Truthtelling in Matching Markets*

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(Preliminary)

Abstract

We analyze a dynamic search and matching model with asymmetric information. Randomly paired agents go through an evaluation phase before they discover each other's types and decide to match or not. Before deciding to enter this phase, agents can send a cheap-talk message about their type to their partner. We provide conditions for this communication to be informative and examine how it impacts search behaviors and the matching that arises in a stationary equilibrium. Early access to truthful information enables agents to avoid spending time in unfruitful evaluation phases and can modify the final matching as it affects how picky agents are. A full characterization of the matching configurations emerging in equilibrium with or without communication is provided. Communication is Pareto improving only when the matching is assortative in the absence of communication and left unchanged by information transmission.

Keywords: Cheap talk; Marriage; Matching; Search.

JEL Classification: C72; C78; D82; D83; J64.

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1 Introduction

In matching markets, partners often have limited information about the quality of their match at the time they meet. Yet they also know that, at some point, all information relevant to their match will eventually be revealed. When partners have the same imperfect information at the time of the meeting (as, e.g., in Jovanovic, 1979), they have no other choice than to collect information by being effectively matched for a sufficiently long period.[1] However, most of the time, partners have private information about their match quality that they could communicate right away. Depending on the situation, they would like to transmit this information rapidly or, on the contrary, they may prefer to hide it for as long as possible. For instance, an employer might want to inform quickly a potential employee about the flexibility of working hours offered in his company. Conversely, this employer might prefer to postpone the discussion about how competitive the ambiance in his team can sometimes be. In both cases, a probation period is likely to help the employee discover the true characteristics of the job he might get. Communication during the job interview, through the job opening announcement or at any other preliminary stage of the potential partnership, is an alternative way to learn such characteristics.[2]

This paper builds on the central observation that partners have private information about the match quality at the time they meet and that this information can be elicited by going through a time-consuming evaluation phase. Because agents do not learn the characteristics of their partners right after they met, some room is left for early and voluntary communication between agents. Precisely, we offer individuals the opportunity to communicate via cheap talk, that is, at no cost and without having to prove their statements with hard evidence. A crucial feature of such contexts is that, if such communication is informative, then everything is as if agents could take informed decisions about matching at the time of their first meeting. This paper analyzes the conditions under which information revelation is truthful and its impact on search behavior and matching (i.e., who matches with whom).

Formally, we analyze a dynamic search and matching model with asymmetric information. Agents can be of two types, high or low, with the same ordinal preference: everyone prefers to be matched with a high-type agent. Before they can enjoy any gain from matching, agents have to go through the following process: first, they are randomly paired; second,

[1]Incomplete but symmetric information about the match-specific dimension has been studied extensively, both theoretically (see, e.g., Pries, 2004, Pries and Rogerson, 2005), and empirically (see, e.g., Nagypal, 2007).

[2]Other examples with preliminary communication in matching markets include men and women filling their profiles on dating websites and going through several "dates" before they consider themselves officially together, or researchers organizing informal meetings before starting to work together.

they communicate bilaterally about their types; third, they decide whether or not to start an evaluation phase during which they cannot search for another partner; last, at the end of the evaluation phase, types are revealed and the informed decision of whether or not to effectively match is taken. We fully characterize the equilibria of this game according to the matching that arises in the steady state and to whether agents truthfully communicate.

When no information is transmitted during the communication phase, decisions to be matched with a known type are taken at the end of the evaluation phase. If communication is informative, everything is as if paired agents immediately learnt each other's type and therefore took final matching decisions at the time of meeting. In both cases, three possible outcomes can emerge in the steady-state: *assortative matching* when agents are matched assortatively by types; *upward matching* when high types accept to match with any other type and low types reject each other; and *random matching* which corresponds to the initial random pairs being finally matched. Informative communication by itself cannot compensate entirely for search frictions and, therefore, the frictionless complete information outcome (assortative matching) does not obtain systematically under informative communication. Said differently, the same outcomes may arise in both cases but under different conditions.

Truthful information revelation is not trivial in our model as agents communicate about vertical characteristics while all competing for the "best" partners. In particular, low-type agents may want to claim being of high type in order to be spotted by high-type agents. However, such a lie reveals unprofitable if it leads a low-type agent to start evaluation phases at the end of which he will be rejected anyway. Indeed, in our model, information transmission can only be used by an agent to convince his partner to start an evaluation phase but nothing prevents the partners from rejecting each other once the truth is known. Our analysis precisely unveils that low-type agents have no interest in lying if high-type agents are patient enough or if high-type agents' chances to be matched quickly with another high-type are big enough. In particular, this shows that truthful communication does not always obtain in equilibrium.

The model also sheds light on the consequences on the functioning of matching markets of having more information about a potential partner at an early stage of a relationship. The question is of interest as the decline in communication costs has made information transmission much easier in modern matching markets.[3] We identify two main effects of informative communication. First, everything else being equal, the opportunity cost of continuing to search is lower when communication is informative because agents no longer

[3]For a general discussion of the impact of the Internet on the functioning of labor markets, see, e.g., Autor (2001).

lose time in unfruitful evaluation phases. Communication therefore provides everyone with stronger incentives to reject lower-types agents. This effect is however less pronounced for the low-type agents since, at the end of the day, they are still constrained by high-type agents' acceptance decision. Put differently, if earlier information makes everyone *want* to be pickier, only the more desired partners *can* indeed always be pickier.

Second, informative communication induces search externalities at the time of meeting. As already noted, more information makes it tempting to immediately reject the less desirable partners. Nevertheless, if everyone does this, the quality of the pool of potential partners deteriorates, incidentally making the strategy of rejecting the lower types right away less attractive. This also suggests that, compared to a situation where early communication is not possible, informative communication may have both a positive and a negative effect on agents' welfare. On the one hand, informative communication allows all agents to avoid spending time in unfruitful time-consuming evaluation phases. On the other hand, informative communication, by making everyone willing to be more selective, may lead to a reduction in the number of available good potential partners, therefore making "single" agents possibly worse off. Whether the positive effect dominates the negative one depends crucially on the matching that arises in the stationary equilibrium. Precisely, we show that high-type agents are strictly better off if communication leads to assortative matching, but strictly worse off if it induces low-type agents to reject each others. On the other side, low-type agents are strictly better off if communication does not change the matching or if it induces them to reject each others, but strictly worse off if it leads to a change to assortative matching. Hence, communication is Pareto improving only when the matching is initially assortative and is left unchanged by informative communication.

Related literature Our paper is related to the literature on search and matching models with heterogeneous agents. The literature separates into two strands depending on whether utility is assumed to be transferable (Sattinger, 1995, Shimer and Smith, 2000, Atakan, 2006) or non-transferable (McNamara and Collins, 1990, Bloch and Ryder, 2000, Eeckhout, 1996, Burdett and Coles, 1997 and Smith, 2006). A common assumption in such models is that all relevant information for the matching is revealed right after the two agents have met. Put differently, there is no evaluation period and, therefore, no room for communication once agents have made contact. As mentioned earlier, we assume that discovering a partner's type is time consuming.

The focus of the literature has been on finding conditions under which the matching that arises in the steady state is "similar" to the matching that would arise in the absence of any search frictions. For instance, under NTU and identical ordinal preferences, it is well known that a perfectly assortative matching arises under complete information. Smith (2006) shows that a similar outcome obtains when there are search frictions if the surplus function is log-supermodular in the partners' types. Our approach is different. We aim at comparing the matching when no information is transmitted to the matching under informative communication. Therefore, we also characterize situations where a matching different from the assortative one arises in equilibrium.

An alternative approach to information transmission in matching markets is to assume that information transmission takes place *before* the matching occurs. Agents may be matched according to costly signals (in the sense of Spence, 1973). This issue has been analyzed extensively by the litterature on matching tournaments (see, e.g., Peters and Siow, 2002, Chiappori et al., 2009, Hoppe et al., 2009, Mailath et al., 2011) which studies how pre-marital investment or investment before trading shape the matching between men and women, buyers and sellers, etc. The informativeness of cheap-talk communication has also be investigated in directed search models (Menzio, 2007, Kim and Kircher, 2011).[4] Last, Lee and Schwarz (2007) look at communication to all the agents of the other side of the market before the interviews in a market design perspective.

The paper is organized as follows. Section 2 describes the model. Then, we characterize the equilibria when there is no communication (Section 3) and when communication is informative (Section 4). Section 5 compares the matching configurations that arise, agents' incentives and welfare under no-communication and informative communication. Section 6 concludes and discusses some extensions to our framework.

2 Model

Matching Environment and Preferences. To fix terminology, we use marriage as a metaphor for the matching problem we analyze. We consider one population of an atomless continuum of agents searching for a marriage partner.[5] The size of the population is normalized to one. It is made up of heterogeneous agents: a proportion $\lambda_h = \lambda \in (0,1)$ of agents are of high type h and a proportion $\lambda_l = 1 - \lambda$ are of low type l. Type is time-invariant and private information to each agent.

[4] Cheap talk was introduced in economic theory by Crawford and Sobel (1982). For a survey, see Sobel (2010).

[5] Our conclusions apply equally in a two populations model as long as the surplus function is symmetric. Most papers in the literature assume symmetry of the surplus function.

Time is discrete and the horizon is infinite. At the beginning of each period, every agent in the market is either *single* or in an *evaluation phase*. An evaluation phase consists of a stochastic number of periods within which two agents stay matched with each other, while both are still unmarried. In every period of the evaluation phase, there is a probability $\beta \in (0,1)$ that the evaluation phase ends before the end of the current period; with probability $1-\beta$ the evaluation phase goes on to the next period. The marriage decisions take place at the end of the evaluation phase. When two agents marry, they leave the market and are immediately replaced by singles of the same types.[6]

When a type-i agent and a type-j agent, $(i,j) \in \{l,h\}^2$, get married, the flow payoff is $u_{ij} > 0$ to type i and $u_{ji} > 0$ to type j. All agents have identical ordinal preferences: they get a higher payoff when married with a high type rather than with a low type agent, i.e., $u_{ih} > u_{il}$, for all $i \in \{l,h\}$. The per-period payoff of an unmarried agent, either in an evaluation phase or single, is assumed to be the same regardless of his type, and is normalized to 0. Agents maximize their discounted expected payoffs at the common rate $\delta \in (0,1)$.

Communication and Trial Strategies. In the beginning of each period, single agents are randomly matched into pairs. When meeting, they do not immediately observe each other's type. Instead, within the period where their match occurred, every pair of agents can strategically communicate through direct cheap talk before deciding whether or not to enter an evaluation phase with each other.

Communication takes the following form: each of the two types of agents sends a costless message from the set $\{h,l\}$ to his partner. Precisely, an agent's steady-state *communication strategy* is a mapping from the set of his possible types $\{h,l\}$ into the set of available messages $\{h,l\}$.[7] The strategy is babbling if the same message is sent by the two types of agents. If all the agents of a given type always send a different message from all agents of the other type, then the communication strategy is fully revealing. Without loss of generality, we consider fully-revealing strategies where type-h agents send message h and type-l agents send message l.

After having sent a cheap-talk message and received one from his partner, each agent takes a decision about whether or not to enter an evaluation phase. For each agent at that step, a (steady-state) *trial strategy* is a mapping from his type and the cheap-talk message he received at that period to a probability to enter or not. We denote by $\tau_{ij} \in [0,1]$ the

[6] This is known in the literature as a "cloning assumption". It simplifies our analysis by limiting the number of possible states in which an agent might be. See, e.g., McNamara and Collins (1990), Bloch and Ryder (2000), Adachi (2003) and the discussion in Smith (2011).

[7] We only study equilibria in pure communication strategies.

trial strategy of a type-i agent who got message j from his partner: τ_{ij} is the probability that type i accepts to enter the evaluation phase after receiving message j from his partner.[8] Both consents to enter the evaluation phase are required for the matched agents to enter the evaluation phase. In case of a refusal, both agents go back to the market as singles. Two agents who have been matched, have exchanged messages and agreed to give a trial to each other finally enter an evaluation phase which ends within that same period with probability β. It is only at the end of this evaluation phase that both agents learn each other's types. No information is released as long as the evaluation phase goes on to a subsequent period.

Marriage Strategies. In each period, agents whose evaluation phase ends finally discover their partner's type and then decide simultaneously whether or not to marry. For each player, a (steady-state) *marriage strategy* then maps his type and the type of his partner into a probability to accept the marriage. We denote by $\mu_{ij} \in [0,1]$ the marriage strategy of a type-i agent who observes the type j of his partner: μ_{ij} is the probability that type i agrees to marry type j at the end of an evaluation phase. When one agent refuses to marry, the marriage cannot occur and both agents go back to the market as singles. When both agents mutually agree to marry, they leave the market and are immediately replaced by singles of the same types. Note that it is a weakly dominated strategy for each type to refuse the marriage with a type-h agents, i.e., $\mu_{ih} = 1$ for all i.[9] Therefore, there are only three possible matching outcomes in pure strategies: Positive Assortative Matching (PAM) when agents of similar types get married together ($\mu_{hl} = 0$, $\mu_{ll} = 1$), Random Matching (RM) when agents end up as if they were randomly married ($\mu_{hl} = 1$, $\mu_{ll} = 1$), and Upward Matching (UM) when low types end up being married only with high types whereas high types marry every type ($\mu_{hl} = 1$, $\mu_{ll} = 0$).

The timing of the game in a typical period is summarized in Figure 1.

Steady-State Equilibrium. We are looking for steady-state (i.e., time invariant) perfect[10] Bayesian equilibria in a steady-state environment. In every period, the (steady) state of the game is given by $\langle n_i, n_{ij} : (i,j) \in \{l,h\}^2 \rangle$, where n_i is the number of single type-i

[8] Since there is a continuum of agents of each type, this is equivalent to the situation in which a proportion τ_{ij} of type-i agents accept to enter the evaluation phase after receiving message j from their partner.

[9] This is the key difference with transferable utility (TU) models. Under TU, we would also have to consider existence of stationary equilibria in which type-h agents only accept type-l and reject type-h agents. See Smith (2011) for a comparison of TU and NTU search and matching models.

[10] Compared to Nash equilibrium, perfection requires that an agent chooses an optimal marriage decision when he realizes, at the end of an evaluation phase, that his partner lied about his type. This only occurs off the equilibrium path, so Nash equilibrium does not make any sequential rationality restriction at such information sets.

