

Product innovation and vertical integration: private and social incentives*

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Abstract

We study the licensing incentives of an independent input producer owning a patented product innovation which allows the downstream firms to improve the quality of their final goods. We consider general two-part tariff contracts for both outside and incumbent innovators and we consider the possibility for the patent holder to set a negative royalty, i.e. a subsidy. Under non-negative royalties, the vertical merger with either downstream firm is always privately profitable and it is welfare improving for large innovations: this implies that not all profitable mergers should be rejected. In contrast, if subsidies are allowed, exclusive licensing always takes place and vertical integration is never profitable.

Keywords: Patent licensing, two-part tariff, negative royalties, vertical differentiation, vertical integration.

JEL Classification: L15, L13, L24.

1 Introduction

There are several examples of innovation in input required for the production of final goods. The introduction of a faster computer chip; the wheat bread flour made in the stone mill as 200 years ago;¹ the Loro Piana innovation in new materials (super-luxury natural textiles).² These input innovations improve the

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¹It is a case of reswitching of techniques to traditional Italian whole wheat breads made 200 years ago when the local stone milled flour was the only option.

²This thread starts with Tasmanian wool in the 1970s, through vicuna (LP was granted 10-year exclusivity), baby cashmere to, most recently, lotus flower cloth (i.e., the fibers of *nelumbo nucifera*, an aquatic perennial more commonly known as the lotus).

quality of the final good and/or possibly create vertical differentiation in the product market.

The soybean seed market makes another case study. As pointed out by Shi and Chavas (2011), recent advances in biotechnology have led agricultural biotech firms (who produce the seeds sold to farmers) to differentiate their seed products through patented genetic materials. The vertical organization of this industry has changed. While biotech firms producing patented genes have relied extensively on licensing their technologies to seed companies, they have recently increased their use of vertical coordination through integration.

We develop a theoretical model to study the incentives of an input innovator to diffuse its innovation either through licensing or through vertical integration. As a result of the technology diffusion decision the final product market is either vertically differentiated or homogeneous. We further wonder whether the private incentives are in line with the social incentives. The social profitability of vertical integration is indeed an important issue with no clearcut results. On one hand theoretical contributions, like Rey and Tirole (2007), point out that vertical merger is welfare detrimental because of market foreclosure.³ Some empirical evidence supports this view. For instance, Shi and Chavas (2011), in the context of the U.S. soybean seed market, develop an empirical investigation of how differentiated seed products are priced in concentrated markets under two alternative vertical structures: vertical integration versus licensing. They use a Cournot model of multi-product firms and find that seeds sold through vertically-integrated structures are priced higher than those that are licensed. On the other hand, however, the merger guidelines do not exclude that vertical merger could increase social welfare for efficiency reasons.

More precisely, we consider two downstream firms producing and selling a final output to heterogeneous consumers and two differentiated inputs in the upstream market, a low quality input provided by competitive firms and a high quality patented input provided by an independent input producer. The quality of the final good depends on the quality of the input. Complete technology diffusion implies a homogeneous final good of high quality, whereas exclusive licensing implies a vertically differentiated market. We consider a general two-part tariff contract for both outside and incumbent innovators. In particular, the set of possible contracts, that are observable, is such that the fixed fee has to be non-negative, whereas the per-unit royalty might also be negative. The motivation for such assumption comes from the observation that, whereas negative fixed fees would be clearly held to be illegal by antitrust authorities, a negative per-unit royalty, that is a per-unit subsidy, cannot a priori be considered welfare detrimental. Most of the literature on licensing focuses on non-negative royalties, however some recent contributions point out the private incentives to subsidize the downstream production (Liao and Sen (2005) and Milliou and Petrakis (2007)), that in some cases are also welfare improving. From a practical point of view, as pointed out by Liao and Sen (2005), per-unit subsidies are rarely observed in licensing contracts, however they might be present implicitly

³We later make a brief review of the literature on vertical integration.

in the form of transmission of knowhow and technical assistance to the licensee. We endogenize market structure allowing the patent holder to vertically integrate with either downstream firm. We also provide the welfare analysis.

In case of *non-negative royalties*, we find that *technology diffusion* critically depends on the nature of market competition (Cournot vs. Bertrand). When firms compete in the quantities, the innovation is sold to all firms thus ensuring complete technology diffusion as well as a high quality homogeneous good in the market. In contrast, when firms compete in the prices, the innovator has no incentive for complete technology diffusion, rather he prefers exclusive licensing which implies a vertically differentiated market and in turn positive industry profit to extract. In particular the internal patent holder does not license its innovation to the rival firm; the external patent holder sells only one license via a fixed fee. The intuition behind this result relies on Bonanno (1986) where the author investigates the effect of the type of competition (Bertrand vs Cournot) on firms' incentives to (vertically) product differentiate. He shows that (with zero production costs) the two firms decide to produce a homogeneous high quality product under Cournot competition and differentiated products under Bertrand competition. The reason is that under vertical differentiation firms have the incentive to improve the quality of their products, since consumers willingness to pay increases with quality. However, under Bertrand competition there is a counteracting tendency as the more similar the products are the tougher the price competition is. Under Cournot competition in contrast this counteracting tendency is not at work so that at equilibrium both firms produce the high quality good.

As far as the *merger profitability* is concerned, we show that under Cournot competition the vertical integration of the upstream inventor with either downstream firm is always privately profitable. This result is in line with the new market foreclosure theory (see Rey and Tirole 2007) according to which vertical integration allows the monopolist upstream producer to protect its monopoly power. This result is in contrast with Sandonis and Fauli-Oller (2006) that consider non-drastring process innovations in a horizontally differentiated Cournot duopoly and find that the merger is privately profitable only for small innovations. They point out the commitment problem faced by the vertical merger (that is the insider innovator) which has only one instrument, the licensing contract to the rival firm rather than two (a licensing contract to each downstream firm), and cannot credibly restrict its output as the new input is transferred at marginal cost. However this result breaks down in our model as the incentive to diffuse the innovation makes homogeneous the downstream market.

As for *social welfare profitability*, we find that under Cournot competition the merger is also welfare improving for large innovations; this implies that not all profitable mergers should be rejected. Indeed on one hand, the merger pushes prices down as it implies the (partial) internalization of the vertical externality; on the other hand, the merger has an anticompetitive effect because the vertically integrated firm is able to (partially) foreclose the rival firm via a positive per-unit royalty. The first effect prevails as long as the quality improvement associated with the innovation is sufficiently large. Thus, a very simple

prescriptions arises: the antitrust authority should approve mergers where large innovations are involved. Our result is in line with the European Commission, who mentions for example in its recent Guidelines on the assessment of non-horizontal mergers (2008): “The Commission may decide that, as a consequence of the efficiencies that the merger brings about, there are no grounds for declaring the merger incompatible with the common market pursuant to Article 2(3) of the Merger Regulation. This will be the case when the Commission is in a position to conclude on the basis of sufficient evidence that the efficiencies generated by the merger are likely to enhance the ability and incentive of the merged entity to act pro-competitively for the benefit of consumers, thereby counteracting the adverse effects on competition which the merger might otherwise have”. Under Bertrand competition, we find a result of equivalence between an external and an internal patent holder, both in terms of private and social welfare profitability. Indeed, an external patent holder optimally sells an exclusive license via a fixed fee, so that there is no distortion due to a positive per-unit royalty (as if the patent holder were vertically integrated). This way the patent holder maximizes the licensee’s profit and extract this profit up to the outside option.

In case of *negative royalties*, the main difference occurs under Cournot competition in the vertical separation scenario, that is when the patent holder remains out of the market. Namely, the outside inventor optimally sells an exclusive license via a two-part tariff contract that specifies a per-unit subsidy. The intuition is that the innovator via the subsidy makes the unique licensee more aggressive in the Cournot market and reap higher profit via the fixed fee (Fershtman and Judd, 1987). Thus exclusive licensing takes place under both Cournot and Bertrand competition. As for the merger profitability, the patent holder always prefers to stay out of the market and private and social incentives always match: vertical integration never takes place.

We extend our analysis modelling the vertical relationship between the U producer and the two D firms through a *bargaining setting* where firms with market power negotiate over a nonlinear (two-part tariff) input price. In this we follow previous work as Milliou and Petrakis (2007). They study incentives for horizontal mergers in the upstream sector of vertically related industries when bargaining is present and contract types are endogenous. In particular, under the upstream merger scenario they model the bargaining process in such a way that the U monopolist bargains *simultaneously and separately* with the two D firms. This key assumption captures the fact that the upstream monopolist has an incentive to behave opportunistically during its negotiations. That is, it cannot commit to each downstream firm that it will not negotiate a more favourable contract with the rival firm. This implies that the per-unit royalty cannot be higher than the marginal cost of input production. This scenario then coincides with the case of private vertical contracts (see also, among others Rey and Vergé (2008) and Rey and Tirole (2007)) to be compared with observable contracts with which we deal in the rest of the paper.

1.1 Related literature

This paper contributes to the literature on licensing a product innovation as well as to the debate on the competitive effects of vertical integration.

From a theoretical viewpoint, most of the literature on optimal licensing focuses on cost-reducing, *process innovations* (see Kamien and Tauman, 1986; Katz and Shapiro, 1986; Kamien, Oren and Tauman, 1992; Sen and Tauman, 2007; Erutku and Richelle, 2007). To our knowledge, little has been done to investigate the issue of licensing a *product innovation*. A first contribution is by Kamien, Tauman and Zang (1988). They consider the introduction of a new product in a Cournot oligopoly focusing on the fixed fee licensing mode. More recently, Lemarie (2005) compare fixed fee versus royalty for a demand-enhancing innovation. We depart from them in what we consider an innovation that improves the quality of a product.⁴ Notice also that this literature focuses on contracts that do not allow for negative per-unit royalties, whereas we extend the analysis to the case of per-unit subsidies.

As far as the non-negative royalty case is concerned, we find that under Cournot competition with homogeneous goods, the external patent holder optimally specifies positive per-unit royalties when the innovation is large. This is in line with the wide prevalence of per-unit royalties over fixed fee in practice (see for instance, Rostoker (1984)). Moreover, Muto (1993) studies optimal licensing of a process innovation and shows that Bertrand competition is a rationale to explain this empirical evidence. We argue that this theoretical result does not hold for a product innovation: in our model under Bertrand competition the external patent holder prefers fixed fee over royalty licensing. The reason for this contrasting result is as follows. Muto (1993)'s general intuition behind the comparison with Cournot competition (where fixed fee always prevails over royalty, see Kamien and Tauman, 1986; Katz and Shapiro, 1986; Kamien, Oren and Tauman, 1992) is that the patentee's profit in the royalty policy depends on the equilibrium total output level, and this amount is much greater in Bertrand competition than in Cournot. In our model in contrast under Bertrand competition the patent holder only sells an exclusive license so that this equilibrium total output advantage is not as strong as in Cournot where complete technology diffusion takes place. Rather under Bertrand, the incentive to set a fixed fee prevails in order to avoid the distorting negative effect of a positive per-unit royalty.

