lambda)(9+\lambda)}$$

Where q^* *is the quantity per firm in the two stage Dupoly benchmark.*

As two firms increase the extent of their overlap, they become less agressive. Figure illustrates the change in market quantities and prices for symmetric firms as MMC increases with linear demand. The total quantity change in the market is 10%. As we discuss below, these numbers are consistent with the empirical effects found for MMC in the existing literature.

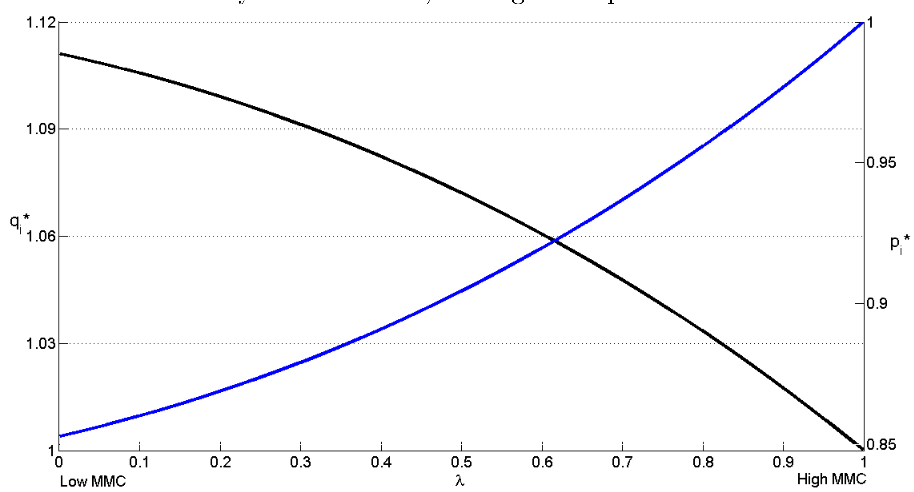
4. DISCUSSION AND APPLICATIONS

4.1. *Mergers and Trade*

The previous sections showed how changes in MMC may generate changes of up to 40% of a firm's quantity and up to 20% of the overall quantity in a competitive market. These changes are a result of a purely competitive effect. However, especially when firms compete in prices, the competitive effect of MMC may be confounded with mutual forbearance. In this section we consider two specific cases in which the implications of the competitive effect of MMC may have alternative policy implications than what current thought may have.

FIGURE 3.2.— Firm's quantity in the overlapping market for different values of MMC

– Symmetric MMC, Strategic Complements –



* Plot shows Firm A's quantity (left axis) and price (right axis) normalized by the appropriate benchmark: $q_A^{\star} = {}^{q_A}/_{\tilde{q}}$ where $\tilde{q} = \frac{3}{10}(2a - bc)$ and $p_A^{\star} = {}^{p_A}/_{\tilde{p}}$ where $\tilde{p} = (\tilde{q} - A)\,\frac{2}{3}b$. Graph is for symmetric MMC: $\lambda_A = \lambda_B = \lambda$. Quantity plot is invariant to A, b, and c. Price plot assumes $A = 10$, $b = 1$, and $c = 1$.

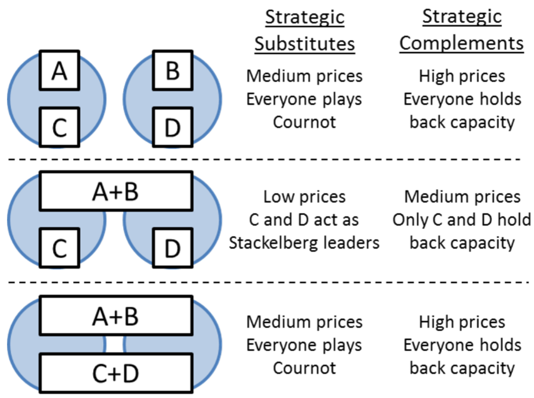


FIGURE 4.1.— Merger example: two markets (circles) and four firms (boxes). The first merger is between firms A and B. After the merger, firm C considers the second market as a private market of firm 'A+B'. The second merger is between firms C and D.

Specifically, we look at meger evaluation and international trade and illustrate how the competitive effect of MMC may be confounded with mutual forebearance and/or economies of scale. We end by suggesting empirical tests that can help distinguish between the competitive effect of MMC and alternative theories.

