Incentives, Employment, and the Division of Labor in Teams

December 19, 2012

Abstract

We develop a theory of incentives and employment for teams with specialization and division of labor. A central insight of the paper is that specialization and division of labor not only improve productivity but also increase effort and the sensitivity of effort to incentives. We show that employment and incentives are substitutes for the budget-breaker when the difficulty of measuring performance increases with team size sufficiently rapidly relative to the positive effects of specialization and division of labor. We provide characterizations of the partnership and the role of the budget-breaker that are quite different from the classical literature.

JEL Classifications: D02, D21, D86, L25, M5.

Keywords: budget-breaker, division of labor, employment, endogenous team size, incentives, moral hazard, partnerships, specialization, teams.

1 Introduction

The positive effects of specialization and division of labor in terms of greater productivity and economic growth have long been recognized. In this paper, we show that there are additional effects under moral hazard and incentive contracting; i.e., specialization and division of labor also generate higher effort and increase the sensitivity of effort to incentives. We show that this simple insight has major implications for both optimal incentives and employment as well as the role of the principal in team settings.

Our starting point is the theory of specialization and division of labor in Becker and Murphy (1992), which we extend to allow for an elastic supply of effort. We first consider the benchmark case where agents' efforts are *contractible* (i.e., observable and verifiable) and the principal chooses efforts and employment to maximize the expected total surplus. We show that effort and employment are strategic *complements* for the principal because an increase in employment leads to greater specialization, division of labor, and expected productivity.[1] The principal therefore requires higher efforts. As in Becker and Murphy, the optimal employment level reflects a tradeoff between greater expected productivity and higher employment transaction costs.

We embed our extension of the Becker-Murphy model of specialization and division of labor in the classical teams framework due to Alchian and Demsetz (1972) and Holmström (1982). We consider two distinct institutions under moral hazard. The first is the *equal-division partnership* (EDP), where revenues and transaction costs are divided equally among partners.[2] The second is the *firm with a budget-breaker* (henceforth referred to as the *firm*), who offers linear contracts based on group performance.[3] In Holmström, the partnership is defined by a balanced-budget sharing rule (an example is equal division as in our paper) which induces free-riding. In his model, the role of the budget-breaker is to correct the free-rider problem by replacing weak balanced-budget incentives

[1]We say that two endogenous variables are strategic *complements* if an increase in one optimally implies an increase in the other for given values of the parameters. Likewise, they are *substitutes* if an increase in one implies a decrease in the other.

[2]Equal-division contracts can be optimal in a variety of circumstances: to prevent sabotage in Bose, Pal, and Sappington (2010), because of inequity aversion in Bartling and von Siemens (2010), and to ensure stability in Levin and Tadelis (2005). Encinosa, Gaynor, and Rebitzer (2007) document the prevalence of equal-division contracts in medical group practices and Lang and Gordon (1995) in law firms in the form of the *lock-step* system.

[3]The assumptions of linear contracts, additive and normally distributed noise, and identical agents with exponential utility are made for tractability but constitute an important limitation of the paper. As is well-known (see Bolton and Dewatripont (2005, Section 4.3)), linear contracts are generally suboptimal under these conditions. Holmström and Milgrom (1987) provide the classic rationale for linear contracts in the case of a single agent; the proof of Proposition 2 below suggests that this rationale extends to our framework. Under limited liability, Bose, Pal, and Sappington (2011) show that the optimal linear contract achieves at least 90% of the expected profit of the optimal nonlinear (i.e., second best) contract. A linear contract will therefore be optimal when transaction costs are sufficiently increasing in contractual complexity.

with stronger incentives that implement the first best (or close in the case of uncertainty). But this characterization contradicts the stylized fact that incentives are weak in firms (see Prendergast (1999)). Absent a budget-breaker, Alchian and Demsetz speculate that partnerships will tend to be small to control free-riding.[4]

The canonical example of specialization and division of labor is the modern assembly line, where production is divided into separate tasks and intermediate outputs are assembled to produce the final product. Such production technologies are inherently Leontief, which leads to a fundamental difference between our model of an EDP and the classical literature. In particular, Legros and Matthews (1993) have shown that there is no free-rider problem in partnerships under general conditions which include a Leontief production function.

Instead, the inefficiency of an EDP in our model revolves around hiring distortions. First, the $1/n$ problem emphasized in Levin and Tadelis (2005) refers to the fact that the representative partner only receives $1/n$ of the expected return from the marginal hire, where n is the number of partners. This biases an EDP towards under-employment. Second, the representative partner only considers her share of the transaction costs and her own individual rather than aggregate risk premium. Finally, we allow the difficulty of measuring performance to depend on team size as in Liang, Rajan, and Ray (2008). In general, one would expect the difficulty of measuring performance to rise with employment but in the present context it may decline because of specialization and division of labor where each worker performs fewer and more specialized tasks. We show that an EDP is smaller than the benchmark case with contractible efforts when the difficulty of measuring performance is increasing in team size. Although this matches the characterization in Alchian and Demsetz, the explanation hinges on the above employment distortions rather than free-riding. But when the difficulty of measuring performance falls sufficiently rapidly with specialization and division of labor we obtain the novel result that an EDP can be *larger* than the benchmark case.

We then turn to the firm, starting with the agents' non-cooperative choice of efforts given the budget-breaker's profile of contracts and the employment level. We show that effort and the sensitivity of effort to incentives are both increasing in employment. This is because the expected marginal benefit of effort is the piece rate times the expected marginal product of effort, which is increasing in employment due to greater specialization and division of labor. These effects tend to make employment and incentives strategic complements for the budget-breaker. A second set of effects arises, however, from risk considerations. In particular, if the difficulty of measuring

[4] A more recent literature identifies conditions under which the partnership achieves the first best or is more efficient than some alternative institution. This literature includes Rasmusen (1987), Legros and Matsushima (1991), Legros and Matthews (1993), Garicano and Santos (2004), and Levin and Tadelis (2005).

performance is increasing in team size then employment and incentives are substitutes in reducing the aggregate risk premium. Ultimately, we show that employment and incentives are substitutes when the difficulty of measuring performance increases sufficiently rapidly with team size relative to the aforementioned effects of specialization and division of labor. We then discuss some empirical evidence which suggests that they are in fact substitutes.

Two examples provide novel characterizations of these institutions. In one example, an EDP has stronger incentives and can be larger than a firm with the same parameters. This contradicts the role of the budget-breaker in Holmström and the characterization of the partnership in Alchian and Demsetz. In this example, an EDP over-employs relative to the firm because the individual risk premium of the representative partner is decreasing in employment, whereas the aggregate risk premium considered by the budget-breaker is increasing. Monteiro and Stewart (2012) present evidence consistent with such relative over-employment. The result that an EDP offers stronger incentives than the budget-breaker follows from the fact that equal-division incentives implement the first best effort under a Leontief production function. In comparison, the budget-breaker offers weaker incentives due to the standard risk-reward tradeoff.[5]

In another example, the firm is larger than the benchmark case with contractible efforts because the aggregate risk premium is strictly convex in incentives but only linear in employment. The budget-breaker therefore offers weak incentives to reduce the agents' exposure to risk but then over-employs to increase effort and expected productivity through greater specialization and division of labor. This combination of weak incentives and excessive specialization and division of labor is characteristic of the bureaucratic organizational form as depicted by the classical sociologist Max Weber (Giddens (1971, p. 158)). Given weak incentives, Weber assigns an important role to duty and institutional loyalty as the main determinants of effort whereas here specialization and the division of labor themselves provide the relevant ancillary motivation.

The plan for the rest of the paper is as follows. In the next section we characterize the benchmark case with contractible efforts. We consider moral hazard in section 3, the EDP in section 4, and the firm in section 5. Section 6 concludes. All proofs are in the appendix.

[5] In our model there is a negative relationship between incentives and risk which is standard in the moral hazard literature. Prendergast (1999) concludes, however, that the empirical evidence is "rather mixed."

2 Benchmark Case

We consider a production process with $n \geq 0$ identical agents, where e_i is the effort of agent i.[6] In this section the agents' efforts are contractible so the principal can specify them as part of the contract. The production function is stochastic Leontief

$$Q = \phi(n) \min_{1 \leq i \leq n} e_i + \epsilon, \tag{1}$$

where ϕ is increasing in employment n and $\phi(0) = 0$.[7] In the Appendix, we show how (1) can be derived from more primitive assumptions about the production process in a framework similar to that in Becker and Murphy (1992). As in their paper, the central theme is that an increase in employment is associated with a more intensive *division of labor* in the sense that each agent performs fewer tasks. This promotes an increase in *specialization* in the sense that agents optimally spend more time per task learning about them, which leads to an increase in expected productivity. The prediction that larger teams tend to be more specialized with a more intensive division of labor is a consistent empirical finding in the management literature; see Carter and Keon (1986) for a survey. Traditionally, specialization and division of labor are associated with increasing returns to scale, so we assume that expected output per head is increasing in team size for a given effort profile.

