Estimating Dynamic Merger Efficiencies with an Application to the 1997 Boeing-McDonnell Douglas Merger*

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Abstract

This paper evaluates the welfare effects of the 1997 merger between Boeing and McDonnell Douglas in the medium-sized wide-bodied aircraft industry. I develop an empirical model of multi-product firms, allowing for both learning-by-doing and product innovation in a dynamic game to quantify merger efficiency. Merger efficiency from learning-by-doing is then disentangled from the impact of innovation and the market power effect. The results show that the primary benefits from the 1997 Boeing-McDonnell Douglas merger come from accelerated learning-by-doing. Taking account of all static and dynamic effects, net consumer surplus is found to have increased by as much as $1.57 billion. In contrast, a static model ignoring dynamic learning-by-doing and innovation predicts a consumer loss of about $20 billion. These results show that ignoring dynamic effects can lead to biased results and erroneous conclusions with regard to the welfare impact of a merger.

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1 Introduction

“A primary benefit of mergers to the economy is their potential to generate significant efficiencies... which may result in lower prices, improved quality, enhanced service, or new products.” (2010 U.S. Horizontal Merger Guidelines)

One of the central duties of the Federal Trade Commission (FTC) and the Antitrust Division of the U.S. Department of Justice (DOJ) is to evaluate the potential impact of a merger between competing firms on the welfare of consumers. Mergers that would make consumers worse off are either to be restructured in order to avoid such detrimental effects or challenged and prevented. In light of the size and number of companies involved in merger activity, the potential welfare impact is significant. As reported in the most recent Hart Scott Rodino Annual Report, there were 1,450 proposed transactions involving large companies in 2011, with a total capitalization of almost one trillion dollars.

In evaluating a prospective merger, the approach is to compare the pre-merger outcome with a forecast of the post-merger outcome. This comparison typically takes the form of comparing current prices with some projected prices if the merger were to occur. To generate that projection, it is common to hold firms’ costs and the quality of their products fixed and to estimate what the impact on price would be if a firm’s assets were acquired by a competitor. While there are some variants to this approach, for example, it may be recognized that some products would be removed or some immediate cost reductions realized, the evaluation still takes the form of a short-run analysis. The fundamental question asked is: what will happen to consumer welfare in the short-run in response to this merger?

It is well recognized, however, that the primary efficiencies from some mergers are likely to be dynamic, as they are realized over time and are endogenous to firms’ decisions in the post-merger environment. Such dynamic efficiencies can come from a reduction in cost because of learning-by-doing or altered incentives to invest in reducing marginal cost, from better products due to investment or adoption of new technologies, and from future entry and exit (perhaps involving additional mergers and acquisitions). For example, the international hard drive disc (HDD) market has experienced a series of major mergers in recent years. Maxtor and Samsung were acquired by Seagate in 2006 and 2011, respectively, and Hitachi was sold to Western Digital in 2012. The most significant impact on consumer welfare from this altered market structure may lie not with how it affects price in the short-run but rather its impact on product cost and quality in the long-run. Will firms have stronger or weaker incentives to invest and innovate? Effectively addressing such questions is central to a proper evaluation of the welfare effects of these mergers.

Though dynamic efficiencies are well-recognized as potentially substantial, they have not played a significant role in merger evaluation by antitrust authorities because of the lack of methods for estimating these efficiencies. Furthermore, while we can speculate as to what the dynamic welfare effects of a past merger might be, there has been little research that actually estimates these effects.

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1 The 2010 U.S. Horizontal Merger Guidelines indicate that dynamic efficiencies, “such as those relating to research and development, are potentially substantial but are generally less susceptible to verification.”
The primary objective of this paper is to contribute to studies on these policy issues by quantifying dynamic efficiencies from the 1997 Boeing-McDonnell Douglas merger. In achieving this, I develop an empirical model targeting the aircraft industry and then compare efficiencies estimated using a dynamic model to those estimated from a static model.

The empirical model encompasses two common dynamic forces relevant to industry performance and thus to the evaluation of a merger: learning-by-doing\(^2\) and improvements in product quality\(^3\). These forces are encompassed in a model with multi-product firms that compete in an infinite-horizon dynamic game. In each period, a firm decides how much to produce (which may be a vector of quantities if it has multiple products), while taking into account its impact on current profit and its future profit stream through the effect of output on the firm’s experience. Experience is a state variable that rises as a firm’s past output accumulates (learning), but also depreciates over time (forgetting). Learning-by-doing is modeled by having unit production cost be a decreasing function of experience. Also, a firm’s production is allowed to have spillover effects with regards to experience accumulation from that firm’s other products and also its competitors’ products; the magnitude of these spillover effects are allowed to depend on ownership and product characteristics. In addition to deciding how much to produce each period, a firm decides whether to invest in improving the quality of its products. These potential product upgrades are exogenously generated from outside of the industry. Adoption of an upgrade incurs a direct cost but also an indirect cost through a setback in experience; for example, Levitt, List, and Syverson (2012) found for the automobile industry that “introducing a new model variant into production does cause productivity setbacks.” For this setting, firms are assumed to behave according to a Markov Perfect Equilibrium in which they decide on production and upgrades in each period given the state variables of firms’ experiences and product qualities, as well as given the stochastic realization of market size, product characteristics, and upgrading costs.

Before moving on to specifics relating to the aircraft industry, let us consider the possible welfare implications of a merger within this framework. A merger might hurt consumers because reduced competition creates the incentive to restrict production and raise price; this is the traditional market power effect. A merger might also generate dynamic efficiencies in several ways through its impact on the evolution of production experience. First, there is an immediate benefit in lowering marginal cost for products of the merged firm because accumulated experience is shared; This is due to possible spillover of experience across products, serving to lower cost, raise output, and elevate consumer welfare. Second, the merged firm may choose to alter its product line, for example,\


\(^3\) Competition in quality improvements is important in high technology industries, such as biotechnology and pharmaceuticals, medical instruments, aircraft, automobiles, computer hardware and software, cell phones, and game consoles. However, there are limited intra-industry empirical studies on relationships between quality improvements and market competition. Examples include Goettler and Gordon (2011) and Nosko (2010) for the CPU market, and Hashmi and Biesebroeck (2010) for automobiles.
by shutting down some of the products of the acquired firm. Fewer products means less variety (which makes consumers worse off) but also more output per product, implying faster experience accumulation, lower unit cost, and lower future prices (which makes consumers better off). Third, future experience might be more effectively shared between different products within the same firm (within-firm spillover) than between different firms (across-firm spillover), which again will produce lower costs after the merger.

A second source of dynamic efficiency comes from altered incentives for quality improvements. The direction of this effect is ambiguous. After a merger, softened competition could discourage innovation but enlarged market share may mean a bigger benefit from a better quality product, which would stimulate incurring the fixed cost to innovate. If quality improvements negatively impact experience and raise unit cost then this will further complicate the evaluation. Assessing how these forces net out in terms of firm behavior and consumer welfare will then require estimating parameters, solving the dynamic model for equilibrium behavior, and simulating the industry path with and without a merger.

Having developed this empirical model, I then apply it to the medium-sized wide-bodied aircraft industry to evaluate the 1997 merger between Boeing and McDonnell Douglas. Prior to the merger, the market was occupied by three firms, Boeing, Airbus, and McDonnell Douglas, who were producing four products (A330, A340, B777, and MD-11) in the medium-sized wide-bodied aircraft market.\(^4\) Immediately after the merger, the new Boeing company shut down production of MD-11. Manufacturing aircraft is labor-intensive and learning-by-doing is commonly recognized as an important feature in the industry.\(^5\) Boeing 777 was introduced only two years before the merger, with submodels of B777 arriving soon after the merger. Thus, by ceasing production of MD-11, Boeing hoped to achieve lower marginal cost more rapidly for its B777. Besides learning-by-doing, innovation through upgrades is another distinct feature of the aircraft industry. New generations of aircraft were introduced of higher quality. This was especially so after the September 11th attacks when petroleum prices skyrocketed and airline demand for more fuel-efficient aircraft accelerated.

To evaluate the welfare effects of the 1997 Boeing-McDonnell Douglas merger, the dynamic model is solved for three different scenarios: (i) merger and the MD-11 is immediately shut down (which is what actually occurred), (ii) merger with continued operation of the MD-11, and (iii) no merger. The time series for price, consumer surplus, profit, and total surplus was computed for all scenarios. To disentangle efficiency resulting from learning-by-doing from efficiency due to quality improvements and the market power effect, I solved an additional model that does not allow for quality improvements and still another that does not allow for either learning-by-doing or quality improvements. The results show that the primary benefits from the 1997 Boeing-McDonnell Douglas merger come from accelerated learning-by-doing rather than from a higher rate of innovation. Furthermore, the dynamic efficiencies generated by the merger are large enough to

\(^4\)The merger of the two companies affects the entire aircraft industry. However, I will focus on its impact on the medium-sized wide-bodied aircraft industry only, which can be viewed as an isolated market from other aircraft industries as discussed in Section 4.1.

\(^5\)The aircraft industry is the market where learning-by-doing was first recognized. See Wright (1936).
exceed the static market power effect, which is about $20 billion. Taking account of all static and dynamic effects, net consumer surplus is found to have increased by as much as $1.57 billion. These results show that ignoring dynamic effects can lead to biased results and erroneous conclusions with regard to the welfare impact of a merger.

This paper is directly related to three lines of research: dynamic effects of a merger, learning-by-doing in the aircraft industry and other industries, and dynamic innovations within an industry. Gowrisankaran (1999) is one of the first papers that theoretically examined the dynamic effects of a merger. Performing numerical analysis within the Ericson-Pakes framework (Ericson and Pakes (1995)), firms were modeled as choosing investment to expand capacities dynamically, with endogenously generated mergers. Gowrisankaran (1999) assumed marginal cost is fixed and common across firms. The impact of a merger on consumer welfare was not a central issue in that paper. Chen (2009) also examined these issues theoretically and had firms make dynamic investment decisions affecting capacity accumulation, which impacted marginal cost over time. That analysis explored the bias in static merger analysis when dynamic investment is ignored. Stahl (2009) estimated cost and revenue parameters for the broadcast television industry, where costs were estimated as residuals of firm behavior unexplained by revenues. That paper focuses on the consolidation process itself rather than evaluating merger-generated efficiencies and thus does not solve the dynamic oligopoly model. Benkard, Bodoh-Creed, and Lazarev (2010) evaluated the medium- and long-run dynamic effects of airline mergers and explored the effect of mergers on market structure rather than consumer welfare. Jeziorski (2011b) and Jeziorski (2011a) studied merger impacts in the U.S. radio industry and took account of the markets being two-sided. Jeziorski (2011b) compared listeners’ welfare increase from product variety with the market power effect. Jeziorski (2011a) endogenized merger decisions and found that total cost savings from mergers outweighed the loss caused by increased market power. Nocke and Whinston (2010) provided a new theoretical framework to model dynamic merger decisions where firms’ choice variables other than merger decisions are assumed to be static. They derived conditions whereby an antitrust authority can maximize the present value of consumer surplus by using a myopic merger review policy. I contribute to this line of research by introducing a model that focuses on the endogenous dynamics of cost and product quality.

The empirical learning-by-doing literature encompasses a wide array of industries. This paper is most closely related to the pioneering research of Benkard (2000) and Benkard (2004). Benkard (2000) introduced the concept of forgetting to explain the rise in cost for the Lockheed L-1011, and Benkard (2004) allowed for a learning curve in a dynamic oligopoly model with four single-product firms, estimating welfare under several counterfactual scenarios with a social planner and a monopoly. This paper follows this methodological path but focuses on merger evaluation. I extend the empirical model to allow for multi-product firms, dynamic quality improvements, and the spillover effect of learning curves. In my model, merger efficiencies are likely to arise either through accumulation of experience due to combining output and the spillover effect or through a higher probability of upgrading products. Although the spillover effect of the learning curve has not been widely investigated for the aircraft industry, it has been modeled and estimated

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6Benkard (2000) modeled a submodel spillover effect among submodels of an aircraft type but no cross-product
for other industries, including semiconductors (Irwin and Klenow (1994)), shipbuilding (Thornton and Thompson (2001)), fuel cell vehicles (Schwoon (2008)), steel (Ohashi (2005)), and health care (Chandra and Staiger (2007)). However, those papers are not targeted at evaluating mergers in the context of a dynamic game, and none of them simultaneously estimated within-firm spillover and across-firm spillover, which could be significant factors in calculating merger efficiencies.

With respect to the empirical literature on innovation, this paper is most closely related to Goettler and Gordon (2011), which examines the microprocessor industry. Both Goettler and Gordon (2011) and my work use the concept of a product’s quality relative to an outside good whose quality is changing over time; this is a modeling device first proposed in Pakes and McGuire (1994). One major difference of my paper from that by Goettler and Gordon (2011) is in the modeling of quality evolution of the outside good. Goettler and Gordon (2011) fixed the difference in quality between the industry frontier product and the outside good. Thus, in their model, the outside good upgrades automatically when the product with the highest quality upgrades. In contrast, I let the quality upgrade of the outside good be exogenous and evolve stochastically. Although it is appealing to endogenize outside good evolution in a single-product firm model as in Goettler and Gordon (2011), their method might not be suitable for multi-product aircraft manufacturers. Aircraft upgrades involve inventions of new patented technology that are more likely to be shared within a firm. Thus, it is less realistic to assume that when the frontier product upgrades, the good outside the market receives the same technology and upgrades automatically while a same-firm product does not. In addition, Goettler and Gordon (2011) focused on dynamic demand while fixing marginal cost for any given relative quality, while I assume static demand and concentrate on cost structure evolution.

In conclusion, I evaluate the 1997 Boeing-McDonnell Douglas merger in terms of its impact on consumer welfare by constructing a dynamic oligopoly model for multi-product firms with learning-by-doing and endogenous quality improvements. Allowing for both the evolution of cost and product quality is new to the literature on dynamic oligopoly equilibrium models. The rest of the paper is organized as follows. Section 2 presents a global view of the structural model. The data used for estimation and calibration is reviewed in Section 3. Section 4 applies the structural model to the aircraft industry. Using equilibrium strategies solved from the dynamic model, merger evaluation is conducted in Section 5. Section 6 concludes the paper.

2 Model

This section describes the general dynamic framework that is the basis for the model to be estimated for the aircraft industry. In describing the framework prior to putting forth the empirical model, the intent is to give readers a global view of the decisions made by firms and consumers and how the or cross-firm spillover effect and a complete within-model spillover is assumed in Benkard (2004). The international trade literature studies knowledge-spillover in the sense of technology transfer across countries and across industries. See Grossman and Helpman (1995) for a review of that literature and Niosi and Zheng (2010) for a review of the aircraft industry specifically.
environment evolves. Then, in Section 4, this framework is populated with the specific structure that will then be estimated.

The model has multi-product firms with differentiated products that compete in both quantities and qualities. Quantity choices affect dynamic market cost structure through the mechanism of learning-by-doing while qualities are improved through innovation decisions to replace old generations of products with the next generation of higher quality products (which are exogenously produced). Thus, improvements in product quality are realized by a generation upgrade. The model is applicable to many industries for which learning-by-doing and innovation are important, including high technology manufacturing industries such as aircraft, computer hardware, tablet, and smart phone.

The industry is composed of \( I \) multi-product firms competing in discrete time over an infinite horizon. Firm \( i \in \mathbb{I} = \{1, ..., I\} \) has a product set \( J_i \) and \( J \) is the union of \( J_i \) for all \( i \). Size of \( J_i \) and \( J \), denoted by \( J_i \) and \( J \), are thus number of products in firm \( i \) and in the industry, respectively. Exit and entry on both firm and product level are assumed away.\(^7\) However, they can be easily incorporated in the model.\(^8\) Quantity of product \( j \) from firm \( i \) at period \( t \) is denoted as \( q_{i,j,t} \), or simply as \( q_{j,t} \) when there is no need to specify to which firm the product belongs.

In the remainder of this section I discuss modeling of the demand function and production cost function to be used when firms are making dynamic decisions. Then, I introduce structures on generation upgrade decisions. The section is concluded with a description of the dynamic game.

### 2.1 Demand Function

Demand is determined by both the market size \( M \) that follows an exogenous stochastic process and characteristics of all products in the market. Characteristics of product \( j \) are classified into 3 categories. \( X_j \) represents all fixed characteristics of product \( j \). \( G_j \) is the relative generation of product \( j \), which measures product quality and evolves following endogenous innovation decisions. (I will explain it more fully later.) Finally, \( \xi_j \) captures characteristics unobserved to econometricians that evolve exogenously, such as product suitability. Let \( X, G \) and \( \xi \) denotes the vector of \( X_j, G_j \) and \( \xi_j \), respectively, of all products.

It is assumed that consumers do not engage in intertemporal substitutions. Their choices of demand are solely based on current period product characteristics. Therefore, I assume that when \( (X, G, \xi) \) and an industry quantity vector \( Q \) is given in a period, the inverse demand function \( P = P(Q; X, G, \xi, M) \) is single valued and taken as given for firms.

### 2.2 Production Cost Function

Production cost of product \( j \) in period \( t \), \( C_{j,t} \), is a function of quantity \( q_{j,t} \) and experience level \( E_{j,t} \). \( C_{j,t} \) is assumed to be increasing in \( q_{j,t} \) and decreasing in \( E_{j,t} \). Thus, experience helps to lower production cost. \( E_{j,t} \) itself is a function of the experience level from last period \( E_{j,t-1} \) and the

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\(^7\)See section 4.5.3 for a more detailed discussion on exit and entry.

\(^8\)See Doraszelski and Pakes (2007) for an example of modeling exit and entry.
quantity vector of last period $Q_{t-1}$. I introduce $E_{j,t}$ so that instead of tracking the entire product history, I can just use $E_{j,t}$ as a state variable in the dynamic game. I restrict $E_{j,t}$ to be increasing in both $E_{j,t-1}$ and any $q_{k,t-1}, \forall k \in J$. This implies that experience accumulates over time both through direct learning from production ($q_{j,t}$) and spillover from production of other goods ($q_{k,t}$, $k \neq j$). Forgetting is incorporated in the model in the form of depreciation of experience $E_{j}$ as $\frac{\partial E_{j,t}}{\partial E_{j,t-1}} < 1$.

2.3 Generation Upgrade

I assume that product innovation can be characterized into discrete generations, with higher generations providing higher utility for consumers. For an industry with everlasting innovations and infinite horizon, it is natural to believe that each product has infinitely many generations $g_j \in \{1, 2, 3, ...\}$. However, since the generation of each product is going to be a state variable in the dynamic game, direct modeling of $g_j \in \{1, 2, 3, ...\}$ will explode the state space and make it empirically intractable. Also, it is too restrictive to assume that there is some maximal level of generation. Therefore, to deal with this dimensionality issue, quality is measured as quality relative to an outside good, where the outside good stochastically improves over time, and the difference in quality between a firms’ product and the outside good is bounded. Formally, relative quality is defined as

$$G_{j,t} = g_{j,t} - g_{0,t}$$

where $g_{0,t}$ is the generation level of the outside good. Relative generation of all products $G_t$ is assumed to contain all of the information of $g_t$ that is relevant in determining the demand function.

