Consumer Search and Price Discrimination*

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2013
Preliminary Draft

Abstract

We compare the case where a monopolist searches for consumers (firm search), to the case where the firm induces consumers to search for products (consumer search). A consumer's preference is private information. We show that, for small or large search costs, firm search is optimal for the firm. For intermediate search costs, however, consumer search can be optimal—by inducing only the consumers with high-preference to search for the products, the firm can price-discriminate more effectively, thus reducing distortions in the optimal outcome. We extend our analysis to the case where the firm can make the full menu available (using different sales channels) to only those who make search efforts. Then, consumer search can be optimal for the firm even for large search costs.

JEL Classification: D82, L11, M21

Key words: Second-Degree Price Discrimination, Screening, Consumer Search.

*We thank Jay Pil Choi, Thomas Jeitschko, Ching-to Albert Ma, Subir Chakrabarti, Henry Mak, Arijit Mukherjee and Huanxing Yang for helpful comments and partcipants at Midwest Economic Theory Meetings, IUPUI, and Michigan State University.

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1 Introduction

In many cases, the likelihood of purchase is increasing in costly effort by either a seller or a buyer. Consider that the purchase can be made only either when the consumer has to visit a store, or when the firm delivers the product to the consumer. Without effort from either side, there may not be market transaction. Consider also skill-based products that require learning how to use. People sometimes need to understand product features and functionalities before purchasing products such as consumer electronics, personal computers, and software applications. For example, although a consumer understands why Apple Mac computers and Mathematica software are useful and places a high value on them, she may not make a purchase without knowing how to use it. Again, the consumer's learning effort or the firm's instructing effort must be made. Consumers expense their own resources and considerable time to know all the features of a product and learn how to use it. On the other hand, firms may operates customer-help-centers to explain product features to consumers, offer free trials of their products, or provide financial incentives for consumers to learn their product features.[1]

To investigate how this friction affects the firm's pricing decision and selling strategy, we look at the following stylized situation. A firm can adopt different approaches to sell a product to its consumers. One way is to directly bear a visiting (transportation) cost of consumers by delivery, or using sales force to inform the consumers how to use the product. The other way is to make the consumers pay search (or learning) costs to purchase the product. In the latter case, the firm does not expend its resources to help the consumers, but instead, it has to indirectly compensate the consumers for their search costs.[2] We will refer to the former strategy as "firm search" and the latter strategy as "consumer search" for the presentation purpose.

The objective of this paper is to see why and when one strategy dominates the other, thus providing a rationale for inducing consumer search for products. We

[1] For instance, university professors are frequently offered a learning opportunity of course management systems with monetary compensation.

[2] The premise that purchasing a product requires incurring the search cost has been adopted by Anderson and Renault (2006), Armstrong and Vickers (2010), etc. In addition, we also allow a firm to pay this cost.

show that a firm can use consumer search as a device to price-discriminate more effectively when a consumer's preference is private information. According to our result, when the search cost is *small or large*, it is optimal for a firm to engage in search for the consumers. When the search cost is *intermediate*, by contrast, it is optimal for the firm to make the consumer search for the product. In addition, the optimal arrangement in the latter case is to induce only the high-preference consumers to engage in the search activity. Since the firm's sales to a low-preference consumer is realized with a smaller probability under such an arrangement, the key trade-off in our paper is '*information extraction* vs. *loss of sales opportunities*.'

We employ a monopoly framework with a second degree price discrimination. The firm can search for consumers for its product, or induce the consumers to search for the product.[3] A consumer's preference (type) can be either high or low in our model, and is her private information. As usual in the model of this type, a high-type consumer has an incentive to mimic a low-type consumer to reap information rent. When the firm searches for the consumer, the optimal outcome for the firm is accompanied by the standard downward distortion in the sales volume for the low-type consumers.

Inducing the consumer to incur the search cost has two merits. First, because a consumer knows her type, unlike when the firm searches for the consumers, making the consumers search allows the firm to (indirectly) incur the search cost only for the consumers of a particular type—as mentioned above, the firm makes only the high-type consumers to engage in search for the product at the optimum. Second, and more importantly, by inducing only the high-type consumers to incur the search cost, the firm can mitigate a consumer's the mimicry incentive, but only when the search cost is intermediate.

When the firm induce only the high-type consumers to search for the product, only the high-type consumers are compensated for their search costs. The firm does so by offering an extra discount only for the high-type consumers (to be exact, only for the consumers choosing the price-quantity bundle for the high-type). Such a discount discourages the high-type consumers from mimicking the

[3]For simplicity, we assume that the firm's search cost to find a consumer and a consumer's search cost to find the product are the same. Different search costs do not change our results if the gap is not too large.

low-type consumers. On the other hand, since the low-type consumers are not induced to make search efforts, the firm's sales to the low-type consumers are realized with a lower probability. Therefore, when the search cost is small, the amount of discount for the high-type consumers is not large enough, and it is better for the firm to directly search for the consumers for a higher sales probability.[4]

As the search cost becomes larger, however, the incentive effect from the discount effect begins to dominate. When the search cost is intermediate, while a high-type consumer can obtain information rent by misrepresenting her type, she is reluctant to do so because she then does not receive the discount. In other words, inducing consumer search allows the firm to create "countervailing incentives" in this range of the search cost. As a result, the firm can price-discriminate more effectively for rent extraction, which in turn allows it to recover the distortion in the optimal sales volume. We show that inducing consumer search can be optimal when the search cost is in the intermediate range.

When the search cost is large, the discount for the high-type consumers for their search effort becomes so large that another incentive problem arises. In particular, if a low-type consumer finds the product (without a search effort), she will mimic the high-type to take advantage of the large discount. To prevent such a mimicry, the firm makes the high-type consumers over-purchase the product. As the reverse incentive problem becomes an issue, it is optimal for the firm to directly incur the search cost when the search cost is large.

Our results mentioned above assume that, when inducing consumer search, the firm offers a 'full menu' to a consumer regardless of the consumer's effort—when a consumer finds the product, she always faces the price-quantity bundles for both types (the full menu strategy). We extend our analysis to the case in which the firm can offer limited menus by setting up different sales channels with respect to a consumer's search effort—the search channels and the regular channels. The firm now offers the price-quantity bundle for the high-type consumers only through the search channels. This can be thought of as hidden discounts or special deals not advertised. For the case of skill-based products, the limited menu is comparable

[4]In a different model, both the firms and the consumers may engage in search at the optimum. It, however, will simply add more cases to our problem without altering the main insight.

to making the product for the high-type consumers complicated. This is more like a professional or mania version of products. Then, we show that inducing consumer search can be optimal even when the search cost is large.

The current paper is most closely related to the literature on second degree price discrimination in monopoly frameworks, pioneered by Mussa and Rosen (1978) and Maskin and Riley (1984). Studies that analyze situations in which consumers' product information include the following papers. Lewis and Sappington (1994) analyze the trade-off between price discrimination versus information rent. In their paper, the firm can provide more information about the product to a consumer for price discrimination, but such an activity makes the consumer realize her private information. Ottaviani and Prat (2001) show that committing to make a consumer's private information public can mitigate the consumer's misrepresenting incentive. Using a aggregate demand function, Johnson and Myatt (2006) extends the first paper to the case where the firm offers a product with different qualities.[5] Unlike our papers, the cost of information is incurred by the firm in these papers.[6] In our paper, the party that directly incurs the search cost is endogenously determined.

Finally, our paper is related to the studies on "countervailing incentives." In their seminar work, Lewis and Sappington (1989) demonstrate that presence of countervailing incentives can improve the principal's welfare. Jullien (2000) provides a general analysis of type-dependent participation constraints with a continuum of types. The optimal mechanism with countervailing incentives and its benefit is applied in our paper. We show that inducing consumers to incur search costs, instead of the firm incurring such costs, generates countervailing incentives, which helps the firm extract a consumer's information rent.

The remainder of the paper is organized as follows. We set up the model in Section 2. In Section 3, the firm's optimal outcome from searching for consumers is presented. The firm's optimal outcome from inducing consumer search is discussed in Section 4. In Section 5, we compare the two cases. In Section 6, we extend our analysis to the case where the firm offers limited menus through differ-

[5] Bar-Issac et al. (2010) study informing consumers without price discrimination.

[6] Persico (2000) analyzes a situation in which the cost of information is incured by consumers.

ent channels. We conclude in Section 7.

2 Model

A monopolist designs a product with new features $q \in \mathbb{R}_+$ and offers it at a price, $p \in \mathbb{R}_+$ to consumers. The population of consumers, for simplicity, is normalized to one. The consumer's preference toward the product is denoted by $i \in \{H, L\}$, where H (L) represents high (low) preference, and $\Delta \equiv H - L > 0$. With a probability φ_i, the consumer is type-i and $\sum_i \varphi_i = 1$. We assume that φ_i is not too small or too large that the firm does not exclude any type. The consumer's type is private information, but the probability distribution is common knowledge. As the revelation principle applies, q_i and p_i denote the quality and the lump sum price that the firm offers to type-i consumer.