Arya and Mittendorf (2006) studies the incentives of an internal patent holder to license a product innovation in the presence of a monopolist input producer. In their model the decision to license the innovation implies giving up the monopoly power on the final good, that is letting a Cournot rival downstream firm enter the market. They find that the licensing incentive is positive as it affects the input pricing terms in a way that is beneficial for the internal

⁴Stamatopoulos and Tauman (2008) analyse optimal licensing for an outside inventor of an innovation that improves the quality of a product and also affects its marginal cost (the innovation can be classified as product and process at the same time). They consider the logit demand framework with price competition.

patent holder. We also find that the internal innovator has incentive to license the new good to the rival, however, the reason is to extract some surplus from the rival firm that is active in the market independently of the licensing strategy.

As for the negative royalty case, to the best of our knowledge the main contributions are Liao and Sen (2005) and Milliou and Petrakis (2007). Our paper is more closely related to Liao and Sen (2005) that study optimal licensing of a process innovation in a Cournot duopoly with homogeneous goods. They show that subsidy-based contracts are optimal under exclusive licensing but not under complete technology diffusion. They conclude that negative royalties are welfare improving with respect to non-negative royalties if the innovation is sufficiently small. They point out the importance of the cost of exclusion: the additional cost that the non-licensee firm has to pay under exclusive licensing. As long as this cost is low, exclusive licensing (that arises with negative royalties) is preferred with respect to complete technology diffusion. We depart from this paper in what we endogenize market structure so that under non-negative royalties vertical integration always takes place whereas under negative royalties vertical integration becomes unprofitable. Therefore, in our model allowing for negative royalties is always welfare improving because vertical integration implies a quasi monopoly outcome to be compared with the exclusive licensing scenario that implements a Stackelberg equilibrium.

As for the competitive effects of vertical integration, there are two opposite views. The Chicago School (e.g., Bork, 1978; Posner, 1976) stresses that, in the absence of efficiency gains, vertical integration could not increase the profitability of merging firms. In contrast the new market foreclosure theory points out that vertical integration can restrict downstream competition. In particular in the recent survey by Rey and Tirole (2007) it is shown that vertical integration is always profitable as the U producer can restore its monopoly power through input foreclosure but vertical integration is anticompetitive. In our framework, under non-negative royalties, vertical integration remains privately profitable but the social welfare profitability depends on the innovation size. In contrast, if we allow for negative royalties, vertical integration becomes privately and socially unprofitable. Reisinger and Tarantino (2012) extend the theory of Rey and Tirole (2007) by analysing the profitability of vertical integration in the presence of complementary input producers and show that vertical integration can be privately unprofitable.

The remainder of the paper is structured as follows. In Section 2, we set up the ante-innovation model. In Section 3 we introduce the product innovation and we study the licensing incentives of an external innovator. In Section 4 we consider the optimal strategy of an internal innovator. In Section 5, we compare the private and social incentives. In Section 6 we extend the analysis to Bertrand competition.

2 Model

2.1 Status quo

We consider two firms producing a homogeneous good and competing à la Cournot.⁵ Final output production requires an essential input provided by a competitive upstream market.

As far as the *demand side* is concerned, we assume that there is a continuum of consumers indexed by θ which is uniformly distributed in the interval $[0, 1]$. Thus, θ is a taste parameter. Each consumer has a unit demand and buys either one unit of a good of quality s at price p or buys nothing at all. Consumer's utility takes the following form:

$$U(\theta) = \begin{cases} \theta s - p, & \text{if consumer type } \theta \text{ buys} \\ 0, & \text{if does not buy} \end{cases}$$

The demand for the good is then

$$Q(p) = 1 - \left(\frac{p}{s}\right) \iff p(Q) = s(1 - Q), \quad (1)$$

where $Q = q_1 + q_2$ and p/s is the fraction of consumers with a taste parameter less than θ , that is the fraction of consumers not buying the good.⁶ For future reference we define the consumer surplus as

$$CS(s) = \int_{\frac{p}{s}}^1 (\theta s - p) d\theta = \frac{(p - s)^2}{2s} \quad (2)$$

As for the *supply side*, the essential input of quality s is produced at zero fixed cost $f_L = 0$ and at constant marginal cost $c = 0$ and it is sold at the competitive price $w = 0$. In this framework quality is assimilated to input. The D firm i profit function is: $\pi_i = pq_i$. D firms compete in the quantities, then the Cournot duopoly equilibrium is (superscript C stands for Cournot):

$$q_i^C (c_i = 0, c_j = 0; s, s) = \frac{1}{3} \quad (3)$$

$$p^C = \frac{s}{3} \quad (4)$$

$$CS^C = \frac{2}{9}s$$

The D firms' price and profits depend on the quality of the input s . The D firms sell $Q = q_1 + q_2 = \frac{2}{3}$ which is the demand for the input faced by the upstream market (perfect vertical complementarity).

⁵Given a homogeneous final good, price competition leads to the Bertrand paradox. We extend the analysis to Bertrand competition in Section 6.

⁶At equilibrium the market is not covered, that is $Q < 1$.

For future reference (and as a benchmark) consider the monopoly outcome for this market:⁷

$$p^m = \frac{s}{2}, q^m = \frac{1}{2}, \pi^m = \frac{s}{4} > 2\pi_i^C.$$

2.2 Innovation

Suppose that an independent input producer obtains a patented product innovation which allows the downstream firms to improve the quality of their final goods. In the upstream market there is now a monopolist selling an input that ameliorates final product quality by $\psi > 1$ that measures the innovation size. Assume the unique production cost is the fixed cost $f_H = f > 0$ for the high quality input. Marginal cost of production is then zero.

We study the licensing incentives of this patent holder. The U firm can sell the new input either to one or both D firms via a two-part licensing contract (r, F) .⁸ We consider the set of possible contracts such that $r \in \mathbb{R}$ and $F \geq 0$, that is we allow for negative royalties. Indeed, whereas negative fixed fees would be clearly held to be illegal by antitrust authorities as they could be a means to strand the rival firm out of the market, a negative per-unit royalty, that is a per-unit subsidy (given that the marginal cost is equal to zero), cannot a priori be considered welfare detrimental. Most of the literature of licensing focuses on non-negative royalties, however some recent contributions point out the private incentives to subsidize the downstream production (Liao and Sen (2005) and Milliou and Petrakis (2007)), that in some cases are also welfare improving.

According to the U innovator licensing incentives, different cases derive:

1. *Complete technology diffusion*: both D firms adopt the new input and we have a homogeneous final good of quality $s\psi > s$. D firms' profits $\pi_i(c_i, c_j; s\psi, s\psi)$ depends on the two part-tariff contracts $c_i = (r_i, F_i)$ with $i = 1, 2$ and $i \neq j$.
2. *Exclusive licensing*: only one of the D firms adopt the new input and we have two final goods of different qualities. The non-innovating firm, say firm 1, produces the low quality good thus incurring zero production costs and gains $\pi_1(0, c_2; s, s\psi)$; while the innovating firm 2 produces the high quality good and gets $\pi_2(c_2, 0; s\psi, s)$.

3 Vertical separation (external patent holder)

Under vertical separation, that is the scenario in which the innovator stays out of the market, we develop a three-stage game: first, the innovator offers a contract to each D firm on a take-it-or-leave-it basis; second, the potential licensees decide whether to accept or reject the contract; finally the D firms

⁷The monopolist maximization problem would be $\max_p [p(1 - p/s)]$.

⁸A two-part licensing contract covers both the case of the royalty and the case of the fixed fee licensing modes: a royalty can be seen as a two-part tariff with $F = 0$; the fixed fee can be seen as a two-part tariff with $r = 0$.

compete. Note that all the contracts are observable.⁹ Solving backwards, we find the subgame perfect Nash equilibrium.

3.1 Exclusive licensing

Suppose that the patent holder decides to sell only one license to firm 2. Therefore, Firm 1 does not adopt the new input and produces a final good of quality $s_1 = s$ at price p_1 ; firm 2 adopts the new input and produces a final good of quality $s_2 = \psi s > s$, at price p_2 with $s_2 - s_1 = s(\psi - 1)$. The demands for the goods are:

$$q_1 = \hat{\theta} - \frac{p_1}{s} \quad (5)$$

$$q_2 = 1 - \hat{\theta}, \quad (6)$$

where

$$\hat{\theta} = \frac{p_2 - p_1}{s(\psi - 1)}.$$

The inverse demands are:

$$\begin{aligned} p_1 &= s(1 - q_2 - q_1) \\ p_2 &= s(\psi - \psi q_2 - q_1) \end{aligned}$$

D firms profits are:

$$\begin{aligned} \pi_1 &= p_1 q_1, \\ \pi_2 &= (p_2 - r_2) q_2 - F_2. \end{aligned}$$

Given the third stage equilibrium quantities,¹⁰ under the assumption of *non-negative royalties*, the U firm chooses the two-part tariff contract for firm 2 (r_2, F_2) such that:

$$\begin{aligned} &\max_{r_2, F_2} \Pi_U \\ s.t. r_2 &\in \left[0, \frac{s(2\psi - 1)}{2} \right] \\ F_2 &\leq \pi_2(r_2, 0; s\psi, s) - \frac{s}{9} \end{aligned}$$

with $\Pi_U = \frac{(2s\psi - s - 2r_2)}{s(4\psi - 1)} r_2 + F_2 - f$. The first constraint comes from the non-negativity of q_2 and the second constraint (binding at equilibrium) ensures that firm 2 has the incentive to get the license rather than the status quo.¹¹ The

⁹This is a usual assumption in patent licensing.

¹⁰More formal details on the solution of this and the other subgames are in Appendix.

