Turning the Page on Business Formats for Digital Platforms: Does Apple’s Agency Model Soften Competition?

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JEL classification:

Keywords: the agency model, resale price maintenance, digital content

Abstract: The agency model applied by Apple and other companies implies that upstream firms (content providers like book publishers and developers of apps) decide retail prices. A distinct feature of this business model is thus Resale Price Maintenance (RPM). A second distinct feature is the revenue-sharing rule, which specifies how the revenue is split between downstream firms (like Apple) and upstream firms. We show that the equilibrium revenue shares determined by downstream firms depend on competition both upstream and downstream. Prices are lower with than without the agency model if the degree of competition is higher at the upstream level than at the downstream level. This seems to be the case in Apple’s App Store, where there is aggressive competition among developers of apps. With asymmetric business formats (where only some downstream firms use
the agency model), we show that a retail Most Favored Nation clause (MFN) may lead to uniform prices that resemble the outcome under industry-wide adoption of RPM.

1 Introduction

The business format that Apple uses towards application providers in App Store and ebook publishers in iBookstore has been labeled the agency model. We scrutinize on two ingredients of this business model. First is the Resale Price Maintenance (RPM) aspect, namely that it is the content providers (upstream firms) who determine the retail prices. Second is the revenue-sharing rule, which specifies that 70% of the revenue that a service generates accrues to the content provider and 30% to Apple. Apple uses the 70/30 revenue sharing rule also towards the music industry in iTunes Store. However, Apple has firmly stayed in control of retail prices. Thus, Apple does not seem to have a preference for RPM per se. On the contrary, from Steve Jobs biography (Isaacson, 2011) there is a clear indication that he viewed the agency model in the publishing sector as a second best solution: He [Jobs] had refused to offer the music companies the agency model and allow them to set their own prices. Why? Because he didn’t have to. But with books he did. “We were not the first people in the books business,” he said. “Given the situation that existed, what was best for us was to do this akido move and end up with the agency model. And we pulled it off”.

Our goal is to analyze the agency model when there is competition between upstream firms (content providers) as well as between downstream firms (such as Apple and Amazon in the example of ebooks). Since the 70/30 split is applied universally across markets like music, ebooks, and apps, we mostly assume that the revenue shares are exogenously given. We show that if the competitive pressure is higher on the upstream level than on the downstream level, prices are lower with than without an industry-wide adoption of the agency model.

It is noteworthy that in order to create entrepreneurship and innovation (letting the thousand flowers bloom) when iPhone was introduced, Apple found it necessary
to allow app developers to set retail prices. The innovation stimulus seem to have come with the cost of low prices, though. The competitive pressure in this market is clearly higher at the upstream level than at the downstream level, and an increasing number of app developers complain that they cannot make profit in Apple’s App Store (Boudreau, 2012). Interestingly, Apple’s perhaps closest rival in the app market, Google Play (formerly Android Market), provides an agency model which is identical to the one employed by Apple. Even the revenue split is the same; 30 % to Google and 70 % to the app developers.¹

Also in the ebook industry the competitive pressure appear to have been higher upstreams (between publishers) than downstreams (where Amazon has had a dominant position). It might thus seem surprising that when Apple introduced the agency model for ebooks, the Department of Justice (2012) conjectured that the key motivation was to stop the low ebook prices set by Amazon, described as "the $9.99-problem" by publishers.² However, the main reason why the agency model has lead to higher prices in this case is presumably the publishers’ incentives to protect profit from the sale of printed books. In the words of Steve Jobs (Isaacson, 2011): [Amazon]started selling them [ebooks] below cost at $9.99. The publishers hated that—they thought it would trash their ability to sell hardcover books at $28.³

To highlight the strategic competitive interactions caused by the presence of

²US antitrust authorities have started an investigation of whether the shift to the agency model was an outcome of explicit collusion among several major publishers and Apple. Department of Justice (2012) revealed that Apple and five major publishers (HarperCollins, Hachette, Macmillan, Penguin and Simon & Schuster) were under investigation for collusion on how the rapid industry-wide adoption of RPM through the agency model was implemented. This is a question we do not go into.
³In contrast to Apple’s business model for music, Amazon did not apply revenue sharing for ebooks. Historically, the dominant business format for printed books in the US has been the wholesale model, where the unit wholesale price is determined as a percentage rate of the list price (the cover price). Before Apple entered the market with iBookstore, the wholesale model was used also for ebooks. As with printed books, the unit wholesale price was a percentage rate of the cover price, and the retailers were free to set the retail price below the cover price.
competition both on the upstream and downstream level, we intentionally abstract from the interplay with adjacent markets like printed books (for the publishers) and tablets, music players and smartphones (for the downstream firms like Apple, Amazon and Google).\footnote{See Abhishek, Jerath and Zhang (2012) for a relevant adjacent market analysis.} In the first part of the paper we thus focus solely on the intrinsic relationship between RPM and retail prices. We then proceed by allowing the revenue shares to be determined by the downstream firms prior to price competition among upstream firms. Thus, the downstream firms offer take-it-or-leave-it revenue sharing contracts to the upstream firms. However, this does not mean that the downstream firms necessarily require revenue shares which are so high that the upstream firms’ participation constraints are binding. On the contrary, we show that the equilibrium revenue shares depend on competition on both levels of the value chain, and that the participation constraints in general will not be binding.

To see why, suppose that we have two downstream firms which are symmetric except that downstream firm I requires a higher revenue share than downstream firm II. Then the upstream firms will have incentives to set relatively high RPM prices at downstream firm I in order to make downstream firm II more competitive. Thereby sales are reallocated to the downstream firm which offers the upstream firms the greatest share of the revenue. This effect will put a stronger downward pressure on the downstream firms’ revenue shares the better substitutes the upstream firms are in the eyes of the consumer. However, the stronger the competition between the upstream firms, the smaller will be their abilities to punish one downstream firm with higher RPM prices than the other. High upstream substitutability therefore tends to increase the downstream firms’ revenue shares.

We also consider an asymmetric structure where one of the downstream firms (“Apple”) uses the agency model, while the other downstream firm (“Amazon”) does not. When Apple entered the market for ebooks, it entailed a retail Most Favored Nation clause (MFN). These MFN clauses require that the publishers do not set a higher RPM price at Apple than what the books are sold for at other retailers, independent of whether the publishers control the prices charged by Apple’s rivals (Department of Justice, 2012). We show that if the rival (“Amazon”) does not use
RPM, then a binding MFN leads to uniform retail prices that resemble the outcome under industry-wide adoption of the RPM.