Type-i consumer values the product with a function $u(q_i, i)$ that is strictly increasing and concave in q_i with $u(0, i) = 0$. The value function also satisfies:

$$u^{\Delta}(q_i) \equiv u(q_i, H) - u(q_i, L) > 0 \quad \text{and} \quad u_q^{\Delta}(q_i) \equiv u_q(q_i, H) - u_q(q_i, L) > 0.$$

Manufacturing the product of quality q_i costs cq_i to the firm, where $c > 0$. The firm's profit and type-i's consumer's payoff from a transaction are respectively:

$$\Pi_i = p_i - cq_i \quad \text{and} \quad U_i = u(q_i, i) - p_i.$$

A search can increase the likelihood that a transaction occurs, and can be organized in the following two ways. The firm can search for the consumer ($\Psi = F$), or alternatively, induce the consumer to search for the product ($\Psi = C$). We denote by $e \in \{0, 1\}$ the effort level of the searching party, depending on $\Psi \in \{F, C\}$. The probability of purchase is given by:

$$\gamma(e) = e + (1 - e)\beta, \quad \beta \in (0, 1).$$ [7]

A transaction can take place when either the firm or the consumer conducts a search ($e = 1$ thus $\gamma = 1$), or the consumer finds the product by accident ($e = 0$

thus $\gamma = \beta$). For example, a well-informed friend may explain the product features. The search cost is given by se for all parties, where $s > 0$.

The timing is as follows:

- The consumer's type $i \in \{H, L\}$ is realized and privately observed by her.
- The firm decides $\Psi \in \{F, C\}$.
- The firm commits to the menu of its offers, $\{q_H, p_H : q_L, p_L\}$, by sending a public advertisement.
- Depending on $\Psi \in \{F, C\}$, either the firm or the consumer makes the search effort $e \in \{0, 1\}$.
- A transaction takes place depending on $\gamma(e)$.

Benchmark: As a benchmark, we present the optimal outcome under full information. The first-best product quality, denoted by q_i^*, is characterized by the following equation:

$$u_q(q_i^*, i) = c, \quad i \in \{H, L\}.$$

Under full information, the optimal outcome satisfies "marginal benefit = marginal cost," with the perfect price discrimination, i.e., the firm leaves no consumer surplus: $p_i^* = u(q_i^*, i)$.

3 The Firm Makes a Searching Effort

In this section, we discuss the firm's optimal offers when the firm searches for the consumer. Since the firm does not know the consumer's type when it decides whether or not to exert a search effort, the firm's choice of search effort is:

$$e = \underset{\widehat{e} \in \{0,1\}}{\arg\max} \gamma(\widehat{e}) \left[\sum_i \varphi_i \left(p_i - c q_i \right) \right] - s\widehat{e}$$

Provided that s is not prohivitively large, a search always takes in this case. Throughout this paper, we will assume that the size of s is not prohibitively large—i.e., $e = 1$

when the firm searches for the consumer. With $e = 1$, the firm's problem becomes a standard non-linear pricing that screens the consumer's type.

With $\Psi = F$, the firm solves the following problem:

$$\underset{q,p}{Max} \sum_i \varphi_i \left(p_i - cq_i\right) - s, \tag{$\mathcal{P}^F$}$$

subject to,

$$u(q_i, i) - p_i \geq 0, \quad i \in \{H, L\}, \tag{PC}$$

$$u(q_i, i) - p_i \geq u(q_j, i) - p_j, \quad i, j \in \{H, L\}. \tag{IC}$$

The first constraints, (PC), is the participation constraint for the consumer, and the second constraints, (IC), assures that the consumer's payoff is higher when she truthfully represents her type.

The following proposition presents the firm's optimal offer when it searches for the consumer.

Proposition 1 *With $\Psi = F$, the the firm's optimal offers are characterized as follows:*

$$q_H^F = q_H^* \quad \textit{and} \quad q_L^F < q_L^*,$$

$$p_H^F < p_H^* \quad \textit{and} \quad p_L^F < p_L^*.$$

The result above is standard. The product quality for type-H consumer is at the first best level, known as "efficiency at the top" in the literature, but the product quality for type-L consumer is distorted downwards. As usual in the model of this type, type-H consumer has an incentive to mimic type-L consumer to reap information rent of $u^{\Delta}(q_L)$. The firm discourages such a mimicry by distorting the product quality for type-L consumer downwards. As a result, the firm must reduce both p_H and p_L from the first best levels ($p_H < p_H^*$ and $p_L < p_L^*$), resulting in an imperfect price discrimination.

4 The Consumer Makes a Searching Effort

We now move on to the case in which the firm induces the consumer to make a search effort. The key difference from the case in the previous section comes from the fact that, when the consumer searches for the product, the firm can induce only a particular type to make a search effort. In other words, the firm manipulates its offers such that the consumer of one type actively searches for the product by incurring the search cost, whereas the other type find the product by accident.

We first establish the following lemma.

Lemma 1 *If $\Psi = C$ ever dominates $\Psi = F$, then the firm induces only type-H consumer to make a search effort.*

It is not difficult to see that inducing both type-H and type-L consumer to incur the search cost simply makes it more costly to the firm, compared to the case in which the firm searches for the consumer. Since type-L consumer gets zero consumer surplus from her purchase, she has no incentive to incur the search cost to find the product. Therefore, to incentivize type-L consumer, the firm must provides her with a strictly positive consumer surplus. Such a surplus to type-L consumer, however, makes type-H consumer misrepresent her preference. As a result, the firm must provide the additional surplus to type-H consumer as well.

Similarly, it is suboptimal for the firm to induce only type-L consumer to make a search effort. To make type-L consumer search for the product, the firm must compensate for the search cost by decreasing the price, which attracts type-H consumer to misrepresent her type as type-L. Again, the firm will simply end up providing more consumer surplus under this arrangement than when it directly searches for the consumer.

Type-H consumer's problem when deciding whether to make a search effort is:

$$\max\{u(q_H, H) - p_H - s, \quad \beta\,[u(q_H, H) - p_H]\},$$

implying that the firm's optimal offer for type-H consumer must satisfy:

$$u(q_H, H) - p_H \geq \frac{s}{1-\beta}. \tag{SC}$$

Since consumer surplus must be non-negative, the firm's maximization problem must satisfy the following participation constraints for the consumer:

$$u(q_H, H) - p_H - s \geq 0, \tag{PC_H}$$

$$\beta\left[u(q_L, L) - p_L\right] \geq 0. \tag{PC_L}$$

Finally, as the revelation principle applies in our model, the firm's offer must satisfy the incentive constraints for the consumer's truthful behavior:

$$u(q_H, H) - p_H - s \geq \max\left\{\begin{array}{c} u(q_L, H) - p_L - s, \\ \beta\left[u(q_L, H) - p_L\right] \end{array}\right\}, \tag{IC_H}$$

$$\beta\left[u(q_L, L) - p_L\right] \geq \max\left\{\begin{array}{c} u(q_H, L) - p_H - s, \\ \beta\left[u(q_H, L) - p_H\right] \end{array}\right\}. \tag{IC_L}$$

The constraints above assure that the consumer's payoff from truthfully representing her type (the left hand sides) is higher than her payoff from misrepresentation (the right hand sides). The right hand sides of (IC_H) and (IC_L) exhibit the consumer's choice of whether or not to search for the product if she decides to misrepresent her type.

When inducing only type-H consumer to incur the search cost, the firm's problem is:

$$\underset{q,p}{Max}\ \varphi_H\left(p_H - cq_H\right) + \beta\varphi_L\left(p_L - cq_L\right), \tag{$\mathcal{P}^C$}$$

subject to $(SC) \sim (IC_L)$. Notice that (SC) implies (PC_H).

We distinguish three different regimes for the optimal outcome when the firm makes only type-H consumer to search for the product. To characterize the optimal outcomes in each regime, we first present the following cutoff levels of the search cost.

Definition 1 *Let* $\underline{s} \equiv (1-\beta)u^{\Delta}\left(q_L^*\right)$ *and* $\overline{s} \equiv (1-\beta)u^{\Delta}\left(q_H^*\right)$.

The following proposition characterizes the firm's optimal offers.

Proposition 2 *With* $\Psi = C$*, the firm's optimal offers are characterized as follows:*

- *When* $s < \underline{s}$*:* $\quad q_H^C = q_H^F$ *and* $q_L^C \lesseqgtr q_L^F$, $\quad p_H^C \gtreqless p_H^F$ *and* $p_L^C \lesseqgtr p_L^F$.
- *When* $s \in [\underline{s}, \bar{s}]$*:* $\quad q_i^C = q_i^* \ \forall i$, $\quad p_H^C = p_H^* - \frac{s}{1-\beta}$, *and* $p_L^C = p_L^*$.
- *When* $s > \bar{s}$*:* $\quad q_H^C > q_H^F$ *and* $q_L^C = q_L^*$, $\quad p_H^C < p_H^F$ *and* $p_L^C \gtreqless p_L^F$.

When the search cost is sufficiently small, type-H consumer's information rent $u^{\Delta}(q_L)$ is large enough that the firm does not need to compensate her for the search cost, and consequently, $p_H^C > p_H^F$. Yet, type-L consumer's lack of search effort leads to a larger the quality distortion in the product for her compared to the one with $\Psi = F$ ($q_L^C < q_L^F$). As a result, the price for type-L consumer also becomes lower ($p_L^C < p_L^F$). As the search cost increases, the firm must incentivize type-H consumer to make a search effort by providing a discount ($p_H^C < p_H^F$). This extra consumer surplus is provided only when type-H consumer truthfully represents her type, allowing the firm to recover some of distortion in the optimal product quality for type-L consumer ($q_L^C > q_L^F$). As a result, the price for type-L consumer becomes higher than when the firm searches for the consumer ($p_L^C > p_L^F$).