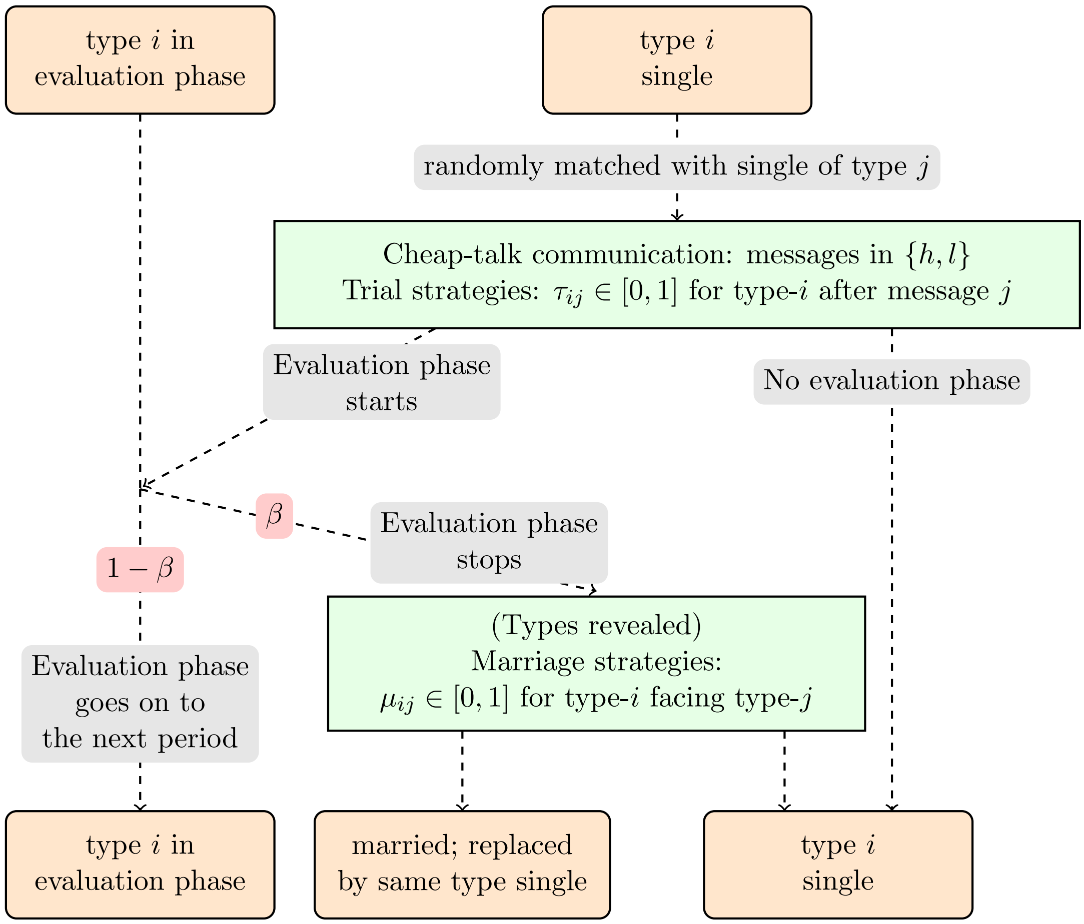

Figure 1: Timing of a Typical Period.

agents at the beginning of the period and $n_{ij} = n_{ji}$ is the number of type-i agents who are in their evaluation phase with a type-j agent as the period starts.

Both the decisions to give a trial or to marry are taken simultaneously and require mutual consent by the two partners. It follows that if one of the agents refuses the evaluation phase or the union, the other agent is indifferent between accepting it or rejecting it. To resolve this ambiguity, we focus on equilibria in which, if matching weakly increases both agents' payoffs, then they both accept the evaluation phase and the union.

An equilibrium in which agents' communication strategy is fully revealing will be called an *informative communication equilibrium*.[11] Such an equilibrium induces the same outcome and must satisfy the same strategic incentive constraints as under complete information, but agents should also have an incentive to reveal their true type when they meet. An equilibrium in which the communication strategy is babbling is called a *no-communication* equilibrium. It induces the same outcome and is characterized by the same incentive constraints as an equilibrium of the game without communication possibilities.

3 No-Communication Equilibria

We first characterize no-communication equilibrium outcomes, i.e., the different matching configurations that can occur in equilibrium when no information is transmitted through cheap-talk messages. In that case, a single agent always accepts to enter an evaluation phase: $\tau_{ij} = 1$ for every $i, j \in \{h, l\}$. Indeed, agents have no information on which to condition their choice of giving a try or not, and we rule out cases in which players coordinated on the dominated equilibrium where no one ever enters an evaluation phase with anyone.

3.1 Steady State

Since agents whose evaluation phase ends either return to the marriage market as singles or are replaced by single agents of the same type, the steady state $\langle n_i, n_{ij} : (i,j) \in \{l,h\}^2 \rangle$ is the same in every no-communication equilibrium. The number of single type-i agents is equal to the number of type-i agents whose evaluation phase ends:

$$n_i = \beta(n_{il} + n_{ih} + n_i). \tag{1}$$

[11]Our definition of "communication equilibrium" is rather narrow since we only consider simultaneous face-to-face communication in pure strategies (it does not correspond to the more general notion introduced by Myerson (1982) which allows general mediated communication).

The number of type-i agents matched with type-j agents in the evaluation phase is equal to the number of such agents whose evaluation phase continues from a period to the next plus the number of single type-i agents that are newly matched with type-j agents:

$$n_{ij} = (1-\beta)\left(n_{ij} + n_i \frac{n_j}{n_i + n_j}\right). \tag{2}$$

For every $i \in \{l, h\}$, type-i agents in the market are either singles or in an evaluation phase with some type-j agent, so we have: $n_i + n_{il} + n_{ih} = \lambda_i$. We use this equation to rearrange (1) and (2) and get:

$$n_i = \beta\lambda_i \text{ and } n_{ij} = (1-\beta)\lambda_i\lambda_j. \tag{3}$$

In particular the proportion of single type-i agents is identical to the overall proportion of type-i agents ($\frac{n_i}{n_l+n_h} = \lambda_i$, $i = l, h$).

3.2 Equilibria

We now look at the conditions for any of the three possible matching configurations (PAM, RM or UM) to occur in equilibrium. Besides, we will show that, generically, there is no equilibrium in which agents use mixed marriage strategies.

In the no-communication case, agents decide to accept or reject a given type at the end of the evaluation phase. At that point of time, agents compare the immediate gain of marriage to the continuation payoff they get when returning to the single status. In what follows, we denote by V_i the continuation equilibrium payoff of a single type-i agent, and byV_{ij} the continuation equilibrium payoff of a type-i agent who is in the evaluation phase with a type-j agent. The corresponding continuation payoffs following an equilibrium deviation by a type-i agent are respectively denoted by $\tilde{V}_i$ and $\tilde{V}_{ij}$.[12] We will also use the following notation

$$\zeta(x) := \frac{x}{1-(1-x)\delta}, \tag{4}$$

with $x \in (0,1)$, and observe that $\zeta(x) \in (0,1)$ is increasing in x and δ, and $\zeta(xy) < \zeta(x)\zeta(y)$ for all $x, y \in (0,1)$.

[12]Notice that since we are looking at steady-state equilibria and since there is a continuum of players (so that a unilateral deviation does not modify the state of the game), any unilateral deviation in a single period is profitable if and only if the corresponding deviation for all remaining periods is profitable.

No-Communication PAM Equilibrium ($\mu_{hl} = 0, \mu_{ll} = 1$). In such an equilibrium, every type-l agent accepts to marry any other agent independently of his type, while every type-h agent accepts to marry only type-h ones. First, note that if $\mu_{hl} = 0$ (i.e., type-h always reject type-l), then in equilibrium type-l agents should indeed always accept type-l agents, i.e., $\mu_{ll} = 1$, as they would obtain 0 in each period otherwise. Therefore $(\mu_{hl} = 0, \mu_{ll} = 1)$ is an equilibrium iff type-h agents are not willing to deviate. A type-h agent who accepts to marry a type-l agent receives a flow payoff u_{hl}; if he refuses to marry a type-l agent, he receives his equilibrium continuation payoff δV_h, where

$$\begin{aligned} V_h &= \lambda \underbrace{(\beta u_{hh} + (1-\beta)\delta V_{hh})}_{V_{hh}} + (1-\lambda) \underbrace{(\beta \delta V_h + (1-\beta)\delta V_{hl})}_{V_{hl}} \\ &= \lambda \frac{\beta}{1-(1-\beta)\delta} u_{hh} + (1-\lambda) \frac{\beta}{1-(1-\beta)\delta} \delta V_h = \zeta(\beta\lambda) u_{hh}. \end{aligned} \tag{5}$$

Therefore, $(\mu_{hl} = 0, \mu_{ll} = 1)$ is an equilibrium iff $u_{hl} \leq \delta V_h$, i.e.,

$$\frac{u_{hl}}{u_{hh}} \leq \delta\zeta(\beta\lambda). \tag{6}$$

No-Communication RM Equilibrium ($\mu_{hl} = 1, \mu_{ll} = 1$). In such an equilibrium, all types of agents accept to marry any type of agent. A type-h agent who marries a type-l agent receives a flow payoff u_{hl}; if he rejects a type-l agent, he receives his continuation payoff $\delta \tilde{V}_h$. Using a similar calculation as for PAM, we get $\tilde{V}_h = \zeta(\beta\lambda) u_{hh}$. A type l who marries a type l receives a flow payoff u_{ll}; if he rejects a type l, he receives his continuation payoff $\delta \tilde{V}_l$, where

$$\tilde{V}_l = \lambda \underbrace{\left(\beta u_{lh} + (1-\beta)\delta \tilde{V}_{lh}\right)}_{\tilde{V}_{lh}} + (1-\lambda) \underbrace{\left(\beta \delta \tilde{V}_l + (1-\beta)\delta \tilde{V}_{ll}\right)}_{\tilde{V}_{ll}} = \zeta(\beta\lambda) u_{lh}.$$

Therefore, $(\mu_{hl} = 1, \mu_{ll} = 1)$ is an equilibrium iff $u_{hl} \geq \delta \tilde{V}_h$ and $u_{ll} \geq \delta \tilde{V}_l$, i.e.,

$$\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\beta\lambda) \quad \text{and} \quad \frac{u_{ll}}{u_{lh}} \geq \delta\zeta(\beta\lambda). \tag{7}$$

No-Communication UM Equilibrium ($\mu_{hl} = 1, \mu_{ll} = 0$). In such an equilibrium, every type-l agent accepts to marry only type-h agents while every type-h agent accepts to marry any type of agent. Following the same logic as before, this is an equilibrium iff

$$\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\beta\lambda) \quad \text{and} \quad \frac{u_{ll}}{u_{lh}} \leq \delta\zeta(\beta\lambda). \tag{8}$$

No-Communication Mixed Strategy Equilibria. In a mixed strategy equilibrium where $\mu_{ij} > 0$ every type-i agent should be indifferent between marrying a type-j agent or not. As already mentioned, in any equilibrium we have $\mu_{hh} = \mu_{lh} = 1$; and from the analysis above, $\mu_{hl} \in (0,1)$ implies $\frac{u_{hl}}{u_{hh}} = \delta\zeta(\beta\lambda)$, and $\mu_{ll} \in (0,1)$ implies $\frac{u_{ll}}{u_{lh}} = \delta\zeta(\beta\lambda)$. Hence, no-communication mixed equilibria only exist for non generic sets of parameters of the game.

The following proposition (illustrated by Figure 2) summarizes the characterization of no-communication equilibria.

Proposition 1. *Generically, there is a unique* no-communication equilibrium. *It is such that $\tau_{ij} = 1$ for $i, j \in \{h, l\}$, and*

- *the matching is Positive Assortative if $\frac{u_{hl}}{u_{hh}} < \delta\zeta(\beta\lambda)$,*
- *the matching is Random if $\frac{u_{hl}}{u_{hh}} > \delta\zeta(\beta\lambda)$ and $\frac{u_{ll}}{u_{lh}} > \delta\zeta(\beta\lambda)$,*
- *the matching is Upward if $\frac{u_{hl}}{u_{hh}} > \delta\zeta(\beta\lambda) > \frac{u_{ll}}{u_{lh}}$.*

For each matching configuration, players' incentives are affected by their continuation value of being single when they reject type-l agents, which increases with the common threshold $\delta\zeta(\beta\lambda)$. This threshold is the same whatever the matching configuration because the proportion of single type-h agents (λ) is not affected by the matching configuration.

The threshold $\delta\zeta(\beta\lambda)$ is increasing in the parameter δ: as the present value of a future marriage increases with δ, the continuation value of being single increases. It also increases with β which determines the expected length of the next evaluation period: the larger is β, the shorter is the expected time to the next potential gain of marriage for a single. In that sense, a large β makes being single relatively more attractive than getting married right away. Finally, there is a positive effect of λ on the threshold $\delta\zeta(\beta\lambda)$. Because every agent accepts type-h agents, the continuation payoff of any single agent who rejects type-l agents increases with the proportion of single type-h agents available in the steady state. Said differently, an increase in λ improves the pool of available potential partners which makes the strategy of rejecting type-l more attractive. For instance, in a UM equilibrium, an increase in the steady-state proportion of single type-h makes the rejection of type-l agents by type-h more attractive and the ratio $\frac{u_{hl}}{u_{hh}}$ preventing this deviation has to be higher. Such an increase makes the acceptation of type-l agents by type-l less attractive and the ratio $\frac{u_{ll}}{u_{lh}}$ preventing this deviation can be higher.