The rest of the paper is organized as follows. In section 2 we provide an overview of related literature. In section 3 we present the model, and analyze the consequences of an industry-wide adoption of the agency model. In section 4 we consider asymmetric structures where only one of the downstream firms uses the agency model, and section 4 provides concluding remarks.

2 Related Literature

Several recent papers are inspired by Apple’s agency model, and they provide insight complementary to that in the present paper. Abhishek, Jerath and Zhang (2012) present an interesting analysis of the interplay between printed and digital publishing. Gans (2012) shows how a hold-up problem might arise if consumers must undertake investments in order to have platform access prior to content providers decide prices. Johnson (2012) sets up a two-period model with lock-ins, and shows how the agency model increases retail prices in the first period but lowers them in the second period compared to a traditional wholesale model (where wholesale terms of trades just consist of a unit wholesale price).

Both Johnson (2012) and Abhishek et al. (2012) compare outcomes under a wholesale model and the agency model. Furthermore, they both assume that the upstream firm offers a take-it-or-leave-it contract under the wholesale model, such that the wholesale price is above marginal costs. Hence, a driving force leading to lower prices under the agency model in their framework is avoidance of double
marginalization.\textsuperscript{5} In contrast, we focus purely on the competitive effects of RPM.\textsuperscript{6} Therefore, we assume revenue sharing regardless of whether upstream or downstream firms determine retail prices.

Our assumption that wholesale terms of trade always are set prior to retail prices contrasts with previous analyses. Dobson and Waterson (2007), for instance, assume that the timing of the game depends on who sets the prices.\textsuperscript{7} If retail prices are determined by downstream firms (no RPM), then they assume that prices are chosen after wholesale prices.\textsuperscript{8} However, they make the opposite timing assumption under RPM. Though such a shift in the timing of the game might be justified in some cases, it does not seem to fit the cases which have motivated this paper. Apple’s 70/30 rule, for instance, is certainly set prior retail prices.\textsuperscript{9}

\textsuperscript{5}Even in the market for printed books it is questionable whether there exists a double marginalization problem under the wholesale model. On a printed book with a $25.00 price printed on the cover, the publisher and the bookstore bargain over the share of the cover price the publisher charges the bookstore. If the share is 50\%, the publisher charges a unit wholesale price of $12.50 on each sale. The bookstore is free to set a retail price that is lower than the cover price, but will in practice find it difficult to charge more. Thus, the wholesale model actually contains a maximum RPM (even though it is not formally binding). This is important to note, since it indicates that potential double marginalization problems to a large extent are solved in the wholesale model adopted in the US for more than 100 years.

\textsuperscript{6}For the same reason we abstract from other potential efficiency gains from using RPM. There are several potential efficiency gains from using RPM in addition to preventing double marginalization (see Posner, 1976). RPM might for instance stimulate inter-brand competition by providing quality certification (Marvel and McCafferty, 2007) or reduce free-riding in presence of unobservable effort, e.g. retail service provision (Telser, 1960). Finally, RPM may maintain retailers’ margins and thereby ensure their incentives to stock and promote products (Deneckere et al., 1996).

\textsuperscript{7}The linear demand system used in the present paper resembles the one in Dobson and Waterson (2007). See also Dobson and Waterson (1996).

\textsuperscript{8}Dobson and Waterson (2007) allow for different distribution of bargaining power, and show how double marginalization arises when downstream firms are in a weak bargaining position.

\textsuperscript{9}Note that we assume that downstream firms can commit to observable wholesale contracts across firms prior to the decision on retail pricing. The outcome changes fundamentally if the firms cannot commit to observable wholesale contracts. O’Brien and Shaffer (1992) were the first to consider how a manufacturer monopolist may use RPM as a means of dampening retail competition when (non-linear) wholesale prices are unobservable across firms.
Johnson (2012) also analyzes the effects of the retail MFN incorporated by Apple into the agency model when entered the ebook market. He proves that MFN has no impact on retail prices under industry-wide adoption of the agency model. In contrast, we show that under an asymmetric business structure, where only one of the downstream firms uses the agency model, such an MFN may restore the outcome under industry-wide adoption of the agency model. In such an asymmetric structure, MFN increases retail prices as long as the competitive pressure is higher at the downstream than the upstream level.

There exists a large literature which focuses on how to find the minimum number of vertical restraints sufficient to maximize total channel profit. Mathewson and Winter (1984), for instance, show how a combination of a two-part tariff and RPM may be used to achieve the integrated channel outcome in a setting where downstream firms undertake market expanding sales effort with potential spillovers. Fixed royalty rates set prior to price competition is then an alternative to lump-sum fixed fees. Lal (1990) shows that revenue-sharing may be used as an additional instrument to a two-part tariff in a context where upstream and downstream firms undertake non-contractible sales efforts (see also Rao and Srinivasan, 1995).

Our approach is different, since we aim to analyze what outcome may be achieved with the instruments actually incorporated in the agency model. Like our paper, Cachon and Lariviere (2005), Dana and Spier (2001), and Mortimer (2008) are motivated by observed contracts.\footnote{These papers focus on revenue-sharing contracts implemented in the video rental industry, and show how revenue-sharing schemes may be used to solve channel coordination problems related to inventory choices.}

An often articulated motivation for delegating retail pricing to upstream firms (content providers) is that these are typically better informed about the demand for their goods than the downstream firms (platform providers). Vertical separation between content provision and distribution, and allowing the content providers to decide retail prices, may then stimulate innovation (letting a thousand flowers bloom) among content providers (see e.g. Boudreau, 2012). Foros, Hagen, and Kind (2009) show how a monopoly platform may balance this trade-off by using a
price-dependent profit sharing rule implemented by Scandinavian mobile providers in the market for mobile content messages. The business format used by mobile providers for such content messages may be considered as the first-generation app stores. We do not incorporate asymmetric demand information between downstream firms and upstream firms into our model, but highlight that the presence of such information asymmetries would imply an efficiency benefit for the agency model.

By delegating retail pricing to the upstream firms, the pre-determined fixed percentage revenue sharing rates set by the downstream firms become strategic delegation devices, and our work is thus related to the literature on strategic delegation (a seminal contribution is Fershtman and Judd, 1987). Shaffer (1991) compares unit wholesale, RPM and two-part tariffs. While we assume that upstream firms supply their products to both downstream firms, Shaffer (1991) constrains the analysis to two pairs of upstream and downstream firms where each upstream firm only offers its product to one of the downstream firms. When upstream firms determine wholesale prices, McGuire and Staelin (1983) show how also linear wholesale tariffs may be used as a strategic delegation device. Similar to Shaffer (1991), McGuire and Staelin (1983) assume that each upstream firm only offers its product to one of the downstream firms.