¹¹We assume that the patent holder decides to sell an exclusive license to firm 2 and if firm 2 does not accept no firm innovates.

solution is a contract such that $r_2^{EL} = 0$ and $F_2^{EL} = \frac{(36\psi^2 - 16\psi + 1)(\psi - 1)s}{9(4\psi - 1)^2}$.¹² Remaining equilibrium variables are (superscript EL stands for exclusive licensing):

$$\begin{aligned} q_1^{EL} &= \frac{\psi}{(4\psi - 1)}, q_2^{EL} = \frac{(2\psi - 1)}{(4\psi - 1)}, Q_{EL} = \frac{(3\psi - 1)}{(4\psi - 1)} \\ p_1^{EL} &= \frac{\psi s}{(4\psi - 1)}, p_2^{EL} = \frac{(2\psi - 1)\psi s}{(4\psi - 1)}, \\ \pi_1^{EL}(s, s\psi) &= \frac{\psi^2 s}{(4\psi - 1)^2} < \pi_2^{EL}(s\psi, s) = \frac{s}{9} \\ \Pi_U^{EL}(s, s\psi) &= \frac{(36\psi^2 - 16\psi + 1)(\psi - 1)s}{9(4\psi - 1)^2} - f. \end{aligned}$$

Π_U^{EL} is the U patent holder equilibrium profit under exclusive licensing, when selling via a two-part tariff, which reduces to a fixed fee, the new input to only one D firm. For completeness we provide equilibrium consumer surplus under exclusive licensing:

$$CS^{EL} = \frac{(\psi + 4\psi^2 - 1)s\psi}{2(4\psi - 1)^2}.$$

Consider now the case of *negative royalties*, the optimal contract under exclusive licensing is then such that:¹³

$$r_2^{Neg} = -\frac{s}{4}, F_2^{Neg} = \frac{1}{36}s(9\psi - 4). \quad (7)$$

Equilibrium variables are:

$$\begin{aligned} \pi_2(r_2, 0; s\psi, s) &= \frac{1}{9}s, \pi_1(0, r_2; s, s\psi) = \frac{1}{16}s \\ q_1(0, r_2; s, s\psi) &= \frac{1}{4}, q_2(r_2, 0; s\psi, s) = \frac{1}{2}, Q^{ELneg} = \frac{3}{4} \\ p_1(0, r_2; s, s\psi) &= \frac{1}{4}s, p_2(r_2, 0; s\psi, s) = \frac{1}{4}s(2\psi - 1), \\ \Pi_U^{ELneg} &= \frac{1}{72}s(72\psi - 17) - f \\ CS^{ELneg} &= \frac{1}{32}(4\psi + 5)s \\ PS^{ELneg} &= \frac{1}{16}s(16\psi - 1) - f \\ SW^{ELneg} &= \frac{3}{32}s(12\psi + 1) \end{aligned}$$

¹²This is because the objective function is decreasing in r_2 , that is the incentive is to set a royalty as low as possible. Given the non-negative constraint, the solution is $r = 0$.

¹³ Π_U is concave wrt r_2 . As $\frac{\partial}{\partial r_2}\Pi_U = \frac{(s+4r_2)(1-2\psi)}{(4\psi-1)^2s} \geq 0 \iff r_2 \leq -\frac{s}{4}$, the objective is maximized at a negative value of r_2 , namely $r_2 = -\frac{s}{4}$.

The equilibrium we get coincide with Stackelberg where the licensee has a first mover advantage. The intuition behind this optimal subsidy relies on the incentive of the U monopolist to make the D licensee more aggressive (strategic delegation literature, Fershtman and Judd, 1987). This is in line with Liao and Sen (2005).

3.2 Complete technology diffusion

Suppose the U firm decides to sell the new input to both D firms via a *two-part tariff* (r, F) . The U firm maximization problem is, under nonnegative royalties:

$$\begin{aligned} \max_{F_1, r_1, F_2, r_2} \quad & \{r_1 q_1(r_1, r_2; s\psi, s\psi) + r_2 q_2(r_1, r_2; s\psi, s\psi) + F_1 + F_2 - f\} \\ \text{s.t.} \quad & \pi_1(r_1, r_2; s\psi, s\psi) - F_1 \geq \pi_1(0, r_2; s, s\psi) \\ & \pi_2(r_2, r_1; s\psi, s\psi) - F_2 \geq \pi_2(0, r_1; s, s\psi) \\ & r_1 \geq 0, r_2 \geq 0, F_1 \geq 0, F_2 \geq 0 \end{aligned}$$

where $\pi_1(0, r_2; s, s\psi) = \frac{(s\psi + r_2)^2}{(4\psi - 1)^2 s}$. Here the outside option for each firm is not buying the new input (thus producing the low quality good at zero marginal cost) given that the rival firm does. Solving the complete technology diffusion subgame, we find that the optimal contract is:

$$\begin{cases} r_1 = r_2 = r^T = \frac{(s\psi - 26s\psi^2 + 16s\psi^3)}{64\psi^2 - 14\psi + 4}, F^T(r^T, r^T) = \frac{(248\psi^2 - 17\psi - 272\psi^3 + 256\psi^4 + 1)s\psi}{4(32\psi^2 - 7\psi + 2)^2} & \text{if } \psi > 1.5856, \\ r_1 = r_2 = 0, F^T(0, 0) = \frac{(\psi - 1)(16\psi - 1)s\psi}{9(4\psi - 1)^2} & \text{if } \psi < 1.5856. \end{cases} \quad (8)$$

where $r^T = \frac{(s\psi - 26s\psi^2 + 16s\psi^3)}{64\psi^2 - 14\psi + 4} \geq 0 \iff \psi \geq 1.5856$. This means that when the innovation is small the inventor's incentive is to set a per-unit price as low as possible, that is the optimal contract is a fixed fee. In other words, given the positive outside option of producing the low quality good at zero marginal cost, for small innovations the inventor cannot set a positive per unit royalty. In contrast for large innovations we have a positive per-unit royalty.¹⁴

If we allow for negative royalties, the optimal contract is the first line of (8) for any innovation size. Then under a non-constrained maximization problem, for a small innovation size ($\psi < 1.5856$), the U monopolist has the incentive to set a negative per-unit royalty, that is to subsidize the production of the new product and to extract market profits via the fixed fees. In this case the explanation is not the incentive to make the D affiliates more aggressive but the presence of the outside option: each D firm can always produce the low quality good at zero marginal cost and make positive profit. To induce them to buy the small innovation the inventor cannot set a positive per unit royalty.

Equilibrium magnitudes are for $\psi > 1.5856$ (superscript T stands for com-

¹⁴This result is in line with Sen and Tauman (2007).

plete technology diffusion), and for any innovation size under negative royalties:

$$\begin{aligned}
q^T(s\psi) &= \frac{(4\psi+16\psi^2+1)}{2(32\psi^2-7\psi+2)}, Q^T = \frac{(4\psi+16\psi^2+1)}{(32\psi^2-7\psi+2)} \\
p^T &= \frac{(16\psi^2-11\psi+1)\psi s}{(32\psi^2-7\psi+2)} \\
\pi^T(s\psi) &= \frac{(4\psi+16\psi^2+1)^2 s\psi}{4(32\psi^2-7\psi+2)^2} \\
\pi^T(s\psi) - F^T(r^T, r^T) &= \frac{25(4\psi-1)^2 s\psi^2}{4(32\psi^2-7\psi+2)^2} \\
\Pi_U^T(s\psi) &= 2r^T q^T(s\psi) + 2F^T(r^T, r^T) - f = \frac{(16\psi^2-16\psi+1)s\psi}{2(32\psi^2-7\psi+2)} - f \\
CS^T &= \frac{(4\psi+16\psi^2+1)^2 s\psi}{2(32\psi^2-7\psi+2)^2}, \\
SW^T &= \frac{3(16\psi^2-6\psi+1)(4\psi+16\psi^2+1)s\psi}{2(32\psi^2-7\psi+2)^2} - f.
\end{aligned}$$

Whereas for $\psi < 1.5856$, in case of non-negative royalties, equilibrium magnitudes are:

$$\begin{aligned}
q_d^T(s\psi) &= \frac{1}{3}, Q^T = \frac{2}{3} \\
p_d^T(s\psi) &= \frac{s\psi}{3} \\
\pi_d^T(s\psi) &= \frac{s\psi}{9} \\
\pi_d^T(s\psi) - F_d^T(0, 0) &= \frac{\psi^2 s}{(4\psi-1)^2} \\
\Pi_{Ud}^T(s\psi) &= 2F_d^T(0, 0) - f = 2\frac{(\psi-1)(16\psi-1)s\psi}{9(4\psi-1)^2} - f, \\
CS_d^T &= \frac{2}{9}\psi s, \\
SW_d^T &= \frac{4}{9}\psi s - f
\end{aligned}$$

Note that this equilibrium coincides with the duopoly outcome as there is no double marginalization (underscript d stands for duopoly).

Comparing equilibrium variables under exclusive licensing and complete technology diffusion, we find the following results.

Proposition 1 *Under Cournot competition, (i) in case of non-negative royalties, the external patent holder always prefers complete technology diffusion, namely, $\Pi_U^T(s\psi) - \Pi_U^{EL}(s\psi) > 0$. The optimal contract is specified in (8). (ii) If we allow for negative royalties, exclusive licensing always prevails over complete technology diffusion, namely, $\Pi_U^T - \Pi_U^{ELneg} < 0$. The optimal contract is specified in (7).*

Under the case of non-negative royalties, two remarks are worthy. First, in contrast with a process innovation, even large product innovations are sold to

both firms as they both survive in the market. A consequence of the innovator's preferences towards technology diffusion is that the downstream market is not vertically differentiated, as only the high quality input is sold to both firms. In other words, the downstream market is characterized by a homogeneous high quality good.

Secondly, note that as the contracts offered to each D firm are observable, the U monopolist could implement the monopoly outcome. In particular, the U monopolist could indeed set a royalty such that each firm produces half monopoly quantity:

$$\begin{aligned} q_1 &= \frac{1}{3s\psi} (s\psi - 2r_1 + r_2) = \frac{1}{4} \\ q_2 &= \frac{1}{3s\psi} (s\psi - 2r_2 + r_1) = \frac{1}{4} \end{aligned}$$

$\Leftrightarrow r_1 = \frac{1}{4}s\psi, r_2 = \frac{1}{4}s\psi$. This implies the following total quantity, price, U profit and industry profit: $Q = \frac{1}{2}$, $p\left(\frac{1}{2}\right) = s\psi\left(1 - \frac{1}{2}\right) = \frac{1}{2}\psi s$; $\Pi_U^m = \frac{1}{4}s\psi\frac{1}{2} + 2\left(\frac{1}{16}\psi s - \frac{25}{16}(4\psi - 1)^{-2}\psi^2 s\right)$, $\Pi = \frac{1}{4}\psi s = \pi_m$. However the U monopolist has not incentive to implement this outcome because of the presence of the D firms' outside options.¹⁵

Under the case of negative royalties, the private incentives about technology diffusion reverse. Subsidizing the unique licensee allows the innovator to conquer most downstream market by selling the high quality good. Negative royalties on one hand make stronger competition because downstream production is subsidized; on the other hand, negative royalties by inducing exclusive licensing make milder competition because vertical product differentiation arises (whereas homogeneous goods prevail under non-negative royalties).

3.3 Bargaining

We next extend our analysis by considering a bargaining process on the vertical contracts. More precisely, the U producer decides how many licenses and then it bargains on the contract with each firm separately.

