Early-Stage Financing and Information Gathering: An Analysis of Startup Accelerators

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Abstract

We study some of the dynamics that are introduced by a startup accelerator program in a competitive market for venture financing. These include inefficiencies that may arise when the accelerator sets an equity fee, chooses a class size, and shares information with investors, as well as efficiencies in terms of granting entrepreneurs better access to investors. We find that the accelerator chooses a class size that is too small relative to the social optimum, but this inefficiency is diminished when the accelerator hires entrepreneurs-in-residence or provides improved access to investors. When the accelerator can strategically decide what type of venture information to release to investors, we show that in order to facilitate the financing of additional program participants, it may choose to transmit only positive information. Finally, we show that when entrepreneurs perceive obtaining external funding as their main objective, an inefficient accelerator may operate in equilibrium, reducing social welfare.

Keywords: Accelerator, entrepreneurship, venture financing, information acquisition

JEL Classifications: G24, L26

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1 Introduction

In the summer of 2005, founders of the technology incubator Y Combinator pioneered a new model of incubating technology startup companies: the accelerator program. Since its early success, an increasing number of seed-stage accelerator programs has emerged in the US (e.g., Techstars, founded in 2006; Excelerate Labs, in 2009) as well as in Europe (e.g., Seedcamp, founded in 2007; Startupbootcamp, in 2010). With their growing popularity,[1] graduating from top-rated seed accelerator programs has become a powerful way to jumpstart a new venture's brand. Although accelerators have become an integral part of the startup ecosystem, they have received relatively little attention in the literature.

Business incubators in general have existed for many decades in various names and forms. For instance, the dot-com boom of the late 1990s was accompanied by a proliferation of what was then called 'network incubators' (see, for instance, Hansen et al. 2000). On the other hand, when studying financial bubbles, the literature has traditionally focused on investors' enthusiasm, neglecting the role of business incubators in supplying a steady stream of high-valuation startup companies. The objective of this paper is to study a presently-prevalent form of early-stage business incubation, focusing on the role of accelerator programs in generating information, and examining some of the market inefficiencies that may arise.

Although there is a wealth of studies on traditional business incubators and venture financing, seed accelerators exhibit some key differences. Whereas business incubators (or science parks) are typically university-based, non-profit organizations, which charge startups for office space and services, seed accelerators make an equity investment in participating startups.[2] Moreover, while business incubators are stereotypically associated with providing basic life support to startup companies, seed accelerators, on the other hand, are associated with high-profile mentors and brand reputation. Further, unlike venture-capital firms,

[1]Techstars, for instance, has an acceptance rate that is lower than one percent. See `http://www.inc.com/magazine/201204/max-chafkin/future-techstars-step-forward.html`.

[2]For instance, Y Combinator invests \$11,000 plus \$3000 per founder in exchange for usually 6-7% equity (`http://ycombinator.com/faq.html`). Other accelerator programs offer comparable base figures.

accelerators fund cohorts or classes of startups rather than one venture at a time.

At the end of an accelerator program, typically, all companies in the same class demonstrate their products to angel investors and early-stage venture-capital firms.[3] Hence, the program can provide a validation benefit to successful graduates. However, it is far from clear whether the accelerator's behavior as an information hub, in terms of gathering and releasing data about program participants, is efficient from the standpoint of social welfare. What appears to be true is that only when sufficiently many graduating startups raise further funding and remain active, will the accelerator be profitable from equity investments. Hence, founders of accelerator programs have a clear incentive to behave strategically when accepting, collecting, and releasing information about program participants.

The framework in this paper is geared towards studying the above dynamics. Our model consists of entrepreneurs (ventures), an accelerator program, and investors. Entrepreneurs may participate in the accelerator program prior to seeking funding, or they can approach investors directly. The benefit from participating is that the accelerator is able to obtain and credibly reveal a signal about the viability of ventures, where, due to increased noise and diminished availability of one-on-one mentorship, the quality of its signals decreases in the number of participants. The accelerator charges entrepreneurs an equity fee, and, after signals are revealed, entrepreneurs attempt to raise a financing round. We extend our base model to consider commitment mechanisms, investor-networking benefits to participants, selective information disclosure about participants, and control-motivated entrepreneurs.

We begin our analysis by studying the class size of participating ventures that an accelerator would choose. The class size of some accelerator programs has in fact been on the rise. For instance, Y Combinator accepted eight startups in 2005, 19 in 2007, 25 in 2009, and 63 in 2011 (Christiansen 2009). One potential reason for this increase, which is supported by our findings, is that seed accelerators suffer from a time-inconsistency problem.

[3]The follow-on funding rate is fairly high, but not all participants successfully graduate. See, for instance, `http://www.huffingtonpost.com/2011/06/09/y-combinator-harvard-silicon-valley_n_874245.html`.

Once an applicant enlists in the program, the accelerator tends to validate too many startups, which lowers an individual firm's validation benefit in fundraising. Thus, the program would choose a larger class size than it would initially indicate to applicants.[4] As long as entrepreneurs fully understand the accelerator's incentives, the class size is larger than the profit-maximizing level. This result is reminiscent of the classical time-inconsistent behavior of central monetary authorities (e.g., Kydland and Prescott 1977; Barro and Gordon 1983). However, whereas in the literature on time inconsistency, social welfare is lower in a rational-expectations equilibrium, the time-inconsistency problem faced by an accelerator program in our model is beneficial from the standpoint of social welfare.[5]

We proceed to incorporate the recent trend of accelerator programs hiring entrepreneurs-in-residence. We argue that this practice can be understood as a solution to the accelerator's time-inconsistency problem. That is, by having experienced entrepreneurs operating in-house in parallel to the accelerator's program, accelerators can achieve the commitment solution, increasing their profits. We next consider entrepreneurs' cost of communicating with investors without participating in the accelerator's program. We show that as this cost increases, the accelerator's chosen class size moves closer to the social optimum. We further show that the accelerator may choose to share only positive information with investors in order to facilitate the financing of more venture participants. Our findings here offer an explanation for why so many graduating participants are funded. We then proceed to study an inefficiency that is introduced when entrepreneurs set fundraising as their objective rather than maximizing the value of their ventures.[6] With this behavioral bias, an accelerator that promotes inefficient information acquisition may exist, reducing social welfare.

The outline for the paper is as follows. Section 2 discusses the relevant literatures. Section 3 sets forth our basic model. Section 4 characterizes the equilibrium and contrasts

[4]This time-inconsistency problem faced by the accelerator program is similar to the incentives faced by younger venture capital firms, which tend to take companies pubic earlier than older venture capital firms do in order to establish a reputation and successfully raise capital for new funds (Gompers 1996).

[5]In a context of monopoly R&D investments, Waldman (1996) shows that the time-inconsistency problem that induces a monopolist to overinvest is also beneficial from the standpoint of social welfare.

[6]See, for instance, `http://techcrunch.com/2011/07/06/raising-most-money-most-valuable/`.

it against two benchmarks, a commitment benchmark and the socially-optimal solution. Section 5 extends the analysis in two ways by incorporating i) entrepreneurs-in-residence, and ii) access benefits to participants. Section 6 consists of two further considerations: i) selective disclosure of signals, and ii) entrepreneurial misperceptions of funding. Section 7 offers concluding remarks.