3 The Model

We consider a market structure with two competing upstream firms, \( j = 1, 2 \) (superscripts on the variables), and two competing downstream firms, \( i = 1, 2 \) (subscripts on the variables). The upstream firms could for instance be publishers or developers of apps and the downstream firms could be platform providers, such as Apple, Google and Amazon.

Assume that the inverse demand curve for good \( j \) at downstream firm \( i \) is given by

\[
P^j_i = 1 - (q^j_i + \beta q^{-j}_i) - \gamma (q^{-j}_i + \beta q^{-j}_i).
\]

By inverting the inverse demand curve we have

\[
P^j_i = 1 - (q^j_i + \beta q^{-j}_i) - \gamma (q^{-j}_i + \beta q^{-j}_i).
\]
\[ q_i^j = \frac{(1 - \gamma)(1 - \beta) - P_i^j + \gamma P_{-i}^j + \beta (P_i^j - \gamma P_{-i}^j)}{(1 - \gamma^2)(1 - \beta^2)} \] (2)

As in Dobson and Waterson (2007), the parameter \( \gamma \in [0, 1) \) captures how similar the consumers perceive goods 1 and 2 to be when sold at the same downstream firm; the goods are demand independent if \( \gamma = 0 \) and perfect substitutes if \( \gamma \to 1 \). The parameter \( \beta \in [0, 1) \) likewise captures the substitutability between the downstream firms if they sell the same goods. For brick-and-mortar retailers (e.g. bookstores) the size of \( \beta \) may reflect a geographical dimension; if \( \beta = 0 \) the downstream firms are so far away from each other that they do not compete, while they are collocated and perceived as perfect substitutes if \( \beta \to 1 \). For digital platforms (selling e.g. ebooks or apps) the size of \( \beta \) may reflect how differentiated their services are. If consumers perceive Apple’s iPad and Amazon’s Kindle to be good substitutes, then \( \beta \) is high, and vice versa. The perceived similarity of good 1 and 2 when sold by different stores is increasing in the interactive term \( \beta \gamma \).

For the sake of simplicity we normalize costs to zero on both the upstream and downstream level.

Profit to downstream firm \( i \) is given by:

\[ \Pi_{Di} = s_i \left[ P_i^1 q_i^1 + P_i^2 q_i^2 \right] \]

while upstream firm \( j \)'s profit is

\[ \Pi^{Uj} = (1 - s_1) P_i^1 q_i^1 + (1 - s_2) P_i^2 q_i^2 \]

\( Di \) keeps a share \( s_i \in [0, 1) \) of the sales revenues it generates and the upstream firms a share \( 1 - s_i \). In our basic model we treat \( s_i \) as exogenously given. The reason is Apple’s "one size fits all" approach, which implies that it uses the same revenue-share across several industries (70/30 split for music, apps, and ebooks).

Below, we compare an outcome where downstream firms determine retail prices (no RPM) to one where upstream firms do so (RPM). Apple used the former business model when entering music distribution with iTunes, while they have used the latter
for apps in App Store and ebooks in iBookstore. We refer to the business format where upstream firms determine prices as the agency model.

As a benchmark, it is straightforward to show that aggregate industry profits are maximized by setting \( P^j_i = P_I = \frac{1}{2} \). The optimal prices from the industry’s point of view are thus independent of how similar the consumers perceive the goods and downstream services to be. However, less diversity typically implies that the total size of the market falls. Inserting for the optimal price into (1) we thus find

\[
q^j_i = q_I = \frac{1}{2(1 + \beta)(1 + \gamma)} \quad \text{and} \quad \Pi_I = \frac{1}{(1 + \beta)(1 + \gamma)} ,
\]

such that aggregate quantity and profits are decreasing in \( \beta \) and \( \gamma \). This is a standard property of quadratic utility functions, and holds quite generally when we have convex preferences/heterogenous consumers.

### 3.1 No RPM

Without RPM, downstream firm \( i \)'s optimization problem is

\[
\{ P^1_i, P^2_i \} = \arg \max D_i = s_i \left[ P^1_i q^1_i + P^2_i q^2_i \right] \tag{3}
\]

Solving (3) yields

\[
P^{no\ RPM} = \frac{1 - \beta}{2 - \beta} \tag{4}
\]

In line with conventional wisdom, this shows that revenue shares do not distort pricing:

**Lemma 1:** Assume no RPM. Retail prices are independent of whether downstream firm 1 requires a different revenue share than downstream firm 2 \(( s_i \neq s_{-i} )\).

The case with revenue-sharing but no RPM resembles Apple’s business format when they entered the music industry. According to Steve Jobs’ biography (Isaacson, 2011) all upstream firms (providers of music) were offered the same 70/30 split of revenues, while Apple decided that the price should be 0.99 cent per song. The retail
price was set low by Apple in order to stimulate the sale of iPods. For simplicity, we have not incorporated complementary goods into the present model (see discussion in the Introduction).

Above, we have implicitly assumed that each downstream firm uses the same revenue-share towards the two upstream firms \( (s_1^i = s_2^i) \). Let us now open up for the possibility that downstream firm \( i \) requires different revenue shares from upstream firm 1 and 2 (in contrast to Apple, which requires 30% from all content providers, big or small). Then, downstream firm \( i \)'s optimization problem becomes

\[
\{ P_1^i, P_2^i \} = \arg \max \Pi_{Di} = s_1^i P_1^i q_1^i + s_2^i P_2^i q_2^i
\]

The FOCs are given by

\[
\frac{d\Pi_{Di}}{dP_1^i} = s_1^i \left[ q_1^i + P_1^i \frac{dq_1^i}{dP_1^i} \right] + s_2^i P_2^i = 0
\]

The optimal level of \( P_1^i \) depends on revenue shares. Hence, we have the following result:

**Proposition 1:** Assume no RPM. Retail prices are then independent of revenue shares if and only if each downstream firm requires a common revenue share from the upstream firms, \( s_1^i = s_2^i = s_i \).

### 3.2 RPM: Industry-wide adoption of the agency model

Let us next consider an industry-wide adoption of the agency model. Then the upstream firms determine retail prices, and they solve

\[
\{ P_1^j, P_2^j \} = \arg \max \Pi^{ij} = (1 - s_1) P_1^j q_1^j + (1 - s_2) P_2^j q_2^j \text{ s.t. } q_1^j \geq 0
\]

This setting is in accordance with the agency model used towards upstream firms in App Store and iBookStore.\(^{11}\) Upstream firms set retail prices, and they all receive the same revenue share from downstream firm \( i \).

