Tariffs, mixing, and collusion

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February 1, 2013
VERY PRELIMINARY

Abstract
This paper analyzes the effect of different pricing schemes on horizontally differentiated firms' ability to sustain collusion. It is shown that collusion at maximum prices is least stable under price discrimination using two-part tariffs compared to linear and fixed prices. Collusion at maximum prices is stablest under linear (fixed) prices for a low (high) degree of differentiation. In a situation where full collusion is not possible, two-part tariffs result in the highest collusive profits when firms are very differentiated.

JEL classification: L13; L41.

Keywords: Collusion; Combinable products; Fixed price; Linear pricing; Mixing; Price discrimination; Two-part tariff.

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1 Introduction

In this paper, I investigate the impact of different pricing schemes on firms' ability to sustain collusive prices in an infinitely repeated game. More precisely, I compare the cases with linear prices, fixed fees, and two-part tariffs (or second-degree price discrimination) in a setup à la Hotelling (1929) where customer have the opportunity to combine (or mix) products from two differentiated firms to achieve a better match of their preferences.

Competition with combinable products was first analyzed by Anderson and Neven (1989) for the case with linear tariffs. The issue was investigated further in Hoernig and Valletti (2006, 2011) who analyze two-part tariffs and nonlinear pricing in general.[1] As these contributions show, the scope of mixing crucially depends on the pricing policy. More precisely, there is no mixing at all if firms charge fixed prices, i.e., firms buy from one firm exclusively: in this case, fixed prices which have to be borne independent of the scope of usage are too high for mixing to be attractive. On the other hand, there is mixing by all customers if they are charged linear prices which means that customers only pay for actual usage. With two-part tariffs and nonlinear pricing in general, some mixing occurs: only those customers whose preferences are met least optimally combine products to achieve a better fit. With two-part tariffs, the fixed part is lower than the prices under a fixed-price regime which means that mixing becomes more affordable. In a static environment, Hoernig and Valletti (2006) stress that the main and robust result is that firms' profits are higher as the number of pricing instruments increases.[2,3]

The possibility of mixing on the customer side and the impact of different pricing

[1] Gal-Or and Dukes (2003) as well as Gabszewicz, Laussel, and Sonnac (2004) use the approach to analyze listeners'/viewers' optimal mix of programs in media markets.

[2] At the same time, customers receive a higher surplus as they save on disutility from imperfectly matched preferences which ultimately results in higher social welfare. Moreover, social welfare is higher under two-part tariffs compared to a fixed-price regime as customers incur less transportation costs. Social welfare is highest under linear tariffs because customers no longer incur any transportation costs.

[3] This result is different from the related literature on mixed bundling which uses mix-and-match models (see Matutes and Regibeau, 1992). The authors show that profits are lower with more instruments, in particular for the case where firms practice mixed bundling compared to the one where products are sold separately.

regimes has so far only been analyzed in a static context where competition drives prices down and yields lower profits which means that firms may try to soften competition and sustain higher prices. Therefore, the present paper investigates the case with infinitely repeated interactions and firms' incentives to collude. Applying grim-trigger strategies, I derive the critical discount factors as a measure of collusive stability in the three scenarios. I show that collusive profits in an infinitely repeated game may be lower with more elaborate pricing techniques: this has to do with the fact that more instruments may render deviation more attractive which decreases collusive stability. More precisely, even if more elaborate tariffs have no impact on collusive outcomes, they—as just mentioned—result in higher competitive (punishment) profits and allow deviating firms to generate higher profits. As a result, collusion at maximum profits can only be sustained for a smaller range of the discount factor. This is exactly what can be found when comparing linear pricing with two-part tariffs where maximum collusive profits are the same. However, this is also the outcome for a comparison of flat fees and two-part tariffs despite the observation that maximum collusive profits are lower under fixed prices which destabilizes the collusive agreement. As it turns out, the destabilizing effects with two-part tariffs dominate compared to the fixed-price scheme which means that collusion can be sustained for a greater range of the discount factor in the latter.

This outcome may be different from a situation where firms have to reduce their collusive prices in order for collusion to be possible whenever the discount factor is below the critical threshold. There, collusive profits may be higher under more sophisticated pricing techniques.