As the search cost increases, while it becomes more costly to induce type-H consumer to make a search effort, her private information becomes less of a problem. Within an intermediate range, the firm's extra discount to type-H consumer to induce her search effort is large enough that the consumer no longer has an incentive to misrepresent her type as type-L. That is, the search cost is large enough that type-L consumer also has an incentive to misrepresent her type as type-H, and the misrepresenting incentives of opposite directions cancel out each other, i.e., "countervailing incentives" arise. Consequently, the firm does not need to distort the product qualities in its optimal offer to extract the consumer's surplus associated with her private information ($q_i^C = q_i^*$, $\forall i$). Within this range of the search cost, although the firm must give type-H consumer a discount for her search effort, the firm's price discrimination is efficient.

As the search cost becomes yet larger, so becomes the discount to type-H consumer, which leads to a "reverse incentive" problem. In this regime, type-H consumer has no misrepresenting incentive, but type-L consumer has such an incentive when she finds the product by chance. As a result, the optimal product quality for type-H consumer is distorted upward ($q_H^C > q_H^F = q_H^*$). The excessive increase in the product quality for type-H also increases the price, which in turn discourages type-L consumer from misrepresenting her type as type-H. While there is no downward distortion in the product quality for type-L consumer (this contributes positively to the price for type-L consumer), the firm must also give type-L consumer a discount to prevent her mimicry. As a result, the price for type-L consumer can be higher or lower compared to the case with $\Psi = F$ ($p_L^C \gtreqless p_L^*$).

In summary, making the consumer search for the product, instead of the firm searching for the consumer, brings about different incentive problems according to the size of the search cost. In the next section, we examine pros and cons of each strategy, and endogenize the firm's choice of $\Psi \in \{F, C\}$ to determine its optimal strategy.

5 The Firm's Optimal Strategy

By directly searching for the consumer ($\Psi = F$), the firm can sell its product with a higher probability, since a search always takes place. This, however, implies that the search cost is always incurred under such an arrangement, regardless of the consumer's type. Moreover, when the firm searches for the consumer, it must always provide type-H consumer with strictly positive information rent.

Making the consumer search for the product ($\Psi = C$) brings more flexibility to the firm — it allows the firm to incur the search cost depending on the consumer's type (inducing only type-H consumer at the optimum). Such a flexibility allows the firm to partially save the search cost, but more importantly, it has a strategic benefit. In particular, when the search cost is not too small or large, making the consumer search for the product brings about the consumer's countervailing incentives, which enables the firm to extract the consumer's information rent.

In the following proposition, we present the firm's optimal sales strategy depending on the search cost.

Proposition 3 *Suppose $\Delta \equiv H - L$ is large enough:*

- *When s is small, $\Psi = F$ is optimal.*
- *When s is intermediate, $\Psi = C$. is optimal.*

- *When s is large,* $\Psi = F$ *is optimal.*

When the search cost is small, directly searching for the consumer is more attractive to the firm since the probability that a transaction takes place is higher. When the search cost is in the intermediate range, making only type-H consumer search for the product enables the firm to extract the consumer's information rent, leading to more efficient price discrimination. This effect dominates when the difference in the consumer's preferences is large enough, and consequently, the firm prefers making the consumer search for the product. When the search cost is large, however, such a strategic merit vanishes since making the consumer search brings about the reverse incentive problem. In such a case, the same discount to type-H consumer for her search effort must also be provided to type-L consumer. Consequently, it becomes too costly to induce the consumer to make a search effort, and the firm prefers searching for the consumer to inducing only type-H consumer search for the product.

6 Limited Menu Strategy

So far, we have considered the case in which the firm always offers the full menu, $\{q_H, p_H : q_L, p_L\}$, to the consumer. This, in fact, must be the case when the firm searches for the consumer ($\Psi = F$). When inducing the consumer's search effort ($\Psi = C$), however, the firm has more flexibility in its strategy. We distinguish two different strategies when $\Psi = C$. The first one is what has been discussed—offering $\{q_H, p_H : q_L, p_L\}$ regardless of the consumer's search effort. We refer this to "the full menu strategy."

The second one is the strategy that makes $\{q_H, p_H\}$ available only for the consumer who makes a search effort ($e = 1$), while making $\{q_L, p_L\}$ available for the consumer regardless of the consumer's effort level. For example, the firm can set up different purchasing channels for the consumer: "The search channel" through which $\{q_H, p_H\}$ is offered only for a consumer who makes an effort to find the sales channel. We refer this to "the limited menu strategy ($\Psi = \widetilde{C}$)."

Figure 1 below illustrates the difference between two cases: $\Psi = C$ and $\Psi = \widetilde{C}$.

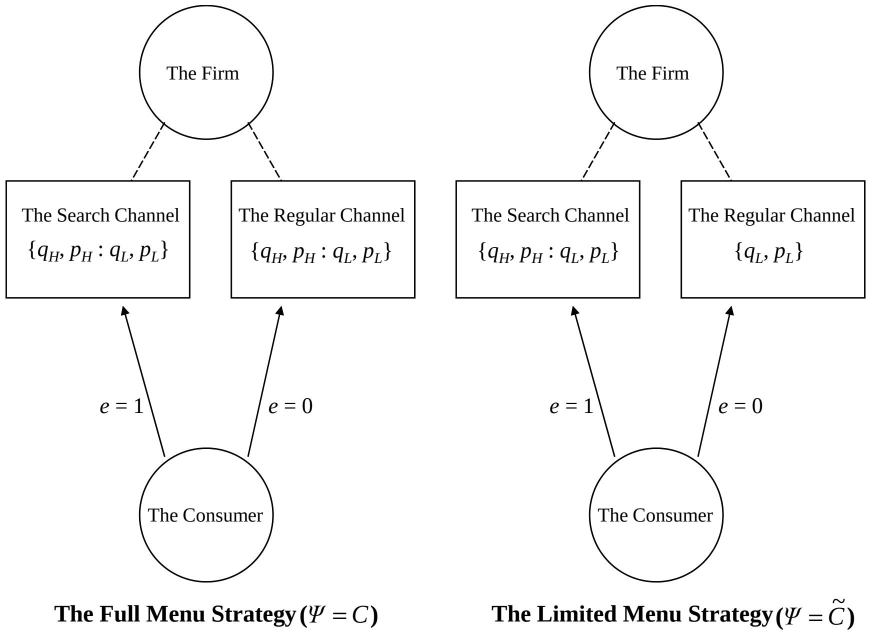

Fig 1. The Full Menu and The Limited Menu Strategy

As illustrated in Figure 1, the difference between the full menu and the limited menu strategy is clear. Unlike the full menu strategy, the limited menu strategy does not allow type-L consumer to mimic type-H without making a search effort. hen purchasing through the search channel, or type-L consumer to mimic type-H consumer when purchasing through the regular channel. This leads to two changes. First, type-H consumer, if she decides not to make a search effort, only the low-quality product is available for her (as before, the chance that she finds the product is β in such a case). Therefore, the search constraint becomes:

$$u(q_H, H) - p_H - s \geq \beta \left[u(q_L, H) - p_L \right]. \qquad (\widetilde{SC})$$

Second, type-L consumer must incur the search cost if she decides to mimic type-H. Therefore, instead of (IC_L) in the full menu strategy, the firm must satisfy the following incentive constraint for type-L consumer's truthful behavior:

$$\beta\left[u(q_L, L) - p_L\right] \geq u(q_H, L) - p_H - s. \qquad (\widetilde{IC}_L)$$

The firm maximizes its expected payoff in $(\mathcal{P}^C)$, subject to (PC_H), (PC_L), $(\widetilde{SC})$, (IC_H) and $(\widetilde{IC}_L)$. To characterize the optimal outcomes in each regime, we first present the following cutoff levels of the search cost.

Definition 2 *Let* $\widetilde{s} \equiv (1-\beta)u^{\Delta}(q_L^F)$.

The firm's optimal offers are characterized in the next proposition.

Proposition 4 *With* $\Psi = \widetilde{C}$, *the firm's optimal offers with the limited menu strategy are characterized as follows:*

- *When* $s < \widetilde{s}$*:* $\widetilde{q}_H^C = q_H^F$ *and* $\widetilde{q}_L^C < q_L^F$, $\widetilde{p}_H^C < p_H^F$ *and* $\widetilde{p}_L^C < p_L^F$.
- *When* $s \geq \widetilde{s}$*:* $\widetilde{q}_H^C = q_H^F$ *and* $\widetilde{q}_L^C = q_L^F$, $\widetilde{p}_H^C \geq p_H^F$, *and* $\widetilde{p}_L^C = p_L^F$.

When s is small enough, the effect is simiar to the full menu strategy. Type-H consumer's information rent is large enough relative to s that the firm does not need to compensate her for the search cost, which leads to $\widetilde{p}_H^C > p_H^F$. Type-$L$ consumer does not make a search effort, which leads to leads to a larger the quality distortion in the product for her compared to the case with firm search ($\widetilde{q}_L^C < q_L^F$). As a result, the price for type-L consumer becomes lower ($\widetilde{p}_L^C < p_L^F$) as well.