Assumption 1 *$\Phi(n) \equiv \phi(n)/n$ is increasing in n for all $n > 0$.*

Following Becker and Murphy, we assume employment transaction costs given by $(1/2)kn^2$, where $k \in K$ is a parameter and K is a compact interval in $\mathbf{R}_{++}$. Examples include healthcare, legal, regulatory, and basic training costs, necessary increases in plant size and other physical costs, and greater coordination costs. Later on we will demonstrate the empirical importance of this parameter. Let $K^{-1} = \{k^{-1} \mid k \in K\}$.

An increase in employment can also affect the variance of output for a variety of reasons. If agents' abilities are uncertain, an increase in team size could increase the variance due to a wider range of abilities. In a moral hazard context, the difficulty of measuring performance is generally increasing in employment. On the other hand, specialization and division of labor should make

[6] We treat $n \geq 0$ as a continuous variable throughout the paper.

[7] Let $\mathbf{R}$ denote the space of real numbers, $\mathbf{R}_+$ the nonnegative reals, and $\mathbf{R}_{++}$ the positive reals. If $x, x' \in \mathbf{R}^m$ then $x \geq x'$ iff $x_i \geq x'_i$ for all $1 \leq i \leq m$ and $x > x'$ iff $x \geq x'$ and $x \neq x'$. If $A \subseteq \mathbf{R}^m$ and $f : A \to \mathbf{R}$ then f is *increasing* if $x > x'$ implies $f(x) \geq f(x')$ and *strictly increasing* if the inequality is strict.

monitoring easier. Following Liang *et al.* (2008), we assume ϵ in (1) is a normally distributed shock with mean zero and variance $\psi(n)\sigma^2$, where ψ is a function of employment n and σ is an exogenous parameter.[8] We do not make any *a priori* assumptions about whether ψ is increasing or decreasing in n.[9] We assume ϕ and ψ are continuous so that solutions to maximization problems exist but do not require differentiability except where indicated.

Let $p \in P$ be the price of output and $N > 0$ the size of the available labor pool, where P is a compact interval in $\mathbf{R}_{++}$ (e.g., bounded by the perfectly competitive and monopoly prices). The principal's problem is to choose team size n and efforts $\{e_i\}$ to maximize the expected total surplus

$$p\phi(n) \min_i e_i - (1/2) \sum_i e_i^2 - (1/2)kn^2 \tag{2}$$

subject to the constraint $0 \leq n \leq N$. The first term in (2) is expected revenue and the second is the sum of quadratic effort costs. Since the optimum necessarily entails $e_i = e$ for all i, we can reduce (2) to

$$p\phi(n)e - (1/2)ne^2 - (1/2)kn^2. \tag{3}$$

For a given level n of employment, optimal effort is

$$e = p\Phi(n). \tag{4}$$

Assumption 1 then implies that effort and employment are strategic complements for the principal in the sense that an increase in one requires an increase in the other for given values of the parameters. As shown in the Appendix, an increase in team size leads to greater specialization and division of labor. This improves expected productivity, so the principal increases effort. This result is new because Becker and Murphy assume that each agent supplies one unit of effort inelastically.

Substituting (4) into (3),

$$V(n, p, k^{-1}) = (1/2) \left[\frac{p^2\phi(n)^2}{n} - \left(k^{-1}\right)^{-1} n^2 \right], \tag{5}$$

[8]This assumption implies that output is negative with positive probability, which may be objectionable. Note that our results would be unaffected under the following alternative assumptions due to Holmström and Milgrom (1991): output $Q = \phi(n) \min_i e_i$ is deterministic but non-contractible because of moral hazard on the part of the principal, it is non-verifiable, or realized too late for contracting. Instead, contracts must be based on the group performance signal $z = \phi(n) \min_i e_i + \epsilon$ (which can legitimately take negative values), where ϵ has the same properties as above.

[9]I thank two anonymous referees for raising these issues.

which we have written in terms of k^{-1} rather than k for reasons that will be clear momentarily. The principal chooses n to maximize (5) subject to the constraint $0 \leq n \leq N$. Since ϕ is continuous, a solution exists.

To state our results requires some definitions from lattice programming and the theory of supermodular games. Let $X \subseteq \mathbf{R}^m$. Given $x, x' \in \mathbf{R}^m$, let

$$x \vee_X x' = \sup_X \{x, x'\} \text{ and } x \wedge_X x' = \inf_X \{x, x'\}. \tag{6}$$

Note that the sup and inf are taken over X. The set X is a *lattice* if $x \vee_X x' \in X$ and $x \wedge_X x' \in X$ for all $x, x' \in X$. It is a *sublattice* if $x \vee_{\mathbf{R}^m} x' \in X$ and $x \wedge_{\mathbf{R}^m} x' \in X$ for all $x, x' \in X$. If X is a lattice and $T \subseteq \mathbf{R}^l$ the function $f(x,t)$ has *(strictly) increasing differences* on $X \times T$ if $t \geq t'$ $(t > t')$ implies $f(x,t) - f(x,t')$ is (strictly) increasing in x. If X and T are open intervals and f is twice continuously differentiable this is equivalent to $\partial^2 f / \partial x \partial t \geq (>)\, 0$ on $X \times T$. Intuitively, an increase in t raises the marginal return to increasing x. A correspondence $\Gamma : T \to X$ is *strongly increasing* if $t > t'$ implies $x \geq x'$ for all $x \in \Gamma(t)$ and $x' \in \Gamma(t')$. The definition for *strongly decreasing* is similar. For comprehensive treatments of these concepts see Topkis (1998) and Vives (1999). We adopt lattice programming techniques because quasi-concavity generally fails under increasing returns in the context of specialization and division of labor, these techniques are analytically more convenient than classical comparative statics methods, and can easily handle corner solutions. Let

$$n_c(p, k^{-1}) = \arg \max_{0 \leq n \leq N} V(n, p, k^{-1}) \tag{7}$$

$$e_c(p, k^{-1}) = \left\{ p\Phi(n) \,|\, n \in n_c(p, k^{-1}) \right\}, \tag{8}$$

the sets of optimal employment and effort levels, respectively. In general, n_c is a correspondence rather than a function because we have not made any assumptions (e.g., strict quasi-concavity) which would ensure uniqueness.

Proposition 1 *Assume Assumption 1 and* $0 \notin n_c(p, k^{-1})$ *for all* $(p, k^{-1}) \in P \times K^{-1}$.

(i) Effort and employment are strategic complements for the principal.

(ii) Expected surplus V *in (5) has strictly increasing differences on* $[0, N] \times \left(P \times K^{-1}\right)$.

(iii) n_c *and* e_c *are strongly increasing in* (p, k^{-1}).

The condition $0 \notin n_c(p, k^{-1})$ rules out a corner solution at zero employment for all possible values of the parameters. The first result records the fact that effort and employment are strategic complements for the principal as discussed previously. The second states that the expected total surplus exhibits strictly increasing differences between employment and the parameters; i.e., an increase in price p or a reduction in the transaction cost parameter k increases the expected marginal return from employment. To understand this result, it is helpful to think in terms of $\partial^2 V/\partial p \partial n \geq 0$. An increase in price raises the expected marginal return to employment because an increase in team size increases both effort (4) and expected productivity ϕ and therefore expected output, whereas a reduction in k lowers the marginal transaction cost of employment. Strictly increasing differences implies the monotone comparative statics in (iii). The optimal employment correspondence n_c is strongly increasing in the parameters (p, k^{-1}), so an increase in either or both shifts the set of optimal employment levels upwards. Since employment and effort are complements, this also produces an upward shift in the set of optimal effort levels. In the special case where the optimal employment level is unique, n_c and e_c are increasing functions in p and k^{-1}.