¹⁵There is also another contract that could implement the monopoly outcome:

$$r = 0, F = \frac{\pi_H^m(0)}{2} - \pi_i(0, 0; s, s\psi) = \frac{\psi s}{8} - \frac{\psi^2 s}{(4\psi - 1)^2} = \frac{(16\psi^2 - 16\psi + 1)s\psi}{8(4\psi - 1)^2}$$

where $\pi_H^m(0)$ is the monopoly profit in case of high quality good and zero marginal cost and the outside option is not buying the innovation given that the rival does. In this case U gets $\Pi_U = 2\left(\frac{(16\psi^2 - 16\psi + 1)s\psi}{8(4\psi - 1)^2}\right)$, the D firms get: $\pi_1 = \pi_2 = (4\psi - 1)^{-2}\psi^2 s$ (which is incentive compatible). Comparing this contract with (8) from U viewpoint we obtain that this contract is superior. However this is not credible because when U offers a zero per-unit royalty, each D firm has the incentive to deviate from half monopoly quantity (prisoner's dilemma) knowing that the rival has zero marginal cost and produces half monopoly quantity. Indeed by deviating it would be able to pay the fixed fee and to get higher profit. The result of the one-shot game would be that they produce the duopoly quantity and they are not able to pay the fixed fee.

Consider first the case of *non-negative royalties*. Suppose U wants to sell the new input to both firms. Negotiations occur simultaneously and independently: U and firm 1 negotiate over the (non-negative) contract (r_1, F_1) while U and firm 2 negotiate over the contract (r_2, F_2) . Each bargaining process is modelled as a two stage game such that they first choose r_i and then F_i .¹⁶ This way the bargaining process is not distortive.¹⁷ Consider the bargaining process of U with firm 1, given the outcome of the bargaining process of U with firm 2 (r_2, F_2) . The objective function is:

$$(r_1 q_1(r_1, r_2; s\psi, s\psi) + r_2 q_2(r_1, r_2; s\psi, s\psi) + F_1 + F_2 - r_2 q_2(r_2, 0; s\psi, s) - F_2)^\alpha (\pi_1(r_1, r_2; s\psi, s\psi) - F_1 - \pi_1(0, r_2; s, s\psi))^{1-\alpha}$$

Solving by backward induction, we first maximize with respect to F_1 and we find $F_1(r_1, r_2) = \frac{A(r_1, r_2) + \alpha B(r_1, r_2)}{9\psi s(4\psi - 1)^2}$, where $A(r_1, r_2)$ and $B(r_1, r_2)$ are long expressions defined in Appendix. We next substitute this solution in the objective function and maximize with respect to r_1 . The symmetric solution is:

$$r_1 = r_2 = r^{Ba} = 0, F_1 = F_2 = F^{Ba} = \alpha \frac{1}{9} \frac{(\psi - 1)(16\psi - 1)s\psi}{(4\psi - 1)^2}.$$

In the bargaining process the optimal contract specifies a zero per-unit royalty versus a positive royalty under full bargaining power. This results in a downstream duopoly. The key assumption to this result is the simultaneity of the bargaining process that introduces an incentive for the U monopolist to behave opportunistically during the separated negotiations with the two D firms. In particular, U is not able to commit to each D firm that it will not negotiate a more favourable contract with the rival D firm. This in turn implies that each D firm will never agree on a per-unit royalty that is higher than the marginal cost of production of the new input. This result is in line with Milliou and Petrakis (2007). Equilibrium price, profits, consumer surplus and welfare are then:

$$\begin{aligned} p^{Ba} &= \frac{1}{3}\psi s \\ \Pi_U^{Ba} &= 2F = 2\alpha \frac{1}{9} \frac{(\psi - 1)(16\psi - 1)s\psi}{(4\psi - 1)^2} - f, \\ \pi_1^{Ba}(0, 0; s\psi, s\psi) - F^{Ba} &= \frac{s\psi(4\psi - 1)^2 - \alpha(16\psi - 1)(\psi - 1)}{9(4\psi - 1)^2}, \\ CS^{Ba} &= \frac{2}{9}\psi s, \\ SW^{Ba} &= \frac{4}{9}\psi s - f, \end{aligned}$$

¹⁶We follow Milliou and Petrakis (2007).

¹⁷Actually, they first determine the size of the pie (setting r_i) and then they divide (setting F_i), however by backward induction, they first choose F for any given r (that is they first decide how to divide the joint surplus) and then they choose r (that is they determine the joint surplus). In a simultaneous bargaining process instead how you decide to divide the pie (F) would affect the size of the pie (and viceversa).

with $\pi_1^{Ba}(0, 0; s\psi, s\psi) - F_1^{Ba} > 0$ always for $\alpha \leq 1$.¹⁸

Consider now the case of *negative royalties*. Under exclusive licensing the U monopolist and firm 2 negotiate over the contract (r_2, F_2) . The objective function is:

$$(r_2 q_2(r_2, 0; s\psi, s) + F_2 - f)^\alpha \left(\pi_2(r_2, 0; s\psi, s) - F_2 - \frac{s}{9} \right)^{1-\alpha}$$

The solution is $r_2 = -\frac{s}{4}$, $F_2 = \frac{1}{72}s(18\alpha\psi - 17\alpha + 9)$. This solution comes from the simultaneous maximization of the above objective w.r.t. r_2 and F_2 . Under exclusive licensing the U objective is to maximize firm 2 downstream profit, because this is its only source of surplus. We could thus expect that the per-unit royalty negotiated under this bargaining process coincide with $r_2^{Neg} = -\frac{s}{4}$, that is the solution of exclusive licensing with full bargaining power. Note that for $\alpha = 1$ the contract with bargaining power coincide with the contract with full bargaining power. Given that exclusive licensing with negative royalties always prevails over complete technology diffusion under full bargaining power, we can extend this result to $\alpha < 1$, given that the equilibrium per-unit royalty is the same.

Extending the private solution to a bargaining process does not change the technology diffusion incentives: under non-negative royalties, complete technology diffusion prevails, whereas under negative royalties, exclusive licensing dominates.

4 Vertical integration (internal patent holder)

We have considered so far, the case of an *external innovator*, i.e. the U firm does not sell the final good in the D market. Suppose now that the U producer and one of the two D firms, say firm 2, merge, in this case the vertically integrated (VI) firm is an internal patent holder and its profit consists of two parts: the profit from selling the new input to the rival D firm 1 (if it decides to license) and the profit from selling the high quality final good 2.

We consider the following three-stage game: first, the patent holder offers a contract to D firm 1, D firm 1 decides whether to accept it and finally market competition takes place.

Proceeding backwards, given the third stage equilibrium quantities, the VI firm offers firm 1 the two-part tariff contract (r_1, F_1) such that:

$$\begin{aligned} \max_{r_1, F_1} \{ & \pi_{VI}(0, r_1; s\psi, s\psi) + r_1 q_1(r_1, 0; s\psi, s\psi) + F_1 \} \\ \text{s.t. } & \pi_1(r_1, 0; s\psi, s\psi) - F_1 \geq \pi_1(0, 0; s, s\psi) \\ & r_1 \in \left[0, \frac{s\psi}{2} \right], F_1 \geq 0 \end{aligned}$$

¹⁸Note that $\frac{(s\psi)^2}{9\psi s} - \alpha \frac{1}{9} \frac{(\psi-1)(16\psi-1)s\psi}{(4\psi-1)^2}$ is incentive compatible: $\frac{(s\psi)^2}{9\psi s} - \alpha \frac{1}{9} \frac{(\psi-1)(16\psi-1)s\psi}{(4\psi-1)^2} \geq \frac{(s\psi)^2}{(4\psi-1)^2 s} \iff \left(-\frac{1}{9}\right)(4\psi-1)^{-2}(\alpha-1)(\psi-1)(16\psi-1)s\psi > 0$.

where $\pi_1(0, 0; s, s\psi) = \frac{\psi^2 s}{(4\psi-1)^2}$, is firm 1 outside option, that is firm 1 profit if it does not buy the new input thus selling the low quality final good and incurring per-unit cost equal to zero. If the VI firm is constrained to nonnegative fees, it will optimally let the nonaffiliate firm to produce a positive quantity as low as possible (up to its outside option) and the optimal contract is then:¹⁹

$$r_{VI}^C = \frac{s(\psi(4\psi-1) - 3\psi\sqrt{\psi})}{2(4\psi-1)} > 0, F_{VI}^C = 0. \quad (9)$$

Equilibrium magnitudes are:

$$\begin{aligned} q_{VI}^C(0, r_{VI}^C; s\psi, s\psi) &= \frac{(4\psi^2 - \psi\sqrt{\psi} - \psi)}{2(4\psi-1)\psi}, \\ q_1^C(r_{VI}^C, 0; s\psi, s\psi) &= \frac{\sqrt{\psi}}{(4\psi-1)}, \\ Q_{VI}^C &= q_{VI}^C(0, r_{VI}^C; s\psi, s\psi) + q_1^C(r_{VI}^C, 0; s\psi, s\psi) = \frac{(4\psi^2 - \psi + \psi^{\frac{3}{2}})}{2(4\psi-1)\psi}, \\ p_{VI}^C &= \frac{\psi(4\psi - \sqrt{\psi} - 1)s}{2(4\psi-1)}, \\ \pi_{VI}^C(0, r_{VI}^C; s\psi, s\psi) &= \frac{(4\psi^2 - \psi\sqrt{\psi} - \psi)^2 s}{4(4\psi-1)^2 \psi}, \\ \pi_1^C(r_{VI}^C, 0; s\psi, s\psi) - F_{VI}^C &= \frac{s\psi^2}{(4\psi-1)^2}, \\ \Pi_{VI}^C &= \pi_{VI}^C(0, r_{VI}^C; s\psi, s\psi) - f + r_{VI}^C q_1^C(r_{VI}^C, 0; s\psi, s\psi) + F_{VI}^C = \frac{(16\psi^2 - 13\psi + 1)s\psi}{4(4\psi-1)^2}, \\ CS_{VI}^C &= \frac{(4\psi + \sqrt{\psi} - 1)^2 s\psi}{8(4\psi-1)^2}, \\ PS_{VI}^C &= \frac{s\psi^2}{(4\psi-1)^2} + \frac{(16\psi^2 - 13\psi + 1)s\psi}{4(4\psi-1)^2}, \\ SW_{VI}^C &= \frac{(4\psi + \sqrt{\psi} - 1)^2 s\psi}{8(4\psi-1)^2} + \frac{s\psi^2}{(4\psi-1)^2} + \frac{(16\psi^2 - 13\psi + 1)s\psi}{4(4\psi-1)^2}, \end{aligned}$$

where PS_{VI}^C denotes the producer surplus, that is industry profit under vertical integration.