\(^{11}\)Cachon and Lariviere (2005), Mortimer (2008) and Dana and Spier (2001) highlight that a limitation of revenue sharing is the costs of monitoring revenues. A practical merit of revenue
Assume that both downstream firms offer both products \((q^j_i > 0)\). The FOC for upstream firm \(j\) when it determines \(D1\)'s sales price (with a similar expression for the price at \(D2\)) is then given by

\[
\frac{d\Pi^{Uj}}{dP^j_1} = (1 - s_1) \left[ q^j_1 + P^j_1 \frac{dq^j_1}{dP^j_1} \right] + (1 - s_2) P^j_2 \frac{dq^j_2}{dP^j_1} = 0
\]  

(5)

From the second term in (5) we see that the marginal profitability of increasing the price \(P^j_1\) is decreasing in \(s_2\) if the consumers perceive the downstream firms as (imperfect) substitutes (i.e., if \(\frac{dq^j_2}{dP^j_1} > 0\)). This means that the optimal level of \(P^j_1\) is lower the higher is \(s_2\). This is because if \(D2\) requires a larger share of the sales revenue, then the upstream firm will have incentives to sell more through \(D1\) and less through the rival. For the same reason \(P^j_1\) is increasing in \(s_1\).

Solving \(\{P^j_1, P^j_2\} = \arg \max \Pi^{Uj}\) for \(j = 1, 2\) we find

\[
P_i = \frac{(1 - \gamma) \left(1 - \beta^2\right) \left[1 - s_i + \beta(1 - s_i)\right]}{d} + (1 - \gamma),
\]

(6)

where \(d \equiv (2 - \gamma)^2 - \beta^2 \gamma^2 - \frac{(1 - \gamma) \beta^2 (2 - s_1 - s_2)^2}{(1 - s_1)(1 - s_2)} > 0\) whenever the second-order conditions hold and \(s_1, s_2 < 1\). We restrict the analysis to parameter values such that \(d > 0\). Note that the term in the square bracket is positive, so that we have a strictly positive price if there is imperfect competition both at the upstream and downstream level; i.e. \(\beta, \gamma \in (0, 1)\).

Since the upstream firms are symmetric, they will charge the same prices at any given outlet. We have therefore skipped the superscript in equation (6). However, in accordance with FOC (5), we find that if one downstream firm has a higher revenue share \(s_i\) than its rival, then its retail prices will also be higher:

\[
P_i - P_{-i} = (s_i - s_{-i}) \frac{(1 - \gamma) \beta (1 - \beta)(2 - s_1 - s_2)}{d (1 - s_1)(1 - s_2)} \geq 0\] if \(s_i \geq s_{-i}\).

Sharing schemes in digital markets as ebooks and apps is that marginal costs are practically equal to zero, such that revenue sharing approaches profit sharing. In most situations it is easier to monitor downstream revenue than downstream profits.

\[12\] Note that if the FOC describes an equilibrium, then the expression in the square bracket of (5) must be negative, since \(\frac{dq^j_2}{dP^j_1} > 0\) for \(\beta > 0\).
From (6) we can verify the following:

**Proposition 2 (the agency model):** Assume RPM. Retail prices differ if the downstream firms require different revenue shares ($P_i \neq P_{-i}$ if $s_i \neq s_{-i}$). An increase in $s_i$ induces the upstream firms to increase $P_i$ and reduce $P_{-i}$. Other things equal, these price changes are greater the larger is the substitution between downstream firms ($\beta$) and the smaller is the substitution between the upstream firms ($\gamma$).

In contrast to the case without RPM, we thus see that if one downstream firm requires a higher revenue share than its rival, then it also has to accept that its retail prices will be higher.

Proposition 2 reflects the fact that an upstream firm which increases the sales price at one of the downstream firms and reduces it at the other need not lose much sales. The main effect might rather be to shift sales to the downstream firm which has become more competitive; this is more likely to be true the closer substitutes the downstream firms are in the eyes of the consumers. For this reason $dP_i/ds_i$ is more negative (and $dP_{-i}/ds_i$ more positive) the larger is $\beta$. Conversely, if the upstream firms produce goods which the consumers perceive as close substitutes, then an upstream firm which unilaterally increases its price will lose much sales to its upstream rival.

Suppose $s_i = s_{-i}$. Then retail prices become

$$P_{rpm | s_i = s_{-i}} = \frac{1 - \gamma}{2 - \gamma}$$

(7)

By comparing (4) and (7) we have the following result:

**Proposition 3:** Assume $s_i = s_{-i}$. Retail prices are lower with RPM (the agency model) than without RPM if the degree of substitution is higher at the upstream level than at the downstream level (i.e. $\gamma > \beta$).

We can further state:

**Corollary 1:** Assume $s_i = s_{-i}$. Other things equal, RPM reduces profits for each firm if $\beta < \gamma$, and increases profits for each firm if $\beta > \gamma$. 
Transferring control of retail pricing to the level where the degree of competition is lowest thus brings prices closer to the ones that maximize total profit (i.e. the cartel prices). This provides interesting management insights when comparing the use of the agency model in the market for apps (App Store) versus the market for ebooks (iBookstore).

For different types of applications (apps) distributed through channels like Apple’s App Store and Google’s Google Play, it seems to be quite clear that transferring the control of retail pricing to upstream firms (developers of apps) involves aggressive price competition among upstream firms. In our context, it is reasonable to assume that $\gamma$ is relatively high compared to $\beta$. This is confirmed by complaints from app developers in Apple’s App Store that they cannot make profit (Boudreau, 2012). Downstream firms such as Apple are in principle in position to set retail prices on their own. Thus, when they choose to let upstream firms determine retail prices, the reason is probably related to factors outside those considered in the present model (most specifically to letting a thousand flowers bloom).

Before Apple entered the market with iBookstore, downstream firms (bookstores) determined prices both for printed and digital books (Department of Justice (DoJ), 2012). After Apple’s entry in the market for ebooks, there was a rapid (and almost) industry-wide transition to the agency model during the spring of 2010. DoJ (2012) conjectures that the key motivation for this transition was to stop the low prices set by Amazon on ebooks, described as "the $9.99-problem" by publishers. DoJ cites Steve Jobs as a support for this conjecture (from Isaacson, 2011): We’ll go to [an] agency model, where you set the price, and we get our 30%, and yes, the customer pays a little more, but that’s what you want anyway. In our model the introduction of the agency model would lead to higher prices only if upstream competition is lower than downstream competition (i.e. $\gamma$ lower than $\beta$). This is unlikely to be the case, since Amazon almost had a monopoly position in the downstream market for ebooks before Apple entered the market. Amazon’s motivation for reducing retail

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13 The previous business model in the US book market (the wholesale model) did not include revenue shares, but instead a unit wholesale price (a percentage rate of the cover price). Downstream firms were free to set the retail price below the cover price.
prices seem to be related to market introduction strategies and to stimulate the sale of Kindle (similar to what Apple did with music as mentioned above).