The following simple example illustrates these comparative statics and will be used later to compare institutions.[10]

Example 1 *If $\phi(n) = n$ there is a unique closed-form solution*

$$n_c = \frac{p^2}{2k} \quad \text{and} \quad e_c = p. \tag{9}$$

3 Moral Hazard

We now extend Becker and Murphy (1992) to the case of moral hazard. We consider two distinct institutions. The first is the firm with a budget-breaker who offers a linear contract $I_i = \alpha_i + \beta_i Q$ to agent i, where I_i is income, α_i the salary, and β_i the piece rate or incentive.[11] The second institution is an EDP where there are no lump-sum transfers and revenue and transaction costs are divided equally among partners. This is the special case where $\alpha_i = 0$ and $\beta_i = p/n$ for all i, where the latter is referred to as the *equal-division incentive.*

[10] Assume $N = \infty$ in all examples throughout the paper.

[11] The assumption of group rather than individual performance contracts may seem inconsistent in a Leontief context where individual performance measures should be readily available since production is divided into tasks assigned to specific agents and where final output entails the assembly of individual outputs in a fixed ratio. In fact, it makes no difference. If individual performance $Q_i = \phi e_i + \epsilon$ were contractible then linear contracts $\alpha_i + \beta_i Q_i$ would implement the unique equilibrium $e_i = \beta_i \phi$. Under Leontief production, it is optimal to equalize efforts and therefore incentives, which leads to the same outcome as (30). Rather than assuming group performance contracts in an EDP and individual performance contracts in the firm, we assume group performance contracts in both for the sake of consistency and simplicity.

The agents' utility functions are negative exponential

$$-\exp\left\{-r[I_i - (1/2)e_i^2]\right\}, \tag{10}$$

where $r > 0$ is the *coefficient of absolute risk aversion* (CARA) and the quadratic term is the cost of effort. The corresponding certainty equivalent payoff is given by[12]

$$U_i = \alpha_i + \beta_i \phi(n) \min_i e_i - (1/2)e_i^2 - (1/2)r\beta_i^2 \psi(n)\sigma^2. \tag{11}$$

The first two terms are expected income as a function of expected output. The final term is the *risk premium*, which is the disutility of risk imposed by the contract and the random shock ϵ. The risk premium is increasing in the CARA r, the incentive β_i (because stronger incentives link income more closely to stochastic final output), and the variance $\psi(n)\sigma^2$ of output. We refer to $r\sigma^2$ as *subjective risk* because it is baseline objective risk σ^2 scaled by the CARA.

The timing of the game is as follows. (i) The budget-breaker chooses the number n of workers and the contracts $\{(\alpha_i, \beta_i)\}$. In an EDP, the sole choice variable is partnership size n because the equal-division incentive $\beta_i = p/n$ is an inherent institutional feature. (ii) Agents who receive offers decide whether or not to accept them. We assume an outside option of zero and that indifferent agents accept. (iii) Those who accept choose their efforts non-cooperatively. (iv) Output is realized and the contract is executed. A budget-breaker chooses n to maximize her expected profits. As in Lang and Gordon (1995) and Levin and Tadelis (2005), an EDP chooses n to maximize the expected payoff of the representative partner. Our first main result characterizes the set of equilibria in efforts in stage (iii) for both institutions.

Proposition 2

(i) Given $n > 0$ and an incentive profile $\{\beta_i\}$ such that $\beta_i > 0$ for all i, there is a continuum of effort equilibria in the third stage of the game. All such equilibria are symmetric and one is Pareto dominant from the perspective of the workers.[13]

(ii) The Pareto dominant equilibrium is given by

$$e = \phi(n) \min_i \beta_i \text{ for all } i. \tag{12}$$

[12] Bolton and Dewatripont (2005, p. 137) provide a detailed derivation.

[13] I.e., all workers are strictly better off as compared with any other equilibrium.

(iii) (Legros and Matthews (1993)) In an EDP, the effort-employment relationship in the Pareto dominant equilibrium (12) is the same as the benchmark case (4).

The existence of a continuum of symmetric effort equilibria in (i) follows from the stochastic Leontief production technology. One such equilibrium is for all agents to exert zero effort. In the proof, we show that (12) is the symmetric equilibrium with the highest level of effort and there is a continuum of effort equilibria in between. From now on, we assume that all players prior to stage (iii) believe that the equilibrium in (12) will obtain because it is Pareto dominant from the perspective of the workers. This is the most reasonable assumption to make, especially in the firm where the budget-breaker also prefers (12) and, as the central party to all contracts, is in a position to provide the necessary coordination.

Under Assumption 1, equilibrium effort (12) is increasing in employment n. This follows from (11), where the expected marginal benefit of effort e_i following an increase in e_j for all $j \neq i$ is the piece rate β_i times the expected marginal product $\phi(n)$ of effort, which is increasing in team size n because of greater specialization and division of labor. This is a central insight of the paper: specialization and division of labor, and therefore employment, have a direct positive effect on effort in a moral hazard context with incentive contracting, in addition to their classical effects on productivity. Note that strictly positive incentives are a necessary condition: specialization and division of labor have no effect on effort under straight-salary compensation.

In Holmström (1982), balanced-budget sharing rules provide low-powered incentives that lead to free-riding. In contrast, in our model the equal-division incentive is high-powered in the sense that it implements the same effort-employment relationship as in the benchmark case; this can be verified by substituting $\beta_i = p/n$ into (12) to obtain (4). This result is a special case of Legros and Matthews (1993, Example B), who show that balanced-budget sharing rules achieve the first best when the production technology is Leontief and employment is exogenous. If $\phi(n)$ is strictly increasing in n then in our model optimal EDP effort is the same as the benchmark iff employment in both institutions is the same. In the next section we show that this is generally not the case, so their results are not robust to the case of endogenous employment.

4 Equal Division Partnerships

In an EDP there are no lump-sum transfers and revenue and transaction costs are shared equally across partners. Substituting $\alpha_i = 0$ and $\beta_i = p/n$ into (11) and subtracting the share of transaction costs, the expected payoff of the representative partner is

$$\frac{p}{n}\phi(n)\min_i e_i - (1/2)e_i^2 - (1/2)r\left(\frac{p}{n}\right)^2\psi(n)\sigma^2 - (1/2)kn = \tag{13}$$

$$p\Phi(n)\min_i e_i - (1/2)e_i^2 - (1/2)rp^2\Psi(n)\sigma^2 - (1/2)kn, \tag{14}$$

where $\Psi = \psi/n^2$. Substituting the Pareto dominant equilibrium effort (4),

$$U^p(n,p,k^{-1}) = (1/2)\left\{p^2\left[\Phi(n)^2 - r\sigma^2\Psi(n)\right] - (k^{-1})^{-1}n\right\} \tag{15}$$

$$= \frac{1}{n}V(n,p,k^{-1}) - (1/2)rp^2\Psi(n)\sigma^2, \tag{16}$$

where V is the expected surplus in the benchmark case (5).[14] An EDP chooses the number n of partners to maximize (16) subject to the constraint $0 \le n \le N$. A solution exists because ϕ and ψ are continuous on $[0, N]$ but as before we cannot ensure uniqueness. Let

$$n_p(p,k^{-1}) = \arg\max_{0\le n\le N} U^p(n,p,k^{-1}) \tag{17}$$

$$e_p(p,k^{-1}) = \left\{p\Phi(n) \,|\, n \in n_p(p,k^{-1})\right\}, \tag{18}$$

the optimal EDP employment and effort correspondences, respectively.

Proposition 3 *Assume* $0 \notin n_p(p,k^{-1})$ *for all* $(p,k^{-1}) \in P \times K^{-1}$.

(i) If

$$\Phi(n)^2 - r\sigma^2\Psi(n) \tag{19}$$

is increasing in n *then the expected payoff* U^p *of the representative partner exhibits strictly increasing differences in* n *and* (p,k^{-1}).

(ii) Furthermore, the correspondences n_p *and* e_p *are strongly increasing in* (p,k^{-1}).

According to (i), an increase in price p or a decrease in the transaction cost parameter k raises the expected marginal return from employment when (19) is increasing in n. This implies the monotone comparative statics in (ii). Consider an increase in p (the discussion for k is the same as before). As in the previous section, an increase in price raises the expected marginal return to employment because expected output is increasing in team size. But in this context an

[14]We do not include subjective risk $r\sigma^2$ as an argument in U^p because it is fixed throughout most of the paper for reasons discussed in the next section.

increase in price is also an increase in the equal-division incentive, which reduces the expected marginal return from employment when the risk premium is increasing in team size; i.e., when ψ increases sufficiently rapidly in n. The expected payoff of the representative partner exhibits strictly increasing differences when the former effect dominates the latter, so (19) is increasing. In particular, the result holds when ψ is decreasing because greater specialization and division of labor make monitoring easier.

We now compare optimal EDP employment n_p and effort e_p with their counterparts n_c and e_c when effort is contractible.

