Demand-enhancing Innovation

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Abstract

This paper studies a monopolist's optimal innovation problem when its innovation spending raises the scale of demand. By reducing a monopolist's incentive to innovate, regulation of the firm's price in such a market may reduce consumer welfare in the long run. I introduce a simple form of price-cap regulation where an antitrust agency allows a fixed markup over marginal cost. I find that the consumer-welfare-maximizing markup allowance is non-zero and dependent on market-specific parameters and that price-cap regulation is beneficial to consumers only in markets where the demand depreciates relatively slowly. In other markets regulation can in the long run be harmful.

JEL codes: L12, L21, L22, L41 & L51

Keywords: innovation, monopoly, antitrust, optimal markup, consumer welfare

1 Introduction

Innovation creates new products and services, and it may also create new product demand. On the other hand, if a firm does not innovate, the value of its existing product or service may decline and its scale of demand may therefore fall. In an effort to raise its profits or even just to maintain its profits, a firm in such an environment may wish to innovate even if it has no competitors. Even if consumers face monopoly prices, in the long run they may on net still benefit from such innovation.

The argument that monopoly can raise welfare dates back to Schumpeter (1947) and has recently been restated by Sidak and Teece (2009). I show that price regulation by an antitrust agency may raise consumer welfare temporarily but that in the long run, it can harm welfare by reducing innovative activity. The net effect of price regulation will thus be positive or negative depending on the nature of market. In general, there exists an welfare-maximizing level price regulation for each market, and understanding the precise nature of this tradeoff requires a dynamic model.

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The model focuses on a monopolist's innovation policy in an infinite-horizon dynamic setting with perfect capital markets. I study a monopolist's optimal innovation decision in a situation in which its innovation spending raises the scale of demand for its product. I first show that it is optimal for the firm to innovate faster and to charge a smaller markup in a market where the value of its product depreciates more quickly. Into this market I then introduce price-cap regulation, and I study the effect on consumer welfare in the short and the long run.

When an antitrust agency attempts to hold the monopolist's price closer to its competitive level, the effect on consumer welfare is twofold. The resulting price-reduction raises consumer welfare in the short run but it also reduces the firm's incentive to innovate, and this leads it to create fewer new products and services in the long run. And since fewer new products are created, the consumers' interest in the monopolist's existing product gradually declines. Price-cap regulation may therefore harm consumer welfare in the long run.

Price-cap regulation in my model takes a simple form – an antitrust agency places a cap on the markup over the marginal cost so as to maximize the present value of the consumer welfare. I derive the consumer-welfare-maximizing markup-allowance and show that it is non-zero and that it depends on market-specific parameters. I then show that in a market where the value of the existing products depreciates slowly, the benefit of price regulation exceeds its harmful effects. On the other hand, in a market where the value of the existing products depreciates quickly, the harm outweighs the benefit. There exists a threshold depreciation rate at which cost and benefit from price-cap regulation are equal. This implies that price-cap regulation is beneficial only for a market that has a depreciation rate slower than this threshold rate. I derive the rate explicitly. For depreciation rates above this cutoff rate, regulation is harmful.

In an appendix, I introduce an imperfect capital market. This raises the possibility of a second harmful effect of price-cap regulation, namely that of depriving the firm of the funds it needs to finance its innovative effort. That is, I study a firm's innovative activity when its innovation is constrained by its internal funds. In this case, regulation slows down or even reduces the firm's access to internal funds and, hence, its ability to finance its innovative activity. This effect only compounds the results obtained in the body of the paper, which therefore hold regardless of whether capital markets are perfect or not.

The rest of the paper is organized as follows. Section 2 introduces the monopolist's optimal innovation problem. Section 3 discusses the effect of price-cap regulation on innovation and consumer welfare. Section 4 studies the optimal price-cap regulation for different markets. Section 5 shows some empirical support of the model's propositions. Section 6 concludes the paper. Some proofs, data tables and additional results are in Appendixes.

2 The model

Consider a monopoly firm (or a dominant firm that has market power) which engages in production and innovation in a perfect capital market. The firm faces a linear downward-sloping demand curve,

$$q_t = d_t - \varepsilon p_t, \tag{1}$$

where q_t is the production and p_t is the product price. The intercept, d_t, represents the scale of demand and $\varepsilon > 0$.

Innovation, x_t, creates new products and increases the scale of demand, d_t. We assume that $x_t \geq 0$ and that the initial scale of demand is given at $d_0 \geq 0$. The product is assumed to become obsolete at a rate, δ, and therefore the scale of demand also depreciates at δ with no innovation effort. That is,

$$\dot{d}_t = x_t - \delta d_t. \tag{2}$$

The cost of innovation is the sum of its direct cost, x_t, and the convex adjustment cost, $\varphi(x_t)$ with $\varphi'(\cdot) > 0$, $\varphi(0) = \varphi'(0) = 0$, and $\varphi''(\cdot) > 0$. When a unit production cost is constant, c, the firm's cash flow at time t is

$$(p_t - c)\, q_t - x_t - \varphi(x_t)\,. \tag{3}$$

Solving the problem

The monopolist's price-setting rule is $(d_t - \varepsilon p_t) - \varepsilon(p_t - c) = 0$ or more explicitly, the monopoly price, p^m, is

$$p^m(d_t) = \frac{d_t + \varepsilon c}{2\varepsilon}, \tag{4}$$

for all t.

Then the firm's gross profit, π, can be defined as a function of the current scale of demand, d_t, only. That is,

$$\pi(d_t) \equiv (p^m(d_t) - c)\{d_t - \varepsilon p^m(d_t)\}\,. \tag{5}$$

From eq. (5), we find that the marginal profit is nonnegative,

$$\pi'(d_t) = \frac{d_t - \varepsilon c}{2\varepsilon} \geq 0, \tag{6}$$

for any $q_t = d_t - \varepsilon p_t \geq 0$ and $p_t \geq c$. The second derivative of eq. (5) is

$$\pi''(d_t) = \frac{1}{2\varepsilon} > 0. \tag{7}$$

That is, $\pi(\cdot)$ has increasing returns to scale.

The monopolist's problem is to maximize its present value, V,

$$\max_{x_t \geq 0} V(d_t, x_t) = \int_0^\infty \{\pi(d_t) - x_t - \varphi(x_t)\} e^{-rt} dt, \tag{8}$$

subject to eq. (2).

Then the current-value Hamiltonian function, H_t, at time t is

$$\begin{aligned} H_t(d_t, x_t) \equiv\ & \pi(d_t) - x_t - \varphi(x_t) \\ & + \gamma_t (x_t - \delta d_t), \end{aligned} \tag{9}$$

where γ_t is the costate variable associated with d_t.

The optimal innovation, x_t, must satisfy

$$1 + \varphi'(x_t) = \gamma_t, \tag{10}$$

for all t. The condition simply states that the marginal benefit from a unit of investment, i.e., the right side of eq. (10), must be equal to its marginal cost, i.e., the left side of eq. (10).

The the law of motion of the costate variable, γ_t, is

$$\dot{\gamma}_t = (r + \delta)\gamma_t - \pi'(d_t), \tag{11}$$

Differentiating eq. (10) with respect to time and equating it and eq. (11) yields the law of motion of x_t,

$$\dot{x}_t = \frac{1}{\varphi''(x_t)} [(r + \delta)\{1 + \varphi'(x_t)\} - \pi'(d_t)]. \tag{12}$$

Finally, we impose the transversality condition,

$$\lim_{t \to \infty} \gamma_t e^{-rt} = 0. \tag{13}$$

The optimal innovation effort

The monopolist's optimal innovative activity, if it exists, must follow the laws of motion, eqs. (2) and (12), and the transversality condition, eq. (13). An optimal innovation policy exists if the steady state is a stable saddle point.

The steady state values, x^* and d^*, solve $\dot{x}_t = \dot{d}_t = 0$. That is

$$(r + \delta)\{1 + \varphi'(x^*)\} = \pi'(d^*), \tag{14}$$

and

$$x^* = \delta d^*. \tag{15}$$

Evaluating the Jacobian matrix, J, in the steady state,

$$J_{x_t=x^*,d_t=d^*} = \begin{bmatrix} r+\delta & -\frac{\pi''(d^*)}{\varphi''(x^*)} \\ 1 & -\delta \end{bmatrix}, \tag{16}$$

where $Tr J_{x_t=x^*,d_t=d^*} = r > 0$, we know that the steady state is a saddle stable point if

$$\det J_{x_t=x^*,d_t=d^*} = -\delta(r+\delta) + \frac{\pi''(d^*)}{\varphi''(x^*)} < 0. \tag{17}$$

Since $\pi'' = (2\varepsilon)^{-1} > 0$ from eq. (7), eq. (17) states that the firm's optimal innovation policy exists if

$$\varphi''(x^*) > \frac{1}{2\varepsilon\delta(r+\delta)}. \tag{18}$$

I assume that eq. (18) is satisfied.

Numerical examples

Of the parameters that affect the monopolist's optimal innovation, the depreciation rate, δ, of the scale of demand probably varies over markets the most. The parameter, δ, may measure the obsolescence rate of the existing products, the speed of product rotation, or the speed of imitation. When consumers' interest in the product erodes quickly (obsolescence) or when potential competitors produce close substitute goods (imitation), δ is large; maintaining its market size then requires the monopolist to innovate faster.[1] In this sense, δ may capture the degree of market's contestability discussed in Baumol (1983). In the subsequent numerical examples, I study the monopolist's optimal innovation in markets characterized by various δs and show that the monopolist actually does innovate faster but charges a smaller markup in markets where δ is larger.

Let $\varphi(\cdot)$ be

$$\varphi(x_t) = x_t^\alpha;\ \alpha > 1. \tag{19}$$

	r	ε	c	α
value	0.05	1.00	1.00	3.00

Table 1: baseline parameters

[1] Obsolescence rates can be measured by patent renewal rates (Pakes and Simpson, 1989; Schankerman, 1998), R&D capiatl depreciation rates (Hall, 2007) or product-price declines (Cummins and Violante, 2002). Product-imitation rates can be measured by diffusion speed (Mansfield, 1968; Romeo, 1977).