As Hoernig and Valletti (2006, 2011) point out, an application of the modeling framework described above are different types of media markets (e.g., television, radio) where firms use various pricing schemes and where mixing by viewers/listeners is very common. When it comes to anti-competitive practices, the business has seen several cases of cartel behavior recently. For example, in the German television broadcasting market, the two largest private media companies, RTL and ProSiebenSat.1, were found guilty of jointly trying to limit access to their to their standard-quality programs and set a higher fee. Both companies were fined a total of $55 mio. by the *Bundeskartellamt* (German Antitrust Authority).[4]

Furthermore, the present analysis has implications for competition policy in a more general context: consumer protection agencies as well as policymakers often criticize firms' complex pricing schedules designed to price discriminate between consumers. They demand that firms reduce the complexity of their pricing schemes in order to make decisions for consumers easier and more transparent. This paper highlights that the implications of such changes are not clearcut: it is true that a smaller number of available contracts, i.e., less instruments to price discriminate among customers, reduces prices customers have to pay in a static context. However, there may be the undesired anti-competitive consequence that collusion may be easier to sustain and firms may end up generating higher supra-competitive profits in a dynamic setting.

Although price discrimination and tariff choice are important features of everyday business practice, the literature on the impact of different pricing schemes on collusion is rather limited. Concerning the relationship between third-degree price discrimination and collusion, Liu and Serfes (2007) investigate the impact of the availability of customer-specific information for market segmentation in a linear-city model on the feasibility to collude using grim-trigger strategies. A higher degree of market segmentation accompanied by a more diversified pricing structure is possible as the quality of customer information increases. For collusive stability, better information has opposing effects: on the one hand, it implies higher collusive profits and harsher punishment; on the other hand, deviation becomes more profitable. The authors show that the latter effect dominates, i.e., collusion is harder to sustain as the firms' ability to segment customers improves. Here, I find a similar result in the context of mixing where—as mentioned above—more elaborate pricing schemes make collusion at maximum prices more difficult. Note, however, that I analyze a situation with second-degree price discrimination as firms cannot distinguish between customers who self-select into their preferred mixing choice.

[4]See, e.g., `http://www.handelsblatt.com/unternehmen/it-medien/verbotene-absprachen-kartellamt-brummt-rtl-und-prosieben-strafen-auf/7567316.html` for details.

A related study to Liu and Serfes (2007) is the one by Colombo (2010): the author allows for different degrees of product differentiation (i.e., firms are not located at the extremes of the linear city and hence are not maximally differentiated) and analyzes perfect price discrimination. With perfect price discrimination, firms may set prices based on the exact location of a customer (so-called delivered pricing), i.e., this is a special case of the analysis in Liu and Serfes (2007) with maximally differentiated firms.[5] The author shows that in contrast to Chang (1991), collusion is easier to sustain the lower transportation costs are and that colluding on discriminatory prices is harder than on a uniform price.[6]

The rest of this paper is organized as follows. In section 2, I present the setup and derive demands. In section 3, I derive profits in the competitive, collusion, and deviation cases for fixed prices (subsection 3.1), linear prices (subsection 3.2), and two-part tariffs (subsection 3.3). The resulting critical discount factors and collusive profits below the critical discount factor are compared in subsection 3.4. The last section concludes.

2 Model

A model of horizontal product differentiation à la Hotelling (1929) with two symmetric firms 1 and 2 is considered: firm 1 is located at $L_1 = 0$ and firm 2 is located at $L_2 = 1$ on the linear city of unit length. Fixed and marginal costs are normalized to zero. Firms discount future profits by the common discount factor δ per period. Customers of mass one are uniformly distributed along the line. Each customers has a total demand of one. Customers incur transportation costs τ per unit of distance between their location and the firms' locations (with $\tau > 0$). These costs reflect the fact that customers' preferences are not fully matched by the firms' products, i.e., a customer located at x incurs transportation costs of τx^2 (or $\tau(1-x)^2$)

[5] Further contributions investigating the implications of delivered pricing on collusion are, among others, Miklós-Thal (2008) and Jorge and Pires (2008).