The upstream firms (publishers), in contrast, wants to protect the profit they earn from selling printed books. This makes upstream competition softer than it would otherwise be, allowing higher prices with than without RPM even if the competitive pressure is higher at the upstream than the downstream level. This seems to be the reason why Apple (Jobs) expected prices to raise when introduced the agency model (Isaacson, 2011): Amazon screwed it up. It paid the wholesale price for some books, but started selling them below cost at $9.99. The publishers hated that—they thought it would trash their ability to sell hardcover books at $28. So before Apple even got on the scene, some booksellers were starting to withhold books from Amazon.

This may as well explain why Apple wanted to use the agency model for ebooks. Ebooks seems to be quite analogous to music. In contrast to the market for apps, the motivation behind transferring control of retail prices to the upstream firms (publishers) was probably not to let the thousand flowers bloom (stimulate more innovation among publishers). As accentuated in the Introduction, according to Steve Jobs the agency model was a second best solution when they entered the market for ebooks (Isaacson, 2011).

3.2.1 The agency model: Downstream firms decide revenue shares

We now analyze how the downstream firms might set revenue shares, if these are determined prior to the upstream firms’ decision on retail prices. Stage 2 equilibrium prices are given from (6), and at stage 1 \( D_i \) solves \( s_i = \arg \max \Pi_{D_i} \), where

\[
\Pi_{D_i} = s_i \left( P_i^j q_i^j + P_i^{-j} q_i^{-j} \right).
\]

Upstream symmetry implies that the upstream firms set the same prices at any given downstream firm \( (P_i^j = P_i^{-j} = P_i) \), so we can write \( D_i \)'s first-order condition as

\[
\frac{d\Pi_{D_i}}{ds_i} = 2P_i q_i + 2s_i \left( P_i \frac{dq_i}{ds_i} + q_i \frac{dP_i}{ds_i} \right) = 0.
\]
From (8) we have the following candidate for a symmetric equilibrium:

\[ s_i = s^* \equiv (1 - \beta^2) \left( 1 + \gamma \frac{\beta}{2 - \gamma (1 + \beta)} \right); \quad \frac{ds^*}{d\beta} < 0 \text{ and } \frac{ds^*}{d\gamma} > 0. \]  

(9)

Clearly, equation (9) cannot describe an equilibrium unless \( s^* < 1 \). It follows that if \( s^* \) is an equilibrium, then the downstream firms voluntarily let the upstream firms keep a share of the revenue. The intuition for why \( s^* \) depends on \( \beta \) and \( \gamma \) is understood from Proposition 2, which de facto tells us that the opportunity costs for a downstream firm of increasing its revenue share are higher the stronger is downstream competition (the higher is \( \beta \)) and the weaker is upstream competition (the smaller is \( \gamma \)).

With \( s_1 = s_2 = s^* \) we find

\[ \Pi^*_D = \frac{2 (1 - \gamma) (1 - \beta)}{[2 - \gamma (1 + \beta)] (2 - \gamma) (1 + \gamma)}. \]

So does \( s^* \) constitute a Nash equilibrium? No, not necessarily. To see why, assume that \( D1 \) has set \( s_1 = s^* \). Inserting for (9) into (2) and (6) we can then write

\[ q_1 = \beta \frac{(1 - \gamma) [2 - (1 + \beta) \gamma] s_2 - (1 - \beta)^2 (2 - \gamma)}{d [2 - \gamma (1 + \beta)] (1 - s^*) (1 + \gamma)}. \]  

(10)

Provided that \( D1 \) sets \( s_1 = s^* \), it will thus be foreclosed from the market if \( s_2 \leq s^*_2 \), where

\[ s^*_2 \equiv \frac{(1 - \beta)^2 (2 - \gamma)}{(1 - \gamma) [2 - \gamma (1 + \beta)]} > 0; \quad \frac{ds^*_2}{d\beta} < 0 \text{ and } \frac{ds^*_2}{d\gamma} > 0. \]

Intuitively, \( sign\frac{ds^*_f}{d\beta} = sign\frac{ds^*_*}{d\gamma} < 0 \) because \( s_1 = s^* \) is decreasing in \( \beta \); a smaller \( s_1 \) requires a smaller \( s_2 \) to foreclose \( D1 \) from the market. Analogously, we have \( sign\frac{ds^*_f}{d\gamma} = sign\frac{ds^*_*}{d\gamma} \). A symmetric equilibrium in pure strategies where downstream firms set revenue shares is consequently more likely to exist the stronger is downstream competition and the weaker is upstream competition.

It now remains to check whether it is profitable for \( D2 \) to foreclose \( D1 \) from the market. With \( s_1 = s^* \) and \( s_2 = s^*_2 \) we find

\[ \Pi^*_{D2} = \frac{(1 - \beta)^2}{[2 - \gamma (1 + \beta)] (2 - \gamma) (1 + \gamma)}. \]
Foreclosure is thus not profitable if

$$\Pi_{D_i}^* - \Pi_{D_2}^* = \frac{2 (1 - \beta)}{[2 - \gamma (1 + \beta)] (2 - \gamma) (1 + \gamma)} (\beta - \gamma) > 0 \text{ if } \beta > \gamma.$$ 

Setting $\beta > \gamma$ into (9) shows that $s^* < 1$ in the relevant area. Remarkably, we thus have a Nash equilibrium in pure strategies if and only if imposition of RPM increases joint profits (see Appendix for proofs):

**Proposition 4:** Assume that $\beta > \gamma$. Then aggregate channel profits are higher with than without RPM, and there exists a symmetric equilibrium where the non-cooperative revenue-share equals $s^* \in (0, 1)$. The revenue share is the decreasing in $\beta$ and increasing in $\gamma$; $s'(\beta) < 0$ and $s'(\gamma) > 0$.

### 3.2.2 Some more implications

Conventionally, royalty rates and fixed fees decided on prior to retail price competition are considered as alternative instruments to redistribute profit. However, using fixed fees would not affect the upstream firms’ choice of retail prices at stage 2.

