Bargaining, Vertical Mergers and Entry*

Geza Sapi†

January 2013

Abstract

This paper analyzes vertical integration incentives in a bilaterally duopolistic industry where upstream producers bargain with downstream retailers on terms of supply. In the applied framework integration does not affect the total output produced, but it affects the distribution of rents among players. Vertical integration incentives depend on the strength of substitutability or complementarity between products and the shape of the unit cost function. I demonstrate furthermore that in contrast to the widely prevailing view in competition policy, vertical integration can under particular circumstances convey more bargaining power to the merged entity than a horizontal merger to monopoly. The model is applied to analyze strategic merger incentives to influence entry decisions. Mergers can facilitate and deter entry. While horizontal mergers to deter entry are never profitable, firms on different market levels may strategically choose to integrate vertically to keep a potential entrant out of the market. I provide conditions for such entry-deterring vertical mergers to occur.

JEL-Classification: L13; L22; L42

Keywords: Bargaining, Vertical Mergers, Entry

---

*This article benefited from helpful comments of seminar and conference participants at Berlin Humboldt University, Düsseldorf Institute for Competition Economics, German Institute for Economic Research (DIW Berlin) and the 2012 EARIE Conference in Rome. I also thank Christian Wey, Ulrich Kamecke and Clémence Christin for detailed discussions and comments. The content of this article does not reflect the official opinion of the European Union. Responsibility for the information and views expressed therein lies entirely with the author.

†European Commission DG COMP - Chief Economist Team and Düsseldorf Institute for Competition Economics (DICE), E-mail: sapi@dice.uni-duesseldorf.de
1 Introduction

Competition policy traditionally looks at vertical and horizontal mergers through different glasses. While horizontal mergers are often regarded to be motivated by the intent to reduce competition, vertical integration is more frequently argued to be driven by efficiencies, for example by eliminating double markups, reducing transaction costs or solving some variant of the holdup problem. This is explicitly stated in paragraph 11 of the EC non-horizontal merger guidelines, recognizing that “[n]on-horizontal mergers are generally less likely to significantly impede effective competition than horizontal mergers.”1 A similar view emerges in the U.S. Department of Justice Merger Guidelines, noting that “[...]non-horizontal mergers are less likely than horizontal mergers to create competitive problems [...]”.2 At least some of this sharp distinction between horizontal and vertical mergers may lie in the tradition of economic analysis to ignore the ability of downstream firms to influence upstream markets. Yet in perhaps most vertically related industries, supply conditions are determined through bargaining, where downstream firms may have the ability to actively shape contracts with suppliers. Much research has been devoted to how horizontal integration can tip bargaining in favor of the merging parties. This research also gave rise to the recent heated debate on buyer-power in the antitrust arena. The question of how vertical integration can affect bargaining outcomes has however remained significantly less studied.

This article intends to make a step towards closing this gap. I investigate the driving forces behind vertical integration, its effects and social desirability while taking into account that supplies arise as a result of bilateral bargaining. I apply a setup in which competition does not change the total industry surplus, only the distribution of rents among actors due to shifting bargaining power across firms. This framework is particularly useful, because it allows to focus on the strategic considerations behind vertical integration in isolation. After identifying these strategic effects, I extend the analysis to incorporate efficiency. To do so, I investigate how vertical integration can be used as a device to deter entry upstream and downstream.

I provide conditions for vertical mergers to take place regarding the strength of substitutability or complementarity between products and the shape of the unit cost function. I show, that

---

1Commission Guidelines on the Assessment of Non-Horizontal Mergers under the Council Regulation on the Control of Concentrations Between Undertakings, 2008 O.J. (C 265) 07, par. 11.
2U.S. Department of Justice Merger Guidelines, 1984, Chapter 4.
in an environment where integration is driven purely by strategic considerations, horizontal and
tvertical merger incentives are very closely related. In the simplest of all settings, the decision to
vertical integrate can be expressed as a mix of horizontal integration incentives upstream and
downstream. I demonstrate furthermore that vertical integration can convey more bargaining
power to the merged entity than a horizontal merger to monopoly.

Finally, I contrast the entry-deterring potential of horizontal and vertical mergers. Contrary
to the broadly prevailing view in competition policy, my results show that vertical mergers can be
more detrimental to welfare than horizontal ones. This is so, because the former can be a more
apt device to deter entry than horizontal integration. If vertical integration prevents entry, this
occurs via sufficiently reducing the rival’s revenues: Vertical integration functions as a structural
barrier to entry either upstream or downstream, by reducing the potential entrant’s expected
revenues below the level necessary to cover its setup costs. In particular, in the proposed model
firms would never merge horizontally just for the purposes of entry deterrence. This implies
that competition on the other level of the supply chain is more important than large size in
bargaining, as has been demonstrated empirically in papers such as Sorensen (2003) and Ellison
and Snyder (2010). I provide a theoretical explanation of this empirical finding.

2 Literature Review

Recent economic theories emphasize that vertical integration may have anticompetitive effects
through various avenues. One such avenue is the foreclosure of necessary inputs or of market
access for rivals. Salinger (1988) and Ordover, Saloner and Salop (1990) are seminal papers in
this strand, which demonstrate that an upstream firm (even without substantial market power)
may have an incentive to integrate vertically and limit supply to downstream rivals. Martin
et al (2001), and Normann (2007) provide empirical an experimental validation for theoretical
predictions on vertical integration incentives.

Hart and Tirole (1990) were first to point out that a dominant upstream firm may be
unable to exert its market power if it sells a product to competing downstream firms. If the
upstream firm makes secret offers and downstream firms hold passive beliefs about the offers
of the competitors, the upstream firm faces a Coasian commitment problem which prevents it
from reaping the full monopoly profit. In this case, despite the market power of the upstream
firm, the produced quantities will correspond to the competitive levels. By vertical integration
the upstream firm can credibly commit to supplying the monopoly quantity. This allows it to foreclose downstream rivals and reduce output, which unambiguously harms welfare.

Much of the literature on vertical integration focuses on the effect of altered ownership structure on broader-defined investment incentives (Hart and Moore 1990; Stole and Zwiebel 1996a and 1996b). In this strand, upstream and downstream competition is typically preceded by a stage in which firms make investment, capacity or technology adoption decisions. Baake et al (2004) extend the model of Hart and Tirole (1990) by allowing the upstream firm to carry out an investment which determines its marginal costs. Downstream competition weakens investment incentives into cutting marginal costs. The resulting welfare loss due to underinvestment may furthermore be larger than the welfare loss arising from the restoration of monopoly power. Also in this strand of literature, Choi and Yi (2000) develop a similar argument as Hart and Tirole (1990) and show that vertical integration can serve as a commitment device, since it may create incentives for a vertically integrated upstream firm to provide a specialized input (i.e. one that can be used by only one downstream firm), although under separation it would offer an input which could be used by all firms downstream.

Turning around the question of when firms integrate vertically, Bonanno and Vickers (1988) investigate incentives of vertically integrated firms to separate. They point out that if suppliers can fully extract the rents of retailers (for example with a franchise fee), and retailers’ decisions are strategic complements, then vertical separation can be profitable. A separated manufacturer can induce all retailers to price higher by increasing the wholesale price, whereas under vertical integration the wholesale price is set to equal production costs. Heavner (2004) similarly concludes that vertical separation may be profitable, when the integrated firm cannot commit to providing equal service quality to an upstream rival.

It has been recognized in industrial organization theory as well as in competition policy, that if delivery conditions between sellers and buyers are determined by bargaining, the resulting outcomes may be markedly different from usual ones (Horn and Wolinsky 1988; Campbell 2007). This article explicitly takes bargaining into account to derive incentives for vertical mergers to occur. In particular, I follow the property rights literature (Grossman and Hart, 1986; Hart and Moore, 1990) and consider a merger as combining two otherwise independently bargaining units into one. Whereas under nonintegration each supplier and retailer bargains separately, under integration the negotiations of the merged entity are controlled by one common agent.
Regarding the way bargaining is modelled, this article has several predecessors. I follow among others, Hart and Moore (1990); Stole and Zwiebel (1996a, 1996b); Rajan and Zingales (1998); Inderst and Wey (2003); Segal (2003); de Fontenay and Gans (2005b) and Montez (2007) use the Shapley value to capture the outcome of bargaining between various actors.

The two articles closest to mine are Inderst and Wey (2003) as well as de Fontenay and Gans (2005b) (in the following, respectively IW and dFG). Both articles focus on bargaining in an industry with two upstream and two downstream firms and use the Shapley value for capturing the outcome of bargaining. Under the assumption that downstream markets are independent, IW analyze horizontal merger incentives upstream and downstream, as well as the choice of a manufacturer between two technologies influencing production costs. They show that upstream merger incentives depend on whether products are substitutes or complements, whereas downstream merger incentives are determined by the shape of the unit cost function.

In turn, dFG focus on vertical merger incentives in a similar bargaining framework and compare outcomes under upstream competition and monopoly. The key modelling difference between IW and dFG is the way mergers are regarded. In IW, with a merger between two firms the integrated entity bargains with other firms as a single party. This is not the case in dFG, who distinguish between the owner and the manager of a firm. After a merger takes place, the manager of a purchased entity remains indispensable in further negotiations, and acts as an independently negotiating party. This creates scope for a rich set of interaction between managers, which allows the authors to distinguish between forward and backward vertical integration.

This article can be regarded as an intersection between IW and dFG. As the latter, I focus on vertical integration incentives, but follow IW to assume that a merged entity has one central management who conducts negotiation with other parties. Doing so yields markedly different results for vertical merger incentives than those in dFG, two of which stand out. First, while in the baseline model of dFG with no downstream competitive externalities, vertical integration (either forward or backward) is always preferred to non-integration, in my model vertical integration may not be profitable. Second, different from dFG, upstream competition in my setup does not always strengthen incentives to vertically integrate.

---

3 These articles derive the Shapley value as the outcome of different bargaining procedures.

4 In the basic model of DFG, downstream firms do not exert competitive externalities on each other. This setup is identical to the one in IW and it is what I apply in this article. DFG introduce downstream competition in Section 4 of their article.
The article proceeds as follows: Section 3 introduces the model. In Section 4 the framework is applied to analyze vertical merger incentives. Section 5 compares horizontal and vertical merger incentives in more detail and derives conditions determining which of these incentives is stronger. In Section 6 I introduce the possibility of entry and compare the deterring potential of horizontal and vertical mergers. Finally, Section 7 concludes. The Annex contains an example to illustrate and verify selected results.

3 Model Setup

Consider an industry in which two upstream suppliers \( s \in S^0 = \{A, B\} \) produce inputs which are turned into final goods by two downstream retailers \( r \in R^0 = \{a, b\} \). The inputs are differentiated with each supplier controlling the production of one input. The input from at least one supplier is necessary for a retailer to produce the final good. The demand at the retailers is independent, hence, there are no competitive externalities downstream.\(^5\) The indirect demand function for the good of supplier \( s \) at retailer \( r \) is denoted by \( p_{sr}(q_{sr}, q_{sr'}) \), where \( s' \) stands for the alternative supplier (similarly, \( r' \) will denote the alternative retailer) and \( q_{sr} \) denotes the quantity of input \( s \) supplied to retailer \( r \). The total costs of supplier \( s \) for providing input quantities \( q_{sr} \) and \( q_{sr'} \) to the retailers are given by \( C_s(q_{sr} + q_{sr'}) \). I will denote the average unit cost of supplier \( s \) for providing quantity \( q \) of the product as \( \bar{C}_s(q) = C_s(q)/q \). The retailers turn inputs into final good costlessly.

Supply contracts between upstream and downstream firms are determined by bargaining. I follow other authors studying the effects of integration in a bargaining framework and adopt the Shapley value as solution concept of the bargaining game (e.g. Hart and Moore 1990, Rajan and Zingales 1998, Inderst and Wey 2003, Segal 2003 and de Fontenay and Gans 2005b).\(^6\) As there are no competitive externalities between retailers, changes in the industry structure affect only the distribution of bargaining power, not the supplied quantities and therefore the surplus generated.

The Shapley value allocates to each independently negotiating party her expected marginal

---

5 We can think for example of retailers operating in different geographic markets, or of ones turning inputs into strongly differentiated final goods.

6 While it is an axiomatic solution concept, the theoretical literature has proposed a number of justifications for the Shapley value as outcome of non-cooperative bargaining processes. See for example Gül (1989), Inderst and Wey (2003), de Fontenay and Gans (2005). Section 8 of Winter (2002) provides an extensive overview.
contribution to coalitions, where the expectation is taken over all coalitions in which the party may belong, with all coalitions assumed to occur with equal probability. More formally, let \( \Psi \) denote the set of independently negotiating parties and \(|\Psi|\) the cardinality of this set, respectively.

The payoff of firm \( \psi \in \Psi \) is given by

\[
U_\psi = \sum_{\tilde{\Psi} \subseteq \Psi | \psi \in \tilde{\Psi}} \left( \left| \tilde{\Psi} \right| - 1 \right)! \left( |\Psi| - |\tilde{\Psi}| \right)! \left( W_{\tilde{\Psi}} - W_{\tilde{\Psi} \setminus \psi} \right),
\]

where \( \tilde{\Psi} \subseteq \Psi | \psi \in \tilde{\Psi} \) represents a set \( \tilde{\Psi} \subseteq \Psi \), such that \( \psi \) is a member of coalition \( \tilde{\Psi} \) and \( W_{\tilde{\Psi}} \) denotes the maximum surplus achieved by the firms in coalition \( \tilde{\Psi} \). For simplicity, I write \( \tilde{\Psi} \setminus \psi \) for \( \tilde{\Psi} \{ \psi \} \). I furthermore denote the set of all firms by \( \Omega = \{A, B, a, b\} \) and define \( W_{\Omega} \) as the maximum industry profit. In the terminology of cooperative game theory \( W(\cdot) \) is often referred to as the characteristic function. \( W_{\Omega'} \) is assumed to be continuous and strictly quasi-concave for all \( \Omega' \subseteq \Omega \). Importantly, since at least one supplier and retailer is necessary for production, \( W_{\tilde{\Psi}} = 0 \) if \( \tilde{\Psi} \) contains less than one firm of each kind. Before proceeding with the analysis, I need some additional definitions and assumptions.

\textbf{Definition 1} The cost function \( C_s(\cdot) \) is said to exhibit strictly increasing (decreasing) unit costs if the unit cost function \( \overline{C}_s(q) \) is strictly increasing (decreasing) on \( q > 0 \).

\textbf{Definition 2} Take any \( s, s' \in S^0 \) with \( s \neq s' \) and \( r \in R^0 \). The two goods are said to be strict substitutes if \( q_{s'r}'' > q_{s'r}' \) and \( p_{sr}(q_{sr}, q_{s'r}) > p_{sr}(q_{sr}, q_{s'r}') \). They are strict complements if \( q_{s'r}'' > q_{s'r}' \) and \( p_{sr}(q_{sr}, q_{s'r}) \neq p_{sr}(q_{sr}, q_{s'r}') \).