2 Relevant Literature

The literature has generally suggested that a third-party certifier can play a beneficial role in markets that are characterized by asymmetric information (e.g., Viscusi 1978; Biglaiser 1993; Lizzeri 1999). In particular, prior literature has provided support for the claim that the certification by venture capitalists lowers a firm's costs of going public (Megginson and Weiss 1991), and startup companies are willing to pay a premium in order to be affiliated with reputable venture-capital firms (Hsu 2004). Here, venture-capital certification serves as an antecedent to the role of seed accelerators in the early stage; however, one difference is that in our analysis, the value of certification does not arise (solely) due to asymmetric information.

Instead, in our model, information about entrepreneurs is initially incomplete and symmetric. The accelerator acquires information about participants in the program and truthfully reveals information to all investors.[7] Thus, the value of the accelerator's certification is to generate public information rather than to reduce informational asymmetry (although we consider private information in Section 6.1). This public signal helps attract investors when entrepreneurs successfully graduate from the accelerator's program. Hence, our paper provides another rationale for the use of staged financing, where staging has certification value rather than serving to control agency problems (e.g., Gompers 1995; Bergemann and

[7] In other words, we basically assume that signals are 'hard' information. If instead signals are not verifiable, then the accelerator becomes privately better informed than the investors. In staged financing, this can lead to other potential biases in the accelerator's behavior. While we think this is less likely an issue for startup accelerators, see Admati and Pfleiderer (1994) and Kim and Wagman (2012) for studies on the signaling by inside investors.

Hege 1998; Wang and Zhou 2004).[8]

Another literature relevant to our study is the literature on financial intermediation and, in particular, the literature focused on the role of intermediaries in providing credible information (e.g., Leland and Pyle 1977; Allen 1990).[9] Campbell and Kracaw (1980) show that intermediaries can profitably emerge where they jointly produce information and other products or services valued by investors. Relatedly, Chan (1983) considers competitive intermediation, where informed agents can increase the welfare of investors by reducing their adverse-selection problem. The role of accelerators in our paper is similar; however, we study the link between the accelerator's class size and the quality of information, focusing on sources of potential (in)efficiencies.

Finally, another related literature is the literature on the optimal portfolios of firms and venture-capital industry equilibria. For instance, Kanniainen and Keuschnigg (2004) consider a venture-capital firm that finances and advises a number of portfolio companies and must allocate limited managerial capacity across firms to maximize profit. They focus on the tradeoff between portfolio size and the quality of advice, and show that the profit-maximizing portfolio size and advice quality are both smaller than what would maximize social welfare. The findings obtained from our base model are similar; however, the difference is that we study a dynamic inconsistency problem that biases upwards the size of accelerator programs, as well as related sources of inefficiencies, where entrepreneurs have the alternative option of approaching investors directly.

[8] The paper in this literature closest to ours is by Ueda (2004), where a venture capitalist can evaluate the entrepreneur's project more accurately than a bank, but can also steal the entrepreneur's idea. In contrast, our analysis focuses on the quality of signals transmitted to investors, and not on expropriation risks.

[9] In this literature, an informed individual faces a reliability problem when he tries to sell his private information, whereas an intermediary can credibly signal its informed status by investing its wealth in assets. The accelerator program in our model is similar except that it earns profits indirectly when buyers of information (investors) make investments in program participants.

3 Model

Consider a world inhabited by a large number of prospective entrepreneurs. Each entrepreneur chooses between applying for an accelerator program and approaching investors directly to finance a new venture. Suppose that the desired amount of funding is $F > 0$. All parties are assumed to be risk neutral. The investment market is assumed competitive; that is, an investor's expected profit from a share of the firm's equity will be zero. Motivated by the scant availability of such programs, particularly in comparison to the number of applications they receive, we consider a situation where there is one (monopsony) accelerator program in the market.

There is uncertainty associated with the success of a new venture. We assume that a successful venture generates revenues v_H and an unsuccessful venture generates revenues v_L, where $v_H > v_L \geq 0$. An entrepreneur i's venture is one of two types, $\theta_i = L$ or $\theta_i = H$, where a type L succeeds with probability λ_L and a type H succeeds with probability λ_H, where $\lambda_H > \lambda_L$. Information is initially assumed symmetric, where the common prior probability that an entrepreneur's venture is of type H is given by $\gamma > 0$. If entrepreneurs had better information about the viability of their ventures, asymmetric information could exist in equilibrium; however, given the pre-seed focus of accelerators, our focus is on information gathered during the accelerator program.[10]

The accelerator chooses how many new ventures to support in a given class, and subsequently acquires information about participants through close monitoring. The marginal cost to support a participant is given by $c > 0$. That is, c includes the cost of providing shared office space and subsistence funding to the participants. Hence, the required capital F indicates funding required beyond what the accelerator may provide. For technical sim-

[10] That is, our focus is on information that is transmitted to investors during entrepreneurs' participation in the accelerator's program, as opposed to the accelerator's program acting as a facilitator for signaling participants' *ex ante* private information. In fact, some participants in accelerator programs change their venture ideas during the program, suggesting that screening *ex ante* private information is not a key component of the potential value added by such programs. Y Combinator, for instance, encourages applications from inspiring entrepreneurs who do not yet have an idea (`http://ycombinator.com/noidea.html`). We consider the possibility of asymmetric information between the accelerator and investors in Section 6.1.

plicity and at no qualitative loss, we normalize the funding gap between what an accelerator may provide relative to outside investors (prior to raising F) to 0. Thus, entrepreneurs would have to raise the same amount of funding F when approaching investors directly without accelerator participation.

The main feature of the information-acquisition technology is that due to limited availability of time and attention, a larger class size in the program entails a noisier signal regarding venture type. Specifically, the accelerator's information technology for gathering investor-pertinent information works as follows. By monitoring program participants, the accelerator program generates a public signal $\sigma_i \in \{0, 1\}$ about each participant i, such that $Pr(\sigma_i = 1|\theta_i = H) = 1$ and $Pr(\sigma_i = 1|\theta_i = L) = \alpha(n)$, where n denotes the participating class size. Here, $\alpha(n)$ denotes a false positive signal and hence the level of noise in obtaining information, which is non-decreasing in n.

Investors subsequently update their beliefs regarding the viability of participants' ventures.[11] To emphasize the informational advantage of participating in the accelerator's program, we assume that aside from the public signal revealed by the accelerator, investors do not obtain new information about program participants. One motivation for this is that a large amount of information is only released at the end of an accelerator's program (during what is often referred to as Demo Day), when participants present their ventures to investors. Relaxing this assumption does not change the qualitative nature of the results, but comes at significant added complexity.

Entrepreneurs can alternatively choose not to participate in the program and to approach investors directly. (At this point, there is no explicit cost associated with approaching investors directly — we introduce such a cost in Section 5.2). If an entrepreneur approaches investors directly, we assume that all investors receive a signal according to the same information technology as the accelerator's, except with noise level $\alpha(k)$, where k is some constant.