As s becomes larger, the firm must incentivize type-H consumer to make a search effort by providing a discount. However, the effect of the limited menu strategy's effect becomes different from the effect of the full menu strategy. First, under the full menu strategy, type-H consumer can truthfully reveal her type regardless of her search effort. Under the limited menu strategy, if type-H consumer decides not to make a search effort, she cannot reveal her true type (only the product for type-L consumer is offered through the regular channel). Second, under the full menu strategy, type-L consumer has an incentive to mimic type-H consumer (without making a search effort) when s is large. This, however, is not the case under the limited menu strategy. Type-L consumer must incur the search cost to mimic type-H. These two effects together make the product quality levels identical to those in firm search ($\widetilde{q}_H^C = q_H^F$ and $\widetilde{q}_L^C = q_L^F$). The only difference from firm search is that p_H is higher ($\widetilde{p}_H^C > p_H^F$ for $s > \widetilde{s}$) since the firm must compensate the consumer for her search effort.

The following proposition presents the firm's optimal strategy when the limited menu strategy is available.

Proposition 5 *Suppose $\Delta \equiv H - L$ is large enough:*

- *When s is small, $\Psi = F$ is optimal.*
- *When s is intermediate, $\Psi = C$. is optimal.*
- *When s is large, $\Psi = \widetilde{C}$ is optimal.*

With the limited menu strategy, type-L consumer's incentive to mimic type-H is not an issue since the high-quality product is available only through the search channel. This has both a negative and a positive side. The negative side is that the consumer's "countervailing incentives" under the full menu strategy vanishes under the limited menu strategy—as mentioned above, the limited menu strategy does not allow type-H consumer to reveal her true type without making a search effort. As a result, with the limited menu strategy, the firm cannot achieve the first best product qualities for both types when the search cost is intermediate. The positive side is that the limited strategy eliminate the "reverse incentive" when the search cost is large. For large search costs, therefore, the limited menu strategy allows the firm to avoid giving a discount to type-L consumer without an upward distortion.

In summary, for intermediate search costs, making consumers search with the full menu strategy enables the firm to price-discriminate more effectively by extracting the consumer's information rent by generating countervailing incentives. For large search costs, the limited menu strategy dominate the full menu strategy because the firm effectively price-discriminate with respect to the consumer's search effort, which cannot be implemented with the full menu strategy. Finally, the limited menu strategy dominates the firm searching for consumers ($\Psi = F$)

because the firm can save the costly search for type-L consumer. The result is illustrated in Figure 2 below.

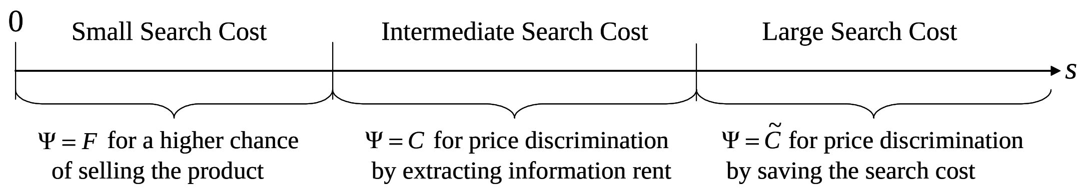

Fig 2. Search Cost and the Firm's Optimal Strategy

We illustrate the findings with the following example: Suppose that $u(q_i, i) = 10i\sqrt{q_i}$, $\varphi_H = 0.4$, $\beta = 0.7$, and $c = 1$. Without the limited menu strategy: $\Psi = F$ is optimal when $s = 3$, $\Psi = C$ (with the full menu strategy) is optimal when $s = 75$, and $\Psi = F$ is optimal again when $s = 250$. If the limited menu strategy is available, then when $s = 250$ the optimality of $\Psi = F$ is replaced with $\Psi = \widetilde{C}$.

7 Conclusion

In this paper, we have shown that a firm can employ consumer search as a device for second degree price discrimination. When a consumer's preference is private information, the firm can extract the consumer's surplus more efficiently by inducing the consumer to make a search effort. According to our result, when the search cost is small or large, it is optimal for the firm to search for the consumer, by directly bearing the search cost. For intermediate search costs, the firm prefers inducing the consumer with a higher preference to bear the search cost and compensate it through the price. We have also shown that if the firm can setup different sales channels and offer a limited menu through each channel, inducing consumer search can dominate even when the search cost is large.

For expositional purpose, we made two simplifying assumptions. First, we assumed that the consumer is risk-neutral. If the consumer is risk averse, then the firm must compensate type-L consumer for the risk she takes by not searching for the product. Although our qualitative result will still hold, the range of the search cost in which inducing consumer search is optimal will become narrower. Second, our result can be extended to the case with a continuum of the consumer's types. Then, unlike the two-type case, inducing consumer search allows the firm to fully extract the consumer's information rent only at a particular level of search cost. Nevertheless, the firm will still prefer consumer search in a intermediate range of the search cost. Also, with a continuum of types, the output distortion becomes more sensitive to the curvature of the search cost.[8]

[8]See Maggi and Rodriguez-Clare (1995) for a formal analysis related ot this issue.

Appendix: Proofs

Proof of Proposition 1.

Since s does not play any role, only (IC) for type-H consumer and (PC) for type-L consumer are binding as in a standard screening problem:

$$p_H = u(q_H, H) - u^{\Delta}(q_L) \quad \text{and} \quad p_L = u(q_L, L). \tag{A1}$$

Substituting for p_H and p_L in the objective function in $(\mathcal{P}^F)$, and we solve:

$$\underset{q_H, q_L}{Max}\ \varphi_H \left[u(q_H, H) - u^{\Delta}(q_L) - cq_H \right] + \varphi_L \left[u(q_L, L) - cq_L \right] - s. \tag{A2}$$

The first order conditions give:

$$u_q(q_H^F, H) = c \quad \text{and} \quad u_q(q_L^F, L) = c + \frac{\varphi_H}{\varphi_L} u_q^{\Delta}(q_L^F),$$

implying that $q_H^F = q_H^*$ and $q_L^F < q_L^*$. From $(A1)$, $p_H^F < p_H^*$ and $p_L^F < p_L^*$.■

Proof of Lemma 1.

We show that, with $\Psi = C$, (i) the case in which the firm induces a learning effort from the consumer regardless of her type and (ii) the case in which the firm only induces type-L consumer's learning effort are both dominated by the firm's optimal outcome with $\Psi = F$. First, suppose, with $\Psi = C$, the firm induces a learning effort from both types. Then, the following learning constraint must be satisfied:

$$u(q_i, i) - p_i - s \geq \beta \left[u(q_i, i) - p_i \right], \quad i \in \{H, L\},$$

which can be rewritten as:

$$u(q_i, i) - p_i \geq \frac{s}{1 - \beta}, \quad i \in \{H, L\}. \tag{A3}$$

The consumer's participation constraint $u(q_i, i) - p_i - s \geq 0$ is implied by $(A3)$ regardless of her type. The constraints that induce the consumer's truthful representation of her type are:

$$u(q_i, i) - p_i - s \geq \max \left\{ \begin{array}{c} u(q_j, i) - p_j - s, \\ \beta \left[u(q_j, i) - p_j \right] \end{array} \right\}, \quad i, j \in \{H, L\}. \qquad (A4)$$

The RHS of $(A4)$ exhibits the consumer's choice of whether or not to exert a learning effort if she decides to misrepresents her type (off the equilibrium path). To simplify $(A4)$, we first present the following lemma.

Lemma 2 *Suppose* $(A3)$ *holds. Then, the inequality below must hold if* $(A4)$ *for type-i consumer is binding:*

$$u(q_j, i) - p_j \geq \frac{s}{1-\beta}, \quad i, j \in \{H, L\}.$$

Proof. Suppose $u(q_j, i) - p_j < \frac{s}{1-\beta}$, implying that $u(q_j, i) - p_j - s < \beta \left[u(q_j, i) - p_j \right]$ in the RHS of $(A4)$. Then, with binding $(A4)$ for type-i consumer can be rewritten as:

$$u(q_i, i) - p_i - \frac{s}{1-\beta} = \beta \left[u(q_j, i) - p_j - \frac{s}{1-\beta} \right],$$

which is a contradiction since the LHS of the equation is positive by $(A3)$, but the RHS is negative. ■

Lemma 2 implies that RHS of $(A4)$ is $u(q_j, i) - p_j - s$. Therefore, $(A4)$ becomes the same as (IC) in the case with $\Psi = F$, since s cancels out with each other in both sides of $(A4)$. The firm's problem then is written as:

$$\underset{q_i, p_i}{Max} \sum_i \varphi_i \left(p_i - c q_i \right),$$

subject to

$$u(q_i, i) - p_i \geq \frac{s}{1-\beta}, \quad i \in \{H, L\}, \qquad (A5)$$

$$u(q_i, i) - p_i \geq u(q_j, i) - p_j, \quad i, j \in \{H, L\}. \qquad (A6)$$

Compared to the case with $\Psi = F$, there are two differences. First, the learning cost s is transferred to the consumer, and second, the consumer's reservation payoff is $\frac{s}{1-\beta}$. As usual, $(A5)$ for type-L and $(A6)$ for type-H consumer are binding at the optimum, and we have expression for p_i, $i \in \{H, L\}$, from these binding constraints. After substituting for the prices in the the firm's objective function, the optimization problem becomes:

$$\underset{q_H, q_L}{Max} \quad \varphi_H \left[u(q_H, H) - u^{\Delta}(q_L) - cq_H\right] + \varphi_L \left[u(q_L, L) - cq_L\right] - \frac{s}{1-\beta}. \qquad (A7)$$

Directly comparing $(A7)$ to $(A2)$ shows that the firm's profit with $\Psi = C$ is strictly lower than its profit with $\Psi = F$.