The model then tracks relative generations instead of absolute ones. This modeling method helps to solve the dimensionality problem for industries where, given an appropriate definition of generation, maximum relative generation is observed to be small. One example of such an industry is the video game console market. A generation of the game console is commonly defined by processor word-length (number of bits), and there has been hardly more than one generation gap between actively produced game consoles at any time in the history of the industry. Note that by treating $G_j$ as a product characteristic, the assumption that relative generation is sufficient in determining demand is consistent with the discrete choice model of the demand system that is widely employed in the literature. Thus, employing relative generation creates no loss of useful information in determining demand.

I assume that $g_{0,t}$ advances each period with probability $p^G$. In the equilibrium, $p^G$ determines

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9See Section 4.4.1 for definition and reasoning for generation upgrades in the aircraft industry.
10Given the assumption that only relative generation matters, $G_j$ can always be normalized by subtracting it from its observed mean.
11See a table of generations of game consoles in Liu (2010).
12If enough generation upgrading decisions at each state are observed, it would be better to let $p^G$ depend on the current state of $G_j$ for all $j$ in order to endogenize outside good evolution.
the long-run industry innovation rate.

\[ g_{0,t} = \begin{cases} 
  g_{0,t-1} + 1 & \text{with probability } p^G \\
  g_{0,t-1} & \text{with probability } 1 - p^G 
\end{cases} \]  

(1)

Evolution of \( g_{j,t} \) is controlled by joint upgrading decisions over all products of firm \( i \) owning product \( j \), denoted as \( U_i \in \{0, 1\}^J \). In each period, \( U_i \) is chosen to maximize total expected value of the firm upon observing realization of a vector of random upgrading cost \( C_{i,t}^G \) for all the products firm \( i \) owns. Let \( u_{j,t} \in \{0, 1\} \) be the indicator of product \( j \) generation upgrading in period \( t \) as a result of joint upgrading decisions, and let \( c_{j,t}^G \) be the realized upgrading cost for product \( j \) in period \( t \). The impact of \( u_{j,t} \) can be summarized by the following equation.

\[
u_{j,t} = \begin{cases} 
  1 \rightarrow \text{pays } c_{j,t}^G; \ g_{j,t} = g_{j,t-1} + 1; \ \ E_{j,t} = \psi(E_{j,t-1}) \\
  0 \rightarrow \text{pays } 0; \ g_{j,t} = g_{j,t-1}; \ \ E_{j,t} = E_{j,t-1} 
\end{cases} \]  

(2)

where \( \psi(x) \) is a given function, with the property \( \psi(x) < x, \forall x \), that models setback in experience level when upgrading a product. Thus, when product \( j \) is upgraded in period \( t \), its generation will increase by 1 while incurring an upgrade cost of \( c_{j,t}^G \) and a setback in experience to \( \psi(E_{j,t-1}) \).

### 2.4 Dynamic Game

For the dynamic game, each product has three states variables: experience level \( E_j \), relative generation \( G_j \), and unobserved characteristics \( \xi_j \). The state of the industry is then characterized by a state profile \( \omega = (E, G, \xi, M) \), where \( M \) is the overall market size. Firm \( i \) makes joint decisions in upgrading all its products, \( U_i \), and in quantity choices of those products, \( Q_i \). Each period in the game can be divided into three stages as follows:

- **(i) Nature Stage**
  - Nature draws shocks on demand (\( M \) and \( \xi \)) and innovation of the outside good (\( g_{0,t} \)). All draws are immediately observed by all firms.

- **(ii) Innovation/Upgrading Stage**
  - **(ii.a)** Firms learn their upgrading cost, which is private information.
  - **(ii.b)** Firms simultaneously make adoption decisions (\( U_i \)). Resulting new generation levels of all products are immediately observed by all firms.

- **(iii) Production and Learning Stage**
  - Firms compete in a simultaneous quantity competition game. Experience level for each product is realized based on quantity choices and is revealed to all firms.

Note that experience state evolves in both stage (ii) and (iii), while generation state changes in stage (i) and (ii). Quantity and upgrading decisions are made in different stages. Thus, expected
future values need to be constructed differently when solving for optimal quantity and upgrading policies. To deal with these complexity, I found it very helpful to be specific about stages for $\omega$. Hereafter, I will denote state profile at the beginning of Stage (ii) as $\omega_i$ and the state profile at the beginning of Stage (iii) as $\tilde{\omega}_i$.

For Stage (ii), since firms do not observe other firms’ realized upgrading costs and upgrading choices when making their own decisions, they have to put probabilities

$$Pr_{k}^{\omega} = \text{Probability of choosing } U_{k}^{\omega}$$
on competitor k’s possible moves. In the following discussion on solving for $Pr_{k}^{\omega}$, I drop superscript $\omega$ on $U_i$ for simplicity and all the discussions are with respect to a given state profile $\omega$. Denote firm i’s expected value, excluding upgrading cost, of choosing $U_i$ as $EV_i^{U_i}$. $EV_i^{U_i}$ is the summation of expected values across all products firm i owns and the expectation is over other firms upgrading probabilities $Pr_{k}^{\omega}$. Let $U_i$ and $U_i'$ be two different vectors of choices from the set $\{0, 1\}^{J_i}$. The vector $U_i$ will be chosen if it gives firm i the largest net continuation value (expected future value less upgrade cost). Thus, the probability of choosing the vector $U_i$ is simply given by the probability of net continuation value with respect to $U_i$ exceeding that with respect to any other choice vector $U_i'$, i.e.

$$Pr_{i}^{U_i} = \text{Prob}[(EV_i^{U_i} - C_i^{G_i} \cdot U_i) \geq (EV_i^{U_i'} - C_i^{G_i} \cdot U_i'), \forall U_i' \neq U_i]$$ (3)

Note that by allowing firms to have multiple products, complications arise in that I need to solve for joint probabilities for each firm, which may have multiple solutions. Fortunately, introducing randomness in a separable form through upgrade cost guarantees a unique solution that can be easily solved for Equation (3). The crucial point is that given $U_i$, $EV_i^{U_i}$ is not a function of any $c_j$. The proof can be found in the Appendix.

With equilibrium $Pr_{i}^{\omega}$ solved from Equation (3), I now turn to equilibrium quantity choices. Since production affects future variable cost through its direct impact on experience accumulation, production decisions for each period could no longer be modeled as static. Quantities enter both the current profit function and the next period value function in the Bellman equation. Aside from this quantity effect on future costs, the per period game is a quantity competition with heterogeneous goods and multi-product firms. The per period payoff (profit) function for product j is

$$\pi_{j}^{\omega} = p_j(Q; X, G, \xi, M)q_j - C_j(q_j; E_j).$$ (4)

Let $\rho$ denote the discount factor. Joint optimal quantity policies for firm i are solved from:

$$\max_{q_j \forall j \in J_i} \left( \sum_{j \in J_i} \pi_{j}^{\omega} + \rho E[V_j(\omega'; \omega, Q)] \right)$$
where next period values are in prime terms. Value function for product \(j\), denoted as \(V_j^{\tilde{\omega}}\), is then defined by the Bellman equation:

\[
V_j^{\tilde{\omega}} = \pi_j^{\tilde{\omega}*} + \rho E[V_j^{\omega'}|\tilde{\omega}, Q^*]
\] (5)

where "*" denotes value based on optimal quantity choices. The transition matrix for calculating \(E[V_j^{\omega'}|\tilde{\omega}, Q]\) is left in the Appendix.

In solving the model numerically, I track \(\Pr_i^{\omega}\) for each state profile \(\omega\) and \(q_j^{\tilde{\omega}}\) and \(V_j^{\tilde{\omega}}\) for each state profile \(\tilde{\omega}\). Note that I utilize the differentiation of \(\omega\) and \(\tilde{\omega}\) here. I find that tracking \(V_j^{\tilde{\omega}}\) instead of \(V_j^\omega\) makes computation much easier.

### 3 Data

Data utilized in this paper is taken from several sources. To obtain evidence on defining medium-sized wide-bodied aircraft as a single market, I utilized route-level aircraft type and traffic data from the Bureau of Transportation Statistics. This data reveals whether medium-sized wide-bodied aircraft are mainly competing with each other on the routes they fly.

In the application to the aircraft industry, I use a nested-logit discrete choice model for the demand function. Annual fleet and deliveries data from the Airline Monitor are taken to construct quantities for each aircraft type each year. Annual average aircraft value data for each type is provided by Avmark and is used as plane prices. Market size is approximated by the total number of used and new wide-bodied aircraft using data from the Airline Monitor. This choice of approximation is based on the resale and rental market assumption discussed in Section 4.2 below. In the discrete choice model, the aircraft are heterogeneous in characteristics, and the characteristics are collected from the official websites of Boeing and Airbus, as well as various online sources. Characteristics include number of seats, maximum range, number of engines, fuselage, empty operating weight, and first flight year.

Prices need to be instrumented in the demand estimation since they are likely to be correlated with unobserved aircraft characteristics, which is the error term in the regression. Assuming that observed characteristics are uncorrelated with the unobserved components, characteristics are taken as one set of instruments. Cost shifters that are assumed to be correlated with price but not with unobserved characteristics are taken as another set of instruments. Cost shifters used include present and lagged terms of U.S. manufacturing wage rates from the Bureau of Labor Statistics, and aluminium prices from IMF’s International Financial Statistics Online Database.

Production cost estimation is decomposed into three steps. First, I estimate labor input as a function of the production rate and experience. I utilize the data on direct man hours incurred by Lockheed in the production of each L-1011 aircraft for labor input;\(^\text{11}\) The Jet Airliner Production List provides the first flight date of every wide-bodied aircraft produced, which is taken as the date

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\(^{13}\)I am grateful to C. Lanier Benkard for making this data available.
of production.\textsuperscript{14} Production rates and experience are constructed using quantity data and date of production. Second, the relationship between total variable costs and labor input is estimated also using data for the L-1011 program taken from Benkard (2004). Third, maintenance costs of the L-1011 plants reported in Lockheed’s annual reports are used to estimate fixed costs.

In labor input estimation, quantities are likely to be correlated with unobserved productivity. Thus, I instrument quantities using a set of cost and demand shifters that are assumed to be correlated with quantities but not with unobserved productivity. Cost shifters are identical to those used in demand estimation. Demand shifters include present and lagged terms of world and regional GDP from IMF’s \textit{International Financial Statistics Online Database} and oil price data from the \textit{Energy Information Administration}.

Generation upgrade-related parameters are calibrated based on data from several difference sources. Fuel efficiency data from the \textit{Airline Monitor} and operating cost difference claims reported in Boeing and Airbus newsletters are used to determine generations of aircraft. Given the definition of generation, average time before generation upgrade can be calculated using differences in first flight year across generations, which was obtained from the \textit{Jet Airliner Production List}. Upgrading probability is then the inverse of this average time. Since generation is included as a characteristic in demand estimation, generation gap is directly obtained from demand estimation. Finally, generation upgrade costs for various aircraft models were collected from news clippings.

\section{Empirical Application}

In this section, I apply the model in Section 2 to the medium-sized wide-bodied aircraft industry. Depending on industry specifics and data availability, demand and cost functions described in Section 2 are parameterized, and parameters in the model can be estimated following at least two different approaches. First, parameters can be estimated directly in the dynamic game. The common solution method is to first build a likelihood function or moment conditions as functions of the parameters based on the data. Typical examples of moments include average firm choices (price, investment, exit and entry, etc.) and covariances between a firm’s choices and a firm’s own states or rival firms’ states. Then one solves a constrained maximization or minimization problem with respect to the likelihood function or moment conditions by treating equilibrium conditions (Equations (3) and (5)) as constraints. Thus, when the optimization problem is solved, optimal parameter values are found together with the corresponding equilibrium of the dynamic game. Second, demand and cost parameters can also be estimated separately in a first stage, and one assumes the structures generating the estimates are unchanged in the dynamic model. The estimates are then taken as primitives in solving for the equilibrium of the dynamic game. This latter approach is computationally less burdensome than the first approach since the dynamic game only needs to be solved once, and there is no parameter searching in solving the dynamic game. However, it also requires more structure assumptions as discussed above.

\textsuperscript{14}The \textit{Jet Airliner Production List} also has ownership history of all wide-bodied aircraft, which can be used to calculate the rate of aircraft resale and rental.
Data availability can be a factor determining which approach is used. When observations are serially correlated, the entire time series of a variable, for example the price of a product, is just one observation of its evolution, which is affected by various shocks. Thus, if there is only one market in the industry, as in the case of the aircraft industry being studied here, there is just one observation for each variable to construct moment conditions or the likelihood function. This limits both credibility and the number of moment conditions that can be constructed. Hence, for this paper, I chose the second approach to evaluate the 1997 Boeing-McDonnell Douglas merger in the aircraft industry.\footnote{For more localized industries containing many geographic markets, the former approach might be more attractive.}

For the rest of this section, some background information is provided regarding market definition. Then, I present the specific empirical model of the demand and cost function for the aircraft industry and discuss the estimated parameters. With demand and cost structure introduced, I turn to discussions of definition and calibration of generation upgrade. I finish this section with further analysis on applying the dynamic game to the medium-sized wide-bodied aircraft industry.

### 4.1 Medium-Sized Wide-Bodied Aircraft as an Industry

A wide-bodied aircraft is a large jet airliner with two passenger aisles. (See Figure 1 for interior arrangements of a typical 3-class-configuration wide-bodied aircraft.) Following the introduction of the first wide-bodied aircraft, Boeing 747, in 1969, only four firms were active in the industry. Of these four firms, Lockheed left the market in 1984. Nine wide-bodied types were in production during the 1990-2010 period, yet they were not all directly competing with each other due to differences in plane size and maximum flying range. Figure 2 suggests that in terms of size and range, these nine aircraft types are clustered into three groups: small (around 250 seats), medium (around 300 seats), and large (around 450 seats). The horizontal line in the figure marks the nautical distance between Beijing and New York, and is used as a benchmark separating transatlantic and transpacific routes. Differences in length of routes are continuous so this benchmark should only be viewed as a guideline rather than a strict rule. However, we can see that, compared with small aircraft, medium and large aircraft have longer range and are more suitable for transpacific routes. The primary impact of the merger on market structure was the elimination of McDonnell Douglas, whose only wide-bodied aircraft then in production was MD-11. Thus, I focus on a sub-market of aircraft that directly competed with MD-11. That is, the medium-sized group, which includes A330, A340, B777, and MD-11.

Other than those nine current types shown in Figure 2, Boeing introduced B787 in 2011 as a replacement of B777 and Airbus answered with A350, an upgrade of A330, that is projected to enter the market in 2014. I treat B777 and A350 as new generation upgrades of B777 and A330 respectively in the model. In this sense, there are more than one aircraft model numbers (e.g. A330, A350) matching the same product in the model due to generation upgrade. I will still call these products B777 and A330 for simplicity when there is no ambiguity. Table 1 provides a summary of the important characteristics of the medium-sized aircraft. MD-11 is the first product in the
medium-sized sector while B777 is the last to enter the market. Number of engines is an important characteristic because it is an indicator of fuel efficiency. Twin-engine aircraft are generally more efficient than aircraft with more engines.

To examine whether medium-sized aircraft can be treated as a single market, I collect route level information and calculate the following ratio for each route:

\[
\text{medium-wide-ratio} = \frac{\text{total number of flights of medium wide-bodied aircraft}}{\text{total number of flights of any wide-bodied aircraft}}
\] (6)

If this ratio is close to 0, then it is a route where the medium-sized aircraft hardly compete with other wide-bodied aircraft (small or large); if this ratio is close to 1, then it is a route where other wide-bodied aircraft (small or large) hardly compete with medium-sized ones. However, if this ratio is close to 0.5, then medium-sized aircraft are actively competing with other wide-bodied aircraft on a given route. As such, a large proportion of routes with the ratio close to either 0 or 1 would be supporting evidence for defining medium-sized as a single market.

I observe monthly the total number of flights for each aircraft sub-model (e.g., Boeing 777-200) on any U.S. domestic and international route during the 1990-2011 period. For each month-aircraft-route observation, I also observe number of passengers, pound of freights, distance of routes, and total flying time. I focus on those routes with at least one flight of medium-sized wide-bodied aircraft and having distances longer than 1000 miles. I merge all of the post-merger years data (1997-2011) and then only keep routes that have, on average, at least 50 flights of any wide-bodied aircraft per year. All these steps are intended to help me focus on medium-sized-related routes where wide-bodied aircraft are flying in a nontrivial frequency. I also drop all non-jet observations, although they are not expected to fly on a route where wide-bodied aircraft are also flying anyway.

I end up with 908 routes. Checking the medium-wide-ratio, I find:

1. 61.5% of the routes with medium − wide − ratio > 0.8 or medium − wide − ratio < 0.2;
2. 74.0% of the routes with medium − wide − ratio > 0.7 or medium − wide − ratio < 0.3;

Figure 3 demonstrates the distribution of the medium-wide-ratio. I also repeated the above steps with several single year data sets and found similar results.

I present some typical examples of routes and their major aircraft:

1. New York, NY – Shanghai, China: A340, 55.0%; B777, 45.0%
2. Miami, FL – Cologne/Dusseldorf, Germany: A330, 73.9%; A340, 11.5%; MD-11, 5.8%
3. Dallas, TX – Osaka, Japan: B777, 82.1%; MD-11, 17.9%

These markets are exclusively served by medium-sized wide-bodied aircraft. In contrast, typical routes with a medium-wide-ratio close to 0.5 are hub-to-hub domestic routes, e.g., Los Angeles to

\footnote{The part of the density outside range \([0, 1]\) in the figure corresponds to observations close to 0 and 1. They are plotting bugs to be fixed. There are in fact many routes with medium − wide − ratio = 0 or 1. The shape of the density would remain the same after fixing the bug, except that it would be a little bit higher at 0 and 1.}
Chicago. Based on their product traits as reported in Figure 2 and demand information as reported in Figure 3, the data supports treating medium-sized wide-bodied aircraft as a well-defined market.