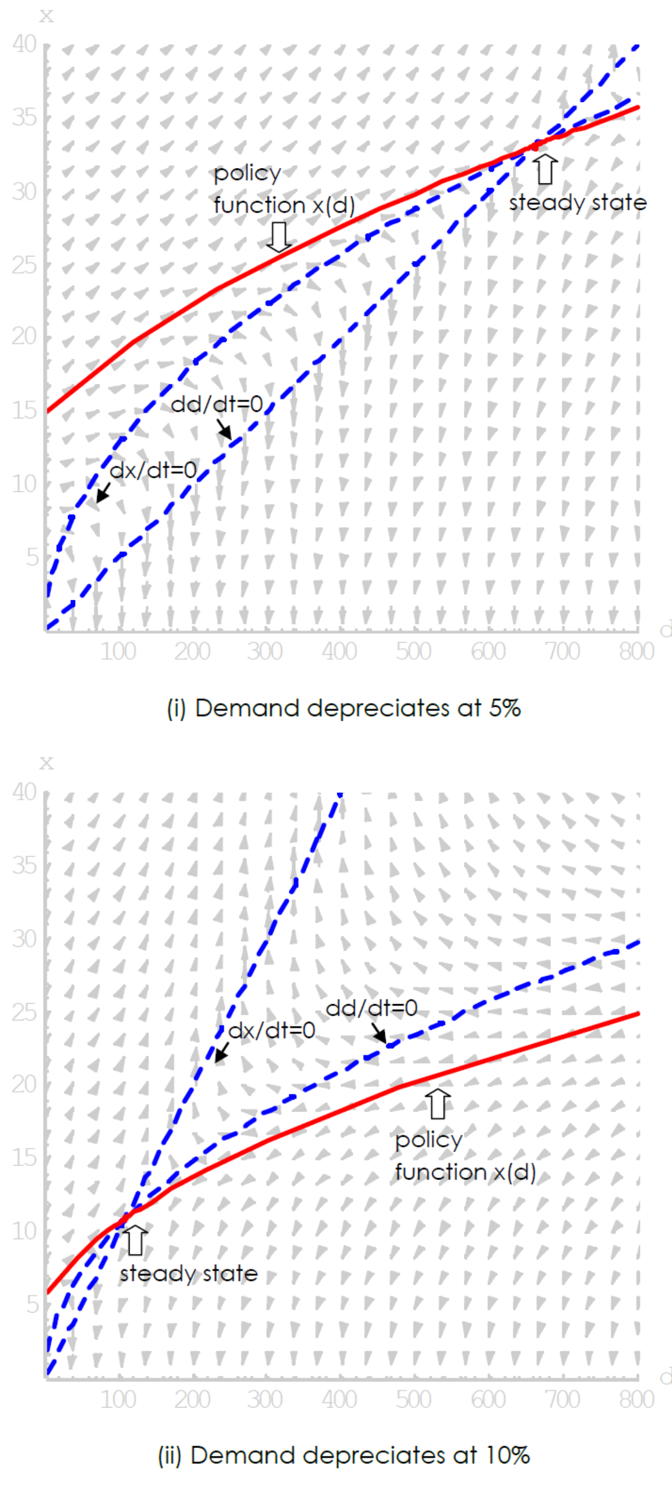

Figure 1: monopolist's optimal innovation in small-δ and large-δ markets

I set the baseline parameters in Table 1. These values are common to examples (i) and (ii). The depreciation rate of the scale of demand is $\delta = .05$ in example (i) and $\delta = .10$ in example (ii). Figure 1 shows the local dynamics of these examples in the (d_t, x_t)-vector-field. The solid line shows the optimal-innovation-policy function, i.e., the optimal x_t for a given scale of demand, d_t, of the market. The dotted lines are the demarcation curves, on which $\dot{x}_t = 0$ and $\dot{d}_t = 0$. The arrows in the vector field shows the law of motion.

ex.	δ	properties
(i)	0.05	x_t is increasing in d_t; steady-state x^*/d^* is low
(ii)	0.10	x_t is increasing in d_t; steady-state x^*/d^* is high

Table 2: numerical examples

First, we find that optimal innovation, x_t, is an increasing function of the scale of demand, d_t. Second, in steady states the larger the depreciation rate, δ, of the scale of demand is, the larger the monopolist's innovation-effort ratio to the scale of demand, x^*/d^*, is. These results are summarized in Table 2.

If we confine our attention to steady states, then it is easy to show analytically that the monopolist innovates faster and charges a smaller markup over production cost in a market where demand depreciates at a faster rate. The following proposition deals with:

$$\begin{aligned} x^*/d^* &= \text{innovation ratio to the scale of demand,} \\ \mu^m &= \text{monopoly markup,} \\ \delta &= \text{depreciation rate of the scale of demand,} \end{aligned}$$

where the markup, μ^m, should satisfy $(1 + \mu^m)\, c = p^m\,(d^*\,(\delta))$.

Proposition 1 *In a market that has a higher δ, the monopolist's x^*/d^* is larger and its μ^m is smaller.*

Proof. In a steady state, eqs. (14) and (15) hold. From eq. (15), obviously $\delta = x^*/d^*$. Thus,

$$\frac{\partial}{\partial \delta}\left(\frac{x^*}{d^*}\right) = 1 > 0. \tag{20}$$

From eq. (4), the steady-state monopoly price, $p^m\,(d_t)$, is an increasing function of d_t. From eqs. (14) and (15), we also know that the effect of an increase in δ on d^* is

$$\frac{\partial d^*}{\partial \delta} = -\frac{1 + \varphi'\,(x^*) + (r + \delta)\,\varphi''\,(x^*)\,d^*}{\delta\,(r + \delta)\,\varphi''\,(x^*) - (2\varepsilon)^{-1}} < 0, \tag{21}$$

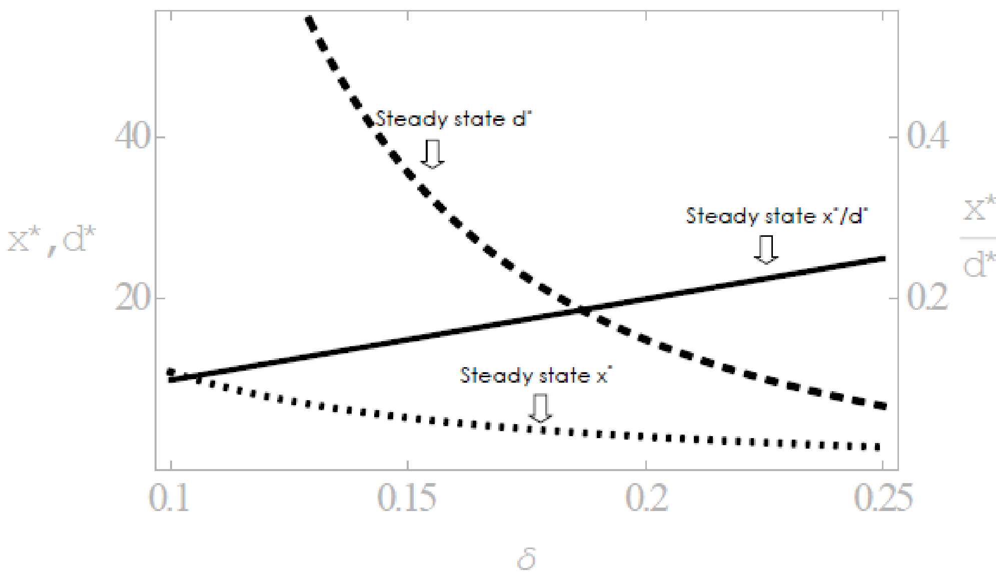

Figure 2: effect of δ on steady-state values of x^*, d^*, and x^*/d^*

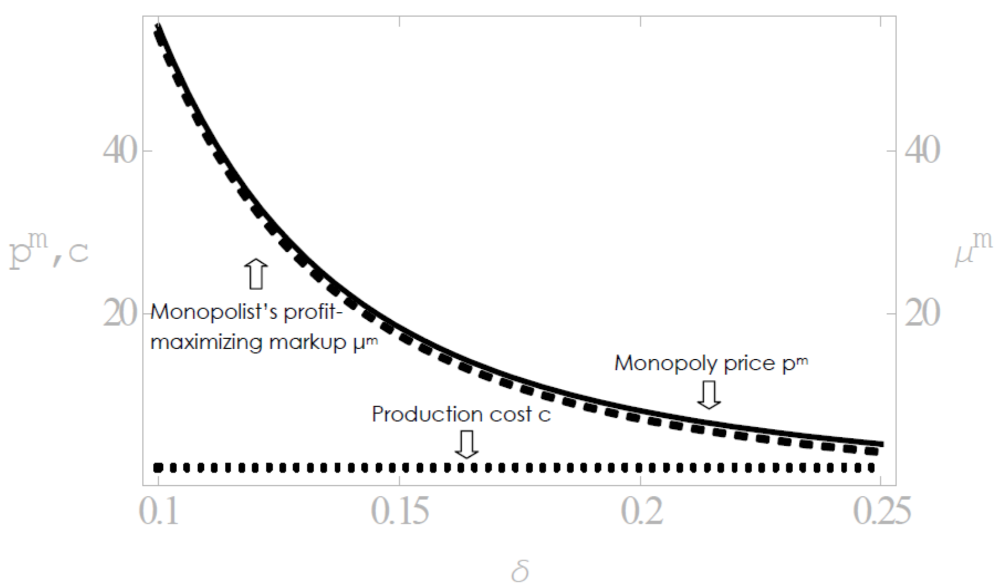

Figure 3: effect of δ on steady-state values of μ^m and p^m

where from eqs. (7) and (18), the denominator in eq. (21) is positive. Therefore, the effect of an increase in δ on μ^m is

$$\frac{\partial \mu^m}{\partial \delta} = \frac{p^{m\prime}(d^*)(\partial d^*/\partial \delta)}{c} < 0. \tag{22}$$

■

Figures 2 and 3 numerically illustrates Proposition 1's claim that x^*/d^* is increasing in δ and that μ^m is decreasing in δ. The simulation uses the baseline parameters shown in Table 1.

Consumer welfare in the monopoly market

When a monopolist sets the profit-maximizing price, eq. (4), the present value of consumer welfare, w, is computed as

$$w \equiv \frac{1}{8\varepsilon}\int_0^\infty (d_t - \varepsilon c)^2 e^{-rt} dt. \tag{23}$$

3 The effect of price-cap regulation

Consider a simple form of price-cap regulation that can reduce the monopolist's excess profit, i.e.,

$$p_t \leq (1+\mu) c, \tag{24}$$

where μ is the markup allowance set by an antitrust agency over the marginal cost.