Next, suppose, with $\Psi = C$, the firm induces a learning effort only from type-L consumer. Then, the following learning constraint must be satisfied:

$$u(q_L, L) - p_L \geq \frac{s}{1-\beta}. \qquad (A8)$$

The constraints that induce the consumer's truthful representation of her type are:

$$\beta \left[u(q_H, H) - p_H\right] \geq \max \left\{ \begin{array}{c} u(q_L, H) - p_L - s, \\ \beta \left[u(q_L, H) - p_L\right] \end{array} \right\}, \qquad (A9)$$

$$u(q_L, L) - p_L - s \geq \max \left\{ \begin{array}{c} u(q_H, L) - p_L - s, \\ \beta \left[u(q_H, L) - p_H\right] \end{array} \right\}. \qquad (A10)$$

Lemma 3 *Suppose $u(q_H, H) - p_H < \frac{s}{1-\beta}$. Then, the inequality below must hold:*

$$u(q_L, H) - p_L < \frac{s}{1-\beta}.$$

Proof. Suppose $u(q_L, H) - p_L \geq \frac{s}{1-\beta}$, implying that $u(q_L, H) - p_L - s \geq \beta \left[u(q_L, H) - p_L\right]$ in the RHS of $(A9)$. Then, $(A9)$ with a simple manipulation gives:

$$\beta \left[u(q_H, H) - p_H - \frac{s}{1-\beta}\right] \geq u(q_L, H) - p_L - \frac{s}{1-\beta},$$

which is a contradiction since the LHS is negative, but the RHS is positive. ■

Lemma 3 implies that the RHS of $(A9)$ is $\beta\left[u(q_L, H) - p_L\right]$. Also by Lemma 2, RHS of $(A10)$ is $u(q_H, L) - p_L - s$. Therefore, $(A9)$ and $(A10)$ become the same as (IC) in the case with $\Psi = F$. For type-L consumer, $(A8)$ implies that the constraint for her participation, $u(q_L, L) - p_L - s \geq 0$, is automatically satisfied. By Lemma 3, $(A9)$ is written as $u(q_H, H) - p_H \geq u(q_L, H) - p_L$, which implies that participation constraint for type-H consumer $u(q_H, H) - p_H \geq 0$ is automatically satisfied. Therefore, the firm's problem is written as:

$$\underset{q,p}{Max}\ \beta\varphi_H\left(p_H - cq_H\right) + \varphi_L\left(p_L - cq_L\right),$$

subject to

$$u(q_L, L) - p_L \geq \frac{s}{1-\beta}, \tag{A11}$$

$$u(q_i, i) - p_i \geq u(q_j, i) - p_j, \quad i, j \in \{H, L\}. \tag{A12}$$

Again, by Lemma 2 and 3, $(A9)$ and $(A10)$ become the constraints in $(A12)$. It can be easily shown that $(A11)$ and $(A12)$ for type-H are binding, and $(A12)$ for type-L consumer is slack. By substituting for p_L and p_H in the objective function, the firm's problem becomes:

$$\underset{q_H, q_L}{Max} \quad \beta\varphi_H\left[u(q_H, H) - u^{\Delta}(q_L) - cq_H - \frac{s}{1-\beta}\right] + \varphi_L\left[u(q_L, L) - cq_L - \frac{s}{1-\beta}\right]. \tag{A13}$$

Clearly, the firm's profit in ($A13$) is even smaller than it's profit in ($A7$). ■

Proof of Proposition 2.

First, the following two lemmas establish that (IC_H) and (IC_L) become:

$$u(q_H, H) - p_H \geq u(q_L, H) - p_L \text{ and } u(q_L, L) - p_L \geq u(q_H, L) - p_H.$$

Lemma 4 *Suppose* (**??**) *holds. Then, the inequality below must hold if* (IC_H) *is binding:*

$$u(q_L, H) - p_L \geq \frac{s}{1-\beta}.$$

Proof. Suppose $u(q_L, H) - p_L < \frac{s}{1-\beta}$, implying that $u(q_L, H) - p_L - s < \beta [u(q_L, H) - p_L]$ in the RHS of (IC_H) Then, binding (IC_H) can be rewritten as:

$$u(q_H, H) - p_H - \frac{s}{1-\beta} = \beta \left[u(q_L, H) - p_L - \frac{s}{1-\beta} \right],$$

which is a contradiction since the LHS is positive by (**??**), but the RHS is negative. ■

The lemma above implies that RHS of (IC_H) is $u(q_L, H) - p_L - s$. Therefore, (IC_H) is rewritten as $u(q_H, H) - p_H \geq u(q_L, H) - p_L$.

Lemma 5 *Suppose $u(q_L, L) - p_L < \frac{s}{1-\beta}$. Then, the inequality below must hold:*

$$u(q_H, L) - p_H < \frac{s}{1-\beta}.$$

Proof. Suppose $u(q_H, L) - p_H \geq \frac{s}{1-\beta}$, implying that $u(q_H, L) - p_H - s \geq \beta [u(q_H, L) - p_H]$ in the RHS of (IC_L). Then, (IC_L) with a simple manipulation gives:

$$\beta \left[u(q_L, L) - p_L - \frac{s}{1-\beta} \right] \geq u(q_H, L) - p_H - \frac{s}{1-\beta},$$

which is a contradiction since the LHS is negative, but the RHS is positive. ■

The lemma above implies that RHS of (IC_L) is $\beta [u(q_L, H) - p_L]$. Therefore, (IC_L) is rewritten as $u(q_L, L) - p_L \geq u(q_H, L) - p_H$.

Since (**??**) implies (PC_H), the Lagrangian of the firm's problem can be written as:

$$\begin{aligned}
\mathcal{L} = \ & \varphi_H (p_H - cq_H) + \beta \varphi_L (p_L - cq_L) \\
& + \lambda_1 \left[u(q_H, H) - p_H - \frac{s}{1-\beta} \right] \\
& + \lambda_2 [u(q_L, L) - p_L] \\
& + \lambda_3 [u(q_H, H) - p_H - u(q_L, H) + p_L] \\
& + \lambda_4 [u(q_L, L) - p_L - u(q_H, L) + p_H].
\end{aligned}$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial p_H} = \varphi_H - \lambda_1 - \lambda_3 + \lambda_4 = 0, \tag{A14}$$

$$\frac{\partial \mathcal{L}}{\partial p_L} = \beta\varphi_L - \lambda_2 + \lambda_3 - \lambda_4 = 0, \tag{A15}$$

$$\frac{\partial \mathcal{L}}{\partial q_H} = -\varphi_H c + (\lambda_1 + \lambda_3) u_q(q_H, H) - \lambda_4 u_q(q_H, L) = 0, \tag{A16}$$

$$\frac{\partial \mathcal{L}}{\partial q_L} = -\beta\varphi_L c + (\lambda_2 + \lambda_4) u_q(q_L, L) - \lambda_3 u_q(q_L, H) = 0, \tag{A17}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = u(q_H, H) - p_H - \frac{s}{1-\beta} \geq 0, \;\; \lambda_1 \frac{\partial \mathcal{L}}{\partial \lambda_1} = 0, \tag{A18}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = u(q_L, L) - p_L \geq 0, \;\; \lambda_2 \frac{\partial \mathcal{L}}{\partial \lambda_2} = 0, \tag{A19}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_3} = u(q_H, H) - p_H - u(q_L, H) + p_L \geq 0, \; \lambda_3 \frac{\partial \mathcal{L}}{\partial \lambda_3} = 0, \tag{A20}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_4} = u(q_L, L) - p_L - u(q_H, L) + p_H \geq 0, \; \lambda_4 \frac{\partial \mathcal{L}}{\partial \lambda_4} = 0. \tag{A21}$$

As will be shown below, the first regime, $s < \underline{s}$ is divided into two sub-regimes, $s < \underline{\underline{s}}$ and $s \in [\underline{\underline{s}}, \underline{s})$, and $s > \overline{s}$ is also divided into two sub-regimes, $s \in (\overline{s}, \overline{\overline{s}}]$ and $s > \overline{\overline{s}}$. The following two lemmas establish the binding constraints in each case.