\textbf{Definition 3} Let \( \Delta_{C}^{\Omega'} := \overline{C}_s(2q_{s'r}) - \overline{C}_s(q_{s'r}) \) and \( \Delta_{p}^{\Omega'} := p_{sr}(q_{s'r}, q_{s''r}) - p_{sr}(q_{s'r}, 0) \), with \( \Omega' \subseteq \Omega \). From definition 1 unit costs are strictly increasing (decreasing) if \( \Delta_{C}^{\Omega'} > 0 \) (\( \Delta_{C}^{\Omega'} < 0 \)). From definition 2 products are strict complements (substitutes) if \( \Delta_{p}^{\Omega'} > 0 \) (\( \Delta_{p}^{\Omega'} < 0 \)).

\textbf{Assumption 1} (superadditivity) \( W(\cdot) \) is superadditive: \( W_{\Omega'} \geq W_{\Omega''} \) for every \( \Omega' \) and \( \Omega'' \) with \( \Omega'' \subset \Omega' \subset \Omega \).

\textbf{Assumption 2} (symmetry) Suppliers and retailers are symmetric: \( C_s(\cdot) = C_s'(\cdot) = C(\cdot) \), \( q_{sr} = q_{sr} \) and \( p_{sr}(\cdot) = p_{sr}(\cdot) \) for any \( s, \tilde{s}, r, \tilde{r} \in S^0 \times R^0 \).
Definitions 1 and 2 are borrowed from IW. While Assumption 1 is put forward throughout this article, Assumption 2 will be necessary only for some results and will be invoked at various segments of the text explicitly.

4 Vertical Merger Incentives

Throughout this paper I refer to a merger as a transaction combining the merging firms into one bargaining unit. This is a realistic way to think about mergers in which the merged firms are united under a common management, which conducts negotiations with other entities. It would happen for example, if the key executives of the acquired company were replaced by the new owner.

We can now calculate equilibrium payoffs under different market structures. Throughout this article I will use the notation \( \{s, s', r, r'\} \) to denote a market structure, where the commas separate non-merged and therefore individually negotiating entities. For example, \( \{AB, a, b\} \) stands for the market structure with an upstream monopoly facing a duopoly of retailers. Similarly, \( \{Aa, B, b\} \) denotes the market structure consisting of supplier A being vertically integrated with retailer a, and supplier B as well as retailer b negotiating independently. I focus on the following market structures: \( \{A, B, a, b\} \) (full separation), \( \{AB, a, b\} \) (upstream monopoly), \( \{A, B, ab\} \) (downstream monopoly), \( \{ABa, b\} \) (vertically integrated upstream monopoly), \( \{Aab, B\} \) (vertically integrated downstream monopoly), \( \{ABab\} \) (full integration), \( \{Aa, B, b\} \) (single vertical integration), \( \{Aa, Bb\} \) (double vertical integration).

Lemma 1. Under the different market structures the payoffs of the actors are as indicated in Table (1).

Proof of Lemma 1. The proof is immediate by applying the Shapley value for the various market structures.

Before proceeding with the analysis of vertical merger incentives I provide a brief interpretation of the payoffs generated by the Shapley value in Table (1). In a well-known interpretation of the Shapley value, players are randomly ordered in a sequence. Since several randomizations are possible, each of them are assumed to be equally probable. Each player gets as payoff its marginal contribution to the coalition formed by the preceding players in the sequence. The Shapley value is the expected payoff taken over all possible sequences.
Take for example the industry structure of upstream monopoly, with $\Psi = \{AB, a, b\}$. In this case six orderings are possible, those displayed in Table (2). I focus on the payoff of supplier $AB$.

<table>
<thead>
<tr>
<th>Ordering</th>
<th>$AB$</th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$AB, a, b$</td>
<td>0</td>
<td>$W_{\Omega \setminus b}$</td>
</tr>
<tr>
<td>2</td>
<td>$AB, b, a$</td>
<td>0</td>
<td>$W_\Omega - W_{\Omega \setminus a}$</td>
</tr>
<tr>
<td>3</td>
<td>$a, AB, b$</td>
<td>$W_{\Omega \setminus b}$</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>$b, AB, a$</td>
<td>$W_{\Omega \setminus a}$</td>
<td>$W_\Omega - W_{\Omega \setminus a}$</td>
</tr>
<tr>
<td>5</td>
<td>$a, b, AB$</td>
<td>$W_\Omega$</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>$b, a, AB$</td>
<td>$W_\Omega$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table (2): Marginal contributions in various orderings.

In orderings 1 and 2, supplier $AB$ comes first. Its marginal contribution is zero, because without a retailer preceding it the supplier cannot produce. It comes second in orderings 3 and 4. In ordering 3 supplier $AB$’s contribution is to enable production with retailer $a$, together creating $W_{\Omega \setminus b}$ of surplus. This is the surplus that can be created without retailer $b$. Similarly, in ordering 4 supplier $AB$ enables production with retailer $b$, and therefore generates $W_{\Omega \setminus a}$ of surplus. In orderings 5 and 6 supplier $AB$ comes last. Since the retailers preceding it are not able to generate value absent a supplier, firm $AB$ receives the entire industry surplus $W_\Omega$ in these orderings. Taking expectations about the orderings, the Shapley value yields as payoff for the supplier

$$U_{AB} = \frac{1}{6}(0) + \frac{1}{6}(0) + \frac{1}{6}W_{\Omega \setminus b} + \frac{1}{6}W_{\Omega \setminus a} + \frac{1}{6}W_\Omega + \frac{1}{6}W_\Omega = \frac{1}{6} \left[ W_{\Omega \setminus b} + W_{\Omega \setminus a} + 2W_\Omega \right].$$

The payoffs of the retailers can be determined in a similar manner.

We can now compare vertical merger incentives for various pre-merger market structures.

**Proposition 1.** Whether a vertical merger between supplier $s \in S^0$ and retailer $r \in R^0$ increases their joint payoff depends on the pre-merger market structure the following way:

(i) If suppliers and retailers are non-integrated ($\Psi = \{A, B, a, b\}$), the joint profit of supplier
$s$ and retailer $r$ weakly increases by vertically merging if

\[(W_{Ω\setminus sr'} - W_{Ω\setminus sr}) + W_{Ω\setminus s} + W_{Ω\setminus r} \geq W_Ω,\]

whereas it decreases if the opposite holds.

(ii) If suppliers are integrated and retailers are separated ($Ψ = \{AB, a, b\}$), the joint profit of supplier $AB$ and retailer $r$ weakly increases by vertically merging if

\[W_{Ω\setminus r'} + W_{Ω\setminus r} \geq W_Ω,\]

whereas it decreases if the opposite holds.

(iii) If suppliers are separated and retailers are integrated ($Ψ = \{A, B, ab\}$), the joint profit of supplier $s$ and retailer $ab$ weakly increases by vertically merging if

\[W_{Ω\setminus a} + W_{Ω\setminus s'} \geq W_Ω,\]

whereas it decreases if the opposite holds.

**Proof of Proposition 1**: See Appendix.

The results formulated in Proposition 1 stand somewhat in contrast to the conventional stance of competition policy on vertical mergers which regards these markedly different from horizontal ones. In my model, where integration affects only the distribution of surplus among the actors, horizontal and vertical mergers are very closely related. Cases (ii) and (iii) (pre-merger upstream and downstream monopoly, respectively) of have particularly interesting implications if they are compared to horizontal merger incentives. These are derived in Inderst and Wey (2003) in the same framework as employed here. Proposition 1 implies that with a monopolist supplier facing competing retailers, vertical merger incentives are identical to horizontal merger incentives between retailers when initially all firms are independent. Similarly, with a monopoly on the retail level facing competing suppliers, vertical integration incentives are identical to horizontal merger incentives between suppliers if prior to the merger all firms are independent. The following corollary proves useful for providing further intuition on these insights.
Corollary 1: Vertical merger incentives depend on the initial market structure, the level of substitutability/complementarity between the products and the shape of the unit cost function in the following way:

(i) With suppliers integrated and retailers separated ($\Psi = \{AB, a, b\}$), a vertical merger between supplier $AB$ and retailer $r$ takes place (does not take place) if both retailers have strictly increasing (decreasing) units costs.

(ii) With suppliers separated and retailers integrated ($\Psi = \{A, B, ab\}$), a vertical merger between supplier $s$ and retailer $ab$ takes place (does not take place) if the products are strict substitutes (complements).

(iii) Invoke Assumption 2 (symmetry) and take the scenario with all firms separated ($\Psi = \{A, B, a, b\}$). Supplier $s$ and retailer $r$ merge (stay separated) if for all $\Omega' \in \Omega$ we have $\Delta_p^{\Omega'} < \Delta_C^{\Omega'}$ ($\Delta_p^{\Omega'} > 0$ and $\Delta_C^{\Omega'} < 0$).

Proof of Corollary 1. See Appendix.

Corollary 1 links vertical integration incentives expressed in Proposition 1 to the primitives of the model. I now provide some additional intuition on vertical merger incentives. Take first the pre-merger case of a monopolist retailer facing separated suppliers. In this situation, vertical integration between the retailer and one supplier is profitable for the merging parties if products are substitutes. Why is this so? It is convenient to focus on the effects of integration on the non-merged supplier: Since only the distribution of payoffs are affected, not overall output, any gains of the merging parties must exactly correspond to the losses of the non-merged supplier.

If products are substitutes, each supplier wants to be first to reach an agreement with the retailer. This is so, because bargaining between a supplier and the retailer revolves around the sharing of the marginal rent generated by the negotiating parties: With products being substitutes, the additional rent generated by the first supplier to reach an agreement with the retailer is larger than that generated by the second supplier. Substitutability of the products implies that the latter generates negative price externalities for the first supplier. Therefore, suppliers prefer negotiating on infra-marginal quantities to bargaining “on the margin.” This explains why with substitutes the non-merging supplier loses if the other market actors integrate vertically. With vertical integration between the retailer and the rival upstream firm, negotiations between
the merged parties cannot break down. Hence, the non-merging supplier cannot be the first to reach an agreement with the retailer, because vertical integration guarantees that an agreement between the rival and the retailer is in place. The non-merging supplier is left with having to bargain at the margin, i.e. about the lower surplus it generates by coming second to the retailer.

The same logic holds if goods are complements. In that case, each supplier prefers to be second in reaching an agreement with the retailer: Complementary products imply that the additional surplus generated by the second supplier to reach an agreement with the retailer is larger than that generated by the first one. Vertical integration with complements would only ensure that the integrated supplier cannot be second to reach an agreement with the retailer. This would benefit the non-merging party and therefore harm the firms considering integration.

Take now the situation in which pre-merger a monopoly supplier negotiates with two retailers. Vertical integration between the supplier and a retailer takes place if unit costs are strictly increasing. The reason is as follows: If unit costs are strictly increasing, each retailer prefers to be first in reaching an agreement with the supplier, i.e. to negotiate about infra-marginal quantities. The retailer coming second faces higher unit costs and is therefore left with a smaller surplus to negotiate about with the supplier. Vertical integration corresponds to a sure agreement between the integrating upstream and downstream firms, leaving the non-merging retailer with the only option to be second. This erodes the bargaining power of the second retailer and therefore benefits the merging parties. If unit costs are strictly decreasing, each retailer prefers to be second in reaching an agreement with the supplier and to negotiate about marginal quantities. Once a supplier-retailer agreement is in place, the additional rent generated by an other retailer is larger, since unit costs are lower. In this case a vertical merger is not attractive, since it forces the integrated supplier to be first.

It was noted above that vertical merger incentives in the pre-merger market structures of downstream and upstream monopoly are identical to horizontal merger incentives between retailers and suppliers respectively, if initially all firms are independent (see IW). I now provide further intuition for why this is the case. Take the initial market structure of full separation and retailer horizontal merger incentives. IW show, that retailers merge if suppliers have strictly increasing unit costs. In this case, each retailer prefers to be first to reach an agreement with a supplier. The retailer coming second to any supplier generates lower marginal surplus, since unit costs for the additional output to be delivered are higher with strictly increasing unit costs.
A horizontal merger between retailers ensure that the merged entity can always come first to each supplier. As was explained above, this is the same logic which drives vertical merger incentives in the downstream monopoly pre-merger market structure. The intuition behind why vertical merger incentives in an upstream monopoly correspond to upstream horizontal merger incentives under full separation is analogous.

I now explain the intuition behind vertical integration incentives under pre-merger full separation. I focus on the most instructive case, namely when all firms are symmetric as assumed in Corollary 1 and postpone discussing the role of asymmetry in the verticals for later. Under such circumstances, vertical integration incentives correspond to a mix of horizontal integration incentives upstream and downstream. IW show that upstream horizontal mergers depend on whether goods are substitutes or complements, while downstream mergers depend on whether unit costs are strictly increasing or decreasing. My results show, that vertical merger incentives are very similar and can be expressed as a mix of horizontal merger incentives. In particular, whether a vertical merger is profitable if initially all firms are separated depends on how strong complements or substitutes the products are compared to how strongly unit costs increase or decrease. This relationship is illustrated in Figure (1). The strength of complementarity/substitutability is captured by $\Delta \Omega_p'$ while the speed with which unit costs increase or decrease is measured by $\Delta \Omega_C'$.

![Figure 1: Vertical integration incentives](image-url)
A vertical merger implies for the integrating firms that they are always first to reach an agreement with each other. If this is what they would want in the absence of the merger, than integration is unambiguously profitable. This is the case when products are substitutes ($\Delta_p^\gamma < 0$) and unit costs are increasing ($\Delta_C^\gamma > 0$). If unit costs are increasing, retailers want to be first to reach an agreement with each supplier. Being second means having to negotiate about the distribution of a lower surplus, because unit costs are higher for the additional output to be supplied. If products are substitutes, also the suppliers prefer to be first in striking an agreement with retailers. The supplier coming second must take into account the negative price externality it imposes on the other supplier already having an agreement in place with the same supplier, and is hence left to negotiate about a lower surplus. Putting these together, with substitute products and strictly increasing unit costs both retailers as well as suppliers prefer to be first to reach an agreement with the other firms. This is exactly what a vertical merger guarantees with the merging partner, and is therefore profitable. The logic is the same for why vertical mergers are not preferred if products are complements ($\Delta_p^\gamma > 0$) and unit costs are strictly decreasing ($\Delta_C^\gamma < 0$). Under such circumstances retailers as well as suppliers prefer to negotiate with firms of the other type once the bargaining partner already has an agreement in place. A vertical merger undermines this opportunity as it in effect guarantees being first to reach agreement, and is therefore not desired.