4.2 Demand Function Estimation

Following Benkard (2004), I model yearly aircraft demand using a nested logit discrete choice model. The demand system is estimated with demand data for the period 1991-2009. A total of 12 aircraft submodels (e.g. Boeing 777-200) were observed over the period, leading to 113 submodel-year observations. Consumer $a$'s utility function of aircraft $j$ at time $t$ is

$$v_{ajt} = \varphi G_{jt} + X_{jt}\beta - \alpha p_{jt} + \xi_{jt} + \zeta_{agt} + (1 - \sigma)\epsilon_{ajt},$$

where $G_{jt}$ is the plane generation level measuring quality. Impact of future generations on demand is then modeled as differences in generations times $\varphi$. $\varphi$ thus represents gaps in quality between generations. $X_{jt}$ are observed characteristics including seats, maximum range, and number of engines. $p_{jt}$ is the average price for aircraft $j$ in year $t$. All prices are converted into 1994 U.S. dollars. $\xi_{jt}$ is the unobserved component affecting demand. Its variation captures variations in consumer preference over brand and plane characteristics. Note that although characteristics are fixed for an aircraft, preference over brand or certain characteristics might change across time due to shocks such as aircraft accidents or expansion of an airline, which prefers a certain aircraft type. Since evolution of $\xi$ is affected by these exogenous shocks, I assume generation upgrade decisions do not affect evolution of $\xi$. $\zeta_{agt}$ and $\epsilon_{ajt}$ are respectively the random group- and plane- specific tastes. $\epsilon_{ajt}$ is an identically and independently extreme value. I allow for two groups in the model, one includes all new medium-sized aircraft and the other includes only the outside good, which stands for small or large wide-bodied aircraft and all of the old wide-bodied aircraft on lease. $\sigma \in [0,1]$ represents the within-group correlation of utilities.

Each year is viewed as a market, and, as in Benkard (2004), the market size $M$ is approximated by the total number of used and new wide-bodied aircraft. This approximation is consistent with the assumption that all old and new aircraft are re-sold or rented out each year.\footnote{Used aircraft trade and rental are very common. For example, almost every MD-11 airliner has changed ownership or is owned by a leasing company.} If a used aircraft did not change ownership in a year, it is viewed as bought by the firm who owned it. In this sense, market size or total transaction each year equals total number of used and new aircraft.

Consumer $a$ chooses product $j \in \{0,1,...,J\}$ in period $t$ if $v_{ajt} > v_{akt}$ for all $k \neq j, k \in \{0,1,...,J\}$. 0 denotes the outside good. Then integrating over the probability of choosing product $j$ for all consumers gives the well-known formula for the market share of product $j$, $s_{jt} = \frac{q_{jt}}{M_t}$ as:

$$s_{jt} = \frac{e^{\varphi G_{jt} + X_{jt}\beta - \alpha p_{jt} + \xi_{jt}}}{D_{gt}\left[\sum_{g}D_{gt}^{(1-\sigma)}\right]},$$
where

\[ D_{gt} \equiv \sum_{j \in \text{group } g} e^{\varphi G_{jt} + X_{jt}\beta - \alpha p_{jt} + \xi_{jt}}. \]

Taking the logarithm and rearranging terms results in the following equation to be estimated using two-stage least squares (2SLS):

\[ s_{\text{share}} \equiv \ln(s_{jt}) - \ln(s_{0t}) = \varphi G_{jt} + X_{jt}\beta - \alpha p_{jt} + \sigma \ln(s_{j/g,t}) + \xi_{jt}, \] (8)

and,

\[ s_{0t} = \frac{M_t - \sum_{k=1}^{J} q_{kt}}{M_t}. \]

Rearranging terms of Equation 8 gives the inverse demand function \( P = P(Q; X, G, \xi, M) \) used in the dynamic game

\[ p_{jt} = \frac{1}{\alpha} \left[ (\varphi G_{jt} + X_{jt}\beta + \xi_{jt}) - (1 - \sigma) \ln(q_{jt}) + \ln \left( M_t - \sum_{k=1}^{J} q_{kt} \right) - \sigma \ln \left( \sum_{k=1}^{J} q_{kt} \right) \right]. \] (9)

Both price and within group share \( \ln(s_{j/g,t}) \) need to be instrumented in the demand estimation since they are likely to be correlated with unobserved aircraft characteristics \( \xi_{jt} \) (the error term in the regression). Used instruments include: observed plane characteristics, characteristics of other planes, hourly wage in manufacturing and its lagged terms, price of aluminum and its lagged terms, and number of other products within the same firm. Firm dummy variables were also tried but adding them did not improve estimation. Observed plane characteristics and characteristics of other planes are taken as instruments with the assumption that observed characteristics are uncorrelated with the unobserved components. Manufacturing wage and aluminum price are cost shifters for price and are assumed to be orthogonal to \( \xi_{jt} \). All these instruments are widely used in the literature except for the number of other products within the same firm. Here I assume that number of other products within the same firm is not correlated with unobserved characteristics of a product. It is correlated with the price of a product because operating cost for an airline (consumer) is generally lower if its fleet consists of a set of planes from the same firm. Thus, a positive externality of products of a firm on other products in the same firm is expected. \( \sigma \) is identified by covariation between the within-group market share of the plane \( s_{j/g,t} \) and its total market share \( s_{jt} \). It is also instrumented by the number of other products within the same firm.

I also tried adding in other independent variables, including fuselage, first delivery year, and firm dummies, but all those variables have very small and insignificant coefficients. Besides, removing and adding them have almost no impact on estimation results.

The estimates for Equation (8) without and with the generation term \( G \) are reported respectively in Table 2 and Table 3. Note that the dependent variable \( s_{\text{share}} \) equals \( \log(\frac{s_{jt}}{s_{0t}}) \), the percentage change of the market share ratio of product \( j \) relative to the outside good. All parameters are significant when generation \( G \) is not included in the regression. Signs of all estimates are as expected. Price has a significant negative influence on market share. Within group utility correlation \( \sigma \) is
close to 1. Together with the comparatively larger price coefficient, it indicates high cross-product elasticities and that inside goods are closer substitutes to each other than to the outside good. The number of engines has negative effect since aircraft with fewer engines generally have higher fuel efficiency. All other factors equal (including price), airlines prefer larger planes and planes with longer range.

The last column in both tables presents standard errors of data observation as indicators of variations in the explanatory variables. Having data variation helps to determine relative importance of characteristics. For instance, in Table 2, the coefficient on maximum range ("range/10000") is estimated to be 2.04, which in absolute value is about three times as large as that of the number of engines. But the ratio of potential variation of the two characteristics is about 2/9th. Putting the coefficient and data variation together, the number of engines generally has a larger contribution to the differences in market share across products than the maximum range does. For example, take the characteristics of A330 and A340 presented in Table 1. The difference in maximum range is 0.2 ten thousand kilometers while the difference in the number of engines is 2, or ten times larger. Hence, combining the information from the first and last column, the number of engines contributes the most to market share differences among characteristics when generation \( G \) is not included. Dominance of the number of engines is understandable since it is correlated with fuel efficiency, which is a major factor in operating cost. This is also supported by the observed trend of twin-engine aircraft replacing those with three or four engines for medium-sized and small-sized wide-bodied aircraft. (With respect to the two non-twin-engine medium-sized aircraft, MD-11 was shut down after the merger and A340 experienced a low production rate in its life and ceased production in 2011.)

When generation \( G \) is taken into account, it explains most variations contributed by the characteristics, rendering them insignificant. The estimate on \( G \) suggests a 12% increase in market share ratio when generation is upgraded. As to be discussed in Section 4.4.1, generation differences represents differences in operating costs for airlines. Therefore, the fact that generation \( G \) has the strongest impact among characteristics (considering data variation) on market share emphasizes the important role of emphasizes the important role that airline’s operating cost concern has in determining competition in aircraft manufacturing.

4.3 Cost Function Estimation

As with many other manufacturing industries, major variations in the unit cost of assembling an aircraft are attributable to variations in labor inputs \( (L) \). Thus, I model total variable cost \( (TVC) \) as a linear function of labor inputs \( L \). Lockheed L-1011 is the only aircraft type that I can observe unit labor cost. I first estimate the learning curve of L-1011 and employ estimates on its total variable cost function from Benkard (2004). Benkard found the wage rate had been quite flat and fixed it at $20/hour. Labor cost is then this wage rate times labor inputs \( L \). Regressing total variable cost on total labor cost gives

\[
TVC_{L-1011} = 36.2 + 0.12L_{L-1011}.
\]
where $TVC_{L-1011}$ is in 1994 dollar millions and $L_{L-1011}$ is labor inputs based on L-1011 estimates and is in 1000 man-hours.

To get the cost function of other products based on that of Lockheed L-1011, I follow the approach in the literature by assuming labor requirements per pound of aircraft is constant across planes. Thus the cost function of product $j$ can be derived from its weight ratio to L-1011, denoted as $r_j$. Total variable cost for product $j$ is then calculated in the model using

$$TVC_j = 36.2 + 0.12r_j L_j.$$ \(^{19}\)

I will discuss the learning curve, $L_j$ as a function of industry quantity vector $Q$ and product experience level $E_j$, in the next section.

Fixed cost is estimated to be $200 million per year based on Lockheed’s annual report on L-1011. It is a strong assumption to speculate that fixed cost is the same across products, but fixed cost has no impact on either prices or consumer surplus in a model without exit and entry. I keep fixed cost in the model only for quantifying firms’ profits.

### 4.3.1 Labor Input Function

The learning curve describes the commonly observed negative relationship between accumulated production and unit labor input requirements in aircraft and many other manufacturing industries. It is decomposed into two equations in my model: labor input as a function of experience and experience as a function of current and past quantities. I will discuss the labor input function in this section and the experience accumulation function in the next one.

Following Benkard (2000), the log unit labor input requirement function for product $j$ produced at time $t$ is estimated based on the following regression:

$$\ln L_{j,t} = \ln A + \gamma_1 \ln E_{j,t} + \gamma_2 \ln S_{j,t} + \epsilon_{j,t}. \quad (10)$$

where $A$ is the intercept and $S = \frac{12}{T} \sum_{\tau=t-3}^{t+3} q_{\tau}$ is the line speed or production rate commonly included in the engineering literature.\(^{20}\) As a summation of recent quantities, line speed $S$ is endogenous and needs to be instrumented. $\gamma_2 > 1$ implies decreasing returns to scale while $\gamma_2 < 1$ implies increasing returns to scale. There is no clear implications of $\gamma_2$ without estimation since productivity of labor depends on the level of capital in the short-run. Dependence of $L$ on experience level $E$ highlights the learning-by-doing feature. The learning, forgetting, and spillover effect on

\(^{18}\)As Benkard (2004) pointed out, although there is no empirical evidence testing whether commercial aircraft share learning curves, literature on military production does suggest that parameters do not vary much across production lines. Further discussion on this issue can be found in Benkard (2004).

\(^{19}\)I also estimated $r_j$ using the first approach described in the beginning of this section for the model without generation upgrade. Specifically, I use difference between estimated and observed average prices as the moment condition. The estimated prices are solved from the dynamic game for each trial of $r_j$ in searching for optimal $r_j$. Minimization is carried out using KNITRO solver with its global multi-start search. I found using the weight ratio as $r_j$ is optimal and cannot be improved.

\(^{20}\)Equation 10 can be derived from a production function with fixed capital taking the Leontief form in labor and materials. See details in Benkard (2000).
marginal cost is then modeled as the impact of industry quantity vector $Q$ on the evolution of experience $E$.

### 4.3.2 Experience Transition Function

When there is no spillover of experience across production, experience accumulation is commonly modeled as

$$E_{j,t+1} = \delta E_{j,t} + q_{j,t}. \quad (11)$$

in the literature, where learning is reflected by the positive relation between $E_{j,t+1}$ and $q_{t}$, and forgetting is modeled as $0 < \delta < 1$. Thus, experience accumulates as more aircraft are produced but also depreciates due to organizational forgetting.

I further allow a spillover effect: experience may also accumulate through production of other products. Thus, $E_{j,t+1}$ will be a function of the entire industry quantity vector $Q_t$. For product $j$, I let the contribution rate of different products on $E_{j,t+1}$ be different in two dimensions: ownership and resemblance in aircraft characteristics. The experience transition function becomes

$$E_{j,t+1} = \delta E_{j,t} + \sum_{j'} \theta_{j'} f(X_j, X_{j'}; \upsilon) q_{j',t}, \quad (12)$$

where

$$\theta_{j'} = \begin{cases} 
1 & \text{if } j = j' \text{ (i.e. on own production)} \\
\theta_1 & \text{if } j' \text{ is a different submodel of } j \\
\theta_2 & \text{if } j' \text{ is a different product in the same firm} \\
\theta_3 & \text{if } j' \text{ is a product from another firm} 
\end{cases} \quad (13)$$

measures the difference of across-firm spillover and within-firm spillover ($\theta_3 - \theta_2$) when products are homogeneous in characteristics. Submodels (for $\theta_1$) are variations of a product. For example, for product A330, there are two variations, A330-200 and A330-300, which have slight differences in seats, range, and other characteristics.

$f(X_j, X_{j'}; \upsilon)$ is a product distance function. I use two characteristics: number of seats and maximum ranges. Specific functional form of $f(X_j, X_{j'}; \upsilon)$ is then

$$f(X_j, X_{j'}; \upsilon) = \frac{|X_{1j} - X_{1j'}|}{\upsilon_1} \frac{|X_{2j} - X_{2j'}|}{\upsilon_2}, \quad (14)$$

where 1 stands for “number of seats” and 2 for “maximum range,” $\upsilon_1, \upsilon_2 \in (0, 1)$; $dx_1$ and $dx_2$ are the maximum difference set to normalize the differences into $[0, 1]$. Note that from Equation (12) and (14), the larger the difference is for a given $\upsilon$, the smaller the spillover effect; $\upsilon_k$ close to 0 implies that characteristic $k$ has a strong impact, while $\upsilon_k$ close to 1 suggests that characteristic $k$ is

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21The spillover effects measured by the parameters here are net effects in the sense that increases in quantities of other aircraft may also spur competition for experienced workers in the labor market. Thus, the parameters represent net effects of experience spillover and labor market competition.

22I tried fuselage and some other characteristics and the results did not change significantly.
has little impact on the spillover rate.

By substituting Equation (12) into (10), I use a GMM method to estimate all the learning curve parameters in these two equations based on monthly data of L-1011.\textsuperscript{23} Note that $\epsilon_{j,t}$ represents the unobserved part of productivity and could be serially correlated. Since productivity interacts with choice of line speed, $S$, and experience accumulation, $E$, $\epsilon_{j,t}$ could also correlate with both $E$ and $S$. Following Benkard (2000), the solution is a GMM-HAC (Heteroskedasticity and Autocorrelation Consistent) estimator suggested by Andrews (1991). The instrument variables are standard: demand shifters include various world GDP measures, the price of oil, and a time trend; cost shifters consist of the world aluminum price and the U.S. manufacturing wage rate.\textsuperscript{24} Other than time trend, all shifters include both present and lagged variables.

Although an instrumental variable that specifically shifts quantity of each aircraft type is not necessary for identification, it would be helpful to have instruments that affect quantities of different types disproportionately. Thus, I include another two sets of instrumental variables. First, I use GDP growth of various regions because different regions have different demands for various aircraft types and brands.\textsuperscript{25} Second, I use the weighted sum of all jet accidents and incidents of a firm for the previous 18 months, with less weight on narrow-bodied aircraft and freighters, divided by total aircraft in service in the same firm. See Figure 4 for an example of the negative correlation of Boeing’s accidents index and quantities produced.

Parameter estimates of the learning curve are given in the first two columns of Table 4. Both characteristics have little impact on spillover ($\nu$ close to 1). Thus, I estimate another learning curve without characteristics as in Equation (15), and the result is given in the last two columns of Table 4.

$$E_{j,t+1} = \delta E_{j,t} + \sum_{j'} \theta_{j'q_{j',t}}.$$  \hspace{1cm} (15)

Estimates are close in both cases since characteristics effects are estimated to be trivial. I drop characteristics and use Equation (15) in the dynamic part. $R^2$ of the estimation is 0.92. Estimated and actual labor input of each L-1011 is plotted in Figure 5. The estimates fit the data well, so I decided it is safe to ignore cost shock $\epsilon_{j,t}$ in the dynamic game. All estimates are significant except for returns to scale. The exponential of the labor cost intercept measures the unit labor requirement for the first aircraft built. As discussed before, I will make this starting level different for different models based on their weight ratios to L-1011. Thus, the shape of the learning curve is assumed to be the same while levels are permitted to be different. There is a 55% labor savings when experience doubles. This seemingly large learning rate is partly offset by a high yearly forgetting rate at 43\% ($= 1 - 0.9549^{12}$). Forty-three percent of experience is lost every year, making it difficult to

\textsuperscript{23}Due to the special connections between L-1011 and McDonnell Douglas’s DC-10, I treat DC-10 as a within firm product for L-1011 in estimation. A detailed discussion on this choice is given in the Appendix.

\textsuperscript{24}For a detailed discussion on choices of these instrumental variables, see Benkard (2000).

\textsuperscript{25}Cited in a Wall Street Journal article "Boeing Ups Forecast For Commercial Aircraft Demand Over 20 Years" published on June 16, 2011, Randy Tinseth, Vice President for marketing at Boeing Commercial Airplanes, said, "Economic growth, world trade and liberalization" are "the fundamental drivers of air travel" and correspondingly aircraft demand.
double experience especially when experience stock is already high. This seemingly high forgetting rate is related to the relatively low aircraft production rate and customized configurations for each aircraft built. In manufacturing aircraft, assembling works repeat at a low rate and tasks are hardly ever identical. In addition, experience measures a firm’s level of human capital rather than skills of each single worker. Thus, frequent turnovers due to layoffs and promotions also imply a high forgetting rate. Both the high learning rate and forgetting rate imply large benefits to produce more oneself and to force one’s rivals to produce less. Dynamically, there will be fierce competition among firms to reach and maintain high output and experience levels, while attempting to force others to be stuck at low output and experience.

Submodel spillover is almost complete (θ₁ = 0.9742). Given this result, along with the fact that demand related characteristics are close among submodels, I decide not to differentiate submodels in the dynamic game. There is almost no cross-firm spillover (θ₃ = 0.0182). This is understandable since experience is believed to be mainly accumulated through repeated practice of workers. Within-firm spillover is about a quarter (θ₂ = 0.2408), indicating that building four aircraft of a different type is as helpful in experience accumulation as assembling one of the same type for a multi-product firm. Note that the large difference between within-firm and cross-firm spillover suggests potential benefits when firms merge and ownership structure changes if the within-firm spillover rate does not vary much on properties beyond ownership.

### 4.3.3 Discretization of Experience

With estimates given in Table 4, next period experience can be calculated using Equation (15) for given experience and quantities of all products in a period. Experience defined in Equation (15) is a continuous variable. To apply it as a state variable in the dynamic game, I discretize the experience variable for each product into 7 grids:

\[ E = \{1, 10, 20, 40, 70, 110, 165\} \]

I use \( E^k \) to denote experience at the \( k \)th grid. (e.g., \( E^2 = 10 \).) With enough grids, the experience process can be approximated arbitrarily well. I will explore that more in Section 4.5.2.

