Competitive Cross-Subsidization*

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Abstract

This paper analyzes competitive cross-subsidization by multiproduct firms, stemming from the heterogeneity of consumer shopping patterns. In a setting where each firm has a comparative advantage on some of the products, although all firms are equally effective in serving one-stop shoppers, cross-subsidy arises in equilibrium, and allows firms to better screen consumers according to their shopping patterns. Firms earn positive profits from multistop shoppers, while fierce price competition dissipates any profit from one-stop shoppers. In this context, banning below-cost pricing leads to higher prices for one-stop shoppers and may or may not reduce consumer surplus as well as total social welfare, depending on the total value of the assortment.

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1 Introduction

Multiproduct firms often engage in cross-subsidization, i.e., pricing some products below costs while subsidizing the loss by the profits from other products. Cross-subsidization as a possible antitrust concern can arise when a firm provides multiple products or services and possesses significant market power in part of the products or services while selling others in competitive markets. For instance, natural monopolists in the industries such as telecommunication and electricity could cross-subsidize services in unregulated markets by their profits from the monopolized segments, and such cross-subsidy might hurt the competitors in the unregulated segments.[1] The analysis of cross-subsidy has been central to the theory and practice of regulation for decades, mostly focusing on the identification and prevention of cross-subsidization and price predation in regulated industries.[2]

Cross-subsidization has been also commonly observed in competitive markets. For instance, large grocery retailers often charge below-cost prices for some staples like milk and bakery and subsidize the loss from other products.[3] The competing retail banks often provide some services below cost, such as zero account fees or free travel insurance, to attract customers for other services. The prevalence of cross-subsidy in competitive markets seems to contradict the existing theory of cross-subsidization. According to the received theory, cross-subsidy arises only when a firm has substantial market power in some markets and is thus able to finance the loss in the competitive market from its more protected revenue source (the so-called "deep pocket" theory). In competitive markets, however, no firm can earn super-marginal profit and thus there is no such a "deep pocket" to subsidize the loss in other markets. Therefore "under competitive conditions, the issue of cross-subsidy simply does not arise", as argued by Faulhaber (2005, pp.442). On the other hand, in the literature of marketing, cross-subsidization has been viewed

[1] Indeed, the concern of the cross subsidization possibility led to the divestiture by *AT&T* of its local telephone companies, and the subsequent restriction of entry by those divested companies into unregulated or complementary markets (*U.S. v. AT&T 1982*)

[2] In the seminal paper by Faulhaber (1975), the concept of cross subsidy was defined rigorously based on microeconomic foundations and the two tests for subsidy-free pricing were presented, which have been widely applied in the theory and practice in regulation and antitrust enforcement. See also Faulhaber (2005) for a recent survey.

[3] Such cross subsidy strategy is well-known as loss leading in grocery retailing. For instance, in its recent report of the investigation on grocery markets, the UK Competition Commission (2008) notes that most large retailers in the UK engage in loss leading, and it finds that the sales of loss leader products represent up to 6% of a retailer's total sales.

as an advertising strategy adopted to attract consumers facing imperfect information of prices;[4] however in many markets, such as the grocery retailing and bank industries, consumers seem to be well informed about the prices due to regular purchases or price search and are unlikely to be misled by below-cost pricing. Hence, it is puzzling why cross-subsidization prevails in competitive markets where consumers are (almost) perfectly informed on prices.[5]

Cross-subsidization involves below-cost pricing and is thus subject to antitrust scrutiny. However, antitrust enforcement against cross subsidization in competitive markets causes hot debates. For instance, in 2000 the German Federal Cartel Office (FCO) ordered the large supermarkets, including Wal-Mart and Aldi, to stop selling below cost staples including milk, sugar, and butter, arguing that this could impair competition and force smaller retailers to exit the market. Wal-Mart thereafter appealed the decision to the Düsseldorf Court of Appeals, arguing that its below-cost selling is simply driven by the competitive pressure from other large retailers. The Court of Appeals ruled for Wal-Mart and reversed the decision of FCO. However, the German Supreme Court finally upheld the FCO's decision in 2002.[6]

According to the German Supreme Court's decision, companies active in Germany should be aware that pricing strategies that may be permissible in the United States (or under EC competition law) might violate Germany's below-cost sales statute, even if they are not in dominant positions. This reflects the discrepancy in competition laws on below-cost sales. In the EU, below-cost selling is banned in Belgium, France, Ireland, Luxembourg, Portugal, and Spain, and is restricted in other countries including Austria, Denmark, Germany, Greece, Italy, Sweden and Switzerland, whereas it is generally allowed in the Netherlands and the UK. In the United States, according to the federal antitrust laws, below cost pricing is only illegal when it is predatory, whereas many states have laws making certain forms of below cost pricing illegal

[4] Lal and Matutes (1994), for example, consider a situation where multi-product firms compete for consumers who are initially unaware of prices, and find that in equilibrium firms may indeed choose to advertise a few loss leaders in order to increase store traffic.

[5] Ambrus and Weinstein (2008) study Bertrand competition among symmetric firms competing for one-stop shoppers. They first show that below-cost pricing cannot arise when consumers have inelastic demand. When demand is elastic, pricing below cost can occur but only under rather specific forms of demand complementarity; in particular, below-cost pricing cannot arise when consumer demand is sufficiently diverse. The scope for below-cost pricing in these settings, as well as its impact on consumers and welfare, still needs to be assessed.

[6] See http://www.wilmerhale.com/files/Publication/d3382527-7acd-45c6-bae6-4ef14157e415/Presentation/PublicationAttachme
21b4-46b0-b356-a96612caf3cf/International%20Competition%20Law%20Update12-2-02.pdf

even absent evidence of predatory intent and the ability to recoup losses at a later time.[7]

Cross-subsidization has been often treated as predatory pricing in antitrust cases, in the absence of specific regulations.[8] However it is inappropriate to treat competitive cross-subsidization as price predation, simply because it is implausible that the "predator" could recoup the losses incurred during the predation phase in competitive markets, either by raising the prices after driving the rival out of the market or by cross-subsidizing from a "deep pocket".[9] This concern is also raised by the UK Competition Commission, which argues in its 2008 report that "We find that the pattern of below-cost selling that we observed by large grocery retailers does not represent behavior that was predatory in relation to other grocery retailers."[10]

This begs the several related questions: what is the rationale for cross-subsidization in competitive markets if it is not predatory? What's the impact on consumer surplus and social welfare if below-cost selling in the competitive markets is banned by competition laws?

This paper sheds a new light on the rationale for competitive cross-subsidization. We develop a model of price competition between two multiproduct firms where both firms offer the same product lines. We abstract away from the above-mentioned efficiency justifications by assuming that consumers are perfectly informed of all prices. Our key modelling feature is to account for the heterogeneity in consumers' shopping costs incurred for sourcing a supplier:[11] some consumers face higher shopping costs, e.g., because of tighter time constraints for visiting a store or adopting a new technology, and thus have a strong preference for one-stop shopping,

[7] See Calvani (2001) for detailed discussion.

[8] See e.g., Bolton, Brodley and Riordan (2000) and Eckert and West (2003) for detailed discussions of how predatory-pricing tests should be designed. In particular, Rao and Klein (1992) and Berg and Weisman (1992) examine the treatment of cross subsidization under U.S. antitrust laws.

[9] The feasibility of recoupment is often a necessary condition for a case of predation; in the U.S., for example, this approach was adopted by the Supreme Court in the *Brooke Group Ltd. v. Brown & Williamson Tobacco Corp*, which involved allegations of predatory pricing by Brown & Williamson against a smaller rival in an effort to discipline the pricing of generic cigarettes. The Court noted that predatory pricing was generally implausible without recoupment conditions, and further stated that intent ought to play no role in assessing whether conduct is predatory.

[10] See Competition Commission (2008) at p. 98. In its 1997 report, the UK Office of Fair Trading argued that, in the analysis of alleged predation in retailing cases, a price-cost comparison is of little use, since pricing below cost on individual items may be profitable without being predatory.

[11] Following Klemperer (1992), we will refer to consumers' real or perceived costs of using a supplier as shopping costs. This modelling feature is widely adopted in the literature of multiproduct competition, including Klemperer (1997) and Armstrong and Vickers (2010).

whereas others have lower shopping costs and can therefore benefit from multistop shopping.

We present the main insights in a stylized setting where two firms supply the same (two) product lines with perfect substitution. Each firm possesses a comparative advantage against its rival in one market, which may stem from the lower unit cost and/or higher consumer value. That is, each firm supplies a strong product in one market, competing with its rival's weak product. For the simplicity of analysis, we assume that each firm's comparative advantage in one product is exactly offset by its comparative disadvantage in another market, so that two firms are symmetric in the sense that they offer the same total values for the assortment of two products.[12] Thus, tough price competition for the assortments between two firms dissipates each firm's profit from selling the bundle: both firms charge the total prices for the bundle at the total costs (thus earn a zero total margin for the bundle).

However, in each individual product market firms are asymmetric in competition, with one firm supplying a strong product at a higher value than its rival's weak product. Purchasing two strong products from two different firms brings a higher value than buying both products from one firm only, but at an extra shopping cost for sourcing an additional supplier. When the additional benefit from this multistop shopping exceeds the extra shopping cost, consumers are indeed willing to do so. This asymmetry in individual markets thus opens a door for cross-subsidization. Each firm could price its weak product below cost and charge a positive margin for its strong product, while still keeps the total margin for the bundle at zero. This cross-subsidizing strategy does not affect one-stop shoppers who only care about the total price for the bundle, however, it indeed allows firms to make a positive profit from multistop shoppers who buy only the strong product from each firm. On the other hand, a firm who prices its weak product below cost imposes a competitive pressure on the rival's strong product, which reduces the rival's margin on the strong product. Solving this trade-off determines the optimal margins for the strong and weak products.

Thus, cross-subsidization acts here as a discriminatory mechanism for screening consumers with different shopping costs, and allows firms to make positive profit from multistop shoppers albeit tough price competition dissipates the profits from one-stop shoppers. We show in the baseline setting that cross-subsidization arises in a unique (symmetric) Nash equilibrium.[13] To examine the robustness of the analysis, we extend the basic setting to incorporate horizon-

[12] No insight will change substantially when this symmetry assumption is dropped.

[13] Since cross-subsidization allows firms to make additional profits by discriminating consumers, they have no incentive to adopt pure and mixed bundling instead.

tal differentiation between firms where consumers have heterogeneous firm-specific preferences. We show that, when horizontal differentiation is relatively weak (so that competition is rather tough), then cross-subsidizing emerges and firms price the weak products below cost in the symmetric equilibrium. Moreover the equilibrium prices converge to these under the baseline setting when the degree of differentiation tends to zero.

To evaluate the welfare effect of cross-subsidization, we compare the social welfare under two regimes regarding on whether below-cost pricing is banned or not.[14] We find that, banning such practice would force each firm to price the week product at cost, which releases the competitive pressure on the strong products. As a result, one-stop shoppers end up with higher prices. Banning below-cost pricing would engender more foot traffics for multistop shopping and thus improve the allocation efficiency, and multi-stop shoppers would face lower expected price when the total value of the assortment is sufficiently large. Finally, banning below-cost pricing would increase the social welfare when the total value of the assortment is sufficiently large, and would reduce the social welfare when the value of the assortment is relatively small.

This paper is a complementary piece to Chen and Rey (2012), which sheds a first light on the exploitative use of loss leading in retail competition, in a model of asymmetric competition between large retailers and small retailers. By contrast, this paper analyzes symmetric competition between two multiproduct firms (offering same product lines) and finds that adopting cross-subsidizing strategy allows firms to discriminate consumers and make profits from multistop shoppers, with no predatory motivation. This finding highlights the new rationale for competitive cross-subsidization, in the absence of any efficiency justification for imperfect information, and thus provides a theoretical foundation for the assessment of cross-subsidization in competitive markets.

2 The model

Two firms, firm 1 and 2, carry the same product lines and compete in a local market. For the simplicity of exposition, we consider only two products, A and B. Supplying product A_i and B_i incur a unit cost c_i^A and c_i^B respectively for firm i, $i = 1, 2$. Each consumer desires at most one unit of A and B, and consuming products A_i and B_i delivers a utility u_i^A and u_i^B respectively

[14] There is no pure-strategy Nash equilibrium when below-cost pricing is banned, in which case we resort to the mixed strategy equilibrium.

while consuming both A_i and B_i yields $u_i^{AB} = u_i^A + u_i^B$.[15]

Consumers incur a shopping cost s for visiting one store, which reflects the opportunity cost of the time spent in traffic, parking, selecting products, checking out, and so forth; it may also account for the consumer's taste for shopping. Our key modelling feature, reflecting the fact that consumers may be more or less time-constrained, or value their shopping experience in different ways, is that the shopping cost s varies across consumers.

A simple example

A numerical example can illustrate the intuition. Assume that consumers value product A at $u^A = \$50$ and product B at $u^B = \$50$. Firm 1 has a cost advantage in market A: $c_1^A = \$10 < c_2^A = \$30$, whereas firm 2 incurs a lower unit cost in market B: $c_2^B = \$10 < c_1^B = \$30$. Assume that half consumers face a high shopping cost $s_H = \$20$ while others incur a low shopping cost $s_L = \$2$. Since both firms provide the same total value of \$100 and incur the same total costs of \$40 for their assortment AB, fierce price competition dissipates their profits from one-stop shoppers and thus their prices for the assortment must be equal to the total costs, i.e.,

$$a_1 + b_1 = a_2 + b_2 = \$40,$$

where a_i (resp. b_i) represents the price for product A_i (resp. B_i).

However, since each firm has a cost advantage in one market, it can offer a lower price than its rival in that market while still keep the total price at total cost to attract one-stop shoppers. Consider for instance the following pricing strategy:

$$\begin{aligned} a_1 &= \$19,\ b_1 = \$21 \\ a_2 &= \$21,\ b_2 = \$19, \end{aligned}$$

that is, each firm sets the price of its weak products below costs, $a_2 = b_1 = \$21 < \$30$, and subsidizes the loss by the profit from the strong product. This pricing strategy does not affect the shopping behavior of high-cost consumers (who still face a total price of \$40), it however engenders additional benefits for multistop shoppers who buy A from 1 and B from 2 since $a_1 + b_2 = \$38 < \$40$ and the price gap offsets the extra shopping cost. Firms then earn a positive profit from multi-stop shoppers who purchase only strong products, which is equal to $(\$19 - \$10)/2 = \$4.5$.[16]

[15] We focus on the case with two independent goods A and B. Indeed, the analysis goes through for partial substitution and complementarity of products A and B, that is, u_i^{AB} could be either lower or higher than $u_i^A + u_i^B$.

[16] It is easy to verify that the above price strategy forms a Nash equilibrium.

Notice that cross-subsidy does not arise if consumers are homogeneous in their shopping costs. Suppose all consumers face high shopping cost, $s_H = \$20$. Then all consumers are one-stop shoppers and the fierce price competition yields zero profit for firms. There is no gain for cross-subsidization since only the the total price for the bundle matters. Suppose in stead that all consumers have low shopping cost, i.e., $s_L = \$2$. Then each firm prices its weak product at cost, $a_2 = b_1 = \$30$, and sets the price of its strong product equal to the cost of rival's weak product minus shopping cost, i.e., $a_1 = b_2 = \$28$. All consumers are multistop shoppers and firms make a profit of \$18.

We now consider more general supply and demand conditions. For the ease of exposition, we denote by $w_i^A \equiv u_i^A - c_i^A$, $w_i^B \equiv u_i^B - c_i^B$, and $w_i^{AB} = u_i^{AB} - c_i^A - c_i^B$ the social value generated by the supply of product A_i, B_i, and the assortment A_iB_i respectively. We focus on the cases where it is socially desirable for each firm to offer products A_i and B_i, as well as the assortment A_iB_i, that is, $w_i^A > 0$ and $w_i^B > 0$, and $w_i^{AB} > 0$, $i = 1, 2$.

Each firm possesses a comparative advantage in one product, say, firm 1 is more efficient in supplying A while firm 2 is more efficient in supplying B, i.e., $w_1^A > w_2^A$ and $w_1^B < w_2^B$. This efficiency gain may be derived from the specialization in different product markets. We will focus on the "symmetric" case where both firms offer the same total value of the assortment A_iB_i: $w_1^{AB} = w_2^{AB} = w$, which implies that each firm possesses the same degree of comparative advantage in one product: $w_1^A - w_2^A = w_2^B - w_2^A \equiv \delta$. This assumption on symmetric total value simplifies the exposition, but the analysis goes through when firms offer asymmetric values.[17] Finally, we allow for general distributions of the shopping cost s, characterized by a cumulative distribution function $F(\cdot)$ with a continuous density function $f(\cdot)$. Intuitively, consumers with a high shopping cost favor one-stop shopping, whereas those with a lower shopping cost can take advantage of multistop shopping; the mix of multistop and one-stop shoppers is however endogenous and depends on firms' prices.