[11]We assume that all investors update their beliefs symmetrically. This assumption is without a loss of generality — without it, Aumann (1976)'s result on common knowledge applies, where information (beliefs) subsequently become symmetric in the competitive-investment stage.

Hence, investors' information-acquisition technology is noisier than the accelerator's, as long as the latter's program supports a smaller number of participants than k.[12] To emphasize the informational benefit of participating in the accelerator's program, we initially assume that on the relevant range on n, to be formalized shortly, $k > n$ is satisfied.

We make a technical assumption to induce interior solutions and, therefore, to permit a meaningful role of the accelerator's program. To be specific, $\alpha(n)$ is twice differentiable and satisfies $2\alpha'(n) + n\alpha''(n) \geq 0$ for $n \leq k$. This assumption has the interpretation that a decrease in the precision of the accelerator's signal from additional participants is either decreasing moderately or is increasing. The assumption is satisfied by common functional forms such as $\alpha(n) = \sqrt{n}$ and $\alpha(n) = ln(n)$, and by any convex function (since accuracy is then reduced at an increasing rate).

We initially assume that ventures for which a negative signal is obtained are not financed. (When we consider asymmetric information in Section 6.1, some ventures that received negative signals do receive funding.) Thus, if only a positive signal is obtained for a venture, then investors make financing offers, which entrepreneurs accept or reject. When indifferent between approaching an investor directly and participating in the accelerator's program, entrepreneurs prefer to participate. This tie-breaking assumption has no qualitative impact on the results because the accelerator's equity fee is a continuous variable.

Except in Section 6.2, we allow entrepreneurs to be fully rational and understand the accelerator's incentives for choosing its class size given the features of its program. The solution concept we employ is a rational-expectations equilibrium, where all agents use backward induction. We let s denote the equity fee (i.e., the share of the venture's equity) that the accelerator requires from each program participant, and let n^e denote the entrepreneur's expectation of the class size when they apply to the accelerator's program.

[12] For simplicity, we 'group up' investors' information-acquisition process. A sufficient and less restrictive setup, which ensures competitive investment rates and gives the same results, requires that an entrepreneur approaches at least two distinct investors who observe the same information about the entrepreneur's venture according to $\alpha(k)$, and other investors learn if this signal turned out negative (i.e., if the venture turned out to be of type L). Such information sharing is motivated by investors' tight networks (e.g., `angellist.com`, `foundersden.com`), and by the common syndication of seed investments (Kerr et al., 2011).

The timing of the game is as follows. First, the accelerator announces the terms of the program (i.e., its equity fee s). Second, entrepreneurs simultaneously decide whether to apply or to approach investors directly. Third, the accelerator selects a subset of applicants to participate, which determines its class size. Fourth, the accelerator reveals a public signal about participants to investors. Investors who were approached directly by entrepreneurs obtain signals about the viability of their ventures. Finally, investors make competitive financing offers to ventures that received positive signals (either from investors directly or from the accelerator's program), which entrepreneurs accept or reject.

4 Characterizing Equilibrium

The model is solved backward starting with the final stage where investors make financing offers. Given that only ventures for which positive signals were obtained are financed, the investor's problem is straightforward. Following a positive signal, the posterior probability that a venture is of type H is given by

$$\tilde{\gamma}(n) = \frac{\gamma}{\gamma + (1-\gamma)\alpha(n)}.$$

Thus, the expected return to a venture i following the accelerator's positive signal is

$$E[v|\sigma_i(n) = 1] = (\tilde{\gamma}\lambda_H + (1-\tilde{\gamma})\lambda_L)v_H + (\tilde{\gamma}(1-\lambda_H) + (1-\tilde{\gamma})(1-\lambda_L))v_L.$$ [13]

Substituting for $\tilde{\gamma}(n)$ and simplifying yields

$$E[v|\sigma_i(n) = 1] = v_L + \lambda_L(v_H - v_L) + \frac{\gamma(\lambda_H - \lambda_L)(v_H - v_L)}{\gamma + (1-\gamma)\alpha(n)}. \quad (1)$$

Hence, in exchange for financing an amount F in a competitive investment market, given a

[13] We drop n in the RHS of the equation for notational simplicity. The derivation is analogous when the entrepreneur goes directly to investors, in which case n is replaced by k.

positive signal, an investor receives a share $F/E[v|\sigma_i(n)=1]$ of the venture.[14]

An entrepreneur's share is diluted both from raising the financing round as well as from participating in the accelerator's program. Specifically, the entrepreneur's expected payoff from going through a program that has a participating class of size n is

$$(\gamma + (1-\gamma)\alpha(n))(1-s)(1 - \frac{F}{E[v|\sigma_i(n)=1]})E[v|\sigma_i(n)=1], \tag{2}$$

whereas his payoff from going directly to investors is given by

$$(\gamma + (1-\gamma)\alpha(k))(1 - \frac{F}{E[v|\sigma_i(k)=1]})E[v|\sigma_i(k)=1]. \tag{3}$$

Breaking indifference in favor of the accelerator's program, an entrepreneur participates in the program if and only if the accelerator's equity fee, s, is less than or equal to a threshold:

$$s_Q(n) = 1 - \frac{(\gamma + (1-\gamma)\alpha(k))(E[v|\sigma_i(k)=1] - F)}{(\gamma + (1-\gamma)\alpha(n))(E[v|\sigma_i(n)=1] - F)}. \tag{4}$$

It can be readily shown that as the number of participants, n, grows, the equity fee threshold diminishes, that is, $s'_Q(n) < 0$. This is because a larger class size entails a less precise signal from the program, diminishing the program's value to participants.

Assuming that the program receives a large number of applicants (which holds in equilibrium), the accelerator chooses a class size n, given an equity fee s per participant,[15] to maximize its expected profit:

[14] Even if investors (and the accelerator) employ convertible debt as the investment instrument, the debt would be converted into equity in subsequent financing rounds (which we do not model in this paper), at which point the financial implications are analogous. Hence, to capture the returns in a tractable manner, we assume that investors acquire shares in the present round. However, we note that a number of authors have proposed alternative explanations of the widespread use of convertible securities (see, e.g., Bascha and Walz 2001; Cornelli and Yosha 2003; Schmidt 2003).