The monopolist's new gross profit, $\hat{\pi}$, is defined as

$$\hat{\pi}(d_t) \equiv \mu c \{d_t - \varepsilon (1+\mu) c\}, \tag{25}$$

where $\hat{\pi}$ exhibits constant returns to scale, i.e.,

$$\hat{\pi}' = \mu c \text{ and } \hat{\pi}'' = 0. \tag{26}$$

The current-value Hamiltonian function, $\hat{H}_t$, at time t is

$$\begin{aligned} \hat{H}_t \equiv\ & \hat{\pi}(d_t) - x_t - \varphi(x_t) \\ & + \nu_t (x_t - \delta d_t), \end{aligned} \tag{27}$$

where ν_t denotes the new costate variable associated with d_t.

The following transversality condition should be met in the optimal path,

$$\lim_{t\to\infty} \nu_t e^{-rt} = 0. \tag{28}$$

The optimal innovation effort under price-cap regulation

Accordingly, the new law of motion of x_t is

$$\dot{x}_t = \frac{1}{\varphi''(x_t)} \left[(r+\delta) \left\{ 1 + \varphi'(x_t) \right\} - \mu c \right]. \quad (29)$$

The steady state values, x^{**} and d^{**}, solve $\dot{x}_t = \dot{d}_t = 0$. That is, from eqs. (2) and (29),

$$(r+\delta) \left\{ 1 + \varphi'(x^{**}) \right\} = \mu c, \quad (30)$$

and

$$x^{**} = \delta d^{**}. \quad (31)$$

If we confine our attention to steady states, then it is easy to show analytically that the monopolist innovates more and that it reaches a higher scale of demand in a market where a larger markup is allowed. Moreover, the monopolist innovates faster in a market where demand depreciates at a faster rate. The following proposition deals with:

$$\begin{aligned} x^{**}/d^{**} &= \text{innovation ratio to the scale of demand (with price cap)}, \\ \mu &= \text{markup allowance set by an agency}, \\ \delta &= \text{depreciation rate of the scale of demand.} \end{aligned}$$

Proposition 2 *In a market where that has a larger μ, the monopolist's x^{**} and d^{**} are larger. In a market that has a higher δ, the monopolist's x^{**}/d^{**} is larger.*

Proof. In a steady state, eqs. (30) and (31) hold. From eq. (30), the effect of an increase in μ on x^{**} is

$$\frac{\partial x^{**}}{\partial \mu} = \frac{c}{(r+\delta)\varphi''(x^{**})} > 0. \quad (32)$$

From eq. (31), the effect of an increase in μ on d^{**} is

$$\frac{\partial d^{**}}{\partial \mu} = \frac{1}{\delta} \frac{\partial x^{**}}{\partial \mu} > 0, \quad (33)$$

and the effect of an increase in δ on x^{*}/d^{**} is

$$\frac{\partial}{\partial \delta} \left(\frac{x^{**}}{d^{**}} \right) = 1 > 0. \quad (34)$$

■

Figures 4 and 5 numerically illustrate Propositions 2's claim that both x^{**} and d^{**} are increasing in μ. The simulation uses the baseline parameters shown in Table 1.

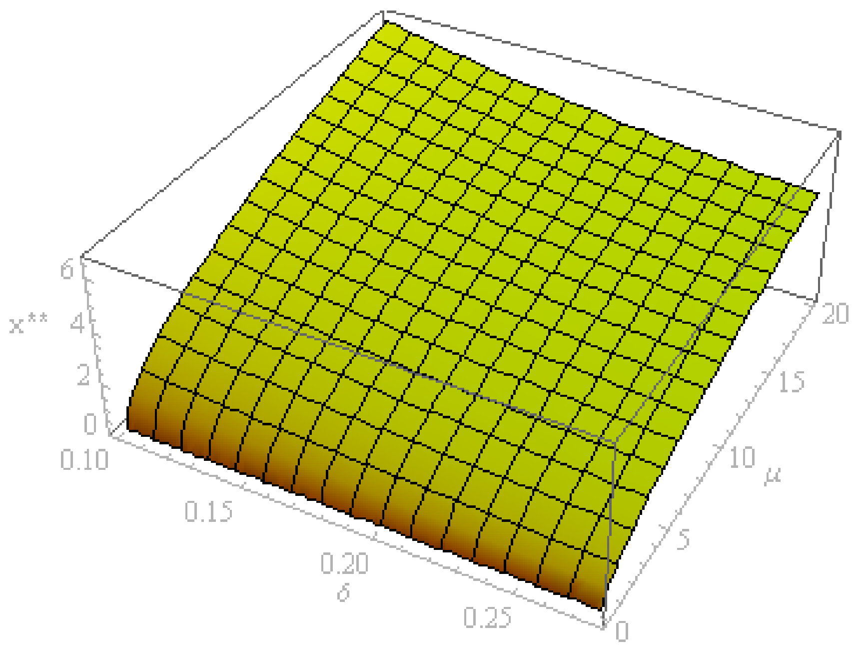

Figure 4: effect of δ and μ on steady state values of x^{**}

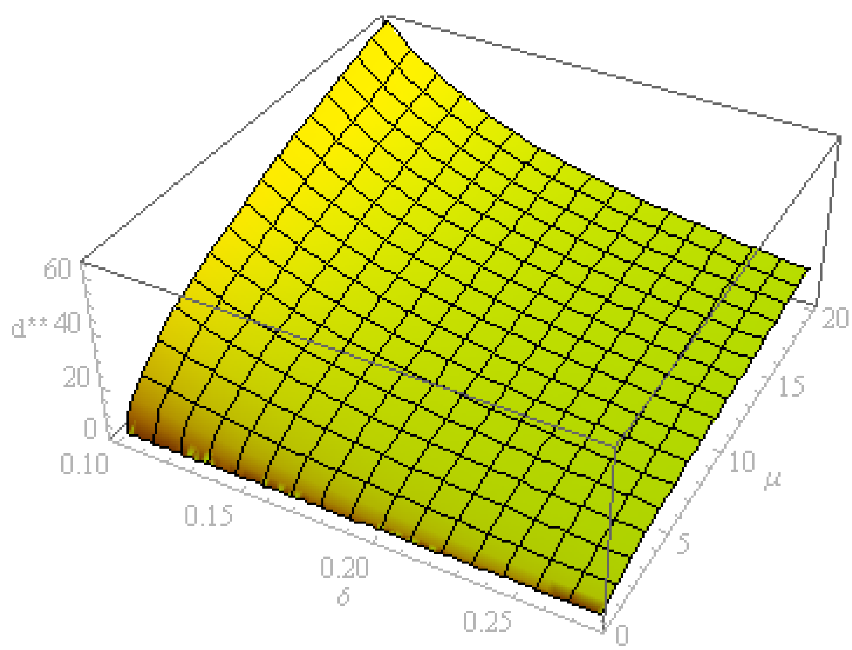

Figure 5: effect of δ and μ on steady state values of d^{**}

Evaluating the Jacobian matrix, $\hat{J}$, of eqs. (2) and (29) in the steady state,

$$\hat{J}_{x_t=x^{**},d_t=d^{**}} = \begin{bmatrix} r+\delta & 0 \\ 1 & -\delta \end{bmatrix}, \tag{35}$$

where $Tr\hat{J}_{x_t=x^{**},d_t=d^{**}} = r > 0$ and $\det \hat{J}_{x_t=x^{**},d_t=d^{**}} = -\delta\left(r+\delta\right) < 0$, we know that the steady state is a saddle-stable point and therefore, the optimal innovation policy exists.

With a price cap, the new law of motion of ν_t is

$$\dot{\nu}_t = \left(r+\delta\right)\nu_t - \mu c. \tag{36}$$

Solving this differential equation, we get[2]

$$\nu_t = \frac{\mu c}{r+\delta}. \tag{37}$$

Thus, ν_t is constant for all t. Then, x_t also must be constant for all t because the optimal x_t must satisfy $1+\varphi'\left(x_t\right) = \nu_t$ and $\varphi\left(\cdot\right)$ is strictly convex. Moreover, the optimal x_t must approach its steady state, x^{**}, to satisfy the transversality condition. From eq. (28),

$$\lim_{t\to\infty}\left\{1+\varphi'\left(x_t\right)\right\}e^{-rt} = 0. \tag{38}$$

Therefore, the optimal innovation effort, x_t, is constant for all t at

$$x_t = x^{**}. \tag{39}$$

Proposition 3 *In a market that has a larger μ, the monopolist's x_t is larger both in and out of the steady state.*

Proof. From eqs. (32) and (39), the effect of an increase in μ on x_t is

$$\frac{\partial x_t}{\partial \mu} = \frac{\partial x^{**}}{\partial \mu} > 0. \tag{40}$$

■

[2]Integrating eq. (36), we get

$$\begin{aligned}\int_t^\infty \left[\dot{\nu}_\tau - \left(r+\delta\right)\nu_\tau\right]e^{-(r+\delta)\tau}d\tau &= -\mu c\int_t^\infty e^{-(r+\delta)\tau}d\tau \\ \lim_{\tau\to\infty}\nu_\tau e^{-(r+\delta)\tau} - \nu_t e^{-(r+\delta)t} &= -\frac{\mu c}{r+\delta}e^{-(r+\delta)t}.\end{aligned}$$

From eq. (28), the first term of the left-hand-side is zero. Therefore, the solution of eq. (36) is

$$\nu_t = \frac{\mu c}{r+\delta}.$$

As eq. (39) finds that the firm's optimal innovation effort is constant at x^{**} for all t, the law of motion of the scale of demand, d_t, eq. (2), can reduce to

$$\dot{d}_t = x^{**} - \delta d_t. \tag{41}$$

Solving eq. (41), we get[3]

$$d_t = d^{**} + (d_0 - d^{**}) e^{-\delta t}, \tag{42}$$

which simply implies that the scale of demand, d_t, asymptotically approaches to its steady state level, d^{**}.