We model price competition as follows: (i) Firm i, $i = 1, 2$, simultaneously set their prices, respectively $(p_i^A,\ p_i^B)$,[18] where p_i^A stands for firm i's price of product A while p_i^B denotes its price for product B. (ii) Consumers then observe all prices and make their shopping decisions.

[17] The main difference is that the firm offering higher total value wins one-stop shoppers who will patronize only one firm for both products.

[18] We first consider stand-alone prices, and show later that allowing for bundled discounts cannot increase firms' profits; see the remark in section 3.

When making these decisions, consumers are fully aware of all prices and take into account the value of the proposed assortments as well as their shopping costs. For the ease of exposition, we will use the retail margins rather than prices in the analysis, which are defined as follows: $r_i^A \equiv p_i^A - c_i^A$ and $r_i^B \equiv p_i^B - c_i^B$.

3 Equilibrium cross-subsidization

A consumer is willing to buy both products A and B from firm i if the value of the assortment A_iB_i exceeds the shopping cost, that is, if

$$\begin{aligned} v_i &\equiv u_i^{AB} - p_i^A - p_i^B = w_i^A - r_i^A + w_i^B - r_i^B \\ &= w - r_i^A - r_i^B \geq s. \end{aligned}$$

Since the product lines of both firms are perfectly substitute, a consumer will patronize firm i only if it offers a better value than its rival, firm j, that is

$$v_i = w - r_i > v_j = w - r_j,$$

where $r_i \equiv r_i^A + r_i^B$ stands for the total margin that firm i charges on the assortment, and this amounts to

$$r_i < r_j.$$

Moreover, consumers may prefer buying both strong products, that is, purchasing A_1 from firm 1 and B_2 from firm 2,[19] rather than patronizing one-store only. Such multistop shopping engenders a gross value

$$v_{12} \equiv w_1^A - r_1^A + w_2^B - r_2^B = w + \delta - r_1^A - r_2^B,$$

at an extra shopping cost s. Thus, consumers favor multistop shopping than patronizing only firm 1 if

$$s \leq \tau_1 \equiv v_{12} - v_1 = \delta + r_1^B - r_2^B,$$

where τ_1 denotes the threshold of shopping cost for which consumers are indifferent between multistop shopping and patronizing firm 1. This threshold indeed reflects the difference of values

[19] We show in the appendix that mix-and-match for both weak products A_2 of firm 2 and B_1 of firm 1 never arises in equilibrium.

in product B offered by two firms. Similarly, consumers prefer multistop shopping to patronizing only firm 2 if

$$s \leq \tau_2 \equiv v_{12} - v_2 = \delta - r_1^A + r_2^A,$$

which reflects the value difference of product A. It follows that the population of multistop shoppers (and thus the demand of strong products A_1 and B_2) is characterized by $F(\tau)$, where $\tau = \min\{\tau_1, \tau_2\}$.

Several configurations may arise depending on the existence of one-stop and multistop shoppers. However, as shown in Appendix A, we can focus on the regime where there always exist both multistop shoppers and one-stop shoppers. Since there are some active one-stop shoppers, competing for these one-stop shoppers dissipates firms' profits from selling the assortment, which implies

$$r_i = r_j = 0.$$

Moreover, it can be shown that multistop shoppers never pick the weak products.

Lemma 1 *In equilibrium:*

- *(i): There exist active multistop shoppers and active one-stop shoppers;*
- *(ii): multistop shoppers buy strong products;*
- *(iii):* $r_i = r_j = 0$.

Proof. See Appendix A. ■

We now consider the regime where both types of shoppers are active. Consumers prefer multistop shopping if their shopping cost is lower than the threshold τ and they prefer one-stop shopping if $\tau \leq s \leq v \equiv \max\{v_1, v_2\}$. Denote by D_1^O the demand from one-stop shoppers, then firm 1's profit can be expressed as

$$\pi_1 = r_1 D_1^O + r_1^A F(\tau) = r_1 \left(D_1^O + F(\tau)\right) - r_1^B F(\tau).$$

By lemma 1, firm 1 must set its total margin to zero in order to attract active one-stop shoppers, i.e., $r_1^A + r_1^B = r_1 = 0$, thus

$$\pi_1 = -r_1^B F(\tau).$$

The optimal pricing strategy thus consists in minimizing $r_1^B F(\tau)$ with respect to r_1^B, which obviously leads to $r_1^B < 0$, that is, firm 1 sells its weak product *below cost*. Similarly firm 2's profit is reduced to $\pi_2 = -r_2^A F(\tau)$, thus the optimal pricing policy yields $r_2^A < 0$. We thus obtain our first insight:

Lemma 2 *In equilibrium, both firms must sell their weak products below cost.*

The intuition is quite simple. While fierce price competition for one-stop shoppers yields zero total margin, firms can still benefit from cross-subsidizing between the strong and weak products. Suppose both firms set $r_i^A = r_i^B = 0$, $i = 1, 2$, then firm 1 offers a better value in its strong product A than firm 2, $w_1^A > w_2^A$, and firm 2 offers a better value in product B: $w_2^B > w_1^B$. Now buying both strong products, A_1 from firm 1 and B_2 from firm 2, brings additional value δ than one-stop shopping in either firm 1 or 2, but at an additional shopping cost s. Therefore consumers with low shopping cost such that $s \leq \delta$ are indeed willing to do so. Accounting for this, firm 1 can now benefit from the following cross-subsidizing strategy: raising its margin on A_1 slightly by ε while reducing its margin on B_1 by the same amount so as to keep the overall margin $r_1 = 0$. Doing so allows the firm to charge a positive margin from multistop shoppers while still keep attracting one-stop shoppers. Firm 1 thus finds it optimal to sell r_1^B below cost.

To characterize further the optimal pricing strategy, in what follows we assume that the inverse hazard rate, $h(\cdot) \equiv F(\cdot)/f(\cdot)$, is strictly increasing, which ensures the quasi-concavity of firm's profit functions. The firms make profits only from multistop shoppers, and their profits are given respectively by $\pi_1 = -r_1^B F(\tau) = r_1^A F(\tau)$ and $\pi_2 = -r_2^A F(\tau) = r_2^B F(\tau)$. Notice that $\tau_1 = \tau_2 = \tau = \delta - r_1^A - r_2^B$ since $r_1 = r_2 = 0$. Firm 1 then chooses r_1^A to maximize $\pi_1 = r_1^A F(\delta - r_1^A - r_2^B)$, and the best response is characterized by the first-order condition, which leads to

$$r_1^A = h(\delta - r_1^A - r_2^B).$$

Similarly, maximizing $\pi_2 = r_2^B F(\delta - r_1^A - r_2^B)$ leads to

$$r_2^B = h(\delta - r_1^A - r_2^B).$$

Thus, the equilibrium margins of the strong products are given by $r_1^{A*} = r_2^{B*} = h(\tau^*)$. Using $\tau^* = \delta - r_1^{A*} - r_2^{B*} = \delta - 2h(\tau^*)$, the equilibrium threshold τ^* is determined by

$$\tau^* = j^{-1}(\delta),$$

where $j(s) = s + 2h(s)$ is strictly increasing in s.

In equilibrium, both firms sell their weak products below cost: $r_2^{A*} = r_1^{B*} = -h(\tau^*) < 0$. This allows them to charge a positive margin on their strong products: $r_1^{A*} = r_2^{B*} = h(\tau^*) > 0$ and still keep attracting one-stop shoppers: $r_1 = r_2 = 0$. This optimal cross-subsidizing strategy thus engenders a positive profit from multistop shoppers, which is given by

$$\pi^* = h(\tau^*)F(\tau^*).$$

Cross-subsidization appears here as discriminatory mechanism, which allows firms to discriminate multistop shoppers from one-stop shoppers and earn a profit from them. Since $h(\tau^*) < \delta$ and $F(\tau^*) \leq 1$, it follows that the equilibrium profit is strictly lower than the efficiency gain: $\pi^* < \delta$. This is because competition for one-stop shoppers dissipates firms' profit from the consumers with $s > \tau^*$.

Summarizing the above analysis leads to:

Proposition 1 *There exists a unique Nash equilibrium where firms compete for one-stop shoppers and also supply to multistop shoppers. In this equilibrium, both firms sell their weak products below cost and cross-subsidize by their strong products, and in this way both firms earn positive profits from multistop shoppers.*

Proof. See Appendix B. ■

Remark: Bundling. Firms may be engaged in pure bundling, that is, tying both products together physically such that consumers cannot unbundle it. In our model, adopting pure bundling cannot improve firm's profit. Notice that, if one firm commits to the pure bundling strategy, the other firm has no choices but to tie also its products, and both firms will adopt pure bundling in equilibrium. In this case all active consumers become one-stop shoppers, and the price competition for one-stop shoppers leads to zero profit. Firms may also adopt mixed bundling, in which case each firm provides a bundle of two products as well as two separate products, and accordingly offers three prices: one for strong product, one for weak product, and one for the bundle. Since one-stop shoppers only purchase the assortment while multistop shoppers only buy the strong product, no consumers will pick the stand-alone weak product. So only two prices matter here: the total price $p_i \equiv p_i^A + p_i^B$ for the bundle and the price for the strong product. Alternatively, these prices can be implemented through stand-alone prices, p_i^A for product A and p_i^B for product B. Therefore, offering an additional bundled discount based on two stand-alone prices could not improve each firm's profit.

Remark: Complementarity. We have focused on two independent products in the basic setting. Indeed, the analysis goes through in exactly the same way when two products are complementary. Suppose now A_i brings a stand alone utility u_i^A for consumers while using A_iB_i gives a utility u_i^{AB}. Defining the additional value that product B_i contributes as $u_i^B \equiv u_i^{AB} - u_i^A$, we can then use the same approach as in the analysis of the independent goods case.

4 Banning below-cost pricing: welfare analysis

Cross-subsidization is allegeable to antitrust scrutiny since it involves below-cost pricing in some product lines. The above analysis shows that cross-subsidization arises in equilibrium as a screening mechanism to discriminate multistop shoppers from one-stop shoppers, without any efficiency justification stemming from incomplete information. To derive further the policy implication for such cross-subsidization, we now examine the impact on the social welfare when below-cost pricing is banned.

Suppose now below-cost pricing is prohibited and firms are forced to charge $r_1^B \geq 0$ and $r_2^A \geq 0$. Notice that the fierce price competition for one-stop shoppers leads to $r_1^A + r_1^B = r_2^A + r_2^B = 0$, and thus firms can only make a profit from serving multistop shoppers, which are given by $\pi_1 = r_1^A F(\delta - r_1^A + r_2^A) = -r_1^B F(\delta + r_1^B - r_2^B)$ and $\pi_2 = r_2^B F(\delta - r_2^B + r_1^B) = -r_2^A F(\delta - r_1^A + r_2^A)$ respectively. But these profit functions are non-positive given that $r_1^B \geq 0$ and $r_2^A \geq 0$, thus firms must charge zero margins on their weak products: $r_1^B = r_2^A = 0$, which implies that the margins on the strong products must be zero too: $r_1^A = r_2^B = 0$. However, these margins cannot support a Nash equilibrium, since increasing the margin on the strong product slightly would make a positive profit from multistop shoppers.

Indeed, no pure-strategy equilibrium exists when below-cost pricing is banned. Suppose there exists such an equilibrium, then price competition for one-stop shoppers must lead to $r_1^A + r_1^B = r_2^A + r_2^B = 0$. Since firms are forced to charge $r_1^B \geq 0$ and $r_2^A \geq 0$, any equilibrium must involve $r_1^A \leq 0$ and $r_2^B \leq 0$. But then each firm will be better off by dropping its weak product and increasing the margin on the strong product, in which way it could make positive profit from multistop shoppers.

However, there does exist a (symmetric) mixed strategy equilibrium where both firms earn the "minmax" profit. Intuitively, the "minmax" profit is obtained when a firm maximizes its profit by accounting for the rival's most aggressive strategy. When below-cost pricing is prohibited, a firm's most aggressive strategy is to charge zero margins on both products. Denoting by ρ_j firm j's margin on its strong product for the ease of exposition, firm j's most aggressive strategy thus involves $r_j = \rho_j = 0$. Consider now firm i's best response (r_i, ρ_i) to firm j's most aggressive strategy.

- Firm i makes no profit from one-stop shoppers given that the rival charges $r_j = 0$.

- Multistop shoppers will not buy weak products: $\hat{\tau} \leq \hat{\tau}_j = -\delta + \rho_j - (r_i - \rho_i) = -\delta - (r_i - \rho_i) < 0$, since the margin of the weak product, which is equal to $r_i - \rho_i$, must be non-negative. It is then optimal to charge zero margin on its weak product, that is, $r_i = \rho_i$.

- Firm i makes a profit from multistop shoppers as given by $\pi_i = \rho_i F(\tau) = \rho_i F(\delta - \rho_i)$.

Therefore, firm i chooses the total margin r_i (which is equal to the margin on its strong product $r_i = \rho_i$) to maximize its profit earned from multistop shoppers, and its best response $\hat{r}$ must solve $\max_r rF(\delta - r)$, which is determined by

$$\hat{r} = h(\delta - \hat{r}). \tag{1}$$

The related threshold triggering multistop shopping is given by

$$\hat{\tau} = l^{-1}(\delta),$$

where $l(x) \equiv x + h(x)$ is strictly increasing in x, and firm's minmax profit is equal to

$$\hat{\pi} = \hat{r}F(\delta - \hat{r}) = h(\delta - \hat{r}) F(\delta - \hat{r}).$$

Firm i's mixed strategy with $\rho_i = r_i$ can be characterized by a distribution function $K(r)$ over some interval $[\underline{r}, \overline{r}]$ ($\overline{r} \leq \delta < w$). Suppose that firm i sets $\rho_i = r_i = r$. Consider consumers' response as a function of the realization of the rival's margin $\rho_j = r_j = \tilde{r}$:

- Consumers buy both goods from firm i if firm i's margins satisfy $s \leq v_i = w - r$ and

  $$r_j = \tilde{r} \geq r_i = r,$$

  and moreover they prefer this to multistop shopping if

  $$s \geq v_{12} - v_i = \delta - \tilde{r}.$$

- Consumers are engaged in multistop shopping if

  $$s \leq \{v_{12} - v_i, v_{12} - v_j\},$$

  which boils down to

  $$s \leq \delta - r \text{ and } s \leq \delta - \tilde{r}.$$

Figure 1 depicts consumers' response.

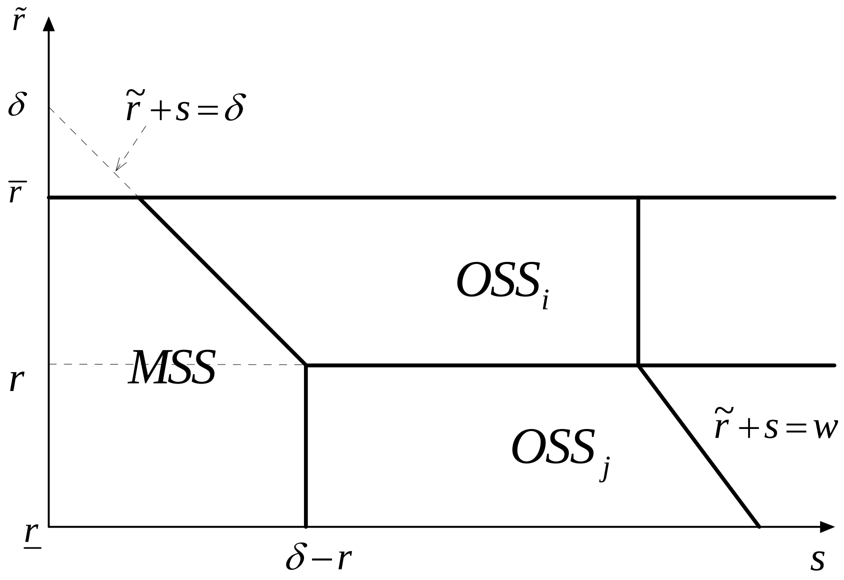

Figure 1

Thus, firm i's expected profit is equal to

$$\pi_i(r) = rE\left[D_i^{OSS} + D^{MSS}\right],$$

where D_i^{OSS} represents the demand of one-stop shoppers going to firm i and D^{MSS} is the demand of multistop shoppers. Notice that firm j's margin $\rho_j = r_j = \tilde{r}$ is distributed according to a distribution function $K(r)$, thus its expected profit can be written as

$$\begin{aligned}\pi_i(r) &= r\left[(1-K(r))F(w-r) + K(r)F(\delta-r)\right]\\ &= r\left[F(w-r) - K(r)(F(w-r) - F(\delta-r))\right].\end{aligned}$$

For firm i to obtain its minmax profit $\hat{\pi}$ through the mixed strategy, it must ensure that, for all r,

$$\pi_i(r) = r\left[F(w-r) - K(r)(F(w-r) - F(\delta-r))\right] = \hat{\pi}.$$

This implies

$$K(r) = \frac{F(w-r) - \frac{\hat{\pi}}{r}}{F(w-r) - F(\delta-r)} = \frac{F(w-r) - \frac{\hat{r}F(\delta-\hat{r})}{r}}{F(w-r) - F(\delta-r)}. \tag{2}$$

In particular, the lower bound $\underline{r}$ and upper bound $\overline{r}$ of the margins must satisfy

$$\pi_i(\underline{r}) = \underline{r}F(w-\underline{r}) = \pi_i(\overline{r}) = \overline{r}F(\delta-\overline{r}) = \hat{\pi}, \tag{3}$$

it thus follows that $\overline{r} = \hat{r}$.

