

The Telecommunications Industry and Economic Growth: How the Market Structure Matters*

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Abstract

This paper presents an endogenous growth model where, in line with the recent empirical evidence, the telecommunications industry (telecom) is an engine of growth. In such a framework, this paper analyzes the channels through which telecom contributes to economic growth and focuses on market structure analysis for telecom, in the light of the recent changes in it. This paper suggests how the market structure of telecom and the competition type in the telecom market can matter for its contribution to economic growth. It also proposes the optimal market structure for telecom from the social welfare perspective. In addition, it suggests the direction of telecom policies which can improve social welfare, and uses its theoretical results for qualitative evaluation of the Telecommunications Act of 1996 and similar policies.

Keywords: Telecommunications industry; Market structure; Economic growth; Policy evaluation

JEL classification: O41; O25; O38; L10

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1 Introduction

A vast empirical literature suggests that the telecommunications industry (telecom) makes a significant contribution to economic growth (e.g., Roller & Waverman, 2001).¹ The infrastructure investments in telecom seem to be the most highlighted driver of that contribution. According to economic theory, these investments can lead to economic growth in various ways. Most intuitively, these investments, while expanding the telecom networks, can increase the availability of telecom products (e.g., wireless and landline services) and motivate higher demand. In addition, according to the conjectures of network economics literature, these investments, while motivating higher demand, can amplify the network externalities. This can increase, for instance, the efficiency of firms in the economy and lead to economic growth (see, for instance, Hardy 1980; Leff, 1984).²

There are also numerous studies which suggest that the market structure of an industry can affect its contribution to economic growth (e.g., van de Klundert & Smulders, 1997; Aghion, Bloom, Blundell, Griffith, & Howitt, 2005). According to these studies, this can happen, for instance, when the products of firms in the industry are imperfect substitutes and the firms have market power. Under such condition, the market structure can determine the R&D effort in the industry. It can also determine the inefficiencies stemming from the market power of the firms. These inefficiencies can alter the demand for the goods produced in that industry, which also can affect its contribution to economic growth.

This type of inefficiencies have motivated, for instance, the Telecommunications Act of 1996 which proposes, and has initiated already, changes in the market structure of telecom in the US (see also the directives 90/388/EEC and 96/19/EEC, in the EU). The policies such as the Telecommunications Act of 1996 aim also at promoting the demand for telecom goods (for the EU see directive 2002/22/EC) and innovation in telecom (for the EU see the directives 2002/22/EC and 2002/58/EC). Policy makers motivate the promotion of the demand, for instance, by the external benefits from the use of telecom goods.

In a general equilibrium framework, this paper analyzes the channels through which telecom contributes to economic growth. Its main focus is on balanced growth

¹See also Jorgenson, Ho, & Stiroh (2005) for a growth accounting exercise in the information and communication technologies sector, which includes telecom.

²It has to be acknowledged that the existence of such externalities, although seems to be intuitive, has limited empirical support. There are studies which seem to provide empirical support for it (e.g., Roller & Waverman, 2001); however, there are also studies which do not find any support (e.g., Stiroh, 2002).

path analysis.³ A seemingly important channel, which is analyzed here and tends to be overlooked in the aggregate level studies related to telecom (e.g., Leff, 1984; Roller & Waverman, 2001; Koutrompis, 2009), is the innovation in telecom which increases the productivity in telecom goods production (e.g., digitalization of telecommunication networks).⁴ Another contribution of this paper is that it allows the telecom firms (e.g., AT&T, in the US) to engage in R&D partnerships and cross-licensing activities. The significance of such partnerships and activities is largely documented for the telecom and other high-tech industries (see, for instance, Hagedoorn, 1993; 2002).⁵ According to empirical studies, it can significantly amplify the innovation in such industries (see, for instance, Belderbos, Carree, & Lokshin, 2004).

In the same framework, this paper focuses on market structure analysis for telecom, while assuming that the products of telecom firms are imperfect substitutes and that these firms have market power. It suggests how the market structure of telecom affects its contribution to economic growth. Given that the market structure matters, the competition type in the telecom market (i.e., Cournot or Bertrand) can also play a role. Therefore, in addition, this paper suggests a link between economic growth and the competition type in the telecom market. In addition, this paper derives the optimal market structure of telecom from the social welfare perspective, which seems to be an open question in the literature related to telecom (see Roller & Waverman, 2001). It suggests also the direction of telecom policies, which can improve the social welfare. In this regard, this paper uses its theoretical results in order to evaluate the implications of the Telecommunications Act of 1996 and similar policies.

In line with the network economics literature, this paper incorporates two types of network externalities (see, for instance, Gandalf, 1995). The first type is the indirect network externalities which stem from the existence of different types of telecom goods, given that a user of a telecom good can access other telecom goods also. These externalities increase the utility of the user with the number of telecom goods.⁶ The second type is the direct network externalities. In the literature, these externalities

³The balanced growth path analysis seems reasonable at least for several OECD countries where there were no significant labor force reallocations between the telecom and the rest of the economy. See Appendix DA.1.

⁴See Valletti & Cambini (2005) and Laffont, Rey, & Tirole (1998) for static microeconomic models of telecom which incorporate quality improvement. See also Bourreau & Dogan (2001) for a discussion of a relationship between telecom regulation and innovation in that industry.

⁵The telecom firms' final outputs are, for instance, telephone calls and the internet. Although part of the innovation/R&D for telecom may not take place in telecom *per se*, in this paper the R&D process is modeled within telecom firms and the cross-licensing activities are modeled across these firms. As long as the innovation is paid its fair price, these assumptions do not drive the results of this paper.

⁶For instance, the increase in the number of telecom goods can create new ways of using the same telecom good and can increase the number of complementary products (e.g., technical support offices).

are those that increase the value of using a telecom good with the number of users. In light of productivity improvements in telecom goods production, however, this paper replaces the number of users by the number of *effective* users, which seems to be novel at least in the aggregate level studies related to telecom.⁷ The intuition for such replacement is as follows: both the number of users and, for instance, the fault rate of lines, can affect the direct network externalities.⁸

The theoretical results of this paper suggest that the entry of telecom firms is one of the channels through which the telecom contributes to economic growth. This channel operates through the indirect network externalities. The other channels include the direct network externalities and the productivity improvements in telecom. The latter seems to need no explanation and, in line with empirical studies, is amplified by cross-licensing activities. The former adds to economic growth when the number of effective users of the telecom goods grows.

Regarding the market structure, the results suggest that in decentralized equilibrium, depending on the economy, the entry into telecom either stops after some number of firms have entered or it continues forever. In the first case, the number of firms in the economy will be always finite, while in the second case it grows permanently, resulting in monopolistic competition in telecom in the long run. The driver of this result are the investments in innovation for productivity improvement, which are fixed costs. The entry of firms erodes the revenues per firm, and these costs can be so high that the new entrant to telecom would have negative profits. Although, the first case seems to be more plausible, it seems hard to rule out the second case. In turn, according to the results, in the social optimum (Social Planner's optimal choice) there is permanent entry since there are no market incentives.

This paper also shows that in decentralized equilibrium the market structure of the telecom matters for the social welfare due to imperfect competition in the telecom market, which creates a resource misallocation compared with the socially optimal one. In case of a finite number of firms, the imperfect substitutability of telecom goods implies an additional link between this misallocation and the market structure of telecom through strategic interactions between telecom firms in the product market. When the market structure changes, the strategic interactions also change, which implies resource reallocation in the economy. Therefore, it implies that the market structure of telecom matters for economic growth. The same applies to the

⁷To my best knowledge this is novel also for micro level studies related to telecom.

⁸The direct network externalities are not endogenously generated and their presence is not driving the main results of this paper. It only suggests one of the channels through which the telecom contributes to economic growth and a reason to observe divergence between the social optimum and the decentralized equilibrium. This divergence seems to be a common observation amongst the studies which analyze network externalities.

competition type in the telecom market.