Interesting situations arise when products are substitutes (complements) and unit costs are strictly decreasing (increasing). In these cases the interests of the suppliers and retailers are not aligned with respect to the desired order to reach an agreement. For example, maintaining the assumption of firms being symmetric, with substitute goods and strictly decreasing unit costs suppliers prefer being first to reach an agreement with retailers, whereas retailers want to be second to agree with suppliers. Since vertical integration implies a sure agreement between the merged parties, it benefits the merging supplier but is contrary to the involved retailer’s interests. The profitability of such a merger therefore depends on whether the gains of the former exceed the losses of the latter. This is the case if products are sufficiently strong substitutes while unit costs are sufficiently slowly decreasing (i.e. if $\Delta_p^\gamma < \Delta_C^\gamma < 0$). The same logic applies if products are complements and unit costs are strictly increasing.

In the discussion of vertical integration incentives under pre-merger full separation I remained silent on the role of asymmetry between firms. I address the issue now. While all of what has
been said so far stays valid, asymmetry between firms has some implications for vertical merger incentives. According to Claim (i) of Proposition 1, vertical integration between supplier \( s \) and retailer \( r \) is profitable if

\[
(W_{\Omega \setminus sr} - W_{\Omega \setminus sr}) + W_{\Omega \setminus s} + W_{\Omega \setminus r} \geq W_{\Omega}. \tag{2}
\]

Under symmetry the term in the brackets cancels out, but it does not do so under asymmetry. Expression (2) connotes that vertical integration is more likely to take place if the merging vertical is relatively large compared to the non-merging one, (i.e. if the difference \( W_{\Omega \setminus sr} - W_{\Omega \setminus sr} \) is larger). This is the case if the vertically integrating firms \( s \) and \( r \) are able to produce a relatively large surplus on their own compared to the surplus produced by the non-merging firms \( s' \) and \( r' \) relying solely on each other. The reason is that infra-marginal rents are greater if the merging vertical is larger. Vertical integration ensures, that the merging parties receive a larger share of the infra-marginal rents. A vertical merger is therefore more likely to take place in a larger vertical.

Finally, it remains to note that in my setup incentives to integrate vertically are not unambiguously greater under upstream competition than under monopoly. This is in contrast to the results derived by dFG, who find that vertical integration incentives are always stronger with competition upstream. To see this, we can compare the conditions for vertical integration in both market structures as given in Claims (i) and (ii) of Proposition (1). Vertical integration incentives are greater under upstream monopoly than under competition if

\[
W_{\Omega \setminus r} + W_{\Omega \setminus r'} > (W_{\Omega \setminus sr} - W_{\Omega \setminus sr}) + W_{\Omega \setminus s} + W_{\Omega \setminus r}, \tag{3}
\]

whereas they are smaller if the opposite holds. To demonstrate that arrangements exists in which vertical integration incentives under upstream monopoly are stronger than under competition, I focus on the case of full symmetry. Condition (3) then reduces to \( W_{\Omega \setminus r} > W_{\Omega \setminus s} \), which holds if an additional retailer increases total surplus by a relatively large amount, while the marginal contribution of a supplier is rather small. This is likely to be the case for example if unit costs are strongly increasing while goods are relatively weak complements. Upstream competition can thus either enhance or reduce the prospective of strategic vertical integration.
5 Comparison of Horizontal and Vertical Merger Incentives

In this section I aim to compare horizontal and vertical merger incentives in more detail. To create a benchmark I assume that one firm, either upstream or downstream, is available for sale by means of an auction. This firm will be referred to as the target firm. The other firms in the market bid to acquire the target, which is sold to the highest bidder. Horizontal integration incentives are said to be stronger (weaker) than vertical integration incentives, if the bidder on the same market level as the target has a higher (lower) willingness to pay for merging with the target than a bidder from the other market level. I consider a very simple two-stage game, where in the first stage firms submit sealed bids for the target. At the end of the stage the highest bidder merges with the latter. In the second stage, the acquirer pays out its bid and supply contracts are negotiated. I assume that the target firm is sold without a reservation price, i.e. it does not have the opportunity to refuse an offer and remain unsold. This is a convenient simplification, which allows us to focus on horizontal and vertical merger incentives arising from the altered bargaining power of the bidders.

In what follows I analyze the first stage of the game, i.e. the auctioning of the target firm. I first turn to the case where a supplier is available for sale. I will than consider the auctioning off of a retailer.

Assume w.l.o.g. that supplier $A$ is available for sale and supplier $B$ and retailer $a$ submit bids $\beta_B$ and $\beta_a$ respectively for acquisition. Let $U^\Psi_{\psi \in \Psi}$ denote the profit of firm $\psi$ in market structure $\Psi$ resulting after stage 1. Then, depending on the outcome of the auction in stage 1, firms make the following profits in stage 2:

(i) Retailer $a$ wins in stage 1 and merges with $A$: $U_A = \beta_a$, $U_B = U_B^{\{Aa,B,b\}}$, $U_a = U_A^{\{Aa,B,b\}} - \beta_a$.

(ii) Supplier $B$ wins in stage 1 and merges with $A$: $U_A = \beta_B$, $U_B = U^{\{AB,a,b\}}_{AB} - b_B$, $U_a = U_a^{\{AB,a,b\}}$.

To determine the winner of the auction, I first derive the maximum possible bids, i.e. those, that leave the bidders indifferent between acquiring the target and not bidding. The indifference
conditions take the form
\[ U^{\{Aa,B,b\}}_B = U^{\{AB,a,b\}}_A - \beta_B, \]
\[ U^{\{Aa,B,b\}}_{Aa} - \beta_a = U^{\{AB,a,b\}}_a. \]

Rearranging yields the maximum bids for acquiring supplier \( A \) as
\[ \beta_a = U^{\{Aa,B,b\}}_{Aa} - U^{\{AB,a,b\}}_a = \frac{1}{6} \left[ W_{\Omega\setminus B} + 2W_{\Omega\setminus Bb} - 2W_{\Omega\setminus Aa} + 2W_{\Omega\setminus a} \right], \]  
\[ \beta_B = U^{\{AB,a,b\}}_{AB} - U^{\{Aa,B,b\}}_B = \frac{1}{6} \left[ 2W_{\Omega\setminus B} + W_{\Omega\setminus Bb} - W_{\Omega\setminus Aa} + W_{\Omega\setminus a} \right]. \] (4)

Assume now that retailer \( a \) is available for sale and supplier \( A \) and retailer \( b \) submit bids \( \beta_A \) and \( \beta_b \) for acquisition, respectively. Depending on the outcome of the auction in stage 1, the profits in stage 2 are:

(i) Retailer \( A \) wins in stage 1 and merges with \( a \): \( U_a = \beta_A, U_A = U^{\{Aa,B,b\}}_{Aa} - \beta_A, U_b = U^{\{Aa,B,b\}}_b \).

(ii) Supplier \( b \) wins in stage 1 and merges with \( a \): \( U_a = \beta_B, U_A = U^{\{A,B,ab\}}_A, U_b = U^{\{A,B,ab\}}_{ab} - \beta_b \).

Bidders are indifferent between acquiring the target and not bidding if the following conditions hold:
\[ U^{\{Aa,B,b\}}_{Aa} - \beta_A = U^{\{A,B,ab\}}_A, \]
\[ U^{\{Aa,B,b\}}_{ab} = U^{\{A,B,ab\}}_{ab} - \beta_b. \]

By rearranging we obtain the maximum bids for acquiring retailer \( a \) as
\[ \beta_A = U^{\{Aa,B,b\}}_{Aa} - U^{\{A,B,ab\}}_A = \frac{1}{6} \left[ 2W_{\Omega\setminus Bb} - 2W_{\Omega\setminus Aa} + 2W_{\Omega\setminus A} + W_{\Omega\setminus B} \right], \]  
\[ \beta_b = U^{\{A,B,ab\}}_{ab} - U^{\{Aa,B,b\}}_b = \frac{1}{6} \left[ W_{\Omega\setminus Bb} - W_{\Omega\setminus Aa} + W_{\Omega\setminus A} + 2W_{\Omega\setminus B} \right]. \] (5)

The following proposition sums up the results on the outcome of the auction.

**Proposition 2:** The auction for takeover has the following outcome:
(i) Assume that supplier A is the target firm. The acquiring firm is supplier B if

\[ W_{\Omega \setminus a} - W_{\Omega \setminus Aa} < W_{\Omega \setminus B} - W_{\Omega \setminus Bb}, \]

whereas it is retailer a if the opposite holds.

(ii) Assume that retailer a is the target firm. The acquiring firm is supplier A if

\[ W_{\Omega \setminus A} - W_{\Omega \setminus Aa} < W_{\Omega \setminus B} - W_{\Omega \setminus Bb}, \]

whereas it is retailer b if the opposite holds.

**Proof of Proposition 2.** Comparing maximum bids in (4) and (5) yields the conditions stated in Proposition 2.

Before providing some intuition to these results, it is helpful to investigate when the conditions stated in Proposition 2 hold. For the case of symmetric firms, Corollary 2 relates incentives to acquire a firm to the primitives of the model.

**Corollary 2:** Under Assumption 2 (symmetry), regardless whether the target firm is a retailer or a supplier, the acquiring firm is a retailer if

\[ -\Delta_{p}' < \Delta_{C}' \quad \text{for all } \Omega' \subseteq \Omega \]

whereas it is a supplier if the opposite holds.

**Proof of Corollary 2.** See Appendix.

Corollary 2 implies, that no matter whether a retailer or a supplier is for sale, the acquiring firm is a supplier if products are substitutes and unit costs are strictly decreasing. It is a retailer if goods are complements and unit costs are strictly increasing. In every other case, which firm acquires the target depends on the relative magnitudes of \( \Delta_{p}' \) and \( \Delta_{C}' \), capturing the strength of complementarity/substitutability between products and the speed to which unit costs increase or decrease. For example, if products are complements and unit costs are decreasing, the acquirer is a retailer if complementarity between products are relatively strong, while unit costs are not decreasing too rapidly. In case products are substitutes and unit costs increase, the acquiring firm is the retailer if unit costs increase quickly while products are relatively weak substitutes.
The intuition is the following. Take first the case where supplier $A$ is for sale, and supplier $B$ as well as supplier $a$ bid for acquisition. The bidders’ incentive to acquire the target stems from the fact that doing so can improve their bargaining power in the subsequent negotiations on delivery conditions. Which firm is willing to pay most for the target therefore depends on whether a merged horizontal supplier or a vertical chain is able to convey more bargaining power to the merged entity. Note that since the target is sold for any positive bid, submitting a bid is always a dominant strategy: acquiring the target for a small but positive bid is always more attractive than giving up on it. As it was explained above, a supplier prefers to merge horizontally if goods are substitutes because bargaining jointly with the other supplier allows them to move away from the margin. Similarly, a retailer prefers to integrate with a supplier if unit costs are strictly increasing. A supplier therefore has a relatively high (low) willingness to pay for its competitor if goods are substitutes (complements) At the same time, the bidding retailer has a relatively high (low) valuation for the target if unit costs are strictly increasing (decreasing). This implies for example, that with goods being complements and unit costs increasing, the retailer can outbid the supplier, i.e. vertical merger incentives are stronger than horizontal ones.

The contrary holds if goods are substitutes and unit costs decrease. Since in this case
upstream horizontal merger incentives are strong while the retailer dislikes vertically merging, the target goes to supplier $B$. If unit costs are increasing and products are substitutes, both bidders have relatively strong incentives to acquire supplier $A$. In this case, which firm gains more by buying the target depends on the relative magnitudes of $\Delta_p\Gamma$ and $\Delta_c\Gamma$, i.e. the relative strength of substitutability/compatibility between the products and the speed with which unit costs increase or decrease. This relationship implies that with the target being an upstream firm, vertical integration is a more apt instrument to extract rents from the non-merging parties compared to upstream horizontal integration, if unit costs increase relatively fast (or decrease relatively slowly) while products are relatively weak substitutes (or are relatively strong complements). In this case, the retailer can outbid the supplier for the acquisition of the upstream target firm.

The intuition behind when a vertical acquisition is preferred to a horizontal in case the target firm is a retailer follows similar lines. Take the case where supplier $a$ is for sale, and supplier $A$ as well as retailer $b$ bid for acquisition, with supplier $B$ being the non-bidding firm. All other things equal, supplier $b$ has a stronger incentive to merge with its horizontal counterpart if unit costs increase, since negotiating jointly with the supplier prevents the retailers from being forced to the margin. At the same time, supplier $A$ values merging vertically more if products are substitutes. In this case having a sure partner in retailer $a$ protects it from having to negotiate about marginal quantities. This relationship reveals that with the target being a downstream firm, vertical integration is a more apt instrument to extract rents from the non-merging parties compared to upstream horizontal integration, if products are relatively strong substitutes (or relatively weak complements), while unit costs decrease relatively fast (or increase relatively slowly). In this case, the supplier can outbid the retailer for the acquisition of the downstream target firm.

To sum up, in my model integration incentives stem from the possibility to extract rents from the non-integrated parties. Acquisition incentives depend on which bidding party can improve its bargaining position more by moving towards infra-marginal quantities, given the shape of unit costs and the level of substitutability and complementarity between products. My results stand somewhat in contrast to the traditional argument that reaching a monopoly position creates strong incentives for horizontal mergers. In particular, by focusing solely on the effects of a merger to increase bargaining power, my model shows that vertical integration may under some circumstances convey more (bargaining) power to the merged entity than horizontally merging.
This is the case for example, if a supplier (retailer) is for sale and unit costs are strictly increasing (decreasing) while products are complements (substitutes).

6 Mergers and Entry

So far I have remained silent on the effects of vertical mergers on efficiency. In this section I address the issue by analyzing how vertical integration can serve as a device to deter entry upstream or downstream. I also compare the entry deterring potential of vertical integration to horizontal mergers. In my framework, deterring entry is always harmful to total welfare. This is due to Assumption 1, which implies that total industry surplus increases in the number of firms in the market.

Consider a situation in which initially three incumbents, $i_1, i_2, i_3 \in \Omega$ are in the market. A potential entrant, $e \in \Omega, e \notin \{i_1, i_2, i_3\}$, considers entering at the market level on which only one firm is active. Entry is costly and involves a fixed investment $I$, which is sunk if $e$ enters. The existence of $e$ and its entry costs $I$ are common knowledge. Incumbents $i_1$ and $i_2$ can integrate to alter the bargaining structure and influence the rents the entrant can expect. If $i_1$ and $i_2$ are of the same type, their merger is a horizontal one, whereas it is vertical if they are of different types.

The game unfolds as follows. In Stage 1, incumbents $i_1$ and $i_2$ decide whether to merge or stay separated. In Stage 2, the potential entrant $e$ decides whether to enter the market or stay out. In Stage 3 firms bargain on the delivery conditions and payments (including the entry costs) are made. The game tree and the resulting market structures are depicted in Figure (3).