However $K(\cdot)$ may not be increasing. Nevertheless, we can derive a new function $\hat{K}(\cdot)$ by modifying the function $K(\cdot)$ such that it is weakly increasing in the range over $[\underline{r}, \hat{r}]$. We show in Appendix C that the mixed strategy with $\rho_i = r_i = r$, which is chosen according to the c.d.f. $\hat{K}(\cdot)$, forms a Nash equilibrium, and this mix-strategy equilibrium yields the positive expected profit $\hat{\pi} > 0$. Since $\hat{\pi} = \max_r rF(\delta - r) = \max_\rho \rho F(\delta - \rho) > \pi^* = \max_\rho F(\delta - \rho^* - \rho)$, firms' expected profit is higher than that when below-cost pricing is allowed.

Banning below-cost pricing leads to positive total margin $r > 0$, since the lower bound of the distribution $\underline{r} > 0$. This implies that one-stop shoppers now face higher price. Moreover, it is straightforward to see that the distribution $K(r)$ increases in the social value of the assortment w and that the lower bound $\underline{r}$ decreases in w whereas the upper bound $\hat{r}$ is irrelevant of w. It thus follows that the expected margin $E[r]$ decreases in w. Since $\underline{r} = \hat{r}$ when $w = \delta$ and $\underline{r}$ converges to zero when w goes to infinity, there must exist some value $\bar{w} > \delta$ such that $E[r] > h(\tau^*)$ if and only if $w < \bar{w}$, which implies that, when below-cost pricing is banned, multistop shoppers end up with higher expected price when the social value of the assortment is sufficiently close to the efficiency gain δ.

Proposition 2 *Suppose below-cost pricing were banned. Then:*

- *There exists a symmetric mixed-strategy equilibrium in which firms price the weak products at cost and choose the margin of the strong product in the range of* $[\underline{r}, \hat{r}]$ *according to a c.d.f.* $\hat{K}(\cdot)$ *modified from* $K(\cdot)$*;*
- *This strategy engenders the higher expected profit than that when below-cost pricing is allowed;*
- *The ex post total margin is positive and thus one-stop shoppers face higher price;*
- *There exists a value* $\bar{w} > \delta$ *such that* $E[r] > h(\tau^*)$ *if and only if* $w < \bar{w}$*, and thus multistop shoppers face higher price if and only if* w *is sufficiently close to* δ*.*

Proof. See Appendix C. ■

Notice that, given a pair of realized total margins r_1, $r_2 \in [\underline{r}, \hat{r}]$, the *ex post* threshold below which consumers prefer multistop shopping is then $\tau = \delta - \max\{r_1, r_2\}$, and thus

$$\tau = \delta - \max\{r_1, r_2\} > \delta - \hat{r} = \hat{\tau} > \tau^*,$$

since $\hat{\tau} = \delta - \hat{r} = l^{-1}(\delta) > \tau^* = j^{-1}(\delta)$. Thus, the threshold that triggers multistop shopping increases when below-cost pricing is banned, which implies that more consumers prefer multistop shopping.

We now compare the social welfare in two regimes regarding on whether below-cost pricing is allowed or not. Notice that, the social welfare when firms adopt cross-subsidization can be expressed by

$$W = \int_0^w (w - s) f(s)ds + \int_0^{\tau^*} (\delta - s) f(s)ds,$$

where the first term is the welfare from supplying products A and B and the second term is additional social welfare from multistop shopping. On the other hand, we can write the *ex post* social welfare in the case of banning below-cost pricing as

$$W^b(r_1, r_2) = \int_0^{w-\min\{r_1,r_2\}} (w - s) f(s)ds + \int_0^{\delta-\max\{r_1,r_2\}} (\delta - s) f(s)ds.$$

Thus, the net gain of social welfare of banning below-cost pricing is equal to

$$\Delta W = W^b(r_1, r_2) - W = \int_{\tau^*}^{\delta-\max\{r_1,r_2\}} (\delta - s) f(s)ds - \int_{w-\min\{r_1,r_2\}}^{w} (w - s) f(s)ds.$$

Notice that the first term is the gain from increased multistop shopping while the second term is the loss from the squeezed demand of one-stop shoppers due to the increase of total margin. So total welfare decreases if the second effect dominates the first, which depends on the difference between w and δ as well as the distribution of consumer shopping cost.

Obviously, if the distribution of shopping costs is bounded above such that $s \leq w - \hat{r} \leq w - \min\{r_1, r_2\}$, then the second term vanishes and thus $\Delta W > 0$. In this case, banning below-cost pricing has no impact on one-stop shoppers since all consumers are willing to patronize one firm, but brings positive effect on social welfare by engendering more traffic of multistop shoppers.

When the distribution of shopping costs is unbounded as we have assumed in the baseline model, then social welfare may or may not increase, depending on the value of w. To be more precise, we focus on the uniform distribution such that $F(s) = s$ and compare the expected net social welfare $E[\Delta W]$. We find that banning below-cost pricing leads to higher expected social welfare if and only if the value of w is sufficiently large.

Summarizing the above analysis leads to:

Proposition 3 *Suppose below-cost pricing were banned. Then*

- *The number of multistop shoppers increases;*

- *If the distribution of shopping costs is bounded above by $w-\hat{r}$, then social welfare increases;*
- *Otherwise, when shopping costs are uniformly distributed, there exists some $\hat{w} > \delta$ and the expected social welfare increases if and only if $w > \hat{w}$.*

Proof. See Appendix D. ■

5 Robustness: Differentiated Firms

So far we have focused on a simple setting where firms' product lines are perfectly substitute and thus fierce price competition leads to zero total margin. Cross-subsidization then allows each firm to earn positive profit from multistop shoppers while still keep attracting one-stop shoppers. When firms are horizontally differentiated in their products, they are able to charge positive total margin and thus earn positive profit from one-stop shoppers. In this case, whether adopting cross-subsidization can increase firms' profits earned from multistop shoppers needs to be examined.

To check the robustness of our analysis, we consider a variant where consumers have firm-specific preferences which vary across persons. Formally, we assume that buying one product from firm 1 incurs a utility shock $-tx/2$ to a particular consumer at location x, and purchasing one product from firm 2 entails a disutility $-t(1-x)/2$ to that consumer. Thus, patronizing firm 1 yields a net value $v_1 - tx - s$ and one-stop shopping in firm 2 brings $v_2 - t(1-x) - s$, whereas multistop shopping yields a net value $v_{12} - t/2 - 2s$. The parameter t reflects the elasticity of demand: the lower t, the faster consumers drop in case of a price increase. We assume that the parameter x is uniformly distributed in $[0, 1]$.

Consumers are willing to patronize firm 1 if $v_1 - tx \geq s$, and they prefer that to one-stop shopping in firm 2 if

$$v_1 - tx - s \geq v_2 - t(1-x) - s,$$

which is equivalent to

$$x \leq \hat{x} \equiv \frac{1}{2} - \frac{1}{2t}(r_1 - r_2).$$

Moreover, consumers prefer multistop shopping than patronizing only firm 1 if

$$v_{12} - \frac{t}{2} - 2s \geq v_1 - tx - s,$$

which amounts to

$$s \leq \lambda_1(x) \equiv \tau_1 - t(\frac{1}{2} - x).$$

Similarly, consumers prefer multistop shopping than patronizing firm 2 if

$$s \leq \lambda_2(x) \equiv \tau_2 - t(x - \frac{1}{2}).$$

Thus, consumers whose preference parameter x and shopping cost s satisfy $x \leq \hat{x}$ and $\lambda_1(x) < s \leq v_1 - tx$ become one-stop shoppers to firm 1, and those with $x > \hat{x}$ and $\lambda_2(x) < s \leq v_2 - t(1-x)$ are attracted to patronize firm 2. The demand for the assortments A_1B_1 and A_2B_2 is then given by

$$D_1^{OSS} \equiv \int_0^{\hat{x}} [F(v_1 - tx) - F(\lambda_1(x))]\, dx,$$

and

$$D_2^{OSS} \equiv \int_{\hat{x}}^1 [F(v_2 - t(1-x)) - F(\lambda_2(x))]\, dx.$$

In addition, consumers whose preference x and shopping cost s are such that $x \leq \hat{x}$ and $s \leq \lambda_1(x)$ as well as $x > \hat{x}$ and $s \leq \lambda_2(x)$ are multistop shoppers, and their demand can be written as

$$D^{MSS} \equiv \int_0^{\hat{x}} F(\lambda_1(x))dx + \int_{\hat{x}}^1 F(\lambda_2(x))dx.$$

The above characterization is demonstrated in Figure 2.

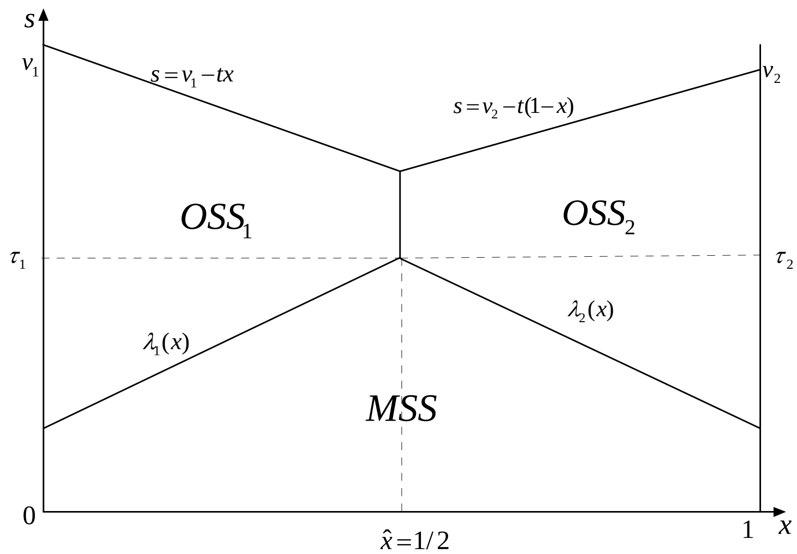

Figure 2

Firm 1 then chooses the margins r_1^A and r_1^B to maximize its profit

$$\Pi_1 = \left(r_1^A + r_1^B\right) D_1^{OSS} + r_1^A D^{MSS}.$$

To characterize the equilibrium, we consider the impact of a small change on r_1^A and r_1^B respectively and then evaluate it at the symmetric candidate equilibrium with $r_1 = r_2$ (notice that $\hat{x} = 1/2$, $\lambda_1(\hat{x}) = \lambda_2(\hat{x}) = \tau$, and $\int_0^{1/2} f(\lambda_1(x))dx = \int_{1/2}^1 f(\lambda_2(x))dx$ in the symmetric candidate equilibrium). Consider first a small change of r_1^A by dr and a small change of r_1^B by $-dr$ such that the total margin on the assortment remains unchanged. Such changes do not affect the behavior of marginal one-stop shoppers who are indifferent between patronizing firm 1 and 2, since $\hat{x} = \frac{1}{2} - \frac{1}{2t}(r_1 - r_2)$ is not affected. However (see Figure 2):

- For $x \in [1/2, 1]$, marginal consumers with $s = \lambda_2(x)$ who were indifferent between multistop shopping or patronizing firm 2, now prefer one-stop shopping in firm 2, therefore firm 1 loses $f(\lambda_2(x))dr$ consumers on which it no longer earns the margin r_1^A. The impact on firm 1's profit is then equal to
$$-r_1^A \int_{1/2}^1 f(\lambda_2(x))dxdr.$$
- In addition, the new strategy alters consumers' choices between one-stop shopping in firm 1 and multistop shopping. For any $x \in [0, 1/2]$, marginal consumers with $s = \lambda_1(x)$, who were indifferent between multistop shopping or patronizing firm 1, now prefer patronizing firm 1 and buy also B_1. The demand of product B_1 increases with $f(\lambda_1(x))dr$ and firm 1 earns an extra profit of
$$r_1^B \int_0^{1/2} f(\lambda_1(x))dxdr.$$
- Moreover, firm 1 increases its margin r_1^A by dr on the mass of multistop shoppers, and thus earns additional profit of $D^{MSS}dr$.

In equilibrium, the overall impact on firm 1's profit by such an adjustment must be offset. This implies
$$\left(r_1^B - r_1^A\right) \int_0^{1/2} f(\lambda_1(x))dx + D^{MSS} = 0. \tag{4}$$

Consider now a small change on r_1^B by dr while keeping r_1^A constant. It does not affect the consumers' choices between multistop shopping or patronizing firm 2. However:

- For $x \in [0, 1/2]$, marginal consumers with $s = \lambda_1(x)$ who were indifferent between multistop shopping and patronizing firm 1 now prefer multistop shopping, and firm 1 loses $f(\lambda_1(x))dr$ consumers who now stop buying B_1, which incurs a loss
$$-r_1^B \int_0^{1/2} f(\lambda_1(x))dxdr.$$

- On the other hand, it alters the decision of marginal consumers (one-stop shoppers) with $x = \hat{x} = 1/2$, who were indifferent between buying from firm 1 or 2 but now are attracted by firm 2, on which firm 1 incurs a loss

$$-\frac{r_1}{2t}\left(F(v_1 - \frac{t}{2}) - F(\tau)\right) dr.$$

This also affects the marginal consumers with $s = v_1 - tx$, who were indifferent between patronizing firm 1 or not but now stop visiting firm 1, on which it entails a loss

$$-r_1 \int_0^{1/2} f(v_1 - tx) dx dr.$$

- Finally, firm 1 gains from the increase of its price r_1^B by dr on the mass D_1^{OSS} of consumers and earns a profit $D_1^{OSS} dr$.

In equilibrium, these effects must be cancelled out and it follows that

$$r_1^B \int_0^{1/2} f(\lambda_1(x)) dx + \frac{r_1}{2t}\left(F(v_1 - \frac{t}{2}) - F(\tau_1)\right) + r_1 \int_0^{1/2} f(v_1 - tx) dx = D_1^{OSS}. \tag{5}$$

First-order conditions for r_2^A and r_2^B can be characterized in a similar way.

Noting that $r_1^A = r_1 - r_1^B$ and using $D^{MSS} = 2\int_0^{1/2} F(\lambda_1(x))dx$ in the symmetric equilibrium, we can rewrite equation (4) as

$$r_1^B = \frac{r_1}{2} - \tilde{h}(\lambda_1),$$

where

$$\tilde{h}(\lambda_1) = \frac{\int_0^{1/2} F(\lambda_1(x)) dx}{\int_0^{1/2} f(\lambda_1(x)) dx}$$

is the expected hazard rate for the marginal multistop shoppers with $s = \lambda_1(x)$. It appears that $r_1^B < 0$ if and only if $r_1 < 2\tilde{h}(\lambda_1)$, which implies that firm 1 will price its weak product below cost if the total margin is sufficiently small. However, it is straightforward to see from (5) that the total margin r_1 must converge to 0 when t goes to zero, otherwise the equation cannot hold, that is, $r_1 = 0$ when $t = 0$.

In particular we have

$$\begin{aligned} \lim_{t \to 0} r_1^B &= r_2^A = -h(\tau^*), \\ \lim_{t \to 0} r_1^A &= r_2^B = h(\tau^*). \end{aligned}$$

That is, the equilibrium margins converge to those under pure price competition, so cross-subsidization arises in equilibrium for t sufficiently small. More precisely, there exists a value $t^0 > 0$ such that $r_1^B < 0$ and $r_2^A < 0$ for all $t < t^0$, in which case each firm charges a price below-cost for its weak product.