The results also suggest that the policies, which can improve the social welfare in a decentralized equilibrium, particularly promote entry into telecom and higher demand for telecom goods. These results support the Telecommunications Act of 1996. However, in addition, the results suggest that such policies would not deliver the highest social welfare. This is because in the decentralized equilibrium the telecom firms have an incentive to under-invest in productivity improvement since that erodes their profit margins. In addition, the entry of firms has a negative effect on the investments in productivity improvement since the returns decline with it. The policies which take into account these two issues, while promoting investments in productivity improvement through subsidies in addition to promoting demand for telecom goods and encouraging entry, can deliver higher social welfare. It seems that this aspect has not been considered in the policies implemented thus far.

There are also policies which promote interconnectedness of networks and mandate number portability (e.g., the Telecommunications Act of 1996). Arguably, such policies can increase the substitutability between telecom goods of different producers. The results of this paper suggest that such policies are not in line with the social optimum. In the social optimum, the welfare decreases *ceteris paribus* with the substitutability. However, depending on the economy these policies may be relevant if the objective is to maximize the GDP growth rate. In the decentralized equilibrium, the GDP growth rate can increase with substitutability since it implies more intensive imperfect/monopolistic competition.

In order to highlight the contribution of telecom to economic growth and examine how that contribution is affected by its market structure this paper models telecom as the growth driving industry. It assumes that the telecom firms are large and long lasting firms which have significant entry costs and models the entry condition to telecom. This condition is needed in order to endogenize the market structure of telecom. In light of the changes in the market structure, modeling the entry decision is important so as to explain the factors that matter in this decision and to explain how entry can affect economic growth. In addition, modeling the entry is important for policy recommendations and evaluation and explains how the telecom market reacts to the entry of a new firm. This paper also models trade of production instructions/patents for the telecom goods (I use the terms "production instructions" and "knowledge" interchangeably). This trade stands for the R&D partnerships and cross-licensing activities.

A general equilibrium framework is used in this paper since that allows explicit accounting for the channels through which telecom can affect the aggregate performance. Moreover, such a framework can be preferred to partial equilibrium frame-

works since it does not fix prices and has a more accurate definition of social welfare. Therefore, it can deliver more accurate inference and policy suggestions for social welfare improvements. Finally, this paper maintains assumptions that guarantee the existence of a balanced growth path.

This paper is related to the endogenous growth literature (e.g., Romer, 1990; Aghion & Howitt, 1992; Jones, 1995) where the positive growth of the economy on a balanced growth path is a result of technological and preference factors. Given that the products of telecom firms can be considered as general purpose technologies (GPT), this paper is also related to the GPT literature (see, for instance, Helpman, 1998, for a collection of essays). Moreover, it is related to the studies which in an endogenous growth framework suggest how the aggregate performance can be affected by imperfect competition in an industry where the firms engage in intra-firm research for productivity improvement (e.g., van de Klundert & Smulders, 1997). It contributes to these streams of studies while showing how the trade of production instructions in such industry can affect the aggregate performance. It also contributes by showing how the market structure and the entry of firms can affect the intra-firm productivity improvement process when there is a trade of production instructions. In addition, it contributes while showing how the direct network externalities can affect economic growth.

Methodologically, this paper is related also to the multi-sector growth literature (e.g., Ngai & Pissarides, 2007; Acemoglu & Guerrieri, 2008), which analyzes the sources and the implications of sectorial growth differences. Primarily it is related to Vourvachaki (2009), which analyzes the impact of information and communication technologies (ICT) on aggregate performance while focusing on inter-sector interactions. In contrast, this paper focuses on the inter-firm interactions and intra-firm productivity improvement process in telecom. The contributions to this literature are the same as those to the endogenous growth theory.

This paper is also closely related to the literature which suggests a positive impact of telecom on the aggregate economy and analyzes/suggests policies for telecom (e.g., Hardy, 1980; Leff, 1984; Madden & Savage, 1998; Economides, 1999; Rolller & Waverman, 2001). It contributes to this literature in several ways. It analyzes the channels through which telecom contributes to economic growth and suggests how the market structure of telecom and the competition type in the telecom market can affect that contribution. It also suggests the market structure of telecom that is socially optimal and policies that can improve welfare in a decentralized equilibrium. Moreover, it evaluates several long run implications of previously implemented policies.

The model presented in this paper is a general endogenous growth model (for

similar models see Romer, 1990, and van de Klundert & Smulders, 1997). Though in this paper the model is specifically applied to the telecommunication industry, for reasons that will become apparent once the model is presented, it can have other applications as well. The only part of the model that can be hard to justify for other industries is the externalities associated with the use of telecom goods. For non-high-tech industries it can be also hard to justify the intra-firm productivity improvement process and the trade of knowledge.

The next section presents the model, defines the decentralized equilibrium and offers the optimal rules. Section 3 analyzes the features of dynamic equilibrium while focusing on a balanced growth path. Section 3 also offers the socially optimal allocations, compares these with the decentralized equilibrium allocations and suggests some comparative statics. Section 4 offers policy suggestions and analysis. Section 5 concludes. The Appendix offers the derivations of growth rates on the balanced growth path and proves the propositions in the text.

2 The model

2.1 Household side

The economy is populated by a continuum of identical and infinitely lived households of mass one. The representative household is endowed with a fixed amount of labor (L). It inelastically supplies the labor to firms which produce a homogenous final good and to telecom firms. The household has a CRRA utility function with an intertemporal substitution parameter θ and discounts the future streams of utility with rate ρ ($\theta, \rho > 0$). The utility gains are from the consumption of C amount of final good, which is the numeraire. The lifetime utility of the household is

$$U = \int_0^{\infty} \frac{C_t^{1-\theta}}{1-\theta} e^{-\rho t} dt. \quad (1)$$

The household finances its expenses through labor income wL and through returns r on its asset holdings A . The household's expenses include the consumption expenditures and the accumulation of assets \dot{A} :

$$\dot{A} = rA + wL - C. \quad (2)$$

2.2 Production side

There are two production sectors which produce the final good and the telecom goods.

2.2.1 Final good production

The household's demand for the final good is served by a representative producer. The production of the final good requires labor L_Y and X , which is a Dixit-Stiglitz composite of the telecom goods x_j , $j = 1, \dots, N$, with an elasticity of substitution ε . The positive indirect network externalities, which are associated with expansion of the telecom good variety, are represented by the "love of variety" in X . In turn, *ceteris paribus* the increasing demand of X creates positive direct network externalities in final good production, which are measured by \tilde{X} and have a scale μ . These externalities increase the productivity of the final good producers.⁹ The production of the final good has a Cobb-Douglas technology and is given by

$$Y = \tilde{X}^\mu X^\sigma L_Y^{1-\sigma},^{10} \quad (3)$$

$$X = \left(\sum_{j=1}^N x_j^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \varepsilon > 1,^{11} \quad (4)$$

$$\begin{aligned} \mu > 0, 0 < \sigma < 1, \\ \text{in equilibrium } \tilde{X} = X. \end{aligned} \quad (5)$$

The representative final good producer's problem can be divided into two stages. In the first stage, the producer decides the optimal amounts of (demand for) the telecom goods x in X , subject to

$$P_X X = \sum_{j=1}^N p_{x_j} x_j, \quad (6)$$

where P_X and p_{x_j} are the prices of X and x_j , respectively. In the second stage, the producer decides the optimal combination of L_Y and X for the production of final good Y .

⁹According to Hardy (1980) and Leff (1984), these externalities can increase the productivity of telecom good users, for instance, through the increased capabilities for search and communication over distances.

¹⁰The way the direct network externalities are modeled here is equivalent to assuming that the value of using a telecom good increases with the volume of use of others. If interpreted so, this way of modeling can be treated as a simplification for the sake of tractability.

¹¹This parameter restriction is a necessary condition for imperfect/monopolistic competition in the telecom market.

2.2.2 Telecom goods production side

At any time t there are $N(t)$ producers in the telecom market. At the same time, a potential producer decides to enter the market. If it enters, it starts producing its distinct type of telecom good.