![Figure 3: Merger and entry decisions.](image-url)
I solve the game by backward induction. If \( e \) enters, the resulting market structure is \( \{i_1i_2, i_3, e\} \) if \( i_1 \) and \( i_2 \) merge in Stage 1, and \( \{i_1, i_2, i_3, e\} \) if they stay separated. With no entry we get the industry structure \( \{i_1i_2, i_3\} \) when \( i_1 \) and \( i_2 \) integrate, and \( \{i_1, i_2, i_3\} \) if they remain separated. I postpone the description of Stage 3 payoffs for later, when I assign roles to variables \( i_1 \) and \( i_3 \) and \( e \).

Depending on the costs of entry, a merger can affect entry incentives in three ways: it can be irrelevant, it can deter entry or foster it. In particular, if \( I < \min\{U_e^{i_1i_2,i_3,e}, U_e^{i_1,i_2,i_3,e}\} \) or \( I > \max\{U_e^{i_1i_2,i_3,e}, U_e^{i_1,i_2,i_3,e}\} \), the merger does not change entry incentives. In the former case the necessary investment is so small, that entry always occurs, irrespective of whether the incumbents have merged or not. In the latter case entry costs are prohibitive.

The most interesting cases arise if entry costs are somewhere in between these polar options. Then, we can have \( U_e^{i_1,i_2,i_3,e} < I < U_e^{i_1i_2,i_3,e} \), in which case the merger between incumbents \( i_1 \) and \( i_2 \) deters an otherwise profitable entry. Alternatively, with \( U_e^{i_1i_2,i_3,e} < I < U_e^{i_1,i_2,i_3,e} \) the merger between \( i_1 \) and \( i_2 \) enables entry, which would not occur if these firms remained separated.

Since they have perfect information, in Stage 1 firms \( i_1 \) and \( i_2 \) can anticipate how their decision to merge or stay separated influences Stage 2 entry. Assume that \( U_e^{i_1,i_2,i_3,e} < I < U_e^{i_1i_2,i_3,e} \), i.e. if the merger takes place it deters entry. Firms \( i_1 \) and \( i_2 \) then choose to integrate if the profit they earn as integrated entity with \( e \) staying out is larger than the sum of their profits in the market structure where they stay separated and \( e \) enters, \( \{i_1, i_2, i_3, e\} \). Formally, an entry-deterring merger is profitable, if

\[
U_e^{i_1,i_2,i_3,e} + U_e^{i_1,i_2,i_3,e} < U_e^{i_1i_2,i_3}. \tag{6}
\]

Assume now that \( U_e^{i_1i_2,i_3,e} < I < U_e^{i_1,i_2,i_3,e} \), i.e. a merger fosters entry. Then, incumbents \( i_1 \) and \( i_2 \) merge if their integrated profit with entry is larger than the profit they realize separately if \( e \) stays out. Formally, an entry-fostering merger is profitable, if

\[
U_e^{i_1,i_2,i_3} + U_e^{i_1,i_2,i_3} < U_e^{i_1i_2,i_3,e} \tag{7}
\]

Having set up the general analytical framework, I am now in the position to become specific about horizontal and vertical mergers and entry. I will distinguish between the effects of horizontal and vertical mergers on entry and start with the former.
Proposition 3. (Horizontal mergers and entry). Invoke Assumption 2 (symmetry) and let $I_u = (1/6)[W_{\Omega,r} - W_{\Omega,s}] + (1/4)W_{\Omega}$, $I_u = (1/6)[2W_{\Omega} - W_{\Omega,s}]$, $I_d = (1/6)[W_{\Omega,s} - W_{\Omega,r}] + (1/4)W_{\Omega}$, $I_d = (1/6)[2W_{\Omega} - W_{\Omega,r}]$. The following relationship holds between horizontal merger incentives and entry.

(Upstream entry)

(i) With $I < \min\{I_u, I_u\}$ ($I > \max\{I_u, I_u\}$), upstream entry takes place (does not take place) regardless whether retailers merge or not.

(ii) With $I \in [\min\{I_u, I_u\}, \max\{I_u, I_u\}]$, and unit costs are strictly increasing (decreasing) retailers stay separated (merge) and accommodate upstream entry.

(Downstream entry)

(iii) With $I < \min\{I_d, I_d\}$ ($I > \max\{I_d, I_d\}$), downstream entry takes place (does not take place) regardless whether suppliers merge or not.

(iv) With $I \in [\min\{I_d, I_d\}, \max\{I_d, I_d\}]$, and products being substitutes (complements) suppliers stay separated (merge) and accommodate downstream entry.

Proof of Proposition 3: See Appendix.

Note first, that a horizontal merger between incumbent suppliers or retailers never deters entry. In fact, the opposite is the case. A horizontal merger which would in the absence of the potential entrant be unprofitable may actually take place, precisely in order to foster entry. This can be the case if the potential entrant is a supplier, and unit costs are decreasing. Broadly speaking, this result is due to the fact that for firms competition on another level of the supply chain is more important than large size in bargaining. This has been recently demonstrated empirically by Sorensen (2003) and Ellison and Snyder (2010). I provide a theoretical explanation of this empirical finding.

With decreasing unit costs retailers otherwise prefer to stay separated and negotiate at the margin with the supplier(s). However, to attract a supplier into the market they may merge. By doing so they reduce their own bargaining power to convey a larger share of the industry surplus to the entrant. If entry costs lie in the appropriate range (in this example $I \in [I_u, I_u]$), this additional surplus can motivate the supplier to enter the market. Although the bargaining power
of the retailers decreases by merging, it is profitable for them to do so. They are compensated for receiving a smaller share of the pie by the increase in the size of the pie due to entry.

Similarly, if the potential entrant is a retailer and goods are complements, suppliers may choose to enter an otherwise unprofitable merger to foster entry downstream. The logic is similar as before. If the products are complements, suppliers prefer to negotiate at the margin and stay separated. However, if passing on a sufficient share of surplus to the potential entrant is necessary to allow it to cover its costs and enter, than they can achieve such a transfer by merging.

Note furthermore, that beside entering an otherwise unprofitable horizontal merger, incumbents may also refrain from an otherwise profitable horizontal merger, in order to induce entry. This can happen if the potential entrant is a supplier and unit costs increase, or if the entrant is a retailer and products are substitutes. They do so to actively reduce their own bargaining power and pass on a share of industry surplus to the entrant, which it needs to cover its entry costs.

Why are incumbents on the same market level always interested in inducing entry? Could not they gain more by increasing their bargaining power and extracting rents from the incumbent, even at the cost that entry does not occur? As it turns out, the benefits of two firms on the same horizontal level from allowing entry always exceed the rents they can extract from a single incumbent. Take for example the case with increasing unit costs and a supplier considering entry. Retailers would absent entry prefer to merge and bargain jointly with the incumbent supplier. By merging none of them is left to be second to reach an agreement with the supplier and can avoid being marginalized. However, if they remain separated and allow entry to happen, each retailer can be first to strike an agreement with a supplier. In addition, they generate rents by being second as well. They are therefore always better off under entry. (Of course, they would be best off if entry occurred and they merged).

Horizontal competitors therefore always act in favor of entry on the other market level. This result qualifies the common claim that buyer power reduces consumer choice. As I demonstrate in the following, vertical mergers are very different in this respect. In contrast to horizontal mergers, vertical integration can under circumstances profitably deter entry. The

---

7See for example EC (1999, p.5): “Concerns have been raised that buyer power abuses of supermarkets have long term consequences for consumers [...] They have negative effects on (long term) consumer interests such as decreasing choice [...] of products [...]”
next proposition summarizes my results on this issue.

**Proposition 4.** (Vertical mergers and entry). Invoke Assumption 2 (symmetry) and let
\[ I_s := (1/6)[W_{\Omega,r} - W_{\Omega,s}] + (1/4)W_{\Omega}, \quad I_r := (1/6)[W_{\Omega,s} - W_{\Omega,r}] + (1/4)W_{\Omega}, \quad I := (1/6)[W_{\Omega,s} - \Delta \Omega'] := p(q', 0) - C(q'). \]
The following relationship holds between vertical merger incentives and entry.

(Upstream entry)

(i) With \( I < \min\{I_s, I_r\} \) (\( I > \max\{I_s, I_r\} \)), upstream entry takes place (does not take place) regardless whether a supplier and a retailer merge or not.

(ii) With \( I \in [\min\{I_s, I_r\}, \max\{I_s, I_r\}] \), vertical merger incentives and upstream entry depend on the strength of substitutability/complementarity between products and the shape of the unit cost function in the following way:

(a) If goods are complements, a vertical merger occurs and upstream entry takes place.

(b) If goods are substitutes and \( \Delta C < 2\Delta p + (1/2)\Delta C \) \( (\Delta C \geq 2\Delta p + (1/2)\Delta C) \) vertical merger does not occur (occurs) and upstream entry takes place (does not take place).

(Downstream entry)

(iii) With \( I < \min\{I_s, I_r\} \) (\( I > \max\{I_s, I_r\} \)), downstream entry takes place (does not take place) regardless whether a supplier and a retailer merge or not.

(iv) With \( I \in [\min\{I_s, I_r\}, \max\{I_s, I_r\}] \), vertical merger incentives and downstream entry depend on the strength of substitutability/complementarity between products and the shape of the unit cost function in the following way:

(a) If unit costs are decreasing a vertical merger occurs and downstream entry takes place.

(b) If unit costs are increasing and \( \Delta C < (1/2)\Delta p + (1/4)\Delta C \) \( (\Delta C \geq (1/2)\Delta p + (1/4)\Delta C) \), vertical merger does not occur (occurs) and downstream entry takes place (does not take place).

**Proof of Proposition 4.** See Appendix.

As opposed to horizontal mergers, vertical integration may actually deter the entry of a supplier and of a retailer. By vertically merging, the involved firms can shift bargaining in their
own favor by an extent that renders it impossible for the entrant to cover its entry costs. In this case the merging parties trade off a smaller pie for a larger slice.

Similarly as in some cases of horizontal mergers, vertical integration may also foster entry by passing on a share of surplus to the entrant, which enables it to cover the costs of entry. While total industry surplus is reduced if entry is deterred, the share of the reduced surplus accruing to the integrating firms can be larger than what they could capture by staying separated and accommodating entry. In the following I provide more detailed intuition for the relationship between vertical integration and entry.

Consider first the entry of a supplier, say $B$, while supplier $A$ and retailers $a$ and $b$ are incumbent. If entry costs are in the appropriate range ($I \in [\min\{L_s, T_s\}, \max\{L_s, T_s\}]$), a vertical merger between supplier $A$ and retailer $a$ can affect the entry decision of supplier $B$. Figure (4) represents regions depending on the level of substitutability/compatibility between goods and the shape of unit costs, where a vertical merger between $A$ and $a$ affects entry in various ways. Table (4) provides further clarification on each region.

![Figure 4: Vertical merger and supplier entry](image-url)
It was shown in Corollary 1 that a vertical merger between a monopoly supplier and a retailer is profitable if unit costs are increasing. In this case, vertically merging allows the integrated retailer to always receive the input in the range with the lowest costs, leaving the other retailer to bargain about deliveries in the range where costs are higher. In regions 1-3 in Figure (4) therefore the merging firms can shift away rents from the other retailer by vertically integrating. However, if they do so they also shift away rents from supplier B now considering entry. If the entry costs lie in the appropriate range (namely in this case $I \in [\bar{I}_s, \underline{I}_s]$), this induces supplier $B$ to stay out of the market. Entry not taking place is costly for the parties considering integration. A new supplier selling a new product generates additional surplus, which is forgone if entry does not take place. The benefits of increased bargaining power must therefore be weighed off against the loss of extra industry surplus. I demonstrate this on each region in Figure (4). Assume in the following, that entry costs lie in the interval $I \in [\min\{\underline{I}_s, \bar{I}_s\}, \max\{\underline{I}_s, \bar{I}_s\}]$. This implies, that the merger decision of supplier $A$ and retailer $a$ can influence retailer $B$’s entry incentives.

In region 1 unit costs are relatively rapidly increasing and goods are rather strong substitutes ($\Delta_C^{\Omega'} \geq 2\Delta_p^{\Omega'} + (1/2)\Delta^{\Omega'}$ and $\Delta_C^{\Omega'} > 0, \Delta_p^{\Omega'} < 0$). Under such circumstances vertical integration takes place and deters entry. The reason behind why this is profitable is that with unit costs rapidly increasing, the merged retailer benefits a lot from not having to negotiate about marginal quantities with its partner. Vertical integration therefore shifts a lot of bargaining power to the merging parties. At the same time, with products being strong substitutes the additional surplus generated by the entrant supplier is relatively low because of the strong negative cross-price effect. In this case, entry-deterring integration is profitable because it conveys a lot of bargaining power to the merging parties and the forgone increase in industry surplus is relatively low.

In region 2 products are relatively weak substitutes and unit costs are slowly increasing.
\( \Delta \alpha C' < 2 \Delta \alpha p' + \frac{1}{2} \Delta \alpha r' \) \( \text{and} \ \Delta \alpha C' > 0, \ \Delta \alpha p' < 0 \). Vertical integration does not take place, and entry can occur. If unit costs are slowly increasing, vertical integration does not improve the bargaining position of the involved parties sufficiently to compensate them for their forgone share of the larger industry surplus they receive if the supplier enters. If goods are weak substitutes, entry increases total surplus by a medium amount, since the negative cross-price effect is not very strong.

In region 3 goods are complements and unit costs are increasing \( (\Delta \alpha C', \Delta \alpha p' > 0) \). Although a vertical merger is profitable with increasing unit costs, and some rent is shifted away from the entrant, entry still occurs. This is because with goods being complements the entrant generates a large additional surplus due to the positive cross-price effect, and captures a sufficient portion of it to cover its entry costs. In fact, the entrant is better off if a vertical merger takes place when goods are complements. Such a merger guarantees that it can negotiate about marginal quantities with the integrated retailer, which are large if goods are complements. If entry costs are in the range \( U_B^{\{A,B,a,b\}} < I < U_B^{\{A,a,B,b\}} \), the vertical merger facilitates entry: the merging parties pass on part of their profits to the entrant which allows it to cover its entry costs.

In region 4 unit costs are relatively weakly increasing while products are rather strong substitutes \( (\Delta \alpha C' \geq 2 \Delta \alpha p' + \frac{1}{2} \Delta \alpha r' \text{ and } \Delta \alpha p' < 0, \ \Delta \alpha C' < 0) \). With unit costs decreasing, absent the possibility of upstream entry no vertical merger would be profitable. Yet with the threat of a supplier entering, the vertical merger takes place and deters entry. The winner is primarily the integrated supplier, since the merger prevents the entry of a competitor offering a strong substitute product. With unit costs decreasing the merging parties lose a bit of bargaining power vis-à-vis the non-integrated retailer, which can always negotiate at the margin with the integrated supplier. However, since products are strong substitutes, the supplier would lose more if entry occurred.