The above analysis is summarized in the following proposition:

Proposition 4 *When horizontally differentiated firms compete fiercely for one-stop shoppers, there exists a symmetric equilibrium where each firm prices its weak product below cost and cross-subsidizes by its strong product.*

Proof. See Appendix E. ■

6 Extensions: Bounded distribution of shopping costs

We have so far considered an unbounded distribution of shopping costs (i.e., $s \in [0, \infty)$), reflecting a widespread heterogeneity in consumer characteristics. To check the robustness of our cross-subsidization result, we now consider bounded distributions of consumer shopping costs.

Suppose first that the distribution of shopping costs is bounded above so that $s \in [0, \overline{s}]$. Notice that consumers are willing to patronize firm i if $s \leq v_i = w - r_i = w$ ($r_i = 0$ in candidate equilibrium), it is thus straightforward to see that this upper bound is irrelevant if $\overline{s} > w$. In this case the previous claims are validated and consumers with shopping costs $s < \tau^*$ are multistop shoppers while those with $\tau^* \leq s \leq w (= v)$ are one-stop shoppers, and firms cross-subsidize their weak products.

Whenever $\tau^* < \overline{s} < w$, consumers with shopping costs $s < \tau^*$ prefer multistop shopping whereas all others (with $\tau^* < s < \overline{s}$) are one-stop shoppers. The total demand from one-stop shoppers is then equal to $F(\overline{s}) - F(\tau^*)$. However, since firms earn zero margins on one-stop shoppers, this change of demand from one-stop shoppers does not affect firms' pricing strategy as long as $\overline{s} > \tau^*$ (i.e. $\overline{s} + 2h(\overline{s}) > \delta$), in which case there are some active one-stop shoppers. It follows that the analysis in the baseline model goes through here and thus cross-subsidization arises in the unique Nash equilibrium.

When $\overline{s} < \tau^*$ (i.e. $\overline{s} + 2h(\overline{s}) \leq \delta$), however, then all consumers are multistop shoppers. It must be that $\tau_1 = \tau_2 = \overline{s}$ in equilibrium, which implies $r_1^A = \delta + r_2^A - \overline{s}$ and $r_2^B = \delta + r_1^B - \overline{s}$. Firms make a profit from multistop shoppers, which are give by $\pi_1 = r_1^A F(\overline{s})$ and $\pi_2 = r_2^B F(\overline{s})$ respectively.

For the above strategies to form an equilibrium, firms must be prevented to deviate. A firm, say, firm 1, may for example transform some multistop shoppers into one-stop shoppers who visit the rival's firm, by increasing its margin on the strong product to $\tilde{r}_1^A$ so that $\tilde{\tau}_2 = \delta + r_2^A - \tilde{r}_1^A < \tau_1 = \overline{s}$, and such deviation yields $\tilde{\pi}_1 \equiv \tilde{r}_1^A F\left(\delta + r_2^A - \tilde{r}_1^A\right)$. To rule out such deviation, the equilibrium margin r_1^A must satisfy $r_1^A \geq \max_{\tilde{r}_1^A} \tilde{\pi}_1$, for all $\tilde{r}_1^A \leq r_1^A$. This implies that the first-order derivative of $\tilde{\pi}_1$ must be non-negative at $\tilde{r}_1^A = r_1^A$, which requires $r_1^A \geq h(\overline{s})$ (similarly $r_2^B \geq h(\overline{s})$).

Alternatively, firm 1 could transform some multistop shoppers into one-stop shoppers who visit its own outlet. This can be done by charging $\tilde{r}_1^A$ and $\tilde{r}_1^B$ such that (i) $\tilde{r}_1 = \tilde{r}_1^A + \tilde{r}_1^B \leq r_2 = r_1$, and (ii) $\tilde{\tau}_1 = \delta + \tilde{r}_1^B - r_2^B < \tau_1 = \tau_2 = \overline{s}$. Such deviation yields a profit

$$\begin{aligned} \pi_1' &= \tilde{r}_1^A F\left(\tilde{\tau}_1\right) + r_1(F(\overline{s}) - F\left(\tilde{\tau}_1\right)) \\ &= r_1 F(\overline{s}) - \tilde{r}_1^B F\left(\delta + \tilde{r}_1^B - r_2^B\right). \end{aligned}$$

It follows that such deviation is not profitable if the equilibrium margin r_1^B solves

$$\max_{\tilde{r}_1^B \leq r_1^B} -\tilde{r}_1^B F\left(\delta + \tilde{r}_1^B - r_2^B\right),$$

which implies $r_1^B \leq -h(\overline{s})$ and thus $r_2^B = \delta + r_1^B - \overline{s} \geq \delta - \overline{s} - h(\overline{s})$ (similarly $r_2^A \leq -h(\overline{s})$ and $r_1^A \geq \delta - \overline{s} - h(\overline{s})$).

Summarizing the above analysis, we have

Proposition 5 *Suppose the distribution of shopping costs is bounded above, i.e., $s \leq \overline{s}$. Then:*

- *If the upper bound is such that $\overline{s} + 2h(\overline{s}) > \delta$, there exists a unique Nash equilibrium which involves cross-subsidization and has the same properties as characterized by Proposition 1;*
- *If instead $\overline{s} + 2h(\overline{s}) \leq \delta$, there exist multiple equilibria in which (i) no cross-subsidy arises, (ii) all consumers are multistop shoppers, and (iii) firms earn a positive margin on its strong product which lies in $[h(\overline{s}), \delta - \overline{s} - h(\overline{s})]$.*

Proof. See Appendix F. ■

Thus, firms price their weak products below cost in all cases. However, cross-subsidization occurs only when consumer shopping costs are sufficiently widespread such that $\overline{s} + 2h(\overline{s}) > \delta$. Conversely, if shopping costs are not sufficiently heterogeneous such that $\overline{s} + 2h(\overline{s}) \leq \delta$, then firms keep offering their weak products below cost but only sell their strong products.

Assume now that shopping costs are bounded below so that $s \in [\underline{s}, +\infty)$, where $\underline{s} < w$ so that it is efficient for a firm to offer the assortment AB. Then there always exist active one-stop shoppers, and the competition for one-stop shoppers drives the total margin down to zero. We show also in the Appendix G that there exist multistop shoppers (those with $s \in [\underline{s}, \tau^*]$) as long as $\underline{s} < \tau^*$ (which boils down to $\underline{s} < \delta/3$), and thus the analysis of the baseline model still goes through. Conversely, multistop shopping does not arise in equilibrium if the lower bound of shopping costs is higher than the comparative advantage derived from the strong product (i.e., $\underline{s} > \delta$).

Finally, we show in Appendix G that when the lower bound $\underline{s}$ lies between $\delta/3$ and δ, both types of equilibria mentioned above exist, and firms earn the positive profit in the first type of equilibrium whereas zero profit in the second type.

Proposition 6 *Suppose the distribution of shopping costs is bounded below, i.e., $s \geq \underline{s}$, then:*

- *If $\underline{s} < \delta/3$, there exists a unique Nash equilibrium, which involves cross-subsidization and has the same property as in the baseline model;*
- *If instead $\underline{s} > \delta$, there are multiple Nash equilibria in which (i) all consumers are one-stop shoppers and (ii) firms make zero profits;*
- *If $\delta/3 < \underline{s} < \delta$, both types of equilibria exist.*

Proof. See Appendix G. ■

7 Conclusions

This paper develops a new model to analyze cross-subsidization by multiproduct firms in competitive markets, taking into account the heterogeneity of consumer shopping patterns. We show that competitive cross-subsidization arises in a unique equilibrium, in the absence of any efficiency justifications stemming from asymmetric information on prices, and acts as a discriminatory mechanism for screening consumers with different shopping patterns. Firms earn positive profits from multistop shoppers, though fierce price competition dissipates away its profit from one-stop shoppers. We show also that banning below-cost pricing leads higher prices for one-stop shoppers and may reduce consumer surplus as well as total social welfare.

Our analysis sheds a new light on the rationale of competitive cross-subsidization and identifies the key factors underlying it: heterogeneity in consumer shopping patterns and asymmetry in comparative advantages, and the insights are robust to variations with product differentiation among firms.

We have furthermore restricted attention to individual unit demands and also neglected any correlation between consumers' valuations for the goods and their shopping costs; whether our insights apply to market environments where consumers' individual demands are elastic, or underlying characteristics (e.g., wealth) affect both shopping costs and willingness to pay, is left to future research.

Appendix

Recall that $v_i = w - r_i$, $i = 1, 2$, and $v_{12} = w + \delta - r_1^A - r_2^B$, and denote by $\hat{v}_{12} = w - \delta - r_1^B - r_2^A$ the value from multistop shopping when consumers pick the weak products from each firm. Let MSS refer to multistop shoppers buying both strong products and $\widehat{MSS}$ refer to multistop shoppers picking both weak products; likewise, let OSS stand for one-stop shoppers and OSS_i for one-stop shoppers patronizing firm i. Notice that the threshold of shopping cost below which consumers prefer multistop shopping for strong products are defined as $\tau_1 = v_{12} - v_1 = \delta + r_1^B - r_2^B$, $\tau_2 = v_{12} - v_2 = \delta - r_1^A + r_2^A$, and $\tau = \min\{\tau_1, \tau_2\}$; likewise the threshold on multistop shopping for the weak products are given by $\hat{\tau}_1 \equiv \hat{v}_{12} - v_1 = r_1^A - r_2^A - \delta$, $\hat{\tau}_2 \equiv \hat{v}_{12} - v_2 = r_2^B - r_1^B - \delta$, and $\hat{\tau} = \min\{\hat{\tau}_1, \hat{\tau}_2\}$. It follows that $\hat{\tau}_1 = -\tau_2$ and $\hat{\tau}_2 = -\tau_1$, and thus $\hat{\tau} = -\tau$. Since τ and $\hat{\tau}$ must be positive, MSS and $\widehat{MSS}$ cannot coexist in equilibrium.

A Proof of Lemma 1

To prove the lemma, we first establish the following claims.

Claim 1: There must be some active consumers in equilibrium.

Proof: Suppose there are no active consumers, which must be the case that $\max\{v_1, v_2\} \leq 0$ and $\max\{v_{12}, \hat{v}_{12}\} \leq 0$, and firms make no profit. Consider firm 1's deviation as follows: charging $\tilde{r}_1^B > 0$ and $\tilde{r}_1^A > 0$ such that $\tilde{r}_1 = \tilde{r}_1^A + \tilde{r}_1^B = w - \varepsilon$. Firm 1 then attracts some consumers (those with $s \leq \tilde{v}_1 = \varepsilon$ are willing to patronize it) and earns positive profit. Thus some consumers must be active in equilibrium. **Q.E.D.**

Claim 2: If there are active one-stop shoppers, then $r_1 = r_2 = 0$.

Proof: Suppose there are some active OSS. We show first that in equilibrium no firms charge negative total margin (i.e., $r_i < 0$, $i = 1, 2$). Suppose firm 1 sets $r_1 < 0$, say, then:

- If $r_1 < r_2$ (thus $v_1 > v_2$), then firm 1 incurs a loss by attracting OSS. Consider firm 1's possible deviations in the following cases:
  - Suppose there are no active multistop shoppers, which must be the case that $\max\{v_{12}, \hat{v}_{12}\} \leq v_1$. Since firm 1 incurs a loss by serving OSS, it would find it profitable to increase r_1.
  - Suppose there are some active multistop shoppers who buy the strong products. This must be the case that $\max\{v_{12}, \hat{v}_{12}\} = v_{12} > v_1\ (\geq v_2)$, which implies $\tau = \tau_1 > 0$ and thus $\hat{\tau} < 0$. If $r_1^A < 0$, then firm 1 makes a loss also from MSS. Then firm 1 can

reduce its loss by raising its margins on both products. If instead $r_1^A \geq 0$, then firm 1 could benefit from raising its margin on weak product only: keep r_1^A constant and charge $\tilde{r}_1^B = -r_1^A = r_1^B - r_1 > r_1^B$ such that $\tilde{r}_1 = 0$. This deviation avoids the loss from OSS, and moreover increases the demand from MSS since $\tilde{\tau}_1 = \delta + \tilde{r}_1^B - r_2^B > \tau_1$ and $\tilde{\tau}_2 = \tau_2$.

  - Suppose there are some active multistop shoppers who buy the weak products. It then must be $\max\{v_{12}, \hat{v}_{12}\} = \hat{v}_{12} > v_1 \geq v_2$, which implies $\hat{\tau} = \hat{\tau}_1 > 0$ and thus $\tau < 0$. If $r_1^B < 0$, then firm 1 also incurs a loss by serving multistop shoppers. It can then reduce the loss by raising its margins on both products. If instead $r_1^B \geq 0$, then firm 1 could avoid the loss from OSS by setting $\tilde{r}_1^A = -r_1^B$ such that $\tilde{r}_1 = 0$. This deviation also increases the demand from $\widehat{MSS}$ since $\hat{\tau}_1$ increases and $\hat{\tau}_2$ remains unchanged.

- If instead $r_1 \geq r_2$ (and thus $r_2 < 0$), then the same argument applies to any firm that attracts OSS (firm 2 if $r_1 > r_2$, and at least one of the firms if $r_1 = r_2$).

Secondly, we show that charging $r_i > 0$, $i = 1, 2$, by both firms cannot be an equilibrium. Suppose firms set $r_1, r_2 > 0$. Then:

- If any firm, say firm 1, charges a higher margin ($r_1 > r_2 > 0$ and $v_2 > v_1 > 0$), it faces no demand from OSS. Consider the profitable deviations in two cases:

  - Suppose there are no active multistop shoppers, i.e., $\max\{v_{12}, \hat{v}_{12}\} \leq v_2$, and thus all active consumers are one-stop shoppers. Firm 1 can then attract all one-stop shoppers by setting $\tilde{r}_1 = r_2 - \varepsilon > 0$ and makes positive profit. Notice that doing so does not transform one-stop shoppers into multistop shoppers since $\tilde{v}_1 = v_2 + \varepsilon > \max\{v_{12}, \hat{v}_{12}\}$.
  - Suppose there are active multistop shoppers, which must be the case $\max\{v_{12}, \hat{v}_{12}\} > v_2 > v_1$. Consider firm 1's deviation as follows: keep the margin on the product purchased by multistop shoppers and reduce the margin on the other product such that $\tilde{r}_1 = r_2 - \varepsilon$. Doing so attracts all OSS at the cost of reducing slightly (by ε) the demand of multistop shoppers, which is obviously profitable when ε is small enough.

- If both firms charge the same margins ($r_1 = r_2 > 0$), then $v_2 = v_1$ and $\tau_1 = \tau_2$. In this case, any firm, say firm 1, who does not obtain more than half of the demand from OSS, can attract all OSS using the same deviations as above (for the case $r_1 > r_2$), and the gain from undercutting the rival offsets the loss from the slight reduction in demand from multistop shoppers (if there are any of them).

Finally, we show that no firm charges a positive margin in equilibrium. Suppose that $r_1 > r_2 = 0$, and thus firm 2 makes zero profit from OSS. Consider the following deviation of firm 2: keep the margin on the product picked by multistop shoppers and increase the margin of the other product by ε such that $\tilde{r}_2 = r_2 + \varepsilon (< r_1)$. It still supplies all OSS, but now makes a profit on them. Doing so also increases slightly the demand from multistop shoppers and thus the profit on them. For instance, when multistop shoppers buy the strong products, then the relevant threshold is $\tau = \tau_2 = \delta - r_1^A + r_2^A$ (since $r_1 > r_2$ implies $\tau_1 > \tau_2$), which increases to $\tilde{\tau}_2 = \delta - r_1^A + \tilde{r}_2^A = \tau_2 + \varepsilon$.

Summarizing the above analysis, we conclude that firms must charge $r_1 = r_2 = 0$ in any equilibrium where there are active one-stop shoppers. **Q.E.D.**

Claim 3: Active multistop shoppers must buy the strong products.

Proof: Suppose there are some active multistop shoppers who purchase the weak products. This must be the case that each firm offers a better value on its weak product than that of the rival's strong product: $v_1^B \equiv w_1^B - r_1^B \geq v_2^B \equiv w_2^B - r_2^B$ and $v_2^A \equiv w_2^A - r_2^A \geq v_1^A \equiv w_1^A - r_1^A$. It follows that $r_2^B \geq r_1^B + w_2^B - w_1^B \geq r_1^B + \delta$ and $r_1^A \geq r_2^A + w_1^A - w_2^A \geq r_2^A + \delta$, that is, each firm's margin on its strong product must be higher than the rival's margin on the same product plus the comparative advantage. We show that such configuration cannot be an equilibrium. Consider two cases:

- Suppose there are only multistop shoppers (who then buy the weak products). To make profits, firms must charge non-negative margins on their weak products, i.e., r_1^B, $r_2^A \geq 0$. This implies that $r_2^B \geq r_1^B + \delta \geq \delta$ and $r_1^A \geq r_2^A + \delta \geq \delta$, that is, each firm's margin on its strong product is higher than its comparative advantage δ. It is then profitable for some firm, say firm 1, to undercut the rival by charging $\tilde{r}_1^A = r_2^A + \delta - \varepsilon > 0$ (keeping r_1^B unchanged). Doing so transforms $\widehat{MSS}$ into OSS_1: Consumers who originally buy A from firm 2 will instead buy both products from firm 1, since firm 1 now offers a better value on its strong product A: $\tilde{v}_1^A = w_1^A - \tilde{r}_1^A = w_1^A - \delta - r_2^A + \varepsilon = v_2^A + \varepsilon$. The deviation may attract additional OSS, on which the firm also makes a profit since $\tilde{r}_1^A, r_1^B \geq 0$.