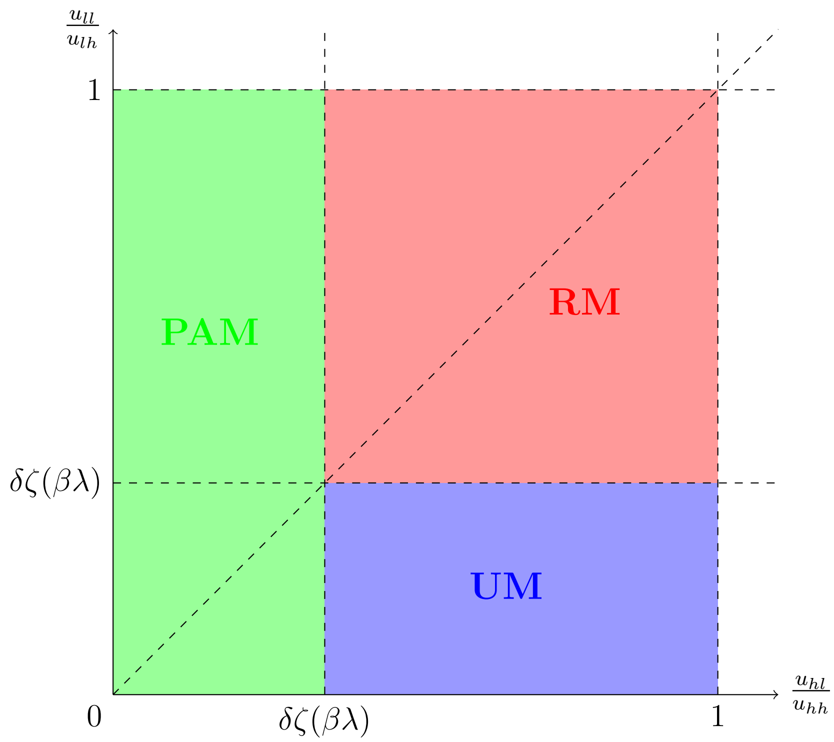

Figure 2: No-communication equilibria.

4 Communication and Incentives for Truthtelling

Before characterizing equilibria under informative communication, we study the equilibrium matching configurations when each agent knows his potential partner's type before deciding whether or not to initiate an evaluation phase. That is, we first characterize trial and marriage equilibrium strategies under complete information, as if the communication strategies were assumed to be fully revealing. Next, we study agents' incentive to reveal their true type before deciding to enter the evaluation phase.

We first make the following important observation: if two agents i and j take the decision to enter the evaluation phase once the real type of each partner has been revealed, then they will decide to marry with probability one at the end of the evaluation phase: $\tau_{ij} > 0 \Rightarrow \mu_{ij} = 1$; and if they never start an evaluation phase ($\tau_{ij} = 0$), then they never have the opportunity to get married. Put differently, when information is fully revealed, everything is as if the final decision about marriage were made as soon as the two agents meet.

4.1 Steady State

We first describe the dynamics of the complete information game as a function of agents' strategies. Note that it is without loss of generality (in terms of equilibrium outcomes) to consider trial strategies such that each type of agent always accepts to start an evaluation phase with a type-h agent, i.e. $\tau_{ih} = 1$ for all i (recall that both consents are required to enter an evaluation phase).

The steady-state number of single type-i agents in any period is equal to the number of type-i agents whose evaluation phase ended in the previous period plus the number of single type-i agents from the previous period who did not enter an evaluation phase because they rejected (or were rejected by) the agent to whom they had been matched (see Figure 3 for an illustration of transitions probabilities at the steady state):

$$n_h = \beta(n_{hl} + n_{hh}) + n_h \left(\frac{n_l}{n_l + n_h}(1 - (1 - \beta)\tau_{hl}) + \frac{n_h}{n_l + n_h}\beta \right), \tag{9}$$

$$n_l = \beta(n_{ll} + n_{lh}) + n_l \left(\frac{n_l}{n_l + n_h}(1 - (1 - \beta)\tau_{ll}^2) + \frac{n_h}{n_l + n_h}(1 - (1 - \beta)\tau_{hl}) \right). \tag{10}$$

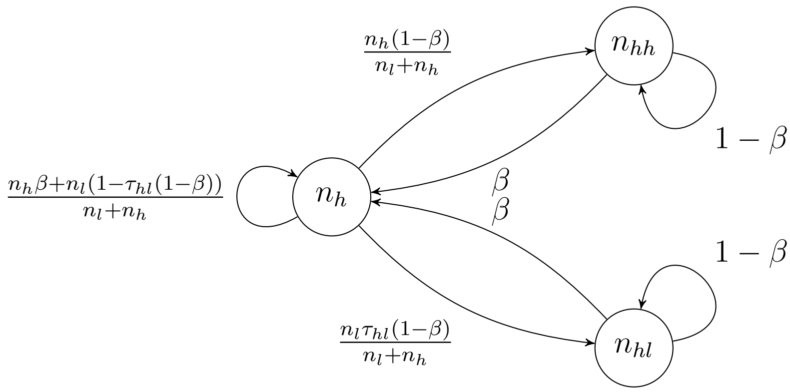

Figure 3: Steady-state transitions probabilities for type-h agents.

The number of type-i agents matched with type-j agents in the evaluation phase is equal to the number of such agents from the previous period who continue their evaluation phase plus the number of single type-i agents from the previous period who entered an evaluation phase with a type-j agent:

$$n_{ij} = (1 - \beta) \left(n_{ij} + n_i \frac{n_j}{n_l + n_h} \tau_{ij}\tau_{ji} \right). \tag{11}$$

For every $i \in \{l, h\}$, a type-i agent in the market is either single or in an evaluation phase with some type-j agent, so we have: $n_i + n_{il} + n_{ih} = \lambda_i$. We use this equation to rearrange (9), (10) and (11) and get:

$$n_h = \beta(\lambda_h - n_h) + n_h \left(\frac{n_h}{n_l + n_h}\beta + \frac{n_l}{n_l + n_h}(1 - (1-\beta)\tau_{hl}) \right), \tag{12}$$

$$n_l = \beta(\lambda_l - n_l) + n_l \left(\frac{n_l}{n_l + n_h}(1 - (1-\beta)\tau_{ll}^2) + \frac{n_h}{n_l + n_h}(1 - (1-\beta)\tau_{hl}) \right). \tag{13}$$

The proportion of single type-h agents now depends on the strategies τ_{hl} and τ_{ll} and on parameters λ and β; it is given by

$$\hat{\lambda}(\tau_{hl}, \tau_{ll}) := \frac{n_h}{n_l + n_h}, \tag{14}$$

where (n_h, n_l) solves (12)–(13). For the case of complete information random matching, we get in particular that $\hat{\lambda}(1,1) = \lambda$. We denote $\hat{\lambda}(0,1) = \hat{\lambda}_{PAM}$ and $\hat{\lambda}(1,0) = \hat{\lambda}_{UM}$.[13] Figure 4 illustrates the values of the complete information proportions of single type-h under PAM and UM as a function of the proportion of type-h agents (λ). Some useful properties of the steady-state proportion $\hat{\lambda}(\tau_{hl}, \tau_{ll})$ of single type-h agents are summarized by Lemma 1 below.

Lemma 1. *The steady-state proportion $\hat{\lambda}(\tau_{hl}, \tau_{ll})$ of single type-h agents has the following properties:*

1. $\hat{\lambda}_{PAM} \geq \lambda$ *iff* $\lambda \leq 1/2$; $\hat{\lambda}_{PAM} \geq \beta\lambda$ *for all* λ;
2. $\beta\lambda \leq \hat{\lambda}_{UM} \leq \hat{\lambda}(1, \tau_{ll}) \leq \lambda$ *for all* λ;
3. $\hat{\lambda}(\tau_{hl}, \tau_{ll})$ *is increasing in* λ.

Proof. See Appendix A.1. □

The difference between the steady-state proportions of singles in the no-communication and complete information cases is due to the fact that, in the absence of communication, some pairs of agents enter the evaluation phase while they will reject each other once their types will be discovered; in the complete information case, such pairs never enter an evaluation phase.

Under upward matching, since type-h agents accept both types, the number of type-h singles is not affected by the information being discovered before the evaluation phase might start. Indeed, both in the no-communication and the complete information cases, the

[13] See the proof of Lemma 1 for the exact formulas.

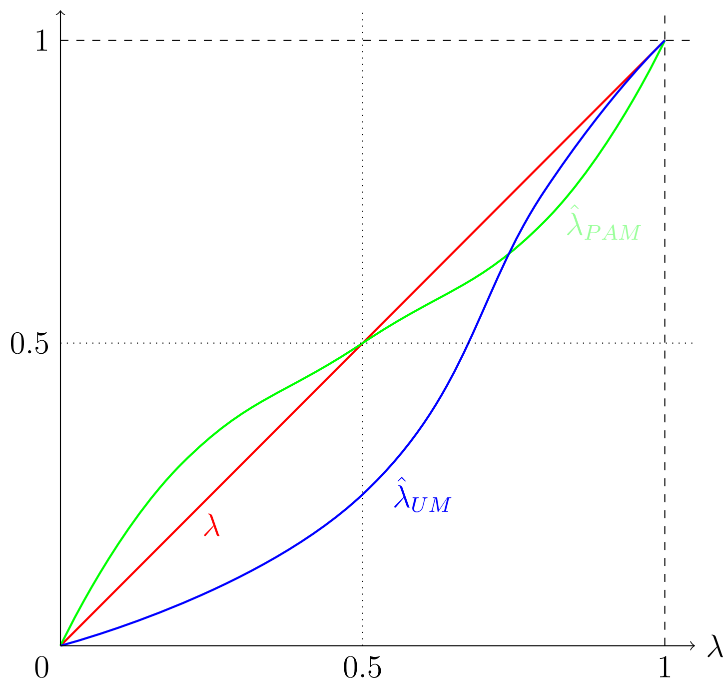

Figure 4: Complete information proportions of single type-h agents under positive assortative matching ($\hat{\lambda}_{PAM}$) and under upward matching ($\hat{\lambda}_{UM}$) as a function of the proportion of type-h agents (λ).

type-h agents directly enter an evaluation phase when they are single. However, the total number of single type-l agents—and hence, the total number of singles—is higher under complete information because a type-l agent is never in an evaluation phase with another type-l. These two effects lead unambiguously to the proportion of single type-h agents being smaller under upward matching with complete information than under upward matching without communication, i.e., $\hat{\lambda}_{UM} \leq \lambda$.

Under positive assortative matching, the proportion of single type-h agents with complete information, $\hat{\lambda}_{PAM}$, is smaller than λ when $\lambda < 1/2$, but it is higher than λ when $\lambda > 1/2$. The intuition of this property is less straightforward, but could be understood as follows. Under positive assortative matching with complete information, both the number of single type-h and type-l are higher compared to a no-communication situation because the evaluation phases involving type l and type h agents no longer occur under complete information. When λ is small ($\lambda < 1/2$), the rise in number of single type-h agents is more pronounced than on number of single type-l agents because, first, there are fewer type h agents than type l agents and, second, single type-h agents search only for type h agents. The opposite logic applies when $\lambda > 1/2$.

4.2 Complete Information Equilibria

We characterize below all possible pure and mixed equilibrium trial strategies under complete information. Notice that $(\tau_{hl} = 0, \tau_{ll} = 0)$ is never an equilibrium since type-l agents always have a strict incentive to start an evaluation phase and get married together when they are rejected by type-h agents.

Complete Information PAM Equilibrium ($\boldsymbol{\tau_{hl} = 0, \tau_{ll} = 1}$). Positive assortative matching is an equilibrium under complete information iff type-h agents are not willing to deviate by entering an evaluation phase with type-l agents (and hence accept to marry at the end of the evaluation phase). A type-h agent who accepts a type-l agent gets

$$\tilde{V}_{hl} = \beta u_{hl} + (1-\beta)\delta\tilde{V}_{hl} = \zeta(\beta)u_{hl},$$

while, if he rejects a type-l agent, he receives his continuation payoff δV_h where

$$\begin{aligned} V_h &= \hat{\lambda}_{PAM} \underbrace{(\beta u_{hh} + (1-\beta)\delta V_{hh})}_{V_{hh}} + (1-\hat{\lambda}_{PAM})\delta V_h \\ &= \frac{\hat{\lambda}_{PAM}\beta}{(1-(1-\hat{\lambda}_{PAM})\delta)(1-(1-\beta)\delta)} u_{hh} = \zeta(\hat{\lambda}_{PAM})\zeta(\beta)u_{hh}. \end{aligned} \tag{15}$$

Therefore, a positive assortative matching equilibrium exists iff $\delta V_h \geq \zeta(\beta)u_{hl}$, i.e.,

$$\frac{u_{hl}}{u_{hh}} \leq \delta\zeta(\hat{\lambda}_{PAM}). \tag{16}$$

Complete Information RM Equilibrium ($\boldsymbol{\tau_{hl} = 1, \tau_{ll} = 1}$). In this case, the dynamic of the game is the same as in the no-communication equilibrium: the steady-state proportion of type-h agents in the population of singles is $\hat{\lambda}(1,1) = \lambda$, and the equilibrium conditions simplify to

$$\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\lambda) \quad \text{and} \quad \frac{u_{ll}}{u_{lh}} \geq \delta\zeta(\lambda). \tag{17}$$

Complete Information UM Equilibrium ($\boldsymbol{\tau_{hl} = 1, \tau_{ll} = 0}$). Similarly, the conditions under which $(\tau_{hl} = 1, \tau_{ll} = 0)$ is a steady-state equilibrium are given by

$$\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\hat{\lambda}_{UM}) \quad \text{and} \quad \frac{u_{ll}}{u_{lh}} \leq \delta\zeta(\hat{\lambda}_{UM}). \tag{18}$$

Complete Information Mixed Strategy Equilibria. In a mixed strategy equilibrium, if $\tau_{hl} \in (0,1)$ then a type-h agent should be indifferent between entering an evaluation phase with a type-l agent or not. Following the same logic as for the PAM and RM equilibria above, the indifference condition is

$$\frac{u_{hl}}{u_{hh}} = \delta\zeta(\hat{\lambda}(\tau_{hl}, \tau_{ll})). \tag{19}$$

If $\tau_{ll} \in (0,1)$ then a type-l agent should be indifferent between entering an evaluation phase with a type-l agent or not. The analysis is slightly more difficult than before, mainly because τ_{hl} also appears in V_l. Indeed, τ_{hl} determines the probability that a single type-l agent starts an evaluation phase when he meets a type-h agent. A type-l agent who accepts a type-l agent gets

$$V_{ll} = \beta u_{ll} + (1-\beta)\delta V_{ll} = \zeta(\beta) u_{ll},$$

while if he rejects a type-l agent he gets δV_l, where

$$\begin{aligned} V_l &= \hat{\lambda}\left(\tau_{hl}\underbrace{(\beta u_{lh} + (1-\beta)\delta V_{lh})}_{V_{lh}} + (1-\tau_{hl})\delta V_l\right) + (1-\hat{\lambda})\delta V_l \\ &= \frac{\tau_{hl}\hat{\lambda}\beta}{(1-(1-\tau_{hl}\hat{\lambda})\delta)(1-(1-\beta)\delta)} u_{lh}. \end{aligned} \tag{20}$$

Therefore, the indifference condition for $\tau_{ll} \in (0,1)$ is

$$\frac{u_{ll}}{u_{lh}} = \delta\zeta(\tau_{hl}\hat{\lambda}(\tau_{hl}, \tau_{ll})). \tag{21}$$

The following proposition summarizes the characterization of all pure and mixed strategy equilibria under complete information.