[15] One might wonder if the equity fee can be renegotiated after n is chosen. However, such renegotiation is difficult because accelerators decide who to fund after each day of interviews and thus entrepreneurs do not know the actual class size or how many have already accepted their offer, when they decide whether to accept the program's offer. Moreover, equity fees may be standardized across participants and non-negotiable. See, for instance, `http://www.thevalleybyte.com/2012/05/09/y-combinator-the-index-fund-of-incubators/`.

$$\sum_{i=1}^{n} Pr(\sigma_i = 1)s\left(1 - \frac{F}{E[v|\sigma_i(n) = 1]}\right)E[v|\sigma_i(n) = 1] - nc. \tag{5}$$

In the following, it would be convenient to denote the expected value of a venture (net of financing F) as $V(n) = (\gamma + (1-\gamma)\alpha(n))(E[v|\sigma_i(n) = 1] - F)$, so that the accelerator's maximization problem is specified by

$$\max_n nsV(n) - nc \tag{6}$$

$$\text{s.t.} \quad s \leq s_Q(n).$$

At the time of application, the equity fee threshold, $s_Q(n)$, depends on entrepreneurs' expectations about the accelerator's class size. Thus, the accelerator optimally responds by setting $s = s_Q(n^e)$, which maximizes its equity fee given n^e. In equilibrium, the accelerator chooses the class size n taking into account entrepreneurs' expectations as well as the unique equilibrium of the ensuing subgame. It follows that (6) can be rewritten as

$$\max_n ns_Q(n^e)V(n) - nc. \tag{7}$$

The profit-maximizing class size, n^*, is then specified by the first-order condition:

$$s_Q(n^e)(V(n^*) + n^*V'(n^*)) = c. \tag{8}$$

The above equation characterizes how the accelerator chooses n^* in response to entrepreneurs' anticipated class size, n^e. In particular, there is a negative relationship between n^* and n^e, whereby the larger entrepreneurs' expectations about the class size, the smaller the class size chosen by the accelerator will be. The reason is that when entrepreneurs expect the class size to be larger, they anticipate that the signal generated by the accelerator will be noisier; consequently, entrepreneurs will be willing to pay a smaller equity fee for participating in the accelerator's program. Given its smaller share of venture firms, the accelerator's expected payoff from operating the program is proportionally diminished, leading the accelerator to choose a smaller class size.[16]

What expectations will entrepreneurs have in equilibrium? In a rational-expectations equilibrium, what entrepreneurs expect should match what the accelerator program ends up choosing. That is, rational-expectations equilibria require $n^* = n^e$. The first-order condition that characterizes the optimal n^* is thus given by

$$s_Q(n^*)(V(n^*) + n^*V'(n^*)) = c. \tag{9}$$

4.1 Commitment Benchmark

To highlight the time-inconsistency problem faced by the accelerator, let us suppose that the accelerator were able to commit to a class size before entrepreneurs apply. That is, due to reputational concerns, the accelerator may choose not to exercise its discretion to choose a class size that is different from its initial announcement. Then the accelerator's problem for choosing an optimal class size is given by

$$\max_n ns_Q(n)V(n) - nc.$$

Let n° denote the accelerator's optimal class size when commitment is possible. The first-order condition that specifies n° is given by

$$s_Q(n^\circ)(V(n^\circ) + n^\circ V'(n^\circ)) + n^\circ s'_Q(n^\circ)V(n^\circ) = c. \tag{10}$$

[16]We note that this does not imply that the accelerator always chooses a smaller class size than what entrepreneurs expect. Since the reaction function is downward sloping, there is in fact a threshold value of n^e, below (above) which the profit-maximizing class size n^* would be larger (smaller) than entrepreneurs' expectations, n^e.

The following proposition provides a comparison between the accelerator's equilibrium class size and benchmark (commitment) solution.

Proposition 1 *When entrepreneurs have rational expectations over class size, the accelerator chooses a larger class size than when it can commit to a class size in advance (i.e., $n^* > n^\circ$). The accelerator's profit is lower relative to the commitment outcome.*

The proofs of Proposition 1 and all proceeding results are relegated to the Appendix. From the above proposition, it is clear that the accelerator suffers from a time-inconsistency problem when choosing its class size. That is, if the accelerator cannot credibly commit to a class size in advance, then the equilibrium class size tends to increase, and the accelerator's profit is lower. The accelerator's time-inconsistent behavior arises because the program gains only from ventures that are subsequently funded, so the accelerator desires a larger number of graduates who receive positive signals. At the same time, a larger number of participants leads to a greater number of false positives in the program's certification, which reduces the program's value to entrepreneurs. Proposition 1 shows that under the model's assumptions, the former incentive dominates the latter in equilibrium.

4.2 The First-Best Class Size

In this subsection, we examine how the welfare-maximizing class size compares with the accelerator's choice. That is, we describe the solution to the planner's problem who decides how many entrepreneurs participate in the accelerator's program (all other aspects of the game remain the same). Since equity fees for participating in the accelerator's program simply constitute transfers, the class size $\hat{n}$ that leads to the highest social surplus solves the following:

$$\max_n nV(n) - nc.$$

Let $\hat{n}$ denote the socially optimal class size, where we assume that $\hat{n} < k$. The first-order condition that characterizes $\hat{n}$ is given by

$$V(\hat{n}) + \hat{n}V'(\hat{n}) = c. \tag{11}$$

A comparison of the two first-order conditions, (8) and (11), shows that the derivative of social welfare with respect to n is strictly greater than the derivative of the accelerator's profit, since $s_Q(n) < 1$. Below, $SW(n)$ denotes the level of social welfare when the program size is n.

Proposition 2 *Let n°, n^*, and $\hat{n}$ denote the commitment, no commitment, and welfare maximizing equilibrium class sizes, respectively. Then the following inequalities are satisfied: (i) $n^\circ < n^* < \hat{n}$, (ii) $SW(n^\circ) < SW(n^*) < SW(\hat{n})$.*

Proposition 2 tells us that the time-inconsistency problem that arises due to the accelerator's incentive to increase its class size serves to increase social welfare. The reason is that the accelerator's private incentive to accept entrepreneurs into the program is too small. That is, given that the program has a superior technology to acquire information about entrepreneurs, it is socially optimal to have as many entrepreneurs go through this certification process; however, as the class size increases, the equity fee the accelerator can charge becomes increasingly small, driving a wedge between the accelerator's chosen class size and the welfare-maximizing class size. It thus follows that entrepreneurs' rational expectations, given the accelerator's inability to commit, are *ex ante* beneficial from a welfare standpoint, despite the accelerator's profit being lower in equilibrium relative to the commitment case.

Although the above analysis specifically focuses on the certifying role of seed-stage startup accelerators, our results shed some light on the supply-side of the late 1990s dot-com bubble as a potential corroborating factor. Specifically, the so-called network incubators marked the era preceding the dot-com bubble, the role of which was similar to that of the seed accelerators studied in this paper. The above propositions imply that from the standpoint of social welfare, these network incubators may have produced a relatively small number of ventures that received positive signals (or certification) and whose valuations were relatively high. That is, by supporting a smaller than optimal number of ventures, incubators generated higher quality signals. Given a competitive capital market, rational investors had an incentive to invest in such ventures at a steep price, as indeed took place.[17]

5 Extensions

In this section, we consider two extensions of our base model. First, we introduce the possibility that at the beginning of the game, the accelerator can hire entrepreneurs-in-residence. Second, we consider a case where entrepreneurs incur a cost if they approach investors directly.

5.1 Commitment Device

The success of the seed-stage accelerator, Y Combinator, which matriculated its first class of participants in 2005, facilitated the establishment of a number of other accelerators, such as TechStars, founded in 2007. Y Combinator and TechStars, however, have different trajectories for their matriculating class sizes, and seem to follow different strategies in dealing with their time-inconsistency problems. In particular, Y Combinator has been steadily increasing its class size. For instance, 43 venture groups participated in 2011, and the number has recently increased to 63 in 2012. On the other hand, TechStars has kept its class size limited to 10 in each of its four US locations. Below, we interpret these variations in the context of the above analysis.