I denote the experience level resulting from Equation (15) as \( E_{j,t+1}^* \), namely,

\[ E_{j,t+1}^* = \delta E_{jt} + \sum_{j'} \theta_{j'} q_{j'} \]

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26 This forgetting rate is close to the 39% forgetting rate estimated in Benkard (2000). See Benkard for further discussion on the high forgetting rate.

27 Several circumstances contribute to a large within-firm spillover effect. First, internal shifts of the workforce are much easier than shifts across firms, and a firm may reallocate workers among different departments to improve efficiency. Second, internal shifts help firms to avoid organizational forgetting by keeping workforce busy assembling other models when demand for a certain model is temporarily low. Furthermore, managerial ability and labor-cost-related production techniques usually can be shared only within a firm, either due to firm differences or the need to keep business secrets.
Then the experience transition process is modeled as

\[ E_{j,t+1} = \begin{cases} 
E_j^u & \frac{E_{j,t+1}^* - E_j^d}{E_j^u - E_j^d} \\
E_j^d & 1 - \frac{E_{j,t+1}^* - E_j^d}{E_j^u - E_j^d} 
\end{cases} \]  

(16)

\( E_j^u \) is the smallest grid in \( E \) larger than \( E_{j,t+1}^* \), and \( E_j^d \) is the largest grid smaller than \( E_{j,t+1}^* \). Thus, both \( E_j^u \) and \( E_j^d \) also depend on quantity \( Q_t \).

### 4.4 Generation Upgrade

In this subsection, I first present how generation is defined in the wide-bodied aircraft industry. Then I discuss employment of relative vs. absolute generation for the medium-sized wide-bodied aircraft industry. Calibration of related parameters are presented at the end.

#### 4.4.1 Generation and Generation Upgrade

Ideally, I would treat each new aircraft type as a new product and allow a specific new vector of characteristics for it. However, it is impossible to do so as we do not observe characteristics of products not yet introduced. Instead, I instead categorize aircraft by generations according to some criteria. Average generation gap \( \varphi \) is estimated in Section 4.2. Note that for the purpose of quantifying the merger effect on expected future welfare and the upgrade rate, knowing the average generation gap is sufficient.

There are many ways to define a generation of a jet airliner. Loosely speaking, a new generation has substantial demand-side advantages attributed to more desirable characteristics over the old generation. Empirically, one simple way is to treat each aircraft submodel (e.g. the Airbus 330-200) as a new generation. However, differences between some types of aircraft are quite distinctive from the differences between other types. New aircraft type and submodels have been introduced to provide longer range, different options in size, higher fuel efficiency, lower \( \text{CO}_2 \) emission, etc. These variations create discrepancies in defining a new generation as any new model introduced. In addition, the demand effect of some new models are small and defining every small changes as a new generation will result in too many possible relative generation levels that again causes the dimensionality problem. Definition of generation needs to be applicable as well as capturing major demand effects. The wide-bodied aircraft industry had evolved for 28 years before the merger and there was no room left for firms to introduce new aircraft with range or plane size meeting the market demand that is not yet covered by an existing type. Models introduced after the 1997 merger were generally driven by concern over operating cost.\(^{28}\) Hence, I define a new generation of aircraft family as one that provides at least 5% lower operating cost for airlines. In the medium-sized wide-bodied aircraft market, introduction of Boeing 777, 787, and Airbus A350 are treated

\(^{28}\)This is confirmed by discussions with Edmund S. Greenslet, an aircraft industry expert, and publisher of The Airline Monitor.
as new generations according to this definition.\textsuperscript{29}

Upgrade in generation may also involve a huge amount of one-time development cost and the lowering of experience level due to adjustments in production procedures. Thus, generation upgrade provides a second source of experience setback other than organizational forgetting. As discussed above, since a product will have different characteristics when its generation is upgraded, its experience level will be lower as workers and technicians will not be familiar with the new specification; it takes practices to figure out new mechanisms that suit the new generation. One such evidence is the temporary rise in labor requirement for the introduction of Lockheed’s L-1011-500.\textsuperscript{30} Figure 6 plots labor input (in 1000 man hours) for each L-1011 built. There were only minor changes in characteristics involved in the introduction of the L-1011-500, which does not even qualified as a new generation according to our definition. However, we can still see a clear cost rise for the first two L-1011-500 aircraft produced in the figure. Following the initial rise, the -500 type required a slightly higher labor input and the difference vanished eventually. Vanished difference suggests that the initial cost rise for the -500 type is not due to a systematically higher cost requirement but resulting from a temporary lowering of experience due to the introduction of new types. The detrimental effect of innovation on experience accumulation is also supported by the work of Levitt, List, and Syverson (2012) in the automobile industry as discussed in the introduction. I will discuss the calibration of upgrading cost and determination of experience setback later in this section.

4.4.2 Relative vs. Absolute Generation

A modeling obstacle of introducing generation upgrade into a dynamic game is that generation itself needs to be a state variable for each product, and it is implausible to assume that the highest generation exists. In fact, even if I assume there is a highest generation level, there are at least two problems. First, existence of the highest level implies that the dynamic mechanism in generation is lost when all products reach the highest level. Second, it is impossible to limit possible generations to a small number, which leads to a dimensionality problem.

I choose to deal with the dimensionality problem by assuming that only relative quality (or generation) matters in determining the individual demand function for each product. Therefore, I can use the pace of evolution of the outside good as a benchmark, and track only the generation difference of each product relative to the outside good.\textsuperscript{31} More precisely, the benchmark is the average difference between the inside goods and the outside good in generation. Thus, generation state $G$ in the model is the difference from that benchmark average difference. Because I observe no more than 2 generation lags among all inside goods (new medium-sized wide-bodied aircraft), I model the generation difference state variable $G_j \in \{-1, 0, 1\}$, where $-1$ denotes one generation behind the average difference between inside and outside goods and 1 denotes one generation ahead.

\textsuperscript{29}B787 and A350 are new generations of B777 and A330, respectively.
\textsuperscript{30}Benkard (2000) also published the same finding.
\textsuperscript{31}The idea of relative generation is originated from relative quality modeled in Pakes and McGuire (1994) and is similar to that in Goettler and Gordon (2011). See further discussion in the introduction.
of it. In this sense, fixing product and product characteristics is equivalent to assuming that the inside goods and the outside good have the same pace of technology improvement. In modeling, this is to fix generation difference state variables for all products at 0.

Theoretically, the outside good is a composition of any products that can be viewed by some consumers as substitutes of the products in the market. Therefore, with respect to the medium-sized wide-bodied aircraft industry, the outside good could consist of old wide-bodied aircraft for sale and new non-medium-sized wide-bodied aircraft and even narrow-bodied aircraft. Different components have different importance in terms of their degree of substitutions to the inside goods.

Since production decisions on non-medium-sized wide-bodied aircraft are also made by Airbus and Boeing, the evolution of the outside good is partially endogenous. However, the event of relative generation downgrade of all inside goods has broader interpretations than the generation upgrade of the outside good. Examining Equation (9) infers that downgrade of \( G_j \) for all inside goods is equivalent to any permanent negative shock on overall demand that lowers prices for all products by \( \frac{\phi}{\alpha} \). For the evolution of demand and generations in an industry, by simply observing the evolution itself, it cannot be determined whether it is driven by innovations in the outside good or by a permanent negative demand shock. In fact, there is no need to make such distinctions in determining generation evolution of the inside goods. For the medium-sized wide-bodied aircraft industry, generation upgrade is mainly driven by macro economic shocks (e.g., the desire for better fuel efficiency due to rising petroleum prices after the September 11th attack), increasing demand for international travel qualities, and increased supply in related markets. For example, Chinese manufacturers recently entered the narrow-bodied aircraft industry. This event is equivalent to the outside good moving to a new generation in the sense that both will permanently shift demand away from the medium-sized wide-bodied aircraft industry and stimulate Boeing and Airbus to innovate for more attractive planes. All these mechanisms are exogenous to the medium-sized wide-bodied aircraft industry; hence, evolution of the market-wide generation downgrade can be treated as exogenous.

4.4.3 Generation Related Parameters

I first specify distribution of upgrading cost \( c^G_j \) and experience setback function \( \psi(E) \) for applying the model to the aircraft industry. \( c^G_j \) is assumed to be drawn from a uniform distribution \( U[C^d, C^u] \).\(^{32}\) The largest and smallest development cost of recent new aircraft models and submodels are chosen as \( C^d = 330 \) and \( C^u = 614 \) (in 1994 dollar millions). The experience setback function \( \psi(E) \) is more difficult as estimating it in the first stage requires observation of a product's unit labor requirement and generation upgrade choices. Experience setback in L-1011-500 provides a lower bound of setback magnitude, but I do not have sufficient data to pin down a specific value.

\(^{32}\)I can also model \( C^G \) as a choice variable rather than a random draw. Randomness is then introduced through probability of success generation advance, which increases with upgrading cost. Although this alternative is more common in the literature, it is less attractive for the aircraft industry since larger investment in aircraft development is realized over time and is generally related to unexpected difficulties in development rather than higher probability of success.
The strategy is to assume generation upgrades setback experience by \( n_G \) grids for the discrete experience state introduced in Section 4.3.3. That is,

\[
E_{j,t} = \psi(E_{j,t-1}) = \min\{E^g - n_G, E^1\},
\]

where \( E^g \) is the discretized grid that \( E_j \) lands on with \( g \in \{1, 2, \ldots, 7\} \). Varying \( n_G \) would demonstrate impact of setback magnitude on firm behaviors. In the merger evaluation, because there are not enough instances of generation upgrade observed to fully estimate the upgrade cost, I set \( n_G = 1 \).\(^{33}\)

Finally, \( p^G \), which also represents the industry long-run innovation rate, is obtained as the inverse of the average years across product before generation advances, which is 10.75. Thus, \( p^G = 1/10.75 = 0.09 \).

4.5 Dynamic Game Specifics

In this sub-section, I explore three issues related to the dynamic game. First, the preference rank state variable is examined in its role to reduce the state space and lessening the computational burden. It can be employed in a dynamic game of any industry with many products that can be grouped into limited categories based on unobserved characteristics. Second, I provide a test with respect to concerns on sensitivity of choices of discretization of the state variables. I conclude this part with arguments on why exit and entry need not to be directly modeled for the medium-sized wide-bodied aircraft industry. Those readers who are not interested in these issues may want to skip to Section 5.

4.5.1 State of Preference Rank

Demand estimation provides a panel data of unobserved characteristics \( \xi \) of all products. Fluctuation in \( \xi \) represents changes in consumer taste driven by exogenous fluctuation in various sources, such as important accidents or technological problems specific to a product or a firm, personnel changes in important airlines, operating-cost-related macroeconomic shocks that lead to preference of twin-engine aircraft, and the temporary spur in international travel driven by the business cycle that makes relatively larger planes more attractive, etc. Ideally, I would model this exogenous fluctuation in \( \xi \) by allowing \( \xi_j \) to be a state variable for each product and then estimate its stochastic process using cell means. However, adding in one more state variable for each product results in the well-known “curse of dimensionality” problem. Figure 7 provides a histogram of percentile prediction error for the market share ratio \( s_{\text{share}} \). Predictions errors are smaller than 3 percent for most observations and smaller than 10 percent for all. This suggests that the error term \( \xi \) is marginal in explaining variations in market share in demand estimation.\(^{34}\) So to save computational power

\(^{33}\)I also tried setting \( n_G \) to 0 or 2 and found no significant differences in quantifying merger efficiencies.

\(^{34}\)The impact of \( \xi \) on quantity prediction is relatively large, but the majority of the prediction error is still less than 15% as in Figure 8.
for more important aspects, I compromise by putting restrictions on joint transitions of all $\xi_j$ and introduce the preference rank state variable discussed below.

For the model of the medium-sized wide-bodied aircraft industry, macroeconomic shocks influencing the entire market are captured by market size state variable $M$; change in product qualities are captured by the generation upgrade decision; observed differences in characteristics are captured by $X$. Suitability is not an important issue when working with the medium-sized market instead of the entire wide-bodied market. Then, variation in $\xi$ is most likely driven by two other factors: variation in preference over the more fuel efficient twin-engine types and variation in preference over firm brands. Therefore, I assume that variation in $\xi_j$, denoted as $\Delta\xi_j$, can be decomposed into two additive parts that evolve independently, with

$$
\xi_j = \bar{\xi}_j + w_j^T \cdot \kappa_j^T + w_j^F \cdot \kappa_j^F,
$$

(18)

where

- $\bar{\xi}_j$ is mean value of time series $\xi_j$;
- $w_j^F$ and $w_j^T$ are given weights;
- $\kappa_j^T$ is variation of preference over twin-engine types; and
- $\kappa_j^F$ is variation of preference between Boeing and Airbus products.

$\kappa_j^T$ and $\kappa_j^F$ are preference rank state variables used in the dynamic game. $\kappa_j^T$ is common among twin-engine types and $\kappa_j^F$ takes the same value for products of the same firm. I denote the vectors of $\kappa_j^T$ and $\kappa_j^F$ for all types and all firms as $\kappa^T$ and $\kappa^F$, respectively. Thus, the lengths of both vectors depend on the number of firms/types instead of the number of products. For example, $\kappa^T$ is of length 2 because I have two types: “twin-engines” and “not-twin-engines.” The two vectors $\kappa^T$ and $\kappa^F$ are preference rank state variables in the dynamic game and evolve stochastically over time. In the dynamic game, I allow each vector to take on two possible values. Specifically, $\kappa^T = \kappa^{T1}$ is the vector for state where twin-engine aircraft are relatively preferred while $\kappa^T = \kappa^{T0}$ is the vector when they are less attractive. Similarly, $\kappa^F = \kappa^{F1}$ when Airbus is preferred while $\kappa^F = \kappa^{F0}$ when Boeing is preferred.

Using the panel data of unobserved characteristics $\xi$, the parameters ($\bar{\xi}_j, \kappa_j^T, \kappa_j^F, w_j^T, w_j^F$) and transitions of $\kappa^T$ and $\kappa^F$ are calibrated as follows:

1. $\bar{\xi}_j$ is calibrated as the mean of time series $\xi_{jt}$ and variation in $\xi$ is calculated as

$$
\Delta\xi_{jt} = \xi_{jt} - \bar{\xi}_j.
$$

$\kappa_j^T$ and $\kappa_j^F$ are preference rank state variables used in the dynamic game. $\kappa_j^T$ is common among twin-engine types and $\kappa_j^F$ takes the same value for products of the same firm. I denote the vectors of $\kappa_j^T$ and $\kappa_j^F$ for all types and all firms as $\kappa^T$ and $\kappa^F$, respectively. Thus, the lengths of both vectors depend on the number of firms/types instead of the number of products. For example, $\kappa^T$ is of length 2 because I have two types: “twin-engines” and “not-twin-engines.” The two vectors $\kappa^T$ and $\kappa^F$ are preference rank state variables in the dynamic game and evolve stochastically over time. In the dynamic game, I allow each vector to take on two possible values. Specifically, $\kappa^T = \kappa^{T1}$ is the vector for state where twin-engine aircraft are relatively preferred while $\kappa^T = \kappa^{T0}$ is the vector when they are less attractive. Similarly, $\kappa^F = \kappa^{F1}$ when Airbus is preferred while $\kappa^F = \kappa^{F0}$ when Boeing is preferred.

Using the panel data of unobserved characteristics $\xi$, the parameters ($\bar{\xi}_j, \kappa_j^T, \kappa_j^F, w_j^T, w_j^F$) and transitions of $\kappa^T$ and $\kappa^F$ are calibrated as follows:

1. $\bar{\xi}_j$ is calibrated as the mean of time series $\xi_{jt}$ and variation in $\xi$ is calculated as

$$
\Delta\xi_{jt} = \xi_{jt} - \bar{\xi}_j.
$$
2. Among the four products in the model, A330 and A340 are of the same firm but only A330 is a twin-engine. Based on Equation (18), differences in series of $\Delta \xi_{A330}$ and $\Delta \xi_{A340}$ come from engine-difference only. $\kappa^T$ and its transition are calibrated as

(a) For each time $t$, if $\Delta \xi_{A330,t} \geq \Delta \xi_{A340,t}$, $\kappa^T_t = \kappa^{T1}$. Otherwise, $\kappa^T_t = \kappa^{T0}$.

(b) Using the time series of $\kappa^T_t$ from (a), its transition matrix is then estimated in the usual way of a Markov chain.

(c) Value of $\kappa^{T1}$ is chosen as the conditional mean ($\langle \Delta \xi_{A330} \rangle | \Delta \xi_{A330,t} \geq \Delta \xi_{A340,t}$). The same applies to $\kappa^{T0}$.

3. $\kappa^F$ and its transition are calibrated similarly using the time series of $\Delta \xi_{A330}$ and $\Delta \xi_{B777}$, both of which have two engines.

4. $w^T_j$ and $w^F_j$ are chosen to minimize the distance between observed and calibrated panel of $\Delta \xi$.

Figure 9 demonstrates the fit of data for $\xi$ calibrated from Equation 18 (labeled as “Rank”) and for having a binary state variable for each $\xi_j$ (labeled as “Cell”). The preference rank approach is able to provide better predictions for transitions and no worse fitting in values, while reducing the size of the state space. Parameters and transition matrices estimated are given in Table 5.

Recall that $\xi$ represents airline preference over brand and characteristics. Although it is reasonable to assume that a merger does not affect airline preference over characteristics, it certainly changes product ownership. This creates problems on how one should adjust $\xi_j$ if it were modeled as a state variable with its transition estimated based on its own time series. However, with the introduction of the preference rank state variable, the merger’s impact on $\xi_j$ through ownership change is directly captured by $\kappa^F_j$ in Equation (18).

4.5.2 Sensitivity of Discretization

Both experience state variable $E$ and market size state variable $M$ are discretized. Solving quantity choices at all state profiles can be viewed as a non-parametric approximation of the underlying equilibrium function from state space to policy space. Then a natural question is whether I have chosen enough number of grids so that the approximation is close to the underlying function. Although it is impossible to test sensibility of the choice of the number of grids by having infinitely many grids, robustness can surely be tested by, for example, doubling the number of grids and comparing resulting policy functions. I tried this on several model specifications and with various denser griding methods and found close equilibrium policy functions. Hence, it is reasonable to believe that the result is robust to the discretization method. While state space has high dimensions, demonstrating policy function for more than three dimensions in one figure is hardly instructive.

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36If there is more than one product of the same type, quantity weighted means can be used instead.

37Value of $\kappa^F_{MD11}$ is also conditional on whether Airbus or Boeing is preferred. I tried to allow preferences rank states over three firms, but did not find much difference.
Thus, I chose to plot one policy variable on two varying state variables while keeping other states fixed. This led to hundreds of figures to cover the entire policy function even for very basic model specifications.