Proposition 4 *Assume a unique interior solution for n_c and n_p and that U^p in (15) is strictly quasi-concave and differentiable on an open interval containing n_p and n_c. (i) We have*

$$n_p \gtreqless n_c \iff r\sigma^2\Psi'(n_c) \lesseqgtr -\frac{2}{p^2 n_c^2} V(n_c, p, k^{-1}), \tag{20}$$

where V is defined in (5). *(ii) If Φ is strictly increasing then*

$$n_p \gtreqless n_c \iff e_p \gtreqless e_c. \tag{21}$$

The comparison between employment levels depends on the term $r\sigma^2\Psi'$, where Ψ' captures the effect of employment on the risk premium. Specifically, $n_p > n_c$ when $r\sigma^2\Psi'$ is sufficiently negative in the sense that $<$ obtains in the second inequality in (20) and $n_c > n_p$ otherwise. If Φ is strictly increasing then optimal effort (4) is strictly increasing in n and the same for both institutions, so the comparison between effort levels follows immediately from the comparison between employment levels.

The intuition is evident from (16). The first term on the right-hand side captures the $1/n$ effect, where the representative partner receives only $1/n$ of the expected surplus generated by the marginal hire. This biases optimal EDP employment down below the benchmark. On the other hand, if ψ does not increase too rapidly with n then $\Psi' < 0$ and the risk premium of the representative partner declines with team size. This dominates the $1/n$ effect when $<$ obtains in the second inequality in (20) and in that case an EDP is larger than the benchmark. The opposite result obtains when the difficulty of measuring performance rises sufficiently fast with employment.

This characterization of an EDP is quite different from the classical literature. In our model the central inefficiency of the EDP is not free-riding but rather distortions in the employment decision. Unlike the classical literature, where the size of the partnership is constrained by the free-rider problem, in our model an EDP can be larger or smaller than the benchmark depending on the interplay between the $1/n$ problem and risk considerations.

The following simple example illustrates most of the main ideas.

Example 2 *If $\phi(n) = \psi(n) = n$ there is a unique closed-form solution*

$$e_p = p \quad \textit{and} \quad n_p = p\left(\frac{r\sigma^2}{k}\right)^{1/2}. \tag{22}$$

In this example

$$\Phi^2 - r\sigma^2\Psi = 1 - \frac{r\sigma^2}{n} \tag{23}$$

is increasing in n so we obtain the same comparative statics for p and k as in Proposition 3. When $\psi = n$ the risk premium of the representative partner is decreasing in n and an increase in subjective risk $r\sigma^2$ leads to an increase in employment. This result, however, is sensitive to the exact specification for ψ and we would obtain the opposite result if ψ increased sufficiently fast with n. Since ϕ is the same as Example 1, we can compare the benchmark and EDP solutions. In this example $e_p = e_c = p$ so there is no shirking in an EDP. Evaluated at the optimum,

$$U^p = (1/2)p\left(p - k^{1/2}r^{1/2}\sigma\right), \tag{24}$$

so an EDP operates if $p^2 \geq kr\sigma^2$. Since

$$n_p \gtreqless n_c \iff r\sigma^2 \gtreqless \frac{p^2}{4k}, \tag{25}$$

we have the following comparison between employment levels:

$$\begin{array}{cc} 0 < r\sigma^2 < \frac{p^2}{4k} & n_c > n_p \\ \frac{p^2}{4k} < r\sigma^2 < \frac{p^2}{k} & n_p > n_c \\ r\sigma^2 > \frac{p^2}{k} & \text{the EDP shuts down.} \end{array} \tag{26}$$

5 The Firm

We now introduce a budget-breaker. The firm's profit equals revenue less the payments to workers and transaction costs

$$pQ - \sum_i I_i - (1/2)kn^2. \tag{27}$$

The budget-breaker's problem is to choose the number n of workers and the contracts $\{(\alpha_i, \beta_i)\}$ to maximize expected profit

$$p\phi(n) \min_i e_i - \sum_i \left[\alpha_i + \beta_i \phi(n) \min_i e_i \right] - (1/2)kn^2 \tag{28}$$

subject to $0 \leq n \leq N$ and the incentive compatibility (12) and participation constraints $U_i \geq 0$.

To state our main comparative statics result, we need to write expected profit as a function of employment n only. We first note that the budget-breaker chooses the lump sums α_i to make the participation constraints bind. Substituting $U_i = 0$ for all i,

$$p\phi(n) \min_i e_i - (1/2) \sum_i e_i^2 - (1/2) r\psi(n)\sigma^2 \sum_i \beta_i^2 - (1/2)kn^2. \tag{29}$$

From (12) it is clear that the optimal incentive β will be the same for each agent[15] so the incentive compatibility constraint reduces to

$$e = \beta\phi(n). \tag{30}$$

Note that an increase in employment not only increases effort but also increases the sensitivity of effort to incentives. Substituting $\beta_i = \beta$ for all i and (30) into (29),

$$p\beta\phi(n)^2 - (1/2)n\beta^2\phi(n)^2 - (1/2)nr\beta^2\psi(n)\sigma^2 - (1/2)kn^2. \tag{31}$$

The optimal incentive for any given positive employment level is

$$\beta = \frac{p\phi(n)^2}{n\left[\phi(n)^2 + r\sigma^2\psi(n)\right]} = pg(n), \tag{32}$$

where

$$g(n) = \frac{\phi(n)^2}{n\left[\phi(n)^2 + r\sigma^2\psi(n)^2\right]}. \tag{33}$$

The expression in (32) already reveals some important comparative statics information. If g is decreasing in n then employment and incentives are strategic substitutes for the budget-breaker in the sense that an increase in n implies a decrease in β for given values of the parameters. Since an increase in team size is associated with greater specialization and division of labor, the latter can also be thought of as substitutes for incentives. Alternatively, employment and incentives are

[15] If $\beta_j > \min_i \beta_i$ a reduction in β_j would reduce the aggregate risk premium with no effect on effort.

complements when g is increasing.[16] By inspection, g is decreasing in n when $\frac{r\sigma^2\psi^2}{\phi^2}$ is increasing or at least not decreasing too rapidly in n; i.e., when ϕ does not increase too rapidly relative to ψ. Intuitively, an increase in employment increases productivity, effort, and the sensitivity of effort to incentives and therefore raises the expected marginal return to increasing incentives. But if ψ is increasing then an increase in team size increases the difficulty of measuring performance and hence raises the expected marginal return to *decreasing* incentives. The relative magnitudes of these effects determine whether employment and incentives are complements or substitutes. Substituting (32) into (30),

$$e = pg(n)\phi(n), \tag{34}$$

so an increase in employment results in an increase in effort when $g\phi$ is increasing in n.

The closest results in the literature are Lin (1997) and Auriol, Friebel, and Pechlivanos (1999). In the case of deterministic output, Lin shows that the incentive which implements the first best effort is weaker when the production function is Leontief rather than linear in the agents' efforts because efforts are complementary under the former. Since Lin characterizes the Leontief technology as production with a division of labor, he interprets this result as stating that incentives are weaker under a division of labor. Likewise, in the case of moral hazard with risk averse agents he shows that the cost of implementing a given level of effort is lower for the Leontief technology. Auriol *et al.* derive an expression for the group incentive (see their equation (4)) which is quite similar to (32). In their model, ψ is constant and ϕ denotes the expected marginal product of helping effort; i.e., effort that increases the expected output of other agents. Although Auriol *et al.* are primarily concerned with other issues, from our perspective they show that group incentives are decreasing in employment when the marginal product of helping effort declines with team size due to increased communication and coordination costs.

Finally, substituting (32) into (31),

$$\Pi(n, p, k^{-1}) = (1/2)\left[p^2\phi(n)^2 g(n) - \left(k^{-1}\right)^{-1} n^2\right], \tag{35}$$

[16]These definitions of complements and substitutes can also be expressed in terms of *supermodularity* (see the aforementioned references on lattice programming and supermodular games). Let $\beta_i = \beta$ and $e_i = e$ for all i in (29) and consider the problem of choosing $e \geq 0$, $\beta \geq 0$, and n to maximize (29) subject to $0 \leq n \leq N$ and (30). We can add (32) to the set of constraints because clearly it will be non-binding. If g is decreasing but $g\phi$ is increasing in n then one can show that the set of all (β^{-1}, e, n) satisfying the constraints is a *chain*, so (29) is trivially supermodular over the restricted constraint set. A similar statement holds when g is increasing.

which is solely a function of n. The budget-breaker's optimal employment correspondence is

$$n_b\left(p, k^{-1}\right) = \arg\max_{0 \le n \le N} \Pi\left(n, p, k^{-1}\right). \tag{36}$$

From (32) and (34), the optimal incentive and effort correspondences are

$$\beta_b\left(p, k^{-1}\right) = \left\{ pg(n) \,\middle|\, n \in n_b\left(p, k^{-1}\right) \right\} \tag{37}$$

$$e_b\left(p, k^{-1}\right) = \left\{ pg(n)\phi(n) \,\middle|\, n \in n_b\left(p, k^{-1}\right) \right\}. \tag{38}$$

Theorem 1 *Assume* $0 \notin n_b\left(p, k^{-1}\right)$ *for all* $(p, k^{-1}) \in P \times K^{-1}$.