- Suppose there exist both OSS and $\widehat{MSS}$. Then price competition for one-stop shoppers leads to $r_1 = r_2 = 0$ by Claim 2, and thus firms make no profit from OSS. Therefore firms must charge non-negative margins on their weak products, i.e., r_1^B, $r_2^A \geq 0$, as otherwise they would incur a loss. But this implies that the margins on the strong products must be non-positive, say, $r_1^A = r_1 -\ r_1^B \leq 0$, which contradicts to the condition $r_1^A \geq r_2^A + \delta \geq \delta$.

Therefore multistop shoppers must pick the strong products in equilibrium. **Q.E.D.**

Claim 4: There must exist active multistop shoppers.

Proof: Suppose all active consumers are OSS. It must be that $\max\{v_1, v_2\} > 0$ and $\max\{v_1, v_2\} \geq \max\{v_{12}, \hat{v}_{12}\}$. Price competition for one-stop shoppers then leads to $r_1 = r_2 = 0$ and firms make zero profit. We show that this configuration cannot be an equilibrium.

Since $v_1 = v_2 \geq \max\{v_{12}, \hat{v}_{12}\}$, it follows that $\tau_1 = \tau_2 \leq 0$ and $\hat{\tau}_1 = \hat{\tau}_2 \leq 0$, and thus $\tau_1 = \tau_2 = 0$ since $\tau_1 = -\hat{\tau}_2$ and $\tau_2 = -\hat{\tau}_1$. Moreover, $v_1 = v_2 \geq v_{12}$ implies $r_1^A + r_2^B - \delta \geq r_1 = r_2 = 0$. It follows that at least one firm, say firm 1, must charge a margin higher than $\delta/2$ on its strong product (i.e., $r_1^A \geq \delta/2$), and it is thus profitable to transform some OSS_1 into MSS. Increasing r_1^B by ε and decreasing r_1^A by ε raise both τ_1 and τ_2 by ε, which engenders some multistop shoppers since $\tau = \varepsilon > 0$. Since $\tilde{r}_1^A = \delta/2 - \varepsilon > 0$, firm 1 makes positive profit on these MSS. **Q.E.D.**

Claim 5: There must exist active one-stop shoppers.

Proof: Suppose there are only multistop shoppers, then they buy the strong products by Claim 3. Notice that consumers are willing to visit both firms only if $2s \leq v_{12}$ (i.e., $s \leq v_{12}/2$) and they prefer MSS to OSS if $s \leq \tau = v_{12} - \max\{v_1, v_2\}$. Then, it must be the case that $v_{12}/2 \leq \tau = v_{12} - \max\{v_1, v_2\}$, that is, $\max\{v_1, v_2\} \leq v_{12}/2$. Suppose $v_{12}/2 > \tau$, which implies $\max\{v_1, v_2\} > v_{12}/2$, then consumers with shopping cost $s \in [\tau, v_{12}/2]$ prefer one-stop shopping since $s \leq v_{12}/2 < \max\{v_1, v_2\}$, which contradicts to the supposition that there are no one-stop shoppers. The demand from MSS is thus $F(v_{12}/2)$. Since multistop shoppers only buy strong products, firms must charge non-negative margins on the strong products in order to make positive profit. Suppose $r_2^B \geq r_1^A \geq 0$. Consider the following deviation for firm 1: keep r_1^A and change r_1^B to $\tilde{r}_1^B = \frac{1}{2}(w - \delta + r_2^B - r_1^A - 2\varepsilon) \geq \frac{1}{2}(w - \delta - 2\varepsilon) > 0$ so that $\tilde{v}_1 = w - r_1^A - \tilde{r}_1^B = \frac{v_{12}}{2} + \varepsilon$ and $\tilde{\tau}_1 = \delta + \tilde{r}_1^B - r_2^B = \frac{v_{12}}{2} - \varepsilon < \tilde{v}_1$. This adjustment transforms some MSS into OSS_1 (those whose shopping cost lies between $\tilde{\tau}_1$ and $\tilde{v}_1$), on which firm 1 acquires an extra profit from selling its weak product (since $\tilde{r}_1^B > 0$). The same logic applies to the case $r_1^A \geq r_2^B$, in which firm 2 can benefit from changing r_2^A in a similar manner. Thus, there must exist active one-stop shoppers in equilibrium. **Q.E.D.**

Therefore, by Claim 4 and 5 there exist active multistop shoppers and one-stop shoppers, and this establishes part (i) of the lemma. Then part (ii) is implied by Claim 3. Finally, since there must exist some one-stop shoppers in equilibrium, Claim 2 then implies zero total margin for each firm, which leads to part (iii) of the lemma.

B Proof of Proposition 1

The candidate equilibrium is characterized by $r_1^{A*} = r_2^{B*} = h(\tau^*)$ and $r_1 = r_2 = 0$, where $\tau^* = j^{-1}(\delta)$. We show now firms cannot benefit from any deviations. Suppose firm 1 charges the margins r_1^A and r_1^B instead of r_1^{A*} and r_1^{B*}. Then:

- it cannot make a profit from one-stop shoppers since, to attract them, it must charge $r_1 \leq r_2^* = 0$.
- it cannot make a profit by offering the weak product to $\widehat{MSS}$, since it would have to charge $r_1^B \leq r_2^{B*} - \delta = h(\tau^*) - \delta < 0$ (as $h(\tau^*) < \delta$) to attract them.
- it can thus make a profit only from MSS, which is equal to $\pi_1^A = r_1^A F(\tau)$. Notice that $\tau = \min\{\delta - h(\tau^*) - r_1^A, \delta + r_1^B - h(\tau^*)\}$, then
$$\pi_1^A = r_1^A F(\tau) \leq r_1^A F(\delta - h(\tau^*) - r_1^A) \leq \pi^*,$$
where the second inequality comes from the fact that the profit function $r_1^A F(\delta - h(\tau^*) - r_1^A)$ is quasi-concave and is maximized for $r_1^A = h(\tau^*)$.

C Proof of Proposition 2

We first characterize the mixed-strategy equilibrium. Consider a candidate equilibrium such that $\rho_i = r_i$, $i = 1, 2$, where r_i is distributed according to the same distribution $K(\cdot)$ over some interval $[\underline{r}, \overline{r}]$, with a continuous density $k(\cdot)$. Without loss of generality, we restrict to $\overline{r} \leq \delta\,(< w)$. Suppose $\overline{r} > \delta$, then choosing $r_i = \overline{r}$ yields zero profit for firm i because (i) no one-stop shoppers patronize firm i (since the rival undercuts with probability 1), and (ii) there are no multistop shoppers (since the relevant threshold is then $\tau = \delta - \overline{r} < 0$). But then choosing r slightly below δ could attract some (multistop and one-stop) shoppers and yield positive profit.

Recall that the function $K(\cdot)$ is defined by (2) as follows:
$$K(r) = \frac{F(w-r) - \frac{\hat{\pi}}{r}}{F(w-r) - F(\delta - r)}.$$
Notice that the lower bound and upper bound of the distribution are given by
$$\pi_i(\underline{r}) = \underline{r} F(w - \underline{r}) = \pi_i(\overline{r}) = \overline{r} F(\delta - \overline{r}) = \hat{\pi},$$

it follows that $\overline{r} = \hat{r}$. Thus we have

$$K(\hat{r}) = \frac{F(w-\hat{r}) - F(\delta-\hat{r})}{F(w-\hat{r}) - F(\delta-\hat{r})} = 1,$$

and

$$K(\underline{r}) = \frac{F(w-\underline{r}) - \frac{\hat{r}F(\delta-\hat{r})}{\underline{r}}}{F(w-\underline{r}) - F(\delta-\underline{r})} = 0.$$

In addition, for any $r < \hat{r}$ it must be $K(r) < 1$. Suppose $K(r) \geq 1$, then

$$\begin{aligned} \pi_i(r) &= r\left[F(w-r) - K(r)\left(F(w-r) - F(\delta-r)\right)\right] \\ &< r\left[F(w-r) - \left(F(w-r) - F(\delta-r)\right)\right] \\ &= rF(\delta-r) < \hat{r}F(\delta-\hat{r}), \end{aligned}$$

which contradicts the condition $\pi_i(r) = \hat{\pi}$.

We now examine some properties of the function $K(\cdot)$ at the upper and lower bound. Differentiating it with respect to r, we have

$$K'(r) = \frac{\left(\frac{\hat{\pi}}{r^2} - f(w-r)\right)\left(F(w-r) - F(\delta-r)\right) - \left(F(w-r) - \frac{\hat{\pi}}{r}\right)\left(f(\delta-r) - f(w-r)\right)}{\left(F(w-r) - F(\delta-r)\right)^2}.$$

Therefore, for $r = \hat{r}$ the numerator boils down to

$$\begin{aligned} &\left(\frac{\hat{r}F(\delta-\hat{r})}{\hat{r}^2} - f(w-\hat{r})\right)\left(F(w-\hat{r}) - F(\delta-\hat{r})\right) - \left(F(w-\hat{r}) - \frac{\hat{r}F(\delta-\hat{r})}{\hat{r}}\right)\left(f(\delta-\hat{r}) - f(w-\hat{r})\right) \\ = \; &\left(F(w-\hat{r}) - F(\delta-\hat{r})\right)\left(\frac{F(\delta-\hat{r})}{\hat{r}} - f(\delta-\hat{r})\right), \end{aligned}$$

which, using the first-order condition (1), boils down to $F(\delta-\hat{r}) = \hat{r}f(\delta-\hat{r})$, it thus follows that $K'(\hat{r}) = 0$. In addition, differentiating further $K'(r)$ with respect to r and then evaluate it at $\hat{r}$ we obtain

$$\begin{aligned} K''(\hat{r}) &= \frac{\left[\left(\frac{\hat{\pi}}{r^2} - f(w-r)\right)\left(F(w-r) - F(\delta-r)\right) - \left(F(w-r) - \frac{\hat{\pi}}{r}\right)\left(f(\delta-r) - f(w-r)\right)\right]'\Big|_{r=\hat{r}}}{\left(F(w-\hat{r}) - F(\delta-\hat{r})\right)^2} \\ &\quad - \frac{2K'(\hat{r})}{F(w-\hat{r}) - F(\delta-\hat{r})}\left(f(\delta-\hat{r}) - f(w-\hat{r})\right) \\ &= \frac{\left(\begin{array}{c} \left(f(\delta-r) - f(w-r)\right)\left(\frac{\hat{\pi}}{r^2} - f(w-r)\right) - \left(F(w-r) - F(\delta-r)\right)\left(f'(w-r) - \frac{2\hat{\pi}}{r^3}\right) \\ - \left(f'(w-r) - f'(\delta-r)\right)\left(F(w-r) - \frac{\hat{\pi}}{r}\right) - \left(\frac{\hat{\pi}}{r^2} - f(w-r)\right)\left(f(\delta-r) - f(w-r)\right) \end{array}\right)_{r=\hat{r}}}{\left(F(w-\hat{r}) - F(\delta-\hat{r})\right)^2} \\ &= \frac{f'(\delta-\hat{r}) - \frac{2f(\delta-\hat{r})}{\hat{r}}}{F(w-\hat{r}) - F(\delta-\hat{r})} \\ &< \frac{\frac{f^2(\delta-\hat{r})}{F(\delta-\hat{r})} - \frac{2f(\delta-\hat{r})}{\hat{r}}}{1 - F(\delta-r)} = \frac{\frac{f(\delta-\hat{r})}{\hat{r}} - 2\frac{f(\delta-\hat{r})}{\hat{r}}}{1 - F(\delta-r)} = -\frac{\frac{f(\delta-\hat{r})}{\hat{r}}}{1 - F(\delta-r)} < 0. \end{aligned}$$

where the first equality uses $K'(\hat{r}) = 0$, the first inequality stems from $h'(.) > 0$ which implies $f'(s) < f^2(s)/F(s)$, and the following equality follows from $F(\delta - \hat{r}) = \hat{r} f F(\delta - \hat{r})$. Since $K''(\hat{r}) < 0$ and $K'(\hat{r}) = 0$, it follows that $K'(r) > 0$ for r close to (and below) $\hat{r}$.

For $r = \underline{r}$ the numerator of $K'(\cdot)$ boils down to:

$$(F(w - \underline{r}) - F(\delta - \underline{r})) \left(\frac{F(w - \underline{r})}{\underline{r}} - f(w - \underline{r}) \right) > 0.$$

Since $\arg\max rF(w-r) > \arg\max rF(\delta-r) = \hat{r} > \underline{r}$, it must be that $F(w - \underline{r}) - \underline{r} f(w - \underline{r}) > 0$, and thus $K'(\underline{r}) > 0$.

Thus, the function $K(\cdot)$ as defined over $[\underline{r}, \hat{r}]$ by (2) and its density $k(\cdot) = K'(\cdot)$ satisfy:

- $K(\hat{r}) = 1$ and $k(\hat{r}) = 0$;
- $K(r) < 1$ for $r < \hat{r}$ and $k(r) > 0$ for r slightly below $\hat{r}$;
- $K(\underline{r}) = 0$ and $k(\underline{r}) > 0$;
- $K(r) > 0$ and $k(r) > 0$ for r slightly above $\underline{r}$.

Equipped with these properties, we now derive a c.d.f. $\hat{K}(\cdot)$ by modifying $K(\cdot)$ over the range $[\underline{r}, \hat{r}]$. If the function $K(\cdot)$ satisfies $K'(r) > 0$ for $r \in [\underline{r}, \hat{r})$, then $\hat{K}(\cdot) = K(\cdot)$. Otherwise $\hat{K}(\cdot)$ is constructed as follows:

- let $\underline{r}_1$ denote the lowest r for which $K'(r) = 0$ and $\overline{r}_1$ denote the lowest $r > \underline{r}_1$ such that $K(\overline{r}_1) = K(\underline{r}_1)$, then define the function $K_1(.)$ such that $K_1(r) = K(\underline{r}_1)$ and it derivative $k_1(r) = 0$ over the interval $[\underline{r}_1, \overline{r}_1]$ and $K_1(r)$ coincides with $K(.)$ otherwise;
- if $K_1'(.) > 0$ for $r > \overline{r}_1$, let $\hat{K}(.) = K_1(.)$. Otherwise, denote by $\underline{r}_2$ the lowest $r > \overline{r}_1$ for which $K'(r) = 0$ and denote by $\overline{r}_2$ the lowest $r > \underline{r}_2$ such that $K(\overline{r}_2) = K(\underline{r}_2)$, then define the function $K_2(.)$ such that $K_2(r) = K_1(\underline{r}_2)$ and $k_2(r) = 0$ over the interval $[\underline{r}_2, \overline{r}_2]$ and $K_2(r)$ coincides with $K_1(\cdot)$ otherwise;
- if $K_2'(.) > 0$ for $r > \overline{r}_2$, let $\hat{K}(.) = K_2(.)$. Otherwise, repeat the same steps until the function so defined is everywhere weakly increasing.

The above algorithm defines a unique and weakly increasing function $\hat{K}$, as illustrated by Figure 3. It is straightforward to see that its density function $\hat{k}(\cdot)$ is continuous.