Representative of that output, Figure 10 plots quantity of Airbus A330 as a function of experience levels of Airbus A330 and A340, fixing experience level of Boeing 777 at the lowest grid and market size at the highest grid.\(^{38}\) The blue plane is solved from the model employed in the paper where \(E\) is discretized into 7 grids while the red plane is solved from a model with a denser grid for \(E\) by adding a grid point between any of the seven original grids for \(E\), i.e.,

\[
E' = \{1, 4.5, 10, 15, 20, 30, 40, 55, 70, 90, 110, 137.5, 165\}.
\]

The two planes are generally close to each other, suggesting the choice of seven grids provides a close approximation to the underlying policy function.

### 4.5.3 Exit and Entry in the Medium-sized Wide-bodied Aircraft Industry

Entry and exit decisions of both firm and product levels are generally assumed away in the model. The only exception is that I allow a firm to switch any of its product to a potential entrant good by setting quantity of that product to 0\(^{39}\) in any period and reverse the process by setting a positive quantity in any future period. Here I elaborate on reasons why there is no need to directly model entry and exit.

There are at least two reasons why entry on the firm level is rare in the wide-bodied aircraft industry. First, it requires huge initial capital and a complete set of frontier technologies to start a new business. Historically, all four firms that participated in the wide-bodied aircraft sector have been active in other sectors of the aircraft industry and are somehow subsidized by powerful governments in their military sectors. Second, the state of the art technologies employed in aircraft design and manufacturing also work as entry barriers. Third, the learning curve feature acts as an entry barrier since it implies that an entrant cannot make any profit until after a long period. Moreover, a firm may incur a potential huge loss if there are not sufficient sales later at the bottom of the learning curve to reimburse pricing below marginal cost in the early stage. Business failure of Lockheed L-1011 stands as a perfect example and a live lesson. It is also the only incidence of exit not resulting from a merger in the industry.

No new firms have entered the wide-bodied market since shortly after the industry spawned about 40 years ago. Given that developmental time needed is at least five years and no entering intention has been revealed by 2012, it is safe to say that there will be no new entrant until at least by 2017, that is, 20 years after the merger studied here. As for exit, for the only two remaining

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\(^{38}\)The equilibrium strategy for Figure 10 is solved from a scenario of post-merger with the MD-11 shut down for the model without the generation upgrade feature. Figures for firm strategies with respect to all states are available upon request.

\(^{39}\)In the empirical model with the demand function defined based on discrete choices, optimal quantity choices are never 0, but can be arbitrarily close to 0. In this case, I call the quantity “effectively” zero since it only has negligible difference from an absolute 0.
firms, Airbus and Boeing, no evidence exists that they will exit the market, particularly considering their important political strategic status.

Although entry and exit on the product level is not directly modeled as a firm choice variable, I allow it in restricted format. First, a product can be switched between a potential entrant and an active good through quantity choices as discussed above. Note that in the model, when quantity of a product is effectively zero, it has no effect on choices of other products or consumer surplus. This is demonstrated by the empirical results on MD-11 shortly after the merger in the scenario where MD-11 is not immediately shut down after the merger. Second, the model is also consistent with the introduction of new aircraft as future generations of the current types. This is because products in the model are captured by characteristics, and their advances are captured by generation upgrade. So introduction of a new generation model replacing the old one is viewed as a quality improvement of a product. Hence, I feel comfortable to assume away exit and entry and instead focus on experience and generation evolution that I believe are much more important in the dynamics of the industry.

5 Results of Dynamic Analysis

Now that the dynamic model for the medium-size wide-bodied aircraft industry has been setup, it can be used to address questions with regard to the impact of the 1997 Boeing-McDonnell Douglas merger. First, I ask questions about the merger effect on consumer welfare. What is the net effect of the merger on consumer surplus? How much efficiency comes from dynamic mechanisms of learning-by-doing and generation upgrade? Was the merger efficiency primarily attributable to learning-by-doing or generation upgrade? If the dynamic mechanisms were ignored and instead a traditional static model were used, how would the answers differ? Second, I examine how the merger affects firm behaviors and market structure. How did the merger affect experience accumulation and generation upgrade? If we had forbid Boeing to shut down the MD-11 immediately after the merger, would Boeing have found it profitable to keep the MD-11 in the long-run? What would have been the impact of keeping the MD-11? I address all of these questions in Section 5.1. Third, recent innovation events suggest that the aircraft upgrade rate and magnitude are likely to be systematically higher in the future. I thus perform comparative static analysis to examine the impact of this possibility. Specifically, what would the net consumer welfare for the merger be if the generation upgrade rate and magnitude were larger? Section 5.2 deals with this question.

To address these questions, I solve three types of games:

- **Game A**: Dynamic game with learning-by-doing and generation upgrade
- **Game B**: Dynamic game only with learning-by-doing
- **Game C**: Static game without learning-by-doing or generation upgrade

\[40\text{For calculation of total discounted values, it is assumed that the same static game repeats in every period.}\]
Game A corresponds to the full model described in Section 2 while Game B and C remove features from the full model to isolate learning-by-doing and market power effects. All dynamic effects are assumed away in Game C, so the merger effect in it reflects only the market power effect. The difference between Game B and C then reveals the impact of dynamic learning-by-doing. The influence of generation upgrade can be studied by comparing results from Game A and B. Finally, Game C is a traditional static model, so comparing it with Game A also reveals potential bias when ignoring dynamic mechanisms.

To evaluate merger efficiency, I solve dynamic models for three different industry scenarios for each of the three games:  

- **Scenario (i):** Boeing merged with McDonnell Douglas and immediately shut down MD-11 (which is what actually occurred)
- **Scenario (ii):** Boeing kept MD-11 after the merger.
- **Scenario (iii):** No merger

The effect of the merger is quantified by comparing Scenario (i) and (iii). The comparison of Scenario (i) and (ii) examines the difference between forcing MD-11 to be shut down immediately after the merger and letting it evolve endogenously after the merger. For each scenario, with the solved equilibrium strategies, I compute the time series of expected values of price, quantity, experience stock, upgrading probability, profit, consumer surplus, and total surplus for 50 years starting from the state of 1997.

Because Hicksian and Marshallian demand functions are identical in the nested logit discrete choice model, consumer surplus can be obtained simply by integrating the demand function. Following the literature (See Small and Rosen (1981) or Trajtenberg (1989).), the formula for consumer surplus is:

\[ CS = M \cdot \ln(1 + \sum_j e^{\frac{e^{G_j} + x_j \beta - \alpha p_j + \xi_j}{1-\sigma}})^{1-\sigma} \]

Note that the CS formula above does not account for consumer benefits from absolute generation upgrades. This is not a problem for the merger evaluation as those benefits will cancel out when comparing the merger scenario with the no-merger scenario. In addition, consumers’ preferences on product qualities also evolve over time. Thus, CS can be viewed as consumer surplus adjusted for demand evolution, with a rate assumed to be the same as the industrial innovation rate \( p^G \).

One-time cost synergy of the merger is modeled as an experience stock transfer from MD-11 to Boeing 777 with a transfer rate \( \tau \). The merger is also likely to have fixed cost synergies, although these cannot be estimated with only one observed merger in the aircraft industry. However, fixed

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41 Discount factor \( \rho \) is set to 0.925.
42 In this scenario, Boeing can set quantity of the MD-11 to 0 and pay the fixed cost. MD-11 then will function as a potential entrant for Boeing that can come back in production at any time.
43 All paths for different scenarios converge within 25 years. The years after reaching convergence have no impact on comparisons across scenarios.
44 Change in CS is a compensation variation, and CS tends to be overestimated in a logit-based model.
cost synergies do not affect price or consumer surplus. Formally, experience transfer follows the equations:

\[
E_{\text{Post-Merger}}^{\text{B777}} = E_{\text{Pre-Merger}}^{\text{B777}} + \tau E_{\text{Pre-Merger}}^{MD11},
\]

\[
E_{\text{Post-Merger}}^{\text{MD11}} = E_{\text{Pre-Merger}}^{\text{MD11}} + \tau E_{\text{Pre-Merger}}^{\text{B777}}.
\]

With \( \tau = 0 \), no experience stock is transferred; with \( \tau = 1 \) all experience stocks are transferred. Under the scenario where MD-11 is kept after the merger, the above equations assume that experience stocks are symmetrically transferred between MD-11 and Boeing 777. Asymmetric transfer rates can be easily incorporated in the model if necessary.

Since experience transfer is essentially a one-time experience spillover across products when product ownership changes, a potential benchmark for \( \tau \) is the estimated difference between the within-firm spillover rate and the cross-firm spillover rate, i.e., \( \theta_2 - \theta_3 \). However, there is not enough evidence to conclude such a relationship since the underlying mechanisms of one-time sharing might be different from experience sharing each period. For example, building an aircraft involves thousands of tasks and different firms might excel in different tasks. A merger then helps sharing advantages in different tasks that might give a larger boost in cost reduction than common experience spillover by having workers perform similar tasks on different planes. I then choose to compute the welfare effect for all \( \tau \in \{0, 0.01, 0.02, ..., 1\} \) to evaluate the effect of the experience transfer rate.

In the rest of this section, I discuss the impact of the 1997 Boeing-McDonnell Douglas merger on consumer welfare and market structure (Section 5.1). In light of the recent spur in innovation for low operating cost aircraft in the 2010s, I then perform a comparative static analysis on the impact of innovation rate and magnitude on merger efficiency (Section 5.2). The comparative static also serves as a sensitivity check for quantified merger efficiency with respect to generation upgrade parameters.

5.1 Merger Evaluation

Tables 6-8\textsuperscript{45} present total discounted surpluses and profits for the three scenarios for game A, B, and C, respectively. For games A and B that have learning-by-doing, the merger scenarios are further categorized into two cases: no experience transfer (\( \tau = 0 \)) and complete experience transfer (\( \tau = 1 \)).

Larger experience transfer helps Boeing to lower its marginal cost. Thus, when \( \tau \) increases from 0 to 1, both Boeing and the consumers should be better off. However, when Boeing has a large cost advantage under \( \tau = 1 \), Airbus products’ might making less profits. These intuitions are confirmed comparing columns for \( \tau = 0 \) with those for \( \tau = 1 \) in Tables 6 and 7. When there

\textsuperscript{45}Negative profits arise in the tables for products that are effectively not active in production. The discrete choice model of the demand function leads to a production level close to, rather than equal to, 0, when a product should be shut down. Since I do not allow exit, the resulting profit is close to total discounted fixed cost. It does not affect estimations of prices or consumer surplus.
are dynamic efficiencies (Tables 6 and 7), the merger lowers consumer surplus when \( \tau = 0 \) but raises it when \( \tau = 1 \). When there is no dynamic efficiency, Table 8 suggests that the merger is detrimental to consumer welfare. Finally, comparing results between Scenario (i) and (ii) in all three tables implies that consumers are always better off when MD-11 is kept after the merger. However, keeping MD-11 lowers Boeing’s total profit.

Analyzing Tables 6-8 leads to the following property with respect to the merger.

**Property 1 (The Welfare Effect Property).** With complete transfer of experience, the merger increases consumer surplus by $1.57 billion while the static equilibrium model predicts a $22.53 billion loss. Merger efficiency mainly comes from learning-by-doing. The presence of generation upgrade raises net merger efficiency at a smaller \( \tau \) but reduces it at a larger \( \tau \). The merger has no impact on long-run consumer welfare.

The last 2 columns of Table 6 indicate that the merger effect on consumer surplus is $1.57 billion when experience transfer is complete and $-1.78 billion when there is no experience transfer. The entire relationship between the net consumer surplus and experience transfer rate \( \tau \) is demonstrated in Figure 11. Values of surpluses, prices, and profits for this and all subsequent figures are in millions of 1994 dollars. The horizontal line in the figure marks zero consumer effect for the merger. The solid and dashed curves plots net consumer surplus for Game A and B, respectively. The solid curve for Game A increase with \( \tau \) and crosses the horizontal line at around \( \tau = 0.2 \), inferring that the merger is beneficial to consumers as long as there is at least a 20% transfer rate. Note that the break-even spillover rate \( \tau = 0.2 \) is smaller than the difference between the within-firm spillover and the cross-firm spillover \( (\theta_2 - \theta_3 \approx 0.22, \text{see Table 4}) \).

The last column of Table 8 shows that abstracting away dynamic efficiencies, the pure market power effect leads to a consumer loss as large as $22.53 billion. Since net consumer surplus in the full model (Game A) is the difference between merger efficiency and the market power effect, a large market power effect implies that absolute efficiency from learning-by-doing and generation upgrade is also large. In addition, Game C corresponds to the traditional static analysis. The large difference in consumer surplus between Table 6 and Table 8 suggests that ignoring dynamic effects can lead to seriously biased results and erroneous conclusions with regard to the welfare impact of the merger.

Comparing the two curves in Figure 11 and the last two columns in Tables 6 and 7 suggests that the presence of generation upgrade raises net merger efficiency at a smaller \( \tau \) but reduces it at larger \( \tau \). On the one hand, by shutting down MD-11, productions are concentrated on other products with higher quality after the merger. This channel of merger efficiency is not captured for the model without generation upgrade (Game B). From this prospective, adding generation upgrade to the model increases consumer surplus after the merger. On the other hand, when experience is higher, quantity is also higher, inferring a larger loss for a given unit cost rise. Since experience is higher with the merger, loss from experience setback due to generation upgrade is then larger with the merger. In this sense, generation upgrade erodes merger benefits from learning-by-doing. Modeling generation upgrade thus leads to two opposite forces on merger efficiency. When there is
no experience transfer, experience is low and the setback effect on it is minimum. Thus, the effect of concentration on the higher quality product dominates, and net consumer surplus is larger for the model with generation upgrade (Game A) at \( \tau = 0 \). As experience transfer rate \( \tau \) increases, the experience setback effect becomes more and more important. At \( \tau = 1 \), the results suggest that experience setback effect dominates and the presence of generation upgrade lowers merger efficiency. Finally, differences in net consumer surplus between Game B and Game C (about $20 billion) is much larger than that between Game A and Game B (about $2 billion). Thus, the primary efficiency comes from the difference between Game B and Game C, that is, the effect of learning-by-doing rather than generation upgrade.

Figure 12 reports the evolution of expected consumer welfare since 1997 for each scenario. The merger has only an intermediate influence on consumer welfare; per period consumer surpluses are the same in the long-run for all scenarios. This is because MD-11 is not in production and market structures converge to the same steady state for all scenarios in the long-run. However, two things needs to be clarified regarding absence of the long-run effect. First, it does not render the dynamic analysis futile. In the absence of the intermediate dynamic efficiency, a static model leads to misleading conclusions on consumer welfare. Second, the long-run effect result here is specific to the Boeing-McDonnell Douglas merger, and is particularly due to the inferiority of MD-11. In general, long-run effect is likely to emerge for a different merger in a dynamic analysis. I will discuss more on this in the next property.

**Property 2 (The Market Structure Effect Property).** Only A330 and B777 are actively in production in the long-run in all scenarios. For the first several years after the merger, the merger accelerates experience accumulation but has no clear implication on the innovation rate. The merger has no impact on long-run firm behavior.

Figures 13-16 and 17-20 report the evolution of expected quantity and experience respectively since 1997 for each scenario for the four products A330, A340, B777, and MD-11. Recall that experience is simply accumulated quantities through the mechanism of learning, forgetting and spillover. The first observation that can be made from the four quantity figures is that only A330 and B777 are in production in the long-run.\(^{46}\) The results suggest A340 and MD-11 are less favored by airlines than A330 and B777 are, and their market shares were gradually eroded by competitors. Learning-by-doing reinforces disadvantages of A340 and MD-11. On the one hand, low production rates of A340 and MD-11 are not enough to cover organizational forgetting in experience, leading to rising marginal cost. On the other hand, the competitive products, A330 and B777, are able to achieve lower marginal cost through learning because of the added production due to the merger. Enlarged differences in marginal cost eventually drive out A340 and MD-11. This result is consistent with the reality where MD-11 was shut down immediately after the merger and A340 phased out in 2011. Exit of A340 comes early in the model because the model prediction provides expected path while the reality path is just one realization possibly affected by several positive shocks.\(^{47}\)

\(^{46}\)Experience stock of A340 and MD-11 (when merged into Boeing) are not 0 because of experience spillover defined in Equation (15). They have no impact on market evolution as long as they are not in production.

\(^{47}\)Recall the discussion in the beginning of Section 4 on the time series being one observation in a dynamic model.
The mechanism of learning and forgetting favors concentrated production from the prospect of reaching and maintaining high experience levels. If product distances are not large enough, it is not profitable to keep two similar products in one firm.

Now turn back to the figures reporting experience. For $\tau = 0$, experience levels of A330, A340, and B777 are slightly higher for a few years if the merger took place. However, the merger benefit of experience accumulation is small since production of MD-11 is low and quickly approaches 0 even without the merger. (See Figure 16.) This explains why learning-by-doing is not large enough to cover the market power effect if $\tau = 0$. However, for $\tau = 1$, B777 would enjoy an intermediate marginal cost advantage if the merger took place. Cost advantage of B777 would be so large that it would lower quantities and experience levels of Airbus products. In the long-run, MD-11 would not be in production whether there was a merger or not. In addition, Boeing’s cost advantage is not large enough to discourage Airbus from catching up. Thus, all scenarios, with or without merger, converge to the same steady state in the long-run.

Figures 21-24 demonstrates path of upgrade probabilities. A340 and MD-11 are not produced in the long-run, so there is no upgrade on them. Long-run upgrade rates for A330 and B777 are equal to the outside good upgrade rate as inherited in the model; firms only upgrade to maintain optimal generation levels in the long-run. Figure 23 indicates that for the first 5-6 years after the merger, upgrade of B777 is more likely to take place earlier for the merger scenario with complete experience transfer than for the no-merger scenario. This is probably because Boeing would have a cost advantage high enough right after the merger that a little setback in experience in exchange for a higher quality is profitable. Generally speaking, the effect of the merger on generation upgrade is ambiguous. After the merger, softened competition could discourage innovation but enlarged market share may mean a bigger benefit from a better quality product, which would stimulate incurring the fixed cost to innovate. In addition, generation upgrade negatively impacts experience and raises unit cost, which further complicates the impact of the merger on upgrade decisions.

Property 3 (The MD-11 Property). With the merger, consumers are worse off with immediate shutdown of MD-11 (Scenario (i)) compared to continuing production of MD-11 (Scenario (ii)). However, total profits of the merged firm would be lower and would need to be subsidized to keep MD-11. In addition, with the merger, MD-11 is phased out faster.