The Folk Theorem suggests that in a repeated, dynamic framework, it may be possible for an accelerator to establish a reputation that prevents time-inconsistent behavior. Thus, given the result that the accelerator's profit is higher if it could credibly commit to a class

[17]Granted, our model only takes the perspective of incubators as certifying venture quality, and does not account for a myriad of potential other reasons for steep venture valuations.

size, TechStars' strategy to maintain a relatively small class size can be interpreted as an attempt to prevent its time-inconsistent behavior. However, even in a repeated setup, the accelerator may choose to deviate if the profit gap is sufficiently high.

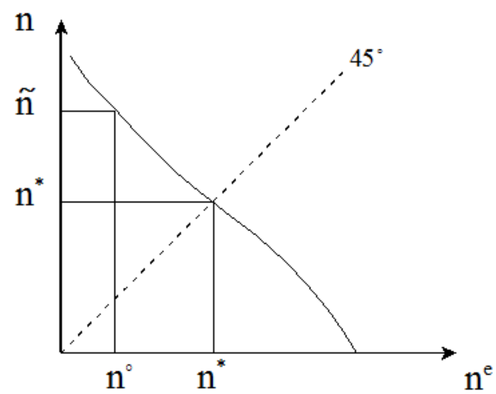

Figure 1: Accelerator's best-response as a function of entrepreneurs' expectation n^e.

Figure 1 illustrates the accelerator's best response to entrepreneurs' expectations, obtained from the first-order condition $s_Q(n^e)(V(n^*) + n^*V'(n^*)) = c$. The location where the 45 degree line goes through the best-response function constitutes the rational-expectations equilibrium. Given $n^\circ < n^*$, if entrepreneurs' expectations were $n^e = n^\circ$, then the best deviation strategy for the accelerator would be to choose $\tilde{n}$ that is even higher than n^*. This is because if entrepreneurs are willing to pay a large equity fee expecting a small class size, then the accelerator can gain by increasing its class size even further. In the case of Y Combinator, due to its early success and limited competition, the incentive to expand its class size is significant.

Recently, Y Combinator announced that it will introduce an entrepreneurs-in-residence program.[18] Such programs are not new to business incubation. Venture-capital firms have traditionally retained experienced founders to provide in-house expertise in the due-diligence process as well as to find new potential investments. Often times, entrepreneurs-in-residence work on launching their own new ventures, while they are paid a stipend and provided with

[18]See, for instance, `http://ycombinator.com/noidea.html` and `http://techcrunch.com/2012/03/13/a-good-idea-y-combinator-now-lets-founders-apply-without-one/`.

office space. In return, their hosting accelerator receives priority in financing their ventures. (In a related fashion, as mentioned previously, Y Combinator proposed to have a separate track for entrepreneurs who do not yet have a business idea.)

Let us extend our analysis in the previous section by allowing the accelerator to hire m entrepreneurs-in-residence (EIRs) before potential participants apply to the program. For simplicity, we assume that the marginal cost of retaining an EIR is the same as that for regular program participants (e.g., the accelerator may be providing a similar stipend and office space). However, EIRs do not have to come up with their venture ideas by the end of the program, so they have more time and access to the accelerator's resources. In other words, the values of their ventures are not tied to the program characteristics such as its class size. For simplicity, let us suppose that the expected net value of an EIR's venture is a constant, $U > 0$, rather than $V(n)$. Further, suppose that the accelerator sets standardized equity fees to all participants.[19]

Proposition 3 *Assuming that it is profitable to have a strictly positive number* $m > 0$ *of entrepreneurs-in-residence (EIRs), there is a value* $m^* > 0$ *such that if the accelerator hires* m^* *EIRs, then it avoids the time-inconsistency problem in the choice of its class size. That is, the accelerator chooses a commitment-level class size of* n° *in equilibrium.*

The logic underlying this result is as follows. EIRs are chosen in addition to an accelerator's class, and the equity fee that EIRs are willing to pay at a later date depends on the chosen class size, rather than on its expectation. Since the equity fee is a decreasing function of the class size, the profit from EIRs at a later stage works to discipline the accelerator to not expand its class size.

Granted, there are likely other important motivations for having EIRs on board (we explore one in the next subsection that focuses on the added value of participating in the

[19]In a world of incomplete contracts, such non-discrimination policy is straightforward to implement and hence serves as a benchmark. Further, this assumption is supported by the fact that Y Combinator allows their entrepreneurs-in-residence to switch status to normal program participants under the same terms and conditions as those for regular program participants.

accelerator's program). One motivation detailed above is that EIRs constitute a commitment device for the accelerator, credibly signaling applicants that the accelerator will not increase the program's class size, since doing so erodes expected profits from EIRs' ventures, thus hurting the accelerator's bottom line.

5.2 Cost of Training and Access

It is well known in the literature that prominent venture-capital firms draw on their networks of service providers and perform a variety of professionalization services beyond simple financial intermediation (e.g., Sahlman 1990; Hellmann and Puri 2002; Hochberg et al. 2006). Some seed-stage accelerators boast large alumni networks where participants are encouraged to seek knowledge and advice. Further, their Demo Days, where a multitude of investors gather to see graduates present their ideas, garner considerable attention. It follows that entrepreneurs who do not participate in accelerator programs may very well be at a disadvantage in gaining access to investors.

In this subsection, we consider how the equilibrium of our model changes in response to a cost asymmetry in gaining access to investors. We extend the base model in a single dimension: entrepreneurs incur a cost $c_e > 0$ when seeking to gain access to potential investors without program participation — a cost that they can avoid by participating in the accelerator's program. This cost can also incorporate the value of any training entrepreneurs acquire when they participate in the accelerator's program. Given the cost c_e, an entrepreneur's expected payoff from going through the accelerator's program is given by

$$(\gamma + (1-\gamma)\alpha(n))(1-s)(1 - \frac{F}{E[v|\sigma_i(n)=1]})E[v|\sigma_i(n)=1], \tag{12}$$

whereas his payoff from going directly to investors is

$$(\gamma + (1-\gamma)\alpha(k))(1 - \frac{F}{E[v|\sigma_i(k)=1]})E[v|\sigma_i(k)=1] - c_e. \tag{13}$$

It follows that the accelerator is able to set a higher equity fee, specified by:

$$s_Q(n) = 1 - \frac{(\gamma + (1-\gamma)\alpha(k))(E[v|\sigma_i(k) = 1] - F) - c_e}{(\gamma + (1-\gamma)\alpha(n))(E[v|\sigma_i(n) = 1] - F)}. \tag{14}$$

The accelerator's share then, in effect, expropriates fees from entrepreneurs for (i) generating more accurate signals regarding their ventures' chances of success, and (ii) for providing training and for introducing entrepreneurs to investors. The following result characterizes how a higher access cost, c_e, influences the accelerator's choice of its class size, and social welfare, provided that this cost is not deadweight loss (that is, when this cost is transferred as surplus to another party).