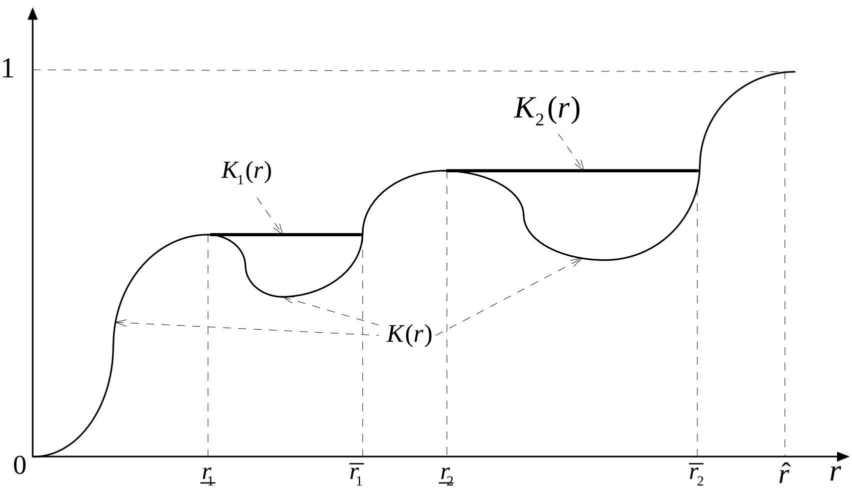

Figure 3

Notice first that the function K, defined by (2), is continuously differentiable. Furthermore, in the range $[\underline{r}, \hat{r}]$, its derivative k is positive for $r = \underline{r}$ as well as in the neighborhood of $\hat{r}$; therefore, it is either positive everywhere in this range (in which case the algorithm yields $\hat{K} = K$) or there exists some $r < \hat{r}$ for which $K'(r) = 0$. Let $\underline{r}_1$ denote the lowest value of such r, then by construction $\underline{r}_1 < \hat{r}$ and thus $K(\underline{r}_1) < K(\hat{r}) = 1$. Let $\overline{r}_1$ denote the lowest $r > \underline{r}_1$ such that $K(\overline{r}_1) = K(\underline{r}_1)$. Then for the region $r \in [\underline{r}_1, \overline{r}_1]$, we simply replace $K(r)$ by a horizontal line $K_1(r)$ as defined such that $K_1(r) = K(\underline{r}_1)$ and $k_1(r) = 0$ over that interval. By construction, the function K_1 defined by the algorithm is continuous, weakly increasing on $[\underline{r}, \overline{r}_1]$ and coincides with K on $[\overline{r}_1, \hat{r}]$. If K is weakly increasing over $[\overline{r}_1, \hat{r}]$, the algorithm stops and $\hat{K} = K_1$; otherwise the algorithm is applied to the restriction of K to $[\overline{r}_1, \hat{r}]$, and so forth.

The function $\hat{K}$ derived above has the following properties in the range $[\underline{r}, \hat{r}]$:

- $\hat{K}(r)$ is continuous and weakly increasing from $\hat{K}(\underline{r}) = 0$ to $\hat{K}(\hat{r}) = 1$;
- $\hat{K}(r) \geq K(r)$;
- its density $\hat{k}(.)$ either coincides with $k(.)$ or is equal to 0.

We now show that the function $\hat{K}$ supports a symmetric mixed strategy equilibrium. In this equilibrium firms price their weak products at cost and charge a margin r on their strong products according to the c.d.f. $\hat{K}$, which satisfies:

- no firm chooses a margin below $\underline{r}$, above $\hat{r}$, or in any interval $[\underline{r}_i, \overline{r}_i]$;

- in the rest of the range $[\underline{r}, \hat{r}]$, the firms choose the margin r according to the density function that coincides with $k = K'$.

To show that this indeed forms an equilibrium, let's suppose that firm j adopts this strategy and check that its rival i cannot benefit from any deviations. Since firms are not allowed to price below cost, we rule out the possibility that $\rho_i > r_i$. Suppose first that firm i chooses margins such that $\rho_i < r_i$. Since no one-stop shoppers would patronize firm i if it charges $r_i > \hat{r}$, it would thus earn less expected profit $\rho F(\delta - \rho) \leq \hat{\pi}$. Thus, we can restrict to the deviations such that $\rho_i < r_i \leq \hat{r}$.

Consider first consumers' response as a function of the realization of the rival's margins $\rho_j = r_j = \tilde{r}$:

- consumers buy both goods from firm i if:

$$\tilde{r} \geq r_i \text{ and } s \geq \tau_i = \delta + r_i - \rho_i - \tilde{r}.$$

- consumers engage in multistop shopping if:

$$s \leq \tau_j = \delta - \rho_i \text{ and } s \leq \tau_i = \delta + r_i - \rho_i - \tilde{r}.$$

Figure 4 depicts consumers' response.

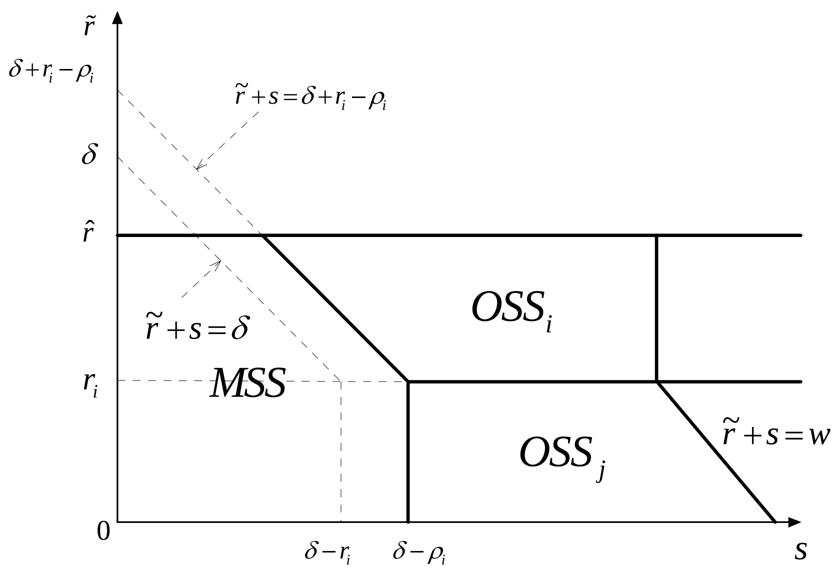

Figure 4

Consider now an increase in ρ_i by $d\rho_i$:

- it increases the profit earned on multistop shoppers (region MSS) by $D^{MSS} d\rho_i$;
- it transforms some multistop shoppers into own one-stop shoppers (transfer from MSS to OSS_i), which yields an additional profit $(r_i - \rho_i) \frac{\partial D_i^{OSS}}{\partial \rho_i} d\rho_i$ on these consumers;
- however, it also transforms some multistop shoppers into one-stop shoppers patronizing the rival (transfer from MSS to OSS_j), which yields a loss $\rho_i \frac{\partial D_j^{OSS}}{\partial \rho_i} d\rho_i$ on these consumers.

The overall impact on firm i's expected profit is thus equal to:

$$
\begin{aligned}
\frac{\partial \pi_i}{\partial \rho_i} &= D^{MSS} + (r_i - \rho_i) \frac{\partial D_i^{OSS}}{\partial \rho_i} - \rho_i \frac{\partial D_j^{OSS}}{\partial \rho_i} \\
&= K(r_i) F(\delta - \rho_i) + \int_{r_i}^{\hat{r}} k(\tilde{r}) F(\delta + r_i - \rho_i - \tilde{r}) d\tilde{r} \\
&\quad + (r_i - \rho_i) \int_{r_i}^{\hat{r}} k(\tilde{r}) f(\delta + r_i - \rho_i - \tilde{r}) d\tilde{r} - \rho_i K(r_i) f(\delta - \rho_i) \\
&> K(r_i) \left(F(\delta - \rho_i) - \rho_i f(\delta - \rho_i) \right),
\end{aligned}
$$

which is positive for any $\rho_i < \hat{r}$. Therefore charging $\rho_i < r_i$ cannot be an equilibrium.

Consider now deviations that are of the type $\rho_i = r_i$, where r_i is outside of the equilibrium range:

- Choosing $r_i < \underline{r}$ attracts all one-stop shoppers and thus yields an expected profit equal to r_i, but this is lower than $\pi_i(\underline{r}) = \hat{\pi}$;
- Choosing $r_i > \hat{r}$ attracts no one-stop shoppers, and thus the expected profit must be lower than $r_i F(\delta - r_i) < \hat{r} F(\delta - \hat{r}) = \max_r r F(\delta - r)$;
- Choosing r_i in one of the intervals $(\underline{r}_i, \overline{r}_i)$ yields an expected profit (since then $\hat{K} > K$):

$$
\begin{aligned}
& r_i \left[F(w - r_i) - \hat{K}(r_i) \left(F(w - r_i) - F(\delta - r_i) \right) \right] \\
< \; & r_i \left[F(w - r_i) - K(r_i) \left(F(w - r_i) - F(\delta - r_i) \right) \right] \\
= \; & \hat{\pi}.
\end{aligned}
$$

Thus, firms cannot benefit from any kinds of deviation.

The above analysis establishes the first part of the proposition, and the second and the third part have been shown in the context. We now show the last part of the statements. Notice that

$$
K(r) = \frac{F(w - r) - \frac{\hat{\pi}}{r}}{F(w - r) - F(\delta - r)} = 1 - \frac{\hat{\pi} - rF(\delta - r)}{r \left(F(w - r) - F(\delta - r) \right)},
$$

it thus appears that $K(r)$ increases in w. We show first that the c.d.f. $\hat{K}(r)$, which is modified from $K(r)$ and satisfies $\hat{K}(r) \geq K(r)$, also increases in w. Suppose there exist some $w' > w$ such that $\hat{K}(r, w') < \hat{K}(r, w)$ for some r. If $\hat{K}(r, w) = K(r, w)$, then it must be that $\hat{K}(r, w') \geq K(r, w') > K(r, w) = \hat{K}(r, w)$, which is a contradiction. If instead $\hat{K}(r, w) > K(r, w)$, then by the construction of $\hat{K}(r, w)$ there exists some $\underline{r}_i < r$ such that $\hat{K}(r, w) = K(\underline{r}_i, w)$. Since $\hat{K}(\cdot)$ increases in r, it must be that $\hat{K}(r, w') \geq \hat{K}(\underline{r}_i, w') \geq K(\underline{r}_i, w') > K(\underline{r}_i, w) = \hat{K}(r, w)$, where the last inequality comes from the fact that $K(r, w)$ increases in w; this contradicts to the supposition that $\hat{K}(r, w') < \hat{K}(r, w)$.

The expected value of margin can be written as

$$\begin{aligned} E[r] &= \int_{\underline{r}}^{\hat{r}} r d\hat{K}(r) = \left[r\hat{K}(r)\right]_{\underline{r}}^{\hat{r}} - \int_{\underline{r}}^{\hat{r}} \hat{K}(r)\, dr \\ &= \hat{r} - \int_{\underline{r}}^{\hat{r}} \hat{K}(r)\, dr. \end{aligned}$$

Since $\hat{r}$ is independent of w and $\hat{K}(r)$ increases in w, it follows that $E[r]$ decreases in w.

It is straightforward to see that $\underline{r} = \hat{r}$ when $w = \delta$ and $\underline{r}$ converges to zero when w goes to infinity, which implies that $E[r] = \hat{r}$ when $w = \delta$ and $E[r]$ tends to zero when w goes to infinity. Thus, there must exist some value $\bar{w} > \delta$ such that $E[r] > h(\tau^*)$ if and only if $w < \bar{w}$.

D Proof of Proposition 3

Suppose consumer shopping costs are uniformly distributed such that $F(s) = s$, in which case we have $\tau^* = \delta/3$ and $\hat{\tau} = \hat{r} = \delta/2$. Then

$$\begin{aligned} \Delta W &= \int_{\tau^*}^{\delta - \max\{r_1, r_2\}} (\delta - s)\, ds - \int_{w - \min\{r_1, r_2\}}^{w} (w - s)\, ds \\ &= \frac{(\delta - \tau^*)^2}{2} - \frac{(\max\{r_1, r_2\})^2}{2} - \frac{(\min\{r_1, r_2\})^2}{2} \\ &= \frac{2}{9}\delta^2 - \frac{r_1^2 + r_2^2}{2}. \end{aligned}$$

Thus

$$E[\Delta W] = \frac{2}{9}\delta^2 - E\left[r^2\right].$$

To calculate the expectation $E\left[r^2\right]$, notice that

$$K(r) = \frac{(w - r) - \frac{\bar{\pi}}{r}}{w - \delta},$$

and

$$k(r) = \frac{\frac{\hat{\pi}}{r^2} - 1}{w - \delta}.$$

Since $\hat{\pi} = \hat{r}(\delta - \hat{r}) = \hat{r}^2 > r^2$, then $k(r) > 0$ for all $r \in [\underline{r}, \hat{r}]$, and thus $\hat{K}(r) = K(r)$ by construction. Therefore

$$\begin{aligned} E\left[r^2\right] &= \int_{\underline{r}}^{\hat{r}} r^2 dK(r) = \left[r^2 K(r)\right]_{\underline{r}}^{\hat{r}} - 2\int_{\underline{r}}^{\hat{r}} rK(r)\,dr \\ &= \hat{r}^2 - 2\int_{\underline{r}}^{\hat{r}} rK(r)\,dr, \end{aligned}$$

where we have used the fact that $K(\underline{r}) = 0$ and $K(\hat{r}) = 1$. Differentiating $E\left[r^2\right]$ with respect to w, we obtain

$$\frac{\partial E\left[r^2\right]}{\partial w} = -2\int_{\underline{r}}^{\hat{r}} r\frac{\partial K(r)}{\partial w} dr.$$

Since

$$\frac{\partial K(r)}{\partial w} = \frac{\hat{\pi} - r(\delta - r)}{r(w - \delta)^2} > 0,$$

it follows that $E\left[r^2\right]$ decreases in w and thus $E[\Delta W]$ increases in w.

Notice that the lower bound $\underline{r}$, as determined by (3), decreases in w. When $w = \delta$, then $\underline{r} = \hat{r}$ and $r_1 = r_2 = \hat{r}$, thus

$$\begin{aligned} \Delta W|_{w=\delta} &= \int_{\tau^*}^{\hat{\tau}} (\delta - s)\,ds - \int_{\hat{\tau}}^{\delta} (\delta - s)\,ds \\ &= \frac{(\delta - \tau^*)^2}{2} - (\delta - \hat{\tau})^2 \\ &= \frac{2}{9}\delta^2 - \frac{1}{4}\delta^2 < 0. \end{aligned}$$

Thus, $E[\Delta W] < 0$ when w is very close to δ.

On the other hand, since

$$E\left[r^2\right] = \int_{\underline{r}}^{\hat{r}} r^2\frac{\frac{\hat{\pi}}{r^2} - 1}{w - \delta} dr = \int_{\underline{r}}^{\hat{r}} \frac{\hat{\pi} - r^2}{w - \delta} dr,$$

then $E\left[r^2\right]$ converges to zero and thus $E[\Delta W] > 0$ as w goes to infinity. It thus follows that there exists some value $\hat{w} > \delta$ such that $E[\Delta W] > 0$ if and only if $w > \hat{w}$.

E Proof of Proposition 4

We show that the equilibrium margins converge to that under pure price competition, so cross-subsidizing arises in equilibrium for t sufficiently small. That is, there exists a value t^0 such that $r_1^B, r_2^A < 0$ for $t < t^0$, in which case each firm charges a price below-cost for its weak product.