Recall that the learning effect is potentially beneficial for the merger, either through concentrating learning on fewer products and reaching the bottom of learning curve faster in the scenario where MD-11 was shut down immediately after the merger, or through within-firm spillover when MD-11 is kept after the merger. Thus, keeping MD-11 after the merger might benefit consumers by enjoying experience spillover while avoiding reduced number of products. Comparing consumer surplus and product profits for Scenarios (i) and (ii) in Tables 6 and 7 shows that keeping MD-11 leads to a higher consumer surplus but lower profits for Boeing and Airbus. Theoretically, keeping MD-11 might also be beneficial for Boeing if there were sufficiently large spillover effects and significant differences in characteristics between MD-11 and Boeing 777. However, the results suggests that Boeing’s total profit would be lower because it would incur fixed costs from MD-11 that could
not be fully covered by revenues from MD-11. Airbus’s profit would also be lower because it would face more competition. Thus, if a policy maker wanted to keep MD-11 for consumers’ benefit, Boeing would need to be subsidized.

In Figure 16, quantity curves for the merger scenarios are lower than the curve for the no-merger scenario for the first 6-7 years after the merger. Namely, MD-11 would phase out faster if it was merged into Boeing. On the one hand, MD-11 receives more experience spillover after the merger. On the other hand, Boeing needs to internalize business stealing of the more promising B777 from production of MD-11. The result shown in Figure 16 indicates that business stealing concerns dominated and Boeing found it more profitable to concentrate on production of Boeing 777.

5.2 Comparative Statics

The 2010s is witnessing a boom of generation upgrades in the entire aircraft industry. For the medium-sized wide-bodied market, Boeing 787 was introduced to replace 777 in 2011 and Airbus responded with the new A350, which is projected to take over A330’s market in 2014. Boeing 787 and Airbus A350 are expected to save much more operating cost than previous innovations.\(^48\) This suggests that estimated industrial innovation rate \(p^G\) and, more importantly, estimated generation quality gap \(\varphi\) based on past data might be too conservative. In fact, estimated \(\varphi\) is only 12\% as shown in Table 3, which implies that upgrading to the next generation only increases a product’s market share by 12\% relative to the outside good. In contrast, industry experts predict that the new Boeing 787 and Airbus A350 will eventually drive out old generation models, indicating a much larger percentage change in market share ratio. Thus, I vary \(p^G\) and \(\varphi\) in all dynamic game scenarios to evaluate their impact on estimated merger efficiency. I find that:

**Property 4 (The Innovation Property).** Higher innovation rate or larger generation gap increases merger efficiency for all \(\tau\). Net merger efficiency is increasing in both the innovation rate \(p^G\) and generation gap \(\varphi\).

I call the dynamic model using parameter values estimated from Section 4 the “base model.” Comparative static analysis is then performed by varying one or more parameters of the base model and resolving the dynamic game. Figure 25 compares merger efficiency (\(\Delta CS\)) across different values for the transfer rate \(\tau\) in the base model, with merger efficiency in a model with doubled \(p^G\) and merger efficiency in a model with doubled \(\varphi\). When \(p^G\) is doubled, consumers benefit more from the merger but not by much, and a smaller experience transfer rate shall be enough for the learning-by-doing effect to offset the market power effect (\(\Delta CS = 0\)). It is probably because higher \(p^G\) implies more frequent generation upgrades and setbacks in experience, favoring a more concentrated market that accumulates experience faster. Furthermore, a doubled \(\varphi\) generates larger consumer benefit than the doubled \(p^G\) does. When MD-11 was active, it was not upgraded to a new generation because it was expected to stop production in the long-run. Therefore, production of

\(^48\)With the first 13 B787 delivered, its launch customer All Nippon Airways said the airplane is 21\% better on fuel consumption than old models. Boeing had also claimed its 787-8 to have about 15\% lower operating cost than A330-200, while Airbus predicted A350-1000 will have 25\% lower fuel burn than B777-300ER.
MD-11 leads to lower consumer surplus under the no-merger scenario. A larger generation quality gap $\varphi$ magnifies the loss from MD-11 production, indicating higher consumer welfare for the merger scenario. Figure 26 plots the net merger consumer surplus $\Delta CS$ at $\tau = 1$ as a function of $\varphi$ for $p^G$ and doubled $p^G$. $\Delta CS$ is found to be increasing in both in $p^G$ and $\varphi$. Thus, if the magnitude or rate of innovation is larger in the future, the merger would be more consumer beneficial.

The analysis here also provides a sensitivity check of the consumer welfare effect of the merger with respect to innovation rate $p^G$ and generation gap $\varphi$. In Figure 25, the difference between $\Delta CS$ curve of the base model and that of the “doubled $p^G$” model is relatively small while net consumer surplus at $\tau = 1$ for the “doubled $\varphi$” model is more than three times larger than that for the base model. Thus, merger efficiency calculated in this paper would be too conservative if one were to believe that generation gaps should be much larger in the future.

6 Summary

In summarizing this paper’s contribution, I will first describe the innovation in terms of model and methods and then describe the policy contribution with respect to evaluating the 1997 Boeing-McDonnell Douglas merger.

A dynamic oligopoly model is constructed that allows for multi-product firms, learning-by-doing and endogenous quality improvements. Allowing for both the evolution of cost and product quality is, to my knowledge, new to the literature on dynamic oligopoly models. Having two product-specific dynamic states (experience and generation) that evolve at multiple and different stages creates complexity in solving the dynamic model. I find that it is helpful to distinguish state profiles at different stages. Joint probabilistic upgrading decisions for a multi-product firm could be very complicated, and I deal with this by introducing randomness in a separable term (upgrading cost) that guarantees a unique analytical solution with given expected future values.

To reduce computational burdens, I also introduced a preference rank state variable to replace the unobserved characteristics state variable for each product. The preference rank state variable is applicable to any dynamic oligopoly models, including those without learning-by-doing or innovation features. Since its size does not depend on number of products, the preference rank state variable is most powerful in reducing size of state space for dynamic games with many products where variations in unobserved characteristics are primarily induced by preference shocks over certain attributes of the products, for example, ownership.

As described, the model is applicable to many industries for which learning-by-doing and quality innovation are relevant. However, the primary purpose of the model was to evaluate the 1997 Boeing-McDonnell Douglas merger in the medium-size wide-bodied aircraft industry. I find that with complete experience transfer, the merger increases consumer surplus by $1.57$ billion. Consumers are better off as long as there is at least a 20% experience transfer rate after the merger. Learning-by-doing is the major source of merger efficiency and is large enough to cover the detrimental market power effect of about $20$ billion. The merger’s impacts on both consumer welfare and market structure are intermediate; it only accelerates experience accumulation towards the
steady state and there is no-merger effect in the long-run. Comparative statics suggest that if future generation gaps were to be larger, merger efficiency would be even greater. Differences in net consumer surplus between the dynamic model and a static model suggest potential caveats in traditional static analysis in antitrust practices.

While the primary purpose of the model was to empirically investigate the aircraft manufacturing industry, the model is applicable to many industries for which learning-by-doing and quality innovation are relevant. For example, the model can be modified to examine the potential impact of the recently turned-down takeover of Seagate by Western Digital in the hard disc drive industry. More generally, the model and methods developed here may prove useful for gaining an improved empirical assessment of the significance of dynamic efficiencies from mergers.
References


A Appendix

A.1 Treating MD-11 as a Within Firm Product for L-1011

There are 5 aircraft with 11 submodels that have overlaps in production life with Lockheed L-1011. To estimate Equation (12), I need to observe different families of aircraft within the Lockheed corporation. However, L-1011 is the only wide-bodied commercial aircraft that Lockheed had ever produced. Hence, I make a further assumption that DC-10 of McDonnell Douglas can be treated as a within-firm product for Lockheed L-1011.49 Seemingly a strong assumption, using an outside product as an inside product tends to under-estimate the within firm spillover effect, which would lead to conservative estimate of merger efficiency. However, the under-estimation problem could be (partially) relieved considering the following arguments. First, L-1011 and DC-10 are probably the pair of most similar aircraft in the entire history of wide-bodied aircraft industry. Both aircraft are fitted with three high-bypass turbofan engines, seat around 300 passengers, and have about the same fuselage diameter and exactly the same wingspan. They are also very similar in many detailed aspects that could not all be covered by the product difference function in Equation (12). Hence, with respect to similarity, DC-10 should have the highest spillover on L-1011, offsetting part of the under-estimation. Second, DC-10 was put into production about a year before L-1011 and stayed in lead during their whole production histories. Combining with the similarities, this implies that Lockheed would enjoy a followers advantage in production techniques and benefit more than average cases in experience sharing from the production of DC-10. Third and probably the most important reasoning, the plants for producing L-1011 and DC-10 both sat in the Los Angeles area while the plants of Boeing and Airbus were far away in Seattle and Europe, respectively. Lockheed’s plant was in Burbank, California while Douglas manufactured in Long Beach, California. The two cities are on the opposite side of Los Angeles City with about 30 miles between them. Producing together in the Los Angeles County since the 1920’s implies an unique close connection between workers of the two companies. When there are layoffs and other mobilities, work force can easily shift from one firm to the other. Experience can be shared in the unions or even in the bars. Since within-firm spillover mainly comes from workforce shifts and experience sharing, the spillover effect between the two plants should be closer to the level of within-firm spillovers.

A.2 Alternative Modeling of Experience Transition

50

The expected value function and therefore the right hand side of the Bellman equation (Equation (5)) is not continuously differentiable in quantities according to the numerical transition rule

49 An alternative choice is to use Lockheed’s military aircraft C-5A Galaxy produced during the period of 1968-1973. C-5A Galaxy has the same number of engines and very similar plane size, range, and other characteristics as Boeing 747. Problems in using C-5A lie in the differences between military and commercial aircraft. For example, C-5A Galaxy does not have a corresponding number of seats characteristic as it is an airlift. In addition, C-5A had shorter overlapping production periods with L-1011 than DC-10 did.

50 I am greatly indebt to C. Lanier Benkard for suggesting this alternative modeling idea.
in Equation (16). This non-differentiability makes it hard to solve for optimal quantities in numerical computation. Here I propose an alternative modeling of experience transition to smooth the expected value function.

The problem of non-differentiability comes from the part that Equation (16) restricts transitions to closest lower and upper bounds/grids only while next period experience could span the entire space for theoretical experience transition (Equation (12)). To overcome this problem, a first thought is to add in a random shock to the process as

$$E_{t+1} = E^k = \text{argmin} |(E_{t+1}^* + \epsilon) - E^k|.$$  

(A-1)

Thus, the discrete experience state $E_{t+1}$ is chosen by using $E_{t+1}^*$ plus a random normal draw $\epsilon$ and then rounding to the nearest grid. For example, if $\epsilon \sim N(0, \sigma)$, then no matter what $E_t$ and $q_t$ are, $(E_{t+1}^* + \epsilon)$ could be any real number and $E_{t+1}$ has positive probability going to any grid. This solves the non-differentiability problem but creates incorrect expectation as expectation of $E_{t+1}$ would then not be equal to $E_{t+1}^*$. To fix this problem, I turn to a mixture of Equation (16) and Equation (A-1). The mixture is accomplished in two steps. First, I define $\tilde{E}$ as

$$\tilde{E} = E^*(q) + \nu,$$  

(A-2)

where $\nu \sim f(\nu)$, with $E(\nu) = 0$ and CDF $F(\nu)$. Then I replace $E_{t+1}^*$ with $\tilde{E}$ in Equation (16). It is easy to see that $\sum_k \Pr(E^k) V^k$ is continuously differentiable under this alternative transition rule. I then show that expectation of $E_{t+1}$ equals $E_{t+1}^*$. Let $N$ be the number of grids for $E$. Then,

$$\Pr(E^1) = \Pr\left(E = E^1 | \tilde{E} < E^1\right) \Pr\left(\tilde{E} < E^1\right)$$

$$+ \Pr\left(E = E^1 | E^1 < \tilde{E} < E^2\right) \Pr\left(E^1 < \tilde{E} < E^2\right)$$  

(A-3)

$$= 1 \cdot F(E^1 - E^*) + \int_{E^1 - E^*}^{E^2 - E^*} \left(1 - \frac{E^* + \nu - E^1}{E^2 - E^1}\right) f(\nu) d\nu$$  

(A-5)

Similarly, for $k = 2, 3, \ldots, (N - 1)$

$$\Pr\left(E^k\right) = \Pr\left(E = E^k | E^{k-1} < \tilde{E} < E^k\right) \Pr\left(E^{k-1} < \tilde{E} < E^k\right)$$

$$+ \Pr\left(E = E^k | E^k < \tilde{E} < E^{k+1}\right) \Pr\left(E^k < \tilde{E} < E^{k+1}\right)$$  

(A-6)

$$= \int_{E^{k-1} - E^*}^{E^k - E^*} \left(\frac{E^* + \nu - E^{k-1}}{E^k - E^{k-1}}\right) f(\nu) d\nu + \int_{E^k - E^*}^{E^{k+1} - E^*} \left(1 - \frac{E^* + \nu - E^k}{E^{k+1} - E^k}\right) f(\nu) d\nu$$  

(A-8)
and

\[ \Pr(E^N) = \Pr(E = E^N | \bar{E} > E^N) \Pr(\bar{E} > E^N) \]

\[ + \Pr(E = E^N | E^{N-1} < \bar{E} < E^N) \Pr(E^{N-1} < \bar{E} < E^N) \]

\[ = 1 \cdot (1 - F(E^N - E^*)) + \int_{E^{N-1} - E^*}^{E^N - E^*} \left( \frac{E^* + \nu - E^{N-1}}{E^N - E^{N-1}} \right) f(\nu) \, d\nu \]

Thus,

\[ \sum_k \Pr(E^k) \, E^k = \frac{1}{(E^1 - E^*)} + \frac{E^N}{(E^1 - E^*)} \]

\[ + \int_{E^1 - E^*}^{E^N - E^*} 1 \cdot f(\nu) \, d\nu \]

\[ - \sum_{k=1}^{N-1} E^k \int_{E^k - E^*}^{E^{k+1} - E^*} 1 \cdot f(\nu) \, d\nu \]

\[ = \left( E^1 - E^* \right) \cdot F \left( E^1 - E^* \right) + \left( E^N - E^* \right) \cdot (1 - F \left( E^N - E^* \right)) \]

\[ + E^* \cdot \int_{E^1 - E^*}^{E^N - E^*} \nu \cdot f(\nu) \, d\nu \]

\[ = E^* + \int_{-\infty}^{E^1 - E^*} (E^1 - E^* - \nu) f(\nu) \, d\nu + \int_{E^N - E^*}^{\infty} (E^N - E^* - \nu) f(\nu) \, d\nu \]

\[ \neq E^* \]

where the last equation used the fact \( E(\nu) = 0 \).

However, note that if I restrict the domain of \( \nu \) to be \((E^1 - E^*, E^N - E^*)\), then

\[ \int_{E^1 - E^*}^{E^N - E^*} \nu \cdot f(\nu) \, d\nu = E(\nu) = 0; \]

\[ F(E^1 - E^*) = 0; \]

\[ F(E^N - E^*) = 1; \]

\[ E^* \cdot \int_{E^1 - E^*}^{E^N - E^*} f(\nu) \, d\nu = E^* \cdot 1 = E^*. \]

Hence,

\[ \sum_k \Pr(E^k) \, E^k = E^*. \]

Note that with Equation (12), next period experience is bounded below by \( \delta E_{t-1} \) but not so under this alternative transition. This problem could be fixed by defining lower bound of next
period $E$ to be $\delta E_{t-1}$. However, this will further complicates the function form to achieve correct expectation.

There are several problems with this alternative modeling though. First, it introduces non-negligible computational burden by employing a more complicated function form. Second, it might create too much variation of experience evolution than the data does. under this alternative transition rule, it is possible for experience to jump from the lowest state to the highest state even when $q = 0$ (or from the highest state to the lowest state even when $q$ is very large). This would cause problems for some simulation paths. However, it won’t be a problem for expected discounted CS since probability of such events will be very small.

A.3 Proof on Uniqueness of $Pr^{U_i}$ for Given $EV^{U_i}$

In Equation (3), $EV^{U_i}$ is not a function of any $c_j$ for given $U_i$. Hence, the difference between $EV^{U_i}$ and $EV^{U'_i}$ is monotonic in one or more $c_j$. For given $C^G_i$, $EV^{U_i} - C^G_i$ is simply a vector of $2^J_i$ numbers and we can always find the largest number. Thus, $C^G_i$ divides the $J_i$-dimension Euclidean space $[C^d_i, C^u_i]^J_i$ into $2^J_i$ areas in a unique way.\(^{51}\) $Pr^{U_i}$ then is unique and equals the proportion of areas of the Euclidean space $[C^d_i, C^u_i]^J_i$.\(^{52}\)

I give an example on finding the unique solution of upgrade probabilities for a two-product firm below. Models with firms that have more than 2 products can be solved similarly. In the example, for subscript $ij$, $i$ and $j$ denotes product 1 and 2 of the firm, respectively. $i$ (or $j$) equals 1 (or 0) if product $i$ (or $j$) is upgraded (not upgraded). There are four possible choices for the firm:

$$U_{11}, U_{10}, U_{01}, U_{00}.$$ 

The problem can be summarized as solving

$$P_{11}, P_{10}, P_{01}, P_{00},$$

for given continuation values

$$EV_{11}, EV_{10}, EV_{01}, EV_{00}.$$ 

The net value from each choice is then:

$$
\begin{align*}
V_{11} &= EV_{11} - c_1 - c_2; \\
V_{10} &= EV_{10} - c_1; \\
V_{01} &= EV_{01} - c_2; \\
V_{00} &= EV_{00}.
\end{align*}
$$

\(^{51}\) Strictly speaking, there might be less than $2^J_i$ areas since some $U_i$ might be so undesirable that it is never chosen for any $c_j \in [C^d_i, C^u_i]^J_i$.

\(^{52}\) Note that the upgrade probabilities $Pr^{U_i}$ are continuous but generally not differentiable (at boundaries between cases) in $EV^{U_i}$.
Comparing $V_{ij}$ defines the following six lines in the $c_1$-$c_2$ plane that marks the boundaries between $V_{ij}$.

$L_1: \ EV_{00} - EV_{10} + c_1 = 0$ (vertical)
$L_2: \ EV_{01} - EV_{11} + c_1 = 0$ (vertical)
$L_3: \ EV_{00} - EV_{01} + c_2 = 0$ (horizontal)
$L_4: \ EV_{10} - EV_{11} + c_2 = 0$ (horizontal)
$L_5: \ EV_{10} - EV_{01} + c_2 - c_1 = 0$ ($45^\circ$)
$L_6: \ EV_{00} - EV_{11} + c_1 + c_2 = 0$ ($-45^\circ$)

It is important to notice that the six boundary lines satisfy the following relationship:

\[
\begin{align*}
L_1 + L_4 & = L_6 \\
L_2 + L_3 & = L_6 \\
L_4 - L_2 & = L_5 \\
L_3 - L_1 & = L_5
\end{align*}
\]

In addition, upgrade choices are indicated in the $c_1$-$c_2$ plane by areas defined by the six boundary lines as follows:

\[
\begin{align*}
U_{11} & : \text{left of } L_2, \text{under } L_4, \text{under } L_6; \\
U_{10} & : \text{left of } L_1, \text{above } L_4, \text{above } L_5; \\
U_{01} & : \text{right of } L_2, \text{under } L_3, \text{under } L_5; \\
U_{00} & : \text{right of } L_1, \text{above } L_3, \text{above } L_6.
\end{align*}
\]

Specific divisions of the $c_1$-$c_2$ plane can be divided into two major cases, with $EV_{00} + EV_{11} - EV_{01} - EV_{10} \geq 0$ and $EV_{00} + EV_{11} - EV_{01} - EV_{10} < 0$ demonstrated, respectively, in Figure 27 and 28.