Proposition 2. *There exists a pure strategy* complete information equilibrium *such that*

- *the matching is Positive Assortative iff* $\frac{u_{hl}}{u_{hh}} \leq \delta\zeta(\hat{\lambda}_{PAM})$,
- *the matching is Random iff* $\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\lambda)$ *and* $\frac{u_{ll}}{u_{lh}} \geq \delta\zeta(\lambda)$,
- *the matching is Upward iff* $\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\hat{\lambda}_{UM}) \geq \frac{u_{ll}}{u_{lh}}$,

There exists a mixed strategy complete information equilibrium *such that*

- $(\tau_{hl} = 1, \tau_{ll} \in (0,1))$ *iff* $\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\hat{\lambda}(1, \tau_{ll})) = \frac{u_{ll}}{u_{lh}}$,

- $(\tau_{hl} \in (0,1), \tau_{ll} = 1)$ *iff* $\frac{u_{hl}}{u_{hh}} = \delta\zeta(\hat{\lambda}(\tau_{hl}, 1))$ *and* $\delta\zeta(\tau_{hl}\hat{\lambda}(\tau_{hl}, 1)) \leq \frac{u_{ll}}{u_{lh}}$,
- $(\tau_{hl} \in (0,1), \tau_{ll} \in (0,1))$ *iff* $\frac{u_{hl}}{u_{hh}} = \delta\zeta(\hat{\lambda}(\tau_{hl}, \tau_{ll}))$ *and* $\delta\zeta(\tau_{hl}\hat{\lambda}(\tau_{hl}, \tau_{ll})) = \frac{u_{ll}}{u_{lh}}$,
- $(\tau_{hl} \in (0,1), \tau_{ll} = 0)$ *iff* $\frac{u_{hl}}{u_{hh}} = \delta\zeta(\hat{\lambda}(\tau_{hl}, 0))$ *and* $\delta\zeta(\tau_{hl}\hat{\lambda}(\tau_{hl}, 0)) \geq \frac{u_{ll}}{u_{lh}}$.

In addition all equilibria are such that $\tau_{ij} > 0 \Rightarrow \mu_{ij} = 1$, *and there is no other equilibrium than the ones described above.*

The ratios $\frac{u_{hl}}{u_{hh}}$ and $\frac{u_{ll}}{u_{lh}}$ for which a pure equilibrium with complete information exists are given by the threshold $\delta\zeta(\hat{\lambda}(\tau_{hl}, \tau_{ll}))$ while they were given by $\delta\zeta(\beta\lambda)$ under incomplete information (see Proposition 1). Hence, the thresholds determining the existence of the different matching conditions under complete information depend on β only through the effect of β on the steady-state proportions of singles. This difference is due to the fact that, in the complete information case, choices to accept or reject an agent are made before the evaluation phase starts. An agent therefore compares returning to a single situation to entering an evaluation phase. In both cases, any potential gain of marriage occur after an evaluation period, which expected length depends on β.

The thresholds determining existence of the different matching conditions under complete information depend on different steady state proportions of singles. It follows that, under complete information, several equilibrium configurations may co-exist. In particular, there are some ratios $\frac{u_{hl}}{u_{hh}}$ and $\frac{u_{ll}}{u_{lh}}$ for which PAM and UM equilibria co-exist, or PAM and RM equilibria co-exist. However, since we always have $\zeta(\hat{\lambda}_{UM}) < \zeta(\lambda)$ (see Lemma 1), UM and RM equilibria never coexist. Overall, three distinct orderings of the thresholds $\delta\zeta(\hat{\lambda}_{PAM})$, $\delta\zeta(\hat{\lambda}_{UM})$ and $\delta\zeta(\lambda)$ are possible. The corresponding pure strategies equilibria are illustrated by Figures 6, 7 and 8 in Appendix A.4.

4.3 Informative Communication Equilibria

We now examine the agents' incentives for telling the truth about their type depending on the matching that arises in the steady state under complete information. Indeed, for an equilibrium of the communication phase to be fully revealing, it has to be that no agent has an interest in misreporting his type when this report is believed and therefore induces the equilibrium trial and marriage strategies of the complete information case.

Recall that the trial strategy τ_{ij} is the probability with which a type-i agent starts an evaluation phase with an agent who *claimed to be* of type-j. It follows that, a type-i agent initially matched with a type-k first plays equilibrium trial strategy τ_{ij} when the type-k

claims to be of type-j, and then plays the equilibrium marriage strategy μ_{ik} at the end of the evaluation phase once the truth is known. When communication is truthful, we still have $\tau_{ij} > 0 \Rightarrow \mu_{ij} = 1$ as in the complete information case. However, if an agent deviates from truthful revelation at the cheap-talk stage, a pair of agents (i, j) who do not agree to enter the evaluation phase ($\tau_{ij} = 0$) might have to decide whether or not to marry after an evaluation phase if one of them lied about its type before starting this phase. That is, even if $\tau_{ij} = 0$, the informative communication equilibrium conditions will depend on the off-equilibrium path decisions $\mu_{ij} = 0$ or $\mu_{ij} = 1$.

Consider first deviation from truthtelling in a PAM equilibrium ($\tau_{hl} = 0, \tau_{ll} = 1$) (and hence $\mu_{ll} = 1$). If a type-h deviates from truthful communication and reveals that his type is l, then he will not start an evaluation phase if he is matched with another type-h agent (who will reject him), and he may start an evaluation phase if he is matched with a type-l agent. In the first case he his clearly worse off; in the second case the deviation is payoff-relevant only if he deviates from his trial strategy $\tau_{hl} = 0$ to $\tau_{hl} = 1$, which is not profitable under the complete information equilibrium conditions of the positive assortative matching outcome. Now consider a deviation by a type-l agent, who claims that his type is h. If $\mu_{hl} = 1$, then this deviation is profitable because he will start an evaluation phase with a type-h agent who will accept to marry him at the end of the evaluation phase. Note that, this type-h would however have rejected a type-l agent before starting the evaluation phase. Hence, to have truthful communication with a positive assortative matching outcome, a type-h agent should prefer to reject a type-l agent at the end of the evaluation phase, i.e., we must have $u_{hl} \leq \delta V_h$, where V_h is the same continuation payoff as in (15): $V_h = \zeta(\hat{\lambda}_{PAM})\zeta(\beta)u_{hh}$. Therefore, there is a fully revealing equilibrium with PAM only if

$$\frac{u_{hl}}{u_{hh}} \leq \delta\zeta(\hat{\lambda}_{PAM})\zeta(\beta). \tag{22}$$

Notice that this condition is strictly stronger than the condition for a positive assortative matching equilibrium to exist under complete information (see Condition (16)) since $\delta\zeta(\beta)\zeta(\hat{\lambda}_{PAM}) < \delta\zeta(\hat{\lambda}_{PAM})$. Hence, (22) is a necessary and sufficient condition for an informative communication equilibrium with positive assortative matching.

Consider now the RM equilibrium $\tau_{hl} = \tau_{ll} = 1$ (and hence $\mu_{hl} = \mu_{ll} = 1$). Since the trial and marriages do not depend on information, truthful information transmission is clearly incentive compatible since it is not influential.

Next, consider the UM equilibrium ($\tau_{hl} = 1, \tau_{ll} = 0$) (and hence $\mu_{hl} = 1$). Here, the conditions for full information transmission are implied by the equilibrium conditions under complete information: if a type-h agent pretends to be a type-l agent, he gets the same payoff as if he told the truth and chose $\tau_{hl} = 0$ instead of $\tau_{hl} = 1$; if a type-l agent pretends to be a type-h agent, he gets the same payoff as if told the truth and chose $\tau_{ll} = 1$ instead of $\tau_{ll} = 0$. Therefore, there is a fully revealing UM equilibrium iff there is an UM equilibrium under complete information. Exactly the same reasoning applies to the mixed UM equilibrium outcome $(\tau_{hl} = 1, \tau_{ll} \in (0, 1))$.

Finally, consider a mixed equilibrium in which $\tau_{hl} \in (0, 1)$; such an equilibrium cannot be implemented with cheap talk because $\tau_{hl} \in (0, 1) \Rightarrow \mu_{hl} = 1$, so a type-$l$ agent will always pretend to be a type-h agent to be able to start an evaluation phase with a type-h agent, who will then accept the type-l agent at the end of the evaluation phase. Indeed, if a type-h agent is indifferent between starting an evaluation phase with a type-l agent $(\tau_{hl} \in (0, 1))$ then he will strictly prefer to marry him at the end of the evaluation phase $(\mu_{hl} = 1)$.

The following proposition (illustrated by Figure 5) summarizes the equilibrium outcomes of the matching game when communication is informative.

Proposition 3. *There exists an* informative communication equilibrium *such that*

- *the matching is Positive Assortative iff* $\frac{u_{hl}}{u_{hh}} \leq \delta\zeta(\beta)\zeta(\hat{\lambda}_{PAM})$,
- *the matching is Random iff* $\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\lambda)$ *and* $\frac{u_{ll}}{u_{lh}} \geq \delta\zeta(\lambda)$,
- *the matching is Upward iff* $\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\hat{\lambda}_{UM}) \geq \frac{u_{ll}}{u_{lh}}$,
- *the matching is Mixed Upward* $(\tau_{hl} = 1, \tau_{ll} \in (0, 1))$ *iff* $\frac{u_{hl}}{u_{hh}} \geq \delta\zeta(\hat{\lambda}(1, \tau_{ll})) = \frac{u_{ll}}{u_{lh}}$.

In addition there is no other informative communication equilibrium than the ones described above.

As can be seen on Figure 5, truthful communication may not always obtain in equilibrium. In particular, if the complete information equilibrium is PAM and the Condition (22) does not hold, type-l agents never reveal their types truthfully.

5 Informative Communication vs. No-Communication

In this section we discuss the conditions for informative communication obtained so far and its impact on the matching configuration, on players' incentives and on welfare.

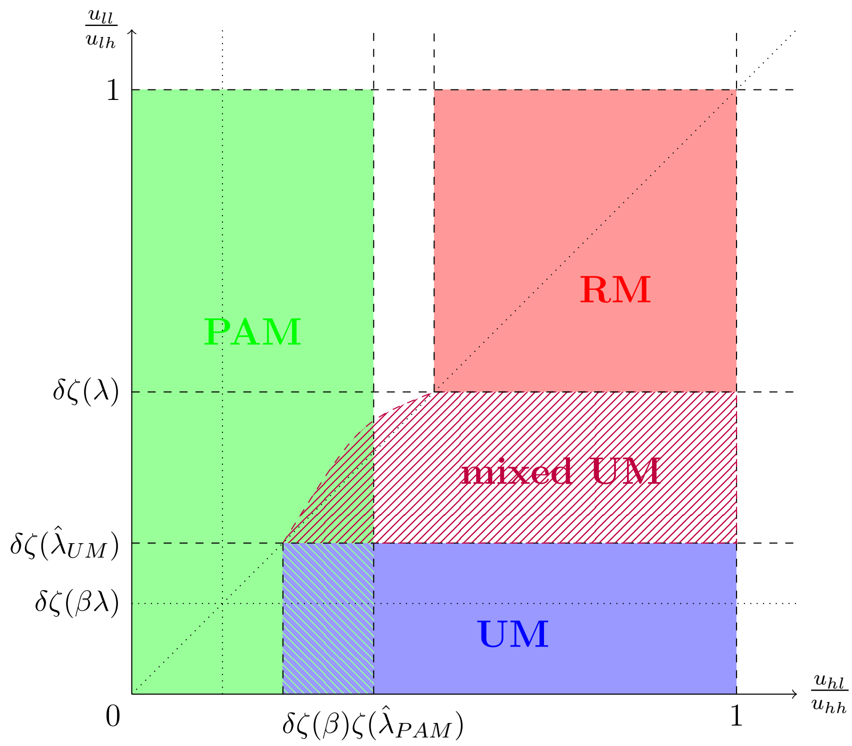

Figure 5: Informative communication equilibria when $\zeta(\beta)\zeta(\hat{\lambda}_{PAM}) > \delta\zeta(\hat{\lambda}_{UM})$.

5.1 Conditions for Truthtelling

Sections 3 and 4 fully characterize the equilibria of our game in the cases of no communication, complete information and informative communication. We saw that incentives to transmit information truthfully crucially depend on the steady-state matching equilibrium that follows. When this equilibrium is random, upward or mixed upward, informative communication is always possible. This results relies on the fact that, in any such matching equilibrium, no type is rejected by a type that he would like to marry. As a consequence, no lie can improve an agent's final matching situation. Under upward matching for instance, type-h agents are finally matched with type-l agents because they both would like to marry each other in equilibrium. It follows that a type-h cannot benefit from claiming his type is low so as to induce rejection from type-l agents.

However, when the matching equilibrium is positive assortative, low types are rejected by high types while they would like to marry them. It follows that low types can have an interest in claiming they are high to enter an evaluation phase with the high types. To prevent such a lie and sustain informative communication, low types should be rejected by high types even at the end of the evaluation phase. This happens under Condition (22) which states that going on searching should be attractive enough for high types. It is the case if high types are patient enough (high δ), if the next evaluation phases are not too long (high β), if there are enough single type-h available (high λ_{PAM}) or if the payoff difference in between marrying a high type and a low type is large enough (small $\frac{u_{hl}}{u_{hh}}$).

5.2 Effect of Informative Communication on Matching Outcomes

When informative communication obtains, it has the crucial effect of advancing the time at which agents take the informed decisions of whether or not to accept a given type. We identify below two effects that early informed decisions have on search behavior and, incidentally, on final matching compared to the case of no communication.

First, early informed decisions have a direct effect on agents' incentives to accept low-type agents. In the no-communication case, agents decide to accept their current partner at the end of the evaluation phase by comparing the immediate gain from marriage with being single again. Under informative communication, informed decisions are taken before an evaluation period may start so that agents do not spend time in unfruitful evaluation phases. It follows that, everything else being equal, the opportunity cost of rejecting the current partner is lower under informative communication, therefore making rejection of low type agents even more appealing. Said differently, informative communication makes every agent want to be "pickier". The lower is β, the stronger is this effect as the expected length of an evaluation phase is longer.