Mergers change the degree of MMC significantly. The recent Delta-Northwest merger, for example, changed the share of Delta's domestic-US routes that are served also by American Airlines from 40% to 33%. However, the follow-up merger between American with US Airways will likely increase the MMC between the firms to over 40%. We discuss airlines in more detail below. For now, suppose an industry consists of two markets, each served by two different firms, as in figure 4.1. For example Delta and American may be serving the first market, and Northwest and US Airways the second. Prior to the merger, neither firm has a private market and all market outcomes follow the benchmark duopoly model of choice. When Delta and Northwest merge, each market is still a duopoly – the number of competitors in each market did not change. Moreover, the merger has no effect on the incentives for mutual forebearance. However, in our model, the partial MMC affects competition. The merged firm has private markets. For American, Northwest's market became Delta's "private" market – Delta can shift capacity from the first market to the second. Whether competition is in strategic complements or substitutes, the smaller firms (American and US Airways) take advantage of their rival's flexibility and set a higher capacity. The merged firm becomes a "partial" Stackelberg follower in both markets. When the two rivals merge as well, the private markets disappear and all markets return to the standard duopoly equilibrium.

The asymmetric phase, after the first merger and prior to the second, is supra-competitive. The second merger reduced competition back to the standard duopoly levels. Regulators may have difficulty rejecting the second merger after approving the first, even though the second merger reduces surplus from the standpoint of the current analysis.

As for distinguishing between mutual forebearance and the competitive effect of MMC, the first merger does not change in the incentives for tacit collusion but does change outcomes do to the competitive effect of MMC. To establish that the merger wave facilitated tacit collusion, the correct ex-post analysis is to compare outcomes prior to first merger with outcomes after the second. Comparing only outcomes before and after the second merger may confound tacit collusion with the competitive effect.

The second main novel implication of the competitive effect of MMC is that demand conditions in private and overlapping markets affect outcomes in all markets. For example, suppose a firm operating in one country expands internationally. If the international market is large and competitive, from the perspective of the firm's domestic rivals, the international market acts as a large "private" market for the firm. As a result, the domestic rivals become more aggressive, making the domestic market more competitive. In contrast, if the local firm is a monopolist and the international market is an oligopoly with larger firms, the local firm may want to be aggressive in the international market, taking advantage of its' rivals flexibility. However, to commit to such aggressive behavior, the firm must increase investments used in the local market, increasing local welfare. Under both scenarios the competitive effect of MMC may partially explain the perceived increase in productivity associated with opening markets to exports.

4.2. *Suggestive Evidence*

Airlines are a perfect industry to apply our model of MMC. Airlines schedule flights from spokes to their hub (or between spokes) well in advance of actually selling tickets on the routes using these flights. Moreover, virtually all flights serve many routes (markets). We show in the appendix that our model can be directly extended to capture the basic airline competition model, allowing for hub-and-spoke and spoke-to-spoke flights.

Past work on MMC has studied how prices are affected in the airline industry with changes in MMC. Evans and Kessides (1994) looked at changes in MMC driven by the surge in entry after deregulation. Gimeno and Woo (1999) used hub formation as the driver of changes in MMC. They both found that higher overlap between carriers was correlated with higher prices. Park and Zhang (2000) study the main North Atlantic alliances in the 1990's (British Airways/USAir, Delta/Sabena/Swissair, KLM/Northwest, and Lufthansa/United Airlines). As predicted by our model, their analysis concludes that a complementary alliance between carriers with low network overlap (e.g., BA/USAir, KLM/NW, LH/UA) is likely to increase total seat miles sold and consumer surplus.

The study by Gimeno and Woo (1999) is particularly akin to our setting, as they divided the extent of multimarket contact into two groups: multimarket contact on markets that shared a common endpoint with the focal market[13], and multimarket contact on all other markets. They defined the first type of multimarket contact as one on markets with *strong resource sharing* opportunities, while the latter was defined as one on markets with *weak resource sharing* opportunities. The study shows how MMC on markets with *strong resource sharing* opportunities was correlated with higher yields (price per mile) but MMC on markets with *weak resource sharing* opportunities was not. Furthermore, increases in the number of rivals' non-overlapping markets with *strong resource sharing* opportunities was correlated with lower yields. The simple model of airline competition that we sketch in the appendix, in which firms commit to flight schedules prior to selling tickets, yields the same empirical patterns as those uncovered by Gimeno and Woo (1999). Committed flight schedules imply the cost of a flight between two cities is sunk, but such flight can be used on multiple routes sharing a common endpoint. In such a setting, increasing MMC on the markets in which the sunk investment is being used would result in higher prices: we would observe a positive correlation between yields and MMC on markets with *strong resource sharing* opportunities. Furthermore, increases in rivals' non-overlapping markets with *strong resource sharing* opportunities would be akin to increasing rivals' private markets in our model, resulting in lower prices.