Since each producer produces a unique good, the number of telecom firms is equal to the number of telecom goods.

2.2.2.1 Firm entry

In order to enter the telecom market and to generate its distinct type of telecom good, the potential producer has to invest. It should borrow the resources for the investment from the household with interest rate r .

This investment generates, for instance, the physical capital/infrastructure of the entrant. It is in terms of the final good and has its productivity η . The creation of the distinct type of telecom good is given by

$$\dot{N} = \eta S, \quad \eta > 0, \quad (7)$$

where \dot{N} is the new telecom good created by the investment S .

2.2.2.2 Telecom goods production

After its entry, the telecom good producer stays in the market forever and discounts the future profit streams π with the interest rate r . The production of any telecom good x_j requires labor input L_{x_j} and has productivity λ_j ,

$$x_j = \lambda_j L_{x_j}.^{12} \quad (8)$$

In order to support a symmetric equilibrium, it is assumed that each and every telecom firm enters the market with the highest productivity available at that date.

The telecom firm can continuously improve its productivity by hiring labor L_{r_j} . The labor force employed in the productivity improvement process uses the previous production instructions of the firm in order to create a better one. Moreover, if the firm decides to buy production instructions from other firms ($u_{i,j}\lambda_i; \forall i \neq j, u_{i,j}$

¹²Olley & Pakes (1996) have a similar structure for the telecommunication equipment producing industry. However, they take productivity as an exogenous state and model capital accumulation at the firm level.

The model presented here can incorporate capital accumulation in incumbent telecom firms in addition to the (capital) investments of entrants. That would be modeled similarly to the entry rule (7) and would particularly replace the infrastructure investments of incumbent telecom firms. However, that would make the analytical results more cumbersome without changing the qualitative results of the model.

is the share of λ_i that would go to the j th firm), the labor force can use a set of composite production instructions for the same purpose. The composite production instructions are a Cobb-Douglas combination of the firm's production instructions with the ones of other firms.¹³ The only essential knowledge input in the productivity improvement process is the one of the firm. The productivity improvement process has an exogenous efficiency level ξ and is given by

$$\begin{aligned} \dot{\lambda}_j &= \xi \left[\sum_{i=1}^N (u_{i,j} \lambda_i)^\alpha \right] \lambda_j^{1-\alpha} L_{r_j}, \\ u_{j,j} &\equiv 1, \xi > 0, 0 < \alpha < 1. \end{aligned} \quad (9)$$

The revenues of the telecom firm are gathered from its supply of telecom good and production instructions ($u_{j,i} \lambda_j; \forall j \neq i$). The costs are the labor compensations and its demand of production instructions. The telecom firm maximizes the present discounted value V of its profit streams. Formally,

$$\begin{aligned} \max_{\substack{\text{Cournot: } L_{x_j}, L_{r_j}, \{u_{j,i}, u_{i,j}\}_{i=1; (i \neq j)}^N \\ \text{Bertrand: } p_{x_j}, L_{r_j}, \{u_{j,i}, u_{i,j}\}_{i=1; (i \neq j)}^N}} V_j(t) &= \int_t^\infty \pi_j(\tilde{t}) \exp \left[-\int_t^{\tilde{t}} r(s) ds \right] d\tilde{t} \\ \text{s.t.} \\ \pi_j &= p_{x_j} x_j + \sum_{i=1, i \neq j}^N p_{u_{j,i} \lambda_j} (u_{j,i} \lambda_j) \\ &\quad - (L_{x_j} + L_{r_j}) w - \sum_{i=1, i \neq j}^N p_{u_{i,j} \lambda_i} (u_{i,j} \lambda_i), \end{aligned} \quad (10)$$

(8), (9),

where t is the entry date, $p_{u_{j,i} \lambda_j}$ and $p_{u_{i,j} \lambda_i}$ are the prices of $u_{j,i} \lambda_j$ and $u_{i,j} \lambda_i$, correspondingly. Under Cournot competition, the telecom firm chooses the supplied quantity of telecom good (i.e., L_{x_j}), given the inverse demand function of its product. In contrast, under Bertrand competition the firm chooses the price of the supplied telecom good (i.e., p_{x_j}) given the demand function of its product. Moreover, all the variables in the profit equation (10) are time dependant. Here and wherever it is relevant in the rest of the text, I have suppressed the time dependence of the variables for the ease of exposition.

Given that telecom firms set prices in the output market, it seems natural to assume that as a seller of production instructions these firms have a right to impose a take-it or leave-it offer. This assumption is maintained in the rest of the text. It implies that the price of production instructions is equal to the buyer's marginal

¹³This assumption ensures a balanced growth path.

valuation. A less plausible assumption would be that the buyer has the right to make a take-it or leave-it offer. Under this assumption, the buyer receives the knowledge at no cost, i.e., there are knowledge externalities. The difference between these assumptions is not crucial for the main results of this paper.

2.3 Definition of equilibrium

The decentralized equilibrium in this model is the paths of the quantities

$$\left\{ C, A, L, Y, X, L_Y, S, \{x_j, L_{x_j}, L_{r_j}\}_{j=1}^N, \{u_{i,j}\lambda_i\}_{i,j=1;(i \neq j)}^N, \{\lambda_j\}_{j=1}^N \right\}$$

and the corresponding prices

$$\left\{ 1, r, w, 1, P_X, w, 1, \{p_{x_j}, w, w\}_{j=1}^N, \{p_{u_{i,j}\lambda_i}\}_{i,j=1;(i \neq j)}^N, \{p_{\lambda_j}\}_{j=1}^N \right\} \text{ such that:}$$

1. Given the prices,
 - a. and the value of A , the household chooses the quantities $\{C, L\}$ to maximize its utility;
 - b. the final good producer chooses the quantities $\left\{ X, L_Y, \{x_j\}_{j=1}^N \right\}$ to maximize its profit; and
 - c. new firms invest $\{S\}$ to enter the telecom market;
2. Given the demand for the telecom good $\{x_j\}$ from the final good producer, the competition type in the telecom market (Cournot or Bertrand), and the value of λ_j , any j th telecom firm chooses quantities $\left\{ L_{x_j}, L_{r_j}, \{u_{j,i}, u_{i,j}\}_{i=1;(i \neq j)}^N \right\}$ or price and quantities $\left\{ p_{x_j}, L_{r_j}, \{u_{j,i}, u_{i,j}\}_{i=1;(i \neq j)}^N \right\}$ to maximize its value;
3. Each telecom firm owns its unique type of good;
4. The initial conditions are given; and
5. All markets clear.

2.4 Equilibrium conditions

In this section I summarize the equilibrium conditions. First, I present the optimal rules. Next, I present the market clearing conditions and other equilibrium rules which need to be highlighted.

2.4.1 Optimal rules

2.4.1.1 Household and final good producer

$$\frac{\dot{C}}{C} = \frac{1}{\theta}(r - \rho), \quad (11)$$

$$wL_Y = (1 - \sigma)Y, \quad (12)$$

$$P_X X = \sigma Y, \quad (13)$$

$$x_j = X \left(\frac{P_X}{p_{x_j}} \right)^\varepsilon. \quad (14)$$

The first equation is the standard Euler equation which follows from household's optimization problem. The rest of the equations are final good producer's optimal rules. The equations (12), (13), and (14) are final good producer's labor demand, demand for telecom goods bundle, and demand for any particular telecom good, correspondingly.