Second, the continuation payoff of being single may not be the same when communication is absent and when it is informative. This stems from the fact that the distribution of types within the pool of singles may be different in the two situations. As already pointed out, agents do not internalize the effect of their marriage strategy on the mix of agents seeking matches. In our model, thanks to the "cloning" assumption, these search externalities play no role when informed decisions are taken at the end of the evaluation phase: the proportions of high type agents in the population of singles and in the whole population coincide (see Section 3). However, when informed decisions are taken at the time of meeting, search externalities are magnified since rejected agents return to the pool of singles right away. It follows that, under informative communication, the proportion of high type agents in the population of singles become very sensitive to agents' decision to form matches. As shown in Section 4, depending on the steady-state matching and on the parameter λ, the proportion of high-type agents in the population of singles can be either lower or higher than in the whole population. This in turn affects incentives: everything else being equal, when the proportion of singles type-h agents increases (decreases), incentives to reject type-l agents are stronger (weaker).

It is the sum of these two (possibly opposite) effects, combined with the incentive constraints for informative communication under positive assortative matching, that determines how equilibrium matching configurations could be sustained and could change with communication. Formally, the impact of communication on equilibrium outcomes relies on the detailed comparisons of the thresholds values obtained in Propositions 1 and 3. These comparisons are provided in Appendix A.2. For example, under random matching, only the direct effect of informative communication is at play as the proportions of singles are the same for the cases of no-communication and informative communication. Hence, the conditions for a RM equilibrium to exist are unambiguously stronger under informative communication than under no-communication (formally, $\delta\zeta(\beta\lambda) \leq \delta\zeta(\lambda)$). In the case of UM, the conditions for a UM to exist are weaker for type-l agents and stronger for type-h agents with informative communication than without (formally, $\delta\zeta(\beta\lambda) \leq \delta\zeta(\hat{\lambda}_{UM})$). Finally, depending on the strength of informational incentive constraints, a PAM equilibrium may be harder or easier to sustain under informative communication compared to no-communication (formally, $\zeta(\beta)\zeta(\hat{\lambda}_{PAM})$ may be smaller or higher than $\zeta(\beta\lambda)$).

Overall, the comparisons of the equilibrium conditions under no-communication and informative communication show that communication can generically induce the seven possible changes of matching configurations represented in the left part of Table 1.

	no com.		informative com.	Type-h agents $\hat{U}_h - U_h$	Type-l agents $\hat{U}_l - U_l$
Fixed Matching	PAM	$\rightarrow$	$\widehat{\text{PAM}}$	$+$	$+$
	UM	$\rightarrow$	$\widehat{\text{UM}}$	$-$	$+$
	RM	$\rightarrow$	$\widehat{\text{RM}}$	$=$	$=$
Changing Matching	RM	$\rightarrow$	$\widehat{\text{PAM}}$	$+$	$-$
	UM	$\rightarrow$	$\widehat{\text{PAM}}$	$+$	$-$
	RM	$\rightarrow$	$\widehat{\text{UM}}$	$-$	$+$
	RM	$\rightarrow$	mixed $\widehat{\text{UM}}$	$-$	$+$

Table 1: Changes in Matching Configurations and Welfare Comparisons.

Interestingly, not all changes in matching configurations are possible when communication becomes informative. A good intuition is obtained by remembering the direct effect of informative communication described above: early informed decisions tend to give stronger incentives to reject type-l agents. As type-h agents are accepted by every type of agent, their own decisions affect their match in equilibrium. Consequently, high-type agents are effectively pickier under informative communication. This implies, for instance, that if the matching is positive assortative without communication, it cannot switch to another matching configuration once agents communicate.

The effect of informative communication on type-l agents' "pickiness" is more subtle. Indeed, their final matching situation depends on whether or not they are accepted by high types. If type-h agents are not strictly pickier with informative communication, then type-l agents have indeed stronger incentives to reject type-l agents. But if type-h agents are strictly pickier with informative communication, it means that type-l agents are then rejected by type-h agents. Type-l agents therefore have no other choice than accepting each others, that is, being less picky. According to this observation, a switch from RM to $\widehat{UM}$ is possible as type-l agents are given some room to be strictly pickier under informative communication by the fact type-h agents accept to marry everybody else. However, if the matching configuration is upward with no communication, type-l agents should be at least as picky under informative communication if type-h agents continue to accept them; that is, type-l agents should continue to reject all type-l agents, meaning that a switch from UM to $\widehat{RM}$ or mixed $\widehat{UM}$ is impossible.

The following proposition summarizes the observations made above about the impact of informative communication on agents' requirement.

Proposition 4.

1. *High type agents are pickier under informative communication:*

$$\hat{\mu}_{hl} \leq \mu_{hl};$$

2. *If informative communication does not make high type agents strictly pickier, then it makes low type agents pickier:*

$$\hat{\mu}_{hl} = \mu_{hl} \;\Rightarrow\; \hat{\mu}_{ll} \leq \mu_{ll};$$

3. *If informative communication makes high type agents strictly pickier, then it makes low type agents less picky:*

$$\hat{\mu}_{hl} < \mu_{hl} \;\Rightarrow\; \hat{\mu}_{ll} \geq \mu_{ll}.$$

5.3 Welfare Effects of Informative Communication

In this section we study the effect of strategic communication on agents' welfare. We denote by

$$U_i := n_i V_i + n_{il} V_{il} + n_{ih} V_{ih}, \tag{23}$$

the total expected welfare of all type-i agents in a stationary equilibrium. Table 1 summarizes the comparisons of the total expected welfare for each type of agent and for each possible equilibrium matching configuration when one moves from a no-communication equilibrium to an informative communication equilibrium (appendix A.3 contains the detailed computations). These welfare comparisons lead to the following proposition.

Proposition 5.

- *High type agents are strictly better off under informative communication iff communication leads to positive assortative matching;*
- *Low type agents are strictly better off under informative communication iff communication leads to upward matching or if it leads to positive assortative matching without changing the matching configuration.*

In particular, Proposition 5 shows that communication is Pareto improving if and only if the matching is assortative in the absence of communication and left unchanged by information transmission.[14]

We provide below some intuitions of these welfare results. For instance, consider a situation in which both the no-communication matching equilibrium and the informative communication matching equilibrium are positive assortative. In this case, all agents (low and high types agents) are always better off in the matching game with informative communication. The direct intuition of this result is simple: when agents reveal their true type before choosing whether to start an evaluation phase or not, low (high, resp.) type agents do not waste their time anymore in an evaluation phase with high (low, resp.) type agents ($\hat{n}_{ij} = 0$ while $n_{ij} > 0$, $i \neq j$). Hence, the number of type-i agents in an evaluation phase with other type-i agents is higher with informative communication than without ($\hat{n}_{hh} > n_{hh}$ and $\hat{n}_{ll} > n_{ll}$), so agents end up being married assortatively more quickly.

However, the result is not so direct because this intuition does not take into account the effect of communication on the proportions of high and low singles in the population ($\hat{\lambda}_{PAM}$

[14] That is, from Propositions 1 and 3, communication is Pareto improving if and only if $\frac{u_{hl}}{u_{hh}} \leq \min\{\delta\zeta(\beta\lambda), \delta\zeta(\beta)\zeta(\hat{\lambda}_{PAM})\}$.

is different from λ, see Figure 4). These proportions have a strong impact on the probability that a single agent is matched with another agent of the same type. Consider for example single type-h agents. When $\lambda \leq 1/2$, we have $\hat{\lambda}_{PAM} \geq \lambda$ (see Lemma 1), so it can be shown that $\hat{V}_h - V_h \geq 0$. But when λ is high enough, single type-h agents may be *worse off* with informative communication. Similarly, when $\lambda \geq 1/2$, we have $\hat{V}_l - V_l \geq 0$, but when λ is low enough single type-l agents are worse off with informative communication. This possible negative proportion effect of communication on single agents is always compensated by the efficiency gained from avoiding non-assortative evaluation phases. This efficiency gain is however absent for high type agents under upward matching when communication does not change the matching configuration. Indeed, information does not affect the number of single high types under UM but it increases the number of single low types. Hence, high types are matched with low types more often with information. This results in a lower expected payoff for both single and in an evaluation phase. Therefore, in this situation, high type agents are unambiguously worse off with information than without.

6 Conclusion

In this article, we propose a model that incorporates cheap-talk communication into a dynamic search and matching model with asymmetric information. The key feature lies in that agents who are randomly paired have to go through a costly evaluation phase to discover each other's types, and finally take an informed decision to be definitely matched or not. We show that information transmission occurring bilaterally right after agents have been paired can be truthful despite every agent's preference to be matched with high-type partners. This result relies on the following simple statement: low-type agents never have an interest to overstate their type if high-type agents refuse to match once the truth is discovered. This condition holds if high type agents are patient enough, evaluation phases are not too long, and the proposition of single high-type agents is large enough.

By considering only two types, the model enables to fully characterize the matching configurations that emerge in equilibrium under no communication and informative communication. One main difference between these two regimes is the moment at which the informed decision is taken: at the end of the evaluation phase in the case of no-communication, and as soon as agents meet with informative communication. For a given set of parameters, it follows that informative communication can significantly modify agents' matching decisions compared to no-communication: high-type agents, for instance, can afford being pickier because they decide whether or not to be matched with a revealed type before having lost time in an evaluation phase. Taking this informed decision earlier makes them more prone to reject low-type agents than without communication. The reasoning is however not as simple as that because communication induces search externalities: when agents refuse to start an evaluation phase with low types, they do not internalize that their decision affect everyone's probability of meeting high types. Our equilibrium analyzes take into account this feedback effect that trial strategies have on the steady-state composition of singles in the population. The comparison of agents' welfare in equilibrium when communication becomes informative reveals that agents who are harmed or benefits from communication depend on the induced equilibrium matching configuration. One main result is that communication is Pareto improving only when the matching is assortative in the absence of communication and left unchanged by truthful information transmission.

This paper is a first attempt to study the role of cheap talk communication when discovering a partner's type is costly. It could be extended in several directions. We have considered a rather elementary opportunity to transmit information as agents communicate only with their partner and about their own type. When a match is not successful, they might transmit to other market participants the information they just acquired about their partner. There could even be room for communication intermediaries who would gather and pass information collected through private search.

A Appendix

A.1 Proof of Lemma 1

$$\hat{\lambda}_{PAM} = \frac{2\lambda}{2\lambda + \beta(1-2\lambda) + \sqrt{\beta^2(1-2\lambda)^2 + 4\lambda(1-\lambda)}},$$

$$\hat{\lambda}_{UM} = \frac{2(1-\beta)\lambda - 1 + \sqrt{1 - 4(1-\beta)\lambda(1-\lambda)}}{2(1-\beta)\lambda}.$$

1. We have:

$$\hat{\lambda}_{PAM} - \lambda = \frac{\lambda(2 - 2\lambda - \beta(1-2\lambda) - \sqrt{A})}{2\lambda + \beta(1-2\lambda) + \sqrt{A}},$$

where $A = \beta^2(1-2\lambda)^2 + 4\lambda(1-\lambda)$. Since, for all $\lambda \in [0,1]$, $2 - 2\lambda - \beta(1-2\lambda) \geq 0$ we have

$$\hat{\lambda}_{PAM} \geq \lambda \;\Leftrightarrow\; (2 - 2\lambda - \beta(1-2\lambda))^2 \geq A.$$

Then, simple calculations show that

$$(2 - 2\lambda - \beta(1-2\lambda))^2 - A = 4(1-\beta)(1-\lambda)(1-2\lambda)$$

which has the sign of $1-2\lambda$. Therefore $\hat{\lambda}_{PAM} \geq \lambda$ iff $\lambda \leq 1/2$.

We also have $\hat{\lambda}_{PAM} \geq \beta\lambda$ iff

$$\frac{2}{2\lambda+\beta(1-2\lambda)+\sqrt{\beta^2(1-2\lambda)^2+4\lambda(1-\lambda)}} \geq \beta$$
$$\Leftrightarrow (2-\beta(2\lambda+\beta(1-2\lambda))^2 \geq \beta^2(\beta^2(1-2\lambda)^2+4\lambda(1-\lambda))$$
$$\Leftrightarrow 1+\beta(1-2\lambda)(1-\beta\lambda) \geq 0,$$

which is always satisfied since $\beta(2\lambda-1) \leq 1$.

2. To show that $\hat{\lambda}_{UM} \leq \hat{\lambda}(1,\tau_{ll}) \leq \lambda$, it suffices to observe that $\hat{\lambda}(1,0) = \hat{\lambda}_{UM}$, $\hat{\lambda}(1,1) = \lambda$ and that

$$\hat{\lambda}(1,\tau_{ll}) = \frac{\sqrt{A}-(1-2(1-\beta)\lambda(1-\tau_{ll}^2))}{2(1-\beta)\lambda(1-\tau_{ll}^2)},$$

where $A = 1-4(1-\beta)\lambda(1-\lambda)(1-\tau_{ll}^2)$, is increasing in τ_{ll}. In addition, we have

$$\hat{\lambda}_{UM} - \beta\lambda = \frac{2\lambda(1-\lambda)(1-\beta)-1+\sqrt{B}}{2(1-\beta)\lambda},$$

where $B = 1-4\lambda(1-\lambda)(1-\beta)$. Since, for all $\lambda \in [0,1]$, $1-2\lambda(1-\lambda)(1-\beta) \geq 0$, we have

$$\hat{\lambda}_{UM} \geq \beta\lambda \;\Leftrightarrow\; B \geq (1-2\lambda(1-\beta\lambda)(1-\beta))^2.$$

Notice that $B-(1-2\lambda(1-\beta\lambda)(1-\beta))^2 = 4\lambda^3(2-\beta\lambda)\beta(1-\beta)^2 \geq 0$. Hence, $\hat{\lambda}_{UM} \geq \beta\lambda$.