5. CONCLUSION

The paper studies the competitive effect of multi-market contact (MMC) in oligopoly settings with sunk, transferable investments. We find that changes in

[13]A focal market is the market under observation: i.e. the market whose price is being regressed on other covariates.

the extent of multi-market contact can have a significant effect on market outcomes, where *increases* in the extent of MMC has ambigous effects on welfare. In particular, we show that when firms compete in strategic substitutes, increasing MMC for that firm increases that firm's output in all of that firm's markets (even its monopolistic markets). The effect on total welfare, though, is ambiguous because of how rivals react to that increase. If, however, competition is in strategic complements, increasing a firm's MMC increases that firm's price in all its markets and rival firms react in such a way that welfare unambiguously decreases in markets where firms overlap.

The analysis highlighted two related forces through which MMC affects competition. The size and elasticity of a firm's private markets determine the firm's *flexibility* – its ability to shift resources across markets. Firms respond to a rival's flexibility by increasing their own aggressiveness, knowing that their rival will have the flexibility to accommodate. This second force – the firm's *commitment power,* measures the extent to which a firm can commit to allocating productive capacity to be used in rivals' markets.

Our analysis relates the most to the analysis done in Bulow et al. (1985), with two critical differences: we allow for all firms to have private markets and we endogenize the diseconomies across markets that Bulow et al. (1985) take as given. Doing so significantly changes the conclusions. We find that changes in one firm's private markets affects how other firms react in their own private markets. We also find that by endogenizing diseconomies across markets changes in MMC may have a non-monotonic effect on consumer welfare (when competing in strategic substitutes).

Previous literature has focused on perceived differences between large and small firms in terms of productivity or mutual forbearance. The competitive effect of MMC can be confounded with either of the two. Thus, an important empirical question is whether it is possible to distinguish between each. A key difference between the competitive effect of MMC and mutual forbearance is that the former relies on specific physical diseconomies across the markets (i.e., sunk, transferable investments) while the latter does not. This opens possibilities for future research to distinguish between the two hypothesis by separating the effect of changes in MMC that use the same capacity (i.e., use of a common distribution center or airline hub) and changes in MMC that do not.

REFERENCES

Bernheim, B. Douglas and Michael D. Whinston, "Multimarket Contact and Collusive Behavior," *The RAND Journal of Economics*, 1990, *21* (1), 1–26.

Bulow, Jeremy I., John D. Geanakoplos, and Paul D. Klemperer, "Multimarket Oligopoly: Strategic Substitutes and Complements," *Journal of Political Economy*, January 1985, *93* (3), 488.

Ciliberto, Federico and Jonathan W. Williams, "Does Multimarket Contact Facilitate Tacit Collusion? Inference on Conduct Parameters in the Airline Industry," 2012.

Davidson, Carl and Raymond Deneckere, "{Long-Run} Competition in Capacity, {Short-Run} Competition in Price, and the Cournot Model," *The {RAND} Journal of Economics*, 1986, *17* (3), 404–415.

Dixit, Avinash, "The Role of Investment in Entry-Deterrence," *The Economic Journal*, March 1980, *90* (357), 95–106. ArticleType: research-article / Full publication date: Mar., 1980 / Copyright © 1980 Royal Economic Society.

Edwards, Corwin D., "Conglomerate Bigness as a Source of Power," in George J. Stigler, ed., *Business Concentration and Price Policy*, Princeton University Press, 1955, p. 329–358.

Evans, William N. and Ioannis N. Kessides, "Living by the "Golden Rule": Multimarket Contact in the U.S. Airline Industry," *The Quarterly Journal of Economics*, May 1994, *109* (2), 341–366.

Fernandez, Nerea and Pedro L. Marin, "Market Power and Multimarket Contact: Some Evidence from the Spanish Hotel Industry," *The Journal of Industrial Economics*, 1998, *46* (3), 301–315.

Gimeno, Javier and Carolyn Y. Woo, "Multimarket Contact, Economies of Scope, and Firm Performance," *The Academy of Management Journal*, June 1999, *42* (3), 239–259.

Jans, Ivette and David I. Rosenbaum, "Multimarket contact and pricing: Evidence from the U.S. cement industry," *International Journal of Industrial Organization*, 1997, *15* (3), 391 – 412.

Kreps, David M. and Jose A. Scheinkman, "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," *The Bell Journal of Economics*, 1983, *14* (2), 326–337.

Park, Jong-Hun and Anming Zhang, "An Empirical Analysis of Global Airline Alliances: Cases in North Atlantic Markets," *Review of Industrial Organization*, June 2000, *16* (4), 367–384.

Parker, Philip M. and Lars-Hendrik Roller, "Collusive Conduct in Duopolies: Multimarket Contact and Cross-Ownership in the Mobile Telephone Industry," *The RAND Journal of Economics*, 1997, *28* (2), pp. 304–322.

Pilloff, Steven J., "Multimarket Contact in Banking," *Review of Industrial Organization*, 1999, *14* (2), 163–182. 10.1023/A:1007779814751.