2.4.1.2 Telecom goods production side

I present the optimal rules of the j th telecom firm; for the rest, the rules are the same.

$$w = \lambda_j p_{x_j} \left(1 - \frac{1}{e_j} \right), \quad (15)$$

$$w = q_{\lambda_j} \frac{\dot{\lambda}_j}{L_{r_j}}, \quad (16)$$

$$u_{j,i} = 1, \quad \forall i \neq j, \quad (17)$$

$$p_{u_{i,j}\lambda_i} = q_{\lambda_j} \xi \alpha \left(\frac{\lambda_j}{u_{i,j}\lambda_i} \right)^{1-\alpha} L_{r_j}, \quad \forall i \neq j, \quad (18)$$

$$\begin{aligned} \frac{\dot{q}_{\lambda_j}}{q_{\lambda_j}} = r_\tau - \left[(N-1) \frac{p_{u_{j,i}\lambda_j}}{q_{\lambda_j}} + \xi L_{r_j} \right. \\ \left. + \xi L_{r_j} (1-\alpha) \sum_{i=1, i \neq j}^N \left(\frac{u_{i,j}\lambda_i}{\lambda_j} \right)^\alpha + \left(1 - \frac{1}{e_j} \right) \frac{p_{x_j} L_{x_j}}{q_{\lambda_j}} \right], \end{aligned} \quad (19)$$

where q_{λ_j} is the shadow value of productivity improvement and e_j is the elasticity of substitution between telecom goods perceived by the telecom firm. The first equation is the labor demand for telecom good production, or the supply of the telecom good. The second equation is the investment in productivity improvement (i.e., wL_{r_j}). The third equation is the supply of knowledge. The fourth equation is the demand for the knowledge stock of any other telecom good producer. The last equation is the internal rate of return on productivity improvement.

The perceived elasticity of substitution (e_j) varies with competition type. Under Bertrand competition

$$e_j \equiv e_j^B = \varepsilon - \left[\frac{(\varepsilon - 1) p_{x_j}^{1-\varepsilon}}{\sum_{i=1}^N p_{x_i}^{1-\varepsilon}} \right], \quad (20)$$

and under Cournot competition

$$e_j \equiv e_j^C = \varepsilon \left\{ 1 + \left[(\varepsilon - 1) \frac{x_j^{\frac{\varepsilon-1}{\varepsilon}}}{\sum_{i=1}^N x_i^{\frac{\varepsilon-1}{\varepsilon}}} \right] \right\}^{-1}, \quad (21)$$

(see Vavra, 2002).¹⁴ The terms in square brackets measure the impact of other telecom firms on the demand of the j th telecom firm. In other words, they measure the extent of strategic interactions between telecom firms. Moreover, these terms indicate the difference between the perceived elasticity of substitution (e) and the actual elasticity of substitution (ε). Therefore, they indicate some of the distortions in the economy which stem from monopolistic competition with a finite number of firms. In a symmetric equilibrium, when the number of firms increases, these distortions tend to zero since the terms in square brackets tend to zero.

2.4.2 Market clearing conditions

The market clearing conditions that are worth highlighting are the following

$$\text{Final good} : \quad Y = C + S, \quad (22)$$

$$\begin{aligned} \text{Production instructions} : \quad & \sum_{j=1}^N \sum_{i=1, i \neq j}^N p_{u_j, i \lambda_j} (u_{j, i} \lambda_j) \\ & = \sum_{j=1}^N \sum_{i=1, i \neq j}^N p_{u_i, j \lambda_i} (u_{i, j} \lambda_i), \end{aligned} \quad (23)$$

$$\text{Labor} : \quad L = L_Y + \sum_{j=1}^N (L_{x_j} + L_{r_j}), \quad (24)$$

$$\text{Entry investments} : \quad V \dot{N} = S. \quad (25)$$

¹⁴In contrast to Vavra (2002), but similar to van de Klundert & Smulders (1997), this paper focuses only on strategic interactions in product markets. The detailed analysis of the strategic interactions of firms between investment and production (pricing) decisions is beyond the scope of this paper; although, such interactions are partly modeled through the trade of production instructions. The detailed analysis of these interactions may be of importance with a relatively small number of firms, as shown in Vavra (2002), and can be considered in later research.

The last market clearing condition equates the entry investment (cost) to the generated value from the entry for any telecom firm. Together with the entry rule (7) it implies that

$$V = \frac{1}{\eta}. \quad (26)$$

This condition states that the incumbents' value is constant and greater than zero.

3 Features of the dynamic equilibrium

I restrict the attention to a symmetric equilibrium in the telecom market. It is instructive to derive the profit function of a telecom good producer first. After tedious algebra one can write

$$\pi = wL_x \Phi(N),$$

where

$$\Phi(N) = \frac{1}{e^k - 1} - \frac{g\lambda}{r - (g_w - \delta g_N)}, \quad k = C, B,$$

$$\delta = 1 \left(\dot{N} \neq 0 \right),$$

and the growth rate of a variable Z is denoted by g_Z .¹⁵

Proposition 1 *The Φ is a decreasing and convex function of the number of firms N .*

Proof. See Appendix A.1. ■

The competition intensifies with the number of firms N . When the strategic interactions in the product market are non-negligible, the intensity of competition and profits are related negatively. The negative relation between N and Φ reflects exactly this point.

Proposition 2 *The growth rate of productivity in telecom good production (or the NL_r) is an increasing and concave function of the number of firms N .*

¹⁵One way of deriving the profit function is by (i) inserting the demand for any other's knowledge (18) into the rate of return on productivity improvement (19); (ii) using the market clearing condition (23) and the supply of knowledge (17) in the resulting equation in order to eliminate $p_{u_{j,i}\lambda_j}$ and $u_{j,i}\lambda_j$; (iii) using the investment in productivity improvement (16) in order to express the growth rate of the shadow value of productivity improvement q_{λ_j} ; (iv) using the supply of the telecom good (15) and the investment in productivity improvement (16) in order to find the ratio of labor force employed in production L_x and in productivity improvement process L_r ; and, finally, (v) using the profit function (10) with the market clearing condition (23).

Proof. See Appendix A.1. ■

This result suggests that the growth rate of productivity in telecom good production can converge to a steady state as the number of firms grows. It will also help in drawing some parallels between the decentralized equilibrium allocations and socially optimal allocations, which will be presented in what follows.

3.1 Transition

Let the economy start with a relatively low number of telecom firms, then the number of telecom firms will grow over time. While the number of firms is growing, there will be resource reallocation in the economy due to the impact of firm entry on the inter-firm strategic interactions. As a result, the GDP growth rate will change over time. Therefore, the economy will experience a transition to a balanced growth path. During that transition the entry will affect economic growth through the indirect network externalities as well.

The transition ends either when there is no entry or when there are so many telecom firms that the new entrant's impact on others' demand is negligible.^{16,17} The Appendix T.1 offers some, additional, description of the transition dynamics. Hereafter, I focus only on the balanced growth path analysis unless stated otherwise.

3.2 Balanced growth

I denote GDP growth rate by g .

Proposition 3 *The constant growth rate of GDP is given by*

$$g = Bg_\lambda, \tag{27}$$

where

$$B = \frac{(\mu + \sigma)(\varepsilon - 1)}{\varepsilon - 1 - \delta(\mu + \sigma)}, \tag{28}$$

$$g_\lambda = \frac{\xi DL - \rho}{(\theta - 1 + \delta)B + D}, \tag{29}$$

$$D = \frac{b\sigma}{b\sigma + 1 - \sigma}, \tag{30}$$

$$b = \frac{e^k - 1}{e^k}, \quad k = C, B. \tag{31}$$

¹⁶This is a shared property of growth models which have household preferences and/or production technology formulated *a la* Dixit & Stiglitz (1977).

¹⁷It has to be emphasized that this paper does not consider collusive behavior that could result in barriers to entry.

Proof. See Appendix A.1. ■

Proposition 4 *The labor force allocations on the balanced growth path are*

$$NL_r = \frac{1}{\xi} g_\lambda, \quad (32)$$

$$NL_x = \frac{[(\theta - 1 + \delta) B g_\lambda + \rho]}{g_\lambda} NL_r, \quad (33)$$

$$L_Y = L - NL_x - NL_r, \quad (34)$$

where the NL_r is the share of labor force employed in the productivity improvement process in telecom. The NL_x is the share of labor force employed in production of telecom goods. The L_Y is the share of labor force employed in production of the final good.