[6] Note that in his setup, firms punish and deviate using discriminatory prices which is different from the present setup where punishment and deviation depends on the pricing instruments available.

when buying from firm 1 (or firm 2). Following Anderson and Neven (1989), the customer may save on these costs by splitting his demand across the two firms to purchase an optimal individual mix of both products. Let $\lambda \leq 1$ denote the share of the overall demand of one which a customer buys from firm 1. Accordingly, a share of $1-\lambda$ is bought from firm 2. Then, mixing leads to transportation costs of $\tau(\lambda L_1+(1-\lambda)L_2-x)^2 = \tau(1-\lambda-x)^2$. The underlying idea is that customers may have different preferences at different points in time and mixing allows customers to save on transportation costs as they get a better match of their preferences.

In what follows, I will analyze the effects on collusion of these three different pricing regimes:

(i) fixed-price scenario (denoted by subscript F): firms compete in fixed prices $f_{i,F}$, i.e., customers pay a flat (subscription) fee independent of actual usage (with $i \in \{1,2\}$) (see section 3.1);

(ii) linear-price scenario (denoted by subscript L): firms compete in prices $p_{i,L}$ per unit purchased (see section 3.2); and

(iii) two-part tariffs (denoted by subscript T): firms compete in tariffs which are made up of a fixed component $f_{i,T}$ and a variable part $p_{i,T}$ charged per unit sold (see section 3.3).

When it comes to customer utility, there are potentially three different types of customers under the assumption that all customers buy at least from one firm (see the assumption below): in the most general case with given prices f_1 and p_1, a customer who is located at x and who buys exclusively from firm 1 (subscript 1) derives a utility of

$$U_1(x) = v - f_1 - p_1 - \tau x^2$$

where v denotes the basic valuation from buying one unit of demand. Similarly, a customer who is located at x and who only buys from firm 2 (subscript 2) derives a utility of

$$U_2(x) = v - f_2 - p_2 - \tau(1-x)^2.$$

Finally, a customers who is located at x and who mixes (subscript m) between both firms gets a utility of

$$U_m(x) = v - f_1 - f_2 - \lambda p_1 - (1-\lambda)p_2 - \tau\left(1-\lambda-x\right)^2.$$

Typically, the focus in models with horizontal product differentiation is on the situation where the market is covered, i.e., no customer along the line does not buy. The same is true for the present setup which means that the following is assumed to hold for customers' transportation costs:

Assumption 1. *Transportation costs are not too high relative to the basic valuation from buying, i.e.,* $0 < \tau \leq 4v/5$.

The assumption guarantees that the whole market is served under any of the collusive outcomes to be considered. Note that this assumption is standard in the literature.

A mixing customer will optimally choose λ to maximize utility depending on the customer's location, i.e.,

$$\frac{\partial U_m}{\partial \lambda} = 0 \Leftrightarrow \lambda(x) = 1 - x - \frac{p_1 - p_2}{2\tau} \tag{1}$$

Given the decision by the mixing customers, it is now possible to derive the customer who is indifferent between buying from firm 1 exclusively and mixing. Denote this customer's location by $\underline{x}$. Similarly, denote the location of the customer who is indifferent between mixing and buying from firm 2 exclusively by $\bar{x}$ and assume that $0 \leq \underline{x} \leq \bar{x} \leq 1$. Then, the location of the first indifferent is the solution to

$$U_1\left(\underline{x}\right) = U_m\left(\underline{x}\right) \Leftrightarrow \underline{x} = \sqrt{\frac{f_2}{\tau}} - \frac{p_1 - p_2}{2\tau}. \tag{2}$$

The location of the second indifferent customer is the solution to

$$U_m\left(\bar{x}\right) = U_2\left(\bar{x}\right) \Leftrightarrow \bar{x} = 1 - \sqrt{\frac{f_1}{\tau}} - \frac{p_1 - p_2}{2\tau}. \tag{3}$$

Given the locations of the indifferent customers, for $0 \leq \underline{x} \leq \bar{x} \leq 1$, subscription demands (subscript s) are as follows

$$D_{1,s}\left(f_1, p_1, f_2, p_2\right) = \bar{x}$$

and

$$D_{2,s}\left(f_1, p_1, f_2, p_2\right) = 1 - \underline{x}.$$

Similarly, variable demands from mixing amount to

$$D_{1,m}\left(f_1, p_1, f_2, p_2\right) = \underline{x} + \int_{\underline{x}}^{\bar{x}} \lambda(x)dx$$

and

$$D_{2,m}\left(f_1, p_1, f_2, p_2\right) = \int_{\underline{x}}^{\bar{x}} 1 - \lambda(x)dx + 1 - \bar{x}.$$

For the demands in those situations where the assumption $0 \leq \underline{x} \leq \bar{x} \leq 1$ is violated, see the appendix.