Spence, A. Michael, "Entry, Capacity, Investment and Oligopolistic Pricing," *The Bell Journal of Economics*, October 1977, *8* (2), 534–544. ArticleType: research-article / Full publication date: Autumn, 1977 / Copyright © 1977 RAND Corporation.

APPENDIX A: PROOFS OF PROPOSITIONS AND LEMMAS

We present the formal proofs to all Lemmas and Propositions. Throughout this section, we denote with subscripts the following partial derivatives: $\rho_{i,1} \equiv \frac{\partial \rho_i}{\partial q_i}$, $\rho_{i,2} \equiv \frac{\partial \rho_i}{\partial q_{-i}}$, $\rho_{i,12} \equiv \frac{\partial^2 \rho_i}{\partial q_i \partial q_{-i}}$.

We briefly discuss some assumptions required in the proofs that impose structure on the inverse demand curves and that hold true in most common inverse demand functions (all hold trivially for linear demand).

Assumption. At the equilibrium, marginal revenue is always affected more by an increase in a firm's output than by a similar decrease in a rival's output: $|\rho_{i,1}| \geq |\rho_{i,2}|$.

In the regular Cournot game, this assumption is equivalent to requiring reaction curves to have a slope of greater than minus one, and thus be at a stable equilibrium.

Assumption. Inverse demand curves are such that $\frac{\partial^2 P}{\partial q_i \partial q_{-i}} \leq \frac{\partial^2 P}{\partial q_{-i}^2}$.

Assumption A holds trivially for homogenous goods and is a statement on how the price drop due to a rival adding output is larger when the firm has a lot of output than when the rival has a lot of output.

Assumption. $\frac{\partial \phi_i}{\partial q_i} \geq 0$, $\frac{\partial \phi_i}{\partial q_{-i}} \geq 0$, $\frac{\partial \sigma_i}{\partial q_{-i}} \leq 0$

This last assumption is a statement on the third derivatives of the demand functions. The first inequality states that a firm's flexibility increases with its own output. This should be expected, since increasing output in the overlapping market implies increasing output in the private market. As quantity in the private market increases, it becomes incresingly more difficult diverge additional output into it. The second inequality follows under the same reasoning. The third inequality simply states that as the rival increases output in the overlapping market, increments in capacity are allocated with a lower proportion into the overlapping market.

LEMMA 1 *In equilibrium, in the second stage:*

1. *The capacity constraint binds:*$k_i = m_o q_i + m_i \hat{q}_i$.
2. *Firm i's marginal revenue is identical in both of it's markets:*$\rho_i = \hat{\rho}_i$.

PROOF: For the first point, assume it does not bind for firm A. Firm A can reduce k_A a an ϵ amount in the first stage, saving $c\epsilon$ on cost. For an ϵ small enough, both firms can allocate the same quantities in the second stage. Since the constraint continues to not bind for firm A, the second stage allocation must be an equilibrium resulting in the same revenues as in the prior equilibrium. As firm A saved $c\epsilon$, the prior equilibrium cannot be an equilibrium and the constraints must bind.

The second point follows directly from the FOC of the subgame (eq. 2.1):

$$\begin{aligned}
\frac{\partial}{\partial q_i}\left(m_o q_i P(q_i + q_{-i}) + m_o \hat{q}_i P(\hat{q}_i)\right) &= m_o \mu_k - \mu_i \\
\frac{\partial}{\partial \hat{q}_i}\left(m_o q_i P(q_i + q_{-i}) + m_i \hat{q}_i P(\hat{q}_i)\right) &= m_i \mu_k - \hat{\mu}_i
\end{aligned}$$

where μ_k is the Lagrangian multiplier on the capacity constraint and $\mu_i, \hat{\mu}_i$ are the Lagrangian multipliers on the non-negativity constraints. If $q_i, \hat{q}_i > 0$, these last multipliers are zero. As the partial derivatives are simply the marginal revenue (MR) curves, the FOCs are $m_o \rho_i = m_o \mu_k$ and $m_i \hat{\rho}_i = m_i \mu_k$, giving the desired result.

In equilibrium both quantities are positive. Suppose not. If one of the quantities is zero, it must be that of the overlapping market as MR in the private market is weakly higher than that of the overlapping market when both markets have zero output allocated to them. If such an equilibrium exists, then the MR in the overlapping market must be lower than that of the private market (else the firm -firm A- would benefit from shifting some output into the overlapping market) and such low MR must be caused by the rival firm -firm B- allocating a large output in the overlapping market. MR in firm A's private market must be weakly lower than c: if it is higher, A can increase capacity and use it in the private market. This would not affect the zero allocation in the overlapping market and so B would not change its capacity nor second stage allocation, but such increase would increase A's profits. This implies the MR in the overlapping market, at zero output, is less than c: meaning the price in the overlapping market is below c. It must be the case that B is selling in both markets (for reasons already described), and so B's MR in both markets must be below c. B could then benefit from reducing capacity slightly and selling this in the private market while continuing to sell the same amount in the overlapping market. This would not affect A's decisions but would profit B. So it cannot be that allocating zero output to at least on market is an equilibrium. *Q.E.D.*