Proof. See Appendix A.1. ■

In order to highlight the effect of entry, I analyze two cases. In the first case, I assume that there are exogenous barriers to entry into telecom (i.e., the number of telecom firms is exogenously fixed). I call this case "Barriers to Entry." The second case I call "Endogenous Entry" and assume no exogenous barriers in that case.

3.2.1 Barriers to entry

With a fixed number of telecom firms the economy grows at constant rates.¹⁸ When there are (exogenous) barriers to entry, the GDP growth is driven only by productivity improvement in telecom good production and $\delta = 0$.

Corollary 5 *In this case, the GDP growth rate is positively related to the elasticity of substitution between telecom goods (ε) and the toughness of competition (Bertrand vs Cournot; Sutton, 1991). Moreover, it does not vary with the share of own knowledge use for productivity improvement ($1 - \alpha$).*

Proof. See Appendix A.2, which also offers comparative statics with respect other parameters. ■

The GDP growth rate does not depend on the share of own knowledge use for productivity improvement since in equilibrium the newly generated knowledge ($\dot{\lambda}$) is linear in the knowledge input (λ) and there are no spillovers assumed.¹⁹ The positive

¹⁸In this case, this model stands close to the one offered by van de Klundert & Smulders (1997).

¹⁹It can be shown that the existence of spillovers from others' knowledge induces the firms to invest less in productivity improvement and the GDP growth rate depends on the scale of those spillovers. The presence of such spillovers would not qualitatively change the results of this paper, unless it alters the linearity of $\dot{\lambda}$ in N in equilibrium.

relation between GDP growth and ε or GDP growth and toughness of competition is implied by more intensive competition. More intensive competition reduces the relative price distortions, which are a result of the price setting behavior of telecom firms [i.e., $\frac{1}{\varepsilon}$ in (15) and (19) decreases]. Since the final good producers are competitive and use labor for production, the relative price distortions matter for growth through labor allocations.²⁰ Lower distortions imply higher growth because they are associated with higher labor force allocation to productivity improvement in telecom (i.e., NL_r increases with lower distortions).

Before proceeding further with the comparative statics analysis, it is useful to consider the function Φ one more time. The Φ on balanced growth path is given by

$$\Phi(N) = \frac{1}{e^k - 1} - \frac{g_\lambda}{(\theta - 1 + \delta) Bg_\lambda + \rho}, \quad k = C, B. \quad (35)$$

The profit of a telecom firm will be non-negative if and only if $\Phi(N)$ is non-negative. In terms of g_λ ,

$$g_\lambda \leq \frac{\rho}{e^k - 1 - (\theta - 1 + \delta) B} \Leftrightarrow \Phi(N) \geq 0, \quad k = C, B. \quad (36)$$

Hereafter, I call the growth rate of the productivity in telecom good production, which satisfies the equality in (36) *ZP*, i.e., zero profit. In turn, I call *CME* the growth rate of the productivity in telecom good production which was derived from the capital market equilibrium [i.e., (29)]. Given N if *CME* is lower (higher) than *ZP* the profit of a telecom firm is positive (negative).

Proposition 6 *The ZP curve is a convex and decreasing function of the number of telecom firms, whereas the CME is a concave and increasing function of the same.*

Proof. See Appendix A.3. ■

The *ZP* declines with the number of firms (N) since the higher growth rate of the productivity in telecom good production erodes the profit margins of the telecom firm. Meanwhile, the higher number of telecom firms increases the growth rate of the productivity in telecom good production since it eliminates part of the distortions in the economy. Thus, there is a positive relation between *CME* and N .

Depending on the household's preferences, final good production technology, and productivity improvement technology there are two cases in the economy. In the first case, *CME* is always lower than *ZP*. While in the second case, there is some N^* number of firms such that when $N > N^*$, *CME* is higher than *ZP*. In the first case

²⁰In an extreme case when the final good producers do not use labor, the b in (31) and the σ are equal to one and the relative price distortions do not matter for growth.

there can be infinitely many telecom firms in the economy since the profit, thus the value V , is always positive. In other words, the fixed number of telecom firms can be arbitrarily large. In contrast, in the second case the maximum number of telecom firms is a finite number or, in other words, the fixed number has a finite upper bound.

The major determinants of these cases are the market power of telecom firms and the cost of productivity improvement. *Ceteris paribus*, the second case would hold under lower market power or higher costs.²¹

Proposition 7 *The upper bound in the second case can be derived from a zero profit condition, which is equivalent to a zero value condition.*

Proof. 1. The equivalence follows from a standard non-arbitrage condition $rV = \pi + \dot{V}$ for any telecom firm and that the value is the sum of discounted profits.

2. The Φ is a continuous and decreasing function of the number of firms. Therefore, the maximum number of firms is given by a zero profit condition.

It is important to highlight the continuity since, according to entry rule (7) and market clearing condition for entry investments (25), the firm N^* for which ZP and CME coincide is not in the telecom market. Therefore, N^* can be treated as an upper bound for the number of firms in the market. ■

These two cases are illustrated in the following figures in (g_λ, N) space, where the maximum number of telecom firms is denoted by a star if it exists.

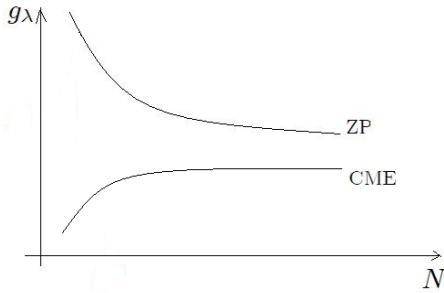


Figure 1

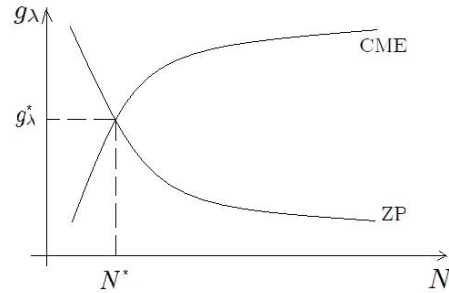


Figure 2

²¹For instance, for a given GDP growth rate the first case holds if $\{\sigma = 0.05; \mu = 0.002; \varepsilon = 2.7; \theta = 4; \rho = 0.025; \xi = 0.9; \eta = 1; L = 1\}$, while the second case holds when $\varepsilon = 2.3, \rho = 0.0242$ and everything else is the same. These parameters are set such that (1) to be close to the share of telecom good consumption in the US implied by the EU KLEMS data; (2) to be close to the suggestion of Roller & Waverman (2001) on the average contribution of telecom to economic growth in the US; and (3) to have low elasticity of substitution between telecom goods in accordance with the suggestions of numerous studies that try to measure that elasticity.

These figures suggest that there are regions of N where the fixed number of telecom firms can be set higher while holding the profit non-negative. If the fixed number of telecom firms is in those regions, then the higher fixed number implies higher growth in productivity of telecom firms, since the later moves along CME . In turn, higher growth in productivity of telecom firms implies higher GDP growth [see (27)].²²

The positive relation between the GDP growth rate and the number of telecom firms can be attributed also to the existence of trade of knowledge for productivity improvement. If there was no trade of knowledge, then under some parameter restrictions and/or minimal number of telecom firms, the growth rate of productivity, and thus the GDP growth rate, would decline with the number of firms.²³ This case is considered by van de Klundert & Smulders (1997). The reason for this negative relation is that the total labor supply is fixed. When the number of firms increases, the supplied labor available for productivity improvement should be shared amongst a greater number of firms; therefore, CME declines with the number of firms.

3.2.2 Endogenous entry

In case there are no exogenous barriers to entry CME and ZP are useful for characterizing the features of the equilibrium.