(i) If $\phi(n)^2 g(n)$ *is strictly increasing in* n *then expected profit* Π *in (35) has strictly increasing differences in* n *and* (p, k^{-1}).

(ii) Furthermore, n_b *is strongly increasing in* (p, k^{-1}).

Consider an increase in price p with the transaction cost parameter k held constant. This increases expected marginal revenue, so the budget-breaker increases employment if it increases expected output. At the optimal effort and incentive, expected output is given by $\phi e = p\phi^2 g$, hence the condition that $\phi^2 g$ is strictly increasing in n. Since ϕ is increasing, this implies that g is not decreasing too rapidly in n; otherwise an increase in employment would reduce the optimal incentive and effort to such an extent that expected output would fall. This in turn requires that ψ and therefore the difficulty of measuring performance not increase too rapidly in n. An increase in k with p held fixed raises the marginal transaction cost of employment, with the obvious comparative static implication.

Given these monotone comparative statics for employment, the comparative statics for effort and incentives follow from (32) and (34) as discussed previously. We now summarize the two main benchmarks, where employment and incentives are either complements or substitutes. Note that there are other possibilities when g is non-monotonic, but in this paper we focus on the following cases.

Corollary 1 *Assume* $0 \notin n_b\left(p, k^{-1}\right)$ *for all* $(p, k^{-1}) \in P \times K^{-1}$.

(i) If $g(n)$ *is decreasing but* $\phi(n)g(n)$ *is increasing in* n *then employment and incentives are substitutes. Furthermore,* e_b *and* n_b *are strongly increasing in* (p, k^{-1}) *and* β_b *is strongly decreasing in* k^{-1}.

(ii) If $g(n)$ is increasing then employment and incentives are complements and β_b, e_b, and n_b are all strongly increasing in (p, k^{-1}).

With one exception this is a straightforward compilation of previous results. According to Theorem 1, an increase in price p produces an upward shift in the set n_b of optimal employment levels. If ϕg is increasing in n, it also produces an upward shift in the set e_b of optimal efforts in (38). But in Corollary 1(i) the effect on incentives is ambiguous because employment and incentives are substitutes: an increase in price tends to shift β_b in (37) upwards but the corresponding increase in employment tends to shift it downwards. This explains why (i) contains no comparative statics statement about the effect of price on incentives. In summary, in (i) the endogenous variables β_b^{-1}, e_b, and n_b are complements and increasing in k^{-1}, while only e_b and n_b are increasing in p. In (ii) all the endogenous variables are complements and increasing in both parameters.

Example 3 *If $\phi(n) = \psi(n) = n$ and*

$$p^2 > kr\sigma^2 \tag{39}$$

there is a unique closed-form solution given by

$$n_b = \frac{p^2 - 4kr\sigma^2 + p\sqrt{p^2 + 8kr\sigma^2}}{4k} \tag{40}$$

$$\beta_b = \frac{4k}{p + \sqrt{p^2 + 8kr\sigma^2}}. \tag{41}$$

This example is valuable in three respects. First, it shows that specifications for ϕ and ψ exist such that g is decreasing but ϕg is increasing in n

$$g = \frac{1}{n\,(1 + r\sigma^2)}, \quad \phi g = \frac{1}{1 + r\sigma^2}, \quad \text{and} \quad \phi^2 g = \frac{n}{1 + r\sigma^2} \tag{42}$$

(recall that we use these terms in their weak senses), so employment and incentives are substitutes. Second, in this example the optimal incentive is decreasing in price whereas in Corollary 1(i) the effect was ambiguous. In contrast, the optimal incentive with one agent with a quadratic cost of effort is

$$\beta = \frac{p}{1 + r\sigma^2} \tag{43}$$

(see Bolton and Dewatripont (2005, p. 139)), so an increase in price induces an *increase* in incentives to generate more effort and expected output. But in our model the budget-breaker has a choice and instead increases employment and then reduces incentives because the two are substitutes. Finally, $r\sigma^2$ breaks this chain of complementarities between the endogenous and exogenous variables

$(\beta_b^{-1}, e_b, n_b, p, k^{-1})$ because an increase in subjective risk leads to a decrease in both incentives (as per the classical risk-reward tradeoff) and employment.[17] As discussed below, parameters such as $r\sigma^2$ cannot be used to test predictions about complementarities, which is why it has been held fixed throughout most of the paper.

We now turn to the empirical implementation of our model. This discussion relies heavily on Holmström and Milgrom (1994). We begin with some definitions. A vector x of random variables is *associated* if $\mathrm{Cov}[f(x), h(x)] \geq 0$ for all increasing real-valued functions f and h. If we take f and h to be the relevant projection maps, this implies that $\mathrm{Cov}(x) \geq 0$ (i.e., all entries of the variance-covariance matrix are nonnegative). If x is associated conditional on any event which is a sublattice then x is *affiliated*. Now consider a theoretical model with a vector y of endogenous variables and a vector $x = (x_1, x_2)$ of exogenous random parameters. After solving the model, we obtain the random variables $(y(x), x)$. If $y(x_1, x_2)$ is increasing in x_1 (but not necessarily in x_2) and x is affiliated then $(y(x), x_1)$ is associated conditional on x_2 fixed at $x_2 = \overline{x}_2$. An empirical analysis of cross-sectional data should therefore find nonnegative correlations between the random variables $(y(x), x_1)$ after controlling for x_2.

To apply these results, we assume that n_b is a function rather than a correspondence (i.e., we assume a unique optimal employment level for all possible values of the parameters). In our model, the vector of exogenous parameters is $x = (p, k^{-1}, r\sigma^2)$, which we assume is affiliated. Note that this allows for independence as well as nonnegative correlations between these parameters. Consider first Corollary 1(ii), where the endogenous variables $y = (\beta_b, e_b, n_b)$ are increasing in $x_1 = (p, k^{-1})$ but not necessarily in $x_2 = r\sigma^2$ as the above example illustrates. We should therefore find nonnegative correlations between the random variables $(\beta_b, e_b, n_b, p, k^{-1})$ after controlling for $r\sigma^2$. This would constitute evidence that employment and incentives are complements. In Corollary 1(i), the effect of price p on the incentive β is ambiguous, so we apply the general result to the endogenous variables (β_b^{-1}, e_b, n_b) with $x_1 = k^{-1}$ and $x_2 = (p, r\sigma^2)$. In this case, the fact that employment and incentives are substitutes can only be ascertained through variation in labor transaction costs after controlling for prices and the difficulty of measuring performance. For example, rising healthcare costs should have the obvious effect on employment but also increase incentives, a prediction which seems less obvious *a priori*. Of course, in a more general model with more exogenous variables there may be other parameters besides k that preserve the complementarities among these endogenous variables.

The empirical evidence on the relationship between employment and incentives is scanty, but consistent with the hypothesis that they are substitutes. Rasmusen and Zenger (1990, Lemma 1)

[17] Auriol and Friebel (p. 12) derive similar comparative statics results with respect to their parameter $r\Sigma^2$.

develop a simple theoretical model that shows that smaller firms are more efficient in a statistical sense at detecting shirking. They therefore predict that smaller firms will compensate more on the basis of performance, while larger firms will emphasize easily observable employee characteristics like tenure. Empirically, they find that the positive relationship between wages and tenure is stronger in large firms. Moreover, regressions of weekly earnings on tenure, outside experience, and education have a larger residual variance for small firms, which is consistent with the hypothesis that small firms reward performance instead. Similarly, Brown and Medoff (1989, p. 1054) find that the standard deviation of wages is smaller in large firms after controlling for the wider range of occupations. If large firms offered stronger incentives, we would expect to observe greater wage variation than in small firms.

We now compare the solutions in Examples 2 and 3.