In region 5 unit costs are relatively rapidly decreasing while products are rather weak substitutes \( (\Delta \alpha C' < 2 \Delta \alpha p' + \frac{1}{2} \Delta \alpha r' \text{ and } \Delta \alpha p' < 0, \ \Delta \alpha C' < 0) \). No merger takes place and supplier B enters. With quickly decreasing unit costs a vertical merger erodes the merging parties’ bargaining position vis-à-vis the second retailer significantly. The rather small benefit to the integrated supplier of keeping a not too strongly substitutable upstream competitor out of the market is not worth paying this price.

Finally, in region 6 goods are complements and unit costs decrease \( (\Delta \alpha C' < 0, \ \Delta \alpha r' > 0) \).
If entry costs are in the range $U_B^{\{A,B,a,b\}} < I < U_B^{\{A,a,B,b\}}$, in this case a vertical merger takes place in order to facilitate entry. Absent potential entry this merger would not occur, and furthermore, absent the merger supplier $B$ would not enter. If goods are complements the entrant supplier benefits from a vertical merger between incumbents, because it guarantees to bargain on marginal quantities with the integrated retailer. Goods being complements means that entry has large benefits for the incumbents as well, which exceed any rents that could be extracted from the second retailer by merging and deterring entry. Therefore, the merger is profitable and it fosters entry.

Consider next the entry of a retailer, say $b$, while suppliers $A$ and $B$ as well as retailer $a$ are incumbent. If entry costs are in the appropriate range ($I \in [\min\{I_r, I_{rg}\}, \max\{I_r, I_{rg}\}]$), a merger between supplier $A$ and retailer $a$ can affect the entry decision of retailer $b$. Figure (5) depicts regions depending on the level of substitutability/compatibility between goods and the shape of unit costs, where a vertical merger between $A$ and $a$ affects entry in various ways.

![Figure 5: Vertical merger and retailer entry](image-url)
Table (5): Figure (5) regions, $I \in [\min\{I_r, I_r\}, \max\{I_r, I_r\}]$

<table>
<thead>
<tr>
<th>Region</th>
<th>Merger</th>
<th>Entry</th>
<th>Merger absent potential entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>5</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>6</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

I demonstrate how vertical merger incentives interact with retailer entry for each region in Figure (5). Assume that $I \in [\min\{I_r, I_r\}, \max\{I_r, I_r\}]$, so that a merger decision is not irrelevant for entry.

In region 1, unit costs are relatively quickly increasing and products are relatively strong substitutes ($\Delta_C^{\sigma'} \geq (1/2)\Delta_p^{\sigma'} + (1/4)\Delta^{\sigma'}, \Delta^{\sigma'}_p < 0$ and $\Delta^{\sigma'}_C > 0$). If this is the case, vertical integration takes place and it deters retailer entry. This is profitable for the following reason. With products being substitutes the integrated supplier benefits from vertical integration, since it does not have to negotiate about marginal quantities with its partner retailer. With unit costs increasing, a second retailer generates relatively little surplus additional to the existing market configuration. Since products are sufficiently strong substitutes and unit costs are sufficiently quickly increasing ($\Delta_C^{\sigma'} \geq (1/2)\Delta_p^{\sigma'} + (1/4)\Delta^{\sigma'}$), the increased rents the merged entity can extract from the incumbent supplier absent entry exceeds the share of the parties of the increased surplus that would be realized with entry. Therefore, a vertical merger to deter the entry of a retailer is profitable.

In region 2, unit costs are strongly increasing and products are complements ($\Delta_C^{\sigma'} \geq (1/2)\Delta_p^{\sigma'} + (1/4)\Delta^{\sigma'}, \Delta^{\sigma'}_C > 0$ and $\Delta^{\sigma'}_p > 0$). A vertical merger takes place and it deters the entry of retailer $b$. Note, that this merger would not be profitable absent the threat of entry: With goods being complements the vertically merging parties actually weaken their bargaining power vis-à-vis the second supplier, because they make sure it can negotiate about marginal quantities, which it prefers. The reason why this merger is still profitable is that with strongly increasing unit costs the incumbent retailer would lose a lot if the other retailer entered. The merger is therefore primarily motivated by the incumbent retailer’s choice of the lesser of two evils: slightly reduced bargaining power but keeping its downstream monopoly position instead of largely decreased
bargaining power and accommodating retailer entry.

In region 3 unit costs are relatively slowly increasing and products are relatively weak substitutes \((\Delta_C^\Omega < (1/2)\Delta_p^\Omega + (1/4)\Delta_p^\Omega, \Delta_C^\Omega > 0 \text{ and } \Delta_p^\Omega < 0)\). No vertical merger takes place and retailer \(b\) enters. Note first, that under such conditions absent the threat of entry a vertical merger would be profitable, since products are (weak) substitutes. Weak substitutability also implies that by merging and deterring entry the involved parties could slightly improve their bargaining power vis-à-vis the second supplier. However, supplier \(A\) benefits a lot if retailer \(b\) enters. Since unit costs are increasing, both retailers want to be first to reach an agreement with the suppliers, which puts the latter into a comfortable bargaining position. A vertical merger is therefore not profitable, because it would make the upstream party worse off by deterring downstream entry.

In region 4 unit costs are relatively slowly increasing and products are complements \((\Delta_C^\Omega < (1/2)\Delta_p^\Omega + (1/4)\Delta_p^\Omega, \Delta_C^\Omega > 0 \text{ and } \Delta_p^\Omega > 0)\). No vertical merger takes place and entry occurs. Since goods are complements a vertical merger which prevents entry does not benefit the involved parties: it guarantees that the non-merging supplier always negotiates at the margin with the retailer, which is precisely what it wants. Staying separated and allowing entry on the other hand allows firms \(A\) and \(a\) to enjoy the benefits of a larger total surplus.

In region 5 unit costs are decreasing and products are substitutes \((\Delta_C^\Omega < 0 \text{ and } \Delta_p^\Omega < 0)\). A vertical merger takes place and it enables entry if \(I \in [\min\{L_r, \bar{I}_r\}, \max\{L_r, \bar{I}_r\}].\) The choice of supplier \(A\) and retailer \(a\) is between merging and inviting entry or staying separated absent entry. The merger fosters entry because it benefits retailer \(b\) by allowing it to negotiate about marginal quantities with the integrated supplier, which it prefers to do with decreasing unit costs.

Finally, in region 6 unit costs are decreasing and products are complements \((\Delta_C^\Omega < 0 \text{ and } \Delta_p^\Omega > 0)\). While absent the threat of entry no vertical merger would take place, now supplier \(A\) and retailer \(a\) merge in order to enable entry. In this case a new retailer increases total surplus by a relatively large amount, of which supplier \(A\) and retailer \(a\) can capture a share which exceeds what they could gain by staying separated and deterring entry.

To sum up, in this section I have contrasted horizontal and vertical mergers in terms of their effect on entry. In general, mergers change the bargaining position of all parties and can either deter entry or facilitate it. Entry deterrence occurs by reducing the potential entrant’s
expected revenues below the level sufficient to cover its entry costs. A merger can facilitate entry by credibly conveying bargaining strength to the entrant, which enables it to generate sufficient revenues to cover its entry costs, which it could not do otherwise. In my framework horizontal mergers to deter entry are never profitable, and they are often entry-facilitating. Vertical mergers however to keep a potential entrant out of the market can pay off. An entry-deterring vertical merger has two effects. First, it changes the bargaining position of all parties, potentially allowing the merging firms to get a larger share of industry surplus. At the same time however, by deterring entry the merger prevents the realization of a higher industry surplus. Broadly speaking, a merger to deter entry may enable the involved firms to get a larger slice of a smaller pie. Horizontal mergers that would deter entry are not profitable because for such mergers the size of the pie matters more, and the total surplus is larger if entry takes place. Vertical mergers are different in this respect. They may shift bargaining in favor of the merged entity so that the additional rents it extracts from the non-merging incumbent exceed the benefits from an increased pie due to entry. When this can occur depends on the level of substitution or complementarity between the products, the shape of the unit cost function and the average markup a pair of supplier and retailer can generate alone. In my setup, vertical mergers are therefore more likely to be harmful for total welfare than horizontal ones.

Although I speak of the entry deterring potential of vertical mergers the intuition applies just as well to forcing an incumbent firm to exit by shifting bargaining power with a vertical merger. Basically, exit has the greatest benefit for the horizontal rival in the level of the industry where it occurs. If a horizontal merger leads to exit in the other level, the benefit to the rival in that other level is an externality that is not internalized by the merging firms, so they do not have an incentive to deter in this way. On the other hand, if a vertical merger leads to exit in one level or another of the market, the merged entity has a unit in each level so a unit that stands to benefit a lot from this exit.

7 Conclusions

I propose a model of a bilaterally duopolistic industry where upstream producers bargain with downstream retailers on supply conditions. In the applied framework integration does not affect the total output produced, but it affects the distribution of rents among players. I make three contributions in this article. First, I identify conditions for vertical mergers to occur and
show, that in a framework in which delivery conditions are determined by bargaining, vertical integration incentives can be regarded as a mix of horizontal merger incentives downstream and upstream. Second, I directly compare the strength of horizontal and vertical merger incentives if either an upstream or a downstream firm is available for sale by means of an auction to the highest bidder. I demonstrate that - as opposed to conventional wisdom - a merger to monopoly may convey less bargaining power to the merged entity than vertical integration. Third, I compare the potential of horizontal and vertical mergers to deter entry. My results show, that while horizontal mergers are never an apt device to deter entry, vertical integration can profitably induce a potential entrant to stay out of the market.

The results presented here on the effects of vertical mergers stand in sharp contrast to several prevailing views in competition policy, which strongly favors vertical mergers over horizontal ones. Taking explicitly into account that deliveries are determined by bargaining between parties, the contrast between horizontal and vertical mergers become less clear. In fact, my insights suggest that in such an environment vertical mergers are likely to be more harmful for welfare than horizontal ones because they are more likely to deter entry. This creates scope for welfare enhancing intervention into such transactions by competition policy.

While many of my results are general, this article has some limitations. In particular, some results are derived under the assumption of symmetry. Imposing this assumption helps identifying the main forces at work, but omits other effects stemming from the asymmetry between firms. Discovering these additional effects could be an interesting avenue for further research, and the first step in this direction is provided in the general formulae derived here.

A further restrictive assumption is that of no competitive externalities downstream. This assumption is necessary to ensure superadditivity, which is required for the application of the Shapley value as allocation rule. Taking into account competitive externalities downstream while maintaining the assumption that the merged firms melt into one bargaining unit could undoubtedly provide valuable insights and extend the applicability of the model to several realistic market scenarios. This could be done for example by applying modifications of the Shapley value to determine the outcome of bargaining, which take into account externalities in the total value generated by various coalitions, as recently suggested for example by De Clippel and Serrano (2008) and Macho-Stadler et al. (2007).

Finally, while this article confines itself to the analysis of vertical merger incentives and its
comparison to horizontal ones, many possible extensions arise naturally. Moving beyond the simple bilateral duopoly setup as well as taking into account investment incentives could be fruitful topics for further research.
8 Appendix

<table>
<thead>
<tr>
<th>Market structure</th>
<th>Payoffs</th>
</tr>
</thead>
</table>
| $\{A, B, a, b\}$ | $U_A = \frac{1}{12} \left[ W_{\Omega \setminus Bb} + W_{\Omega \setminus Ba} + W_{\Omega \setminus b} - W_{\Omega \setminus Ab} + W_{\Omega \setminus a} - W_{\Omega \setminus Aa} + W_{\Omega \setminus B} - 3W_{\Omega \setminus A} + 3W_{\Omega} \right]$
|                 | $U_B = \frac{1}{12} \left[ -W_{\Omega \setminus Bb} - W_{\Omega \setminus Ba} + W_{\Omega \setminus b} + W_{\Omega \setminus Ab} + W_{\Omega \setminus a} - 3W_{\Omega \setminus B} + W_{\Omega \setminus A} + 3W_{\Omega} \right]$
|                 | $U_a = \frac{1}{12} \left[ W_{\Omega \setminus Bb} - W_{\Omega \setminus Ba} + W_{\Omega \setminus b} + 3W_{\Omega \setminus Aa} + W_{\Omega \setminus Ab} + W_{\Omega \setminus B} + W_{\Omega \setminus A} + 3W_{\Omega} \right]$
|                 | $U_b = \frac{1}{12} \left[ -W_{\Omega \setminus Bb} + W_{\Omega \setminus Ba} - 3W_{\Omega \setminus B} + W_{\Omega \setminus a} + W_{\Omega \setminus Aa} + W_{\Omega \setminus Ab} + W_{\Omega \setminus B} + W_{\Omega \setminus A} + 3W_{\Omega} \right]$
| $\{AB, a, b\}$  | $U_{AB} = \frac{1}{6} \left[ W_{\Omega \setminus a} + W_{\Omega} \right]$ |
|                 | $U_a = \frac{1}{6} \left[ W_{\Omega \setminus b} - 2W_{\Omega \setminus a} + 2W_{\Omega} \right]$ |
|                 | $U_b = \frac{1}{6} \left[ -2W_{\Omega \setminus b} + W_{\Omega \setminus a} + 2W_{\Omega} \right]$ |
| $\{ABa, b\}$    | $U_{ABa} = \frac{1}{2} \left[ W_{\Omega \setminus b} + W_{\Omega} \right]$ |
|                 | $U_b = \frac{1}{2} \left[ -W_{\Omega \setminus b} + W_{\Omega} \right]$ |
| $\{A, B, ab\}$  | $U_A = \frac{1}{6} \left[ W_{\Omega \setminus B} - 2W_{\Omega \setminus A} + 2W_{\Omega} \right]$ |
|                 | $U_B = \frac{1}{6} \left[ -2W_{\Omega \setminus B} + W_{\Omega \setminus A} + 2W_{\Omega} \right]$ |
|                 | $U_{ab} = \frac{1}{6} \left[ W_{\Omega \setminus B} + W_{\Omega \setminus A} + 2W_{\Omega} \right]$ |
| $\{Aab, B\}$    | $U_{Aab} = \frac{1}{2} \left[ W_{\Omega \setminus B} + W_{\Omega} \right]$ |
|                 | $U_B = \frac{1}{2} \left[ -W_{\Omega \setminus B} + W_{\Omega} \right]$ |
| $\{ABab\}$     | $U_{ABab} = W_{\Omega}$ |
| $\{Aa, B, b\}$  | $U_{Aa} = \frac{1}{6} \left[ 2W_{\Omega \setminus Bb} + W_{\Omega \setminus b} + W_{\Omega \setminus B} - 2W_{\Omega \setminus Aa} + 2W_{\Omega} \right]$ |
|                 | $U_B = \frac{1}{6} \left[ -W_{\Omega \setminus Bb} + W_{\Omega \setminus B} - 2W_{\Omega \setminus Ab} + W_{\Omega \setminus Aa} + 2W_{\Omega} \right]$ |
|                 | $U_b = \frac{1}{6} \left[ -W_{\Omega \setminus Bb} - 2W_{\Omega \setminus B} + W_{\Omega \setminus B} + W_{\Omega \setminus Aa} + 2W_{\Omega} \right]$ |
| $\{Aa, Bb\}$    | $U_{Aa} = \frac{1}{2} \left[ W_{\Omega \setminus Bb} - W_{\Omega \setminus Aa} + W_{\Omega} \right]$ |
|                 | $U_{Bb} = \frac{1}{2} \left[ -W_{\Omega \setminus Bb} + W_{\Omega \setminus Aa} + W_{\Omega} \right]$ |