Proposition 8 *Since CME increases with the number of firms and ZP declines with it, the equilibrium is always stable.*

Proof. See Appendix A.3 for stability conditions. ■

Moreover, depending on the household's preferences, final good production technology and the telecom firm's productivity improvement technology, there are two cases when the economy grows at constant rates. In the first case there are so many telecom firms that the new entrant's impact on others' demand is negligible. Whereas in the second case, the next entrant will have negative profit streams (i.e., there are endogenous barriers to entry).²⁴

In the first case, CME is always lower than ZP . On the balanced growth path there are infinitely many telecom firms and there is permanent entry. The GDP growth rate in this case is the same as in the case of exogenous barriers to entry

²²Given that there is continuous entry during transition, for a given country this comparison is for different time horizons. In that regard, it has to be emphasized that the statement is not conditional on the time horizon.

²³The Appendix NT.3 offers sufficient conditions for the negative relation between GDP growth rate and the number of firms when there is no trade of knowledge.

²⁴This ordering is possible given that Φ is negatively related to the number of firms and the investments in productivity improvement are fixed costs.

(27), but δ is equal to one. Therefore, the GDP growth rate depends on the same factors as in the exogenous barriers case. However, neither the number of telecom firms nor the type of competition affect the GDP growth rate. This is the case since here the number of firms is so high that the strategic interactions are negligible and the perceived elasticities of substitution (e) are equal to the actual one (ε).

Proposition 9 *In contrast to the exogenous barriers case, in this case the effect of higher ε on the GDP growth rate is ambiguous and depends on model parameters.*

Proof. See Appendix A.2. ■

This is the case since higher ε implies more intensive competition and a lower contribution of entry to the GDP growth [i.e., the $\frac{\varepsilon}{\varepsilon-1}$ in (4) decreases with ε]. More intensive competition implies a higher growth rate since it reduces the relative price distortions. However, the combined effect is ambiguous.

Corollary 10 *It turns out that when there are exogenous barriers to entry and infinitely many telecom firms, then the GDP growth rate is higher than the GDP growth rate when there is entry.*

Proof. See Appendix A.4. ■

This is a general equilibrium result. It can be partially explained by declining returns on productivity improvement due to the entry, which can be shown by substituting (15), (16), (18) into (19) and using (9). The returns decline with entry since the entry decreases the demand for the telecom good, while the investments for productivity improvement were increasing it. When the returns decline, the firms cut their investments in productivity improvement and the growth rate of productivity declines [see δ in (29)]. The GDP growth rate is proportional to the growth rate of productivity; therefore, it also declines.

In the second case of Endogenous Entry, let the N^{**} ($< \infty$) be the last telecom firm that will have non-negative profit streams if it enters.

Proposition 11 *There is no entry after N^{**} (i.e., $\dot{N} = 0$) and N^{**} is determined from the intersection of the CME and ZP curves [point (g_λ^*, N^*) in Figure 2].*

Proof. There is no entry after N^{**} since for any $N > N^{**}$ the value V would be negative.²⁵ When there is no entry, the economy is on a balanced growth path; therefore, N^{**} is determined from the intersection of CME and ZP curves. ■

²⁵Strictly speaking, the firm that has zero profits invests zero; therefore, according to (7), it also does not enter. Therefore, N^{**} is an upper bound for the number of firms in the telecom market. However, since Φ is a continuous function of the number of firms, N^{**} is exactly the number of firms in the telecom market.

Proposition 12 *Under Cournot and Bertrand competitions, the number of firms are*

$$N^C = (\varepsilon - 1) \frac{\xi L [(\theta - 1)(\sigma + \mu) + 1]}{\varepsilon (\xi L - \frac{\rho}{\sigma}) - \xi L [(\theta - 1)(\sigma + \mu) + 1]}, \quad (37)$$

and

$$N^B = (\varepsilon - 1) \frac{\xi L - \frac{\rho}{\sigma}}{\varepsilon (\xi L - \frac{\rho}{\sigma}) - \xi L [(\theta - 1)(\sigma + \mu) + 1]}, \quad (38)$$

respectively.

Proof. See Appendix A.5. ■

Corollary 13 *The GDP growth rate is the same under both types of competition and it depends only on model parameters. However, under Bertrand competition the number of telecom firms is less than under Cournot competition.*

Proof. See Appendix A.5, which offers also comparative statics for the number of firms. ■

It turns out that the effect of change in almost any parameter on the GDP growth rate depends on the model parameters. The only exceptions are μ and ξ . The GDP growth rate increases with these two parameters. Figure 2 can illustrate why this is the case for ξ . The *CME* increases with ξ while *ZP* does not change. Therefore, the growth rate of the productivity in telecom good production, as well as the GDP growth rate, increase with ξ . The Appendix A.7 shows analytically that the GDP growth rate increases with μ and ξ .

3.3 Social optimum

In this section I compare the decentralized equilibrium allocations and growth rates with those of the Social Planner's solution and analyze the sources of any difference. The Social Planner selects a feasible path that maximizes the lifetime utility of the household. In the social optimum, symmetry holds in telecom due to symmetry in knowledge for production of telecom goods. Moreover, there is no market for that knowledge, but it is shared across the telecom firms. The Appendix S.1 presents the Social Planner's problem.

Proposition 14 *In the social optimum the economy grows at constant rates.*

Proof. See Appendix S.1. ■

This is the case since there are no historically determined rigidities and in the social optimum there are no strategic interactions.

Proposition 15 *The socially optimal GDP growth rate is given by*

$$g^S = B^S g_\lambda^S, \quad (39)$$

where

$$B^S = \frac{(\varepsilon - 1)(\sigma + \mu)}{\varepsilon - 1 - (\sigma + \mu)}, \quad (40)$$

$$g_\lambda^S = \frac{\xi D^S L - \rho}{(\theta - 1)B^S + D^S}, \quad (41)$$

$$D^S = \frac{\sigma + \mu}{1 + \mu}. \quad (42)$$

Proof. See Appendix S.1. ■

Proposition 16 *The socially optimal labor allocations are*

$$\begin{aligned} NL_r^S &= \frac{1}{\xi} g_\lambda^S, \\ NL_x^S &= D^S [L - NL_r^S], \\ L_Y^S &= L - NL_x^S - NL_r^S, \end{aligned}$$

where the NL_r^S is the share of labor force employed in the productivity improvement process in telecom. The NL_x^S is the share of the labor force, which is employed in the production of the telecom goods. The L_Y^S is the share of the labor force which is employed in the production of the final good.

Proof. See Appendix S.1. ■

Corollary 17 *The socially optimal GDP growth rate decreases with the elasticity of substitution between telecom goods (ε).*

Proof. See Appendix S.2, which offers comparative statics with respect to other parameters as well. ■

The socially optimal GDP growth rate decreases with ε since in the social optimum there is permanent entry and a higher ε implies a lower contribution of entry to growth [i.e., in (4) the $\frac{\varepsilon}{\varepsilon-1}$ decreases with ε]. Meanwhile, there are no relative price distortions as in the decentralized equilibrium.

The permanent entry result is due to the absence of market incentives in the social optimum. It stands in contrast to the decentralized equilibrium result where it may be the case that there are endogenous barriers to entry. From the Social Planner's problem it can be shown also that the analogue of the profit function in

the social optimum is constant and greater than zero. If such a relation would hold in a decentralized equilibrium, then there would also be permanent entry.²⁶

Corollary 18 *In the social optimum the share of labor force allocated to productivity improvement is higher than that in the decentralized equilibrium, both on a balanced growth path and during transition in the decentralized equilibrium (i.e., $NL_r^S > NL_r$).*

Proof. See appendices S.3 and S.4. ■

This result holds out of the balanced growth path as well. However, in order to offer intuition for it, I refer to the balanced growth path results in the decentralized equilibrium. On the balanced growth path, it can be easily shown that this result follows from three factors. First, in the decentralized equilibrium the telecom firms have an incentive to under-invest in productivity improvement since that erodes their profit margins (i.e., for any given number of firms, higher CME implies lower profit). Second, the telecom firms are price setters, which creates relative price distortions and alters the labor allocations. A comparison of D^S and D from (42) and (30) can reveal these two factors. Third, in the decentralized equilibrium the rate of return on productivity improvement declines with the entry; thus, when there is permanent entry the firms cut their investments [see δ in (29)]. In contrast, the Social Planner has no market incentives; therefore, it does not have any incentive to cut investments in productivity improvement.