Example 4 *Let* $\phi(n) = \psi(n) = n$ *and assume (39).*

$$\beta_b < \frac{p}{n_p} \tag{44}$$

$$n_b \gtreqless n_p \iff p^3 \gtreqless 2kr\sigma^2 \left[2p + \left(kr\sigma^2\right)^{1/2}\right] \tag{45}$$

$$e_b < e_p. \tag{46}$$

In the classical literature the partnership is an institution characterized by weak balanced-budget incentives, free-riding, and under-employment to limit shirking but in this example we obtain the opposite. The optimal incentive β_b in the firm is weaker than the equal-division incentive p/n_p in an EDP which is consistent with the well-known stylized fact that incentives are weak in firms (see Prendergast (1999)). Effort is also higher in an EDP. From (45), employment n_b in the firm can be higher or lower than n_p in an EDP depending on the parameters.

We are unaware of any other model where a partnership can be larger than a capitalist firm with the same parameters. In their preliminary analysis of a comprehensive data set of Portuguese firms, Monteiro and Stewart (2012) note that the average size of a labor-managed firm is 22 employees, versus only 10 for the average capitalist firm (excluding sole proprietorships, which tend to be even smaller). Moreover, labor-managed firms are capable of operating on a relatively large scale: up to 800 employees in their data set and as many as 3,000 in others. Our result therefore seems plausible at least in certain circumstances.

To understand this result, we compare objective functions. First, the representative partner only considers her own share of employment transaction costs, which explains why an EDP is larger than the firm in (45) when k is sufficiently large. Another difference is that the budget-breaker has two available instruments (employment and incentives) whereas an EDP has only one, which allows the budget-breaker to raise one instrument to increase expected output and reduce the other to lower the aggregate risk premium. Finally, the budget-breaker takes into account the *aggregate* risk premium whereas the representative partner only considers her own *individual* risk premium, which can bias an EDP towards over-employment. In this example, the individual risk premium in (13) is *decreasing* in n, whereas the aggregate risk premium in (31) is *increasing* in n for fixed incentives β. As a result, n_p is increasing in $r\sigma^2$ in Example 2 whereas n_b is decreasing in Example 3, so there exists a threshold value for subjective risk in (45) such that $n_p < n_b$ below the threshold and $n_b > n_p$ above.

In this example the role of the budget-breaker is quite different from that in Holmström (1982), where the budget-breaker addresses the free-rider problem in the partnership by strengthening weak balanced-budget incentives. In our model the budget-breaker is less concerned about incentives and effort (which are lower in the firm) and more about employment distortions connected to risk and transaction costs. When subjective risk or transaction costs are high an EDP is excessively large to reduce the representative partner's individual risk premium or because the representative partner does not take into account the full transaction costs of employment. In this case, the role of the budget-breaker is to downsize the EDP. When subjective risk is low the firm is larger than an EDP but with weaker incentives and the primary role of the budget-breaker is to increase productivity by promoting greater specialization and division of labor.

In the previous example, a straightforward comparison shows that n_b is less than n_c in Example 1 for all values of the parameters. But the next example shows that optimal employment in the firm can exceed that in the benchmark case.

Example 5 *If* $\phi(n) = n$, $\psi(n) = 1$, *and*

$$p^4 > 4k^2 r\sigma^2 \tag{47}$$

(i) g *is strictly decreasing in* n *on the interval* $\left[\sqrt{r\sigma^2}, \infty\right)$ *which always contains the optimum* n_b, *(ii)* ϕg *and* $\phi^2 g$ *are strictly increasing in* n, *and (iii)* $n_b > n_c$.

In this example, g is not globally decreasing in n but it is decreasing over a certain region which always contains the optimum. Employment and incentives are therefore effectively substitutes. The results in (i) and (ii) show that case (i) of the above Corollary characterizes this example. Under the condition in (47), employment is greater in the firm than in the benchmark case. The intuition is clear by inspection of the aggregate risk premium $(1/2)nr\beta^2\psi\sigma^2$. If $\psi = n$ as in the previous example, the risk premium is $(1/2)n^2r\beta^2\sigma^2$ and employment and incentives are equally effective in reducing the agents' exposure to risk. But if $\psi = 1$ as in the present example, the risk premium $(1/2)nr\beta^2\sigma^2$ is linear in employment but strictly convex in incentives. The budget-breaker therefore reduces the incentive significantly to reduce the risk premium and increases employment to increase effort, productivity, and expected output. Since employment is greater in the firm, specialization and division of labor are also greater in the firm than in the benchmark case.

Other papers have found similar results. For example, Stole and Zwiebel (1996) show that the principal over-employs relative to the neoclassical firm to reduce workers' bargaining power. But as far as we know, the only literature that predicts an excessive division of labor in capitalist firms is the Marxist literature on labor processes initiated by Braverman (1974). Braverman's main contention is that capitalists use machinery and division of labor not only to improve productivity, but also to reduce wages and wrest greater control over the production process from a union or skilled craftsmen. But in this example the negative relationship is between division of labor and *incentives* rather than *wages*. In fact, we can use the participation constraint $U_i = 0$ to solve for expected income

$$\alpha + \beta\phi e = (1/2)e^2 + (1/2)r\beta^2\psi\sigma^2. \tag{48}$$

Substituting (30) and (32) into the right-hand side,

$$\alpha + \beta\phi e = (1/2)p^2 g(n)^2[\phi(n)^2 + r\psi(n)\sigma^2], \tag{49}$$

so expected income rises with employment, specialization, and division of labor unless g is rapidly decreasing in n. In this example,

$$\alpha + \beta\phi e = \frac{n^2p^2}{2\,(n^2 + r\sigma^2)}, \tag{50}$$

which is increasing in employment.

6 Conclusion

In contract theory the production technology is usually an exogenous "black box" whose effects on incentive contracts are one-way and causal.[18] In particular, the increase in productivity flowing from specialization and division of labor has long been recognized but not the existence of feedback effects to and from incentive contracting. In this paper, these feedback effects occur through two main channels: (i) the effects of specialization and division of labor on expected productivity, effort, and the sensitivity of effort to incentives which operate through the production process and (ii) the effects of employment and incentives operating through the agents' risk premia. There are therefore important two-way links between such endogenous variables as employment, incentives, and task assignments, so one cannot legitimately study incentive mechanisms in isolation.

In this paper we focused on the relationship between employment and incentives, two important decision variables in firms that are rarely analyzed together, either empirically or theoretically.[19] We showed that whether employment and incentives are strategic complements or substitutes for the budget-breaker depends on the relative strengths of the two main channels discussed above. If (ii) dominates then employment and incentives are substitutes and if (i) does they are complements. This leads to the novel prediction that an increase in the transaction costs of employment (e.g., stricter regulation or an increase in healthcare costs) would not only have the obvious effect of reducing employment but should also lead to stronger incentives.

To understand the role of the budget-breaker, we compared the solution in the firm to those in an EDP and the benchmark case with contractible efforts. In comparison with an EDP, the role of the budget-breaker has less to do with incentives and effort as in Holmström (1982) and more to do with correcting distortions in EDP employment connected to risk and transactions costs. In comparison with the benchmark, we showed that the firm can be inefficiently large with weak incentives but an excessive degree of specialization and division of labor, in which case the firm resembles the bureaucratic organizational form described by Weber.

[18] An exception is the literature on task assignments and job design, which includes Holmström and Milgrom (1991), Itoh (1994), Schmitz (2005), and Schöttner (2009).

[19] Exceptions include Auriol *et al.* (1999) and Liang *et al.* (2008) as discussed earlier and Schaefer (1998) in the empirical literature, who finds that pay-performance sensitivity declines with firm size for teams of executives.

7 Appendix

Derivation of Equation 1. We show that the production function in (1) can be derived in a framework similar to that in Becker and Murphy (1992). The main difference between their model and ours is that here we allow for an elastic supply of effort because we are interested in incentive contracting. Consider a production process with a continuum $S = [0, 1]$ of tasks. We assume each task can be performed by at most one agent. Let $S_i \subseteq S$ be the set of tasks assigned to agent i and ρ_i the Lebesgue measure of S_i. Output in task $s \in S_i$ is given by

$$q_{is} = \Gamma(l_{is})e_{is}, \tag{A1}$$

where l_{is} is the time spent by i learning about task s, Γ is a strictly increasing function of l_{is}, and e_{is} is actual production effort. Final output is given by

$$Q = \inf_{s \in S} q_s + \epsilon, \tag{A2}$$

where q_s is output in task s and ϵ is the same as in the text. Let

$$e_i = \int_{s \in S_i} e_{is}\, ds \quad \text{and} \quad l = \int_{s \in S_i} l_{is}\, ds \tag{A3}$$

denote total production and learning efforts across all tasks in S_i, respectively.