LEMMA 2 *In equilibrium,*

$$\phi_i = -\frac{\frac{\partial \rho_i}{\partial q_{-i}}}{\frac{\partial \rho_i}{\partial q_i} + \frac{m_o}{m_i}\frac{\partial \hat{\rho}_i}{\partial \hat{q}_i}} \in \left[\overline{\phi}_i, 0\right] .$$

If the demand curve is linear,

$$\phi_i = -\frac{1}{2} \cdot \frac{m_i}{m_i + m_o}$$

PROOF: Define $\hat{\phi}_i \equiv \frac{d\hat{q}_i}{dq_{-i}}$. The second stage FOCs for firm A can be characterized by

$$\rho_A = \hat{\rho}_A \quad \text{and} \quad m_o q_A + m_A \hat{q}_A = k_A$$

Taking full derivatives of these two equations with respect to rival's quantity gives the desired result:

$$\rho_{A,1}\phi_A + \rho_{A,2} - \hat{\rho}_{A,1}\hat{\phi}_A = 0 \qquad \text{and} \qquad m_o\phi_A + m_A\hat{\phi}_A = 0$$

$$\begin{aligned} 0 &= \rho_{A,1}\phi_A + \rho_{A,2} - \hat{\rho}_{A,1}\left(-\frac{m_o}{m_A}\phi_A\right) \\ \phi_A &= \frac{-\rho_{A,2}}{\rho_{A,1} + \frac{m_o}{m_A}\hat{\rho}_{A,1}} \end{aligned}$$

and since $\hat{\rho}_{A,1} \leq 0$ and $\rho_{A,1} \leq 0$, $\phi_A \geq \overline{\phi}_A = \frac{\rho_{A,2}}{\rho_{A,1}}$. The second result points follows immediately from substituting in $\rho_{A,1} = \hat{\rho}_{A,1} = -2b$ and $\rho_{A,2} = -b$. *Q.E.D.*

Definition 5 Let $\hat{\sigma}_i \equiv m_i \frac{d\hat{q}_i}{dk_i}$, $\eta_i \equiv m_o \frac{dq_{-i}}{dk_i}$, and $\hat{\eta}_i \equiv m_{-i} \frac{d\hat{q}_{-i}}{dk_i}$.

Lemma 3 *In equilibrium,*

$$\sigma_i = \frac{\frac{\partial \hat{\rho}_i}{\partial \hat{q}_i}}{m_i \left(\frac{\partial \rho_i}{\partial q_i} + \frac{\partial \rho_i}{\partial q_{-i}} \phi_{-i} \right) + m_o \frac{\partial \hat{\rho}_i}{\partial \hat{q}_i}} \in [0,1]$$

If the demand curve is linear

$$\sigma_i = \frac{1 + \frac{m_{-i}}{m_o}}{1 + \frac{m_i}{m_o} + \frac{m_{-i}}{m_o} + \frac{3}{4} \frac{m_i}{m_o} \frac{m_{-i}}{m_o}}$$

Proof: The second stage FOCs for firm B can be characterized by:

$$\rho_B = \hat{\rho}_B \quad \text{and} \quad m_o q_B + m_A \hat{q}_B = k_B$$

Taking full derivatives of these two equations with respect to A's capacity gives that $\eta_A = \sigma_A \phi_B$:

$$\rho_{B,1} \frac{1}{m_o} \eta_A + \rho_{B,2} \frac{1}{m_o} \sigma_A - \hat{\rho}_{B,1} \frac{1}{m_B} \hat{\eta}_A = 0 \qquad \text{and} \qquad \eta_A + \hat{\eta}_A = 0$$

$$\eta_A = \frac{-\frac{1}{m_o} \sigma_A \rho_{B,2}}{\frac{1}{m_o} \rho_{B,1} + \frac{1}{m_B} \hat{\rho}_{B,1}} = \sigma_A \phi_B$$

We now take full derivatives of A's FOC with respect to k_A:

$$\rho_{A,1} \frac{\sigma_A}{m_o} + \rho_{A,2} \frac{\eta_A}{m_o} - \hat{\rho}_{A,1} \frac{\hat{\sigma}_A}{m_A} = 0 \qquad \text{and} \qquad \sigma_A + \hat{\sigma}_A = 1$$

Substituting in $\eta_A = \sigma_A \phi_B$ and solving for σ_A gives the desired result:

$$\sigma_A = \frac{\hat{\rho}_{A,1}}{\frac{m_A}{m_o} \left(\rho_{A,1} + \phi_B \rho_{A,2} \right) + \hat{\rho}_{A,1}}$$

To show that $\sigma_A \in [0,1]$, note that $\hat{\rho}_{A,1} \leq 0$ and $\rho_{A,1} + \phi_B \rho_{A,2} \leq 0$, where the latter follows from $|\phi_B| \leq 1$ and $\left|\rho_{A,1}\right| \geq \left|\rho_{A,2}\right|$ (given by assumption A).