A.2 Comparisons of no-communication and informative communication equilibrium conditions

Denote $\frac{u_{hl}}{u_{hh}}$ by r_h and $\frac{u_{ll}}{u_{lh}}$ by r_l. To compare the existence conditions of no communication and informative communication equilibria given by Propositions 1 and 3, we use the two following statements:

(i) $\delta\zeta(\beta\lambda) \leq \delta\zeta(\lambda)$ since $\beta\lambda \leq \lambda$ and $\delta\zeta(.)$ is an increasing function;

(ii) $\delta\zeta(\beta\lambda) \leq \delta\zeta(\hat{\lambda}_{UM}) \leq \delta\zeta(\hat{\lambda}(1,\tau_{ll}))$ as Lemma 1 states $\beta\lambda \leq \hat{\lambda}_{UM} \leq \hat{\lambda}(1,\tau_{ll})$ for all λ;

The following list gives the set of parameters for which one can have existence of a no-communication and an informative communication equilibria:

PAM and $\widehat{RM}$: from (i), they never coexist since it would require that $r_h = \delta\zeta(\beta\lambda) = \delta\zeta(\lambda)$;

PAM and $\widehat{UM}$: from (ii), they never coexist since it would require thatr $r_h = \delta\zeta(\beta\lambda) = \delta\zeta(\hat{\lambda}_{UM})$;

PAM and mixed $\widehat{UM}$: from (ii), they never coexist since it would require that $r_h = r_l = \delta\zeta(\beta\lambda) = \delta\zeta(\hat{\lambda}_{UM})$;

PAM and $\widehat{PAM}$: they both exist if $r_h \leq \text{Min}\{\delta\zeta(\beta\lambda), \delta\zeta(\beta)\zeta(\hat{\lambda}_{PAM})\}$;

RM and $\widehat{RM}$: from (i), they both exist if $r_h, r_l \geq \delta\zeta(\lambda)$;

RM and $\widehat{UM}$: from (ii), they both exist if $\delta\zeta(\hat{\lambda}_{UM}) \geq r_l \geq \delta\zeta(\beta\lambda)$ and $r_h \geq \delta\zeta(\hat{\lambda}_{UM})$;

RM and mixed $\widehat{UM}$: from (ii), they both exist if $r_h \geq \delta\zeta(\lambda(1, \tau_{ll})) = r_l$;
RM and $\widehat{PAM}$: they both exist if $\delta\zeta(\beta)\zeta(\hat{\lambda}_{PAM}) \geq r_h \geq \delta\zeta(\beta\lambda)$ and $r_l \geq \delta\zeta(\beta\lambda)$;
UM and $\widehat{RM}$: from (i), they never coexist since it would require that $r_l = \delta\zeta(\beta\lambda) = \delta\zeta(\lambda)$;
UM and $\widehat{UM}$: from (ii), they both exist if $r_h \geq \delta\zeta(\hat{\lambda}_{UM})$ and $r_l \leq \delta\zeta(\beta\lambda)$;
UM and mixed $\widehat{UM}$: from (ii), they never coexist since it would require that $r_h = r_l = \delta\zeta(\beta\lambda) = \delta\zeta(\hat{\lambda}_{UM})$;
UM and $\widehat{PAM}$: they both exist if $\delta\zeta(\beta)\zeta(\hat{\lambda}_{PAM}) \geq r_h \geq \delta\zeta(\beta\lambda)$ and $r_l \leq \delta\zeta(\beta\lambda)$.

A.3 Welfare Comparisons

Proposition 6. *The total expected welfare of both low and high type agents is higher in an informative communication PAM equilibrium than in a no-communication PAM equilibrium.*

Proof. Let $\hat{U}_i$ and U_i denote respectively the type-i agents total expected welfare in an informative communication and in a no-communication PAM equilibrium. We show that (a) $\hat{U}_h \geq U_h$ and (b) $\hat{U}_l \geq U_l$.

(a) $\hat{U}_h \geq U_h$**.** We have:

$$\hat{U}_h - U_h = \hat{n}_{hh}\hat{V}_{hh} + \hat{n}_h\hat{V}_h - n_{hh}V_{hh} - n_hV_h - n_{hl}V_{hl}. \tag{24}$$

Notice that $\hat{V}_{hh} = V_{hh} = \zeta(\beta)u_{hh}$. Rearranging terms in equation (24), we obtain:

$$\begin{aligned} \hat{U}_h - U_h &= (\hat{n}_{hh} - n_{hh})V_{hh} + \hat{n}_h\hat{V}_h - (n_h + n_{hl})V_h + n_{hl}(V_h - V_{hl}), \\ &= (\hat{n}_{hh} - n_{hh})(V_{hh} - \hat{V}_h) + (\lambda - n_{hh})(\hat{V}_h - V_h) + n_{hl}(V_h - V_{hl}), \end{aligned} \tag{25}$$

where the second equality stems from the fact that $n_h + n_{hl} + n_{hh} = \lambda$ and $\hat{n}_{hh} + \hat{n}_h = \lambda$.

Notice that $V_{hh} - \hat{V}_h \geq 0$, a type h is better off being engaged with a type-h agent than being single, and $V_h - V_{hl} \geq 0$, a type h is better off being single than being engaged with a type-l agent that he will reject in the end. To show that $\hat{U}_h - U_h \geq 0$ we proceed in two steps: we show that (i) $\hat{n}_{hh} - n_{hh} \geq 0$ and (ii) $(\lambda - n_{hh})(\hat{V}_h - V_h) + n_{hl}(V_h - V_{hl}) \geq 0$.

(i) Recall that

$$n_{hh} = (1-\beta)\lambda^2 \text{ and } \hat{n}_{hh} = (1-\beta)\lambda - \frac{\beta}{2(1+\beta)}\left(\sqrt{\beta^2 + 4\lambda(1-\lambda)(1-\beta)} - \beta\right),$$

so that

$$\hat{n}_{hh} - n_{hh} = \lambda(1-\lambda)(1-\beta) - \frac{\beta}{2(1+\beta)}\left(\sqrt{\beta^2 + 4\lambda(1-\lambda)(1-\beta)} - \beta\right).$$

Therefore

$$\begin{aligned} \hat{n}_{hh} - n_{hh} \geq 0 \quad &\Leftrightarrow \quad \left(\tfrac{2(1+\beta)}{\beta}\lambda(1-\lambda)(1-\beta) + \beta\right)^2 - \beta^2 - 4\lambda(1-\lambda)(1-\beta) \geq 0, \\ &\Leftrightarrow \quad \tfrac{4(1-\beta^2)^2\lambda^2(1-\lambda)^2}{\beta^2} \geq 0, \end{aligned}$$

which concludes step (i).

(ii) Recall that $\lambda - n_{hh} = \lambda - (1-\beta)\lambda^2$ and $n_{hl} = (1-\beta)\lambda(1-\lambda)$. Tedious but straightforward calculations show that:

$$\begin{aligned} & (\lambda - n_{hh})(\hat{V}_h - V_h) + n_{hl}(V_h - V_{hl}) \\ = & \lambda \Big((1-(1-\beta)\lambda)(\zeta(\beta)\zeta(\hat{\lambda}_{PAM}) - \zeta(\beta\lambda)) + (1-\beta)(1-\lambda)\zeta(\beta\lambda)(1-\delta\zeta(\beta)) \Big) u_{hh}, \\ = & \lambda\zeta(\beta)\zeta(\beta\lambda)\frac{(1-\delta)}{\beta}\frac{2(1-\lambda) - \beta(1-(1-\beta)\delta)(\sqrt{A} - \beta(2\lambda-1))}{(1-\delta)(\sqrt{A} - \beta(2\lambda-1)) + 2\lambda} u_{hh}, \end{aligned} \tag{26}$$

where $A = \beta^2 + 4\lambda(1-\lambda)(1-\beta^2)$. Notice first that $\sqrt{A} - \beta(2\lambda - 1) \geq \sqrt{A} - \beta \geq 0$, so that the denominator is positive. This also shows, in particular, that the numerator is (linearly) increasing in δ. It is equal to $2(1-\lambda) - \beta(\sqrt{A} - \beta(2\lambda - 1))$ when $\delta = 0$. To conclude, notice that:

$$\begin{aligned} 2(1-\lambda) - \beta(\sqrt{A} - \beta(2\lambda-1)) \geq 0 \quad & \Leftrightarrow \quad (2(1-\lambda) + \beta^2(2\lambda-1))^2 - \beta^2 A \geq 0, \\ & \Leftrightarrow \quad 4(1-\beta^2)(1-\lambda)^2 \geq 0 \end{aligned}$$

which always holds.

(b) $\hat{U}_l \geq U_l$. The proof is similar to part (a). From $V_{ll} = \hat{V}_{ll} = \zeta(\beta)u_{ll}$ and $1 - \lambda = n_l + n_{lh} + n_{ll} = \hat{n}_l + \hat{n}_{ll}$, we get:

$$\hat{U}_l - U_l = (\hat{n}_{ll} - n_{ll})(V_{ll} - \hat{V}_l) + (1 - \lambda - n_{ll})(\hat{V}_l - V_l) + n_{lh}(V_l - V_{lh}). \tag{27}$$

In a PAM equilibrium, a type l is never married with type-h agents, he is better off being engaged with a type l agent than being single, i.e. $V_{ll} - \hat{V}_l \geq 0$, and he is better off being single than being engaged with a type-l agent, i.e. $V_l - V_{lh} \geq 0$. In the following, we show that (i) $\hat{n}_{ll} - n_{ll} \geq 0$ and (ii) $(1-\lambda-n_{ll})(\hat{V}_l - V_l) + n_{lh}(V_l - V_{lh}) \geq 0$.

(i) We have

$$\hat{n}_{ll} - n_{ll} = \frac{2(1-\lambda)\lambda + \beta^2(1 - 2(1-\lambda)\lambda) - \beta\sqrt{A}}{2(1+\beta)},$$

so $\hat{n}_{ll} - n_{ll} \geq 0 \Leftrightarrow 4(1-\beta^2)^2(1-\lambda)^2\lambda^2 \geq 0$, which is always satisfied.

(ii) Tedious but straightforward calculations show that:

$$\begin{aligned} & (1-\lambda-n_{ll})(\hat{V}_l - V_l) + n_{lh}(V_l - V_{lh}) \\ = & (1-\lambda) \Big((1-(1-\beta)(1-\lambda))(\zeta(\beta)\zeta(1-\hat{\lambda}_{PAM}) - \zeta(\beta(1-\lambda))) + (1-\beta)\lambda\zeta(\beta(1-\lambda))(1-\delta\zeta(\beta)) \Big) u_l \\ = & \lambda\zeta(\beta)\zeta(\beta(1-\lambda))\frac{(1-\delta)}{\beta}\frac{\sqrt{A} - \beta(1 - 2(1-\beta)\delta(1-\lambda)^2)}{\sqrt{A} - \beta(2\lambda-1) + 2\lambda(1-\delta)} u_{ll}. \end{aligned}$$

Notice that $\beta(1-2(1-\beta)\delta(1-\lambda)^2) \leq \beta$. Hence, since $\sqrt{A} \geq \beta$, the numerator is positive. Then, notice that $\sqrt{A}-\beta(2\lambda-1)+2\lambda(1-\delta) \geq \sqrt{A}-\beta \geq 0$ which shows that the denominator is positive and concludes the proof. $\square$

Proposition 7. *The total expected welfare of high type (low type, respectively) agents is lower (higher, respectively) in an informative communication UM equilibrium than in a no-communication UM equilibrium.*

Proof. Let $\hat{U}_i$ and U_i denote respectively the type-i agents total expected welfare in an information communication and in a no-communication UM equilibrium. We show that (a) $\hat{U}_h \leq U_h$ and (b) $\hat{U}_l \geq U_l$.

(a) $\hat{U}_h \leq U_h$. In a no-communication UM equilibrium we have

$$n_h = \beta\lambda,\ n_{hh} = (1-\beta)\lambda^2,\ n_{hl} = (1-\beta)(1-\lambda)\lambda,$$

$$V_{hh} = \zeta(\beta)u_{hh},\ V_{hl} = \zeta(\beta)u_{hl},\ V_h = \zeta(\beta)\left(\lambda u_{hh} + (1-\lambda)u_{hl}\right).$$

Hence:

$$\begin{aligned} U_h &= n_h V_h + n_{hl}V_{hl} + n_{hh}V_{hh}, \\ &= \zeta(\beta)\lambda(\lambda u_{hh} + (1-\lambda)u_{hl}). \end{aligned} \tag{28}$$

Similarly, in a fully revealing UM equilibrium, we have

$$\hat{n}_h = \beta\lambda,\ \hat{n}_{hh} = (1-\beta)\lambda\hat{\lambda}_{UM},\ \hat{n}_{hl} = (1-\beta)\lambda(1-\hat{\lambda}_{UM}),$$

$$\hat{V}_{hh} = V_{hh} = \zeta(\beta)u_{hh},\ \hat{V}_{hl} = V_{hl} = \zeta(\beta)u_{hl},\ \hat{V}_h = \zeta(\beta)(\hat{\lambda}_{UM}u_{hh} + (1-\hat{\lambda}_{UM})u_{hl}).$$

Hence:

$$\begin{aligned} \hat{U}_h &= \hat{n}_h\hat{V}_h + \hat{n}_{hl}\hat{V}_{hl} + \hat{n}_{hh}\hat{V}_{hh}, \\ &= \zeta(\beta)\lambda(\hat{\lambda}_{UM}u_{hh} + (1-\hat{\lambda}_{UM})u_{hl}). \end{aligned} \tag{29}$$

By Lemma 1, we have $\hat{\lambda}_{UM} \leq \lambda$. Then, Equations (28) and (29) together yield $\hat{U}_h \leq U_h$.

(b) $\hat{U}_l \geq U_l$. By analogy with the proof of Proposition 6, we rewrite $\hat{U}_l - U_l$ as follows:

$$\hat{U}_l - U_l = (\hat{n}_{lh} - n_{lh})(V_{lh} - \hat{V}_l) + (1-\lambda-n_{lh})(\hat{V}_l - V_l) + n_{ll}(V_l - V_{lh}). \tag{30}$$

Let us prove first that the first term in the rhm of Equation (30) is positive. Notice first that $V_{lh} \geq \hat{V}_l$: in a UM equilibrium, low types are only matched with high types, hence, a low type is better off being engaged with a high type than being single. Then, notice that

$$\begin{aligned} \hat{n}_{lh} \geq n_{lh} &\Leftrightarrow \tfrac{1}{2}(1-\sqrt{1-4\lambda(1-\lambda)(1-\beta)}) \geq \lambda(1-\lambda)(1-\beta), \\ &\Leftrightarrow (1-2\lambda(1-\lambda)(1-\beta))^2 \geq 1-4\lambda(1-\lambda)(1-\beta), \\ &\Leftrightarrow (2\lambda(1-\lambda)(1-\beta))^2 \geq 0. \end{aligned}$$

Therefore, the first term in the rhs of Equation (30) is positive: $(\hat{n}_{lh} - n_{lh})(V_{lh} - \hat{V}_l) \geq 0$.