Table 1: Payoffs in various market structures

Proof of Proposition 1: The proof is immediate by comparing the change in payoffs of the merging parties as summarized in Table (3).
### Table 3: Change in payoffs by vertical integration

<table>
<thead>
<tr>
<th>Change in market structure</th>
<th>Change in payoffs of vertically merging parties ($\Delta U$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>${A, B, a, b} \rightarrow {Aa, B, b}$</td>
<td>$[U_A + U_a]<em>{{A, B, a, b}} = \frac{1}{6} [3W</em>\Omega - W_{\Omega\setminus A} + W_{\Omega\setminus B} - W_{\Omega\setminus a} + W_{\Omega\setminus b}]$</td>
</tr>
<tr>
<td></td>
<td>$[U_{Aa}]<em>{{Aa, B, b}} = \frac{1}{6} [2W</em>{\Omega\setminus B} + W_{\Omega\setminus b} + W_{\Omega\setminus A} - 2W_{\Omega\setminus Aa} + 2W_{\Omega\setminus b}]$</td>
</tr>
<tr>
<td></td>
<td>$\Delta U_{Aa} = \frac{1}{6} [(W_{\Omega\setminus B} - W_{\Omega\setminus Aa}) + W_{\Omega\setminus a} + W_{\Omega\setminus a} - W_{\Omega\setminus b}]$</td>
</tr>
<tr>
<td>${AB, a, b} \rightarrow {ABa, b}$</td>
<td>$[U_{AB} + U_a]<em>{{AB, a, b}} = \frac{1}{6} [4W</em>\Omega - W_{\Omega\setminus a} + 2W_{\Omega\setminus b}]$</td>
</tr>
<tr>
<td></td>
<td>$[U_{ABa}]<em>{{ABa, b}} = \frac{1}{2} [W</em>{\Omega\setminus B} + W_{\Omega\setminus b}]$</td>
</tr>
<tr>
<td></td>
<td>$\Delta U_{ABa} = \frac{1}{6} [W_{\Omega\setminus a} + W_{\Omega\setminus B} - W_{\Omega\setminus b}]$</td>
</tr>
<tr>
<td>${A, B, ab} \rightarrow {Aab, B}$</td>
<td>$[U_A + U_{ab}]<em>{{A, B, ab}} = \frac{1}{6} [4W</em>\Omega - W_{\Omega\setminus A} + 2W_{\Omega\setminus B}]$</td>
</tr>
<tr>
<td></td>
<td>$[U_{Aab}]<em>{{Aab, B}} = \frac{1}{2} [W</em>{\Omega\setminus B} + W_{\Omega\setminus b}]$</td>
</tr>
<tr>
<td></td>
<td>$\Delta U_{Aab} = \frac{1}{6} [W_{\Omega\setminus A} + W_{\Omega\setminus B} - W_{\Omega\setminus b}]$</td>
</tr>
</tbody>
</table>

Q.E.D.

**Proof of Corollary 1.** We proceed by proving each claim separately, starting with Claim (i).

**Claim (i)** With suppliers integrated and retailers separated ($\Psi = \{AB, a, b\}$), the condition for a vertical merger between supplier $AB$ and retailer $r$ to take place is given by Claim (ii) in Proposition 1. This is identical to the condition for a horizontal merger between retailers to take place in IW (2003). The proof of Claim (i) is immediate from Corollary 1(ii) of the same article.

**Claim (ii)** With suppliers separated and retailers integrated ($\Psi = \{A, B, ab\}$), the condition for a vertical merger between supplier $s$ and retailer $ab$ to take place is given by Claim (iii) of Proposition 1. This is identical to the condition for a horizontal merger between suppliers to take place in IW (2003). The proof of Claim (ii) is immediate from Corollary 1(i) of the same article.

**Claim (iii)** Under Assumption 2 (symmetry) the condition for a vertical merger to take place in Claim (i) of Proposition 1 reduces to

$$W_{\Omega\setminus a} + W_{\Omega\setminus r} > W_{\Omega}. \quad (8)$$

I focus w.l.o.g. on a merger between supplier $A$ with retailer $a$. The proof for any other supplier-retailer combination would proceed analogously. I first show that if the products are substitutes and unit costs are strictly increasing a vertical merger takes place. Let $q_{sir}^r$ denote the quantities
of supplier $s$ at retailer $r$ if the subset $\Omega' \subseteq \Omega$ of firms participate. Condition (8) for supplier $A$ and retailer $a$ to merge can be written as

$$
\sum_{r \in R^0} p_{Br}(q_{Br}, 0)q_{Br} - C_B(q_{Br} + q_{Br'}) + \sum_{s \in S^0} p_{sb}(q_{sb}, q_{sb}')q_{sb} - \sum_{s \in S^0} C_s(q_{sb}) > 0.
$$

(9)

Note that the sum of payoffs on the LHS in Expression (8) does not increase if the optimal quantities $q_{rs}$ and $q_{rs}'$ are replaced by $q_{rs}'$. It follows, that (8) holds if

$$
\sum_{r \in R^0} \sum_{s \in S^0} p_{sr}(q_{sr}^\Omega, q_{sr}')q_{sr} - \sum_{s \in S^0} C_s(q_{sr} + q_{sr}') > 0.
$$

Under Assumption 2 (symmetry), this inequality can be written as

$$
4p(q^\Omega, q^\Omega)q^\Omega - 2C(2q^\Omega) > 2p(q^\Omega, 0)q^\Omega - C(2q^\Omega) + 2p(q^\Omega, q^\Omega)q^\Omega - 2C(q^\Omega).
$$

Dividing by $2q^\Omega$ and rearranging yields

$$
p(q^\Omega, q^\Omega) - p(q^\Omega, 0) < \overline{C}(2q^\Omega) - \overline{C}(q^\Omega),
$$

or identically,

$$
\Delta^\Omega_p < \Delta^\Omega_C.
$$

(10)

The RHS is positive by Definition 2 if unit costs are strictly increasing, while the LHS is by Definition 1 negative if the goods are substitutes. Consequently, if the products are substitutes and unit costs are strictly increasing, Condition (8) holds.

I next show that if products are complements and unit costs are strictly decreasing, no vertical merger takes place. A vertical merger does not occur if inequality (9) is reversed, such
that
\[ \left[ \sum_{r \in R^0} p_{Br} (q_{Br}^{\Omega \setminus A}, 0) q_{Br}^{\Omega \setminus A} - C_B (q_{Br}^{\Omega \setminus A} + q_{Br}^{\Omega \setminus A}) \right] + \left[ \sum_{s \in S^0} p_{sb} (q_{sb}^{\Omega \setminus a}, q_{sb}^{\Omega \setminus a}) q_{sb}^{\Omega \setminus a} - \sum_{s \in S^0} C_s (q_{sb}^{\Omega \setminus a}) \right] < \\
\left[ \sum_{s \in S^0} \sum_{r \in R^0} p_{sr} (q_{sr}^{\Omega}, q_{sr}^{\Omega}) q_{sr}^{\Omega} - \sum_{s \in S^0} C_s (q_{sr}^{\Omega} + q_{sr}^{\Omega}) \right]. \]

(11)

Under Assumption 2 (symmetry) this can be written as
\[ \left[ 2p(q^{\Omega \setminus A}, 0) q^{\Omega \setminus A} - C(2q^{\Omega \setminus A}) \right] + \left[ 2p(q^{\Omega \setminus a}, q^{\Omega \setminus a}) q^{\Omega \setminus a} - 2C(q^{\Omega \setminus a}) \right] < \\
\left[ 2p(q^{\Omega}, q^{\Omega}) q^{\Omega} - C(2q^{\Omega}) \right] + \left[ 2p(q^{\Omega}, q^{\Omega}) q^{\Omega} - C(2q^{\Omega}) \right]. \]

Each bracket on the RHS corresponds to half of the industry surplus if all firms participate, which supplier B maximizes. Therefore, the relationship does not change if I replace \( q^{\Omega} \) by \( q^{\Omega \setminus A} \) and \( q^{\Omega \setminus a} \) in each bracket on the RHS. Doing so yields
\[ 2p(q^{\Omega \setminus A}, 0) q^{\Omega \setminus A} - 2C(q^{\Omega \setminus a}) < 2p(q^{\Omega \setminus A}, q^{\Omega \setminus A}) q^{\Omega \setminus A} - C(2q^{\Omega \setminus a}). \]

By rearranging and dividing both sides by \( 2q^{\Omega \setminus a} \) I get
\[ \left[ p(q^{\Omega \setminus A}, 0) - p(q^{\Omega \setminus A}, q^{\Omega \setminus A}) \right] \frac{q^{\Omega \setminus A}}{q^{\Omega \setminus a}} < C(q^{\Omega \setminus a}) - C(2q^{\Omega \setminus a}), \]

which by Definition 3 is equivalent to
\[ \Delta_{C}^{\Omega \setminus a} < \Delta_{\Omega \setminus A}^{\Omega \setminus A} \frac{q^{\Omega \setminus A}}{q^{\Omega \setminus a}}. \]

(12)

The LHS of (13) is negative if unit costs are strictly decreasing, while the RHS is positive when products are complements. We can conclude that if products are complements and unit costs are strictly decreasing no vertical merger between a supplier and a retailer takes place.

\( \text{Q.E.D.} \)

**Proof of Corollary 2.** We first consider the case where supplier A is available for sale. We then turn to the case where retailer a is the target firm.
Assume that supplier $A$ is the target firm. Retailer $a$ can make a higher bid than supplier $B$ if $\beta_a > \beta_B$. Analogously, supplier $B$ can outbid retailer $a$ if the opposite holds. Under Assumption 2 (symmetry), from Expression (4) we have, that $\beta_a > \beta_B$ ($\beta_a < \beta_B$) if $W_{\Omega \setminus A} > W_{\Omega \setminus B}$ ($W_{\Omega \setminus a} > W_{\Omega \setminus B}$). Consider first the condition $W_{\Omega \setminus a} > W_{\Omega \setminus B}$. This can be written as

$$\sum_{s \in S^a} p_{sb}(q_{sb}^{\Omega \setminus a}, q_{sb}^{\Omega \setminus B}) q_{sb}^{\Omega \setminus a} - \sum_{s \in S^a} C_s(q_{sb}^{\Omega \setminus a}) > \sum_{r \in R^a} p_{Ar}(q_{Ar}^{\Omega \setminus B}, 0) q_{Ar}^{\Omega \setminus B} - C_A(q_{Ar}^{\Omega \setminus B} + q_{Ar'}^{\Omega \setminus B}).$$

Under Assumption 2 (symmetry), the RHS remains unchanged if we replace the quantity $q_{Ar}^{\Omega \setminus B}$ by $q_{Ar}^{\Omega \setminus A}$. Furthermore, the LHS remains unchanged if we replace the quantities $q_{Ar}^{\Omega \setminus A}$ and $q_{Ar}^{\Omega \setminus B}$ by $q_{Ar}^{\Omega \setminus A}$ and $q_{Ar}^{\Omega \setminus B}$ respectively. Doing so and dividing both sides by $2q_{\Omega \setminus s}$ yields

$$p(q_{\Omega \setminus s}^{\Omega \setminus a}, q_{\Omega \setminus a}^{\Omega \setminus s}) - C(q_{\Omega \setminus s}^{\Omega \setminus a}) > p(q_{\Omega \setminus s}^{\Omega \setminus a}, 0) - C(2q_{\Omega \setminus s}),$$

which can be rearranged to get

$$-\Delta_{\Omega \setminus s}^{\Omega \setminus a} < \Delta_{\Omega}^{\Omega \setminus s}. \tag{14}$$

Therefore, if the target firm is supplier $A$, the acquirer is retailer $a$ if $-\Delta_{\Omega \setminus a}^{\Omega \setminus a} < \Delta_{\Omega}^{\Omega \setminus a}$ holds for every $\Omega' \subseteq \Omega$. The argument for the condition $W_{\Omega \setminus a} < W_{\Omega \setminus B}$ is analogous.

Consider next the case where retailer $a$ is the target firm. Retailer $b$ can make a higher bid than supplier $A$ if $\beta_b > \beta_A$. Analogously, supplier $A$ can outbid retailer $b$ if the opposite holds. Under Assumption 2 (symmetry), we have from Expression (5), that $\beta_b > \beta_A$ ($\beta_b < \beta_A$) if $W_{\Omega \setminus b} > W_{\Omega \setminus A}$ ($W_{\Omega \setminus b} < W_{\Omega \setminus A}$). Consider first the condition $W_{\Omega \setminus b} > W_{\Omega \setminus A}$. This can be written as

$$\sum_{s \in S^a} p_{sa}(q_{sa}^{\Omega \setminus b}, q_{sa}^{\Omega \setminus A}) q_{sa}^{\Omega \setminus b} - \sum_{s \in S^a} C_s(q_{sa}^{\Omega \setminus b}) > \sum_{r \in R^a} p_{Br}(q_{Br}^{\Omega \setminus A}, 0) q_{Br}^{\Omega \setminus A} - C_B(q_{Br}^{\Omega \setminus A} + q_{Br'}^{\Omega \setminus A}). \tag{15}$$

Since firms are assumed to be symmetric, Condition (15) is identical to Condition (14). Therefore, if the target firm is retailer $a$, the acquirer is retailer $b$ if $-\Delta_{\Omega \setminus b}^{\Omega \setminus b} < \Delta_{\Omega}^{\Omega \setminus b}$ holds for every $\Omega' \subseteq \Omega$. The argument for the condition $W_{\Omega \setminus b} < W_{\Omega \setminus A}$ is analogous. *Q.E.D.*

**Proof of Proposition 3.** Table (8) contains payoffs for the market structures \{A, ab\} and \{AB, a\} determined by applying the Shapley value, which will be useful in this proof.
I prove each claim separately, starting with Claim (i). Invoke Assumption 2 and assume w.l.o.g. that \( i_1 = a, i_2 = b, i_3 = A, e = B \), i.e. supplier \( B \) is the potential entrant, while retailers \( a \) and \( b \) consider merging horizontally. Note that \( I_u = U_B^{(A,B,a,b)} \) and \( T_u = U_B^{(A,B,ab)} \). If \( I < \min\{U_B^{(A,B,ab)}, U_B^{(A,B,a,b)}\} \), the entrant supplier \( B \) can cover its entry costs regardless whether firms \( a \) and \( b \) merge or not. Conversely, if \( I > \max\{U_B^{(A,B,ab)}, U_B^{(A,B,a,b)}\} \), firm \( B \) cannot make enough profits to cover its entry costs. Claim (i) follows immediately.