Before analyzing the three different pricing regimes and their impact on collusion, a note on the measure for collusive stability seems in order. I will derive the critical discount factor for the different scenarios. To this end, I focus on the standard grim-trigger strategies defined by Friedman (1971). Thus, collusion is profitable as long as the discounted profits from colluding, $\pi^c/(1-\delta)$, are higher than those from deviation, π^d, and the ensuing punishment phase, $\delta\pi^*/(1-\delta)$, i.e., if

$$\frac{\pi^c}{1-\delta} \geq \pi^d + \frac{\delta\pi^*}{1-\delta} \tag{4}$$

is satisfied. Solving this equation for δ gives the critical discount factor denoted by $\bar{\delta}$ and ensures a stable collusive agreement as long as

$$\delta \geq \bar{\delta} := \frac{\pi^d - \pi^c}{\pi^d - \pi^*} \tag{5}$$

holds. All things equal, a lower (higher) punishment or deviation profit leads to a stabilization (destabilization) of the collusive agreement whereas the opposite is true for a change in the collusive profit.

I briefly comment on the choice of grim-trigger strategies instead of optimal punishments following Abreu (1986, 1988) and Abreu, Pearce, and Stacchetti (1986) (so-called stick-and-carrot strategies). In the context of a setup à la Hotelling (1929) with quadratic transportation costs and symmetric firms, Häckner (1996) shows that applying optimal punishments gives qualitatively the same results regarding the impact of product differentiation on the collusive price as with grim-trigger strategies analyzed in Chang (1991). As grim-trigger strategies typically lead to more tractable results, it appears reasonable to stick to these strategies.[7]

3 Analysis

In what follows, I will derive the profits in the punishment, collusion, and deviation cases for all three pricing regimes which are then used to calculate and compare the critical discount factors. Note that I will derive collusive profits at maximum prices (full collusion) and will consider collusion at prices which are lower than these maximum prices when full collusion is not sustainable (partial collusion) in subsection 3.4.

I start with the case where firms set fixed prices.

3.1 Fixed prices

The competitive outcome is a special case of the analysis in Hoernig and Valletti (2006). As they show, mixing by any of the customers cannot be an equilibrium. As a result, in the fixed-pricing regime, competitive prices and profits are given by

$$f_F^* = \tau \tag{6}$$

and

$$\pi_F^* = \frac{\tau}{2}. \tag{7}$$

[7]As Liu and Serfes (2007) report, their main result is also robust to modeling changes involving stick-and-carrot punishments.

With this competitive price, indeed no customer along the linear city mixes as paying the relatively high fixed fee twice is not compensated by the potential savings on transportation costs.

Turning to the collusion scenario, I first observe that mixing cannot occur either. If firms find it optimal to set competitive equilibrium prices that are so high that no customer wants to mix, this is also true for the collusion case: as a firm's subscription demand in case mixing occurs only depends on its own fixed price (see expressions (2) and (3) for $p_1 = p_2 = 0$), increasing the price until mixing no longer occurs is profitable for competitive firms—and thus also for colluding firms. As a consequence, prices are set such that the only indifferent customer is located in the middle of the linear city (or $\underline{x}_F^c = \bar{x}_F^c = 1/2$) and derives zero utility. Hence, I arrive at the following result:

Proposition 1. *Under a fixed-price scenario, collusive prices and profits are given as*

$$f_F^c = v - \frac{\tau}{4} \tag{8}$$

and

$$\pi_F^c = \frac{v}{2} - \frac{\tau}{8}. \tag{9}$$

As customers have to possibility to mix, I need to check whether a deviating firm has an incentive to set a price that is sufficiently low such that mixing occurs. However, it can be shown that this is not the case (i.e., $0 < \underline{x}_F^d = \bar{x}_F^d \leq 1$) which means that the optimal deviation price corresponds to the one known from a situation without the possibility of mixing:

Proposition 2. *With fixed prices, deviation prices and profits are given by*

$$f_F^d = \begin{cases} v - \frac{5\tau}{4} & \text{if } 0 < \tau \leq \frac{4v}{13} \\ \frac{v}{2} + \frac{3\tau}{8} & \text{if } \frac{4v}{13} < \tau \leq \frac{4v}{5} \end{cases} \tag{10}$$

and

$$\pi_F^d = \begin{cases} v - \frac{5\tau}{4} & \textit{if } 0 < \tau \le \frac{4v}{13} \\ \frac{(4v+3\tau)^2}{128\tau} & \textit{if } \frac{4v}{13} < \tau \le \frac{4v}{5} \end{cases} \tag{11}$$

Note that for sufficiently low transportation costs (i.e., $0 < \tau \le 4v/13$), a deviating firm captures the whole market.

From the analysis of fixed prices it follows that mixing does not occur in any of the three cases which means that the critical discount factor is the same as the one in Chang (1991) for the case where firms are located at the two extremes. More precisely, the critical factor can be written as

$$\bar{\delta}_F = \begin{cases} \frac{4v-9\tau}{2(4v-7\tau)} & \text{if } 0 < \tau \le \frac{4v}{13} \\ \frac{4v-5\tau}{4v+11\tau} & \text{if } \frac{4v}{13} < \tau \le \frac{4v}{5} \end{cases} \tag{12}$$

As Chang (1991) shows, a higher degree of product differentiation results in greater collusive stability as the firms find it harder to profitably deviate, i.e., $\partial\bar{\delta}_F/\partial\tau < 0$.

I now turn to a situation in which firms choose linear prices per unit purchased.

3.2 Linear pricing

The results for the punishment phase are due to Anderson and Neven (1989). With linear pricing, competitive prices and profits are the same as with fixed prices, i.e.,

$$p_L^* = p_F^* = \tau \tag{13}$$

and

$$\pi_L^* = \pi_F^* = \frac{\tau}{2}. \tag{14}$$

Compared to the previous case, there is an important difference, though: at this competitive price, all customers along the linear city mix and fully save on transportation costs, i.e., they get a perfect bundle of the products.

The case of collusion among the two firms corresponds to the monopoly case in

Hoernig and Valletti (2011) which they use a benchmark. The authors assume that a monopolist offers both products at either extreme of the line which is the same as the case of colluding firms here. The firms—just like the monopolist—have an incentive to maximize customer surplus (i.e., induce the efficient allocation) which is then fully extracted by setting an optimal linear price. As a result, under linear pricing, collusive prices and profits are given by

$$p_L^c = v \tag{15}$$

and

$$\pi_L^c = \frac{v}{2}. \tag{16}$$

Hence, linear pricing allows colluding firms to make higher profits compared to the case with fixed prices.

Now if a firm deviates from the collusive path, it needs to offer a lower price to attract customers as all customers get zero utility under collusion and will not buy at an even higher price. Suppose firms 1 deviates: this means that some customers located sufficiently close to firm 1 find it more attractive to buy exclusively from firm 1, i.e., $\underline{x} > 0$, and all other customers keep mixing, i.e., $\bar{x} = 1$. Define $A := \sqrt{v^2 - 4v\tau + 28\tau^2}$. Then, the deviation case can be characterized as follows:

Proposition 3. *With linear pricing, deviation prices and profits are given by*

$$p_L^d = \frac{2v - 4\tau + A}{3} \tag{17}$$

and

$$\pi_L^d = \frac{(-2v + 4\tau - A)(v^2 - vA - 4v\tau + 2\tau A - 20\tau^2)}{108\tau^2}. \tag{18}$$

Making use of the profits under linear pricing, the critical discount factor can now be characterized as follows:

$$\bar{\delta}_L = \frac{v^3 - v^2 A - 6v^2\tau + 4v\tau A - 6v\tau^2 - 28\tau^2 A + 136\tau^3}{v^3 - v^2 A - 6v^2\tau + 4v\tau A - 60v\tau^2 - 28\tau^2 A + 190\tau^3} \tag{19}$$

The following proposition analyzes the impact of a change in the degree of product differentiation on the critical discount factor:

Proposition 4. *It holds that* $\partial\bar{\delta}_F/\partial\tau < 0 \ \forall 0 < \tau \leq 4v/5$*, i.e., an increase in differentiation increases collusive stability and vice versa.*

Hence, as is the case with fixed prices, more differentiated firms find it easier to sustain collusion for a higher degree of differentiation. Note that—unlike in the previous case—with linear prices, a change in transportation costs does not influence firms' collusive profits which are higher compared to the fixed-price scheme. However, these higher profits make deviation even more attractive compared to fixed prices when transportation costs are reduced. At the same time, a reduction in transportation costs makes punishment harsher—by the same magnitude as under fixed pricing. The proposition highlights that the first effect dominates.