The last result follows from substituting in $\hat{\rho}_{i,1} = \rho_{i,1} = -2b$, $\rho_{i,2} = -b$, and $\phi_i = -\frac{1}{2} \frac{m_i}{m_i + m_o}$. *Q.E.D.*

Proposition 1 *The equilibrium of the game is characterized by the following equation*

$$c-\rho_i = c-\hat{\rho}_i = q_i P^{'}\left(q_i+q_{-i}\right)\cdot\phi_{-i}\cdot\sigma_i > 0 \qquad i\in\{A,B\}\ .$$

Firms behave more aggresively under MMC than they would have under standard oligopoly.

If $\frac{m_i}{m_o}\to\infty$ *and* $\frac{m_{-i}}{m_o}\to 0$ *, then the equilibrium in the overlapping market is the Stackelberg equilibrium with firm* i *the Stackelberg follower.*

Proof: The FOC in the first stage is given by the partial derivative of equation 2.2 with respect to k_i:

$$\frac{\partial R_i(k_i,k_{-i})}{\partial k_i}-c=0$$

$$\begin{aligned}\frac{\partial R_i(k_i,k_{-i})}{\partial k_i} &= m_o\rho_i\frac{dq_i}{dk_i}+m_o q_i\frac{\partial P}{\partial q_{-i}}\frac{dq_{-i}}{dk_i}+m_i\hat{\rho}_i\frac{d\hat{q}_i}{dk_i}\\ &= \rho_i\sigma_i+q_i\frac{\partial P}{\partial q_{-i}}\sigma_i\phi_{-i}+\hat{\rho}_i\hat{\sigma}_i\end{aligned}$$

where $\hat{\sigma}_i$, σ_i, and ϕ_{-i} are imputed from their definitions. Since in any equilibrium, $\rho_i=\hat{\rho}_i$ and $\sigma_i+\hat{\sigma}_i=1$, the FOC can be written as

$$c=\rho_i+q_i\frac{\partial P}{\partial q_{-i}}\sigma_i\phi_{-i}$$

The last term is positive as $\frac{\partial P}{\partial q_{-i}}\leq 0$, $\phi_{-i}\leq 0$, and $\sigma_i\geq 0$.

For the second part, when $\frac{m_i}{m_o}\to\infty$, then $\phi_i\to\overline{\phi}_i$ and $\sigma_i\to 0$. When $\frac{m_{-i}}{m_o}\to 0$, then $\phi_{-i}\to 0$ and $\sigma_{-i}\to 1$. The FOCs of the two players are then

$$c=\rho_i \qquad \text{and} \qquad c=\rho_{-i}+q_{-i}\frac{\partial P}{\partial q_i}\overline{\phi}_i$$

which are exactly the same FOC as the Stackerlberg game with firm i as the follower. *Q.E.D.*

Proposition 2 *If demand is linear, consumer and total surplus is always at least as high in industries with MMC as without. Total industry profit is always at most as high in industries with MMC as without.*

Proof: Consumer and total surplus are a monotonic function of total market output. It is enough to prove that total market output in every market is larger with MMC than without it. That such holds in the private market follows from the fact that the strategic effect,

$q_i \frac{\partial P}{\partial q_{-i}} \sigma_i \phi_{-i}$, is positive. To show that is holds in the overlapping market, note that the FOC for the linear demand has the form $c - q_i \frac{\partial P}{\partial q_i} + \alpha = P(Q)$ where $\frac{\partial P}{\partial q_i} = -b$ and α is the strategic effect ($\alpha \geq 0$). Since $P(Q)$ and $\frac{\partial P}{\partial q_i}$ is the same for both firms, adding up the two FOC gives: $2c - Q(-b) - \alpha_i - \alpha_{-i} = 2P(Q)$. Solving for Q gives:

$$Q = \frac{2}{3} \frac{A - c + \frac{\alpha_i + \alpha_{-i}}{2}}{b}$$

When $\alpha_i = \alpha_{-i} = 0$ the Cournot output is achieved. As the above function is an increasing function of α_i and α_{-i}, any MMC that makes at least one of the two positive results in a higher level of total output in the overlapping market than there would have been with no MMC (with $\alpha_i = \alpha_{-i} = 0$).