At first glance, the comparison of D^S and D can suggest that the share of direct network externalities (μ) may be a fourth factor. The presence of these externalities also creates resource misallocations through relative price distortions, which are due to the unrealized value from using telecom goods. However, the effect of μ on NL_r relative to the same effect on NL_r^S is ambiguous, and the proposition $NL_r^S > NL_r$ holds for any μ .

In the remaining part of this section I offer additional comparisons between the socially optimal and decentralized equilibrium allocations and growth rates. Given that I have not fully presented the transition stage of the decentralized equilibrium, these comparisons do not offer a complete picture. They are only for the balanced growth path results of the decentralized equilibrium. Nevertheless, they are informative and can help to gain some intuition for the differences between socially optimal and decentralized equilibrium allocations and growth rates.

Corollary 19 *The comparison of other labor allocations is not so straight forward and depends on the model parameters. The only straight forward inference is in the*

²⁶See ν in the next section.

case where there is no entry. In such case, in the decentralized equilibrium the labor force employed in production of telecom goods is less than that in the social optimum.

Proof. See Appendix S.4. ■

Given that in the social optimum the share of labor force employed in productivity improvement in telecom is greater than that in the decentralized equilibrium, the growth rate of productivity in the social optimum is higher than that in the decentralized equilibrium (i.e., $g_\lambda^S > g_\lambda$). Since in the social optimum there is permanent entry, and in the decentralized equilibrium there can be endogenous barriers to entry, the B^S from (40), which is the ratio of the GDP growth to productivity growth, is greater than or equal to the B from (28).²⁷ Therefore, the GDP growth rate is also higher in the social optimum (for analytical derivations, see Appendix S.5).

It is worth noting that when there is no entry due to exogenous barriers, the labor force employed in the production of telecom goods increases with the number of telecom firms and toughness of competition (see Appendix S.4). Therefore, in this case, increasing the number of telecom firms or motivating a tougher competition not only implies higher GDP growth rate, but also implies labor force allocations that are closer to the socially optimal ones. Consequently, these actions lead to higher social welfare.

4 Policy inference

In this section I derive policy inference from the results above. First, I offer policies which can lead to socially optimal allocations in the decentralized equilibrium. Second, I analyze the social welfare impact of a policy (action) which can change the elasticity of substitution between telecom goods (ε). Third, I offer a comparison of cases when there is trade of knowledge and when there is no trade.

4.1 Policies leading to the social optimum

In this part I consider a government which tries to set policies that can deliver the socially optimal allocations as a decentralized equilibrium outcome. The policy instruments available to government are taxes and subsidies, and market structure regulation in telecom. In view of the results above, I suggest these "optimal" policies and discuss the implications of a policy which is in the spirit of the Telecommunications Act of 1996. Given that there is no transition in the social optimum, under these policies there will be no transition in the decentralized equilibrium.

²⁷The B can be interpreted as a measure of the contribution of productivity growth to GDP growth.

In search of optimal policies the sources of the differences between the socially optimal and decentralized equilibrium outcomes should be taken into account. In order to highlight the direction of an optimal policy, the following list briefly summarizes these sources. In the decentralized equilibrium

1. due to the price setting behavior of the telecom firms, there are relative price distortions;
 - a. the telecom firms under-invest in productivity improvement.
Moreover, *ceteris paribus*, on a balanced growth path they under-invest more when there is entry of telecom firms since their returns decline with entry;
2. the direct network externalities are not internalized, which also creates relative price distortions;
3. there can be endogenous barriers to entry, in contrast to the social optimum where there is permanent entry.

In order to eliminate these differences, the optimal policy would fix the resource misallocations through corrections in relative prices. As a market structure regulation, it would allow free entry into telecom and would subsidize the entry, if needed.

A policy, under which the decentralized equilibrium allocations are the same as the socially optimal ones, subsidizes telecom goods production and the investments in productivity improvement, in terms of labor force compensation. It transfers to telecom firms subsidies proportional to the value of knowledge for production. Moreover, if there are endogenous barriers to entry, it transfers lump-sum subsidies to telecom firms in order to keep the profits marginally greater than zero. It finances these subsidies through a lump-sum tax imposed on the household. The Appendix DE.1 derives this policy. Formally, it is given by

$$\begin{aligned}
\tau_{L_x} = \tau_{L_r} &= 1 - b \left(\frac{\sigma}{\sigma + \mu} \right), \\
\tau_\lambda &= \frac{N}{\alpha} \left(B - \frac{g_b}{g_\lambda^S} \right), \\
T_\pi : \pi \equiv \nu &= \frac{\theta}{\eta} (g^S + \rho), \\
G &= N [(\tau_{L_x} w L_x + \tau_{L_r} w L_r) + \tau_\lambda p_\lambda \lambda + T_\pi],
\end{aligned} \tag{43}$$

where τ_{L_x} , τ_{L_r} , and τ_λ are in percentage terms. The first two are the subsidies to a telecom good production and to investments in productivity improvement, correspondingly. The last one is the subsidy to a telecom firm proportional to the value of knowledge. The subsidy to a telecom firm, which keeps the profit of the telecom firm

constant and greater than zero in order to guarantee permanent entry, is denoted by T_π . Formally, ν is equal to $r^S V$, where r^S is the return on assets in the social optimum.²⁸ The value of the firm's knowledge λ is denoted by p_λ (p_λ and $p_{u\lambda}$ are equal in symmetric equilibrium when $u \equiv 1$). Finally, the lump-sum tax imposed on the household is denoted by G .

The subsidy to a telecom firm proportional to the value of knowledge (τ_λ) increases with the number of firms in order to cancel the decline of the rate of return on productivity improvement. In other words, τ_λ eliminates the δ in (29) given that there is entry of firms. In its turn, τ_{L_x} insures that the marginal value of the telecom good is equal to the marginal cost of its production. The term in brackets in (43) takes into account the direct network externalities. Without the term in brackets, or when there are no direct network externalities, τ_{L_x} equates the supply price of the telecom good to the marginal cost. This follows from an analogous equation to the supply of the telecom good (15), where the policy has been implemented:

$$p_x = \frac{wL_x}{xb} (1 - \tau_{L_x}), \quad (44)$$

given that the marginal cost of producing x is

$$MC_x = \frac{wL_x}{x}.$$

That, with the term in brackets, τ_{L_x} equates the marginal cost to the marginal value can be shown by using the demand for telecom goods bundle (13), together with (6) and (44). Moreover, from (44) and (43) it follows that this policy reduces the price of the telecom good; therefore, it motivates a higher demand for the telecom good.

A critical aspect of this policy is that the subsidies to telecom good production (τ_{L_x}) and to investments for productivity improvement (τ_{L_r}) are equal in percentage terms. It is possible to equate τ_{L_x} and τ_{L_r} since the subsidies to investments and subsidies proportional to the value of knowledge for production (τ_λ) have the same target. They are motivating higher investments in productivity improvement.

A different policy can deliver socially optimal allocations as a decentralized equilibrium outcome without transferring payments proportional to the value of knowledge. However, this policy would disproportionately subsidize the telecom good production and the investments for productivity improvement. Appendix DE.1 derives this policy as well. The only aspect which is worth emphasizing for this policy is that the subsidy to the telecom good production (τ_{L_x}) is the same as in the first

²⁸The expression $r^S V$ is the analogue of the profit function of a telecom firm in the social optimum.

policy. This is the case since in both policies τ_{L_x} fixes the labor force misallocation in the telecom good production on the condition that the labor force allocation to productivity improvement (NL_r) is the socially optimal one.²⁹

There are other optimal policies also. For instance, an optimal policy could subsidize the demand instead of subsidizing the supply of telecom goods. The important similarity between the optimal policies is that they all share at least three interdependent, but not perfectly dependent, components. The two components fix the two resource misallocations given that there is permanent entry into the telecom market, and the third component motivates the permanent entry, if needed.