Finally, note that $C^d$ and $C^u$ define a square box, $[C^d, C^u] \times [C^d, C^u]$, in the $c_1$-$c_2$ plane. The upgrade probabilities $P_{ij}$ then equal their corresponding percentage of areas within the square box $[C^d, C^u] \times [C^d, C^u]$. Specifically, depending on the size of the square box and its relative position to the six boundary lines, the division of the square box can be categorized into the following 24 cases. The upgrade probabilities $P_{ij}$ are solved according to the following formulas in each case:

- For $EV_{00} + EV_{11} - EV_{01} - EV_{10} \geq 0$
  - Case I(i)
    - Condition:
      \[
      C^d > EV_{11} - EV_{10}; \\
      C^u \leq EV_{10} - EV_{00}
      \]
* Probability: \[ P_{10} = 1 \]

- Case I(ii)
  * Condition:
    \[
    C^d > EV_{11} - EV_{10}; \\
    C^d \leq EV_{10} - EV_{00} < C^u
    \]
  * Probability:
    \[
    P_{10} = \frac{EV_{10} - EV_{00} - C^d}{C^u - C^d}; P_{00} = 1 - P_{10}
    \]

- Case I(iii)
  * Condition:
    \[
    EV_{00} - EV_{11} + 2C^d \geq 0; \\
    EV_{10} - EV_{00} < C^d; \\
    EV_{01} - EV_{00} < C^d
    \]
  * Probability:
    \[ P_{00} = 1 \]

- Case I(iv)
  * Condition:
    \[
    C^d \leq EV_{11} - EV_{10} < C^u; \\
    C^u \leq EV_{10} - EV_{00}
    \]
  * Probability:
    \[
    P_{11} = \frac{EV_{11} - EV_{10} - C^d}{C^u - C^d}; P_{10} = 1 - P_{11}
    \]

- Case I(v)
  * Condition:
    \[
    C^d \leq EV_{10} - EV_{00} < C^u; \\
    C^d \leq EV_{11} - EV_{10} < C^u
    \]
\[ P_{10} = \frac{EV_{10} - EV_{00} - C^d}{C^u - C^d} \cdot \frac{C^u - (EV_{11} - EV_{10})}{C^u - C^d} \]
\[ S_{11} = \frac{EV_{10} - EV_{00} - C^d}{C^u - C^d} \cdot \frac{EV_{11} - EV_{10} - C^d}{C^u - C^d} \]
\[ S_{00} = \frac{C^u - (EV_{10} - EV_{00})}{C^u - C^d} \cdot \frac{C^u - (EV_{11} - EV_{10})}{C^u - C^d} \]
\[ S_{\text{joint}} = 1 - P_{10} - S_{11} - S_{00} \]

if,

\[ EV_{00} - EV_{11} + C^u + C^d \geq 0 \]

\[ S_{\text{joint,11}} = \frac{(EV_{11} - EV_{10} - C^d)^2}{2 (C^u - C^d)^2} \]
\[ P_{11} = S_{11} + S_{\text{joint,11}} \]
\[ P_{00} = S_{00} + S_{\text{joint}} - S_{\text{joint,11}} \]

eles if,

\[ EV_{00} - EV_{11} + C^u + C^d < 0 \]

\[ S_{\text{joint,00}} = \frac{(EV_{10} - EV_{00} - C^u)^2}{2 (C^u - C^d)^2} \]
\[ P_{11} = S_{11} + S_{\text{joint}} - S_{\text{joint,00}} \]
\[ P_{00} = S_{00} + S_{\text{joint,00}} \]

- Case I(vi)

\* Condition:

\[ EV_{00} - EV_{11} + 2C^u < 0; \]
\[ EV_{11} - EV_{10} \geq C^u; \]
\[ EV_{11} - EV_{01} \geq C^u \]

\* Probability:

\[ P_{11} = 1 \]

- Case I(vii)
* Condition:

\[ C^d > EV_{10} - EV_{00}; \]
\[ C^d > EV_{01} - EV_{00}; \]
\[ C^u \leq EV_{11} - EV_{01}; \]
\[ C^u \leq EV_{11} - EV_{10} \]

* Probability:

\[ P_{11} = \frac{(EV_{11} - EV_{00} - 2C^d)^2}{2(C^u - C^d)^2}; \quad P_{00} = 1 - P_{11} \quad \text{if} \ EV_{00} - EV_{11} + C^d + C^u \geq 0 \]
\[ P_{00} = \frac{(2C^u - EV_1 + EV_0)^2}{2(C^u - C^d)^2}; \quad P_{11} = 1 - P_{00} \quad \text{otherwise} \]

- Case I(viii)

* Condition:

\[ C^d \leq EV_{11} - EV_{01} < C^u; \]
\[ C^d \leq EV_{01} - EV_{00} < C^u \]

* Probability:

\[ P_{01} = \frac{EV_{01} - EV_{00} - C^d}{C^u - C^d} \cdot \frac{C^u - (EV_{11} - EV_{01})}{C^u - C^d} \]
\[ S_{11} = \frac{EV_{01} - EV_{00} - C^d}{C^u - C^d} \cdot \frac{EV_{11} - EV_{01} - C^d}{C^u - C^d} \]
\[ S_{00} = \frac{C^u - (EV_{01} - EV_{00})}{C^u - C^d} \cdot \frac{C^u - (EV_{11} - EV_{01})}{C^u - C^d} \]
\[ S_{joint} = 1 - P_{01} - S_{11} - S_{00} \]

if,

\[ EV_{00} - EV_{11} + C^d + C^u \geq 0 \]

\[ S_{joint,11} = \frac{(EV_{11} - EV_{01} - C^d)^2}{2(C^u - C^d)^2} \]
\[ P_{11} = S_{11} + S_{joint,11} \]
\[ P_{00} = S_{00} + S_{joint} - S_{joint,11} \]

eles if,

\[ EV_{00} - EV_{11} + C^d + C^u < 0 \]
$$S_{\text{joint,00}} = \frac{(EV_{01} - EV_{00} - C^u)^2}{2(C^u - C^d)^2}$$

$$P_{11} = S_{11} + S_{\text{joint}} - S_{\text{joint,00}}$$

$$P_{00} = S_{00} + S_{\text{joint,00}}$$

– Case I(ix)

* Condition:

$$C^d > EV_{11} - EV_{01};$$

$$C^d \leq EV_{01} - EV_{00} < C^u$$

* Probability:

$$P_{01} = \frac{EV_{01} - EV_{00} - C^d}{C^u - C^d}; P_{00} = 1 - P_{01}$$

– Case I(x)

* Condition:

$$C^d \leq EV_{11} - EV_{01} < C^u;$$

$$C^u \leq EV_{01} - EV_{00}$$

* Probability:

$$P_{11} = \frac{EV_{11} - EV_{01} - C^d}{C^u - C^d}; P_{01} = 1 - P_{11}$$

– Case I(xi)

* Condition:

$$C^d > EV_{11} - EV_{01};$$

$$C^u \leq EV_{01} - EV_{00}$$

* Probability:

$$P_{01} = 1$$

– Case I(xii)

* Condition:

$$EV_{11} - EV_{10} < C^u$$

$$EV_{01} - EV_{00} \geq C^d$$

$$EV_{11} - EV_{01} < C^u$$

$$EV_{10} - EV_{00} \geq C^d$$

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\* Probability:

\[
P_{01} = \frac{EV_{01} - EV_{00} - C^d}{C^u - C^d} \cdot \frac{C^u - (EV_{10} - EV_{11})}{C^u - C^d}
\]

\[
P_{10} = \frac{EV_{01} - EV_{00} - C^d}{C^u - C^d} \cdot \frac{C^u - (EV_{10} - EV_{01})}{C^u - C^d}
\]

\[
S_{11} = \frac{EV_{11} - EV_{01} - C^d}{C^u - C^d} \cdot \frac{EV_{11} - EV_{10} - C^d}{C^u - C^d}
\]

\[
S_{00} = \frac{C^u - (EV_{01} - EV_{00})}{C^u - C^d} \cdot \frac{C^u - (EV_{10} - EV_{00})}{C^u - C^d}
\]

\[
S_{\text{joint}} = P_{10} + P_{01} + S_{11} + S_{00} - 1
\]

then

\[
P_{11} = S_{11} - \frac{1}{2}S_{\text{joint}}
\]

\[
P_{00} = S_{00} - \frac{1}{2}S_{\text{joint}}
\]

- For $EV_{00} + EV_{11} - EV_{01} - EV_{10} < 0$,
  - Case II(i)
    - * Condition:
      \[
      C^d > EV_{10} - EV_{00};
      \]
      \[
      C^d > EV_{01} - EV_{00}
      \]
    - * Probability:
      \[
      P_{00} = 1
      \]
  - Case II(ii)
    - * Condition:
      \[
      C^d > EV_{01} - EV_{00};
      \]
      \[
      C^d \leq EV_{10} - EV_{00} < C^u
      \]
    - * Probability:
      \[
      P_{10} = \frac{EV_{10} - EV_{00} - C^d}{C^u - C^d}; P_{00} = 1 - P_{10}
      \]
  - Case II(iii)
* Condition:

\[ EV_{10} - EV_{01} + C^d - C^u \geq 0; \]
\[ EV_{10} - EV_{00} \geq C^u; \]
\[ EV_{11} - EV_{10} < C^d \]

* Probability:

\[ P_{10} = 1 \]

– Case II(iv)

* Condition:

\[ C^d \leq EV_{01} - EV_{00} < C^u; \]
\[ C^d > EV_{10} - EV_{00} \]

* Probability:

\[ P_{01} = \frac{EV_{01} - EV_{00} - C^d}{C^u - C^d}; P_{00} = 1 - P_{01} \]

– Case II(v)

* Condition:

\[ C^d \leq EV_{10} - EV_{00} < C^u; \]
\[ C^d \leq EV_{01} - EV_{00} < C^u \]

* Probability:

\[ P_{00} = \frac{C^u - (EV_{10} - EV_{00})}{C^u - C^d} \cdot \frac{C^u - (EV_{01} - EV_{00})}{C^u - C^d} \]
\[ S_{10} = \frac{EV_{10} - EV_{00} - C^d}{C^u - C^d} \cdot \frac{C^u - (EV_{01} - EV_{00})}{C^u - C^d} \]
\[ S_{01} = \frac{C^u - (EV_{10} - EV_{00})}{C^u - C^d} \cdot \frac{EV_{01} - EV_{00} - C^d}{C^u - C^d} \]
\[ S_{\text{joint}} = 1 - P_{00} - S_{10} - S_{01} \]

if,

\[ EV_{10} - EV_{01} + C^d - C^d = EV_{10} - EV_{01} \geq 0 \]

\[ S_{\text{joint},01} = \frac{(EV_{01} - EV_{00} - C^d)^2}{2(C^u - C^d)^2} \]
\[ P_{01} = S_{01} + S_{\text{joint},01} \]
\[ P_{10} = S_{10} + S_{\text{joint}} - S_{\text{joint},01} \]
eles if,

\[ EV_{10} - EV_{01} + C^d - C^d = EV_{10} - EV_{01} < 0 \]

\[ S_{\text{joint},10} = \frac{(EV_{10} - EV_{00} - C^d)^2}{2(C^u - C^d)^2} \]

\[ P_{01} = S_{01} + S_{\text{joint}} - S_{\text{joint},10} \]

\[ P_{10} = S_{10} + S_{\text{joint},10} \]

- Case II(vi)
  
  * Condition:

\[ EV_{10} - EV_{01} + C^u - C^d < 0; \]

\[ EV_{01} - EV_{00} \geq C^u; \]

\[ EV_{11} - EV_{01} < C^d \]

* Probability:

\[ P_{01} = 1 \]

- Case II(vii)
  
  * Condition:

\[ C^d > EV_{11} - EV_{01}; \]

\[ C^d > EV_{11} - EV_{10}; \]

\[ C^u \leq EV_{10} - EV_{00}; \]

\[ C^u \leq EV_{01} - EV_{00} \]

* Probability:

\[ P_{01} = \frac{(C^u - (EV_{10} - EV_{01}) - C^d)^2}{2(C^u - C^d)^2}; P_{10} = 1 - P_{01} \text{ if } EV_{10} - EV_{01} \geq 0 \]

\[ P_{10} = \frac{(C^u + (EV_{10} - EV_{01}) - C^d)^2}{2(C^u - C^d)^2}; P_{01} = 1 - P_{10} \text{ otherwise} \]

- Case II(viii)
  
  * Condition:

\[ C^d \leq EV_{11} - EV_{01} < C^u; \]

\[ C^d \leq EV_{11} - EV_{10} < C^u \]
\* Probability:

\[
P_{11} = \frac{EV_{11} - EV_{10} - C^d}{C^u - C^d} \cdot \frac{EV_{11} - EV_{01} - C^d}{C^u - C^d}
\]

\[
S_{10} = \frac{EV_{11} - EV_{01} - C^d}{C^u - C^d} \cdot \frac{C^u - (EV_{11} - EV_{10})}{C^u - C^d}
\]

\[
S_{01} = \frac{C^u - (EV_{11} - EV_{01})}{C^u - C^d} \cdot \frac{EV_{11} - EV_{10} - C^d}{C^u - C^d}
\]

\[
S_{joint} = 1 - P_{11} - S_{10} - S_{01}
\]

if,

\[
EV_{10} - EV_{01} + C^u - C^u = EV_{10} - EV_{01} \geq 0
\]

\[
S_{joint_{01}} = \frac{(EV_{11} - EV_{01} - C^u)^2}{2(C^u - C^d)^2}
\]

\[
P_{01} = S_{01} + S_{joint_{01}}
\]

\[
P_{10} = S_{10} + S_{joint} - S_{joint_{01}}
\]

eles if,

\[
EV_{10} - EV_{01} + C^u - C^u = EV_{10} - EV_{01} < 0
\]

\[
S_{joint_{10}} = \frac{(EV_{11} - EV_{10} - C^u)^2}{2(C^u - C^d)^2}
\]

\[
P_{01} = S_{01} + S_{joint} - S_{joint_{10}}
\]

\[
P_{10} = S_{10} + S_{joint_{10}}
\]

- Case II(ix)

\* Condition:

\[
C^u \leq EV_{11} - EV_{01};
\]

\[
C^d \leq EV_{11} - EV_{10} < C^u
\]

\* Probability:

\[
P_{11} = \frac{EV_{11} - EV_{10} - C^d}{C^u - C^d}; P_{10} = 1 - P_{11}
\]

- Case II(x)

\* Condition:

\[
C^u \leq EV_{11} - EV_{10};
\]

\[
C^d \leq EV_{11} - EV_{01} < C^u
\]
* Probability:
\[ P_{11} = \frac{EV_{11} - EV_{01} - C^d}{C^u - C^d}; \quad P_{01} = 1 - P_{11} \]

– Case II(xi)
* Condition:
\[ C^u \leq EV_{11} - EV_{10}; \]
\[ C^u \leq EV_{11} - EV_{01} \]

* Probability:
\[ P_{11} = 1 \]

– Case II(xii)
* Condition:
\[ EV_{10} - EV_{00} < C^u \]
\[ EV_{11} - EV_{10} \geq C^d \]
\[ EV_{01} - EV_{00} < C^u \]
\[ EV_{11} - EV_{01} \geq C^d \]

* Probability:
\[ P_{11} = \frac{EV_{11} - EV_{01} - C^d}{C^u - C^d} \cdot \frac{EV_{11} - EV_{10} - C^d}{C^u - C^d} \]
\[ P_{00} = \frac{C^u - (EV_{10} - EV_{00})}{C^u - C^d} \cdot \frac{C^u - (EV_{01} - EV_{00})}{C^u - C^d} \]
\[ S_{10} = \frac{EV_{10} - EV_{00} - C^d}{C^u - C^d} \cdot \frac{C^u - (EV_{11} - EV_{10})}{C^u - C^d} \]
\[ S_{01} = \frac{C^u - (EV_{11} - EV_{01})}{C^u - C^d} \cdot \frac{EV_{01} - EV_{00} - C^d}{C^u - C^d} \]
\[ S_{\text{joint}} = P_{11} + P_{00} + S_{10} + S_{01} - 1 \]

then
\[ P_{10} = S_{10} - \frac{1}{2} S_{\text{joint}} \]
\[ P_{01} = S_{01} - \frac{1}{2} S_{\text{joint}} \]

### A.4 Expected Value Function and State Transition

First I describe the state transition with experience state variables only. In this case, define

\[ \eta_j (h; q_j) = \left( \frac{E_{j,t+1}^* (q_j) - E_{j,d} (q_j)}{E_{j,u} (q_j) - E_{j,d} (q_j)} \right)^h \left( 1 - \frac{E_{j,t+1}^* (q_j) - E_{j,d} (q_j)}{E_{j,u} (q_j) - E_{j,d} (q_j)} \right)^{1-h} \]
where $h$ is either 0 or 1. Then,

$$
EV_j (\bar{q}) = EV_j (E') = EV_j \left( E_j' ; E_{-j}' \right)
$$

$$
= \eta_j (1) V_j \left( E_u ; E \left[ E_{-j} (\bar{q}) \right] \right) + \eta_j (0) V_j \left( E_d ; E \left[ E_{-j} (\bar{q}) \right] \right)
$$

$$
= \sum_{h_1=0,1} \cdots \sum_{h_k=0,1} \cdots \sum_{h_j=0,1} \prod_k \eta_k (h_k) V_j (E_1, h_1, \ldots E_k, h_k, \ldots) (A-26)
$$

where experience state transition $Pr (E'|E, Q)$ is simply

$$
Pr (E'|E, Q) = \sum_{h_1=0,1} \cdots \sum_{h_k=0,1} \cdots \sum_{h_j=0,1} \prod_k \eta_k (h_k)
$$

Then I add in market size state variable $M_t$. Note that given $Q_t$ and $E_t$, $E_{t+1}$ does not depend on $M_t$ or $M_{t+1}$, i.e.

$$
Pr (E', M'|E, M, Q) = Pr (E'|E, Q) \cdot Pr (M'|M)
$$

Then

$$
EV_j = \sum_{M'} \sum_{E'} V_j (E', M') \cdot Pr (E', M'|E, M, Q) \quad (A-27)
$$

$$
= \sum_{M'} \left[ \sum_{E'} V_j (E', M') Pr (E'|E, Q) \right] \cdot Pr (M'|M) \quad (A-28)
$$

$$
= \sum_{M'} EV_{j}^{M'} \cdot Pr (M'|M) \quad (A-29)
$$

where $EV_{j}^{M'}$ is just $EV_j$ in Equation A-26.