Consumer welfare under price regulation

The present value of consumer welfare, $\hat{w}$, under price-cap regulation is computed as

$$\hat{w} = \frac{1}{2\varepsilon} \int_0^{\infty} \{d_t - \varepsilon (1+\mu) c\}^2 e^{-rt} dt. \tag{43}$$

4 Optimal price-cap regulation

Should antitrust enforcement aim to maximize the consumer welfare only, or to maximize the aggregate economic welfare?[4] This still is a much debated question. In case of merger, for example, which is the most debated context, the argument can be summarized as follows: while a merger can reduce consumer welfare (as a result of less competition and a post-merger price increase), it can increase shareholders' welfare (as a result of an efficiency increase of the merged firms). On net, aggregate economic welfare may then increase. This implies that under the two different welfare standards, opposite policy conclusions can be drawn, so that maximization of consumer welfare calls for blocking the merger, whereas maximizing aggregate economic welfare would call for allowing the merger.

[3] Integrating eq. (41), we get

$$\begin{aligned}
\int_0^t \left[\dot{d}_\tau + \delta d_\tau\right] e^{\delta\tau} d\tau &= x^{**} \int_0^t e^{\delta\tau} d\tau \\
\left[d_\tau e^{\delta\tau}\right]_0^t &= x^{**} \left[\frac{e^{\delta\tau}}{\delta}\right]_0^t \\
d_t e^{\delta t} - d_0 &= \frac{x^{**}}{\delta}\left(e^{\delta t} - 1\right).
\end{aligned}$$

From eq. (31), $x^{**}/\delta = d^{**}$. Thus,

$$d_t = d^{**}\left(1 - e^{-\delta t}\right) + d_0 e^{-\delta t}.$$

[4] See Salop (1995, 2005), Farrell and Katz (2006), Pittman (2007), and Kaplow (2011).

In case of monopoly, regulating the price reduces shareholders' welfare. Unlike the merger case, however, its effect on consumer welfare is not necessarily positive in a dynamic situation. Consumers, in general, benefit from both a lower price and from more innovation; regulating a monopolist's price can reduce its incentive to innovate, implying that shareholders' and consumer welfare may *both* decline. Price regulation then reduces aggregate welfare.

In this section, I show the above-stated points analytically and numerically. That is, even under the pure consumer welfare standard, bringing the market's price to its competitive level is never optimal. Moreover, in some high-δ markets, no intervention is optimal. Although I do not analyze how these results may change under the aggregate economic welfare standard, one would conjecture that when shareholders' welfare is also taken into account, even less regulation should be optimal for all markets.

Price-cap regulation benefits consumers only if $w < \hat{w}$, otherwise it harms them in the long run. From eqs. (23) and (43), we may have $w \lesseqgtr \hat{w}$ depending on parameters. Let us now focus on two parameters, δ and μ, as key determinants of w and $\hat{w}$, with all other parameters being held constant. The effect of an increase in the markup allowance, μ, in a given-δ market, on consumer welfare, $\hat{w}$, is twofold. A higher markup allowance immediately increases the monopolist's product price and thus reduces consumer welfare in the short run. On the other hand, a higher markup allowance can raise the monopolist's incentive to innovate. As innovation enhances the scale of demand, in the long run, consumer welfare may rise.

I also show that there exists a unique markup allowance rate, μ^o, for a given δ, that maximizes $\hat{w}$. However, imposing the markup allowance, μ^o, may not benefit for consumers if it does not improve consumer welfare. I show that there exists a threshold depreciation rate, $\tilde{\delta}$, so that price-cap regulation is beneficial only for a market that has a depreciation rate slower than $\tilde{\delta}$ and that otherwise it is harmful.

To simplify the argument, I assume that at the moment that regulation is imposed market is in its steady state, i.e., $d_0 = d^*$. If the regulation is not imposed, the present value of consumer welfare, eq. (23), is

$$w(\delta) \equiv \frac{1}{8\varepsilon} \int_0^\infty \left(d^*(\delta) - \varepsilon c\right)^2 e^{-rt} dt. \tag{44}$$

On the other hand, if the regulation is imposed, from eq. (42), the present value of consumer welfare, eq. (43), is

$$\hat{w}(\delta, \mu) = \frac{1}{2\varepsilon} \int_0^\infty \left\{d_t - \varepsilon(1+\mu)c\right\}^2 e^{-rt} dt. \tag{45}$$

Suppose now that an antitrust agency represents consumer welfare only. Then its problem is to find the markup-allowance, μ^o, that maximizes the consumer welfare, i.e.,

$$\mu^o = \arg\max \text{eq. } (45). \tag{46}$$

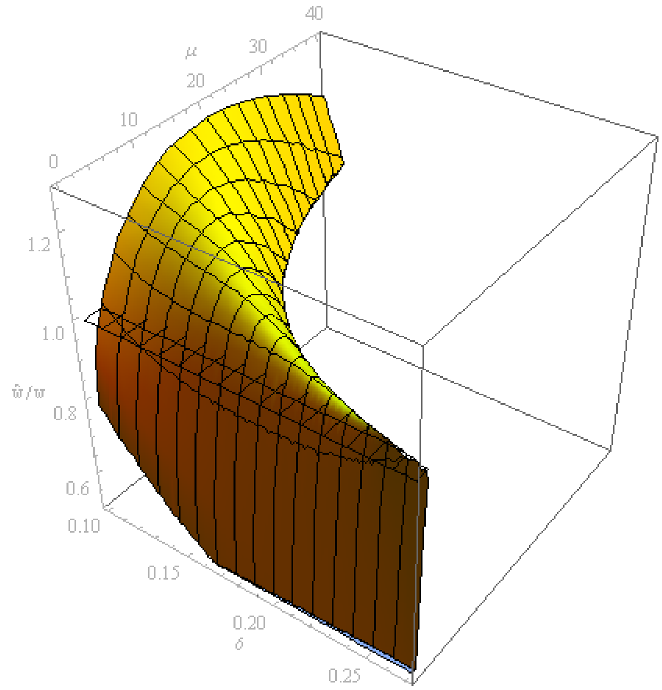

Figure 6: consumer welfare ratio Γ ($\hat{w}/w$, with and w/o price-cap regulation)

The first-order necessary condition for maximization of consumer welfare is $\partial\hat{w}/\partial\mu = 0$. Rearranging the condition, we get

$$\underbrace{\frac{\partial d^{**}(\delta,\mu^o)}{\partial\mu}\frac{1}{\varepsilon}\int_0^\infty \{d_t - \varepsilon(1+\mu^o)c\}\left(1-e^{-\delta t}\right)e^{-rt}dt}_{\equiv\Delta^1(\delta,\mu)} \tag{47}$$

$$= \underbrace{c\int_0^\infty \{d_t - \varepsilon(1+\mu^o)c\}e^{-rt}dt}_{\equiv\Delta^2(\delta,\mu)}.$$

That is, the markup allowance, μ, in a given-δ market, should be raised until its marginal benefit, Δ^1, from demand enhancement becomes equal to its marginal cost, Δ^2, due to the price increase. The next proposition deals with:

$$\begin{aligned} \hat{w}(\delta,\mu) &= \text{present value of consumer welfare (with price cap)}, \\ \mu^o &= \text{markup allowance that maximizes } \hat{w}. \end{aligned}$$

Proposition 4 *There exists μ^o, for a given δ, that maximizes $\hat{w}(\delta,\mu)$.*

Proof. See Appendix 1. ■

Figure 6 numerically illustrates Proposition 4's claim. The simulation uses the baseline parameters shown in Table 1. The vertical axis is the consumer-welfare ratio, defined as $\hat{w}(\delta, \mu)/w(\delta) \equiv \Gamma(\delta, \mu)$. The welfare-maximizing markup, μ^o, for a given δ-market, occurs along the peak of the function $\Gamma(\delta, \mu)$. The price-cap regulation is beneficial to consumers only if $\Gamma(\delta, \mu^o(\delta)) > 1$. Note that the domain of the function $\Gamma(\delta, \mu)$ is restricted to be $\mu < \mu^m(\delta)$, where $\mu^m(\delta)$ is the monopolist's profit-maximizing markup.[5] This is simply because any price-cap regulation that allows a higher markup than $\mu^m(\delta)$ will no longer be binding and therefore $\Gamma(\delta, \mu) = 1$ for $\mu \geq \mu^m(\delta)$. In Figure 6, a unique value, $\tilde{\delta} > 0$, at which $\Gamma\left(\tilde{\delta}, \mu^o\left(\tilde{\delta}\right)\right) = 1$ exists. This implies that, in a market where demand depreciates at a faster rate than $\tilde{\delta}$, price-cap regulation no longer raises consumer welfare. Moreover, reducing the monopolist's markup in this case will harm the consumer welfare. The next proposition will clarify this point. We deal with:

$$\begin{aligned}
\Gamma(\delta, \mu^o(\delta)) &\equiv \frac{\hat{w}(\delta, \mu^o(\delta))}{w(\delta)} \\
&= \frac{\text{consumer welfare (with } \textit{optimal} \text{ price-cap)}}{\text{consumer welfare (w/o regulation)}}, \\
\tilde{\delta} &= \text{threshold-depreciation rate.}
\end{aligned}$$

Proposition 5 *There exists* $\tilde{\delta}$ *at which* $\Gamma\left(\tilde{\delta}, \mu^o\left(\tilde{\delta}\right)\right) = 1$.

Proof. See Appendix 2. ■

Figure 7 numerically confirms Proposition 5's claim. The simulation uses the baseline parameters shown in Table 1. Price-cap regulation is beneficial only for a market that has a depreciation rate less than $\tilde{\delta}$. Figure 8 shows the optimal rate of reduction, $\alpha = 1 - (\mu^o/\mu^m)$, of the monopolist's markup. It confirms that a smaller markup reduction is optimal in a market where demand depreciates at a faster rate.[6]

5 Some evidence

The purpose of this section is to report some empirical observations which are relevant to the model's predictions, but not to give a formal empirical analysis.

[5] The monopolist's markup, μ^m, satisfies $p^m(d^*) = (1 + \mu^m)c$, where from eq. (4), $p^m(d^*) = \{d^*(\delta) + \varepsilon c\}/2\varepsilon$. By solving this for μ^m, we get $\mu^m(\delta) = \{d^*(\delta) - \varepsilon c\}/2\varepsilon c$.