Since the downstream firms are symmetric, they will in equilibrium require the same revenue share. We know from above that when $s_i = s_{-i}$ the equilibrium price is given by (7); i.e. $P^{rpm} = (1 - \gamma) / (2 - \gamma)$. The analysis above makes it clear that greater substitutability (less differentiation) between the upstream goods reduces equilibrium prices ($P'(\gamma) < 0$) as well as the size of the market. Total industry profit is therefore unambiguously decreasing in $\gamma$. However, this does not necessarily mean that downstream profits are decreasing in $\gamma$. The reason is that a higher $\gamma$ increases the downstream firms’ revenue shares; $s'(\gamma) > 0$. From the downstream firms’ perspective we thus have the following trade-off with respect to the degree of differentiation among upstream firms:

$$\frac{d \Pi_D}{d \gamma} = 2 \left\{ s[P'(\gamma)q + Pq'(\gamma)] + s'(\gamma)Pq \right\} < 0.$$ 

We have the following result:
Proposition 5: Less upstream differentiation (higher $\gamma$) might increase downstream firm profit, since $s'(\gamma) > 0$.

Proposition 5 reflects a novel feature that is consistent with common sense: The lower the degree of upstream competition, the higher is the share of the revenues that downstream firms have to offer upstream firms. Innovation into more unique content (smaller $\gamma$) at the upstream level might therefore harm downstream firms, even if it should increase both the size of the market and equilibrium prices.

In the case at hand it seems reasonable to assume that wholesale terms of trade (revenue sharing) is decided prior to the decision of retail prices (both with and without RPM). Without RPM we often observe that wholesale terms of trade consist of a unit whole sale price. One example is the conventional business format for books in the US (the wholesale model) where the publishers determine a unit whole price prior to the bookstores decision on retail prices. In contrast, in several European book markets RPM is used. Then the wholesale terms of trade include revenue sharing instead of unit wholesale prices. Thus, the business format in European countries where RPM is used resembles the agency model. So why switch from a unit wholesale price to revenue sharing when transferring the control of retail prices from the downstream firms to the upstream firms? We now show the following:

Proposition 6: Assume that downstream firms use a unit wholesale price instead of revenue sharing in the agency model. Then, even if the downstream firms are in a position to offer the unit wholesale price as a take-it-or-leave-it contract, all profits are captured by the upstream firms.

Proof: For the sake of simplicity, let us abstract from competition ($\beta = 0$ and $\gamma = 0$). At stage 2 the upstream firms then solves $P = \arg \max \ Wq$, where $q = 1 - P$ s.t. $P \geq w$ (the downstream firm’s participation constraint). Hence, the upstream firms sets $P = w$.

This is in sharp contrast to the case considered above, where the downstream firm uses revenue sharing at stage 1 to earn positive profits unless there is perfect downstream competition. Furthermore, this reveals an advantage from the down-
stream firms’ point of view of using revenue sharing rather than unit wholesale prices in presence of RPM.

Dobson and Waterson (2007) do not consider revenue sharing, and the only tariff structure they consider is one with a fixed unit wholesale fee. Without RPM Dobson and Waterson assume that retail prices are determined after the wholesale prices, while with RPM retail prices are set (by the upstream firms) before the wholesale prices. Hence, there is a switch in timing with respects to wholesale terms of trade in their model when control of retail prices is transferred to upstream firms.

Remark (downstream firms decide both revenue shares and retail prices): Assume that downstream firms decide \( s_i \) at stage 1 and retail prices at stage 2. Then the upstream firms’ participation constraints are binding \( (s_i = 0) \) if and only if \( \beta = 0 \). If \( \beta = 0 \) the downstream firms have no incentives to deviate from this, since an increase in \( s_i \) has no impact on the rival downstream firm’s prices. By the same token, the upstream firms have no incentives to reject the offers since the downstream firms’ sales are independent when \( \beta = 0 \). Assume in contrast that \( \beta \in (0, 1) \) and that the downstream firms set \( s_i = 0 \). A deviation from \( D_{-i} \) such that \( s_{-i} = \varepsilon > 0 \) induces the upstream firms to reject \( D_i \)’s offer, such that all sales are directed to \( D_{-i} \) where the upstream firms make a positive profit. Hence, there does not exist any pure-strategy Nash equilibrium where both upstream firms accept the offers from both downstream firms when \( \beta > 0 \). However, if we allow for an bargaining process as in Dobson and Waterson (2007) the outcome would be \( s_i = 0 \) also with \( \beta > 0 \) (when downstream firms have all the bargaining power).

3.3 Asymmetric business formats: Only one downstream firm uses the agency model

We now consider the outcome in a case where \( D_1 \) uses RPM (the agency model) but \( D_2 \) does not. This means that \( D_2 \) decides \( P^1_2 \) and \( P^2_2 \) while upstream firm \( j \) decides \( P^j_1 \) \( (j = 1, 2) \). We shall assume that the downstream firms’ revenue shares are the
same \((s_1 = s_2 = s)\). The maximization problems are thus given by

\[
\max_{P_i} \Pi^{U_j} = \sum_{i=1}^{2} (1 - s) P_i^j q_i^j \quad \text{and} \quad \max_{P_2} \Pi^{D_2} = s \sum_{j=1}^{2} P_2^j q_2^j.
\]  

(11)

Solving (11) s.t. \(q_i^j > 0\), and subsequently using that \(P_i^j = P_i^{-j} \equiv P_i\), we find

the following reaction functions:\n
\[
P_1 = \frac{(1 - \gamma) (1 - \beta)}{(2 - \gamma)} + \beta P_2.
\]  

(12)

and

\[
P_2 = \frac{1 - \beta}{2} + \frac{\beta P_1}{2}.
\]  

(13)

\(D2\) does not at the outset care about upstream substitutability when he sets end-user prices; he is only concerned about how fiercely he competes with his rival.$^{16}$ Therefore reaction function (13) is a function only of \(P_1\) and \(\beta\). However, the upstream firms are concerned about both upstream and downstream competition when they set end-user prices at \(D1\). This explains why reaction function (12) also is a function of \(\gamma\), with \(\partial P_1/\partial \gamma < 0\).

Combining (12) and (13) we have

\[
P_{rpm}^1 = (1 - \beta) \frac{2 - \beta \gamma + 2 (\beta - \gamma)}{(2 - \beta^2) (2 - \gamma)}
\]  

(14)

\[
P_2 = (1 - \beta) \frac{2 - \beta \gamma + (\beta - \gamma)}{(2 - \beta^2) (2 - \gamma)}
\]  

(15)

From (14) and (15) we find

\[
P_{rpm}^1 - P_2 = (\beta - \gamma) \frac{1 - \beta}{(2 - \beta^2) (2 - \gamma)}
\]  

(16)

\footnote{In this asymmetric case there does not exist any pure-strategy equilibrium where the downstream firms decide \(s_1\) and \(s_2\).}

\footnote{All SOCs and stability conditions are satisfied for \(\gamma < 1, \beta < 1\); see Appendix.}

\footnote{Note that \((2 - \gamma - s_1 - s_2 + \gamma s_1) > 0\). Too see this, assume \(s_2 = 1\). Then \((2 - \gamma - s_1 - s_2 + \gamma s_1) = (1 - s_1) (1 - \gamma) > 0\).}
We can state:

**Proposition 7**: Assume that only $D_1$ adopts RPM, and suppose that $\beta > \gamma$.

(i) Retail prices at $D_1$ are higher than at $D_2$ ($P_{rpm}^1 > P_2$).