Similar as the market size state variable, the feature that preference rank state variables $(\kappa^T, \kappa^F)$ evolves separately from other state variables make computation easier. Since

$$
Pr (E', M', \kappa^{T'}, \kappa^{F'}|E, M, \kappa^{T}, \kappa^{F}, Q) = Pr (E'|E, Q) \cdot Pr (M'|M) \cdot Pr (\kappa^{T'}|\kappa^{T}) \cdot Pr (\kappa^{F'}|\kappa^{F})
$$

then

$$
EV_j = \sum_{\kappa^{T'}, \kappa^{F'}} \sum_{M'} \sum_{E'} V_j (E', M', \kappa^{T'}, \kappa^{F'}) \cdot Pr (E', M', \kappa^{T'}, \kappa^{F'}|E, M, \kappa^{T}, \kappa^{F}, Q) \quad (A-30)
$$

$$
= \sum_{\kappa^{T'}, \kappa^{F'}} \left\{ \sum_{M'} \left[ \sum_{E'} V_j (E', M') Pr (E'|E, Q) \right] \cdot Pr (M'|M) \right\} \cdot Pr (\kappa^{T'}|\kappa^{T}) \cdot Pr (\kappa^{F'}|\kappa^{F}) \quad (A-31)
$$

$$
= \sum_{\kappa^{T'}, \kappa^{F'}} \left\{ \sum_{M'} EV_{j}^{M'} \cdot Pr (M'|M) \right\} \cdot Pr (\kappa^{T'}|\kappa^{T}) \cdot Pr (\kappa^{F'}|\kappa^{F}) \quad (A-32)
$$

$$
= \sum_{\kappa^{T'}, \kappa^{F'}} EV_{j}^{\kappa^{T'}, \kappa^{F'}} \cdot Pr (\kappa^{T'}|\kappa^{T}) \cdot Pr (\kappa^{F'}|\kappa^{F}) \quad (A-33)
$$

where $EV_{j}^{\kappa^{T'}, \kappa^{F'}}$ is just $EV_j$ in Equation A-27.
Finally, I add in generation difference state variables. I need to compute
\[
\Pr(\tilde{\omega}'|\tilde{\omega}) = \Pr(\tilde{\omega}'|\omega') \cdot \Pr(\omega'|\omega').
\]

Since \((M, \kappa^T, \kappa^F)\) evolve exogenously, I only need to specify transitions of \(E\) and \(G\). Note that \(G^\omega\) is only updated in Stage (i) and (ii) governed by Equation (1) and (2). Transition of \(G\) does not depend on transition of \(E\) but not the other way around. For given \(\tilde{\omega}\), denote \(G_{\text{down}}\) and \(G_{\text{stay}}\) as the states of \(G\) at the beginning of Stage (ii) in the next period when the event \textit{Outside Good Generation Upgrade} took place and otherwise, respectively. For product \(j\) and given \(\Pr_{i}^{U}\), let \(\Pr_{j}^{\text{down}}, \Pr_{j}^{\text{stay}}\) and \(\Pr_{j}^{\text{up}}\) denote probability of generation difference \(G_j\) of product \(j\) after Stage (ii) in the next period decreases, remains the same and increases, respectively. Then,
\[
\begin{align*}
\Pr_{j}^{\text{down}} &= p^G \cdot \Pr_{i}^{u} \cdot G_{\text{down}} + (1 - p^G) \cdot \Pr_{i}^{u} \cdot G_{\text{stay}} \quad (A-34) \\
\Pr_{j}^{\text{up}} &= p^G \cdot \Pr_{i}^{u} \cdot G_{\text{stay}}. \quad (A-36)
\end{align*}
\]

The above equations describe the transition of \(G\) part of \(\Pr(\tilde{\omega}'|\omega')\). Given \(\Pr_{j}^{\text{down}}\), the transition of \(E_j\) part is simply
\[
E_j' = \begin{cases} 
E_j^u \text{ downgrade by } n_G & \text{ with prob. } \Pr_{j}^{\text{down}} \cdot \eta_j(1) \\
E_j^d \text{ downgrade by } n_G & \text{ with prob. } \Pr_{j}^{\text{down}} \cdot \eta_j(0) \\
E_j^u & \text{ with prob. } (1 - \Pr_{j}^{\text{down}}) \cdot \eta_j(1) \\
E_j^d & \text{ with prob. } (1 - \Pr_{j}^{\text{down}}) \cdot \eta_j(0)
\end{cases}
\]

A.5 Computational Algorithms

This section describes algorithms used in solving policy and value functions over the state space for given primitives of the dynamic model. I use a parallel Gauss-Seidel policy iteration developed from Pakes and McGuire (1994). \textit{To be finished.}

\footnote{I omit special cases of hitting the smallest and largest grids of \(E\) and \(G\) in notations here for simplicity.}
### Table 1: Aircraft Characteristics

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>A330</th>
<th>A340</th>
<th>B777</th>
<th>MD-11</th>
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<tbody>
<tr>
<td>Aircraft ID</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
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<tr>
<td>seats</td>
<td>270</td>
<td>326</td>
<td>325</td>
<td>293</td>
</tr>
<tr>
<td>range (km)</td>
<td>12378</td>
<td>14312</td>
<td>14067</td>
<td>12670</td>
</tr>
<tr>
<td>No. of engines</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

### Table 2: Demand Function Estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>S.E.</th>
<th>t</th>
<th>p &gt;</th>
<th>Data S.E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-3.59</td>
<td>0.22</td>
<td>-16.51</td>
<td>0.00</td>
<td>N/A</td>
</tr>
<tr>
<td>seats/100</td>
<td>0.11</td>
<td>0.06</td>
<td>1.91</td>
<td>0.06</td>
<td>0.36</td>
</tr>
<tr>
<td>range/10000</td>
<td>2.04</td>
<td>1.07</td>
<td>1.91</td>
<td>0.06</td>
<td>0.20</td>
</tr>
<tr>
<td>No. of engines</td>
<td>-0.07</td>
<td>0.02</td>
<td>-2.73</td>
<td>0.01</td>
<td>0.91</td>
</tr>
<tr>
<td>price/100</td>
<td>-0.52</td>
<td>0.16</td>
<td>-3.25</td>
<td>0.00</td>
<td>0.17</td>
</tr>
<tr>
<td>InGroup Corr. (σ)</td>
<td>0.98</td>
<td>0.04</td>
<td>23.81</td>
<td>0.00</td>
<td>1.13</td>
</tr>
</tbody>
</table>

### Table 3: Demand Function Estimates (with Generation)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>S.E.</th>
<th>t</th>
<th>p &gt;</th>
<th>Data S.E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
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<td>0.22</td>
<td>-15.46</td>
<td>0.00</td>
<td>N/A</td>
</tr>
<tr>
<td>Generation</td>
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<td>0.06</td>
<td>2.02</td>
<td>0.05</td>
<td>0.49</td>
</tr>
<tr>
<td>seats/100</td>
<td>0.07</td>
<td>0.06</td>
<td>1.25</td>
<td>0.21</td>
<td>0.36</td>
</tr>
<tr>
<td>range/10000</td>
<td>0.15</td>
<td>0.11</td>
<td>1.43</td>
<td>0.16</td>
<td>0.20</td>
</tr>
<tr>
<td>No. of engines</td>
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<td>0.03</td>
<td>-0.80</td>
<td>0.42</td>
<td>0.91</td>
</tr>
<tr>
<td>price/100</td>
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<td>0.13</td>
<td>-5.90</td>
<td>0.00</td>
<td>0.17</td>
</tr>
<tr>
<td>InGroup Corr. (σ)</td>
<td>0.97</td>
<td>0.02</td>
<td>50.26</td>
<td>0.00</td>
<td>1.13</td>
</tr>
</tbody>
</table>
### Table 4: Learning Curve Parameters

<table>
<thead>
<tr>
<th>Explanation</th>
<th>Value</th>
<th>Std</th>
<th>Value</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>lnA Labor Cost Intercept</td>
<td>9.2590</td>
<td>(3.2885)</td>
<td>9.3113</td>
<td>(3.1696)</td>
</tr>
<tr>
<td>$\gamma_2$ Return to Scale</td>
<td>0.3178</td>
<td>(0.5904)</td>
<td>0.3141</td>
<td>(0.5552)</td>
</tr>
<tr>
<td>$\gamma_1$ Learning Parameter</td>
<td>-1.1462</td>
<td>(0.1374)</td>
<td>-1.1523</td>
<td>(0.1275)</td>
</tr>
<tr>
<td>Implied Learning Rate</td>
<td>55%</td>
<td></td>
<td>55%</td>
<td></td>
</tr>
<tr>
<td>$\delta$ Depreciation of E</td>
<td>0.9546</td>
<td>(0.0014)</td>
<td>0.9549</td>
<td>(0.0012)</td>
</tr>
<tr>
<td>$\theta_1$ In-family Spillover</td>
<td>0.9999</td>
<td>(0.0239)</td>
<td>0.9742</td>
<td>(0.0198)</td>
</tr>
<tr>
<td>$\theta_2$ In-firm Spillover</td>
<td>0.2383</td>
<td>(0.0029)</td>
<td>0.2408</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>$\theta_3$ Across-firm Spillover</td>
<td>0.0138</td>
<td>(0.0017)</td>
<td>0.0182</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>$\upsilon_1$ Seats Diff.</td>
<td>0.9998</td>
<td>(0.0037)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\upsilon_2$ Maximum Range Diff.</td>
<td>0.9998</td>
<td>(0.0032)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: Implicit learning rate is calculated as $1 - 2^{\gamma_1}$, which measure percent of labor saving when experience doubles.*

### Table 5: Market Size and Preference Rank Parameters

| $\xi_j$ = (-1.5E-9, 6.0E-10, 0.0, -1.2E-9) | $\kappa_T^{T1}$ = (0.0995, 0.0590) |
| $\kappa_T^{T0}$ = (-0.0885, -0.0524) | $\kappa_F^{T1}$ = (-0.0156, -0.0754) |
| $\kappa_F^{F0}$ = (0.0286, 0.2074) |

$T$ transition: 

$$
\begin{pmatrix}
0.4286 & 0.5556 \\
0.5714 & 0.4444
\end{pmatrix}
$$

$F$ transition: 

$$
\begin{pmatrix}
0.8182 & 0.4 \\
0.1818 & 0.6
\end{pmatrix}
$$

$M$ grids: 

$$
(2823, 2966, 3100)
$$

$M$ transition: 

$$
\begin{pmatrix}
0.8462 & 0.2143 & 0.0000 \\
0.1538 & 0.7143 & 0.1429 \\
0.0000 & 0.0714 & 0.8571
\end{pmatrix}
$$
Table 6: Merger Effect for Game A (Full Model)

<table>
<thead>
<tr>
<th>Value</th>
<th>Scenario (i)</th>
<th>Scenario (ii)</th>
<th>Scenario (iii)</th>
<th>(i)-(iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a) ( \tau = 0 )</td>
<td>(b) ( \tau = 1 )</td>
<td>(a) ( \tau = 0 )</td>
<td>(b) ( \tau = 1 )</td>
</tr>
<tr>
<td>CS</td>
<td>154.19</td>
<td>157.55</td>
<td>154.83</td>
<td>157.61</td>
</tr>
<tr>
<td>( \pi_{all} )</td>
<td>30.43</td>
<td>36.30</td>
<td>27.41</td>
<td>33.62</td>
</tr>
<tr>
<td>TS</td>
<td>184.62</td>
<td>193.85</td>
<td>182.25</td>
<td>191.23</td>
</tr>
<tr>
<td>( \pi_{A330} )</td>
<td>18.27</td>
<td>15.13</td>
<td>17.87</td>
<td>15.17</td>
</tr>
<tr>
<td>( \pi_{A340} )</td>
<td>-2.23</td>
<td>-2.35</td>
<td>-2.27</td>
<td>-2.35</td>
</tr>
<tr>
<td>( \pi_{B777} )</td>
<td>14.39</td>
<td>23.52</td>
<td>14.19</td>
<td>23.35</td>
</tr>
<tr>
<td>( \pi_{MD11} )</td>
<td>N/A</td>
<td>N/A</td>
<td>-2.38</td>
<td>-2.54</td>
</tr>
</tbody>
</table>

All values are total discounted expected values in billions of 1994 U.S. dollar.

Table 7: Merger Effect for Game B (without Generation Upgrade)

<table>
<thead>
<tr>
<th>Value</th>
<th>Scenario (i)</th>
<th>Scenario (ii)</th>
<th>Scenario (iii)</th>
<th>(i)-(iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a) ( \tau = 0 )</td>
<td>(b) ( \tau = 1 )</td>
<td>(a) ( \tau = 0 )</td>
<td>(b) ( \tau = 1 )</td>
</tr>
<tr>
<td>CS</td>
<td>177.89</td>
<td>184.06</td>
<td>178.36</td>
<td>184.06</td>
</tr>
<tr>
<td>( \pi_{all} )</td>
<td>58.99</td>
<td>63.64</td>
<td>56.09</td>
<td>61.02</td>
</tr>
<tr>
<td>TS</td>
<td>236.87</td>
<td>247.69</td>
<td>234.46</td>
<td>245.08</td>
</tr>
<tr>
<td>( \pi_{A330} )</td>
<td>24.04</td>
<td>20.69</td>
<td>23.71</td>
<td>20.69</td>
</tr>
<tr>
<td>( \pi_{A340} )</td>
<td>-1.45</td>
<td>-1.97</td>
<td>-1.52</td>
<td>-1.98</td>
</tr>
<tr>
<td>( \pi_{B777} )</td>
<td>36.40</td>
<td>44.92</td>
<td>36.27</td>
<td>44.92</td>
</tr>
<tr>
<td>( \pi_{MD11} )</td>
<td>N/A</td>
<td>N/A</td>
<td>-2.36</td>
<td>-2.61</td>
</tr>
</tbody>
</table>

All values are total discounted expected values in billions of 1994 U.S. dollar.

Table 8: Merger Effect for Game C (the Static Game)

<table>
<thead>
<tr>
<th>Value</th>
<th>Scenario (i)</th>
<th>Scenario (ii)</th>
<th>Scenario (iii)</th>
<th>(i)-(iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS</td>
<td>63.05</td>
<td>82.99</td>
<td>85.58</td>
<td>-22.53</td>
</tr>
<tr>
<td>( \pi_{all} )</td>
<td>51.91</td>
<td>45.77</td>
<td>43.68</td>
<td>8.23</td>
</tr>
<tr>
<td>TS</td>
<td>114.96</td>
<td>128.76</td>
<td>129.26</td>
<td>-14.30</td>
</tr>
<tr>
<td>( \pi_{A330} )</td>
<td>26.56</td>
<td>17.22</td>
<td>16.18</td>
<td>10.38</td>
</tr>
<tr>
<td>( \pi_{A340} )</td>
<td>23.45</td>
<td>15.50</td>
<td>14.63</td>
<td>8.82</td>
</tr>
<tr>
<td>( \pi_{B777} )</td>
<td>1.90</td>
<td>-0.65</td>
<td>-0.25</td>
<td>2.15</td>
</tr>
<tr>
<td>( \pi_{MD11} )</td>
<td>N/A</td>
<td>13.69</td>
<td>13.13</td>
<td>-13.13</td>
</tr>
</tbody>
</table>

All values are total discounted expected values in billions of 1994 U.S. dollar.

Recall that \( \tau = 0 \) corresponds to no experience share after the merger while \( \tau = 1 \) matches complete experience share. The 3 scenarios are:

- **Scenario (i):** Boeing merged with McDonnell Douglas and immediately shut down MD-11
- **Scenario (ii):** Boeing kept MD-11 after the merger.
- **Scenario (iii):** No merger
2.4.1 INTERIOR ARRANGEMENTS - TRI-CLASS CONFIGURATION

MODEL 777-200

D6-58329

JULY 1998

328 PASSENGERS

24 FIRST CLASS AT 60-IN PITCH
61 BUSINESS CLASS AT 58-IN PITCH
243 ECONOMY CLASS AT 32-IN PITCH
(60 SEATS AT 31-IN PITCH)

Figure 1: Interior Arrangements of a Typical Boeing 777-200 (3-Class)

Figure 2: Seats and Range of All Wide-bodied Aircraft in Production since 1990
Figure 3: Distribution of medium-wide-ratio

Figure 4: Correlation of Quantities and Past Accident Index
Figure 5: Fit of Labor Input of L-1011

Figure 6: L-1011 Generation Impact on Experience Level

A slight setback in experience when -500 type is first introduced.
Figure 7: Distribution of Percentile Difference between Actual and Estimated Market Share Ratio

Figure 8: Distribution of Percentile Difference between Actual and Estimated Quantity
Figure 9: $\xi$ Approximation Performance Comparison

Figure 10: Demonstration of Robustness of Discretization

Quantity of A330 at $E_{5777}=1$ and $M=3100$
Figure 11: Comparison of $\Delta CS$ for Game A and B when $\tau$ Varies

![Graph showing comparison of $\Delta CS$ for Game A and B when $\tau$ Varies.]

Figure 12: CS Path Comparison since 1997

![Graph showing CS path comparison since 1997.]

65
Figure 13: Quantity Path Comparison of A330 since 1997

Figure 14: Quantity Path Comparison of A340 since 1997
Figure 15: Quantity Path Comparison of B777 since 1997

Figure 16: Quantity Path Comparison of MD-11 since 1997
Figure 17: Experience Path Comparison of A330 since 1997

Figure 18: Experience Path Comparison of A340 since 1997
Figure 19: Experience Path Comparison of B777 since 1997

Figure 20: Experience Path Comparison of MD-11 since 1997
Figure 21: Paths of Expected Upgrading Prob. for A330 since 1997

Figure 22: Paths of Expected Upgrading Prob. for A340 since 1997
Figure 23: Paths of Expected Upgrading Prob. for B777 since 1997

Figure 24: Paths of Expected Upgrading Prob. for MD-11 since 1997
Figure 25: Comparison of $\Delta CS$ when $\tau$ Varies for Different Models

Figure 26: Merger Efficiency for different $p^G$ and $\varphi$ when $\tau = 1$
Figure 27: $EV_{00} + EV_{11} - EV_{01} - EV_{10} \geq 0$

Figure 28: $EV_{00} + EV_{11} - EV_{01} - EV_{10} < 0$