To see this, consider the case where firms compete for one-stop shoppers, that is, $t \leq 2(v-\tau)$ such that $v - \frac{t}{2} \geq \tau$. We focus on the symmetric equilibrium with $v_1 = v_2 = v$, and denote by $r_1^A = r_2^B = \rho$ the margin for the strong product and by $r_2^A = r_1^B = \mu$ the margin of the weak product. Since $\lambda_1(x) = \lambda_2(1-x)$, then $D^{MSS} = 2\int_0^{1/2} F(\lambda_1(x))dx$ and $D_1^{OSS} = D_2^{OSS} = \int_0^{1/2} \left[F(v_1 - tx) - F(\lambda_1(x))\right] dx$. We can then rewrite (4) and (5) as

$$(\rho - \mu)\int_0^{1/2} f(\lambda_1(x))dx = 2\int_0^{1/2} F(\lambda_1(x))dx, \tag{6}$$

and

$$\begin{aligned}
&(\rho + \mu)\left[\frac{1}{2t}\left(F(v - \frac{t}{2}) - F(\tau)\right) + \int_0^{1/2} f(v - tx)dx\right] + \mu\int_0^{1/2} f(\lambda_1(x))dx \\
= &\int_0^{1/2} \left[F(v - tx) - F(\lambda_1(x))\right] dx.
\end{aligned}$$

Solving for μ from these two equations, we obtain

$$\begin{aligned}
&\mu\int_0^{1/2} f(\lambda_1(x))dx\left(F(v - \frac{t}{2}) - F(\tau) + 2\int_0^{1/2} tf(v - tx)dx + \int_0^{1/2} tf(\lambda_1(x))dx\right) \\
= &\left(\int_0^{1/2} \left[F(v - tx) - F(\lambda_1(x))\right] dx\right)\int_0^{1/2} tf(\lambda_1(x))dx \\
&-\int_0^{1/2} F(\lambda_1(x))dx\left[\left(F(v - \frac{t}{2}) - F(\tau)\right) + 2\int_0^{1/2} tf(v - tx)dx\right].
\end{aligned} \tag{7}$$

Denoting by

$$\begin{aligned}
H(t) \equiv &\left(\int_0^{1/2} \left[F(v - tx) - F(\lambda_1(x))\right] dx\right)\int_0^{1/2} tf(\lambda_1(x))dx \\
&-\int_0^{1/2} F(\lambda_1(x))dx\left[\left(F(v - \frac{t}{2}) - F(\tau)\right) + 2\int_0^{1/2} tf(v - tx)dx\right],
\end{aligned}$$

and

$$Q\,(t) \equiv \int_0^{1/2} f(\lambda_1(x))dx\left(F(v - \frac{t}{2}) - F(\tau) + 2\int_0^{1/2} tf(v - tx)dx + \int_0^{1/2} tf(\lambda_1(x))dx\right),$$

we can rewrite (7) as

$$\mu(t) = \frac{H(t)}{Q\,(t)}.$$

Using

$$\begin{aligned}
\int_0^{1/2} tf(\lambda_1(x))dx &= \int_0^{1/2} dF(\lambda_1(x)) = F(\tau) - F(\tau - \frac{t}{2}) \\
\int_0^{1/2} tf(v - tx)dx &= -\int_0^{1/2} dF(v - tx) = F(v) - F(v - \frac{t}{2}),
\end{aligned}$$

we obtain

$$\begin{aligned} H(t) &\equiv \int_0^{1/2} F(v-tx)dx\left(F(\tau)-F(\tau-\frac{t}{2})\right)-\int_0^{1/2} F(\lambda_1(x))dx\left(2F(v)-F(\tau-\frac{t}{2})-F(v-\frac{t}{2})\right) \\ &= \int_0^{1/2}\left[F(v-tx)\left(F(\tau)-F(\tau-\frac{t}{2})\right)-F(\lambda_1(x))\left(2F(v)-F(\tau-\frac{t}{2})-F(v-\frac{t}{2})\right)\right]dx, \end{aligned}$$

and

$$Q(t) \equiv \int_0^{1/2} f(\lambda_1(x))dx\left(2F(v)-F(\tau-\frac{t}{2})-F(v-\frac{t}{2})\right),$$

Since $Q(t) > 0$, it follows that $\mu < 0$ if and only if $H(t) < 0$. Notice that $\lambda_1(x) = \tau$ when $t = 0$, thus

$$\mu(0) = \frac{H(0)}{Q(0)} = -h(\tau).$$

It follows that there exists some $t_0 > 0$ such that $\mu(t) < 0$ for $t \leq t_0$. Substituting $\mu(0) = -h(\tau)$ into equation (6), we obtain $\rho(0) = h(\tau)$.

F Proof of Proposition 5

Assume that consumer shopping costs are bounded above, that is, $s \leq \overline{s}$. It is straightforward to see that the upper bound is irrelevant if $\overline{s} > w$. In this case the analysis in the baseline model goes through exactly here. Thus, consumers with shopping costs $s < \tau^*$ are multistop shoppers while those with $\tau^* \leq s \leq w(= v)$ are one-stop shoppers, and firms cross-subsidize their weak products.

If instead $\overline{s} < w$, however, the upper bound is relevant. It is straightforward to check that the first four claims in the proof of Lemma 1 still hold; that is, in any equilibrium, there exist active multistop shoppers who buy the strong products; in addition, if there are active one-stop shoppers, then $r_1 = r_2 = 0$. Intuitively, whether there are still some one-stop shoppers depends on if $\overline{s}$ is higher or lower than τ^*. Notice that τ^* is defined by $j(\tau^*) = \tau^* + 2h(\tau^*) = \delta$ and that $j(\cdot)$ is strictly increasing, then $\overline{s} > \tau^*$ if and only if $\overline{s} + 2h(\overline{s}) > \delta$.

We now proceed to prove the second part of the proposition.

Claim 5: When $\overline{s} + 2h(\overline{s}) \leq \delta$, then all active consumers are multistop shoppers.

Proof: Suppose there exist some one-stop shoppers, which requires $\tau < \min\{\max\{v_1, v_2\}, \overline{s}\}$. Competition for these *OSS* leads to $r_1 = r_2 = 0$, and thus $\tau_1 = \tau_2 = \delta - r_1^A - r_2^B < \overline{s}$, which implies $r_1^A + r_2^B > \delta - \overline{s} > 2h(\overline{s})$. Therefore, at least one firm's equilibrium margin for its strong product must exceed $h(\overline{s})$. Suppose $r_1^A > h(\overline{s})$; then $r_1^A > h(\overline{s}) > h(\tau)$, since $\overline{s} > \tau$ and $h(\cdot)$ is strictly increasing. Consider now the following deviation: decrease r_1^A to $\tilde{r}_1^A$ and increase r_1^B to $\tilde{r}_1^B$ such that the total margin $\tilde{r}_1 = \tilde{r}_1^A + \tilde{r}_1^B$ is the same as r_1. This does not affect the profit from one-stop shoppers (which remains equal to zero), but yields now a profit from multistop shoppers, equal to $\tilde{\pi}_1 = \tilde{r}_1^A F(\tilde{\tau})$, where $\tilde{\tau} = \delta - \tilde{r}_1^A - r_2^B$. Since $d\tilde{\pi}_1/d\tilde{r}_1^A|_{\tilde{r}_1^A = r_1^A} = -f(\tau)(r_1^A - h(\tau))$, which is strictly negative as $r_1^A > h(\tau)$, such deviation is profitable. Hence, all active consumers must be MSS. **Q.E.D.**

We now characterize the equilibria where all consumers are MSS. Consumers are willing to visit both firms only if $s \leq v_{12}/2$, and they prefer MSS to OSS_i if $s \leq \tau_i = v_{12} - v_i$. We show first that the equilibrium configuration must satisfy $v_{12}/2 \geq \overline{s}$. Suppose $v_{12}/2 < \overline{s}$, then it must be $\max\{v_1, v_2\} \leq v_{12}/2$ since all consumers are MSS. This holds because, if $v_1 > v_{12}/2$, say, then $\tau_1 = v_{12} - v_1 < v_{12}/2 < \min\{v_1, \overline{s}\}$, and consumers with $s \in (\tau_1, \min\{v_1, \overline{s}\})$ prefer one-stop shopping, which is a contradiction. Notice that $\tau_1 = v_{12} - v_1 \geq v_{12}/2$, so the demand for MSS is equal to $F(v_{12}/2)$ and firm 1 then earns positive profit $\pi_1 = r_1^A F(v_{12}/2)$. Suppose $(r_1^A,\ r_1^B)$ and $(r_2^A,\ r_2^B)$ form a candidate equilibrium with $r_2^B \geq r_1^A$ (one can show the case $r_2^B \leq r_1^A$ in a similar way). Consider the change of r_1^B such that $\tilde{v}_1 = v_{12}/2 + \varepsilon$, which implies $\tilde{r}_1^B = \frac{1}{2}(w^{AB} - \delta + r_2^B - r_1^A - 2\varepsilon) \geq \frac{1}{2}(w^{AB} - \delta - 2\varepsilon) > 0$. Such deviation transforms some MSS into OSS_1 (those with shopping cost such that $\tilde{\tau} = v_{12}/2 - \varepsilon < s < v_{12}/2$), on which firm 1 makes an extra profit by selling its weak product (since $\tilde{r}_1^B > 0$). In addition, firm 1 also sells the assortment to extra consumers at a positive margin $\tilde{r}_1^B + r_1^A$ (those with shopping cost such that $v_{12}/2 < s < \tilde{v}_1 = v_{12}/2 + \varepsilon$). Thus, such deviation increases firm 1's profit and the configuration with $v_{12}/2 < \overline{s}$ cannot be an equilibrium.

To ensure that all active consumers are MSS, it must be that $\overline{s} \leq \tau_1,\ \tau_2$, because if $\overline{s} > \tau_i = v_{12} - v_i (> 2\overline{s} - v_i)$ then $v_i > \overline{s} > \tau_i$ and consumers with $s \in [\tau_i, \overline{s}]$ would favor OSS_i. Conversely, $\overline{s} \leq \min\{\tau_1, \tau_2, v_{12}/2\}$ implies that all active consumers are MSS.

Thus, both firms sell their strong products to all consumers (who are multistop shoppers) and accordingly firms make profits $\pi_1 = r_1^A F(\overline{s})$ and $\pi_2 = r_2^B F(\overline{s})$ respectively. While it is tempting for both firms to raise the margins on their strong products as much as possible, excessively high margins cannot form an equilibrium since the rival will undercut. Intuitively, there exist multiple equilibria and the equilibrium margins must be bounded within a parameter region.

First of all, excessively high margins on weak products such that $r_2^A \geq w_2^A$ and $r_1^B \geq w_1^B$ cannot be an equilibrium. Suppose that one firm, say, firm 2, charges an "excessive" margin for its weak product: $r_2^A \geq w_2^A$. Notice that firm 1's margin on the strong product must be bounded above such that $r_1^A \leq w_1^A - \overline{s}$, as otherwise $v_1 \geq \overline{s}$ and no consumers will buy A from firm 1. Since $r_2^A \geq w_2^A$ implies $v_2^A \leq 0$, firm 2's margin on weak product places no competitive constraints on firm 1's strong product. Therefore firm 1 will charge $r_1^A = w_1^A - \overline{s}$ (and thus $v_1^A = \overline{s}$) and sell A to all consumers. But then firm 2 could undercut the rival by selling its weak product A at the margin (slightly less than) $\tilde{r}_2^A = r_1^A - \delta + \overline{s} = w_1^A - \delta = w_2^A > 0$. Applying the same logic shows that $r_1^B \geq w_1^B$ cannot hold in equilibrium.

Thus, it must be that $r_1^B < w_1^B$ and $r_2^A < w_2^A$. Notice that $\overline{s} \leq \tau_1 = \delta + r_1^B - r_2^B < \delta + w_1^B - r_2^B = w_2^B - r_2^B$, it then implies $r_2^B < w_2^B - \overline{s}$; likewise, $\overline{s} \leq \tau_1$ implies $r_1^A < w_1^A - \overline{s}$. It follows that $r_1^A + r_2^B < w_1^A + w_2^B - 2\overline{s}$, which amounts to $\overline{s} < v_{12}/2$. Therefore, the constraint $\overline{s} < v_{12}/2$ is implied by other two constraints $\overline{s} \leq \tau_1$ and $\overline{s} \leq \tau_2$ and is thus irrelevant here.

Since the only relevant constraints are $\overline{s} \leq \tau_1, \tau_2$, they must be binding in equilibrium. Suppose $\tau_1 = \delta + r_1^B - r_2^B > \overline{s}$, then firm 2 could increase its profit by raising its margin on the strong product r_2^B by ε such that $\tau_1 - \varepsilon \geq \overline{s}$, without losing any consumers. Similarly $\tau_2 > \overline{s}$ cannot hold in equilibrium. Hence, it must be that $\tau_1 = \tau_2 = \overline{s}$, which amounts to $r_1^A = \delta + r_2^A - \overline{s}$ and $r_2^B = \delta + r_1^B - \overline{s}$, and thus $r_1 = r_1^A + r_1^B = r_2 = r_2^A + r_2^B$.

To characterize the candidate equilibria, one must examine possible deviations. For firm 1, for instance, it could transform some MSS into OSS_1 by charging $\tilde{r}_1^A$ and $\tilde{r}_1^B$ such that $\tilde{r}_1 = \tilde{r}_1^A + \tilde{r}_1^B \leq r_2 = r_1$ and $\tilde{\tau}_1 = \delta + \tilde{r}_1^B - r_2^B < \tau_1 = \tau_2 = \overline{s}$. Obviously, given $\tilde{r}_1^B$ it is best to set $\tilde{r}_1^A = r_1 - \tilde{r}_1^B$, and firm 1 thus earns a profit equal to

$$\begin{aligned}\pi_1 &= \tilde{r}_1^A F(\tilde{\tau}_1) + r_1(F(\overline{s}) - F(\tilde{\tau}_1)) \\ &= r_1 F(\overline{s}) - \tilde{r}_1^B F\left(\delta + \tilde{r}_1^B - r_2^B\right).\end{aligned}$$

To rule out such deviation, r_1^B must satisfy

$$r_1^B = \arg\max_{\tilde{r}_1^B \leq r_1^B} -\tilde{r}_1^B F\left(\delta + \tilde{r}_1^B - r_2^B\right),$$

which, given the monotonicity of $h(\cdot)$, amounts to

$$r_1^B \leq -h(\overline{s}).$$

Alternatively, firm 1 could transform some MSS into OSS_2 by charging $\tilde{r}_1^A$ and $\tilde{r}_1^B$ such that $\tilde{r}_1 \geq r_2 = r_1$ and $\tilde{\tau}_2 = \delta + r_2^A - \tilde{r}_1^A < \tau_2 = \tau_1$, which yields a profit

$$\pi_1 = \tilde{r}_1^A F(\tilde{\tau}_2).$$

To form an equilibrium, r_1^A must therefore solve

$$\max_{\tilde{r}_1^A \leq r_1^A} \tilde{r}_1^A F\left(\delta + r_2^A - \tilde{r}_1^A\right),$$

which implies that the first-order derivative must be non-negative at $\tilde{r}_1^A = r_1^A$:

$$r_1^A \geq h(\overline{s}).$$

Using the same logic, we obtain $r_2^A \leq -h(\overline{s})$ and $r_2^B \geq h(\overline{s})$. The candidate equilibria must therefore satisfy $-h(\overline{s}) \geq r_1^B = r_2^B - \delta + \overline{s} \geq h(\overline{s}) - \delta + \overline{s}$, which implies $\overline{s} + 2h(\overline{s}) \leq \delta$. Conversely, when this condition holds, any margins such that r_1^A, $r_2^B \in [h(\overline{s}), \delta - \overline{s} - h(\overline{s})]$, $r_2^A = r_1^A - \delta + \overline{s}$ and $r_1^B = r_2^B - \delta + \overline{s}$ constitute an equilibrium, in which all consumers buy A from firm 1 and B from firm 2.

It is now easy to establish the first part of the proposition.

Claim 6: When $\overline{s} + 2h(\overline{s}) > \delta$, there must exist active one-stop shoppers.

Proof: Suppose there are only MSS (who buy the strong products by Claim 3). Then from the above characterization, the candidate equilibrium margins must satisfy $-h(\overline{s}) \geq r_1^B = r_2^B - \delta + \overline{s} \geq h(\overline{s}) - \delta + \overline{s}$, and thus $\overline{s} + 2h(\overline{s}) \leq \delta$, a contradiction. **Q.E.D.**

Therefore, when the upper bound is sufficiently high, two types of consumers coexist in equilibrium and the analysis of the baseline model again applies: consumers with $s \leq \tau$ are multistop shoppers while those with $\tau < s \leq \min\{\max\{v_1, v_2\}, \overline{s}\}$ are one-stop shoppers. Since the competition for one-stop shoppers leads to $r_1 = r_2 = 0$, firms only make profits from multistop shoppers: $\pi_1 = r_1^A F(\tau)$ and $\pi_2 = r_2^B F(\tau)$ respectively. Therefore, the equilibrium margins are given by $r_1^A = r_2^B = h(\tau^*)$ and $r_1^B = r_2^A = -h(\tau^*)$, where $\tau^* < \overline{s}$, and firms cross-subsidize their weak products.

G Proof of Proposition 6

Suppose that consumer shopping costs are bounded below, i.e., $s \geq \underline{s}$, with $\underline{s} < w$. We first establish the following claims.

Claim 1: There must exist some active consumers in equilibrium.

Proof: Suppose there are no active consumers. It must be the case that $\max\{v_1, v_2\} \leq \underline{s}$ and $\max\{v_{12}, \hat{v}_{12}\} \leq 2\underline{s}$. By deviating and charging $\tilde{r}_1^B > 0$ and $\tilde{r}_1^A > 0$ such that $\tilde{r}_1 = \tilde{r}_1^A + \tilde{r}_1^B = w - \underline{s} - \varepsilon$, firm 1 can attract some consumers (those with $s \leq \tilde{v}_1 = \underline{s} + \varepsilon$ are willing to patronize it) and make a profit since both margins are positive. This deviation is thus profitable. **Q.E.D.**

Claim 2: When there are active one-stop shoppers, then $r_1 = r_2 = 0$.

Proof: Suppose there are some active OSS.