I next analyze the case where firms set two-part tariffs.

3.3 Two-part tariffs

The results for the punishment phase are a special case (extreme locations) of the more general analysis in Hoernig and Valletti (2006). They derive the competitive equilibrium prices and profits which are given as

$$f_T^* = \frac{\left(7 - 3\sqrt{5}\right)\tau}{2}, \tag{20}$$

$$p_T^* = \frac{\left(3\sqrt{5} - 5\right)\tau}{2}, \tag{21}$$

and

$$\pi_T^* = \frac{\left(13\sqrt{5} - 27\right)\tau}{4}. \tag{22}$$

With this competitive price, not all customers along the linear city mix as $0 < \underline{x}_T^* = (3 - \sqrt{5})/2 < (\sqrt{5} - 1)/2 = \bar{x}_T^* < 1$.

For the case with collusion, I can state the following result:

Proposition 5. *With two-part tariffs, collusive prices and profits are given by*

$$f_T^c = 0, \tag{23}$$

$$p_T^c = p_L^c = v \tag{24}$$

and

$$\pi_T^c = \pi_L^c = \frac{v}{2}. \tag{25}$$

In the case of collusion at maximum prices, firms set a fixed component equal to zero and only set an optimal linear price, i.e., the pricing structure is the same under linear pricing. The reason behind this result is that if no customers incurs any transportation costs, customer surplus is maximized which is then taken away from customers through the symmetric variable price. Charging a fixed fee on top of that would make some customers stay away from mixing which would ultimately reduce the scope of customer surplus which the colluding firms can appropriate. Note, however, that colluding firms may set strictly positive fixed prices when full collusion is not possible (see *Corollary 2*).

I next analyze firm's deviating behavior given that the competitor sticks to the collusive agreement:

Proposition 6. *With two-part tariffs, deviation prices and profits are given by*

$$\begin{cases} f_T^d = \tau, p_t^d = v - 2\tau & \text{if } 0 < \tau \leq \frac{v}{4} \\ f_T^d = \frac{(v-\tau)^2}{9\tau}, p_t^d = \frac{v}{3} + \frac{2\tau}{3} & \text{if } \frac{v}{4} < \tau \leq \frac{2v}{5} \\ f_T^d = \frac{\tau}{4}, p_t^d = \frac{v}{2} + \frac{\tau}{4} & \text{if } \frac{2v}{5} < \tau \leq \bar{\tau} \end{cases} \tag{26}$$

and

$$\pi_T^d = \begin{cases} v - \tau & \text{if } 0 < \tau \leq \frac{v}{4} \\ \frac{-v^3+12v^2\tau+6v\tau^2+10\tau^3}{54\tau^2} & \text{if } \frac{v}{4} < \tau \leq \frac{2v}{5} \\ \frac{4v^2+8v\tau+5\tau^2}{32\tau} & \text{if } \frac{2v}{5} < \tau \leq \bar{\tau}. \end{cases} \tag{27}$$

Note that given the deviation prices, it holds that $\underline{x}_T^d = \bar{x}_T^d = 1 \; \forall 0 < \tau \leq v/4$ and $0 < \underline{x}_T^d < \bar{x}_T^d = 1 \; \forall v/4 < \tau \leq 2v/5$.