Proposition 4 *A higher access cost c_e leads the accelerator to accept a larger class size n^* and thus brings the outcome closer to the social optimum.*

Proposition 4 shows that a higher cost of training and gaining access to potential investors for nonparticipants increases the efficiency of the accelerator's program. The logic follows from the fact that entrepreneurs become more dependent on accelerators for receiving training and for meeting potential investors, which enables the accelerator to increase its class size without significantly diminishing its equity fees (if at all). This gives rise to the prediction that, other things being equal, programs that provide more networking and training opportunities to participants would tend to have larger classes and lower signal qualities. We further note that while the accelerator's signaling efficiency improves as a result of $c_e > 0$, from the perspective of entrepreneurs who do not participate in the program, this gain comes at the cost of incurring c_e. Hence, if this cost entails a deadweight loss in surplus, the net impact on social welfare is ambiguous.

6 Further Results

In the above, we presented a model and extensions focusing on the relationship between the accelerator's class size and the accuracy of its certification feature. In this section, we discuss other potential sources of inefficiencies when the accelerator operates in a noisy environment, and when entrepreneurs' main objective is to successfully raise funding rather than maximizing the value of their expected venture equity.

6.1 Asymmetric Information and Selective Signaling

In this subsection, we allow for the possibility of asymmetric information between the accelerator and investors. Specifically, let us assume that the accelerator privately learns whether each participant receives a positive or a negative signal, and then decides which signal(s) to release to investors. (We set $c_e = 0$ to more easily contrast our findings with the base model.) Further, suppose that with some small probability $\rho > 0$, the accelerator obtains no signal (which we denote as $\sigma_i(n) = \phi$) about the prospects of a participating venture.[20] This means that ventures that did not receive a positive signal may still seek financing from investors by approaching them after the accelerator's program.

Thus, there are four types of entrepreneurs: Those who participated in the accelerator's program and (i) received a positive signal, (ii) received a negative signal, (iii) received no signal; and (iv) those who did not participate. We note that once revealed, the signals are verifiable; hence, the accelerator cannot manipulate the signals. However, the accelerator may strategically decide not to release negative signals. (It will be evident that the accelerator has no incentive to withhold positive signals.) From the perspective of investors, entrepreneurs who received negative signals and those who received no signals are pooled

[20] If $\rho = 0$, then withholding negative signals does not change the previous analysis because investors can perfectly infer that the signal was negative.

together. Hence, the posterior probability that a venture is of type H given no signal is

$$\gamma_\rho(n) = \frac{\gamma \cdot \rho}{\rho + (1-\rho)(1-\gamma)(1-\alpha(n))}. \quad (15)$$

Using parallel notations to those used in the base model, let $V(k|\gamma_\rho)$ denote the expected proceeds (net of financing costs F) from a venture for which no signal was revealed (by the accelerator) and investors obtained a positive signal. Investors will then make competitive financing offers as long as the venture's expected value is positive, that is, as long as

$$\begin{aligned} V(k|\gamma_\rho) &= (\gamma_\rho + (1-\gamma_\rho)\alpha(k))(E[v|\sigma_i(k)=1, \sigma_i(n)=\phi] - F) \quad (16) \\ &= (\gamma_\rho + (1-\gamma_\rho)\alpha(k))(v_L + \lambda_L(v_H - v_L) - F) + \gamma_\rho(\lambda_H - \lambda_L)(v_H - v_L) > 0, \end{aligned}$$

where the second equality makes use of (1) and (15).

From the accelerator's perspective, let $V_N(k|\gamma_\rho)$ denote the expected proceeds (net of financing costs F) from a participant that in fact received a negative signal. The accelerator privately knows that such ventures are of type L, but it also recognizes that they would receive financing with probability $\alpha(k)$ if the negative signal is not revealed. Since investors do not know whether a negative or no signal was received by the accelerator, conditional on obtaining their own positive signal, they would offer financing F in exchange for an equity share of $F/E[v|\sigma_i(k)=1, \sigma_i(n)=\phi]$, where $E[v|\sigma_i(k)=1, \sigma_i(n)=\phi] = v_L + \lambda_L(v_H - v_L) + \frac{\gamma_\rho(\lambda_H-\lambda_L)(v_H-v_L)}{\gamma_\rho+(1-\gamma_\rho)\alpha(k)}$. Then $V_N(k|\gamma_\rho)$ is given by:

$$V_N(k|\gamma_\rho) = \alpha(k)(1-\gamma-(1-\gamma)\alpha(n))(1-\frac{F}{E[v|\sigma_i(k)=1, \sigma_i(n)=\phi]})(v_L+\lambda_L(v_H-v_L)). \quad (17)$$

Letting s_ρ denote the accelerator's chosen equity fee for program participants, its expected profit is given by

$$ns_\rho[(1-\rho)(V(n) + V_N(k|\gamma_\rho)) + \rho V(k|\gamma_\rho)] - nc. \quad (18)$$

The accelerator has no incentive to withhold positive signals from investors. If the accelerator were to withhold both positive and negative signals, then it provides no new information to investors; hence, it cannot charge a positive equity fee for program participation. If only negative signals are withheld, then more (low-type) participants will be financed by investors. On the other hand, if both positive and negative signals are revealed, then the expected value of ventures that received no signal is increased relative to their value when they are pooled with low-type ventures. Thus, the decision to withhold negative signals depends on the tradeoff between these two effects. It is intuitive that when the proportion of ventures that received no signal is small, the former effect dominates and the accelerator chooses to withhold negative signals.

Proposition 5 *There exists* $\bar{\rho} > 0$ *such that for any* $\rho \in (0, \bar{\rho})$ *there is a* $\underline{\lambda}_{L,\rho} < \frac{F - v_L}{v_H - v_L}$, *where, given parameters* ρ *and* $\lambda_L > \underline{\lambda}_{L,\rho}$, *the accelerator is better off under* $\rho > 0$ *with negative signals being withheld than under* $\rho = 0$ *where all signals are revealed.*

We first emphasize that Proposition 5 immediately implies that the accelerator is better off withholding negative signals than disclosing them when the conditions of the proposition are satisfied and $\rho > 0$. This follows from the fact that the accelerator is better off under $\rho = 0$ than under $\rho > 0$ if all signals are disclosed (else its equity fee must be reduced, since participants pay an equity fee for a potentially uninformative signal).

In the equilibrium described in Proposition 5 under $\rho > 0$, investors take a second look and invest in some accelerator participants who did not receive a positive signal. This means that there are two sources of inefficiencies. One source is that the accelerator's program may lack incentives to improve its information-acquisition technology. That is, while it is more efficient to have $\rho = 0$, if the accelerator has to invest in generating more informative signals about participants (i.e., invest in reducing ρ), it may lack the incentive to do so. This is because the accelerator can benefit from pooling ventures that received uninformative signals with ventures that received negative signals. The other inefficiency rises from the fact that in an environment with $\rho > 0$ as described above, the accelerator knowingly enables low-type ventures to receive financing (with some probability), which reduces social welfare. Hence, while the added noise in the information-acquisition process may work to the benefit of the accelerator, it detracts from social welare.