Given fixed values for e_i and l, we first consider their optimal allocations across tasks in S_i. For any arbitrary Borel measurable e_{is} and l_{is} we have

$$\inf_{s \in S_i} \Gamma(l_{is})e_{is} \le \Gamma(l_{is})e_{is} \text{ for all } s \in S_i \implies \rho_i \inf_{s \in S_i} \Gamma(l_{is})e_{is} \le \int_{s \in S_i} \Gamma(l_{is})e_{is}\, ds. \tag{A4}$$

Consider the calculus of variations problem

$$\max_{e_{is}, l_{is}} \int_{s \in S_i} \Gamma(l_{is})e_{is}\, ds \tag{A5}$$

subject to (A3). The necessary conditions for e_{is} and l_{is} require them to be constant (see Kamien and Schwartz (1981, p. 43-47)), so $e_{is} = e_i/\rho_i$ and $l_{is} = l/\rho_i$ from (A3). It follows that

$$\inf_{s \in S_i} \Gamma(l_{is})e_{is} \le (1/\rho_i) \int_{s \in S_i} \Gamma(l_{is})e_{is}\, ds \le \Gamma(l/\rho_i)(e_i/\rho_i), \tag{A6}$$

so the optimal solution is to allocate e_i and l equally across tasks.

Consider the benchmark case with contractible efforts. Assume each i has the same fixed stock l of learning time to allocate across tasks in S_i but can increase e_i at cost $(1/2)e_i^2$. We can therefore write the principal's problem as: choose n, $\{e_i\}$, and $\{\rho_i\}$ to maximize the expected total surplus

$$p \min_i \Gamma(l/\rho_i)(e_i/\rho_i) - (1/2)\sum_i e_i^2 - (1/2)kn^2. \tag{A7}$$

For any fixed team size n, we now show that the principal can improve upon any $\{\tilde{e}_i\}$ and $\{\tilde{\rho}_i\}$ with the alternative $\hat{\rho}_i = 1/n$ (an equal division of labor) and $\hat{e}_i = E/n$ for all i, where

$$E = \sum_i \tilde{e}_i. \tag{A8}$$

Since effort costs are strictly convex, replacing $\{\tilde{e}_i\}$ with its average $\hat{e}_i$ will reduce aggregate effort costs, so we turn to its effect on expected output. As before,

$$\min_i \Gamma(l/\rho_i)(e_i/\rho_i) = \sum_i \rho_i \min_j \Gamma(l/\rho_j)(e_j/\rho_j) \leq \sum_i \rho_i \Gamma(l/\rho_i)(e_i/\rho_i), \tag{A9}$$

so we consider the problem: choose $\{e_i\}$ and $\{\rho_i\}$ to maximize

$$\sum_i \Gamma(l/\rho_i)e_i \tag{A10}$$

subject to (A8) and $\sum_i \rho_i = 1$. The first-order conditions require e_i and ρ_i to be constant, so $e_i = E/n$ and $\rho_i = 1/n$. It follows that

$$\min_i \Gamma(l/\rho_i)(e_i/\rho_i) \leq \Gamma(nl)n\hat{e}_i, \tag{A11}$$

so $\hat{e}_i$ and $\hat{\rho}_i = 1/n$ at least weakly increase expected output.

We can therefore restrict attention to the case where there is an equal division of labor and production efforts e_i are the same for all i. Note that an increase in employment n is associated with greater specialization $l/\rho_i = nl$ (more learning per task) and a more intensive division of labor $\rho_i = 1/n$ (fewer tasks per agent) as asserted in the text. The principal's problem is now to choose e and n to maximize

$$p\Gamma(nl)ne - (1/2)ne^2 - (1/2)kn^2. \tag{A12}$$

Equivalently, choose n and $\{e_i\}$ to maximize

$$pn\Gamma(nl)\min_i e_i - (1/2)\sum_i e_i^2 - (1/2)kn^2 = p\phi(n)\min_i e_i - (1/2)\sum_i e_i^2 - (1/2)kn^2, \qquad \text{(A13)}$$

where $\phi(n) = n\Gamma(nl)$. ■

Proof of Proposition 1. The problem is to choose n from the compact chain $[0, N] \subseteq \mathbf{R}_+$ to maximize (5). If we can show that (ii) holds then (iii) follows from an application of theorem 2.3 in Vives (1999). Let $(p_1, k_1^{-1}) > (p_2, k_2^{-1})$, which implies $p_1 \geq p_2$ and $k_2 \geq k_1$ with at least one strict. Since

$$V(n, p_1, k_1^{-1}) - V(n, p_2, k_2^{-1}) = (1/2)\left[\left(p_1^2 - p_2^2\right)\frac{\phi(n)^2}{n} + (k_2 - k_1)n^2\right] \qquad \text{(A14)}$$

is strictly increasing in n, we are done. ■

Proof of Proposition 2. We first note that

$$\alpha_i + \beta_i\phi e_i - (1/2)e_i^2 - (1/2)r\beta_i^2\psi\sigma^2 \qquad \text{(A15)}$$

is maximized by

$$e_i = \beta_i\phi. \qquad \text{(A16)}$$

Since expected output is Leontief, it is clear from (11) that all equilibria are necessarily symmetric. Given $e_j = e$ for all $j \neq i$, the best response for i is (A16) when $\beta_i\phi < e$ and $e_i = e$ otherwise. It follows that e is a symmetric equilibrium iff $e \leq \beta_i\phi$ for all i, or $e \leq \phi\min_i\beta_i$. Clearly, (12) is the Pareto dominant equilibrium because it is the one closest to the maximizers (A16). Since (12) is determined by (A15), we can think of the Pareto dominant equilibrium as generated by a single agent. This suggests that the Holmström and Milgrom (1987) rationale for linear contracts extends to our framework as noted in footnote 3. ■

Proof of Proposition 3. Let $(p_1, k_1^{-1}) > (p_2, k_2^{-1})$. Since

$$U^p(n, p_1, k_1^{-1}) - U^p(n, p_2, k_2^{-1}) = (1/2)\left\{\left(p_1^2 - p_2^2\right)\left[\Phi(n)^2 - r\sigma^2\Psi(n)\right] + (k_2 - k_1)n\right\} \qquad \text{(A17)}$$

is strictly increasing in n, we are done. ■

Proof of Proposition 4. Let $O \subseteq (0, N)$ be an open interval such that U^p is strictly quasi-concave on O and $n_c, n_p \in O$. It follows that U^p increases up to n_p and declines thereafter on O, so

$$n_p \gtreqless n_c \iff U_n^p(n_c, p, k^{-1}) \gtreqless 0, \tag{A18}$$

where subscripts indicate partial derivatives. Differentiating and evaluating the result at $n = n_c$,

$$U_n^p = \frac{1}{n}V_n - \frac{1}{n^2}V - (1/2)rp^2\sigma^2\Psi' = -\frac{1}{n^2}V - (1/2)rp^2\sigma^2\Psi' \tag{A19}$$

because $V_n(n_c, p, k^{-1}) = 0$. ■

Proof of Theorem 1. If $(p_1, k_1^{-1}) > (p_2, k_2^{-1})$ then

$$\Pi(n, p_1, k_1^{-1}) - \Pi(n, p_2, k_2^{-1}) = (1/2)\left\{\left(p_1^2 - p_2^2\right)g\phi^2 + (k_2 - k_1)n^2\right\} \tag{A20}$$

is strictly increasing in n. ■

Proof of Example 3. Substituting $\phi = \psi = n$ into (35),

$$\Pi = \frac{n^2\left[p^2 - k\left(n + r\sigma^2\right)\right]}{2\left(n + r\sigma^2\right)}. \tag{A21}$$

Differentiating with respect to n,

$$\Pi' = \frac{n\left[2r\sigma^2\left(p^2 - kr\sigma^2\right) + n\left(p^2 - 4kr\sigma^2\right) - 2kn^2\right]}{2\left(n + r\sigma^2\right)^2}. \tag{A22}$$

Π therefore has critical points at $n = 0$ and (40). Given (39), the quadratic expression in n in square brackets is positive at $n = 0$. It follows that Π is strictly increasing up to (40) and strictly decreasing thereafter, so (40) is the unique global maximizer. Substituting $\phi = \psi = n$ and (40) into (32), we obtain (41). ■