Lemma 6 *$\lambda_3\lambda_4 = 0$, i.e., (IC_H) and (IC_L) cannot be simultaneously binding.*

Proof. Suppose $\lambda_3 > 0$ and $\lambda_4 > 0$. Then, from $(A14)$ and $(A15)$, $\lambda_3 = \varphi_H - \lambda_1 + \lambda_4$ and $\lambda_4 = \beta\varphi_L - \lambda_2 + \lambda_3$. Also, from $(A16)$ and $(A17)$, $\lambda_3 = \frac{\varphi_H c}{u_q(q_H,H)} - \lambda_1 + \frac{u_q(q_H,L)}{u_q(q_H,H)}\lambda_4$ and $\lambda_4 = \frac{\beta\varphi_L c}{u_q(q_L,L)} - \lambda_2 + \frac{u_q(q_L,H)}{u_q(q_L,L)}\lambda_3$. These equations imply that $u_q(q_H, H) = u_q(q_L, L) = u_q(q_H, L) = u_q(q_L, H)$, which is not possible. ■

Lemma 7 *If $\lambda_4 = 0$ $(\lambda_3 = 0)$, then $\lambda_2 > 0$ $(\lambda_1 > 0)$ and $q_H^C = q_H^*$ $(q_L^C = q_L^*)$. In other words, (i) if (IC_H) is non-binding, then (**??**) is binding and $q_H^C = q_H^*$, and (ii) if (IC_L) is non-binding, then (PC_H) is binding and $q_L^C = q_L^*$.*

Proof. Suppose $\lambda_4 = \lambda_2 = 0$. $(A17)$ gives $\lambda_3 < 0$, which is a contradiction. From $(A14)$ and $(A16)$ with $\lambda_4 = 0$, we obtain $u_q(q_H, H) = c$, and thus $q_H^C = q_H^*$. Similarly, $\lambda_3 = \lambda_1 = 0$ yields $\lambda_4 < 0$ in $(A16)$, which is a contradiction. Also, $(A15)$ and $(A17)$ with $\lambda_3 = 0$, we obtain $u_q(q_L, L) = c$, and thus $q_L^C = q_L^*$. ∎

By Lemma 6 and 7, we can confine our attention to the five cases below. Case I, in which $\lambda_1 = 0$, $\lambda_2 > 0$, $\lambda_3 > 0$ and $\lambda_4 = 0$ ((PC_L) and (IC_H) are binding), Case II, in which $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 > 0$ and $\lambda_4 = 0$ ((SC), (PC_L) and (IC_H) are binding), Case III, in which $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 = 0$ and $\lambda_4 = 0$ ((SC) and (PC_L) are binding), Case IV, in which $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 = 0$ and $\lambda_4 > 0$ ((SC), (PC_L) and (IC_L) are binding), and Case V, in which $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_3 = 0$ and $\lambda_4 > 0$ ((SC) and (IC_L) are binding). We show that Case I and II belong to the regime of $s < \underline{s}$, Case III to the regime of $s \in [\underline{\underline{s}}, \underline{s})$, and Case IV and V to the regime of $s > \bar{s}$.

$$\underline{\underline{q_L}} \text{ by } u_q(\underline{\underline{q_L}}, L) = c + \frac{\varphi_H}{\beta \varphi_L} u_q^{\Delta}(\underline{\underline{q_L}}), \tag{A22}$$

$$\underline{q_L} \text{ by } u^{\Delta}(\underline{q_L}) = \frac{s}{1-\beta}, \tag{A23}$$

$$\overline{q_H} \text{ by } u^{\Delta}(\overline{q_H}) = \frac{s}{1-\beta}, \text{ and} \tag{A24}$$

$$\overline{\overline{q_H}} \text{ by } u_q(\overline{\overline{q_H}}, H) = c - \frac{\beta \varphi_L}{\varphi_H} u_q^{\Delta}(\overline{\overline{q_H}}). \tag{A25}$$

Case I: $\lambda_1 = 0$, $\lambda_2 > 0$, $\lambda_3 > 0$ and $\lambda_4 = 0$. Suppose s is close to zero. Then, the problem becomes a standard screening problem in which only (IC_H) and (PC_L) are binding. From $(A14)$ and $(A16)$ with $\lambda_4 = 0$, we have $u_q(q_H, H) = c$, and thus $q_H^C = q_H^F$ $(= q_H^*)$. To find q_L^C, we insert $\lambda_2 = \varphi_H + \beta \varphi_L$ into $(A17)$. We have:

$$u_q(q_L, L) = c + \frac{\varphi_H}{\beta \varphi_L} u_q^{\Delta}(q_L),$$

which implies $q_L^C = \underline{\underline{q_L}} < q_L^F$. Accordingly, $p_H^C = u(q_H^*, H) - u^{\Delta}(\underline{\underline{q_L}}) > p_H^F$ and $p_L^C = u(\underline{\underline{q_L}}, L) < p_L^F$. The non-binding constraint associated with λ_1 is written as:

$$u^{\Delta}(\underline{\underline{q_L}}) - \frac{s}{1-\beta} > 0.$$

As s increases, the constraint will eventually be binding at $s = \underline{\underline{s}}$, where $\underline{\underline{s}} = (1-\beta)u^{\Delta}(\underline{\underline{q_L}})$.

Case II: $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 > 0$ and $\lambda_4 = 0$. As s becomes $\underline{\underline{s}}$, the constraint linked to λ_1, (SC), begins to bind as well. In this case, the binding $(A18)$, $(A19)$, and $(A20)$ give:

$$u^{\Delta}(q_L) = \frac{s}{1-\beta},$$

which implies $q_L^C = \underline{q_L} \lesseqgtr q_L^F$, depending on the size of s. From $(A14)$ and $(A16)$ with $\lambda_4 = 0$, we have $q_H^C = q_H^F$ $(= q_H^*)$. From the binding constraints, we have $p_H^C = u(q_H^*, H) - \frac{s}{1-\beta} \gtreqless p_H^F$ and $p_L^C = u(\underline{q_L}, L) \lesseqgtr p_L^F$. Case II is now valid when $s \in [\underline{\underline{s}}, \underline{s})$, where $\underline{s} = (1-\beta)u^{\Delta}(q_L^*)$. If s becomes greater than $\underline{s}$, the constraint linked to λ_3, (IC_H), becomes no longer binding.

Case III: $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 = 0$ and $\lambda_4 = 0$. Only (SC) and (PC_L) are binding in this sub-regime. As in the above two cases, $\lambda_4 = 0$ implies that $q_H^C = q_H^*$. From $(A15)$ and $(A17)$ with $\lambda_3 = 0$ we have $q_L^C = q_L^*$. From the binding constraints, we have $p_H^C = u(q_H^*, H) - \frac{s}{1-\beta} \left(= p_H^* - \frac{s}{1-\beta}\right)$ and $p_L^C = u(q_L^*, L) = p_L^*$. The non-binding constraint related with λ_4, (IC_L), is written as:

$$u^{\Delta}(q_H^*) > \frac{s}{1-\beta}.$$

As s becomes larger, this constraint will bind at $s = \bar{s}$, where $\bar{s} = (1-\beta)u^{\Delta}(q_H^*)$.

Case IV: $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 = 0$ and $\lambda_4 > 0$. As $s > \bar{s}$, we consider the case where the constraint linked to λ_4, (IC_L), begins to bind. Note that q_H^C is no longer q_F^* because of $\lambda_4 > 0$. Binding $(A18)$, $(A19)$, and $(A21)$ give:

$$u^{\Delta}(q_H) = \frac{s}{(1-\beta)},$$

which implies $q_H^C = \overline{q_H} > q_H^*$. As in Case III, $\lambda_3 = 0$ implies that $q_L^C = q_L^*$. From the binding constraints, $p_H^C = u(\overline{q_H}, H) - \frac{s}{1-\beta}$ $(< p_H^F)$ and $p_L^C = u(q_L^*, H) - \frac{s}{1-\beta}$ $\left(\gtreqless p_L^F\right)$. As s increases, p_L^C decreases and eventually the constraint linked to λ_2 become no longer binding at $s = \bar{\bar{s}}$, where $\bar{\bar{s}} = (1-\beta)u^{\Delta}(\overline{\overline{q_H}})$.

Case V: $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_3 = 0$ and $\lambda_4 > 0$. Solving $(A14)$ and $(A15)$ together with $\lambda_2 = \lambda_3 = 0$, we obtain $\lambda_1 = \varphi_H + \beta\varphi_L$ and $\lambda_4 = \beta\varphi_L$. Thus, $(A16)$ is rewritten as:

$$u_q(q_H, H) = c - \frac{\beta\varphi_L}{\varphi_H} u_q^{\Delta}(q_H),$$

which implies $q_H^C = \overline{\overline{q_H}} > q_H^F = q_H^*$. As in Case III and IV, $\lambda_3 = 0$ implies that $q_L^C = q_L^*$. From the binding constraints, we have $p_H^C = u(\overline{\overline{q_H}}, H) - \frac{s}{1-\beta}$ $(< p_H^F)$ and $p_L^C = u(q_L^*, L) - u_q^{\Delta}(\overline{\overline{q_H}}) - \frac{s}{1-\beta} \left(\gtreqless p_L^F \right)$.

It follows that Case I and II belong to the regime of $s < \underline{\underline{s}}$, Case III to the regime of $s \in [\underline{\underline{s}}, \underline{s})$, and Case IV and V to the regime of $s > \overline{s}$. ■

Proof of Proposition 3.