Consider next Claim (ii). Two cases are possible: either \( I_u \leq I \leq T_u \) or \( T_u \leq I \leq L_u \). I first investigate when each of these conditions hold. Assume that \( I_u \leq I \leq T_u \). With \( L_u = U_B^{(A,B,a,b)} \) and \( T_u = U_B^{(A,B,ab)} \), for the interval \([L_u, T_u]\) to be non-empty we must have \( U_B^{(A,B,a,b)} < U_B^{(A,B,ab)} \).

Under symmetry, by plugging in the corresponding values from Table (1) this is equivalent to
\[
\frac{1}{12} \left[ 2W_{\Omega}^{(r)} - 2W_{\Omega} - 3W_{\Omega} \right] < \frac{1}{6} \left[ 2W_{\Omega} - W_{\Omega \setminus s} \right],
\]
which can be rearranged to get
\[
2W_{\Omega}^{(r)} < W_{\Omega}.
\]
From Proposition 2 of IW, this relationship holds if unit costs are strictly decreasing.

\[
U_B^{(A,B,a,b)} \leq I \leq U_B^{(A,B,ab)}
\]
implies that a merger between retailers \( a \) and \( b \) makes an otherwise unprofitable entry of supplier \( B \) profitable. For the horizontal merger to occur, it must also be profitable for the merging parties, i.e. we must have \( U_a^{(A,a,b)} + U_b^{(A,a,b)} < U_{ab}^{(A,B,ab)} \). Plugging in the corresponding values from Tables 2 and 6 yields the profitability condition \( W_{\Omega} > W_{\Omega \setminus s} - W_{\Omega \setminus sr} \). This relationship is fulfilled under Assumption 1. Therefore, with \( L_u \leq I \leq T_u \) and unit costs strictly decreasing, retailers merge and accommodate upstream entry.

Assume now that \( T_u \leq I \leq L_u \). With the same logic as above, the interval \([T_u, L_u]\) is non-empty if unit costs are strictly increasing. \( U_B^{(A,B,ab)} \leq I \leq U_B^{(A,B,a,b)} \) implies that a merger between retailers \( a \) and \( b \) makes an otherwise profitable entry of supplier \( B \) unprofitable. Such a merger is therefore entry-deterring. For it to take place, it must also be profitable for the merging parties, i.e. we must have \( U_{ab}^{(A,ab)} > U_a^{(A,B,a,b)} + U_b^{(A,B,a,b)} \). Plugging in the corresponding values from Tables 2 and 6 yields the profitability condition \( W_{\Omega \setminus s} + 2W_{\Omega \setminus r} > 3W_{\Omega} \). This relationship violates Assumption 1. Therefore, with \( T_u \leq I \leq L_u \) and unit costs strictly increasing, retailers
Plugging in the corresponding values from Tables 2 and 6 yields the profitability condition with $I > \min\{U_b^{AB,a,b}, U_b^{AB,A,b}\}$, the entrant retailer $b$ can cover its entry costs regardless whether firms $A$ and $B$ merge or not. Conversely, if $I > \max\{U_b^{AB,a,b}, U_b^{AB,b,a}\}$, firm $b$ cannot make enough profits to cover its entry costs. Claim (iii) follows immediately.

Consider next Claim (iv). Again two cases are possible: either $L_d \leq I \leq \bar{T}_d$ or $\bar{T}_d \leq I \leq L_d$. First investigate when each of these conditions hold. Assume that $L_d \leq I \leq \bar{T}_d$. With $L_d = U_b^{AB,A,b}$ and $\bar{T}_d = U_b^{AB,b,a}$, for the interval $[L_d, \bar{T}_d]$ to be non-empty we must have $U_b^{AB,A,b} < U_b^{AB,b,a}$. Under symmetry, by plugging in the corresponding values from Table (1) this is equivalent to $\frac{1}{\Omega} [2W_{\Omega,s} - 2W_{\Omega,r} + 3W_\Omega] < \frac{1}{\Omega} [2W_\Omega - W_{\Omega,r}]$, which can be rearranged to get $2W_{\Omega,s} < W_\Omega$. From Proposition 2 of IW, this relationship holds if the products are complements.

$U_b^{AB,A,b} \leq I \leq U_b^{AB,b,a}$ implies that a merger between suppliers $A$ and $B$ makes an otherwise unprofitable entry of retailer $b$ profitable. For the horizontal merger to occur, it must also be profitable for the merging parties, i.e. we must have $U_{AB}^{AB,a,b} > U_A^{AB,a,b} + U_B^{AB,b,a}$. Plugging in the corresponding values from Tables 2 and 6 yields the profitability condition $W_\Omega > W_{\Omega,s} - W_{\Omega,r}$. This relationship is fulfilled under Assumption 1. Therefore, with $L_d \leq I \leq \bar{T}_d$ and products being complements, suppliers merge and accommodate downstream entry.

Assume now that $\bar{T}_d \leq I \leq L_d$. With the same logic as above, the interval $[\bar{T}_d, L_d]$ is non-empty if products are substitutes. $U_b^{AB,A,b} \leq I \leq U_b^{AB,b,a}$ implies that a merger between suppliers $A$ and $B$ turns an otherwise profitable entry of retailer $b$ unprofitable. Such a merger is therefore entry-deterring. For it to take place, it must also be profitable for the merging parties, i.e. we must have $U_{AB}^{AB,a,b} > U_A^{AB,a,b} + U_B^{AB,b,a}$. Plugging in the corresponding values from Tables 2 and 6 yields the profitability condition $2W_{\Omega,r} + 4W_{\Omega,s} > 6W_\Omega$. This relationship violates Assumption 1. Therefore, with $\bar{T}_d \leq I \leq L_d$ and products being substitutes, retailers stay separated and accommodate downstream entry. Q.E.D.

**Proof of Proposition 4.** I prove each claim separately starting with upstream entry and Claim (i). Invoke Assumption 2 and assume w.l.o.g. that $i_1 = A$, $i_2 = a$, $i_3 = b$, $e = B$, i.e. supplier $B$ is the potential entrant, while supplier $A$ and retailer $a$ consider merging vertically.
If supplier $B$ enters, the corresponding Stage 3 payoffs are contained in the rows $\{A, B, ab\}$ and $\{A, B, a, b\}$ of Table (1). If it does not enter, applying the Shapley value yields the following payoffs, depending on firm $A$’s and $a$’s merger decision:

<table>
<thead>
<tr>
<th>Market structure</th>
<th>Payoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td>${A, a, b}$</td>
<td>$U_A = \frac{1}{6}[2W_{\Omega\setminus B} + W_{\Omega\setminus Bb} + W_{\Omega\setminus Ba}]$</td>
</tr>
<tr>
<td></td>
<td>$U_a = \frac{1}{6}[2W_{\Omega\setminus B} + W_{\Omega\setminus Bb} - 2W_{\Omega\setminus Ba}]$</td>
</tr>
<tr>
<td></td>
<td>$U_b = \frac{1}{6}[2W_{\Omega\setminus B} - 2W_{\Omega\setminus Bb} + W_{\Omega\setminus Ba}]$</td>
</tr>
<tr>
<td>${Aa, b}$</td>
<td>$U_{Aa} = \frac{1}{2}[W_{\Omega\setminus B} + W_{\Omega\setminus Bb}]$</td>
</tr>
<tr>
<td></td>
<td>$U_{b} = \frac{1}{2}[W_{\Omega\setminus B} - W_{\Omega\setminus Bb}]$</td>
</tr>
</tbody>
</table>

Table (9): Payoffs with only one supplier and vertical merger.

If $I < \min\{U_B^{\{Aa,B,b\}},U_B^{\{A,B,a,b\}}\}$, the entrant supplier $B$ can cover its entry costs regardless whether firms $A$ and $a$ merge or not. Conversely, if $I > \max\{U_B^{\{Aa,B,b\}},U_B^{\{A,B,a,b\}}\}$, firm $B$ cannot make enough profits to cover its entry costs. Note that $I_s = U_B^{\{A,B,a,b\}}$ and $I_s = U_B^{\{Aa,B,b\}}$. Claim (i) is therefore straightforward.

Consider next Claim (ii). Two cases are possible: either $I_s \leq I \leq T_s$ or $T_s \leq I \leq I_s$. I first investigate when each of these conditions hold. Assume first that $I_s \leq I \leq T_s$. With $L_s = U_B^{\{A,B,a,b\}}$ and $T_s = U_B^{\{Aa,B,b\}}$, for the interval $[I_s, T_s]$ to be non-empty we must have $U_B^{\{A,B,a,b\}} < U_B^{\{Aa,B,b\}}$. Under symmetry, by plugging in the corresponding values from Table (1) this is equivalent to $\frac{1}{12}[2W_{\Omega\setminus r} - 2W_{\Omega\setminus s} + 3W_\Omega] < \frac{1}{6}[W_{\Omega\setminus r} - 2W_{\Omega\setminus s} + 2W_\Omega]$, which can be rearranged to get $2W_{\Omega\setminus s} < W_\Omega$. From Proposition 2 of IW, this relationship holds if the products are strict complements.

$U_B^{\{A,B,a,b\}} \leq I \leq U_B^{\{Aa,B,b\}}$ implies that a merger between supplier $A$ and retailer $a$ makes an otherwise unprofitable entry of supplier $B$ profitable. For the vertical merger to occur, it must also be profitable for the merging parties, i.e. we must have $U_A^{\{A,a,b\}} + U_a^{\{A,a,b\}} < U_{Aa}^{\{Aa,B,b\}}$. Plugging in the corresponding values from Tables (1) and (9) yields the profitability condition $W_{\Omega\setminus r} + 2W_\Omega > 3W_{\Omega\setminus s} + W_{\Omega\setminus sr}$. This relationship is fulfilled under the assumption $2W_{\Omega\setminus s} < W_\Omega$ (complements), which proves Claim (ii.a).

Assume now that $T_s \leq I \leq L_s$. With $L_s = U_B^{\{A,B,a,b\}}$ and $T_s = U_B^{\{Aa,B,b\}}$, for the interval $[T_s, L_s]$ to be non empty we must have $U_B^{\{A,B,a,b\}} > U_B^{\{Aa,B,b\}}$. Plugging in the corresponding values from Table (1) this relationship holds if $2W_{\Omega\setminus s} > W_\Omega$. From Proposition 2 of IW, this is the case if the products are strict substitutes. $U_B^{\{Aa,B,b\}} \leq I \leq U_B^{\{A,B,a,b\}}$ implies that a
merger between supplier A and retailer a renders the otherwise profitable entry of supplier B unprofitable and is therefore entry-deterring. For the vertical merger to occur, it must also be profitable for the merging parties, i.e. we must have \( U_{Aa}^{(Aa,b)} > U_a^{(A,B,a,b)} + U_A^{(A,B,a,b)} \). Plugging in the corresponding values from Tables (1) and (9) yields the profitability condition \( W_\Omega < W_{\Omega \setminus s} + W_{\Omega \setminus sr} \). This can be written as

\[
4p(q^\Omega, q^\Omega)q^\Omega - 2C(2q^\Omega) < [2p(q^{\Omega \setminus s}, 0)q^{\Omega \setminus s} - C(2q^{\Omega \setminus s})] + [p(q^{\Omega \setminus sr}, 0)q^{\Omega \setminus sr} - C(q^{\Omega \setminus sr})].
\]

Note that the above relationship remains valid if on the RHS we plug in \( q \) for \( q^{\Omega \setminus s} \) and \( q^{\Omega \setminus sr} \). Doing so and simplifying yields \( \Delta \Omega \geq 2\Delta p + (1/2)\Delta \Omega \). A vertical merger profitably deters upstream entry if this condition is fulfilled, where it is unprofitable if the opposite holds. Claim (ii.b) follows immediately.

Consider now the entry of a retailer. Assume w.l.o.g. that \( i_1 = A, i_2 = a, i_3 = B, e = b \), i.e. retailer b is the potential entrant, while supplier A and retailer a consider merging vertically. If b enters, the corresponding Stage 3 payoffs are contained in the rows \( \{Aa, B, b\} \) and \( \{A, B, a, b\} \) of Table (1). If b does not enter, applying the Shapley value yields the following payoffs, depending on firm A’s and a’s merger decision:

<table>
<thead>
<tr>
<th>Market structure</th>
<th>Payoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td>{A, B, a}</td>
<td>( U_A = \frac{1}{6}[2W_{\Omega \setminus b} + W_{\Omega \setminus Bb} - 2W_{\Omega \setminus Ab}] )</td>
</tr>
<tr>
<td></td>
<td>( U_a = \frac{1}{6}[2W_{\Omega \setminus b} + W_{\Omega \setminus Bb} + W_{\Omega \setminus Ab}] )</td>
</tr>
<tr>
<td></td>
<td>( U_B = \frac{1}{6}[2W_{\Omega \setminus b} - 2W_{\Omega \setminus Bb} + W_{\Omega \setminus Ab}] )</td>
</tr>
<tr>
<td>{Aa, B}</td>
<td>( U_{Aa} = \frac{1}{2}[W_{\Omega \setminus b} + W_{\Omega \setminus Bb}] )</td>
</tr>
<tr>
<td></td>
<td>( U_B = \frac{1}{2}[W_{\Omega \setminus b} - W_{\Omega \setminus Bb}] )</td>
</tr>
</tbody>
</table>

Table (10): Payoffs with only one retailer and vertical merger.

If \( I < \min\{U_b^{(Aa,B,b)}, U_{b}^{(A,B,a,b)}\} \), the entrant retailer b can cover its entry costs regardless whether firms A and a merge or not. Conversely, if \( I > \max\{U_b^{(Aa,B,b)}, U_{b}^{(A,B,a,b)}\} \), firm b cannot make enough profits to cover its entry costs. Note that \( L_r = U_b^{(A,B,a,b)} \) and \( T_r = U_{b}^{(Aa,B,b)} \). Claim (iii) is therefore straightforward.