(ii) All retail prices are lower than in a symmetric RPM equilibrium, and higher than in a symmetric no-RPM equilibrium ($P_{rpm}^1 > P_{rpm}^1 > P_2 > P_{no \ rpm}$).

If $\gamma \geq \beta$ we have $P_{no \ rpm} \geq P_2 \geq P_{rpm}^1 \geq P_{rpm}$.

**Proof**: (i): follows from (16). (ii) Using (15) and (4) we find:

\[
P_2 - P_{no \ rpm} = (\beta - \gamma) \frac{\beta (1 - \beta)}{(2 - \beta) (2 - \beta^2) (2 - \gamma)} > 0 \text{ if } \beta > \gamma,
\]

while (14) and (7) give us

\[
P_{rpm}^1 - P_{rpm} = - (\beta - \gamma) \frac{\beta}{(2 - \beta^2) (2 - \gamma)} < 0 \text{ if } \beta > \gamma.
\]

To see the intuition for equation (16), suppose first that $\beta = 0$. Since we then have no downstream competition, $D_2$ will simply set monopoly prices; $P_2(\beta = 0) = 1/2$.

Retail prices at $D_1$, on the other hand, will reflect the competitive pressure between the upstream firms, so that we have $P_1(\beta = 0) = \frac{1 - \gamma}{2 - \gamma}$ (i.e., the same price as would result from an industry-wide adoption of RPM). This means that $P_2(\beta = 0) > P_1(\beta = 0)$. More generally, prices at $D_1$ will be relatively closely related to upstream competition, and prices at $D_2$ to downstream competition. $D_2$ thus has stronger incentives to undercut $D_1$’s retail prices the stronger is downstream competition compared to upstream competition, and for $\beta > \gamma$ we have $P_2 < P_1$. Note, however, that both retail prices are driven towards the marginal costs if $\beta \to 1$. Even if retail prices at $D_1$ are determined by the upstream firms, the Bertrand paradox cannot be avoided if the downstream firms are perceived to be undifferentiated. A retailer which is more expensive than his rival in this case would lose all his sale, regardless of who has set the prices.

From Corollary 1 we know that it is profitable for the downstream firms to impose RPM in the symmetric equilibrium if $\beta > \gamma$ and unprofitable if $\beta < \gamma$, other
things equal. So a natural question to ask is what determines whether it is the firm which has imposed RPM or its rival which is most profitable in the asymmetric equilibrium. In the Appendix we show the following:

**Proposition 8:** Let $0 < \beta_0 \equiv \frac{\sqrt{3-4\gamma}-(3-2\gamma)}{2(2-\gamma)} < \gamma$, and assume that only $D_1$ uses RPM. $D_1$ makes a higher profit than its rival if and only if $\beta \in (\beta_0, \gamma)$.

Proposition 8 is illustrated in Figure 1; $\Pi_{1D} - \Pi_{2D} > 0$ only for $\beta \in (\beta_0, \gamma)$. Consider first the point $\beta = \gamma$; here the two downstream firms have the same prices and profits. Intuitively, this is because upstream- and downstream competition are equally strong. Next, consider a small change in $\beta$ around the point $\beta = \gamma$. From its reaction curve, we know that $D_2$ only indirectly takes upstream competition into account when it sets prices. Since a small reduction in $\beta$ leads to a somewhat higher importance of upstream competition, profits for $D_2$ will consequently fall relative to the rival’s profit. An increase in $\beta$, on the other hand, makes downstream competition relatively stronger. This is directly taken into account by $D_2$’s reaction curve. We thus have $\Pi_{D_2} > \Pi_{D_1}$ if $\beta$ is larger than $\gamma$, and $\Pi_{D_2} < \Pi_{D_1}$ if $\beta$ is somewhat smaller. To see why $D_2$ nonetheless makes higher profits than $D_1$ if $\beta$ is sufficiently small, consider the extreme $\beta = 0$. Then, as noted above, $D_2$’s retail prices reflect its monopoly position. The same is not true for $D_1$; since it has delegated the pricing decisions to competing upstream firms, it is not able to utilize its downstream monopoly power. It follows immediately that $\Pi_{D_2}(\beta = 0) > \Pi_{D_1}(\beta = 0)$. 

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3.3.1 Retail MFN as an ancillary restraint

Apple’s agency model towards ebook publishers contains an ancillary restraint labeled "Most Favored Nation clause" (retail MFN) by DoJ (2012). This clause prevents any publisher from selling his books at higher retail prices through Apple’s iBookstore than the books are sold for elsewhere, independent of whether Apple’s rivals use RPM.\footnote{The clause is described in the following way by DoJ (2012): "[T]he MFN here required each publisher to guarantee that it would lower the retail price of each e-book in Apple’s iBookstore to match the lowest price offered by any other retailer, even if the Publisher Defendant did not control that other retailer’s ultimate consumer price."} So let us hold on to the assumption that only $D_1$ employs RPM.

If $\gamma > \beta$, we have from (16) that $D_1$’s prices are lower than $D_2$’s prices, and the MFN would not be binding. In contrast, when $\beta > \gamma$, an MFN clause does bind. We then have the following result:

\textbf{Proposition 9:} Assume that only $D_1$ employs RPM and has an MFN clause. If $\beta > \gamma$, the prices equal the outcome under industry-wide adoption of RPM:

![Figure 1: $\Pi_{1D} - \Pi_{2D}$ when only $D_1$ uses RPM.](image)
\[ p^{MFN}_{asym} = p^{rpm} = \frac{1 - \gamma}{2 - \gamma} \]

The MFN implies that \( D2 \) has no ability to undercut the prices the upstream firms determine for sale through \( D1 \). Thus, \( D2 \) has no incentives to reduce its prices as long as the prices in \( D1 \) are not above the cartel price. In contrast, it follows from the upstream firms’ reaction functions (still given by equation (12)) that the upstream firms would undercut each other until \( P = (1 - \gamma)/(2 - \gamma) \).