Together with the profits from the punishment and collusive phases, I can now characterize the critical discount factor:

$$\bar{\delta}_T = \begin{cases} \frac{2(v-2\tau)}{4v+23\tau-13\sqrt{5}\tau} & \text{if } 0 < \tau \leq \frac{v}{4} \\ \frac{(10\tau-v)(v-\tau)^2}{-2v^3+24v^2\tau+12v\tau^2+749\tau^3-351\sqrt{5}\tau} & \text{if } \frac{v}{4} < \tau \leq \frac{2v}{5} \\ \frac{4v^2-8v\tau+5\tau^2}{4v^2+8v\tau+221\tau^2-104\sqrt{5}\tau^2} & \text{if } \frac{2v}{5} < \tau \leq \bar{\tau} \end{cases} \tag{28}$$

I define $\tau' := (2(329 + 65\sqrt{5} - \sqrt{44503 + 3718\sqrt{5}})/751)v \approx 0.65120v$. The following proposition characterizes the relationship between the critical discount factor and the degree of product differentiation:

Proposition 7. *It holds that $\partial\bar{\delta}_F/\partial\tau \leq 0 \; \forall 0 < \tau \leq \tau'$ and $\partial\bar{\delta}_F/\partial\tau > 0 \; \forall \tau' < \tau \leq \bar{\tau}$, i.e., if transportation costs are already high, a further increase in transportation costs reduces collusive stability.*

This is different from the other two cases where an increase in transportation costs always leads to a lower critical discount factor, i.e., collusion is stabilized. The difference can be explained as follows: under all three pricing schemes, deviation is not very attractive for high transportation costs as attracting additional customers requires rather drastic price cuts. At the same time, punishment is not very harsh. The latter, destabilizing effect is relatively more pronounced under two-part tariffs as competitive profits are higher and becomes more and more important as transportation costs increase. As a result, the destabilizing effect dominates for high degrees of differentiation.

I next discuss the implications of the results so far for the stability of collusion in the three different pricing scenarios.

3.4 Comparisons

From the observation that punishment is less harsh and deviation profits are higher under two-part tariffs than under linear prices, I can draw the following conclusion:

Corollary 1. *A comparison of the critical discount factors under linear pricing and two-part tariffs reveals that $\bar{\delta}_L \leq \bar{\delta}_T$, i.e., collusion is never easier to sustain under two-part tariffs than under linear pricing.*

Both pricing regimes lead to the same collusive profit for the two firms but deviation incentives are greater under two-part tariffs which means that collusion at maximum prices can only be sustained for a smaller range of the discount factor.

A full characterization of the order of the critical discount factors is a little bit more involved due to the different effects on profits in the three phases. The following proposition provides an answer:

Proposition 8. *A comparison of the critical discount factors reveals that*

(i) *there exists a* τ'' *such that* $\bar{\delta}_L \leq \bar{\delta}_F \; \forall 0 < \tau \leq \tau''$ *and* $\bar{\delta}_L > \bar{\delta}_F \; \forall \tau'' < \tau \leq 4v/5$ *as well as*

(ii) $\max\{\bar{\delta}_L, \bar{\delta}_F\} \leq \bar{\delta}_T \; \forall 0 < \tau \leq 4v/5.$

Compared to standard case with fixed prices, linear pricing yields the same Nash punishment profits but has two opposing effects concerning collusive stability: on the one hand, monopoly profits are higher which makes collusion more attractive for firms. On the other hand, higher collusive prices make deviation easier which results in higher deviation profits. For low transportation costs, deviation requires only moderate price cuts which means that the first effect dominates, i.e., higher monopoly profits stabilize collusion under linear pricing. Now as transportation costs increase, the second effect becomes more important because deviating is harder. As a consequence, full collusion is sustainable for a greater range of the discount factor under fixed prices.

Compared to case with fixed prices, under two-part tariffs, there are two opposing effects with respect to collusive stability: on the one hand, monopoly prices and profits are higher. At the same time, deviation profits are higher and competitive profits are higher. The second effect dominates. *Figure 1* illustrates these findings.

The results in *Proposition 8* are derived under the assumption that firms collude on maximum prices. For any discount factor below the respective critical threshold, these prices cannot be supported in equilibrium. However, in a situation where firms are horizontally differentiated, firms can still charge prices which are higher than