Let us show that the sum of the second and third terms in the rhs of Equation (30) are also positive: $(1-\lambda-n_{lh})(\hat{V}_l - V_l) + n_{ll}(V_l - V_{lh}) \geq 0$. It is immediate that this is equivalent to showing that:

$$\Delta := (1-(1-\beta)\lambda)(\zeta(\beta)\zeta(\hat{\lambda}_{UM}) - \zeta(\beta\lambda)) + (1-\beta)(1-\lambda)\zeta(\beta\lambda)(1-\delta\zeta(\beta)) \geq 0.$$

Denote by $B = 1-4\lambda(1-\lambda)(1-\beta)$. Replacing function $\zeta(.)$ by its expression in the above equation shows that Δ rewrites $\Delta = (1-\delta)N/D$, where

$$N(\lambda,\beta,\delta) = \beta\left(-1+\sqrt{B}-(1-\beta)\lambda\left(-3+\sqrt{B}+\lambda(2-\beta\delta(\sqrt{B}-1))\right)\right), \tag{31}$$

$$D(\lambda,\beta,\delta) = (1-(1-\beta)\delta)(1-(1-\beta\lambda)\delta)\left(2(1-\beta)\lambda-\delta(1-\sqrt{B})\right). \tag{32}$$

To conclude the proof, let us prove that (i) $D(\cdot) \geq 0$ and (ii) $N(\cdot) \geq 0$.

(i) $D(\cdot) \geq 0$**.** By equation (32), we have to prove that $F(\lambda) := 2(1-\beta)\lambda - \delta(1-\sqrt{B}) \geq 0$. The first and second derivatives of $F(.)$ wrt λ are given by:

$$F'(\lambda) = \frac{2(1-\beta)((2\lambda-1)\delta+\sqrt{B})}{\sqrt{B}} \text{ and } F''(\lambda) = \frac{4\beta(1-\beta)\delta}{B^{3/2}} \geq 0.$$

Therefore $F'(.)$ is nondecreasing in λ. Then, notice that $F'(0) = 2(1-\beta)(1-\delta)$ so that $F'(\lambda) \geq 0$ for all λ, i.e. $F(.)$ is nondecreasing in λ. To conclude, notice that $F(0) = 0$.

(ii) $N(\cdot) \geq 0$**.** Notice that $\partial N/\partial\delta = \beta(1-\beta)\lambda^2(1-\sqrt{B}) \geq 0$. Therefore, showing that $N(.) \geq 0$ is equivalent to showing $N(\lambda,\beta,\delta=0) \geq 0$. We have:

$$N(\lambda,\beta,0) = -1+\sqrt{B}-(1-\beta)\lambda(2\lambda-3+\sqrt{B}),$$

and

$$\frac{\partial N(\lambda,\beta,0)}{\partial\lambda} = \frac{1-\beta}{\sqrt{B}}(2(1-\beta)\lambda-(1-\sqrt{B}))(4\lambda-3)$$

Notice that, if $G(\lambda) := 2(1-\beta)\lambda-(1-\sqrt{B}) \geq 0$, then, $\partial N(\lambda,\beta,0)/\partial\lambda$ has the sign of $4\lambda-3$, i.e. $N(\lambda,\beta,0)$ is increasing in λ on $[0,3/4]$ and decreasing in λ on $[3/4,1]$. Since $N(0,\beta,0) = 0$ and $N(1,\beta,0) = 0$, this would prove that $N(\lambda,\beta,0) \geq 0$ and, therefore conclude the proof. Then, let us show that $G(\lambda) \geq 0$. The first and second derivatives of $G(.)$ wrt λ are given by

$$G'(\lambda) = \frac{2(1-\beta)(2\lambda-(1-\sqrt{B}))}{\sqrt{B}} \text{ and } G''(\lambda) = \frac{4\beta(1-\beta)}{B^{3/2}} \geq 0.$$

Therefore $G'(.)$ is nondecreasing in λ. Then, notice that $G'(0) = 0$ so that $G'(\lambda) \geq 0$ for all λ, i.e. $G(.)$ is nondecreasing in λ. To conclude, notice that $G(0) = 0$. □

Proposition 8. *When one moves from a no-communication RM equilibrium to an informative communication PAM equilibrium,*

(a) low type agents are worse off under informative communication: $\hat{U}_l^{PAM} - U_l^{RM} < 0$*;*

(b) high type agents are better off under informative communication: $\hat{U}_h^{PAM} - U_h^{RM} > 0$.

Proof. **(a)** $\hat{U}_l^{PAM} - U_l^{RM} < 0$**.** Recall that:

$$\hat{U}_l^{PAM} = \hat{n}_l \hat{V}_l + \hat{n}_{ll} V_{ll} = \hat{n}_l \zeta(\beta)\zeta(1-\hat{\lambda}_{PAM}) u_{ll} + \hat{n}_{ll}\zeta(\beta) u_{ll},$$

$$U_l^{RM} = n_l V_l + n_{lh} V_{lh} + n_{ll} V_{ll} = (1-\lambda)\zeta(\beta)(\lambda u_{lh} + (1-\lambda) u_{ll}).$$

Then, notice that $\hat{n}_{ll} = 1 - \lambda - \hat{n}_l$ and, after rearranging terms, we obtain:

$$\hat{U}_l^{PAM} - U_l^{RM} = -\zeta(\beta)\left(\hat{n}_l\left(1-\zeta(1-\hat{\lambda}_{PAM})\right)\frac{u_{ll}}{u_{lh}} + \lambda(1-\lambda)(1-\frac{u_{ll}}{u_{lh}})\right)u_{lh} < 0.$$

(b) Sign of $\hat{U}_h^{PAM} - U_h^{RM}$**.** Similar calculations show that:

$$\hat{U}_h^{PAM} - U_h^{RM} = \zeta(\beta)\left(\lambda(1-\lambda)(1-\frac{u_{hl}}{u_{hh}}) - \hat{n}_h(1-\zeta(\hat{\lambda}_{PAM}))\right)u_{hh},$$

where $\frac{u_{hl}}{u_{hh}} := \frac{u_{hl}}{u_{hh}} \in (\delta\zeta(\beta\lambda), \delta\zeta(\beta)\zeta(\hat{\lambda}_{PAM}))$. In particular, $\frac{u_{hl}}{u_{hh}} \leq \delta\zeta(\hat{\lambda}_{PAM})$. Since $\hat{U}_h^{PAM} - U_h^{RM}$ is decreasing in $\frac{u_{hl}}{u_{hh}}$, we would like to show that $\Delta = \lambda(1-\lambda)(1-\delta\zeta(\hat{\lambda}_{PAM})) - \hat{n}_h(1-\zeta(\hat{\lambda}_{PAM})) \geq 0$. Replacing $\hat{\lambda}_{PAM}$ and $\hat{n}_h$ by their expressions, we find:

$$\Delta = -\frac{(1-\delta)\lambda\left(\beta\lambda(\sqrt{A}+\beta(1-2\lambda)) + (1-\lambda)(\beta-2\lambda-\sqrt{A})\right)}{(1+\beta)(2\lambda+(1-\delta)(\sqrt{A}+\beta(1-2\lambda)))},$$

where $A = \beta^2(1-2\lambda)^2 + 4\lambda(1-\lambda)$. Notice then that $\sqrt{A} \geq |\beta(1-2\lambda)|$ so that the denominator in Δ is positive. Let us show that the term in parenthesis in the numerator is negative, which will conclude the proof. Notice first that $\sqrt{A} \geq \beta$, so that:

$$\beta\lambda(\sqrt{A}+\beta(1-2\lambda)) + (1-\lambda)(\beta-2\lambda-\sqrt{A}) \leq \beta\lambda(\sqrt{A}+\beta(1-2\lambda)) - (1-\lambda)2\lambda =: F(\lambda,\beta).$$

Let us show that $F(\lambda,\beta) \leq 0$ for all (λ,β). Then,

$$\frac{\partial F}{\partial \beta}(\lambda,\beta) = \frac{\lambda(\beta(1-2\lambda)+\sqrt{A})^2}{\sqrt{A}} \geq 0$$

so that, for all λ, function $\beta \mapsto F(\lambda,\beta)$ is increasing in β. Now, notice that $F(\lambda,1) = 0$, and therefore $F(\lambda,\beta) \leq 0$ for all (λ,β).

□

Proposition 9. *When one moves from a no-communication RM equilibrium to an informative communication UM equilibrium,*

(a) low type agents are better off under informative communication: $\hat{U}_l^{UM} - U_l^{RM} > 0$;

(b) high type agents are worse off under informative communication: $\hat{U}_h^{UM} - U_h^{RM} < 0$.

Proof. **(a)** $\hat{U}_l^{UM} - U_l^{RM} > 0$**.** Direct calculations show that:

$$\hat{U}_l^{UM} - U_l^{RM} = \zeta(\beta)\left((1-\lambda)^2(1-\frac{u_{ll}}{u_{lh}}) - \hat{n}_l(1-\zeta(\hat{\lambda}_{UM}))\right)u_{lh},$$

where $\frac{u_{ll}}{u_{lh}} \leq \delta\zeta(\hat{\lambda}_{UM})$. Since $\hat{U}_l^{UM} - U_l^{RM}$ is decreasing in $\frac{u_{ll}}{u_{lh}}$, we would like to show that $\Delta = (1-\lambda)^2(1-\zeta(\hat{\lambda}_{UM})) - \hat{n}_l(1-\zeta(\hat{\lambda}_{UM}))$ is positive. Replacing $\hat{n}_l$ and $\hat{\lambda}_{UM}$ by their expressions, we obtain:

$$\Delta = -\frac{(1-\delta)\lambda(2(1-\beta)\lambda(1-\lambda)-1+\sqrt{B})}{2(1-\beta)\lambda-\delta(1-\sqrt{B})},$$

where $B = 1-4\lambda(1-\lambda)(1-\beta)$. The denominator in Δ is positive (see the proof of Proposition 7). Let $N(\lambda,\beta) = 2(1-\beta)\lambda(1-\lambda)-1+\sqrt{B}$. Let us show that $N(\lambda,\beta)\leq 0$ for all (λ,β). Notice that

$$\frac{\partial N}{\partial\lambda}(\lambda,\beta) = \frac{2(1-\beta)(1-\sqrt{B})(2\lambda-1)}{\sqrt{B}},$$

which has the sign of $2\lambda-1$ since $1-\sqrt{B}\geq 0$. Therefore, for all β, function $\lambda\mapsto N(\lambda,\beta)$ is decreasing (resp. increasing) in λ on the interval $[0,1/2]$ (resp. $[1/2,1]$). In particular, $N(\lambda,\beta)\leq \max\{N(0,\beta),N(1,\beta)\}$. To conclude the proof, notice that $N(0,\beta)=N(1,\beta)=0$.

(b) $\hat{U}_h^{UM} - U_h^{RM} < 0$**.** Notice that $U_h^{RM} = U_h^{UM}$. The result then follows immediately from Proposition 7.

□

Proposition 10 (mixed UM informative communication equilibrium vs. RM no-communication equilibrium)**.** *When one moves from a no-communication RM equilibrium to an informative communication mixed UM equilibrium,*

(a) high type agents are worse off under informative communication: $\hat{U}_h^{UM} < U_h^{RM}$*;*

(b) low type agents are better off under informative communication: $\hat{U}_l^{UM} > U_l^{RM}$*.*

Proof. Let $\tau_{ll} = \tau \in (0,1)$ the probability that a low type agent accepts to start the evaluation phase when he receives message l. Let $\hat{\lambda}(\tau)$ the proportion of type h agents in the population of singles in the associated mixed UM equilibrium:

$$\hat{\lambda}(\tau) = \frac{\sqrt{A}-(1-2(1-\beta)\lambda(1-\tau^2))}{2(1-\beta)\lambda(1-\tau^2)},$$

where $A = 1-4(1-\beta)\lambda(1-\lambda)(1-\tau^2)$.

(a) $\hat{U}_h^{UM} < U_h^{RM}$**.** Recall that:

$$U_h^{RM} = \zeta(\beta)\lambda(\lambda u_{hh}+(1-\lambda)u_{hl}),$$

$$\hat{U}_h^{UM} = \zeta(\beta)\lambda(\hat{\lambda}(\tau)u_{hh} + (1-\hat{\lambda}(\tau))u_{hl}).$$

Comparing the two previous equations, it is therefore sufficient to show that, $\forall \tau \in (0,1)$, $\hat{\lambda}(\tau) \leq \lambda$. To conclude, notice then that:

$$\lambda - \hat{\lambda}(\tau) = \frac{(1-\lambda)(2-(1+\sqrt{A}))}{1+\sqrt{A}} \geq 0,$$

where the inequality comes from the fact that $A \leq 1$.

(b) $\hat{U}_l^{UM}(\tau) > U_l^{RM}$. Recall that

$$U_l^{RM} = \zeta(\beta)(1-\lambda)(\lambda u_{lh} + (1-\lambda)u_{ll}). \tag{33}$$

$$\hat{U}_l^{UM}(\tau) = \hat{n}_l(\tau)\hat{V}_l(\tau) + \hat{n}_{ll}(\tau)\hat{V}_{ll} + \hat{n}_{lh}(\tau)\hat{V}_{lh}, \tag{34}$$

where $\hat{V}_{ll} = \zeta(\beta)u_{ll}$, $\hat{V}_{lh} = \zeta(\beta)u_{lh}$ and

$$\begin{array}{rcl} \hat{V}_l(\tau) & = & \hat{\lambda}(\tau)\hat{V}_{lh} + (1-\hat{\lambda}(\tau))(\tau\hat{V}_{ll} + (1-\tau)\delta\hat{V}_l), \\ & = & \frac{1}{1-(1-\tau)(1-\hat{\lambda}(\tau))\delta}(\lambda(\tau)u_{lh} + (1-\lambda(\tau))\tau u_{lh})\zeta(\beta). \end{array}$$

Plugging this into equation (34) and rearranging, we obtain:

$$\begin{array}{rcl} \hat{U}_l^{UM}(\tau) & = & \Big\{\Big(\frac{(1-\hat{\lambda})\tau}{1-(1-\tau)(1-\hat{\lambda}(\tau))\delta}\hat{n}_l(\tau) + \hat{n}_{ll}(\tau)\Big)u_{ll} \\ & & + \Big(\frac{\hat{\lambda}}{1-(1-\tau)(1-\hat{\lambda}(\tau))\delta}\hat{n}_l(\tau) + \hat{n}_{lh}(\tau)\Big)u_{lh}\Big\}\zeta(\beta) \end{array} \tag{35}$$

Equations (33) and (35) together show that sufficient conditions for having $\hat{U}_l^{UM}(\tau) > U_l^{RM}$ are:

$$f(\tau) := \frac{(1-\hat{\lambda})\tau}{1-(1-\tau)(1-\hat{\lambda}(\tau))\delta}\hat{n}_l(\tau) + \hat{n}_{ll}(\tau) \leq (1-\lambda)^2,$$

$$g(\tau) := \frac{\hat{\lambda}}{1-(1-\tau)(1-\hat{\lambda}(\tau))\delta}\hat{n}_l(\tau) + \hat{n}_{lh}(\tau) \geq \lambda(1-\lambda).$$

In the following, we show that (i) function $f(.)$ is increasing in τ and $f(1) = (1-\lambda)^2$, and (ii) function $g(.)$ is decreasing in τ and $g(1) = \lambda(1-\lambda)$, from which we derive, in particular, the above sufficient conditions. For sake of conciseness, we only establish (ii). Claim (i) proves similarly. We proceed in two steps.