As to the second point, as total industry profits are declining in output when output is greater than the monopoly output, and the total output with MMC is larger than the Cournot output, which itself is larger than the monopoly output, total industry profits are less with MMC than without it. *Q.E.D.*

Proposition 3 *If demand is linear, let* $\lambda_i = \frac{m_o}{m_o + m_i}$*, then the equilibrium quantities are...:*

Proof: Plug in the above definitions into the FOC of 1 and solve for quantities. Online Appendix contains the Mathematica notebook we used for this proposition. *Q.E.D.*

Proposition 4 *A firm's equilibrium quantity in both markets* ***increases*** *with the firm's MMC and* ***decreases*** *with the rival's MMC*

$$\frac{dq_i}{dm_i} < 0 \quad \frac{d\hat{q}_i}{dm_i} \leq 0 \qquad \frac{dq_i}{dm_{-i}} > 0 \quad \frac{d\hat{q}_i}{dm_{-i}} \geq 0$$

Firm's total profits are increasing in it's own MMC and decreasing in rival's MMC.

Proof: The proof proceeds in 4 steps. We first show that you can change the variable of the first stage optimization from k_i to q_i. We then show that the FOC is decreasing in m_i, holding the rival's output fixed. Third, we show that the FOC is increasing in m_{-i}, holding the rival's output fixed.. Finally, we show that strategic substitution is preserved even with the dynamics. The first step shows that increasing m_i (a decrease in MMC) has a direct effect of decreasing q_i, holding rival's outcomes fixed. The second and third steps show how the rival firm reacts by increasing output and that there are two drivers for this: the direct effect of m_{-i} and the strategic substitution effect. Since strategic substitution is preserved (step 3), all these effects reinforce each other, proving that $\frac{dq_i}{dm_i} < 0$ and that $\frac{dq_i}{dm_{-i}} > 0$.

The results that output in the private market reacts in the same direction as output in the overlapping market ($\frac{d\hat{q}_i}{dm_i} \leq 0 \iff \frac{d\hat{q}_i}{dm_{-i}} \geq 0$) is shown within step one of the proof.

Step 1. Change of variables.

The second stage equilibrium provides the mapping between $(q_i, \hat{q}_i)$ and k_i, k_{-i}. The second stage equilibrium is characterized by the two equations equating marginal revenue and by the two constraints. We apply the implicit function theorem on these four equations. In doing so we can express k_i, $\hat{q}_i$, and q_{-i} as a function of (q_i, k_{-i}). Denote these functions by κ_i ,$\hat{\vartheta}_i$, and ϑ_{-i} so that $\kappa_i(q_i, k_{-i}) = k_i$, $\tilde{\vartheta}_i(q_i, k_{-i}) = \hat{q}_i$, and $\vartheta_{-i}(q_i, k_{-i}) = q_{-i}$. The derivatives of these functions is also given by the implicit function theorem: let $\mathbf{G}(\mathbf{q}, \hat{\mathbf{q}}, \mathbf{k})$ be a 4x1 vector that stacks the equations charaterizing the second stage equilibrium such that $\mathbf{G}(\mathbf{q}, \hat{\mathbf{q}}, \mathbf{k}) = 0$ and partition the variables such that $x = (\hat{q}_i, k_i, q_{-i}, \hat{q}_{-i})$ and $y = (q_i, k_{-i})$. The Jacobian of the 4x1 vector of functions $v = (\tilde{\vartheta}_i, \kappa_i, q_{-i}, \hat{q}_{-i})'$, given with respect to the two input variables (q_i, k_{-i}) is given by the solution to the system of equations: $\nabla_x \mathbf{G} \cdot \nabla_y v = \nabla_y \mathbf{G}$, (∇_x is a differential operater, such that the j-th colum contains $\frac{d}{dx_j}$). By solving this system, one obtains the derivatives:

$$\frac{d\hat{\vartheta}_i}{dq_i} = \frac{m_o}{m_i}\frac{1-\sigma_i}{\sigma_i} \ , \ \frac{d\kappa_i}{dq_i} = \frac{m_o}{\sigma_i} \ , \ \frac{d\vartheta_{-i}}{dq_i} = \phi_{-i}$$

As $\sigma_i \geq 0$, $\frac{d\hat{\vartheta}_i}{dq_i} \geq 0$. This shows that output in the private market reacts in the same direction as output in the overlapping market.