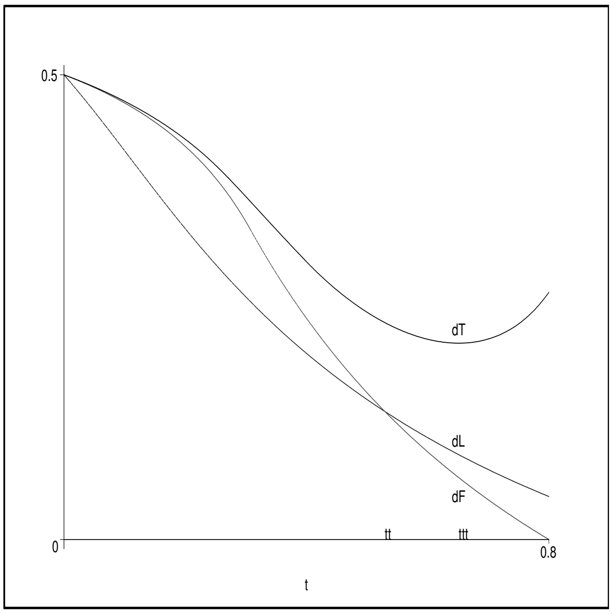

Figure 1: Comparison of the critical discount factors in the three pricing scenarios (for $v = 1$ and $0 < \tau \leq 4/5$; $\tau'' \approx 0.52935$ and $\tau' \approx 0.65120$).

the competitive prices in a collusion equilibrium as long as the discount factor is strictly positive as shown in Chang (1991). To this end, firms reduce their initial maximum collusive prices and set δ-adjusted prices such that the incentive to stick to the collusive agreement is (just) met. As the discount factor converges to zero, collusive profits converge to competitive profits for which $\pi_F^* = \pi_L^* < \pi_T^*$ holds. Taking this observation together with the result for maximum collusive profits that $\pi_F^c < \pi_L^c = \pi_T^c$ and the approach by Chang (1991) gives the following proposition:

Proposition 9. *A comparison of the collusive profits reveals that*

(i) *for* $0 < \tau \leq \tau''$ *and* $\delta \in [\bar{\delta}_L, \bar{\delta}_F]$, *it holds that* $\pi_F^c(\delta) \leq \pi_L^c$,

(ii) *for* $\tau'' < \tau \leq 4v/5$ *and* $\delta \in [0, \bar{\delta}_L]$, *it holds that* $\pi_L^c(\delta) \leq \pi_F^c$,

(iii) $\pi_T^c(\delta) \leq \pi_L^c \ \forall \delta \in [\bar{\delta}_L, \bar{\delta}_T]$, *and*

(iv) *there exists a* $\delta' \in [0, \bar{\delta}_L]$ *such that* $\pi_L^c(\delta) \leq \pi_T^c(\delta)$ *if* $\delta \leq \delta'$ *and* $\pi_T^c(\delta) < \pi_L^c(\delta)$ *if* $\delta > \delta'$.

Proof. Follows the proof of *Proposition 3* in Rasch and Wambach (2009). □

As the proposition highlights, the fact that one specific collusive pricing regime is less stable compared to other schemes when it comes to full collusion does not mean that it results in least damage for customers in any collusive situation. For instance, maximum collusive two-part tariffs are relatively hard to sustain but given the higher profit in the competitive case, colluding on two-part tariffs may yield higher profits (and hence higher prices) under partial collusion.

From the profits under partial collusion when firms set two-part tariffs the following result can be obtained:

Corollary 2. *There exists a $\delta'' \in [0, \bar{\delta}_T]$ such that $f_T^c(\delta) = 0$ if $\delta \geq \delta''$ and $f_T^c(\delta) > 0$ if $\delta < \delta''$, i.e., colluding firms do not always abstain from setting a strictly positive fixed fee.*

Hence, different from the case where firms are able to set maximum collusive prices, firms find it optimal to charge a strictly positive fixed component when only partial collusion on two-part tariffs is feasible. An immediate conclusion from this result is that just because firms actually make use of fixed fees when they can does not imply that they are not colluding.

4 Conclusion

In this paper, I consider the influence of three different pricing regimes on firms' ability to collude. It is shown that full collusion is easiest to sustain with linear pricing as long as firms are not too differentiated. For high degrees of differentiation, collusion at maximum prices can be sustained for the greatest range of the discount factor under fixed pricing. Full collusion with two-part tariffs is always hardest to maintain. However, two-part tariffs may result in the highest collusive profits when partial collusion is considered.

An aspect which I have not analyzed is the strategic choice of the pricing schedule employed by firms. If firms decide on the tariff before they collude, they may use other pricing techniques when they choose to deviate.

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