[6] A possibile extension of the model would be to rewrite the antitrust agency's welfare-maximizing problem in terms of finding, for each market, an optimal patent length, instead of an optimal markup allowance. For early studies on optimal patent life, see Nordhaus (1969) and Scherer (1972). Given the model's features, one would expect that similar conclusions would obtain. That is, tougher regulation would take the form of a shorter patent life, and it then would be optimal in a market which has a slower depreciation of demand. Conversely, in markets where demand depreciates faster, it would be optimal to have little or no regulation which in that formulation would translate into a longer or an infinite patent life allowance. The formal analysis of such is left for future study.

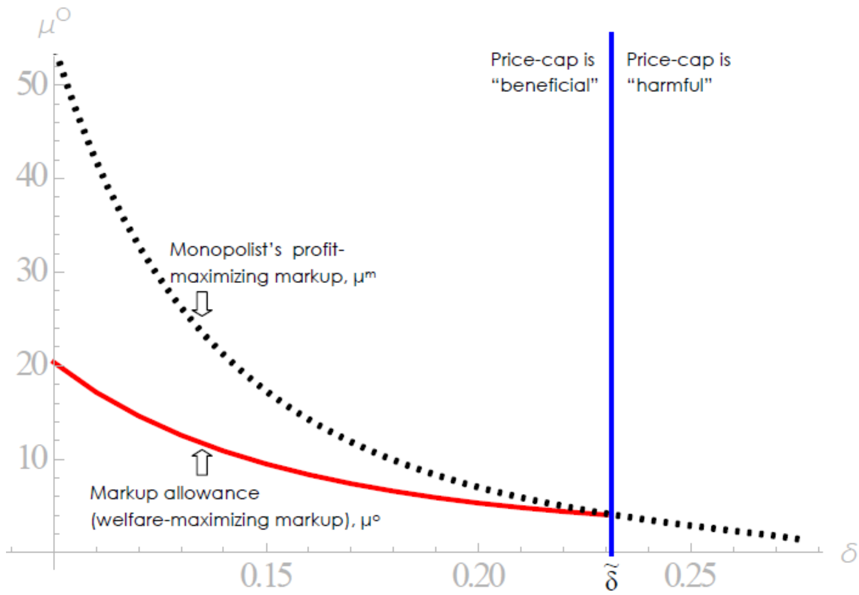

Figure 7: optimal markup allowance (μ^o) and threshold ($\tilde{\delta}$)

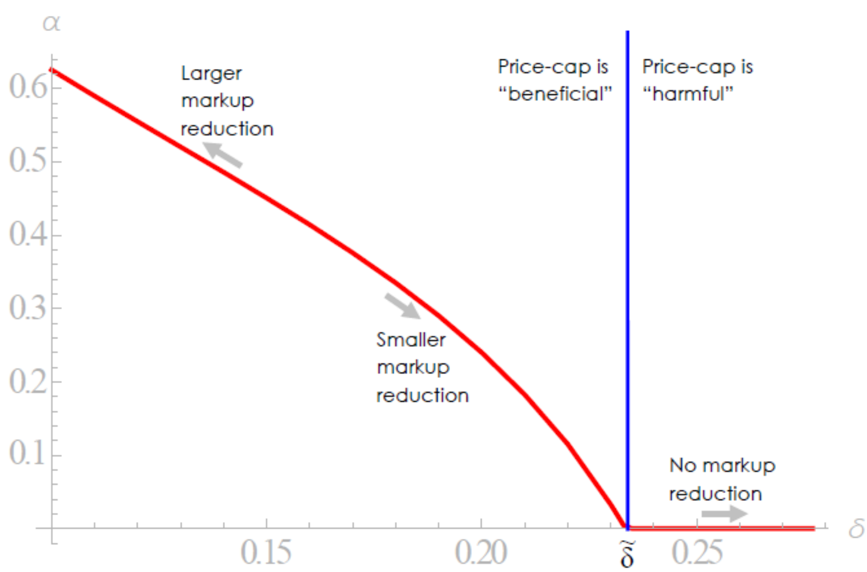

Figure 8: optimal markup reduction (α) and threshold ($\tilde{\delta}$)

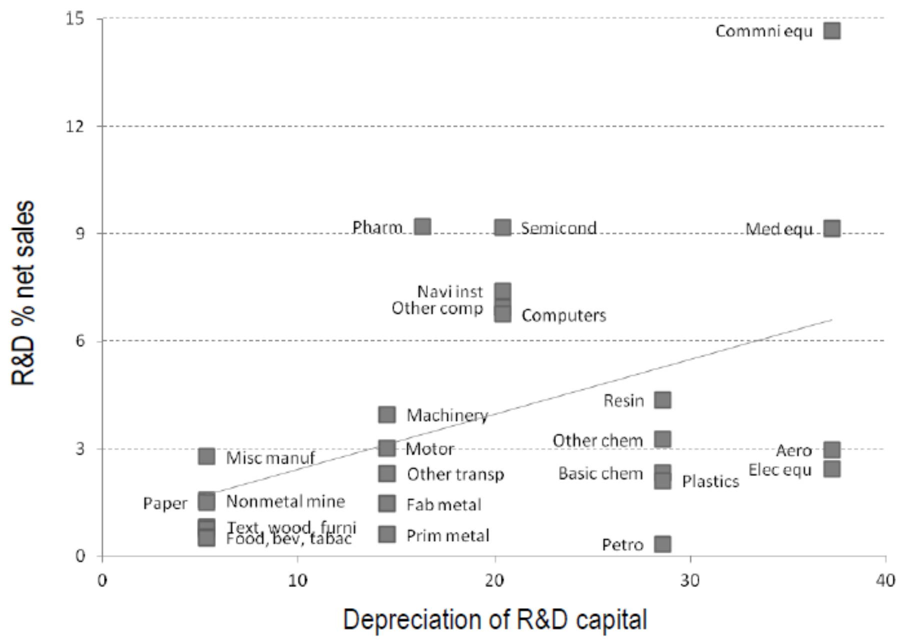

Figure 9: R&D capital depreciation rate and R&D % of net sales

As shown in Proposition 1, the model predicts a positive relation between δ and i^*/d^*. That is, in markets that have a higher depreciation of demand, a monopolist should innovate at a faster rate. Since the monopolist's current scale of demand is as a result of its cumulative past innovation effort, we may measure the scale of demand by the firm's R&D capital and δ by the depreciation rate of that capital. Hall (2007) estimates the depreciation rate of the R&D capital using firm market value for six U.S. manufacturing sectors (chemicals, rubber, and plastics; pharmaceuticals and medical instruments; electrical machinery; computers and scientific instruments; metals, machinery, and transport equipment; and miscellaneous manufacturing) for the period 1999-2003. I shall use Hall's depreciation rate as a proxy for the model's δ. As a proxy for i^*/d^*, I use average R&D funds (company and other nonfederal) for the period 1999-2003 as a percent of net sales. The data are available for the U.S. manufacturing industries at the North American Industry Classification System (NAICS)'s 4-digit-level from the NSF's Industrial R&D Information System. Table A1 in Appendix 4 provides the summary of the data. Figure 9 shows the relation between the constructed depreciation rates and the innovation ratios.

As Proposition 1 also shows, the model predicts a negative relation between δ and μ^*, that is a monopolist in markets that have a higher depreciation rate should charge a smaller markup. Hall (1988) estimates markup ratios at the Standard Industrial Classification (SIC)'s 2-digit-level U.S. manufacturing industry for the period 1953-1984. Roeger (1995) re-estimates them in a different way to overcome identification problems recognized in Hall (1988). I use Roeger (1995)'s markup estimates as a