Note that the optimal contract is the same if we allow for negative royalties, as the VI firm has always incentive to set a positive royalty.

We gather the above results as follows.

Proposition 2 *Under Cournot competition, the internal patent holder always sells the innovation to the rival firm. The optimal contract is the two-part tariff specified in (9).*

¹⁹In the Appendix we show that the VI firm if unconstrained would set a negative fixed fee.

Proposition 2 contains two results. The first concerns the firm's incentive to license the product innovation thus preferring a homogeneous high quality good market rather than a vertically differentiated market in which the internal patentee would be the high quality firm competing with a low quality producer. The intuition relies on Bonanno (1986) where the author investigates the effect of the type of competition (Bertrand vs Cournot) on firms' incentives to (vertically) product differentiate. He shows that (with zero production costs) the two firms decide to produce a homogeneous high quality product under Cournot competition and differentiated products under Bertrand competition.²⁰ Arya and Mittendorf (2006) studies the incentives of an internal patent holder to license an innovation on the final good in the presence of a monopolist input producer. In their model the decision to license the innovation implies giving up the monopoly power on the final good, that is letting a Cournot rival firm enter the market. They find that the licensing incentive is positive as doing so the internal patent holder creates a "weak" rival (as the rival has the additional marginal cost equal to the per-unit royalty) that induces the external supplier to make better pricing conditions. We also find that the VI (input) innovator has incentive to license the new good to the rival, however, the reason is not to affect the input pricing terms but to extract some surplus from the rival firm that is active in the market independently of the licensing strategy.

The second result of Proposition 2 concerns the optimal licensing mode of the internal patent holder. San Martin and Saracho (2010) consider a non drastic process innovation and show that in a Cournot duopoly with homogeneous goods the optimal licensing mode is an ad valorem royalty, $\alpha \in [0, 1]$ that is a profit sharing agreement. This result does not hold in our model: as we prove in the Appendix the ad valorem licensing mode is dominated by the two-part tariff contract defined in (9). The intuition behind this result is as follows. In San Martin and Saracho (2010) the ad valorem royalty is optimal as the internal patentee introduces the process innovation that increases total quantity (it let the rival firm to produce with a more efficient technology) and then appropriates the rival's profit up to its outside option. In our model if the innovation is diffused via an ad valorem royalty total output does not increase (as under homogeneous goods it is independent of the quality, see the status quo equilibrium quantity, expression 3). This implies that industry profits correspond to the duopoly profits that are shared according to α . Whereas under two-part tariff the internal patentee can at least partially internalize the negative externality coming from competition and approach the monopoly outcome.

²⁰This result also explains the patent holder's incentives under Bertrand competition, Section 6.

5 Private and social profitability of vertical integration

We next consider the merger profitability as well as the social welfare comparing the vertical integration scenario with the vertical separation scenario (i.e., external patent holder) where complete technology diffusion takes place.

As far as the private profitability is concerned, the merger is always profitable under non-negative royalties, namely:

$$\Pi_{VI}^C - (\pi^T(s\psi) - F^T(r^T, r^T) + \Pi_U^T(s\psi)) = \begin{cases} \frac{5(256\psi^4 - 57\psi^2 - 112\psi^3 - 7\psi + 1)s\psi^2}{4(32\psi^2 - 7\psi + 2)^2(4\psi - 1)^2} > 0 \text{ for } \psi > 1.5856, \\ \frac{(\psi - 1)(16\psi - 1)s\psi}{36(4\psi - 1)^2} > 0 \text{ for } \psi < 1.5856. \end{cases}$$

This result is in line with the new market foreclosure theory according to which vertical integration allows the U producer monopolist to protect its monopoly power. More precisely Rey and Tirole (2007) point out that vertical integration is a device to restore the U monopoly power that cannot be exerted under vertical separation when contracts are observable. In our model, contracts are observable, however due to the strategic outside option ($\pi_i(0, r_j; s, s\psi)$) the external patent holder cannot implement the monopoly outcome. Under the case of negative royalties, however, the private incentives reverse, namely:

$$\Pi_U^{ELneg} - \Pi_{VI}^C(s\psi) = \frac{(190\psi - 614\psi^2 + 864\psi^3 - 17)s}{72(4\psi - 1)^2} > 0.$$

As for the social profitability of vertical integration, we make the following comparisons. For $\psi < 1.5856$, $CS_{VI}^C - CS^T = \frac{(4\psi + \sqrt{\psi - 1})^2 s\psi}{8(4\psi - 1)^2} - \frac{2}{9}\psi s = \psi s \left(\frac{(4\psi + \sqrt{\psi - 1})^2}{8(4\psi - 1)^2} - \frac{2}{9} \right) < 0$. For $\psi > 1.5856$, $CS_{VI}^C - CS^T = s\psi \left(\frac{(4\psi + \sqrt{\psi - 1})^2}{8(4\psi - 1)^2} - \frac{(4\psi + 16\psi^2 + 1)^2}{2(32\psi^2 - 7\psi + 2)^2} \right) > 0 \iff \psi > 3.4078$. We can conclude that for $\psi > 3.4078$ both the industry profit and consumer surplus are higher under VI, that is vertical integration is welfare improving. In contrast for $\psi < 3.4078$, the result is ambiguous as consumer surplus is lower but producer surplus is higher under VI rather than the non-merger case. However direct computations of social welfare ($SW = CS + PS$) reveal the following.

$$SW_{VI}^C - SW^T < 0 \iff \psi < 3.4078.$$

Compare next the two vertical scenarios, considering the case that, under vertical separation, the external patent holder does not have full bargaining power:

$$\begin{aligned} \Pi_{VI}^C - (\pi^{Ba} - F^{Ba} + \Pi_U^{Ba}) &= \frac{(5 - 4\alpha)(\psi - 1)(16\psi - 1)s\psi}{36(4\psi - 1)^2} > 0, \\ SW_{VI}^C - SW^{Ba} &= \frac{(31\psi^2 - 5\psi - 80\psi^3 - 18\psi^{\frac{3}{2}} + 72\psi^{\frac{5}{2}})s}{72(4\psi - 1)^2} < 0. \end{aligned}$$

We gather our results in the following proposition.

Proposition 3 (i) *Under non-negative royalties, vertical integration is always privately profitable. However, as long as the patent holder has full bargaining power, VI is welfare improving if and only if the innovation is sufficiently large, namely $\psi > 3.4078$; in contrast, in case the external patent holder does not have full bargaining power, then VI turns out to be welfare detrimental for any innovation size.* (ii) *Under negative royalties, vertical integration becomes privately and socially unprofitable.*

Consider first the case of *non-negative royalties* and full bargaining power. Sandonis and Fauli-Oller (2006) consider non-drastring process innovations in a horizontally differentiated Cournot duopoly and study the patentee's incentives to merge with either firm in the market. They show that the merger is privately profitable for small innovations and it is welfare improving for large innovations. They argue that all profitable mergers are welfare detrimental, this also holds for homogeneous goods. More precisely, Sandonis and Fauli-Oller (2006) individuate the following trade-off: an internal patentee (VI case) better internalizes market profit with respect to an external one, however the internal patentee can only use one instrument (a contract for the other firm in the market) rather than two (a contract for each firm in the market). An external patentee has two instruments but it has to care about firms' outside option which depends on the royalty, in particular it increases with the royalty and decreases with the innovation size. They find that the balance of these two effects depend on the innovation size: the merger is privately profitable for small innovations, in fact for large innovations the outside option faced by the external patentee is low and so it has little relevance with respect to the availability of two instruments. In our model the merger is profitable for any innovation size ψ . In particular, in contrast with Sandonis and Fauli-Oller (2006) the internal patentee approaches the monopoly outcome as the innovation size ψ increases. In fact, the VI firm has incentive to reduce as much as possible the quantity produced by the non-affiliate, in order to internalize as much as possible the vertical externality (in the limit, if allowed, the VI firm would completely foreclose the rival firm compensating it via a negative fixed fee). However the VI firm is constrained by the nonaffiliate outside option: the higher is ψ the lower is this outside option and so the lower the quantity that the nonaffiliate produces and in turn the more the VI firm approaches the monopoly outcome. The outside option for the external patentee is $\pi_i(0, r_j; s, s\psi)$ decreasing in ψ and increasing in r_j . The outside option for the internal patentee is $\pi_1(0, 0; s, s\psi) = \frac{\psi^2 s}{(4\psi - 1)^2}$ independent of r and decreasing in ψ . For low ψ , both outside options are large. There is the same negative effect of Sandonis and Fauli-Oller (2006) related to the external patentee and so vertical integration dominates vertical separation. For high ψ , both outside options are small, however also in this case vertical integration dominates as the internal patentee approaches the monopoly outcome.

As for social welfare, in contrast with Sandonis and Fauli-Oller (2006) we find that, under homogeneous goods, vertical integration is privately and socially profitable for high quality improvements. Vertical integration has two opposite effects on prices: on one hand, VI pushes prices down as it implies

the (partial) internalization of the vertical externality; on the other hand, VI has an anticompetitive effect because the VI firm is able to (partially) foreclose the rival firm via a positive per-unit royalty. The first effect prevails for ψ high (and we have $p_{VI}^C - p^T < 0$), the opposite effect prevails for ψ low (and we have $p_{VI}^C - p^T > 0$).

Whenever the external patent holder does not have full bargaining, as one could expect the private profitability of VI continues to hold. However, from the social welfare point of view, VI becomes detrimental. Indeed, in this case under vertical separation, complete technology diffusion and the duopoly equilibrium occurs for any value of ψ , which is always preferred to a close-monopoly solution, $p_{VI}^C - p^{Ba} < 0$.

Under *negative royalties* the private and social incentives change. From a private point of view, the U monopolist by vertically integrating (at least partially) internalizes the vertical externality, but on the other hand, it reduces the set of licensing policies given that under VI it transfers internally the input at the marginal cost. At equilibrium vertical separation and exclusive licensing always prevails because the Stackelberg equilibrium arises and the innovator gets the first mover advantage. As for the social welfare, under vertical separation, the double marginalization problem disappears because the innovating firm is subsidized and the non-innovating firm buys the input from competitive firms, however goods are vertically differentiated; in contrast under VI the market is characterized by a homogeneous high quality good, however the VI firm forecloses the rival and approaches the monopoly outcome. In other words, the Stackelberg equilibrium is always preferred with respect to a quasi-monopoly. Therefore private and social incentives match when negative royalties are allowed. We conclude that *allowing for negative royalties is always welfare improving*. Liao and Sen (2005) study licensing of a process innovation and show that negative royalties are welfare improving with respect to non-negative royalties if the innovation is sufficiently small. They point out the importance of the cost of exclusion: the additional cost that the non-licensee firm has to pay under exclusive licensing. As long as this cost is low, exclusive licensing (that arises with negative royalties) is preferred with respect to complete technology diffusion.