Step 1: $\hat{n}_{lh}(\tau)$ is decreasing in τ. Simple calculations show that:

$$\frac{\partial \hat{n}_{lh}}{\partial \tau} = -\frac{\tau(1-2(1-\beta)\lambda(1-\lambda)(1-\tau^2)-\sqrt{A})}{(1-\tau)^2\sqrt{A}},$$

which has the opposite sign of the numerator. Then, notice that $2(1-\beta)\lambda(1-\lambda)(1-\tau^2) \leq 1/2$, so that, in particular, $B = 1 - 2(1-\beta)\lambda(1-\lambda)(1-\tau^2) \geq 0$. Then, we have

$$B^2 - A = 4(1-\beta)\lambda^2(1-\tau^2)(\beta + (1-\beta)\tau^2) \geq 0.$$

Therefore, $\partial \hat{n}_{lh}/\partial \tau \leq 0$ and $\hat{n}_{lh}(\tau)$ is decreasing in τ.

Step 2: $h(\tau) := \frac{\hat{\lambda}}{1-(1-\tau)(1-\hat{\lambda}(\tau))\delta}\hat{n}_l(\tau)$ is decreasing in τ. Tedious calculations show that

$$h'(\tau) = -\frac{2\beta\lambda(1-2(1-\beta)\lambda(1-\lambda)(1-\tau^2)-\sqrt{A})(2(1-\beta)\tau+\delta\sqrt{A})}{(1-\tau^2)\sqrt{A}(\delta-2(1-\beta)\lambda(1+\tau)-\delta\sqrt{A})^2},$$

which has the opposite sign of the numerator. From step 1, the numerator is positive so that $h(.)$ is decreasing in τ. □

Proposition 11 (PAM informative communication equilibrium vs. UM no-communication equilibrium)**.** *When one moves from a no-communication UM equilibrium to an informative communication PAM equilibrium,*

(a) high type agents are better off under informative communication: $\hat{U}_h^{PAM} > U_h^{UM}$*;*

(b) low type agents are worse off under informative communication: $\hat{U}_l^{PAM} < U_l^{UM}$*.*

Proof. (a) We have

$$\hat{U}_h^{PAM} = \hat{n}_h\hat{V}_h + \hat{n}_{hh}\hat{V}_{hh} = \zeta(\beta)(\lambda - \hat{n}_h(1-\zeta(\hat{\lambda}_{PAM})))u_{hh},$$

where we used the condition $\hat{n}_h + \hat{n}_{hh} = \lambda$. From the proof of Proposition 7, Equation (28), we know that

$$U_h^{UM} = \zeta(\beta)\lambda(\lambda + (1-\lambda)\frac{u_{hl}}{u_{hh}})u_{hh}.$$

Using the conditions for a fully revealing PAM equilibrium to exist, we have $\frac{u_{hl}}{u_{hh}} \leq \delta\zeta(\beta)\zeta(\hat{\lambda}_{PAM}) \leq \delta\zeta(\hat{\lambda}_{PAM})$. Therefore, to prove the announced result, it suffices to show that

$$\lambda - \hat{n}_h(1-\zeta(\hat{\lambda}_{PAM})) \geq \lambda(\lambda + (1-\lambda)\delta\zeta(\hat{\lambda}_{PAM})).$$

Tedious but straightforward calculations then show that

$$\begin{aligned}&\lambda - \hat{n}_h(1-\zeta(\hat{\lambda}_{PAM})) - \lambda(\lambda + (1-\lambda)\delta\zeta(\hat{\lambda}_{PAM}))\\ =&\frac{\lambda(1-\delta)\left(\beta^2(2\lambda-1)\lambda + (2\lambda-\beta)(1-\lambda) + (1-\lambda(1+\beta))\sqrt{A}\right)}{(1+\beta)(2\lambda + (1-\delta)(\sqrt{A} - \beta(2\lambda-1)))},\end{aligned} \tag{36}$$

where $A = \beta^2(2\lambda-1)^2 + 4\lambda(1-\lambda)$. Since $\sqrt{A} \geq \sqrt{\beta^2(2\lambda-1)^2} \geq \beta(2\lambda-1)$, the denominator in equation (36) is positive. Therefore, it remains to show that the numerator in (36) is positive. Let $\Delta = \beta^2(2\lambda-1)\lambda + (2\lambda-\beta)(1-\lambda) + (1-\lambda(1+\beta))\sqrt{A}$. Since $A \geq \beta^2$, we have $(1-\lambda)\sqrt{A} \geq (1-\lambda)\beta$, so that

$$\Delta \geq \beta^2(2\lambda-1)\lambda + (2\lambda-\beta)(1-\lambda) + (1-\lambda)\beta - \lambda\beta\sqrt{A} := F(\lambda,\beta).$$

Let us show that $F(\lambda, \beta) \geq 0$. Notice first that, for all $\lambda \in [0, 1]$, $F(\lambda, 1) = 0$. Then, to conclude the proof, let us show that, for all $\lambda \in [0, 1]$, $\beta \mapsto F(\lambda, \beta)$ is decreasing in β. Taking the derivative of $F(.)$ wrt β, we find

$$\frac{\partial F}{\partial \beta}(\lambda, \beta) = -\frac{(\sqrt{A} - \beta(2\lambda - 1))^2}{\sqrt{A}} \leq 0.$$

(b) In a no-communication UM equilibrium, the total welfare of low type agents is given by

$$\begin{aligned} U_l^{UM} &= n_l V_l + n_{lh} V_{lh} + n_{ll} V_{ll} \\ &= \beta(1-\lambda)\zeta(\lambda\beta)u_{lh} + (1-\beta)\lambda(1-\lambda)\zeta(\beta)u_{lh} + (1-\beta)(1-\lambda)^2\delta\zeta(\beta)\zeta(\lambda\beta)u_{lh} \\ &= (1-\lambda)\zeta(\lambda\beta)u_{lh}. \end{aligned} \tag{37}$$

In a fully revealing PAM equilibrium, the total welfare of low type agents is given by

$$\begin{aligned} \hat{U}_l^{PAM} &= \hat{n}_l \hat{V}_l + \hat{n}_{ll} \hat{V}_{ll} \\ &= \hat{n}_l \zeta(\beta)\zeta(1 - \hat{\lambda}_{PAM})u_{ll} + \hat{n}_{ll}\zeta(\beta)u_{ll}. \end{aligned} \tag{38}$$

Hence, we have $\hat{U}_l^{PAM} < U_l^{UM}$ iff

$$\frac{u_{ll}}{u_{lh}}\zeta(\beta)(\hat{n}_l\zeta(1-\hat{\lambda}_{PAM}) + \hat{n}_{ll}) < (1-\lambda)\zeta(\beta\lambda).$$

The equilibrium conditions for a no-communication UM equilibrium imply $\delta\zeta(\beta\lambda) > \frac{u_{ll}}{u_{lh}}$, so it suffices to show that

$$\delta\zeta(\beta)(\hat{n}_l\zeta(1-\hat{\lambda}_{PAM}) + \hat{n}_{ll}) < (1-\lambda).$$

This inequality is always satisfied since $\delta\zeta(\beta) < 1$ and $\hat{n}_l\zeta(1-\hat{\lambda}_{PAM}) + \hat{n}_{ll} \leq \hat{n}_l + \hat{n}_{ll} = 1 - \lambda$. □

A.4 Illustrations of pure strategies equilibria under complete information.

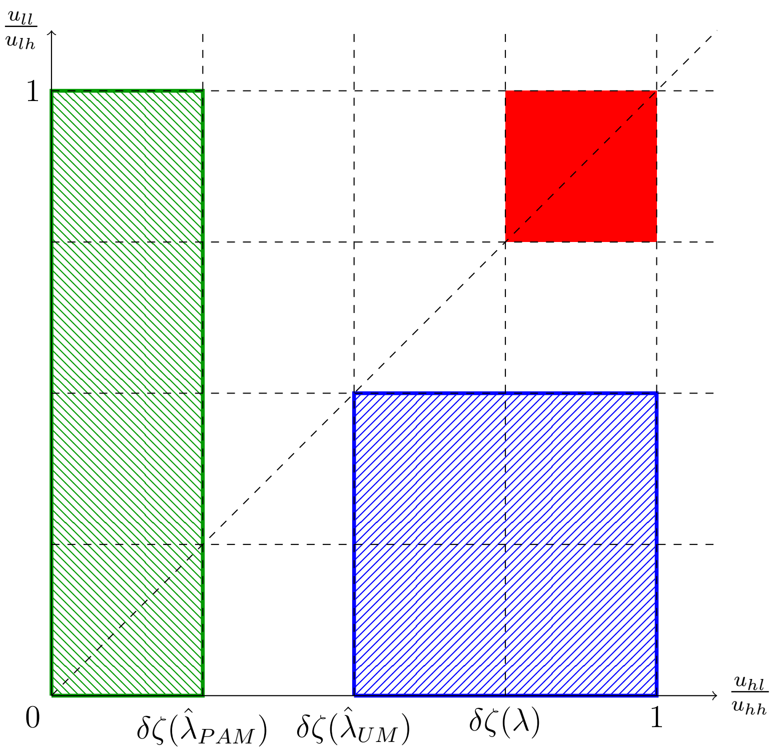

Figure 6: Pure strategy equilibria under complete information when $1/2 < \hat{\lambda}_{PAM} < \hat{\lambda}_{UM} < \lambda$. Green: PAM equilibria; Red: RM equilibria; Blue: UM equilibria.

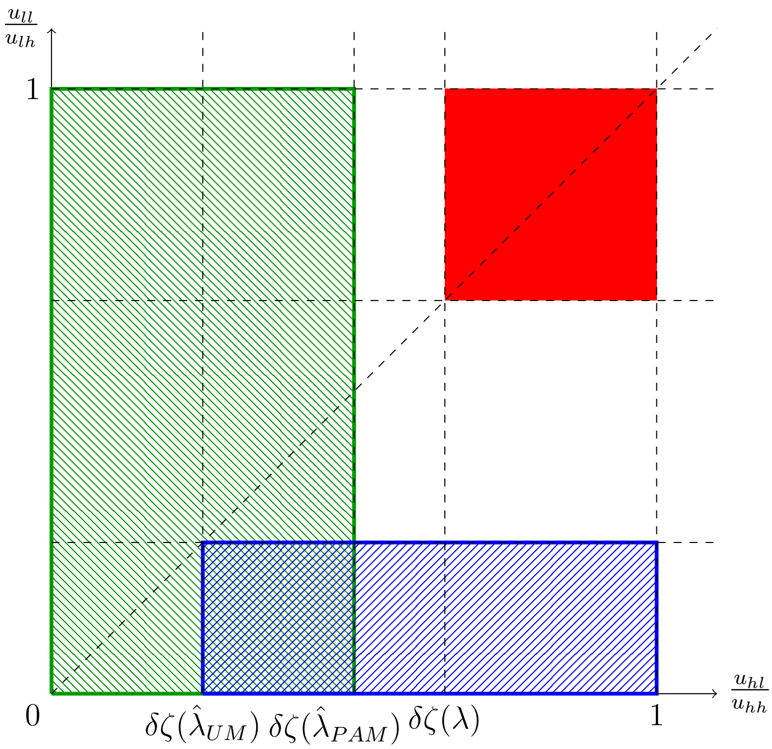

Figure 7: Pure strategy equilibria under complete information when $\hat{\lambda}_{UM} < 1/2 < \hat{\lambda}_{PAM} < \lambda$. Green: PAM equilibria; Red: RM equilibria; Blue: UM equilibria.

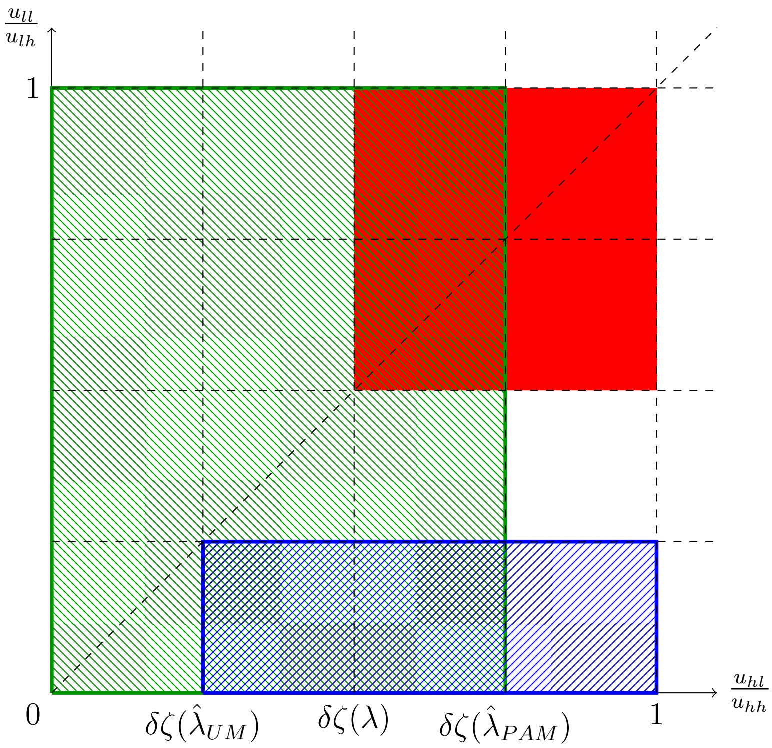

Figure 8: Pure strategy equilibria under complete information when $\hat{\lambda}_{UM} < \lambda < \hat{\lambda}_{PAM} < 1/2$. Green: PAM equilibria; Red: RM equilibria; Blue: UM equilibria.

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