Proposition 5 also implies that under private information and selective signaling, from the perspective of investors, ventures that receive financing are divided into three segments: (i) ventures who participated in the program and received positive signals, (ii) ventures who initially approached investors directly and were screened, and (iii) ventures who participated for whom no signal was revealed and were then screened by investors. A corollary to Proposition 5 is that the groups (i)-(iii) are ranked in descending order according to the average success likelihood of ventures in each segment, and also in descending order of expected payoffs for entrepreneurs in these segments. While we are not aware of formal studies documenting the performance difference between accelerator participants and nonparticipants, this prediction is consistent with recent evidence presented by Kerr et al. (2011), where firms that obtained angel financing are overall more successful than those that pitched to an angel group but did not receive financing.[21]

6.2 When Entrepreneurs Interpret Funding as Success

In all preceding sections, the accelerator could profitably run a program because its ability to acquire information about ventures was superior to that of investors. If the accelerator were to choose a class size n larger than k, then entrepreneurs would not pay a positive equity fee to participate in the accelerator's program. In this subsection, we show that under a plausible behavioral hypothesis, an inefficient accelerator program can profitably exist. More specifically, we consider the possibility that entrepreneurs in the early stage of their ventures want, first and foremost, to raise funding. In other words, entrepreneurs' objectives are not necessarily to maximize the expected value of their ventures (more specifically, their

[21] To be more specific, using ventures that fall within a narrow quality range, Kerr et al. (2011) find that firms that received angel financing are more likely to survive for at least four years, undergo a successful exit (IPO or acquisition), grow faster in terms of employment size and web traffic, and raise additional financing.

equity stakes), but rather, to make sure that their ventures are financed.[22]

To be precise, let us consider a setup identical to the base model, except that entrepreneurs value raising funding at a certain amount $R > 0$ (which maps — for simplicity, we set it equal — to a valuation of their ventures), independent of the signal generated by the accelerator or by investors. It follows that $E[v|\sigma_i(\cdot) = 1] = R$, whereby entrepreneurs do *not* Bayesian-update their beliefs about venture valuation conditional on the properties of a signal. Instead, entrepreneurs' goal is simply to obtain funding F at the best available financing terms (i.e., in exchange for the smallest equity share), whether F is raised after participating in the accelerator's program or obtained directly from investors. We emphasize that neither the accelerator nor investors have a bias as entrepreneurs do, and they correctly update the expected valuation of ventures.

Since the accelerator charges a positive equity fee, a necessary condition (with an equity fee of 0) for an entrepreneur to choose the accelerator is given by:

$$(\gamma + (1-\gamma)\alpha(k))(1 - \frac{F}{E[v|\sigma_i(k) = 1]}) < (\gamma + (1-\gamma)\alpha(n))(1 - \frac{F}{E[v|\sigma_i(n) = 1]}). \quad (19)$$

Rearranging yields the following result.

Proposition 6 *Given that entrepreneurs perceive funding as success, an inefficient accelerator program (i.e., $n > k$) can exist if the prior probability of high-type ventures, γ, is sufficiently low, and the precision of the information acquisition technology is sufficiently high (i.e., $\alpha(k)$ and $\alpha(n)$ are small).*

Proposition 6 shows that entrepreneurs whose primary objective is to successfully raise funding may apply to participate in accelerator programs, even if the signal quality of such programs is lower than that of investors. Under such conditions, entrepreneurs choose the accelerator to obtain funding with a higher probability, albeit at worse financing terms. It

[22] Entrepreneurs may also care more about the early survival of their ventures if they develop a personal attachment to their ideas or believe they can increase value down the road. Overconfidence of entrepreneurs has been well documented in the literature — see, for instance, Cooper et al. 1988.

thus follows that a greater number of low-type venture participants would receive funding, which detracts from social welfare.

7 Conclusions

In this paper, we provided a simple framework for studying some of the dynamics that are introduced by an accelerator program. We investigated inefficiencies that are introduced by the accelerator in terms of the participating class size, the equity fee charged to participating ventures, and information sharing with investors. We also studied some of the benefits of the accelerator, in terms of improving the quality of information, providing training, and facilitating access to investors, and discussed the potential inefficiencies when the accelerator can selectively disclose signals, and when entrepreneurs perceive receiving funding as success.

This paper is a small step towards understanding an emerging form of business incubation, which has established itself as an important actor in startup financing. Directions for future research include allowing for private information on the side of entrepreneurs, allowing for competing accelerator programs, and factoring in additional value-added features of accelerator programs, such as gaining brand-name recognition and matching entrepreneurs with investors best suited to support their ventures. It would also be interesting to empirically investigate the predictions of our model by studying the performance of accelerator participants.

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Appendix

Proof of Proposition 1. We remind that $V(n) = (\gamma + (1-\gamma)\alpha(n))(E[v|\sigma_i(n) = 1] - F) > 0$. If not, the accelerator cannot profitably operate. Substituting in for $E[v|\sigma_i(n) = 1]$ from equation (1) yields

$$V(n) = (\gamma + (1-\gamma)\alpha(n))(v_L + \lambda_L(v_H - v_L) - F) + \gamma(\lambda_H - \lambda_L)(v_H - v_L).$$

Differentiating $V(n)$ with respect to n, we get $V'(n) = (1-\gamma)\alpha'(n)(v_L+\lambda_L(v_H-v_L)-F)$. By the model's assumption, $\alpha'(n) > 0$ and $\lambda_L v_H + (1-\lambda_L)v_L - F < 0$. The latter is true because we assumed that ventures for which a negative signal is obtained are not financed. Thus, if the venture is surely of type L, then the expected profit must be negative. Give these, it follows that $V'(n) < 0$. Differentiating $V'(n)$ with respect to n, we have $V''(n) = (1-\gamma)\alpha''(n)(v_L + \lambda_L(v_H - v_L) - F)$. Since $\alpha''(n) > 0$ by assumption, it also follows that $V''(n) < 0$. Also, $s_Q(n)$ can be rewritten as $s_Q(n) = 1 - V(k)/V(n)$. It follows from $V'(n) < 0$ that $s'_Q(n) < 0$.

The first-order condition that characterizes the accelerator's optimal choice n^* under rational expectations is given by equation (9):

$$s_Q(n)(V(n) + nV'(n)) = c.$$

The first-order condition that characterizes the accelerator's commitment solution n° is given by equation (10):

$$s_Q(n)(V(n) + nV'(n)) = c - ns'_Q(n)V(n).$$

Notice that the RHS of equation (10) is greater than the RHS of equation (9) because $s'_Q(n) < 0$.

Since the LHS of both equations (9) and (10) has positive sign when evaluated at $n = 0$, both n^* and n° are at the interior and we can restrict attention to the relevant range, for which $V(n) + nV'(n) > 0$, so that n is positive. Further, the LHS of both equations (9) and (10) is decreasing in n. Differentiating $V(n) + nV'(n)$ yields $2V'(n) + nV''(n)$, which is negative since $2\alpha'(n) + n\alpha''(n) \geq 0$ on this range. Together with the fact that $s'_Q(n) < 0$, this establishes the claim. Therefore, it follows that n° must be smaller than n^*.