Hence, while Johnson (2012) show that MFN has no impact on downstream prices under an industry-wide adoption of the agency model (and thus has no value), we show that MFN increase prices and make prices uniform under an asymmetric structure when \( \beta > \gamma \). This makes MFN a binding constraint. When iPad was launched (January 2010) Steve Jobs was asked by Wall Street Journal why someone should by a book from Apple for $14.99 if the same book was offered for $9.99 from Amazon. Steve Jobs responded (Isaacson, 2011): That won’t be the case .... The price will be the same.

4 Conclusion

We set up a model with competition among upstream firms (content providers like publishers and developers of apps) and downstream firms (platform providers like Apple, Google and Amazon). In contrast to much of the existing literature, we assume that each downstream firm may supply products (ebooks, apps, and so forth) from all upstream firms, and that all upstream firms may serve all downstream firms. Independent of business format we presuppose that wholesale terms of trade consist of revenue sharing contracts. First, we treat the revenue sharing contract as exogenously given. We show that retail prices are lower with than without the agency model if the upstream competitive pressure is higher than the downstream competitive pressure. Then we assume that downstream firms (like Apple, Google and Amazon) are in position to offer revenue shares as take-it-or-leave-it contracts prior to upstream firms’ determination of retail prices. This is consistent with the
widespread presumption that firms like Amazon and Apple have higher bargain-
ing power over upstream firms than brick-and-mortar retailers (e.g. bookstores) had previously. Even if downstream firms offer take-it-or-leave-it revenue sharing contracts, a key outcome from our model is that the upstream firms’ participation constraints will in general not be binding. Thus, upstream firms capture positive shares of the profit. We show that equilibrium revenue shares determined by down-
stream firms like Apple depend on competition on both levels. Under asymmetric adoption of business formats (not all use the agency model) we show that upstream firms may make positive profits even if their products are perceived to be (almost) perfect substitutes.

An interesting implication of how the size of the revenue shares is determined is that it creates an ambiguous relationship between product variety and downstream profits. On the one hand, greater product variety (less upstream substitutability) implies that retail prices with RPM (prices decided by upstream firms) will be relatively high, i.e. closer to the ones we would observe under a cartel. In isolation, this is good for the downstream firms. On the other hand, it reduces upstream com-
petition, and this forces the downstream firms to increase the share of the revenues transferred to the upstream firms. This is bad for the downstream firms. Put dif-
ferently, greater product variety might increase equilibrium prices and even the size of the market, but could nonetheless reduce downstream profits. As a consequence, product development that increases product variety and consumers’ willingness to pay is not necessarily profitable for the downstream firms.

Finally, we emphasize that we have not endogenized the choice of business for-
mat. To scrutinize on the strategic interactions among downstream firms (decide the revenue split) and upstream firms (decide retail prices), we have with intention abstracted from other potential factors of the business format decision. In particu-
lar, both in the academic literature and among market players, it is argued that a key motivation for delegating retail pricing to content providers (upstream firms) is the existence of asymmetric information about demand; content providers are typi-
cally better informed about demand for their goods than are platform providers like Apple. Vertical separation between content provision and distribution, and allowing
the content providers to decide retail prices may then stimulate innovation (letting a thousand flowers bloom). If we consider different types of applications (apps) distributed through channels like Apple’s App Store and Google’s Google Play, it seems to be quite clear that vertical separation between content and distribution comes with the problem of destructive price competition among content providers. Obviously, platform providers such as Apple are in principle in position to set retail prices on their own. Thus, when they choose to let content providers determine retail prices, despite widespread complaints over destructive competition, the reason is probably related to factors outside those considered in the present model (most specifically to letting a thousand flowers bloom).

5 Appendix

5.1 SOCs and stability conditions, Section 3.3

The second-order conditions for \( D_2 \) are fullfilled: \( \frac{d^2 \Pi_{D2}}{d(P_2^2)^2} = \frac{d^2 \Pi_{D2}}{d(P_2^2)^2} = \frac{2s}{(1-\beta^2)(1-\gamma^2)} < 0 \) for \( \beta, \gamma < 1 \). For the upstream firms we likewise have \( \frac{d^2 \Pi_{Ui}}{d(P_i^2)^2} = \frac{2(1-s)}{(1-\beta^2)(1-\gamma^2)} < 0 \) for \( \beta, \gamma < 1 \) and \( s < 1 \).

The reaction function for \( U_i \) equals \( P_i^1 = \frac{(1-\beta)(1-\gamma)+\beta P_2^2+\gamma P_2^{-1}-\beta^\gamma P_2^{-1}}{2} \), so that we have \( \frac{dP_i^1}{dP_2} = \beta < 1, \frac{dP_i^1}{dP_1} = \gamma < 1 \) and \( \left| \frac{dP_i^1}{dP_2^{-1}} \right| = \beta \gamma < 1 \). For \( D_2 \) we find \( P_i^2 = \frac{s(1-\beta)(1-\gamma)+s^2 \beta P_2^2+2s \gamma P_2^{-1}-s \beta^\gamma P_2^{-1}}{2} \), where again all price derivatives are smaller than one in absolute value. All stability conditions are thus satisfied.

5.2 Proof of Proposition 8

Inserting for (14) and (15) into the profit functions we find

\[
\Pi_{1DM} = \frac{2s(1-\beta)(2-\beta^2)(2-\beta^2)}{D_1}, \quad \Pi_{2D} = \frac{2s(1-\beta)(2-2\beta^2+(\beta^2-\gamma)^2)}{D_1},
\]

\[
\Pi^{Ui} = \frac{(1-s)(1-\beta)(8(1-\gamma)(1+\beta)-(\beta^2-(\gamma+\beta)^2))(2-\gamma)^2}{D_1},
\]
where \( D_1 = (1 + \beta) (1 + \gamma) (2 - \beta^2)^2 (2 - \gamma)^2 \). This implies that

\[
\Pi_{1D}^{RPM} - \Pi_{2D} = \frac{2s (1 - \beta^2) \left( \frac{\sqrt{9 - 4\gamma} + 3 - 2\gamma}{2(2 - \gamma)} + \beta \right)}{(1 + \gamma) (2 - \beta^2)^2 (2 - \gamma) (1 + \beta)^2 (\beta - \gamma) (\beta_0 - \beta)},
\]

where \( \beta_0 = \frac{\sqrt{9 - 4\gamma} - (3 - 2\gamma)}{2(2 - \gamma)} \). We thus have \( \Pi_{1D}^{RPM} - \Pi_{2D} \) for \( \beta_0 < \beta < \gamma \). Q.E.D.

6 References