(i) We show first that no firm charges $r_i < 0$, $i = 1, 2$. Suppose $r_1 < 0$, say; then:

- If $r_1 < r_2$ (thus $v_1 > v_2$), then firm 1 incurs a loss by attracting OSS. Consider firm 1's possible deviations in the following cases:

  - Suppose there are no active multistop shoppers, which must be the case that $\max\{v_{12}, \hat{v}_{12}\} - \underline{s} \le v_1$. Since firm 1 incurs a loss by serving OSS, it would find it profitable to increase r_1.
  - Suppose there are some active multistop shoppers who buy the strong products. This must be the case that $\max\{v_{12}, \hat{v}_{12}\} - \underline{s} = v_{12} - \underline{s} > v_1\,(\ge v_2)$, which implies $\tau = \tau_1 > \underline{s}$ and thus $\hat{\tau} < -\underline{s} \le 0$. If $r_1^A < 0$, firm 1 also makes a loss from MSS. Then firm 1 can reduce its loss by raising its margins on both products. If instead $r_1^A \ge 0$, then firm 1 could benefit from raising its margin on weak product only: keep r_1^A constant and charge $\tilde{r}_1^B = -r_1^A = r_1^B - r_1 > r_1^B$ such that $\tilde{r}_1 = 0$. This deviation avoids the loss from OSS, and moreover increases the demand from MSS since $\tilde{\tau}_1 = \delta + \tilde{r}_1^B - r_2^B > \tau_1$ and $\tilde{\tau}_2 = \tau_2$.
  - Suppose there are some active multistop shoppers who buy the weak products. It then must be $\max\{v_{12}, \hat{v}_{12}\} - \underline{s} = \hat{v}_{12} - \underline{s} > v_1 \ge v_2$, which implies $\hat{\tau} = \hat{\tau}_1 > \underline{s}$ and thus $\tau < -\underline{s} \le 0$. If $r_1^B < 0$, then firm 1 also incurs a loss by serving multistop shoppers. It can then reduce the loss by raising its margins on both products. If $r_1^B \ge 0$, then maintaining r_1^B while charging $\tilde{r}_1^A = -r_1^B$ (so that $\tilde{r}_1 = 0$) would avoid the loss from OSS, and also increase the demand from $\widehat{MSS}$ since $\hat{\tau}_1$ increases and $\hat{\tau}_2$ remains unchanged.

- If instead $r_1 \ge r_2$ (and thus $r_2 < 0$), then the same argument applies to any firm that attracts OSS (firm 2 if $r_1 > r_2$, and at least one of the firms if $r_1 = r_2$).

(ii) Secondly, we show that charging $r_i > 0$, $i = 1, 2$, by both firms cannot be an equilibrium. Suppose $r_1, r_2 > 0$; then:

- If any firm, say firm 1, charges a higher margin ($r_1 > r_2 > 0$ and $v_2 > v_1 > 0$), it faces no demand from OSS. Consider the profitable deviations in two cases:

  - Suppose there are no active multistop shoppers, i.e., $\max\{v_{12}, \hat{v}_{12}\} - \underline{s} \le v_2$, and thus all active consumers are one-stop shoppers. Firm 1 can then attract all one-stop shoppers by setting $\tilde{r}_1 = r_2 - \varepsilon > 0$ and makes positive profit. Notice that doing so does not transform one-stop shoppers into multistop shoppers since $\tilde{v}_1 = v_2 + \varepsilon > \max\{v_{12}, \hat{v}_{12}\}$.

– Suppose there are active multistop shoppers, which must be the case that $\max\{v_{12}, \hat{v}_{12}\} - \underline{s} > v_2 > v_1$. Consider firm 1's deviation as follows: keep the margin on the product purchased by multistop shoppers and reduce the margin on the other product such that $\tilde{r}_1 = r_2 - \varepsilon$. Doing so attracts all OSS at the cost of reducing slightly (by ε) the demand of multistop shoppers, which is obviously profitable when ε is small enough.

- If both firms charge the same margins ($r_1 = r_2 > 0$), then $v_2 = v_1$ and $\tau_1 = \tau_2$. In this case, any firm, say firm 1, who does not obtain more than half of the demand from OSS, can attract all OSS using the same deviations as above (for the case $r_1 > r_2$), and the gain from undercutting the rival offsets the loss from the slight reduction in demand from multistop shoppers (if there are any of them).

(iii) Finally, we show that no firm charges a positive margin in equilibrium. Suppose that $r_1 > r_2 = 0$, and thus firm 2 makes zero profit from OSS. Consider the following deviation of firm 2: keep the margin on the product picked by multistop shoppers and increase the margin of the other product by ε such that $\tilde{r}_2 = r_2 + \varepsilon (< r_1)$. It still supplies all OSS, but now makes a profit on them. Doing so also increases slightly the demand from multistop shoppers and thus the profit on them. For instance, when multistop shoppers buy the strong products, then the relevant threshold is $\tau = \tau_2 = \delta - r_1^A + r_2^A$ (since $r_1 > r_2$ implies $\tau_1 > \tau_2$), which increases to $\tilde{\tau}_2 = \delta - r_1^A + \tilde{r}_2^A = \tau_2 + \varepsilon$.

Summarizing the above analysis, we conclude that firms must charge $r_1 = r_2 = 0$ in any equilibrium where there are active one-stop shoppers. **Q.E.D.**

Claim 3: Active multistop shoppers must buy the strong products.

Proof: Suppose there are some active multistop shoppers who purchase the weak products. This must be the case that each firm offers a better net value on its weak product than that of the rival's strong product: $v_1^B - v_2^B = \left(w_1^B - r_1^B\right) - \left(w_2^B - r_2^B\right) > \underline{s}$ and $v_2^A - v_1^A = \left(w_2^A - r_2^A\right) - \left(w_1^A - r_1^A\right) > \underline{s}$. It follows that $r_2^B \geq r_1^B + w_2^B - w_1^B + \underline{s} \geq r_1^B + \delta + \underline{s}$ and $r_1^A \geq r_2^A + w_1^A - w_2^A + \underline{s} \geq r_2^A + \delta + \underline{s}$. We show that such configuration cannot be an equilibrium. Consider two cases:

- Suppose there are only multistop shoppers (who then buy the weak products). To make profits, firms must charge non-negative margins on their weak products, i.e., r_1^B, $r_2^A \geq 0$. This implies that $r_2^B \geq r_1^B + \delta + \underline{s} \geq \delta + \underline{s}$ and $r_1^A \geq r_2^A + \delta + \underline{s} \geq \delta + \underline{s}$, that is, each firm's margin on its strong product is higher than its comparative advantage δ plus $\underline{s}$. It is then profitable for some firm, say firm 1, to undercut the rival by charging $\tilde{r}_1^A = r_2^A + \delta + \underline{s} - \varepsilon > 0$ (keeping r_1^B unchanged). Doing so transforms $\widehat{MSS}$ into OSS_1: Consumers who originally buy A from firm 2 will instead buy both products from firm 1, since firm 1 now offers a better value on its strong product A (saving extra shopping cost for at least $\underline{s}$): $\tilde{v}_1^A = w_1^A - \tilde{r}_1^A = w_1^A - \delta - r_2^A - \underline{s} + \varepsilon = v_2^A - \underline{s} + \varepsilon$. The deviation may attract additional OSS, on which the firm also makes a profit since $\tilde{r}_1^A, r_1^B \geq 0$.

- Suppose there exist both OSS and $\widehat{MSS}$. Then price competition for one-stop shoppers leads to $r_1 = r_2 = 0$ by Claim 2, and thus firms make no profit from OSS. Therefore firms must charge non-negative margins on their weak products, i.e., r_1^B, $r_2^A \geq 0$, as otherwise they would incur a loss. But this implies that the margins on the strong products must be non-positive, say, $r_1^A = r_1 - r_1^B \leq 0$, which contradicts to the condition $r_1^A \geq r_2^A + \delta + \underline{s} \geq \delta + \underline{s} > 0$.

Therefore multistop shoppers must pick the strong products in equilibrium. **Q.E.D.**

Claim 4: There must exist active one-stop shoppers.

Proof: Suppose there are only multistop shoppers, then they buy the strong products by Claim 3. Notice that consumers are willing to visit both firms only if $2s \leq v_{12}$ (i.e., $s \leq v_{12}/2$) and they prefer MSS to OSS if $s \leq \tau = v_{12} - \max\{v_1, v_2\}$. Then, it must be the case that $v_{12}/2 \leq \tau = v_{12} - \max\{v_1, v_2\}$, that is, $\max\{v_1, v_2\} \leq v_{12}/2$. Suppose $v_{12}/2 > \tau$, which implies $\max\{v_1, v_2\} > v_{12}/2$, then consumers with shopping cost $s \in [\tau, v_{12}/2]$ prefer one-stop shopping since $s \leq v_{12}/2 < \max\{v_1, v_2\}$, which contradicts to the supposition that there are no one-stop shoppers. The demand from MSS is thus $F(v_{12}/2)$. Since multistop shoppers only buy strong products, firms must charge non-negative margins on the strong products in order to make positive profit. Suppose $r_2^B \geq r_1^A \geq 0$. Consider the following deviation for firm 1: keep r_1^A and change r_1^B to $\tilde{r}_1^B = \frac{1}{2}(w - \delta + r_2^B - r_1^A - 2\varepsilon) \geq \frac{1}{2}(w - \delta - 2\varepsilon) > 0$ so that $\tilde{v}_1 = w - r_1^A - \tilde{r}_1^B = \frac{v_{12}}{2} + \varepsilon$ and $\tilde{\tau}_1 = \delta + \tilde{r}_1^B - r_2^B = \frac{v_{12}}{2} - \varepsilon < \tilde{v}_1$. This adjustment transforms some MSS into OSS_1 (those whose shopping cost lies between $\tilde{\tau}_1$ and $\tilde{v}_1$), on which firm 1 acquires an extra profit from selling its weak product (since $\tilde{r}_1^B > 0$). The same logic applies to the case $r_1^A \geq r_2^B$, in which firm 2 can benefit from changing r_2^A in a similar manner. Thus, there must exist active one-stop shoppers in equilibrium. **Q.E.D.**

We now proceed to show the main results of the proposition.

Claim 5: If $\underline{s} < \delta/3$, there must exist active multistop shoppers in equilibrium.

Proof: Suppose all active consumers are OSS. Then competition for OSS leads to $r_1 = r_2 = 0$ by Claim 2, and thus $\tau_1 = \tau_2 = \delta - r_1^A - r_2^B$. In addition, it must be that $\tau, \hat{\tau} \leq \underline{s}$, which implies $r_1^A + r_2^B \leq \delta + \underline{s}$. Suppose that firm 2, say, is the one that charges less on its strong product, that is, $r_1^A \geq r_2^B$, and thus $r_2^B \leq \frac{\delta+\underline{s}}{2}$. Consider the following deviation by firm 1: charge $\tilde{r}_1^A = \varepsilon$ and $\tilde{r}_1^B = -\varepsilon$ such that the total margin keeps unchanged. Since $\delta > 3\underline{s}$, it follows that $\tilde{\tau} = \tilde{\tau}_1 = \tilde{\tau}_2 = \delta - \tilde{r}_1^A - r_2^B \geq \delta - \varepsilon - \frac{\delta+\underline{s}}{2} \geq \frac{\delta-\underline{s}}{2} - \varepsilon > \underline{s} - \varepsilon$, and $\tilde{\tau} > \underline{s}$ for sufficiently small ε sufficiently small. Thus firm 1 attracts some MSS and makes a profit on them. **Q.E.D.**

Therefore, the analysis in the baseline model goes through in the case of $\underline{s} < \delta/3$. Consumers with shopping cost such that $\underline{s} < s < \tau$ prefer multistop shopping while those with $\tau < s < \max\{v_1, v_2\}$ become one-stop shoppers. The equilibrium is characterized exactly as in the baseline model. This establishes the first part of the proposition.

Claim 6: If $\underline{s} > \delta$, there are no multistop shoppers in equilibrium.

Proof: Suppose there are some multistop shoppers, then multistop shoppers must buy the strong products by Claim 3. This must be the case that $\tau = \min\{\tau_1, \tau_2\} > \underline{s}$ and consumers such that $\underline{s} < s < \tau$ are MSS. There must be also some OSS by Claim 4, and the competition for these consumers leads to $r_1 = r_2 = 0$. It follows that $\tau = \delta - r_1^A - r_2^B > \underline{s} > \delta$, which implies $r_1^A + r_2^B < 0$. Thus, at least one firm, say, firm 1, must charge a negative margin on its strong product and this firm must incur a loss from serving MSS. This cannot be an equilibrium since firm 1 would then increase r_1^A to avoid the loss. **Q.E.D.**

We now characterize the equilibrium margins. Notice that the competition for one-stop shoppers leads to $r_1 = r_2 = 0$, and thus $\tau_1 = \tau_2 = \tau = \delta - r_1^A - r_2^B$. Consider possible deviations by firm 1:

- (i) To attract some MSS, it must charge $\tilde{r}_1^A$ such that $\tilde{\tau}_2 = \delta - \tilde{r}_1^A + r_2^A = \delta - \tilde{r}_1^A - r_2^B > \underline{s}$, which implies $\tilde{r}_1^A < \delta - \underline{s} - r_2^B$. Ruling out such deviation then requires $r_2^B \geq \delta - \underline{s}$.
- (ii) Alternatively, firm 1 could attract some $\widehat{MSS}$ by charging $\tilde{r}_1^B$ such that $\tilde{\tilde{\tau}}_2 = -\delta + r_2^B - \tilde{r}_1^B > \underline{s}$, which implies $\tilde{r}_1^B < r_2^B - \delta - \underline{s}$. Preventing such deviation then requires $r_2^B \leq \delta + \underline{s}$.

The same logic applies to firm 2's possible deviations. In addition, the margins on strong products, r_1^A and r_2^B, should not exceed their social values, w_1^A and w_2^B, respectively, and the margins on the weak products, $r_1^B = -r_1^A$ and $r_2^A = -r_2^B$, should not exceed w_1^B and w_2^A, respectively.

Conversely, any margins that satisfy (i) $r_1^A + r_1^B = r_2^A + r_2^B = 0$, (ii) $\delta - \underline{s} \leq r_1^A, r_2^B, r_1^A + r_2^B \leq \delta + \underline{s}$, and (iii) $-w_1^B \leq r_1^A \leq w_1^A$ and $-w_2^A \leq r_2^B \leq w_2^B$ constitute an equilibrium, in which all active consumers are OSS and both firms earn zero profits. This establishes the second part of the proposition.

Finally, we show the last part of the proposition.

Claim 7: If $\delta/3 < \underline{s} < \delta$, there are two types of equilibria, in which either all active consumers are OSS or both types of shoppers coexist.

Proof: Suppose $\underline{s} < \delta$. If there exists an equilibrium where all active consumers are OSS, then from the above analysis equilibrium margins must satisfy $\delta - \underline{s} \leq r_1^A, r_2^B$, which implies $r_1^A + r_2^B \geq 2\delta - 2\underline{s}$. Moreover, from the above characterization the equilibrium margins must satisfy $r_1^A + r_2^B \leq \delta + \underline{s}$. Combining these two constraints lead to $2\delta - 2\underline{s} \leq \delta + \underline{s}$, that is $\delta/3 < \underline{s}$, and thus such equilibrium exists only if $\delta/3 < \underline{s} < \delta$.

Conversely, when $\delta/3 < \underline{s} < \delta$, any margins satisfying $r_1 = r_2 = 0$ and $\delta - \underline{s} \leq r_1^A, r_2^B, r_1^A + r_2^B \leq \delta + \underline{s}$ constitute an equilibrium where all active consumers are OSS. Any firm, say firm 1, cannot profitably deviate by attracting MSS, as it would have to charge $\tilde{r}_1^A < \delta - \underline{s} - r_2^B \leq 0$. Moreover, it does not pay for attracting $\widehat{MSS}$, as it would have to charge $\tilde{r}_1^B < r_2^B - \delta - \underline{s} \leq 0$.

Consider now the candidate equilibrium with both OSS and MSS. In such candidate equilibrium consumers with shopping cost such that $\underline{s} < s < \tau = \delta - r_1^A - r_2^B$ are multistop shoppers and those with $\tau < s < w$ are one-stop shoppers. Applying the same approach as in the baseline model, the equilibrium margins are then characterized by $r_1^A = r_2^B = -r_2^A = -r_1^B = h(\tau^*)$, where τ^* satisfies $j(\tau^*) = \tau^* + 2h(\tau^*) = \delta$. Notice that $j(\cdot)$ is strictly increasing and that $j(\underline{s}) = \underline{s} + 2h(\underline{s}) = \underline{s}$, thus $\underline{s} < \tau^*$ if and only if $\underline{s} < \delta$. It follows that such candidate equilibrium does exist when $\underline{s} < \delta$. Conversely, these margins constitute indeed an equilibrium. By construction, no firm can make a profit on OSS due to fierce price competition, and the profit from MSS that are given by $r_1^A F(\tau)$ for firm 1 and $r_2^B F(\tau)$ for firm 2 are maximized. **Q.E.D.**

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