By Proposition 1, the firm's expected profit with $\Psi = F$ is:

$$\Pi^F = \varphi_H \left[u(q_H^*, H) - cq_H^* - u^{\Delta}(q_L^F) \right] + \varphi_L \left[u(q_L^F, L) - cq_L^F \right] - s.$$

Also, by Proposition 2, the firm's expected profit with $\Psi = C$ is:

$$\Pi^C = \begin{cases} \varphi_H \left[u(q_H^*, H) - cq_H^* - u^{\Delta}(\underline{\underline{q_L}}) \right] + \beta\varphi_L \left[u(\underline{\underline{q_L}}, L) - c\underline{\underline{q_L}} \right] & \text{for } s < \underline{\underline{s}}, \\ \varphi_H \left[u(q_H^*, H) - cq_H^* - \frac{s}{1-\beta} \right] + \beta\varphi_L \left[u(\underline{q_L}, L) - c\underline{q_L} \right] & \text{for } s \in [\underline{\underline{s}}, \underline{s}), \\ \varphi_H \left[u(q_H^*, H) - cq_H^* - \frac{s}{1-\beta} \right] + \beta\varphi_L \left[u(q_L^*, L) - cq_L^* \right] & \text{for } s \in [\underline{s}, \overline{s}], \\ \varphi_H \left[u(\overline{q_H}, H) - c\overline{q_H} - \frac{s}{1-\beta} \right] + \beta\varphi_L \left[u(q_L^*, L) - cq_L^* - \frac{s}{1-\beta} \right] & \text{for } s \in (\overline{s}, \overline{\overline{s}}], \\ \varphi_H \left[u(\overline{\overline{q_H}}, H) - c\overline{\overline{q_H}} - \frac{s}{1-\beta} \right] + \beta\varphi_L \left[u(q_L^*, L) - cq_L^* - u_q^{\Delta}(\overline{\overline{q_H}}) - \frac{s}{1-\beta} \right] & \text{for } s > \overline{\overline{s}}. \end{cases}$$

Lemma 8 Π^C *is non-increasing and weakly concave in s.*

Proof. Applying the envelope theorem to the Lagrangian, we have $\frac{\partial \Pi^C}{\partial s} = -\frac{\lambda_1(s)}{1-\beta} \leq 0$. Now, we need to show

$$sign\left(\frac{\partial^2 \Pi^C}{\partial s^2}\right) = sign\left(-\frac{\partial \lambda_1(s)}{\partial s}\right) \leq 0.$$

Let us find $\lambda_1(s)$. From $(A14)$, $\lambda_1 = \varphi_H - \lambda_3 + \lambda_4$. Solving $(A15)$ and $(A17)$, we obtain $\lambda_3 = \beta\varphi_L \frac{u_q(q_L,L)-c}{u_q^\Delta(q_L)}$. Similarly, solving $(A14)$ and $(A16)$, we obtain $\lambda_4 = \max\left\{\varphi_H \frac{c-u_q(q_H,H)}{u_q^\Delta(q_H)}, 0\right\}$. As a result,

$$\lambda_1(s) = \varphi_H - \beta\varphi_L \frac{u_q(q_L,L)-c}{u_q^\Delta(q_L)} + \max\left\{\varphi_H \frac{c-u_q(q_H,H)}{u_q^\Delta(q_H)}, 0\right\}.$$

It is immediate that $\frac{\partial\lambda_1(s)}{\partial s} = 0$ when $s < \underline{\underline{s}}$, $s \in [\underline{s}, \bar{s}]$, and $s > \bar{\bar{s}}$, because q_H^*, $\overline{\overline{q_H}}$, q_L^*, and $\overline{\overline{q_L}}$ is independent of s. In other words, in these three regimes, Π^C is linearly decreasing. However, when $s \in [\underline{\underline{s}}, \underline{s})$ or $s \in (\bar{s}, \bar{\bar{s}}]$, we have to investigate the sign of $\frac{\partial\lambda_1(s)}{\partial s}$. First, when $s \in [\underline{\underline{s}}, \underline{s})$, $\lambda_1(s) = -\beta\varphi_L \frac{u_q(\underline{q_L},L)-c}{u_q^\Delta(\underline{q_L})}$. Thus, a simple calculation gives

$$\frac{\partial\lambda_1(s)}{\partial s} = -\beta\varphi_L \frac{\left[u_{qq}(\underline{q_L},L)u_q^\Delta(\underline{q_L}) - u_{qq}^\Delta(\underline{q_L})(u_q(\underline{q_L},L)-c)\right]}{u_q^\Delta(\underline{q_L})^2} \frac{\partial \underline{q_L}}{\partial s} > 0.$$

The terms in the bracket is negative under well-accepted, $\frac{u_{qq}(q,L)}{u_q(q,L)} \leq \frac{u_{qq}(q,H)}{u_q(q,H)}$. By applying the implicit function theorem to $u^\Delta(\underline{q_L}) = \frac{s}{1-\beta}$, we have $\frac{\partial \underline{q_L}}{\partial s} = \frac{1}{(1-\beta)u_q^\Delta(\underline{q_L})} > 0$. Likewise, it can be easily checked that for $s \in (\bar{s}, \bar{\bar{s}}]$, $\lambda_1(s) = \varphi_H \frac{c-u_q(q_H,\overline{H})}{u_q^\Delta(q_H)}$ is decreasing in s. ■

Π^F is linearly decreasing, while Π^C is concavely decreasing in s. Also, note that $\Pi^F(s=0) > \Pi^C(s=0)$, while $\Pi^F(s=\bar{\bar{s}}) > \Pi^C(s=\bar{\bar{s}})$. Thus, if we find any s such that $\Pi^F(s) < \Pi^C(s)$, there should exist s_l and s_h so that $\Pi^F(s) \geq \Pi^C(s)$ for $s \leq s_l$ and $s \geq s_h$, but $\Pi^F(s) < \Pi^C(s)$ for $s \in (s_l, s_h)$. Next, consider $\underline{s} \equiv (1-\beta)u^\Delta(q_L^*)$. Then, we have:

$$\begin{aligned}\Pi^C(s=\underline{s}) - \Pi^F(s=\underline{s}) = {} & (1-\beta)u^\Delta(q_L^*) + \beta\varphi_L\left[u(q_L^*,L) - cq_L^*\right] \\ & - \varphi_H\left[u^\Delta(q_L^*) - u^\Delta(q_L^F)\right] - \varphi_L\left[u(q_L^F,L) - cq_L^F\right]\end{aligned}$$

The first two terms are positive, whereas the last two terms are negative. Given L, as H increases (hence Δ increases), both $u^\Delta(q_L^*)$ and $u^\Delta(q_L^F)$ increase. On the other hand, $u(q_L^F, L) - cq_L^F$ become smaller due to a greater downward distortion in q_L^F. Therefore:

$$\lim_{H\to\infty} \Pi^C\left(s = \underline{s}\right) - \Pi^F\left(s = \underline{s}\right) > 0,$$

which implies that $\Pi^F(s) < \Pi^C(s)$ for $s \in (s_l, s_h)$, where $s_l < \underline{s} < s_h$. ■

Proof of Proposition 4.

First, (PC_H) is implied by $(\widetilde{SC})$ and (PC_L):

$$\begin{aligned} u(q_H, H) - p_H - s &\geq \beta\left[u(q_L, H) - p_L\right] \\ &\geq \beta\left[u(q_L, L) - p_L\right] \geq 0 \end{aligned}$$

Thus, (PC_H) is not binding. Next we show that $(\widetilde{IC}_L)$ is non-binding. After rearranging $(\widetilde{SC})$, we have: $s \geq u(q_H, H) - p_H - \beta\left(u(q_L, H) - p_L\right)$. Similarly, $(\widetilde{IC}_L)$ is written as $s \geq u(q_H, L) - p_H - \beta\left[u(q_L, L) - p_L\right]$. These two inequalities imply that the latter is always satisfied because $u^\Delta(q_H) > \beta u^\Delta(q_L)$ regardless of $(\widetilde{SC})$. Since (PC_H) and $(\widetilde{IC}_L)$ are non-binding, the Lagrangian of the firm's problem can be written as:

$$\begin{aligned} \mathcal{L} = {} & \varphi_H\left(p_H - cq_H\right) + \beta\varphi_L\left(p_L - cq_L\right) \\ & + \mu_1\left[u(q_H, H) - p_H - s - \beta\left(u(q_L, H) - p_L\right)\right] \\ & + \mu_2[u(q_L, L) - p_L] \\ & + \mu_3\left[u(q_H, H) - p_H - u(q_L, H) + p_L\right] \end{aligned}$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial p_H} = \varphi_H - \mu_1 - \mu_3 = 0, \tag{A26}$$

$$\frac{\partial \mathcal{L}}{\partial p_L} = \beta\varphi_L + \beta\mu_1 - \mu_2 + \mu_3 = 0, \tag{A27}$$

$$\frac{\partial \mathcal{L}}{\partial q_H} = -\varphi_H c + \left(\mu_1 + \mu_3\right) u_q(q_H, H) = 0, \tag{A28}$$

$$\frac{\partial \mathcal{L}}{\partial q_L} = -\beta\varphi_L c + \mu_2 u_q(q_L, L) - (\mu_1\beta + \mu_3)\, u_q(q_L, H) = 0, \qquad (A29)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_1} = u(q_H, H) - p_H - s - \beta\,(u(q_L, H) - p_L) \geq 0,\ \ \mu_1 \frac{\partial \mathcal{L}}{\partial \mu_1} = 0, \qquad (A30)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_2} = u(q_L, L) - p_L \geq 0,\ \ \mu_2 \frac{\partial \mathcal{L}}{\partial \mu_2} = 0, \qquad (A31)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_3} = u(q_H, H) - p_H - u(q_L, H) + p_L \geq 0,\ \mu_3 \frac{\partial \mathcal{L}}{\partial \mu_3} = 0, \qquad (A32)$$

As will be shown below, the first regime, $s < \widetilde{s}$, is divided into two sub-regimes: $s < \underline{\underline{s}}$ and $s \in [\underline{\underline{s}}, \widetilde{s})$.