Proof of Example 4. Substituting n_p from Example 2,

$$\Pi'\big|_{n=n_p} = \frac{p\left[p^3 - 4kpr\sigma^2 - 2\left(kr\sigma^2\right)^{3/2}\right]}{2\left(p + \sqrt{k}\sqrt{r}\sigma\right)^2}. \tag{A23}$$

Since

$$n_b \gtreqless n_p \iff \Pi'\big|_{n=n_p} \gtreqless 0, \tag{A24}$$

(45) follows. To prove (44), substitute (41) and n_p into $\beta_b \lesseqgtr p/n_p$. After some algebra, this reduces

to $p^2 \gtreqless kr\sigma^2$. From (32),

$$e_b = \beta_b \phi = \beta_b n_b < p = e_p, \tag{A25}$$

which proves (46). ■

Proof of Example 5. Substituting $\phi = n$ and $\psi = 1$ into (35),

$$\Pi = \frac{n^2 \left[np^2 - k\left(n^2 + r\sigma^2\right)\right]}{2\left(n^2 + r\sigma^2\right)}. \tag{A26}$$

Given (47), Π has three zeros

$$n_1 = 0, \quad n_2 = \frac{p^2 - \sqrt{p^4 - 4k^2 r\sigma^2}}{2k}, \quad \text{and} \quad n_3 = \frac{p^2 + \sqrt{p^4 - 4k^2 r\sigma^2}}{2k}. \tag{A27}$$

Since the expression in square brackets in (A26) is negative at $n = 0$, $\Pi < 0$ on $(0, n_2)$, $\Pi > 0$ on (n_2, n_3), and $\Pi < 0$ on (n_3, ∞). Differentiating with respect to n,

$$\Pi' = \frac{n\left[np^2\left(n^2 + 3r\sigma^2\right) - 2k\left(n^2 + r\sigma^2\right)^2\right]}{2\left(n^2 + r\sigma^2\right)^2}. \tag{A28}$$

Let

$$\Lambda = np^2\left(n^2 + 3r\sigma^2\right) - 2k\left(n^2 + r\sigma^2\right)^2. \tag{A29}$$

Differentiating with respect to n,

$$\Lambda' = \left(3p^2 - 8kn\right)\left(n^2 + r\sigma^2\right). \tag{A30}$$

Λ therefore increases up to $n = 3p^2/8k$ and decreases thereafter. Furthermore, $\Lambda < 0$ at $n = 0$, $\Lambda > 0$ at $n = n_2$, and $\Lambda < 0$ for all n sufficiently large. Altogether, we see that Λ is at first negative, then positive, and then negative again, with zeros at n_4 and n_5, where $n_4 < n_5$. It follows that Π decreases to n_4, which is a local minimum, and then increases to n_5, the global maximum n_b, and declines thereafter. To prove (i) and (ii),

$$g = \frac{n}{n^2 + r\sigma^2} \tag{A31}$$

$$\phi g = \frac{n^2}{n^2 + r\sigma^2} \tag{A32}$$

$$\phi^2 g = \frac{n^3}{n^2 + r\sigma^2}. \tag{A33}$$

The latter two expressions are strictly increasing in n and g is strictly decreasing for all $n \geq \sqrt{r\sigma^2}$. Evaluating (A28) at $n = \sqrt{r\sigma^2}$, we obtain

$$\Pi'\Big|_{n=\sqrt{r\sigma^2}} = \frac{1}{2}\left(p^2 - 2k\sqrt{r\sigma^2}\right), \tag{A34}$$

which is positive under (47). It follows that $n_b > \sqrt{r\sigma^2}$. Evaluating (A28) at $n = n_c$, we obtain

$$\Pi'\Big|_{n=n_c} = \frac{2k^2p^2r\sigma^2\left(p^4 - 4k^2r\sigma^2\right)}{\left(p^4 + 4k^2r\sigma^2\right)^2} > 0, \tag{A35}$$

so $n_b > n_c$. ■

8 References

Alchian, A.A., Demsetz, H., 1972. Production, information costs, and economic organization. Amer. Econ. Rev. 62(5) 777-795.

Auriol, E., Friebel, G., Pechlivanos, L. 1999. Teamwork management in an era of diminishing commitment. CEPR Discussion Paper # 2281.

Bartling, B., von Siemens, F.A., 2011. Equal sharing rules in partnerships. J. Instit. Theor. Econ. 166(2) 299-320.

Becker, G.S., Murphy, K.M., 1992. The division of labor, coordination costs, and knowledge. Quart. J. Econ. 107(4) 1137-1160.

Bolton, P., Dewatripont, M., 2005. Contract Theory. MIT Press, Cambridge, Mass.

Bose, A., Pal, D., Sappington, D.E.M., 2011. On the performance of linear contracts. J. Econ. Man. Strat. 20(1) 159-193.

Bose, A., Pal, D., Sappington, D.E.M., 2010. Equal pay for unequal work: limiting sabotage in teams. J. Econ. Man. Strat. 19(1) 25-53.

Braverman, H., 1974. Labor and Monopoly Capital. Monthly Review Press, New York, NY.

Brown, C., Medoff, J., 1989. The employer size-wage effect. J. Polit. Econ. 97(5) 1027-1059.

Carter, N.M., Keon, T.L., The rise and fall of the division of labour, the past 25 years. 1986. Organ. Stud. 7(1) 57-74.

Encinosa, W.E., Gaynor, M., Rebitzer, J.B., 2007. The sociology of groups and the economics of incentives: theory and evidence on compensation systems. J. Econ. Behav. Org. 62(2) 187-214.

Garicano, L., Santos, T., 2004. Referrals. Amer. Econ. Rev. 94(3) 499-525.

Giddens, A. 1971. Capitalism and Modern Social Theory: An Analysis of the Writings of Marx, Durkheim, and Max Weber. Cambridge University Press, Cambridge, UK.

Holmström, B., 1982. Moral hazard in teams. Bell J. Econ. 13(2) 324-340.

Holmström, B., Milgrom, P., 1987. Aggregation and linearity in the provision of intertemporal incentives. Econometrica 55(2) 303-328.

Holmström, B., Milgrom, P., 1991. Multitask principal-agent analyses: incentive contracts, asset ownership, and job design. J. Law Econ. Org. 7(1) 24-52.

Holmström, B., Milgrom, P., 1994. The firm as an incentive system. Amer. Econ. Rev. 84(4) 972-991.

Itoh, H., 1994. Job design, delegation and cooperation: a principal-agent analysis. Europ. Econ. Rev. 38(3-4) 691-700.

Lang, K., Gordon, P-J., 1995. Partnerships as insurance devices: theory and evidence. RAND J. Econ. 26(4) 614-629.

Legros, P., Matsushima, H., 1991. Efficiency in partnerships. J. Econ. Theory 55(2) 296-392.

Legros, P., Matthews, S.A., 1993. Efficient and nearly-efficient partnerships. Rev. Econ. Stud. 60(3) 599-611.

Levin, J., Tadelis, S., 2005. Profit-sharing and the role of professional partnerships. Quart. J. Econ. 120(1) 131-171.

Liang, P.J., Rajan, M.V., Ray, K., 2008. Optimal team size and monitoring in organizations. The Acc. Rev. 83(3) 789-822.

Lin, Y.J., 1997. Division of labor in teams. J. Econ. Man. Strat. 6(2) 403-423.

Milgrom, P., Roberts, J., 1990. Rationalizability, learning, and equilibrium in games with strategic complementarities. Econometrica 58, 1255-77.

Monteiro, N.P., Stewart, G., 2012. New comparative evidence on labor-managed and capitalist production. Working paper.

Prendergast, C., 1999. The provision of incentives in firms. J. Econ. Lit. 37(1) 7-63.

Rasmusen, E., 1987. Moral hazard in risk averse teams. RAND J. Econ. 18(3) 428-435.

Rasmusen, E., Zenger, T., 1990. Diseconomies of scale in employment contracts. J. Law Econ. Org. 6(1) 65-92.

Schaefer, S., 1998. The dependence of pay-performance sensitivity on the size of the firm. Rev. Econ. Stat. 80(3) 436-443.

Schmitz, P.W., 2005. Allocating control in agency problems with limited liability and sequential hidden actions. RAND 36(2) 318-336.

Schöttner, A., Relational contracts, multitasking, and job design. J. Law Econ. Org. 24(1) 138-162.

Stole, L.A., Zwiebel, J. 1996. Organizational design and technology choice under intrafirm bargaining. Amer. Econ. Rev. 86(1) 195-222.

Topkis, D.M., 1998. Supermodularity and Complementarity. Princeton University Press, Princeton.

Vives, X., 1999. Oligopoly Pricing. MIT Press, London.