6 Bertrand competition

We next extend our analysis to Bertrand competition.²¹ Ex-ante, as goods are homogeneous, equilibrium price is equal to marginal cost, set to zero. Therefore, the market is covered, i.e., the demand is equal to one; firms make zero profit and social welfare coincide with consumer surplus that is:

$$SW^B(s) = CS^B(s) = \int_0^1 (\theta s) d\theta = \frac{s}{2}.$$

²¹We focus on the case of full bargaining power as we expect the same differences to arise under partial bargaining power as in Cournot.

Under Bertrand competition, the external patent holder will sell the new input to only one firm (otherwise the Bertrand paradox applies). The external innovator has no incentive to sell its product innovation to both firms, as under Bertrand competition with homogeneous goods, market profits are equal to zero and therefore he could not extract any surplus from the licensees. In contrast, the internal patent holder will not sell any license. We analyse the optimal contract considering in turn the two scenarios.

Suppose as before that firm 1 is the non-innovating firm that produces a final good of quality $s_1 = s$ at price p_1 ; firm 2 is the innovating firm that produces a final good of quality $s_2 = \psi s > s$, at price p_2 . Solving the game for the *external patent holder* we find that the optimal contract is:

$$r_2^B = 0, F_2^B = \frac{(\psi - 1) s \psi}{(4\psi - 1)}. \quad (10)$$

A product innovation is sold via a fixed fee, this is due to the fact that the rival firm has zero marginal cost.²² This result is in contrast with a process innovation (Muto, 1993). Here the per-unit royalty is set to zero in order to avoid the distorting negative effect of a positive royalty on the innovating firm and profits are extracted via a positive fixed fee. Remaining equilibrium variables are:

$$q_1^B = \frac{\psi}{(4\psi - 1)}, q_2^B = \frac{2\psi}{(4\psi - 1)}, Q_B = \frac{3\psi}{(4\psi - 1)} \quad (11)$$

$$p_1^B = \frac{(\psi - 1) s}{4\psi - 1}, p_2^B = \frac{2\psi s (\psi - 1)}{4\psi - 1}, \quad (12)$$

$$\pi_1(s, s\psi) = \frac{(\psi - 1) s \psi}{(4\psi - 1)^2} = \pi_2(s\psi, s) = \frac{(\psi - 1) s \psi}{(4\psi - 1)^2}$$

$$\Pi_U^B(s, s\psi) = \frac{(\psi - 1) s \psi}{(4\psi - 1)} - f.$$

$$PS^B = \frac{(\psi - 1)(4\psi + 1) s \psi}{(4\psi - 1)^2} - f \quad (13)$$

$$CS^B = \frac{(4\psi + 5) s \psi^2}{2(4\psi - 1)^2}. \quad (14)$$

Π_U^B is the U patent holder profit under exclusive licensing, when selling via a two-part tariff, which reduces to a fixed fee, the new input to only one D firm.

Consider next the case of an *internal patentee*, in particular assume that the U producer and firm 2 merge. The VI firm does not sell the innovation to the rival so that we have an equilibrium such that the VI firm produces the high quality good at zero costs and compete with the rival firm 1 that produces the low quality good at zero cost. Equilibrium quantities, prices, PS and CS are as in (11), (12), (13) and (14), equilibrium profits are:

$$\pi_1^B = \frac{(\psi - 1) s \psi}{(4\psi - 1)^2}, \pi_{VI}^B = 4 \frac{(\psi - 1) s \psi^2}{(4\psi - 1)^2}.$$

²²If the rival firm had a positive marginal cost of production, say $c > 0$, (so that the innovation would include the product as well as the process) then the optimal contract would be $r_2 = \frac{c}{2} > 0$, $F_2 = \frac{4\psi s^2(\psi - 1) + 4cs\psi + c^2(1 - 4\psi)}{4(4\psi - 1)s}$.

As for the merger private and social profitability under Bertrand competition we find that:

$$\pi_{VI}^B - (\pi_2(s\psi, s) - F_2^B + \Pi_U^B(s\psi)) = 0, SW_{VI}^B - SW^B = 0.$$

The patent holder is indifferent between staying out of the market and vertically integrate. The same holds from the social welfare point of view.

Extending the analysis to negative royalties would not change our results because under Bertrand competition, the U innovator has no incentive to subsidize the downstream production.

The above results are gathered in the following proposition.

Proposition 4 *Under Bertrand competition the product innovation is never completely diffused. (i) An external patent holder prefers to sell an exclusive licensing via a fixed fee rather than a royalty (the optimal contract is specified in (10)). (ii) An internal patent holder does not license its innovation. (iii) The equilibrium prices, quantities and social welfare are independent of whether the patent holder stays out of the market or vertically integrate with either firm.*

We conclude our analysis by comparing the *private and social profitability of Cournot vs Bertrand* competition. In case of non-negative royalties, under Cournot competition, at equilibrium vertical integration takes place and technology diffusion is complete, so that we have a homogeneous high quality good in the market; in contrast under Bertrand competition, at equilibrium exclusive licensing occurs, so that the market is vertically differentiated. We find that from the firms' point of view, the producer surplus is higher under Cournot than under Bertrand competition, namely:²³

$$\begin{aligned} \Pi_{VI}^C - \pi_{VI}^B &= \left(\frac{(16\psi^2 - 13\psi + 1)s\psi}{4(4\psi - 1)^2} - 4 \frac{(\psi - 1)s\psi^2}{(4\psi - 1)^2} \right) = \frac{s\psi}{4(4\psi - 1)^2} (3\psi + 1) > 0, \\ PS_{VI}^C - PS^B &= \frac{s\psi}{4(4\psi - 1)^2} (3\psi + 5) > 0. \end{aligned}$$

As for consumer surplus and welfare, the comparison depends on the quality improvement, in particular total social welfare is higher under Cournot than under Bertrand competition for ψ sufficiently high:

$$SW_{VI}^C - SW^B = \frac{(11\psi - 21\psi^2 - 2\psi^{\frac{3}{2}} + 8\psi^{\frac{5}{2}})s}{8(4\psi - 1)^2} > 0 \iff \psi > 6.28$$

This result is clearly linked to the fact that on one hand, under Cournot, competition is milder than under Bertrand where both qualities stays on sale; on the other hand, under Cournot, complete technology diffusion arises so that the average quality of the final good is higher than under Bertrand competition. In this static setting traditional conclusions about price and quantity competition (Singh and Vives, 1984) may be reversed. This is in line with Arya, et al. (2008)

²³This is in line with Singh and Vives (1984).

that show that the vertically integrated producer may set a higher input price under Bertrand competition than under Cournot competition. In our framework however this result relies on the technology diffusion incentives which in turn determine the average quality of the final good.

Finally, comparing the private and social profitability of Cournot vs Bertrand competition in the light of *negative royalties*, we find that Cournot always prevails over Bertrand competition. Under negative royalties, exclusive licensing arises in both competition modes, therefore the quality improving effect which is at work with non-negative royalties when complete technology diffusion takes place under Cournot is absent. However, under Cournot competition the equilibrium outcome coincides with Stackelberg and Cournot competition is more profitable than Bertrand. In particular:

$$\begin{aligned}
PS^{ELneg} - PS^B &= \frac{1}{16}s(16\psi - 1) - \frac{(\psi-1)(4\psi+1)s\psi}{(4\psi-1)^2} = \frac{(40\psi-96\psi^2+192\psi^3-1)s}{16(4\psi-1)^2} > 0 \\
CS^{ELneg} - CS^B &= \frac{1}{32}(4\psi + 5)s - \frac{(4\psi+5)s\psi^2}{2(4\psi-1)^2} = \left(-\frac{1}{32}\right)(4\psi - 1)^{-2}(8\psi - 1)(4\psi + 5)s < 0 \\
SW^{ELneg} - SW^B &= \frac{3}{32}s(12\psi + 1) - \left(\frac{(4\psi+5)s\psi^2}{2(4\psi-1)^2} + \frac{(\psi-1)(4\psi+1)s\psi}{(4\psi-1)^2}\right) = \frac{(44\psi-224\psi^2+384\psi^3+3)s}{32(4\psi-1)^2} > 0
\end{aligned}$$

The producer surplus is higher under Cournot than under Bertrand, and this is in line with traditional results. Consumer surplus is instead higher under Bertrand, this is also in line with the idea that Bertrand implies tougher competition than Cournot. However, overall, Cournot is preferred with respect to Bertrand: the industry profitability overcompensate the loss of consumer surplus.

7 Conclusion

We have analysed the optimal licensing strategy of an upstream input innovator producing a new input which improves the quality of the final goods. We have considered a duopoly downstream market and two-part tariff contracts with non-negative fixed fees and either non-negative per-unit royalties or per-unit subsidies. In case of *non-negative royalties*, we have shown that, under Cournot competition complete technology diffusion takes place and the innovator always prefers to be inside the market as the vertical merger with either downstream firm is always privately profitable. It is also welfare improving for large innovations. Extending the analysis to the case in which the external patent holder does not have full bargaining power, the private profitability of vertical integration continues to hold, whereas vertical integration turns out to be welfare detrimental. Under Bertrand competition, in contrast with Cournot, complete technology diffusion never takes place and we find an equivalence result between vertical integration and vertical separation from both the private and social welfare point of view. Allowing for per-unit *subsidies* completely reverse our results. Exclusive licensing takes place under both modes of competition (Bertrand and Cournot), and vertical integration becomes privately and socially unprofitable.

In the licensing game we have assumed that firms' outside option in case of an external patent holder and complete technology diffusion is such that if the firm does not get the innovation the rival firm does. This means that the outside option depends on the royalty rate. Things are much different if we assume that either both firms get the innovation or neither does.²⁴ Indeed, in this case the outside option is constant with respect to the per-unit royalty so that the upstream innovator via a two-part tariff contract can implement the monopoly outcome. This result has been proved by Inderst and Shaffer (2009) studying optimal two-part tariff contracts in vertical relations with independent asymmetric firms. Li and Wang (2010) obtains the same result for the specific framework of patent licensing. Clearly, the vertical integration would not be privately profitable anymore as under vertical separation industry profit would be maximal. In contrast, vertical integration would result to be welfare improving as a quasi monopoly would be better than a monopoly.

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²⁴Somewhat in line with Gonzales and Ayala (2012) this could be a scenario where the downstream firms act cooperatively when purchasing the new input from the upstream innovator.