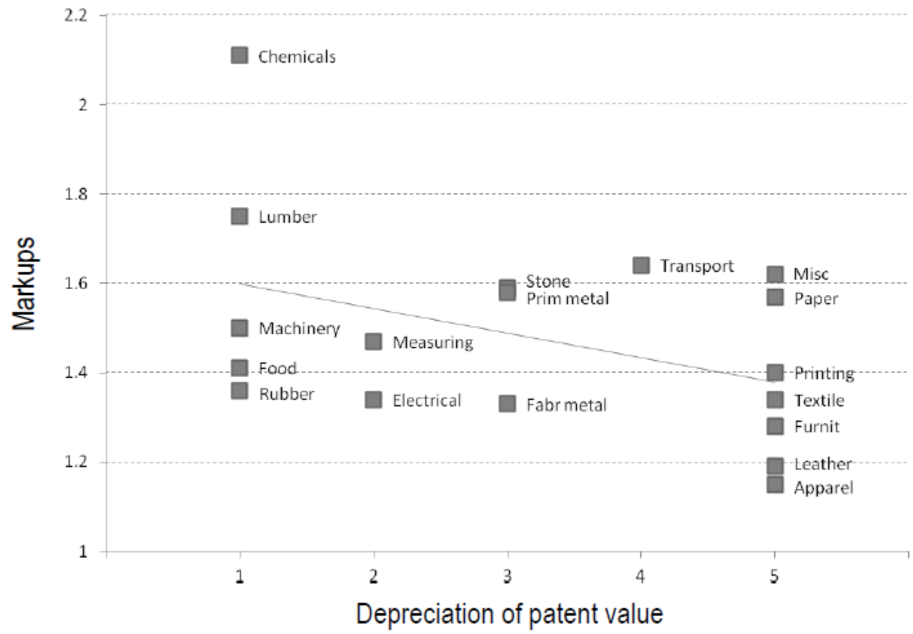

Figure 10: patent depreciation rate and markup

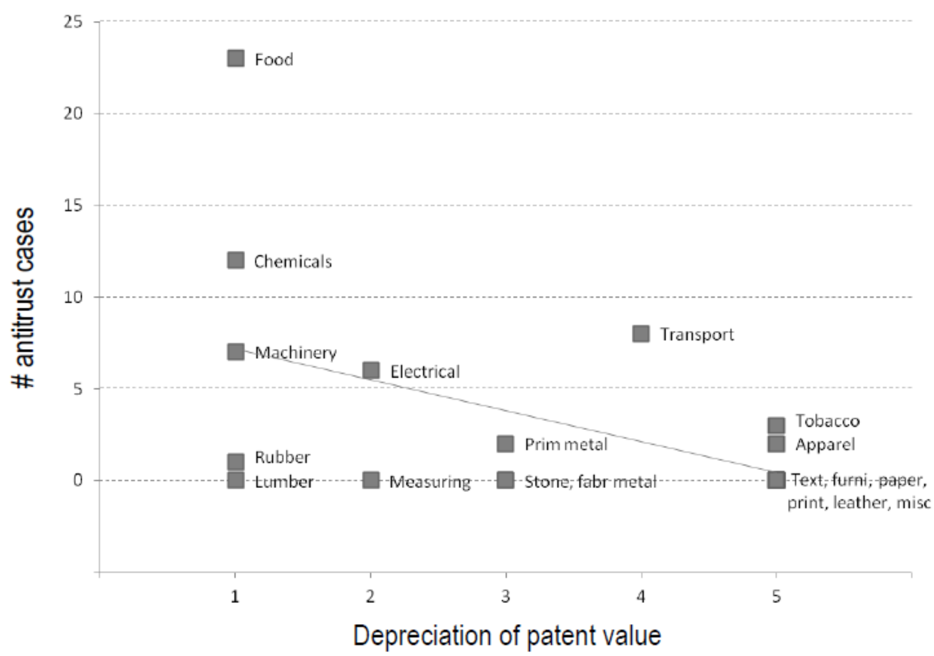

Figure 11: patent depreciation rate and antitrust cases

proxy for the model's μ^*. As a proxy for δ, I use a patent renewal rate as a negative indicator of the rate at which R&D capital depreciates.[7] Pakes and Simpson (1989) estimate patent renewal rates using Finnish manufacturing data at the 2-digit level for the period 1969-1987. The period they study appears to have a significant overlap with Roeger's. Pakes and Simpson cluster industries into the highest-patent-renewal-rate Group 1, and so on down to the lowest-patent-renewal rate Group 5. I interpret high rates of renewal to mean low rates of depreciation, and low renewal rates to mean high depreciation of patent value. Column 2 and 4 of Table A2 in Appendix 4 provides the summary of the data. Figure 10 shows the relation between the depreciation rate and the markup ratio. The depreciation of patent value is on the x-axis and is ranked from 1 (the lowest depreciation) to 5 (the highest depreciation).

Finally, Propositions 4 and 5 imply that reduction of a monopolist's markup should be larger in markets that have a slow depreciation of demand and that no intervention is optimal in other markets. If the current antitrust practice already recognizes this feature of optimal price-cap regulation, markets that have a high depreciation rate of demand should have fewer antitrust cases. I use U.S. Supreme Court antitrust cases filed under federal antitrust laws; the Sherman Antitrust Act of 1890, 15 U.S.C. 1, the Clayton Act, 15 U.S.C. 12, and the Federal Trade Commission Act, 15 U.S.C. 41, both passed in 1914, for the period 1895-2004. From those, I select those antitrust cases that involved manufactured products, and classify them using their 2-digit-level SIC codes. I use Pakes and Simpson (1989)'s depreciation rates as a proxy for δ. Column 2 and 5 of Table A2 in Appendix 4 provides the summary of the data. Figure 11 shows that the relation between the depreciation rate and the number of antitrust cases. The frequency of antitrust cases seems higher in low-δ industries, which implies that U.S. antitrust agencies seem to be aware of the importance of δ.

6 Conclusions

This paper has analyzed the effect of price-cap regulation on a monopolist's innovative effort and its effect on consumer welfare in an infinite-horizon dynamic model. Price-cap regulation in the model takes a simple form – an antitrust agency allows a certain markup over the marginal cost. Such regulation may improve consumer welfare temporarily. However, it also can reduce the monopolist's incentive to innovate. In that case, consumers will receive less long-run benefit from future products.

The paper studied price-cap regulation that maximizes consumer welfare. The finding is that a greater reduction of the monopolist's markup is optimal in markets where demand depreciates at a slower rate and that a smaller or even a zero reduction of the monopolist's markup is optimal in markets where demand depreciates at a

[7] I do not use, in this case, Hall (2007)'s market-value-based depreciation rate as a proxy for δ because Hall's study period, 1999-2003, has no overlap with Roeger's study period, 1953-1984.

faster rate. The paper also found that there is a unique threshold rate of depreciation of demand that describes the nature of the optimal policy. That is, regulation is on net beneficial if market demand depreciates more slowly than the threshold rate: If demand depreciates more slowly than this rate, price regulation raises welfare, and the antitrust agency should therefore step in to regulate the monopolist's price. On the other hand, if the market's demand depreciates faster than the threshold rate, price-cap regulation reduces consumer welfare.

References

[1] Baumol, W. J. 1983. Contestable Markets and the Theory of Industry Structure. *The Journal of Political Economy* 91(6): 1055-1066.

[2] Cummins, J. G. and Violante, G. L. 2002. Investment-Specific Technical Change in the US (1947-2000): Measurement and Macroeconomic Consequences. *Review of Economic Dynamics* 5(2): 243-284.

[3] Farrell, J. and Katz, M. L. 2006. The Economics of Welfare Standards in Antitrust. *CPI Journal* 2(2): 3-28.

[4] Hall, B. H. 2007. Measuring the Returns to R&D: The Depreciation Problem. NBER Working Paper No. 13473.

[5] Hall, R.E. 1988. The Relation between Price and Marginal Cost in U.S. Industry. *Journal of Political Economy* 96(5): 921-947.

[6] Himmelberg, C. P. and Petersen, B. C. 1994. R&D and internal finance: a panel study of small firms in high-tech industries. *The Review of Economics and Statistics* 76(1): 38-51.

[7] Kaplow, L. " On the Choice of Welfare Standards in Competition Law." *The Harvard John M. Olin Discussion Paper Series* No. 693.

[8] Mansfield, E. 1968. *The Economics of Technical Change*, New York, WW Norton.

[9] Mead, Charles I. 2007. "R&D Depreciation Rates in the 2007 R&D Satellite Account." *Bureau of Economic Analysis/ National Science Foundation 2007 R&D Satellite Account Background Paper.*

[10] Nordhaus, W. 1969. Invention, *Growth and Welfare: A Theoretical Treatment of Technological Change.* Cambridge, MIT Press.

[11] Scherer, F. M. 1972. Nordhaus' Theory of Optimal Patent Life: A Geometric Reinterpretation. *American Economic Review* 62(3): 422-427.

[12] Pakes, Ariel and Margaret Simpson. 1989. "Patent Renewal Data." *Brookings Papers and Economic Activity*: *Microeconomics*: 331-410.

[13] Pittman, Russell. 2007. "Consumer Surplus as the Appropriate Standard for Antitrust Enforcement." *U.S. Department of Justice Antitrust Division Economic Analysis Group Discussion Paper* EAG 07-9.

[14] Roeger, W. 1995. Can Imperfect Competition Explain the Difference between Primal and Dual Productivity Measures? Estimates for U.S. Manufacturing. *Journal of Political Economy* 103(2): 316-330.

[15] Salop, S. C. 1995. Efficiencies in Dynamic Merger Analysis. Federal Trade Commission Hearings on Global and Innovation-based Competition. http://www.ftc.gov/opp/global/saloptst.shtm.

[16] Salop, S. C. 2005. Question: What is the Real and Proper Antitrust Welfare Standard? Answer: The True Consumer Welfare Standard. *Georgetown University Law Center Working Paper Series.*

[17] Schankerman, Mark. 1998. "How Valuable is Patent Protection? Estimates by Technology Field." *The RAND Journal of Economics*, 29(1): 77-107.

[18] Schumpeter, J. A. 1947. *Capitalism, Socialism and Democracy.* 2nd Ed. New York and London: Harper & Brothers Publishers.

[19] Romeo, A. A. 1977. The Rate of Imitation of a Capital-Embodied Process Innovation. *Economica* 44(173): 63-69.

[20] Schworm, W. E. 1980. Financial Constraints and Capital Accumulation. *International Economic Review* 21(3): 643-660.

[21] Sidak, J. G. and Teece, D. 2009. Dynamic Competition in Antitrust Law. *Journal of Competition Law & Economics* 5(4): 581-631.

Appendix 1: Proof of proposition 4

Proof. We know that $\hat{w}$ has a local maximum at μ^o if $\hat{w}$ is concave down at μ^o, i.e.,

$$\text{if } \frac{\partial^2 \hat{w}}{\partial \mu^2} = \frac{\partial \Delta^1}{\partial \mu} - \frac{\partial \Delta^2}{\partial \mu} < 0 \Rightarrow \hat{w} \text{ has a local maximum at } \mu^o. \tag{a1}$$

Taking a derivative of the marginal benefit, Δ^1, with respect to μ gives

$$\begin{aligned} \frac{\partial \Delta^1}{\partial \mu} &= \underbrace{\frac{\partial^2 d^{**}(\delta, \mu^o)}{\partial \mu^2} \frac{1}{\varepsilon} \int_0^\infty \{d_t - \varepsilon (1 + \mu^o) c\} \left(1 - e^{-\delta t}\right) e^{-rt} dt}_{term\ 1} \\ &\quad \underbrace{+ \frac{\partial d^{**}(\delta, \mu^o)}{\partial \mu} \frac{1}{\varepsilon} \int_0^\infty \left\{ \left(1 - e^{-\delta t}\right) \frac{\partial d^{**}(\delta, \mu^o)}{\partial \mu} - \varepsilon c \right\} \left(1 - e^{-\delta t}\right) e^{-rt} dt.}_{term\ 2} \end{aligned} \tag{a2}$$

From eqs. (32) and (33), $\partial^2 d^{**}/\partial \mu^2$ in *term* 1 is negative.[8] Thus, *term* 1 of eq. (a2) is negative.

Let *term* 2 of eq. (a2) be expanded as[9][10]

$$term\ 2 = \frac{\partial d^{**}(\delta, \mu^o)}{\partial \mu} \frac{1}{\varepsilon} \frac{2\delta^2}{r(r+\delta)(r+2\delta)} \left\{ \frac{\partial d^{**}(\delta, \mu^o)}{\partial \mu} - \frac{r + 2\delta}{2\delta} \varepsilon c \right\}. \tag{a3}$$

where from eq. (33), $\partial d^{**}/\partial \mu > 0$ and thus the sign of *term* 2 depends solely on the sign of $\{\cdot\}$ in eq. (a3).