Case I: $\mu_1 = 0, \mu_2 > 0$, and $\mu_3 > 0$. Suppose s is small enough. Then, the problem is reduced to a standard problem. We obtain $\widetilde{q}^C_H = q^*_H$ and $\widetilde{q}^C_L = \underline{\underline{q_L}}$ as in Case I in the full menu case. The non-binding constraint associated with μ_1 is written as:

$$u^{\Delta}(\underline{\underline{q_L}}) - \frac{s}{(1-\beta)} > 0.$$

Accordingly, $\widetilde{p}^C_H = u(q^*_H, H) - u^{\Delta}(\underline{\underline{q_L}}) > p^F_H$ and $p^C_L = u(\underline{\underline{q_L}}, L) < p^F_L$. As s becomes large, the constraint will be binding at $s = \underline{\underline{s}}$, where $\underline{\underline{s}} \equiv (1-\beta)u^{\Delta}(\underline{\underline{q_L}})$.

Case II: $\mu_1 > 0, \mu_2 > 0$, and $\mu_3 > 0$. As s becomes $\underline{\underline{s}}$, the three constraints, $(A30)$, $(A31)$, and $(A32)$, simultaneously binding. Thus, in this case, $q^{\widetilde{C}}_H = q^*_H$, and $\widetilde{q}^C_L$ is characterized as:

$$u^{\Delta}(\widetilde{q}^C_L) = \frac{s}{(1-\beta)}.$$

This case is valid when $s \in [\underline{\underline{s}}, \widetilde{s})$, where $\widetilde{s} = (1-\beta)u^{\Delta}(q^F_L)$ and $\underline{\underline{s}}$ is defined in the Proof of Proposition 2. From the binding constraints, we have $\widetilde{p}^C_H = u(q^*_H, H) - \beta u^{\Delta}(\underline{q_L}) - s$ and $p^C_L = u(\widetilde{q}^C_L, L)$. If s becomes larger than $\widetilde{s}$, then (IC_H) becomes no longer binding.

Case III: $\mu_1 > 0, \mu_2 > 0$, and $\mu_3 = 0$. Next, when $s > \widetilde{s}$, only $(\widetilde{SC})$ and (PC_L) are binding. Substituting $(\widetilde{SC})$ and (PC_L) into the firm's objective function, we obtain

$$\varphi_H \left(u(q_H, H) - s - \beta u^{\Delta}(q_L) - c q_H \right) + \beta\varphi_L \left(u(q_L, L) - c q_L \right).$$

The first order conditions give $\widetilde{q}_H^C = q_H^*$ and $\widetilde{q}_L^C = q_L^F$. Note that β does not affect the choice of $\widetilde{q}_L^C$, which is characterized by: $u_q(\widetilde{q}_L^C, L) = c + \frac{\varphi_H}{\varphi_L} u_q^\Delta(\widetilde{q}_L^C)$. From the binding constraints, $\widetilde{p}_H^C = u(q_H^*, H) - \beta u^\Delta(q_L^F) - s$ and $p_L^C = u(q_L^F, L)$.■

Proof of Proposition 5.

By proposition 4, the firm's expected profit with $\Psi = \widetilde{C}$ is:

$$\Pi^{\widetilde{C}} = \begin{cases} \varphi_H \left[u(q_H^*, H) - cq_H^* - u^\Delta(\underline{\underline{q_L}})\right] + \beta\varphi_L \left[u(\underline{\underline{q_L}}, L) - c\underline{\underline{q_L}}\right] & \text{for } s < \underline{\underline{s}}, \\ \varphi_H \left[u(q_H^*, H) - cq_H^* - \frac{s}{1-\beta}\right] + \beta\varphi_L \left[u(\underline{q_L}, L) - c\underline{q_L}\right] & \text{for } s \in [\underline{\underline{s}}, \widetilde{s}), \\ \varphi_H \left[u(q_H^*, H) - cq_H^* - \beta u^\Delta(q_L^F) - s\right] + \beta\varphi_L \left[u(q_L^F, L) - cq_L^F\right] & \text{for } s \geq \widetilde{s}. \end{cases}$$

Like $\Pi^C(s)$, it can be shown that $\Pi^{\widetilde{C}}(s)$ is non-increasing and weakly concave in s. It is clear that $\Pi^C(s) = \Pi^{\widetilde{C}}(s)$ for $s \leq \widetilde{s}$.

Note that $\Pi^{\widetilde{C}}(s)$ is decreasing in s more slowly than $\Pi^C(s)$ and $\Pi^F(s)$ for $s > \underline{s}$, where $\underline{s}$ is defined in Definition 1. Thus, if we find $\Pi^{\widetilde{C}}(s) < \Pi^C(s)$ at a certain s, then there must exist $\widetilde{s}_h < s_h$ such that $\Pi^{\widetilde{C}}(s) > \max\{\Pi^C(s), \Pi^F(s)\}$ for $s > \widetilde{s}_h$. Below we provide a sufficient condition for it. We compare $\Pi^{\widetilde{C}}(s)$ and $\Pi^C(s)$ at $s = \underline{s}$ to find:

$$\Pi^{\widetilde{C}}(s = \underline{s}) - \Pi^C(s = \underline{s}) < 0 \Leftrightarrow \frac{\varphi_H}{\varphi_L} < \frac{\left[(u(q_L^*, L) - cq_L^*) - \left(u(q_L^F, L) - cq_L^F\right)\right]}{\left[u^\Delta(q_L^*) - u^\Delta(q_L^F)\right]}.$$

Therefore, the firm's choice of $\Psi = \widetilde{C}$ generates a greater profit when s is sufficiently large. ■

References

[1] **Anderson, S. and Renault, R. (2006)**, "Advertising Content," *American Economic Review*, 96, 93-113.

[2] **Armstrong, M and Vickers, J. (2010)**, "Competitive Non-linear Pricing and Bundling," *Review of Economic Studies*, 77, 30-60

[3] **Armstrong, M. et al. (2009)**, "Prominence and Consumer Search," *Rand Journal of Economics*, 40, 209–233.

[4] **Bar-Issac, H. et al. (2010)**, "Information Gathering and Marketing," *Journal of Economic and Management Strategy*, 19, 375–401.

[5] **Braverman, A. (1980)**, "Consumer Search and Alternative Market Equilibria," *Review of Economic Studies*, 47, 487–502.

[6] **Carlson, J. and McAfee, P. (1983)**, "Discrete Equilibrium Price Dispersion," *Journal of Political Economy*, 91, 480–493.

[7] **Janssen, M. and Moraga-Gonzalez, J. (2004)**, "Strategic Pricing, Consumer Search and the Number of Firms," *Review of Economic Studies*, 71, 1089–1118.

[8] J**ohnson, J. and Myatt, D. (2006)**, "On the Simple Economics of Advertising, Marketing and Product Design," *American Economic Review*, 96, 756–784.

[9] **Jullien, B. (2000)**, "Participation Constraints in Adverse Selection Model," *Journal of Economic Theory*, 93, 1–47.

[10] **Lewis, T. and Sappington, D. (1989)**, "Countervailing Incentives in Agency Problems," *Journal of Economic Theory*, 49, 294–313.

[11] **Lewis, T. and Sappington, D. (1994)**, "Supplying Information to Facilitate Price Discrimination," *International Economic Review*, 35, 309–327.

[12] **Maggi, G. and Rodriguez-Clare, A. (1995)**, "On Countervailing Incentives," *Journal of Economic Theory*, 66, 238–263.

[13] **Maskin, E. and Riley, J. (1984)**, "Monopoly with Incomplete Information," *Rand Journal of Economics*, 15, 171–196.

[14] **Morgan, P. and Manning, R.(1985)**, "Optimal Search," *Econometrica*, 53, 923–944.

[15] **Mussa, M. and Rossen, S. (1978)**, "Monopoly and Product Quality," *Journal of Economic Theory*, 18, 301–317.

[16] **Ottaviani, M. and Prat, A. (2001)**, "The Value of Public Information in Monopoly," *Econometrica*, 69, 1673–1683.

[17] **Persico, N. (2000)**, "Information Acquisition in Auctions," *Econometrica*, 68, 135–148.

[18] **Reinganum, J. (1979)**, "A Simple Model of Equilibrium Price Dispersion," *Journal of Political Economy*, 87, 851–858.

[19] **Salop, S. and Stiglitz, J. (1982)**, "The Theory of Sales: A Simple Model of Equilibrium Price Dispersion with Identical Agents," *American Economic Review*, 79, 1121–1130.

[20] **Stahl, D. (1989)**, "Oligopolistic Pricing with Sequential Consumer Search," *American Economic Review*, 79, 700–712.

[21] **Stigler, G. (1961)**, "The Economics of Information," *Journal of Political Economy*, 69, 213–225.

[22] **Stiglitz, J. (1979)**, "Equilibrium in Product Markets with Imperfect Information," *American Economic Review*, 69, 339–345.

[23] **Varian, H. (1980)**, "A Model of Sales," *American Economic Review*, 70, 651–659.