Consider next Claim (iv). Again two cases are possible: either \( L_r \leq I \leq T_r \) or \( T_r \leq I \leq L_r \). I first investigate when each of these conditions hold. Assume first that \( L_r \leq I \leq T_r \). With \( L_r = U_b^{(A,B,a,b)} \) and \( T_r = U_{b}^{(Aa,B,b)} \), for the interval \([L_r, T_r] \) to be non-empty we must have
$U_b^{\{A,B,a,b\}} < U_b^{\{Aa,B,b\}}$. Under symmetry, by plugging in the corresponding values from Table (1) this is equivalent to $\frac{1}{12} [2W_{\Omega,s} - 2W_{\Omega,r} + 3W_\Omega] < \frac{1}{12} [2W_{\Omega,s} - 4W_{\Omega,s} + 4W_\Omega]$, which can be rearranged to get $2W_{\Omega,r} < W_\Omega$. From Proposition 2 of IW, this relationship holds if unit costs are strictly decreasing.

$U_b^{\{A,B,a,b\}} \leq I \leq U_b^{\{Aa,B,b\}}$ implies that a merger between supplier $A$ and retailer $a$ makes an otherwise unprofitable entry of retailer $b$ profitable. For the vertical merger to occur, it must also be profitable for the merging parties, i.e. we must have $U_A^{\{A,B,a\}} + U_a^{\{A,B,a\}} < U_{Aa}^{\{A,B,a\}}$.

Plugging in the corresponding values from Tables (1) and (10) yields the profitability condition $W_{\Omega,s} + 2W_\Omega > 3W_{\Omega,r} + W_{\Omega,sr}$. This relationship is fulfilled under the assumption $2W_{\Omega,r} < W_\Omega$ (unit costs strictly decreasing), which proves Claim (iv.a).

Assume now that $I < I^*$. With $I = U_b^{\{A,B,a,b\}}$ and $I^* = U_b^{\{Aa,B,b\}}$, for the interval $[I, I^*]$ to be non empty we must have $U_b^{\{A,B,a,b\}} > U_b^{\{Aa,B,b\}}$. Plugging in the corresponding values from Table (1) this relationship holds if $2W_{\Omega,r} > W_\Omega$. From Proposition 2 of IW, this is the case if the unit costs are strictly increasing. $U_b^{\{Aa,B,b\}} \leq I \leq U_b^{\{A,B,a,b\}}$ implies that a merger between supplier $A$ and retailer $a$ renders the otherwise profitable entry of retailer $b$ unprofitable and is therefore entry-deterring. For the vertical merger to occur, it must also be profitable for the merging parties, i.e. we must have $U_A^{\{Aa,B\}} > U_a^{\{A,B,a\}} + U_a^{\{A,B,a\}}$.

Plugging in the corresponding values from Tables (1) and (10) yields the profitability condition $W_\Omega < W_{\Omega,r} + W_{\Omega,sr}$. This can be written as

$$4p(q^\Omega, q^\Omega)q^\Omega - 2C(2q^\Omega) < [2p(q^\Omega, q^\Omega)q^\Omega, q^\Omega, q^\Omega, q^\Omega] + [p(q^\Omega, 0)q^\Omega, q^\Omega, q^\Omega, q^\Omega].$$

Note that the above relationship remains valid if on the RHS we plug in $q^\Omega$ for $q^\Omega,r$ and $q^\Omega, sr$. Doing so and simplifying yields $\Delta^\Omega \geq \Delta^\Omega / 2 + (1/4)\Delta^\Omega$. A vertical merger profitably deters downstream entry if this condition is fulfilled, where it is unprofitable if the opposite holds. Claim (iv.b) follows immediately. Q.E.D.
9 Appendix II: An Example

This section provides a simple, discrete example to illustrate and verify selected results derived earlier in this article. Assume that suppliers and retailers are symmetric, and each supplier can provide either one unit of the product at a retail outlet or none. Indirect demand for product \( s \) at retailer \( r \) is
\[
p_{sr} = \begin{cases} 
  p & \text{if } q_{sr'} = 0 \\
  p + \gamma & \text{if } q_{sr'} = 1 \\
  \forall s, r \in S^0 \times R^0,
\end{cases}
\]
with \( \gamma > 0 \) (\( \gamma < 0 \)) if products are complements (substitutes). The cost function of supplier \( s \) is given by
\[
C_s(q_{sr} + q_{sr'}) = \begin{cases} 
  0 & \text{if } q_{sr} + q_{sr'} = 0 \\
  c_1 & \text{if } q_{sr} + q_{sr'} = 1 \\
  2(c_1 + \kappa) & \text{if } q_{sr} + q_{sr'} = 2 \\
  \forall s, r \in S^0 \times R^0,
\end{cases}
\]
The resulting unit cost function is then
\[
\overline{C}_s(q_{sr} + q_{sr'}) = \begin{cases} 
  0 & \text{if } q_{sr} + q_{sr'} = 0 \\
  c_1 & \text{if } q_{sr} + q_{sr'} = 1 \\
  c_1 + \kappa & \text{if } q_{sr} + q_{sr'} = 2 \\
  \forall s, r \in S^0 \times R^0,
\end{cases}
\]
where \( \kappa > 0 \) (\( \kappa < 0 \)) captures increasing (decreasing) unit costs. I focus on the symmetric equilibrium, in which it is trivially always optimal for each supplier to provide one unit of the good to each retailer. The resulting industry surpluses under various market configurations are the following:
\[
\begin{align*}
W_{s,r} &= p - c_1, \\
W_{s} &= 2p - [2(c_1 + \kappa)], \\
W_{r} &= 2(p + \gamma) - 2c_1, \\
W &= 4(p + \gamma) - 2[2(c_1 + \kappa)].
\end{align*}
\]
Assumption 1 (superadditivity) and the exclusion of corner solutions imply, that the following restrictions need to hold:
\[
\begin{align*}
W_{\Omega \setminus sr} < W_{\Omega \setminus r} & \iff 0 < p - c_1 + 2\gamma \\
W_{\Omega \setminus sr} < W_{\Omega \setminus s} & \iff 0 < p - c_1 - 2\kappa \\
W_{\Omega \setminus s} < W_{\Omega} & \iff 0 < 2(p - c_1) - 2\kappa + 4\gamma \\
W_{\Omega \setminus r} < W_{\Omega} & \iff 0 < 2(p - c_1) - 4\kappa + 2\gamma \\
0 < W_{\Omega \setminus sr} & \iff 0 < p - c_1
\end{align*}
\] (17)

I first verify vertical merger incentives under full separation stated in Proposition 1 and Corollary 1. According to Proposition 1, supplier \(s\) and retailer \(r\) merge, if \((W_{\Omega \setminus sr} - W_{\Omega \setminus sr}) + W_{\Omega \setminus s} + W_{\Omega \setminus r} \geq W_{\Omega}\), which after plugging in the values from (16) becomes simply \(\kappa > \gamma\). This immediately verifies Corollary 1, according to which a vertical merger takes place if \(\Delta_C^\Omega > \Delta_P^\Omega\). Note that from Definition 3 in this example \(\Delta_C^\Omega = \gamma \) and \(\Delta_P^\Omega = \kappa\), for every \(\Omega\).

I next turn to vertical integration and entry, and focus on the case where entry can occur upstream, with one supplier and two retailers being incumbent. Let \(s'\) be the potential entrant and supplier \(s\) as well as retailer \(r\) consider vertical integration to deter or facilitate upstream entry. The game unfolds according to Figure (4): In Stage 1, incumbents \(s\) and \(r\) decide whether to merge or stay separated. In Stage 2, the potential entrant \(s'\) decides whether to enter the market or stay out. In Stage 3 firms bargain on the delivery conditions and payments are made.

A vertical merger can deter the entry of a supplier if \(U_{s'}^{(sr,s',r')} < I < U_{s'}^{(s,s',r,r')}\). Using Table 1 this corresponds to

\[
W_{\Omega \setminus r}/6 - W_{\Omega \setminus s}/3 + W_{\Omega}/3 < I < W_{\Omega \setminus r}/6 - W_{\Omega \setminus s}/6 + W_{\Omega}/4.
\] (18)

Note, that this interval is non-empty if \(2W_{\Omega \setminus s} > W_{\Omega}\). As was discussed above, this relationship holds if products are substitutes. We can plug in the values from Expression (16) into (18) to get the interval of entry costs for which a vertical merger in Stage 1 deters entry as

\[-2\kappa/3 - c_1 + 5\gamma/3 + p < I < -2\kappa/3 - c_1 + 4\gamma/3 + p.\]

A vertical merger to deter the entry of a supplier is profitable if

\[
U_{sr}^{(sr)} - \left[U_{s}^{(s,s',r,r')} + U_{s'}^{(s,s',r,r')}\right] > 0.
\] (19)
Using the values from Tables (1) and (4), this equivalent to $W_{s,s} + W_{sr} > W_\Omega$, which after plugging in Expression (16) becomes

$$\kappa > (p - c_1)/2 + 2\gamma.$$ 

Observe that this corresponds to the condition stated in Proposition (4), Claim (II.b).

A vertical merger fosters supplier entry if $U_{s,s'} < I < U_{s',s'}$, i.e. if $W_{s,s}/6 + W_{s}/4 < I < W_{s,s}/6 - W_{s}/3 + W_{s}/3$. This interval is non-empty if $2W_{s,s} < W_\Omega$, and therefore, as discussed above, if products are complements. Plugging in Expression (16) yields the interval of entry costs in which a merger can foster entry as

$$-2\kappa/3 - c_1 + 4\gamma/3 + p < I < -2\kappa/3 - c_1 + 5\gamma/3 + p.$$ 

A vertical merger that enables supplier $s'$ to enter is profitable if

$$U_{s,s',s'} - \left[U_{s,s} + U_{s',s'}\right] > 0. \quad (20)$$

This can be simplified to get $W_{s,s} + W_{s,s} > W_{s,s'}$, which always holds due to superadditivity.

Table (6) contains examples for parameter values for each region in Figure (4). It is assumed, that entry may occur upstream while the incumbent supplier and a retailer consider merging vertically. For the example I take $p = 1$ and $c_1 = 1/8$, implying that $\Delta = p - c_1 = 7/8$.

<table>
<thead>
<tr>
<th>Figure (4) region</th>
<th>Potential effect of the merger</th>
<th>Benefit/loss from merging</th>
<th>$\gamma$</th>
<th>$\kappa$</th>
<th>Expression (19) or (20)</th>
<th>$I \in [\ldots]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>deter</td>
<td></td>
<td>$-1/10$</td>
<td>$1/4$</td>
<td>$1/85$</td>
<td>$[13/23, 23/40]$</td>
</tr>
<tr>
<td>2</td>
<td>deter</td>
<td></td>
<td>$-1/10$</td>
<td>$1/10$</td>
<td>$-1/80$</td>
<td>$[77/120, 27/40]$</td>
</tr>
<tr>
<td>3</td>
<td>foster</td>
<td></td>
<td>$1/10$</td>
<td>$1/10$</td>
<td>$137/230$</td>
<td>$[113/120, 29/40]$</td>
</tr>
<tr>
<td>4</td>
<td>deter</td>
<td></td>
<td>$-44/160$</td>
<td>$-1/10$</td>
<td>$1/85$</td>
<td>$[29/60, 23/40]$</td>
</tr>
<tr>
<td>5</td>
<td>deter</td>
<td></td>
<td>$-1/10$</td>
<td>$-1/10$</td>
<td>$-27/80$</td>
<td>$[31/40, 27/40]$</td>
</tr>
<tr>
<td>6</td>
<td>foster</td>
<td></td>
<td>$1/10$</td>
<td>$-1/10$</td>
<td>$51/80$</td>
<td>$[43/40, 133/120]$</td>
</tr>
</tbody>
</table>

$p = 1, c_1 = 1/8, \Delta = p - c_1 = 7/8.$

Table (6): Example for upstream entry and vertical merger incentives.
The same exercise can be performed for the case where entry can occur downstream. Consider retailer \( r' \) as the potential entrant while suppliers \( s \) and \( s' \) as well as retailer \( r \) are incumbent. A vertical merger between the incumbent retailer \( r \) and supplier \( s \) is entry-deterring if \( U_{sr}^{(s,s',r')} < I < U_{sr}^{(s,s',r')}, \) whereas it fosters entry if \( U_{sr}^{(s,s',r')} < I < U_{sr}^{(s,s',r')} \). The respective conditions for a vertical merger that deters or fosters entry to be profitable are

\[
U_{sr}^{(sr,s')} - \left[ U_{sr}^{(s,s',r')} + U_{sr}^{(s,s',r')} \right] > 0 \quad (21)
\]

and

\[
U_{sr}^{(sr,s')} - \left[ U_{sr}^{(s,s')} + U_{sr}^{(s',s')} \right] > 0. \quad (22)
\]

Table (7) provides a similar example for each region in Figure (5), when entry may occur downstream while the incumbent supplier and a retailer consider merging vertically.

<table>
<thead>
<tr>
<th>Figure (5) region</th>
<th>Potential effect of the merger</th>
<th>Benefit/loss from merging</th>
<th>Expression (21) or (22)</th>
<th>( I ) in [..,..]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>deter</td>
<td>( \gamma )</td>
<td>( \kappa )</td>
<td>( [23,47] )</td>
</tr>
<tr>
<td>2</td>
<td>deter</td>
<td>( \frac{1}{10} )</td>
<td>( \frac{10}{32} )</td>
<td>( [101,21] )</td>
</tr>
<tr>
<td>3</td>
<td>deter</td>
<td>( \frac{1}{10} )</td>
<td>( \frac{1}{10} )</td>
<td>( [27,40] )</td>
</tr>
<tr>
<td>4</td>
<td>deter</td>
<td>( \frac{1}{10} )</td>
<td>( \frac{1}{10} )</td>
<td>( [31,97] )</td>
</tr>
<tr>
<td>5</td>
<td>foster</td>
<td>( \frac{1}{10} )</td>
<td>( \frac{1}{10} )</td>
<td>( [137,240] )</td>
</tr>
<tr>
<td>6</td>
<td>foster</td>
<td>( \frac{1}{10} )</td>
<td>( \frac{1}{10} )</td>
<td>( [43,133] )</td>
</tr>
</tbody>
</table>

\[ p = 1, c_1 = 1/8, \Delta = p - c_1 = 7/8. \]

Table (7): Example for downstream entry and vertical merger incentives.

The benefits from merging in Tables (6) and (7) can be compared with the corresponding prediction for a vertical merger to take place in Tables (4) and (5), respectively. With possible entry occurring upstream, a vertical merger takes place for the parameter combinations in regions 1,3,4 and 5 of Figure (4). If entry can occur downstream, a vertical merger is profitable for the parameter combinations 1,2,5 and 6 of Figure (5). The example therefore complies with the general theoretical results derives in the previous Sections.
References