That the accelerator's profit is lower under rational expectations is straightforward because given the same expression for the objective function the commitment solution yields the most the accelerator can get. Thus, as long as the solution under rational expectations is different from the commitment solution, this proves that the accelerator's profit is lower in a rational expectations equilibrium. ■

Proof of Proposition 2. As long as the accelerator chooses a strictly positive number of participants, the total surplus of the program is also strictly positive at a strictly positive class size. The first-order condition that characterizes the rational expectations equilibrium is given by equation (3):

$$s_Q(n)(V(n) + nV'(n)) = c.$$

The first-order condition that characterizes the planner's solution is given by equation (5):

$$V(n) + nV'(n) = c.$$

Since $s_Q(n) = 1 - V(k)/V(n)$, it follows that $s_Q(n^*) < 1$. Also, $k > n^*$ because otherwise the accelerator cannot charge a positive equity fee. Suppose $\hat{n} \leq n^*$. From the proof of Proposition 1, $V(n) + nV'(n)$ is a decreasing function of n. This implies $s_Q(n^*)(V(n^*) + n^*V'(n^*)) < V(\hat{n}) + \hat{n}V'(\hat{n})$, which is a contradiction. Therefore, $\hat{n} > n^*$.

By differentiating the planner's objective function twice yields $2V'(n) + nV''(n) < 0$. Hence, the value function $SW(n)$ is concave, and given that $\hat{n}$ is the global maximum, $n^\circ < n^* < \hat{n}$ implies $SW(n^\circ) < SW(n^*) < SW(\hat{n})$. ■

Proof of Proposition 3. Notice that the analysis is restricted to parametrizations such that the accelerator hires at least one entrepreneurs-in-residence. Given this, U denotes the expected value of the startup firm the entrepreneurs-in-residence propose to the accelerator, which is sufficiently large. Since the accelerator obtains $s_Q(n)$ share of the companies proposed by the entrepreneurs-in-residence, the problem faced by the accelerator on equilibrium path is

$$\max_n n s_Q(n^e)V(n) - (n + m)c + m s_Q(n)U.$$

The first-order condition is

$$s_Q(n^e)(V(n) + nV'(n)) + m s'_Q(n)U = c,$$

and the rational expectations require $n^e = n$.

The first-order condition that characterizes the commitment solution is given by equation (4):

$$s_Q(n)(V(n) + nV'(n)) + n s'_Q(n)V(n) = c.$$

Given $V(n^\circ) > 0$, a comparison of equations (4) and (6) yields that by setting $m^* = nV(n)/U > 0$, the accelerator chooses n° in rational expectations equilibrium. ■

Proof of Proposition 4. From the first-order condition of the accelerator's problem, we have $s_Q(n)(V(n) + nV'(n)) = c$. From (14), it is straightforward to find that $\frac{\partial s_Q(n)}{\partial c_e} > 0$. Since $s_Q(n)(V(n)+nV'(n))$ is decreasing in n, it immediately follows that the accelerator will increase n^* as c_e increases. Since the accelerator cannot own an entire venture in equilibrium (raising financing F requires handing a stake to investors), it follows that the outcome moves closer to the social optimum as c_e increases but never reaches it. ■

Proof of Proposition 5. First, for ventures to be financed given no signal, inequality (16) must be satisfied. Since the only negative term in the inequality is $v_L + \lambda_L(v_H - v_L) - F$, it follows that for any given $\rho > 0$, there exists a $\underline{\lambda}_{L,\rho} \in (0, \frac{F-v_L}{v_H-v_L})$ such that (16) holds for all $\lambda_L \geq \underline{\lambda}_{L,\rho}$. Moreover, when (16) is satisfied, $F < E[v|\sigma_i(k) = 1, \gamma_\rho]$. From equation (17), it thus follows that $V_N(k|\gamma_\rho)$ is positive.

Contrasting the accelerator's expected profit in the base model (when $\rho = 0$), for arbitrarily given class size n and equity fee s, to its profit when $\rho > 0$ and only positive signals are revealed, we see that the expected profit is higher when $\rho V(n) < (1-\rho)V_N(k|\gamma_\rho)+\rho V(k|\gamma_\rho)$, where the same inequality also ensures that entrepreneurs are better off (given the same s and n). Hence, the accelerator can set a higher fee s_ρ when $\rho > 0$. On the other hand, since $V_N(k|\gamma_\rho)$ is positive on the range of λ_L described above, the above inequality is satisfied as $\rho \to 0$. Therefore, the accelerator's maximized profit is higher when $\rho > 0$, under these conditions. ■

Proof of Proposition 6. To reduce the number of terms, let $\psi(k) = \gamma + (1-\gamma)\alpha(k)$ and $\psi(n) = \gamma + (1-\gamma)\alpha(n)$. Let $a = v_L + \lambda_L(v_H - v_L)$ and $b = \gamma(\lambda_H - \lambda_L)(v_H - v_L)$. Also let $x = E[v|\sigma_i(k) = 1] = a + \frac{b}{\psi(k)}$ and $y = E[v|\sigma_i(n) = 1] = a + \frac{b}{\psi(n)}$.
Given $n > k$, a necessary and sufficient condition for an inefficient accelerator to exist is given by:

$$(\gamma + (1-\gamma)\alpha(k))(1 - \frac{F}{E[v|\sigma_i(k) = 1]}) < (\gamma + (1-\gamma)\alpha(n))(1 - \frac{F}{E[v|\sigma_i(n) = 1]}).$$

This is because when the above inequality holds, an accelerator with a class size that satisfies $n > k$ can set an arbitrarily low equity fee that still induces entrepreneurs to participate in its program. The above inequality can be rewritten as $\psi(k)(1-\frac{F}{x}) < \psi(n)(1-\frac{F}{y})$. Rearranging, we get

$$F < \frac{xy(\psi(n) - \psi(k))}{x\psi(n) - y\psi(k)}$$

Substituting for x and y and simplifying, we obtain

$$F < a + \frac{b^2}{a\psi(k)\psi(n) + b(\psi(k) + \psi(n))}$$

After substituting for a and b and rearranging terms, the above inequality reduces to

$$F < v_L + \lambda_L(v_H - v_L) + \gamma(\lambda_H - \lambda_L)(v_H - v_L)K$$

where K is defined as:

$$K = \frac{\gamma(\lambda_H - \lambda_L)(v_H - v_L)}{(v_L + \lambda_L(v_H - v_L))\psi(k)\psi(n) + \gamma(\lambda_H - \lambda_L)(v_H - v_L)(\psi(k) + \psi(n))}$$

Since $F > E[v] = v_L + \lambda_L(v_H - v_L) + \gamma(\lambda_H - \lambda_L)(v_H - v_L)$ by assumption, the above inequality requires that K is sufficiently large. This occurs when $\psi(k) + \psi(n) = 2\gamma + (1 - \gamma)(\alpha(k) + \alpha(n))$ is sufficiently small, which takes place when the prior probability of high-quality ventures, γ, is relatively low, and when the precision of the information-acquisition technology is sufficiently high (i.e., $\alpha(k)$ and $\alpha(n)$ are small). ■