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8 Appendix

8.1 Exclusive licensing subgame

Given demands (5) and (6), *Cournot competition* leads to the following third stage quantity and price equilibrium:

$$\begin{aligned} q_1(0, r_2; s, s\psi) &= \frac{s\psi + r_2}{s(4\psi - 1)}, \\ q_2(r_2, 0; s\psi, s) &= \frac{(2s\psi - s - 2r_2)}{s(4\psi - 1)}, \\ p_1(0, r_2; s, s\psi) &= \frac{(s\psi + r_2)}{(4\psi - 1)}, \\ p_2(r_2, 0; s\psi, s) &= \frac{(2\psi - 1)(s\psi + r_2)}{(4\psi - 1)}, \end{aligned}$$

with $q_2(r_2, 0; s\psi, s) > 0 \iff \frac{s(2\psi-1)}{2} > r_2$. Firm1 profit is then:

$$\pi_1(0, r_2; s, s\psi) = \frac{(s\psi + r_2)^2}{(4\psi - 1)^2 s} \quad (15)$$

As for firm 2:

$$\pi_2(r_2, 0; s\psi, s) = \frac{\psi(s - 2s\psi + 2r_2)^2}{(4\psi - 1)^2 s}$$

The U firm chooses the two-part tariff contract for firm 2 (r_2, F_2) such that:

$$\begin{aligned} &\max_{r_2, F_2} \Pi_U \\ \text{s.t. } r_2 &\in \left[0, \frac{s(2\psi - 1)}{2} \right] \\ F_2 &\leq \pi_2(r_2, 0; s\psi, s) - \frac{s}{9} \end{aligned}$$

with $\Pi_U = \frac{(2s\psi - s - 2r_2)}{s(4\psi - 1)}r_2 + F_2 - f$. The solution is a contract such that $r_2^{EL} = 0$ and $F_2^{EL} = \frac{(36\psi^2 - 16\psi + 1)(\psi - 1)s}{9(4\psi - 1)^2}$.²⁵

²⁵This is because $\partial/\partial r_2 \left(\frac{(2s\psi - s - 2r_2)}{s(4\psi - 1)}r_2 + \pi_2(r_2, 0; s\psi, s) - \frac{s}{9} - f \right) < 0$, that is the incen-

8.2 Complete technology diffusion subgame

The U firm maximization problem is:

$$\begin{aligned} \max_{F_1, r_1, F_2, r_2} \{ & r_1 q_1(r_1, r_2; s\psi, s\psi) + r_2 q_2(r_1, r_2; s\psi, s\psi) + F_1 + F_2 - f \} \quad (16) \\ \text{s.t. } & \pi_1(r_1, r_2; s\psi, s\psi) - F_1 \geq \pi_1(0, r_2; s, s\psi) \\ & \pi_2(r_2, r_1; s\psi, s\psi) - F_2 \geq \pi_2(0, r_1; s, s\psi) \\ & r_1 \geq 0, r_2 \geq 0, F_1 \geq 0, F_2 \geq 0 \end{aligned}$$

where $\pi_1(0, r_2; s, s\psi)$ is defined in (15). Here the outside option for each firm is not buying the new input given that the rival firm does. $\pi_i(r_i, r_j; s\psi, s\psi)$ and $q_i(r_i, r_j; s\psi, s\psi)$ denote the third stage equilibrium D firm i profit and quantity when both firms produce the high quality good, namely:

$$\pi_i(r_i, r_j; s\psi, s\psi) = \frac{(s\psi - 2r_i + r_j)^2}{9\psi s}, \quad (17)$$

$$q_i(r_i, r_j; s\psi, s\psi) = \frac{1}{3s\psi} (s\psi - 2r_i + r_j). \quad (18)$$

$$p(r_i, r_j; s\psi, s\psi) = s\psi \left(1 - \frac{1}{3s\psi} (s\psi - 2r_i + r_j) - \frac{1}{3s\psi} (s\psi - 2r_j + r_i) \right)$$

As the two constraints are binding at equilibrium, we have

$$\begin{aligned} F_1(r_1, r_2) &= \pi_1(r_1, r_2; s\psi, s\psi) - \pi_1(0, r_2; s, s\psi), \\ F_2(r_1, r_2) &= \pi_2(r_2, r_1; s\psi, s\psi) - \pi_2(0, r_1; s, s\psi), \end{aligned}$$

with $\pi_i(0, r_j; s, s\psi) = \frac{(s\psi + r_j)^2}{(4\psi - 1)^2 s}$. The maximization problem, thus becomes:

$$\max_{r_1, r_2} \{ r_1 q_1(r_1, r_2; s\psi, s\psi) + r_2 q_2(r_2, r_1; s\psi, s\psi) + F_1(r_1, r_2) + F_2(r_1, r_2) - f \}.$$

8.3 Bargaining

Consider the bargaining process of U with firm 1, given the outcome of the bargaining process of U with firm 2 (r_2, F_2). The objective function is:

$$\begin{aligned} & (r_1 q_1(r_1, r_2; s\psi, s\psi) + r_2 q_2(r_1, r_2; s\psi, s\psi) + F_1 + F_2 - r_2 q_2(r_2, 0; s\psi, s) - F_2)^\alpha \\ & (\pi_1(r_1, r_2; s\psi, s\psi) - F_1 - \pi_1(0, r_2; s, s\psi))^{1-\alpha} \end{aligned}$$

Solving by backward induction, we first maximize with respect to F_1 and we find

$$F_1(r_1, r_2) = \frac{A(r_1, r_2) + \alpha B(r_1, r_2)}{9\psi s(4\psi - 1)^2}, \text{ where } A(r_1, r_2) = 3(4\psi - 1)(-2)(r_1^2 - r_1 r_2 + r_2^2) + \psi^2 2s3(4\psi - 1)(r_2 - 2r_1) + \psi 3(4\psi - 1)(sr_1 - 2sr_2 - 8r_1 r_2 + 8r_1^2 + 2r_2^2) \text{ and } B(r_1, r_2) = 2r_1 r_2 - 2r_1^2 - 5r_2^2 + 16s^2 \psi^4 + \psi^3 s(8r_2 - 16r_1 - 17s) + \psi(16r_1^2 - 4sr_2 - 16r_1 r_2 - sr_1 + 13r_2^2) +$$

time is to set a royalty as low as possible. Given the non-negative constraint, the solution is $r = 0$.

$\psi^2 (8sr_1 - 4sr_2 + 32r_1r_2 + s^2 - 32r_1^2 - 8r_2^2)$. We next substitute this solution in the objective function and maximize with respect to r_1 . The symmetric solution $r_1 = r_2$ is:²⁶

$$r^B = \frac{s}{(8\psi - 5)(\psi - 1)} \left(\psi \left(-\frac{1}{2} \right) (8\psi^2 - 4\psi + 5) - \frac{3}{2} \sqrt{\psi^2 (4\psi^2 - 8\psi + 5) (4\psi - 1)^2} \right) < 0$$

So that we conclude that the equilibrium royalty is $r_B = 0$.

8.4 Vertical integration

Consider the quantity competition between the VI firm and firm 1 producing the high quality final good. The VI firm has zero variable production costs as the new input is transferred at the marginal cost $c_2 = 0$, whereas firm 1 incurs marginal cost r_1 . The third stage equilibrium quantities and profits are:

$$\begin{aligned} q_{VI}(0, r_1; s\psi, s\psi) &= \frac{1}{3s\psi} (s\psi + r_1) \\ q_1(r_1, 0; s\psi, s\psi) &= \frac{1}{3s\psi} (s\psi - 2r_1) \geq 0 \iff r_1 \leq \frac{s\psi}{2} \\ p_{VI} &= s\psi (1 - q_{VI}(0, r_1; s\psi, s\psi) - q_1(r_1, 0; s\psi, s\psi)) = \frac{1}{3} (s\psi + r_1) \\ \pi_{VI}(0, r_1; s\psi, s\psi) &= \frac{(s\psi + r_1)^2}{9s\psi} - f \\ \pi_1(r_1, 0; s\psi, s\psi) &= \frac{(s\psi - 2r_1)^2}{9\psi s} \end{aligned}$$

where $q_{VI}(0, r_1; s\psi, s\psi)$ and $q_1(r_1, 0; s\psi, s\psi)$ are obtained from expression (18) substituting properly r_i and r_j ; $\pi_{VI}(0, r_1; s\psi, s\psi)$ and $\pi_1(r_1, 0; s\psi, s\psi)$ are obtained from expression (17) substituting properly r_i and r_j . The VI firm offers firm 1 the two-part tariff contract (r_1, F_1) such that:²⁷

$$\begin{aligned} \max_{r_1, F_1} & \{ \pi_{VI}(0, r_1; s\psi, s\psi) + r_1 q_1(r_1, 0; s\psi, s\psi) + F_1 \} \\ \text{s.t.} & \pi_1(r_1, 0; s\psi, s\psi) - F_1 \geq \pi_1(0, 0; s, s\psi) \\ & r_1 \in \left[0, \frac{s\psi}{2} \right], \end{aligned}$$

²⁶The sign of the derivative w.r.t. r_1 is the sign of $\frac{(4\psi-1)^2(2r+s\psi)}{\psi^2 s^2 (1-16\psi)(\psi-1) + r^2(8\psi-5)(\psi-1) + r\psi s(8\psi^2-4\psi+5)} > 0 \iff s^2(1-16\psi)(\psi-1) + r^2(8\psi-5)(\psi-1) + r\psi s(8\psi^2-4\psi+5) > 0 \iff r < r_-, r > r_+$, with $r_- = \frac{s}{(8\psi-5)(\psi-1)} \left(\psi \left(-\frac{1}{2} \right) (8\psi^2 - 4\psi + 5) - \frac{3}{2} \sqrt{\psi^2 (4\psi^2 - 8\psi + 5) (4\psi - 1)^2} \right) < 0$ and $r_+ = \frac{s}{(8\psi-5)(\psi-1)} \left(\psi \left(-\frac{1}{2} \right) (8\psi^2 - 4\psi + 5) + \frac{3}{2} \sqrt{\psi^2 (4\psi^2 - 8\psi + 5) (4\psi - 1)^2} \right) > 0$.