The objective function can now be expressed in terms of q_i: $F_i(q_i) = m_o q_i P(q_i + \vartheta_{-i}(q_i, k_{-i})) + m_i \hat{\vartheta}_i P(\hat{\vartheta}_i) - c\kappa_i$. And the FOC is

$$\begin{aligned}
\frac{\partial F_i}{\partial q_i} = F_{q_i} &= m_o\rho_i + m_o q_i \frac{\partial P}{\partial q_{-i}}\frac{d\vartheta_{-i}}{dq_i} + m_i\hat{\rho}_i\frac{d\hat{\vartheta}_i}{dq_i} - c\frac{d\kappa_i}{dq_i} \\
&= m_o\rho_i + m_o q_i \frac{\partial P}{\partial q_{-i}}\phi_{-i} + m_o\hat{\rho}_i\frac{1-\sigma_i}{\sigma_i} - \frac{cm_o}{\sigma_i} \\
&= \frac{m_o}{\sigma_i}\left(\rho_i - c + q_i\sigma_i\phi_{-i}\frac{\partial P}{\partial q_{-i}}\right)
\end{aligned}$$

where the last equality used the fact that in all subgame equilibria $\rho_i = \hat{\rho}_i$.

To simplify on notation, let $\tilde{F}_{q_i} = \rho_i - c + q_i\sigma_i\phi_{-i}\frac{\partial P}{\partial q_{-i}}$ and know that in equilibrium, $\tilde{F}_{q_i} = 0$

Step 2. The FOC is decreasing in m_i when holding q_{-i} fixed.

The sign of the derivative of the FOC when holding q_{-i} fixed is given by the sign of $\frac{\partial}{\partial m_i}\tilde{F}_{q_i}$. Within $\tilde{F}_{q_i}$, the only function that involves m_i is σ_i, which is decreasing in m_i (cfr. lemma 3). As ϕ_{-i} and $\frac{\partial P}{\partial \vartheta_{-i}}$ are negative, the whole term is negative.

Step 2. The FOC is increasing in m_{-i} when holding q_{-i} fixed.

The sign of the derivative of the FOC when holding q_{-i} fixed is given by the sign of $\frac{\partial}{\partial m_{-i}}\tilde{F}_{q_i}$. Within $\tilde{F}_{q_i}$, the only functions involving m_{-i} are σ_i and ϕ_{-i}. The latter is decreasing in m_{-i} (cfr. lemma 2). The former is increasing in m_{-i} as it is decreasing in ϕ_{-i} (and ϕ_{-i} is decreasing in m_{-i}). Thus, the sign of the derivative stated above is given by the sign of

$$q_i \frac{\partial P}{\partial \vartheta_{-i}} \left(\phi_{-i} \frac{\partial \sigma_i}{\partial m_{-i}} + \sigma_i \frac{\partial \phi_{-i}}{\partial m_{-i}} \right)$$

which is positive: the term outside the parenthesis is negative and both terms inside the parenthesis are negative.

Step 3. Strategic substitution is preserved: $\frac{\partial^2 F_i}{\partial q_i \partial q_{-i}} \leq 0$

As before, it is sufficient to sign $\frac{\partial}{\partial q_{-i}}\tilde{F}_{q_i}$:

$$\begin{aligned}
\frac{\partial}{\partial q_{-i}}\tilde{F}_{q_i} &= \rho_{i,2} + q_i \frac{\partial}{\partial q_{-i}} \left(\sigma_i \phi_{-i} \frac{\partial P}{\partial q_{-i}} \right) \\
&= \rho_{i,2} + \sigma_i \phi_{-i} q_i \frac{\partial^2 P}{\partial q_{-i}^2} + q_i \frac{\partial P}{\partial q_{-i}} \left(\sigma_i \frac{\partial \phi_{-i}}{\partial q_{-i}} + \phi_{-i} \frac{\partial \sigma_i}{\partial q_{-i}} \right) \\
&\leq \rho_{i,2} + \sigma_i \phi_{-i} q_i \frac{\partial^2 P}{\partial q_{-i}^2}
\end{aligned}$$

The last line followed from the fact that the term in parenthesis is positive by assumption A. Expanding the $\rho_{i,2}$ term and collecting terms gives

$$\begin{aligned}
\frac{\partial}{\partial q_{-i}}\tilde{F}_{q_i} &\leq \frac{\partial P}{\partial q_{-i}} + q_i \left(\frac{\partial^2 P}{\partial q_{-i} \partial q_i} + \sigma_i \phi_{-i} \frac{\partial^2 P}{\partial q_{-i}^2} \right) \\
&\leq \frac{\partial P}{\partial q_{-i}} + q_i \left(\frac{\partial^2 P}{\partial q_{-i} \partial q_i} - (1) \cdot \frac{\partial^2 P}{\partial q_{-i}^2} \right)
\end{aligned}$$

as $\phi_{-i} \geq \overline{\phi}_{-i} \geq -1$ and $\sigma_i \leq 1$. Since $\frac{\partial P}{\partial q_{-i}}$ is negative and so is the term in parentheses by assumption A, the whole term is negative.

This concludes the proof. *Q.E.D.*