Now, recall the first-order necessary condition, eq. (47) and rewrite it as

$$\frac{\partial d^{**}(\delta, \mu^o)}{\partial \mu} = \Phi \varepsilon c, \tag{a4}$$

[8] From eqs. (32) and (33),

$$\begin{aligned} \frac{\partial^2 d^{**}}{\partial \mu^2} &= \frac{-c\varphi'''}{\delta (r+\delta) \varphi''^2} \frac{\partial x^{**}}{\partial \mu} \\ &= \frac{-c^2 \varphi'''}{\delta (r+\delta)^2 \varphi''^3} < 0. \end{aligned}$$

[9] Rewrite *term* 2 in eq. (a2) as $\frac{\partial d^{**}(\delta,\mu^o)}{\partial \mu} \frac{1}{\varepsilon} \left[\frac{\partial d^{**}(\delta,\mu^o)}{\partial \mu} \int_0^\infty \left(1 - e^{-\delta t}\right)^2 e^{-rt} dt - \varepsilon c \int_0^\infty \left(1 - e^{-\delta t}\right) e^{-rt} dt \right]$. Then use expansions listed in footnote 6.

[10] $\int_0^\infty e^{-rt} dt = \frac{1}{r}$. $\int_0^\infty e^{-(r+\delta)t} dt = \frac{1}{r+\delta}$. $\int_0^\infty \left(1 - e^{-\delta t}\right) e^{-rt} dt = \frac{\delta}{r(r+\delta)}$. $\int_0^\infty \left(1 - e^{-\delta t}\right) e^{-(r+\delta)t} dt = \frac{\delta}{(r+\delta)(r+2\delta)}$. $\int_0^\infty \left(1 - e^{-\delta t}\right)^2 e^{-rt} dt = \frac{2\delta^2}{r(r+\delta)(r+2\delta)}$.

where

$$\Phi \equiv \frac{\displaystyle\int_0^\infty \{d_t - \varepsilon(1+\mu^o)c\} e^{-rt}dt}{\displaystyle\int_0^\infty \{d_t - \varepsilon(1+\mu^o)c\}(1-e^{-\delta t})e^{-rt}dt} > 1. \tag{a5}$$

Then by plugging eq. (a4) into eq. (a3), we get

$$term\ 2 = \frac{\partial d^{**}(\delta,\mu^o)}{\partial \mu}\frac{1}{\varepsilon}\frac{2\delta^2}{r(r+\delta)(r+2\delta)}\left\{\Phi - \frac{r+2\delta}{2\delta}\right\}\varepsilon c. \tag{a6}$$

We find that $term\ 2$ is negative if $\frac{r+2\delta}{\delta} > \Phi$.

Then we may state that:

$$\text{if } \frac{r+2\delta}{\delta} > \Phi \Rightarrow \frac{\partial \Delta^1}{\partial \mu} < 0. \tag{a7}$$

By the way, Φ in eq. (a5) can be further expanded as[11]

$$\Phi = \frac{d^{**}(\delta,\mu^o)\left(\frac{\delta}{r(r+\delta)}\right) + d^*(\delta)\left(\frac{1}{r+\delta}\right) - \varepsilon(1+\mu^o)c\left(\frac{1}{r}\right)}{d^{**}(\delta,\mu^o)\left(\frac{2\delta^2}{r(r+\delta)(r+2\delta)}\right) + d^*(\delta)\left(\frac{\delta}{(r+\delta)(r+2\delta)}\right) - \varepsilon(1+\mu^o)c\left(\frac{\delta}{r(r+\delta)}\right)} \tag{a8}$$

Dividing both sides of eq. (a8) by $\frac{r+2\delta}{\delta}$ gives

$$\begin{aligned}\frac{\Phi}{\frac{r+2\delta}{\delta}} &= \frac{d^{**}(\delta,\mu^o)\left(\frac{\delta}{r(r+\delta)}\right) + d^*(\delta)\left(\frac{1}{r+\delta}\right) - \varepsilon(1+\mu^o)c\left(\frac{1}{r}\right)}{d^{**}(\delta,\mu^o)\left(\frac{2\delta}{r(r+\delta)}\right) + d^*(\delta)\left(\frac{1}{r+\delta}\right) - \varepsilon(1+\mu^o)c\left(\frac{r+2\delta}{r(r+\delta)}\right)} \\ &\equiv \frac{f_1}{f_2},\end{aligned} \tag{a9}$$

where

$$f_1 - f_2 = \frac{-\delta}{r(r+\delta)}\{d^{**}(\delta,\mu^o) - \varepsilon(1+\mu^o)c\} < 0. \tag{a10}$$

Note that $\{\cdot\} > 0$ and it is the monopolist's steady-state production level.

Thus at μ^o,

$$\frac{\Phi}{\frac{r+2\delta}{\delta}} = \frac{f_1}{f_2} < 1 \Rightarrow \frac{r+2\delta}{\delta} > \Phi. \tag{a11}$$

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[11]Rewrite Φ in eq. (a6) as $\frac{d^{**}(\delta,\mu^o)\int_0^\infty (1-e^{-\delta t})e^{-rt}dt + d^*(\delta)\int_0^\infty e^{-(r+\delta)t}dt - \varepsilon(1+\mu^o)c\int_0^\infty e^{-rt}dt}{d^{**}(\delta,\mu^o)\int_0^\infty (1-e^{-\delta t})^2 e^{-rt}dt + d^*(\delta)\int_0^\infty (1-e^{-\delta t})e^{-(r+\delta)t}dt - \varepsilon(1+\mu^o)c\int_0^\infty (1-e^{-\delta t})e^{-rt}dt}$.
Then use expansions listed in footnote 6.

Then, from the statement (a7), we may conclude that at μ^o,

$$\frac{\partial \Delta^1}{\partial \mu} < 0. \tag{a12}$$

Similarly, taking a derivative of the marginal cost, Δ^2, with respect to μ gives

$$\frac{\partial \Delta^2}{\partial \mu} = c\int_0^{\infty} \left\{ \left(1 - e^{-\delta t}\right) \frac{\partial d^{**}(\delta, \mu^o)}{\partial \mu} - \varepsilon c \right\} e^{-rt} dt. \tag{a13}$$

We can expand eq. (a14) as[12]

$$\frac{\partial \Delta^2}{\partial \mu} = \frac{c\delta}{r(r+\delta)} \left\{ \frac{\partial d^{**}(\delta, \mu^o)}{\partial \mu} - \frac{r+\delta}{\delta} \varepsilon c \right\}. \tag{a14}$$

By plugging the first-order necessary condition, eq. (a4), into eq. (a14), we get

$$\frac{\partial \Delta^2}{\partial \mu} = \frac{c\delta}{r(r+\delta)} \left\{ \Phi - \frac{r+\delta}{\delta} \right\} \varepsilon c. \tag{a15}$$

Then we may state that:

$$\text{if } \frac{r+\delta}{\delta} < \Phi \Rightarrow \frac{\partial \Delta^2}{\partial \mu} > 0. \tag{a16}$$

Dividing both sides of eq. (a8) by $\frac{r+\delta}{\delta}$ gives

$$\begin{aligned} \frac{\Phi}{\frac{r+\delta}{\delta}} &= \frac{d^{**}(\delta, \mu^o)\left(\frac{\delta}{r(r+\delta)}\right) + d^*(\delta)\left(\frac{1}{r+\delta}\right) - \varepsilon(1+\mu^o)c\left(\frac{1}{r}\right)}{d^{**}(\delta, \mu^o)\left(\frac{2\delta}{r(r+2\delta)}\right) + d^*(\delta)\left(\frac{1}{r+2\delta}\right) - \varepsilon(1+\mu^o)c\left(\frac{1}{r}\right)} \\ &\equiv \frac{f_3}{f_4}. \end{aligned} \tag{a17}$$

where

$$f_3 - f_4 = \frac{\delta}{(r+\delta)(r+2\delta)} \left\{ d^*(\delta) - d^{**}(\delta, \mu^o) \right\} > 0. \tag{a18}$$

Note that the monopolist's steady-state scale of demand, d^*, without price-cap regulation is always greater than d^{**} with price-cap regulation. Thus $\{\cdot\}$ is always positive.

Thus at μ^o,

$$\frac{\Phi}{\frac{r+\delta}{\delta}} = \frac{f_3}{f_4} > 1 \Rightarrow \frac{r+\delta}{\delta} < \Phi. \tag{a19}$$

Then, from the statement (a16), we may conclude that at μ^o,

$$\frac{\partial \Delta^2}{\partial \mu} > 0. \tag{a20}$$

From the two findings, (a12) and (a20), we may conclude that $\partial^2 \hat{w}/\partial \mu^2 < 0$ and thus that $\hat{w}$ has a local maximum at μ^o. ■

[12] Rewrite eq. (a13) as $c\left[\frac{\partial d^{**}(\delta,\mu^o)}{\partial \mu}\int_0^{\infty}\left(1-e^{-\delta t}\right)e^{-rt}dt - \varepsilon c\int_0^{\infty} e^{-rt}dt\right]$. Then use expansions listed in footnote 6.

Appendix 2: Proof of proposition 5

Proof. The first-order condition, eq. (47), should hold also at $\tilde{\delta}$, i.e., $\Delta^1\left(\tilde{\delta}, \mu^o\left(\tilde{\delta}\right)\right) = \Delta^2\left(\tilde{\delta}, \mu^o\left(\tilde{\delta}\right)\right)$. At $\tilde{\delta}$, by definition, $\hat{w}\left(\tilde{\delta}, \mu^o\left(\tilde{\delta}\right)\right) = w\left(\tilde{\delta}\right)$, $p^m\left(d^*\left(\tilde{\delta}\right)\right) = \left(1+\mu^o\left(\tilde{\delta}\right)\right)c$ and $d^*\left(\tilde{\delta}\right) = d^{**}\left(\tilde{\delta}, \mu^o\left(\tilde{\delta}\right)\right)$. Then from eqs. (32) and (33) and footnote 6, the first-order condition at $\tilde{\delta}$ reduces to[13]

$$\varphi''\left(x^*\left(\tilde{\delta}\right)\right) = \frac{1}{\varepsilon\left(r+\tilde{\delta}\right)^2}. \tag{a21}$$

A unique value of $\tilde{\delta}$ solves eq. (a21). ∎

Appendix 3: The case with imperfect capital market

In this appendix, I study the firm's innovative activity in an imperfect capital market in which its innovation is constrained by its internal funds.[14] Under this assumption, the firm's accumulation of internal funds from monopoly profit can slow down or even decline when the antitrust agency attempts to keep the monopolist's price close to its competitive level. This then reduces the firm's ability to finance its innovative activity. I follow Schworm (1980) and assume that the cost of innovation can be financed only through the firm's accumulated retained earnings, w_t, but not through borrowing. Given the firm's initial endowment of retained earnings, $w_0 > 0$, the law of motion of w_t is

$$\dot{w}_t = (p_t - c)\, q_t - x_t - \varphi(x_t) + r w_t, \tag{a22}$$

where no borrowing is allowed, i.e.,

$$-w_t \leq 0, \tag{a23}$$

but the firm can lend its funds at the rate of return, r.