²⁷In the following maximization problem we do not restrict the VI firm to set nonnegative fees. This allows us to make clear its incentives. We next solve the maximization problem constrained to nonnegative fees.

where $\pi_1(0, 0; s, s\psi) = \frac{\psi^2 s}{(4\psi-1)^2}$, obtained from (15) is firm 1 outside option. As the first constraint is binding at equilibrium, we have:

$$\max_{r_1} \{ \pi_{VI}(0, r_1; s\psi, s\psi) + r_1 q_1(r_1, 0; s\psi, s\psi) + \pi_1(r_1, 0; s\psi, s\psi) - \pi_1(0, 0; s, s\psi) \}$$

The optimal contract is then:

$$r_1^* = \frac{s\psi}{2}, F_1^* = -\frac{\psi^2 s}{(4\psi-1)^2}$$

If we let the VI firm to set negative fees, the vertical merger implements the monopoly outcome by inducing the nonintegrated firm to produce a nil quantity (foreclosure) and compensating it for the outside option. Equilibrium magnitudes are:

$$\begin{aligned} q_{VI}(0, r_1^*; s\psi, s\psi) &= \frac{1}{2} = q^m \\ q_1(r_1^*, 0; s\psi, s\psi) &= 0 \\ p_{VI} &= \frac{1}{2}\psi s = p^m \\ \pi_{VI}(0, r_1^*; s\psi, s\psi) &= \frac{1}{4}\psi s \\ \pi_1(r_1^*, 0; s\psi, s\psi) - F_1^* &= \frac{\psi^2 s}{(4\psi-1)^2} \end{aligned}$$

$$\Pi_{VI} = \pi_{VI}(0, r_1^*; s\psi, s\psi) - f + r_1^* q_1(r_1^*, 0; s\psi, s\psi) + F_1^* = \frac{1}{4}\psi s - \frac{\psi^2 s}{(4\psi-1)^2} - f$$

However negative fees would be clearly held to be illegal by antitrust authorities. It is clear from the analysis above that the VI firm wants to restrict as much as possible the quantity produced by the non affiliate firm so as to (at least) partially internalize the vertical externality.

If the VI is constrained to nonnegative fees, it will optimally let the nonaffiliate firm to produce a positive quantity $q_1(r_1) : \pi_1(r_1) = \frac{\psi^2 s}{(4\psi-1)^2}$. Given the Cournot equilibrium quantities, we have

$$\begin{aligned} \left(\frac{1}{3}(s\psi + r_1) - r_1 \right) \frac{1}{3s\psi} (s\psi - 2r_1) &= \frac{\psi^2 s}{(4\psi-1)^2} \\ \Leftrightarrow r_1 &= \frac{(\psi s (4\psi - 1) - 3s\psi\sqrt{\psi})}{2(4\psi - 1)}. \end{aligned}$$

8.5 Ad valorem royalty

Suppose (under Cournot competition) the internal patentee (the VI firm) decides to sell the innovation via an ad valorem royalty, that is a profit sharing agreement. Firms' profits are then: $\pi_1^{PS} = (1 - \alpha)pq_1$, $\pi_{VI}^{PS} = pq_{VI} + \alpha pq_1$

where $\alpha \in [0, 1]$ is the ad valorem royalty. The equilibrium ad valorem royalty is the α such that the licensee is indifferent between buying and not buying the innovation, i.e.:

$$\begin{aligned} \alpha & : & (1 - \alpha) \frac{s\psi}{9} &= \frac{\psi^2 s}{(4\psi - 1)^2} \\ \iff \alpha^* & = & \frac{(\psi - 1)(16\psi - 1)}{(4\psi - 1)^2} &\in (0, 1). \end{aligned}$$

Industry profit is $2\frac{s\psi}{9}$, $\pi_{VI}^{PS} = \frac{s\psi}{9} + \frac{(\psi-1)(16\psi-1)}{(4\psi-1)^2} \frac{s\psi}{9}$ and $\Pi_{VI}^* - \pi_{VI}^{PS} = \frac{(\psi-1)(16\psi-1)s\psi}{36(4\psi-1)^2} > 0$.

8.6 Social welfare comparisons

For completeness we provide equilibrium social welfare under exclusive licensing and non-negative royalties:

$$SW^{EL} = PS^{EL} + CS^{EL} = \left(\frac{(36\psi^2 - 16\psi + 1)(\psi - 1)s}{9(4\psi - 1)^2} + \frac{\psi^2 s}{(4\psi - 1)^2} + \frac{s}{9} \right) + \frac{(\psi + 4\psi^2 - 1)s\psi}{2(4\psi - 1)^2}$$

where $CS^{EL} = \int_{\frac{p_1}{s}}^{\frac{p_2 - p_1}{s(\psi - 1)}} (\theta s - p_1) d\theta + \int_{\frac{p_2 - p_1}{s(\psi - 1)}}^1 (\theta s\psi - p_2) d\theta$. The remaining social

welfare comparisons are: for $\psi < 1.5856$, $SW^T - SW^{EL} = s \left(\frac{2}{9}\psi + \left(2\frac{\psi^2}{(4\psi - 1)^2} + 2\frac{(\psi - 1)(16\psi - 1)\psi}{9(4\psi - 1)^2} \right) \right) - s \left(\left(\frac{(36\psi^2 - 16\psi + 1)(\psi - 1)}{9(4\psi - 1)^2} + \frac{\psi^2}{(4\psi - 1)^2} + \frac{1}{9} \right) + \frac{(\psi + 4\psi^2 - 1)\psi}{2(4\psi - 1)^2} \right) > 0$; for $\psi > 1.5856$, $SW^T - SW^{EL} = s \left(\psi \frac{(4\psi + 16\psi^2 + 1)^2}{2(32\psi^2 - 7\psi + 2)^2} + \left(2\frac{25(4\psi - 1)^2\psi^2}{4(32\psi^2 - 7\psi + 2)^2} + \frac{(16\psi^2 - 16\psi + 1)\psi}{2(32\psi^2 - 7\psi + 2)} \right) \right) - s \left(\left(\frac{(36\psi^2 - 16\psi + 1)(\psi - 1)}{9(4\psi - 1)^2} + \frac{\psi^2}{(4\psi - 1)^2} + \frac{1}{9} \right) + \frac{(\psi + 4\psi^2 - 1)\psi}{2(4\psi - 1)^2} \right) > 0$. $SW_{VI}^C - SW^{EL} = s \left(\frac{(4\psi + \sqrt{\psi} - 1)^2\psi}{8(4\psi - 1)^2} + \frac{\psi^2}{(4\psi - 1)^2} + \frac{(16\psi^2 - 13\psi + 1)\psi}{4(4\psi - 1)^2} \right) - s \left(\left(\frac{(36\psi^2 - 16\psi + 1)(\psi - 1)}{9(4\psi - 1)^2} + \frac{\psi^2}{(4\psi - 1)^2} + \frac{1}{9} \right) + \frac{(\psi + 4\psi^2 - 1)\psi}{2(4\psi - 1)^2} \right)$.

For high values of ψ ($\psi > 3.4078$), social welfare ranking is $SW_{VI}^C > SW^T > SW^{EL}$. For $\psi < 3.4078$, we have $SW^T > SW_{VI}^C$ and $SW^T > SW^{EL}$.

In case of negative royalties,

$$\begin{aligned} SW^{ELneg} - SW^T &= \frac{3}{32} \frac{(4\psi - 127\psi^2 + 1548\psi^3 - 3840\psi^4 + 8192\psi^5 + 4)s}{(32\psi^2 - 7\psi + 2)^2} > 0 \\ CS^{EL} - CS^T &= \frac{5}{32} \frac{(129\psi^2 - 28\psi - 460\psi^3 + 256\psi^4 + 4)s}{(32\psi^2 - 7\psi + 2)^2} > 0 \iff \psi > 1.5059 \\ PS^{ELneg} - PS^T &= \frac{(76\psi - 513\psi^2 + 3472\psi^3 - 6400\psi^4 + 12288\psi^5 - 4)s}{16(32\psi^2 - 7\psi + 2)^2} > 0 \end{aligned}$$

8.7 Bertrand competition

The demands for the goods are the same as in (5) and (6), in particular:

$$q_1 = \frac{p_2 - p_1}{s(\psi - 1)} - \frac{p_1}{s}, q_2 = 1 - \frac{p_2 - p_1}{s(\psi - 1)}.$$

D firms profits are:

$$\pi_1 = p_1 q_1, \pi_2 = (p_2 - r_2) q_2 - F_2.$$

Bertrand competition leads to the following third stage prices and quantity equilibrium:

$$\begin{aligned} p_1(0, r_2; s, s\psi) &= \frac{s\psi - s + r_2}{4\psi - 1}, \\ p_2(r_2, 0; s\psi, s) &= \frac{2\psi(s\psi - s + r_2)}{4\psi - 1} \\ q_1(0, r_2; s, s\psi) &= \frac{(s\psi - s + r_2)\psi}{(\psi - 1)(4\psi - 1)s} \\ q_2(r_2, 0; s\psi, s) &= \frac{2\psi s(\psi - 1) - r_2(2\psi - 1)}{(\psi - 1)(4\psi - 1)s} \end{aligned}$$

with $q_2(r_2, 0; s\psi, s) > 0 \iff r_2 < \frac{2\psi s(\psi - 1)}{(2\psi - 1)}$.²⁸ Firm 1 profit is then:

$$\pi_1(0, r_2; s, s\psi) = \frac{(s\psi - s + r_2)^2 \psi}{(4\psi - 1)^2 (\psi - 1) s}$$

As for firm 2:

$$\pi_2(r_2, 0; s\psi, s) = \frac{(2\psi s(\psi - 1) - r_2(2\psi - 1))^2}{(4\psi - 1)^2 (\psi - 1) s} - F_2$$

The U firm chooses the two-part tariff contract for firm 2 (r_2, F_2) such that:

$$\begin{aligned} &\max_{r_2, F_2} \Pi_U \\ &s.t. r_2 \in \left[0, \frac{2\psi s(\psi - 1)}{(2\psi - 1)} \right] \\ &F_2 \leq \frac{(2s\psi - r_2 + 2\psi r_2 - 2s\psi^2)^2}{(4\psi - 1)^2 (\psi - 1) s} - \frac{(s\psi - s + r_2)^2 \psi}{(4\psi - 1)^2 (\psi - 1) s} \end{aligned}$$

with $\Pi_U = \frac{2\psi s(\psi - 1) - r_2(2\psi - 1)}{(\psi - 1)(4\psi - 1)s} r_2 + F_2 - f$. The first constraint comes from the non-negativity of q_2 and the second constraint (binding at equilibrium) ensures that firm 2 has the incentive to get the license rather than the outside option, that we assume to be equal to $\pi_1(0, r_2; s, s\psi) > 0$.²⁹ $\frac{\partial}{\partial r_2} \Pi_U = -\frac{2r_2\psi}{s(\psi - 1)(4\psi - 1)} \geq 0 \iff r_2 \leq 0$.

²⁸This inequality is more stringent than the corresponding one under Cournot competition. Namely, $\frac{s(2\psi - 1)}{2} > \frac{2\psi s(\psi - 1)}{(2\psi - 1)}$.

²⁹In fact firm 2 can always refuse the offer of the patent holder knowing that he will make the offer to the rival firm. Therefore the outside option is not zero (the status quo profit), rather it is positive and equal to the low quality firm's profit.