[13] From eqs. (32) and (33) and footnote 6,

$$\Delta^1\left(\tilde{\delta}, \mu^o\left(\tilde{\delta}\right)\right) = \frac{c}{\varepsilon r\left(r+\tilde{\delta}\right)^2 \varphi''\left(x^*\left(\tilde{\delta}\right)\right)}\left\{d^*\left(\tilde{\delta}\right) - \varepsilon p^m\left(d^*\left(\tilde{\delta}\right)\right)\right\},$$

and

$$\Delta^2\left(\tilde{\delta}, \mu^o\left(\tilde{\delta}\right)\right) = \frac{c}{r}\left\{d^*\left(\tilde{\delta}\right) - \varepsilon p^m\left(d^*\left(\tilde{\delta}\right)\right)\right\}.$$

[14] A number of studies find a statistically significant relationship between R&D investment and internal finance, e.g., Himmelberg and Petersen (1994).

The non-negativity constraint can be rewritten as

$$z_t \equiv -w_t \Longrightarrow \dot{z}_t = -\dot{w}_t = -\left\{(p_t - c)\, q_t - x_t - \varphi(x_t) + r w_t\right\}, \tag{a24}$$

and

$$\dot{z}_t \leq 0 \text{ whenever } z_t = 0. \tag{a25}$$

The monopoly firm's new problem is to maximize its present value, eq. (8), subject to eqs. (2) and (a22)-(a25).

The current-value Lagrangian function, L_t, at time t is

$$\begin{aligned} L_t \equiv\ & \{\pi(d_t) - x_t - \varphi(x_t)\} e^{-rt} \\ & + \gamma_t (x_t - \delta d_t) \\ & + (\eta_t + \theta_t)\{\pi(d_t) - x_t - \varphi(x_t) + r w_t\}, \end{aligned} \tag{a26}$$

where η_t is the shadow value of w_t and θ_t is the Lagrangian multiplier that is active only when $w_t = 0$, i.e.,

$$-w_t \leq 0 \quad \theta_t w_t = 0. \tag{a27}$$

The optimal innovation, x_t, must satisfy the condition, $\partial L_t / \partial x_t = 0$, or

$$\{1 + \varphi'(x_t)\}\left(e^{-rt} + \eta_t + \theta_t\right) = \gamma_t, \tag{a28}$$

for all t. The condition simply states that the marginal benefit from a unit of investment, i.e., the right side of eq. (a28), must be equal to its marginal cost, i.e., the left side of eq. (a28).

The the law of motion of the costate variable, γ_t, is

$$\dot{\gamma}_t = \delta \gamma_t - \pi'(d_t)\left(e^{-rt} + \eta_t + \theta_t\right), \tag{a29}$$

and the law of motion of the costate variable, η_t, is

$$\dot{\eta}_t = -r(\eta_t + \theta_t). \tag{a30}$$

Differentiating eq. (a28) with respect to time and inserting it into eq. (a29) yields

$$\begin{aligned} \dot{\gamma}_t &= \{1 + \varphi'(x_t)\}\left(-r e^{-rt} + \dot{\eta}_t + \dot{\theta}_t\right) + \varphi''(x_t)\left(e^{-rt} + \eta_t + \theta_t\right)\dot{x}_t \\ &= -r\{1 + \varphi'(x_t)\}\left(e^{-rt} + \eta_t + \theta_t\right) + \{1 + \varphi'(x_t)\}\dot{\theta}_t + \varphi''(x_t)\left(e^{-rt} + \eta_t + \theta_t\right)\dot{x}_t. \end{aligned} \tag{a31}$$

By equating eqs. (a29) and (a31), we get

$$\begin{aligned} & \delta\{1 + \varphi'(x_t)\}\left(e^{-rt} + \eta_t + \theta_t\right) - \pi'(d_t)\left(e^{-rt} + \eta_t + \theta_t\right) \\ = \ & -r\{1 + \varphi'(x_t)\}\left(e^{-rt} + \eta_t + \theta_t\right) + \{1 + \varphi'(x_t)\}\dot{\theta}_t + \varphi''(x_t)\left(e^{-rt} + \eta_t + \theta_t\right)\dot{x}_t. \end{aligned} \tag{a32}$$

Rearranging eq. (a32) gives the law of motion of x_t,

$$\dot{x}_t = \frac{1}{\varphi''(x_t)} \left[(r+\delta) \{1 + \varphi'(x_t)\} - \pi'(d_t) \right] + \frac{1+\varphi'(x_t)}{\varphi''(x_t)} \frac{-\dot{\theta}_t}{e^{-rt} + \eta_t + \theta_t}. \quad \text{(a33)}$$

In the case when the monopolist's retained earnings are positive, i.e., $w_t > 0$, the financial constraint is not active, i.e., $\theta_t = \dot{\theta}_t = 0$. Then eq. (a33) reduces to eq. (12). The monopolist in this case behaves the same as the one in a perfect capital market. On the other hand, when the retained earnings dry up, i.e., $w_t = 0$, the financial constraint is active, i.e., $\partial L_t / \partial \theta_t = 0$. Then, the optimal innovation is

$$x_t = \pi(d_t) - \varphi(x_t). \quad \text{(a34)}$$

That is, the optimal innovation, x_t, is just as large as the firm's current gross profit. In this case, it is clear that any price-cap regulation has an immediate and negative effect on x_t.

Appendix 4: Data

Industry sector	Hall (1997)'s depreciation rats	SIC codes	NAICS codes	Industry	R&D % net sales
Chemicals	28.6	28	3252	-Resin, synthetic rubber, fibers, and filament	4.4
		28	other 325	-Other chemicals	3.3
		28	3251	-Basic chemicals	2.3
		30	326	Plastics and rubber products	2.1
		29	324	Petroleum and coal products	0.3
Computers & inst	20.4	367	3344	-Semiconductor and other electronic components	9.2
		38	3345	-Navigational, measuring, electromedical, and control instruments	7.4
		365	other 334	-Other computer and electronic products	6.9
		357	3341	-Computers and peripheral equipment	6.8
Drugs & med inst	16.3	283	3254	-Pharmaceuticals and medicines	9.2
Electrical	37.2	366	3342	-Communications equipment	14.7
		384	3391	-Medical equipment and supplies	9.2
		372 376	3364	-Aerospace products and parts	3.0
		36	335	Electrical equipment, appliances, and components	2.4
Metals & machinery	14.5	35	333	Machinery	4.0
		371	3361–63	-Motor vehicles, trailers, and parts	3.0
		37	other 336	-Other transportation equipment	2.3
		34	332	Fabricated metal products	1.5
		33	331	Primary metals	0.6
Miscellaneous	5.3	39	other 339	-Other miscellaneous manufacturing	2.8
		32	327	Nonmetallic mineral products	1.6
		26 27	322, 323	Paper, printing, and support activities	1.5
		22 23 31	313–16	Textiles, apparel, and leather	0.8
		25	337	Furniture and related products	0.8
		24	321	Wood products	0.8
		20 21	312	Beverage and tobacco products	0.5
		20	311	Food	0.5

Table A1: R&D capital depreciation rates and R&D % of net sales

Column 1: names of six manufacturing sectors studied in Hall (2007)
Column 2: market-value-based depreciation rates of R&D capital in Hall (2007)
Column 3: SIC codes
Column 4: NAICS codes
Column 5: names of industries
Column 6: R&D funds as a percent net sales

PS (1989)'s patent renewal rates		SIC codes	Roeger (1995)'s markups	# antitrust cases
Group 1	Food and kindred products	20 Food And Kindred Products	1.5	23
	Chemicals and allied products	28 Chemicals And Allied Products	2.11	7
	Machinery	35 Industrial And Commercial Machinery And Computer Equipment	1.41	7
	Drugs and medicines	28 Chemicals And Allied Products	2.11	5
	Rubber and plastic products	30 Rubber And Miscellaneous Plastics Products	1.36	1
	Lumber, wood, and paper	24 Lumber And Wood Products, Except Furniture	1.75	0
Group 2	Communication equipment	36 Electronic And Other Electrical Equipment And Components, Except Computer Equipment	1.34	6
	Professional, scientific, and electrical equipment	38 Measuring, Analyzing, And Controlling Instruments; Photographic, Medical And Optical Goods; Watches And Clocks	1.47	0
Group 3	Primary metals	33 Primary Metal Industries	1.58	2
	Fabricated metals	34 Fabricated Metal Products, Except Machinery And Transportation Equipment	1.33	0
	Stone, clay, and glass	32 Stone, Clay, Glass, And Concrete Products	1.59	0
Group 4	Farm, motor, and air	37 Transportation Equipment	1.64	8
Group 5	Other	21 Tobacco Products	excl.	3
	Other	23 Apparel And Other Finished Products Made From Fabrics And Similar Materials	1.15	2
	Other	31 Leather And Leather Products	1.19	0
	Other	25 Furniture And Fixtures	1.28	0
	Other	26 Paper And Allied Products	1.57	0
	Other	27 Printing, Publishing, And Allied Industries	1.4	0
	Other	39 Miscellaneous Manufacturing Industries	1.62	0
	Textiles, apparel, and leather	22 Textile Mill Products	1.34	0

Table A2: patent renewal rates, antitrust cases, and markups

Column 1: patent renewal rate Group 1 (highest) to Group 5 (lowest) in Pakes and Simpson (1989)

Column 2: names of industries

Column 3: SIC codes

Column 4: markup ratios in Roeger (1995)[15]

Column 5: number of antitrust cases

[15] Tobacco (SIC code: 21) and petroleum (SIC code: 29) are excluded as high taxes on these products make the markup estimates for these industires more complicated.