Proposition 20 *A policy will fail to deliver the socially optimal allocations as a decentralized equilibrium outcome if it lacks at least one of the components which fix the different resource misallocations.*

Proof. See Appendix DE.2. ■

Proposition 21 *The component which motivates permanent entry is needed only in the case of endogenous barriers and for equalizing the decentralized equilibrium and the socially optimal GDP growth rates.*

Proof. See Appendix DE.2. ■

4.1.1 Discussion of a policy in the spirit of the Telecommunications Act of 1996

Many recently implemented policies seem to have a structure which is similar to the one of the suggested optimal policies. The similarities are that those policies promote demand for telecom goods and as a regulation they motivate entry. A prominent example of such implemented policy is the Telecommunications Act of 1996.

Despite these similarities, these policies tend to lack important components. They tend not to consider the incentive of telecom firms to under-invest in productivity improvement and the negative effect of entry on the rate of return on that investment.³⁰ They also do not incorporate subsidies which could allow permanent entry, if needed.

The subsidies allowing permanent entry are relevant only if there are endogenous barriers to entry.

²⁹Formally, τ_{L_x} equalizes the ratio $\frac{L_Y}{NL_x}$ to the ratio $\frac{L_Y^S}{NL_x^S}$, conditional that $NL_r = NL_r^S$.

³⁰It has to be acknowledged that, for instance, the Federal Communications Commission (FCC) in the US envisions the need to foster innovation in telecom (see, for instance, FCC Strategic Plan 2009-2014). However, the FCC tries to foster innovation by the means of more competition in telecom by motivating free entry.

Proposition 22 *In any case, whether there is or there is no entry, there is a policy that can over-perform the model analogue of the implemented policies in terms of social welfare. In addition to the targets of the implemented policies, this policy would at least subsidize the investments for productivity improvement.*

Proof. The Appendix DE.2 derives two policies which subsidize only the investments in productivity improvement and the telecom good production. It shows that these deliver socially optimal allocations as a decentralized equilibrium outcome. ■

This result is due to the incorporation of two different tools/subsidies for different labor force allocations. In case there is no entry, the only failure of such a policy is that the GDP growth rate is lower in the decentralized equilibrium.

4.2 Changing the elasticity of substitution

In this part I analyze the impact of a policy (action) which can affect the elasticity of substitution between telecom goods (ε). I assume that there are some exogenous mechanisms which may allow changing this parameter. I do not try at all to model these mechanisms and base my inference on comparative statics analysis, while assuming no costs for changing ε .

The Telecommunications Act of 1996 mandates number portability and interconnectedness of networks (see Economides, 1999). Given that the substitutability of telecom goods can depend on these factors, the Telecommunications Act of 1996 can affect the elasticity of substitution between telecom goods.

In the next section I discuss the incentives of telecom good producers and the choice of the Social Planner.

4.2.1 Incentives of telecom goods producers and the actions of the Social Planner

The telecom goods producers, due to their price setting behavior, are better off when product differentiation is greater. In other words, their profit is higher in a market where ε is lower since the competition is less intensive in such a market (see Appendix A.8). This means that the telecom goods producers have an incentive and would try to decrease ε if they had mechanisms to do so. The presented model can suggest such actions if the telecom good producers were to select ε at time zero and after setting the prices for any ε .

Similar to telecom good producers, the Social Planner prefers lower ε (see Appendix S.1). However, in contrast to telecom good producers' profit seeking behavior,

the Social Planner prefers lower ε simply because it implies higher GDP growth rate through higher contribution of entry.

Since a policy similar to the Telecommunications Act of 1996 is more likely to increase ε , it does not seem to be in line with the Social Planner's choice along this dimension.

4.3 Trade of knowledge for production

From the demand for the knowledge for production (18) follows that the value of that knowledge tends to infinity when the demanded amount tends to zero,

$$\lim_{u_{i,j} \rightarrow 0} p_{u_{i,j} \lambda_i} = +\infty, \forall i, j; (i \neq j).$$

Therefore, if there are policies which ban the trade of knowledge, then the model suggests removal of those policies. Otherwise, if the market for the knowledge should exist only under some specific policies, then the model encourages adoption of such policies.³¹

When the trade of knowledge is banned, under some reasonable parameter restrictions and/or minimal number of telecom firms., the *CME* curve slopes down in the (g_λ, N) space (see section 3.3. and p. 109 in van de Klundert & Smulders, 1997). Moreover, in such case on a balanced growth path either there is no entry or there is no productivity improvement [van de Klundert & Smulders (1997) consider the case when there is no entry]. For a given number of firms, if there are exogenous barriers to entry, the growth rate of productivity g_λ and hence the GDP growth rate g are always lower than g_λ and g when there is trade of knowledge. In addition, the same relationship holds if the number of telecom firms is sufficiently high in both cases, if there are no barriers to entry in case there is trade of knowledge, and when *CME* declines with the number of firms when there is no trade. Since the profit is negatively related to g_λ [see (35)], when there are endogenous barriers to entry the number of firms is always higher if there is no trade of knowledge.³²

In the social optimum there is permanent entry and positive growth in productivity (see Appendix S.1). However, when there cannot be a knowledge transfer, as considered in van de Klundert & Smulders (1997), there also cannot be permanent entry. This is the case since otherwise the labor market clearing condition would be violated. Therefore, it is socially beneficial to have a mechanism for knowledge transfer.

³¹The discussion of such policies is beyond the scope of this paper, although throughout the paper it is assumed that the market for knowledge exists.

³²The Appendix NT.3 proves these propositions.

5 Conclusions

The model presented in this paper suggests channels through which the telecommunication industry contributes to economic growth. On a balanced growth path these channels include the productivity improvement in telecom and the generated externalities. When there are no barriers to entry both the direct and indirect network externalities add to growth. Otherwise, only direct network externalities matter. In light of productivity improvements in telecom, in this paper the direct network externalities stem from the *effective* number of telecom users. This seems to be novel for the aggregate level studies related to telecom.

When there are exogenous barriers to entry the model suggests that the GDP growth rate and social welfare increase with the number of telecom firms and with the toughness of competition. In contrast, when the barriers to entry are endogenously generated, the GDP growth rate does not depend on the toughness of competition and the number of telecom firms. The same holds on a balanced growth path in the case when there are no barriers to entry.

These decentralized equilibrium outcomes are far from the social optimum according to the model. There are four reasons to observe this divergence. First, there are relative price distortions, thus resource misallocations, in the decentralized equilibrium due to the imperfect competition in the telecom market and because the competitive forces do not internalize the direct network externalities. Second, the telecom firms under-invest in productivity improvement since that erodes their profit margins. Third, the rate of return on productivity improvement declines with entry in the decentralized equilibrium, which is not the case in the social optimum. Fourth, there is permanent entry in the social optimum, which may not be the case in the decentralized equilibrium. The last two take place because the market incentives are neglected in the social optimum. These four aspects imply that the socially optimal GDP growth rate is always higher than the decentralized equilibrium one and that there is an under-investment in productivity improvement in the decentralized equilibrium.

Given these observations, the policies which can increase the social welfare in a decentralized equilibrium would (1) subsidize the production of telecom goods or, equivalently, subsidize the demand for telecom goods; (2) subsidize the investments for productivity improvements in telecom; and (3) allow entry and, if needed, subsidize it.

These policies would be similar to the Telecommunications Act of 1996. However, they would avoid increasing the substitutability between telecom goods and, in addition to the targets of that policy, they would subsidize the investments in

productivity improvement and the entry, if needed. If policies leading to permanent entry cannot be implemented, subsidizing investments still can lead to higher social welfare.

In a general equilibrium framework, this paper also suggests that the existence of mechanisms for the transfer of knowledge for production is socially optimal. The results suggest removal of policies that can ban the trade of that knowledge, if any. Moreover, if the market for knowledge should exist only under certain policies, then the results suggest adoption of such policies.

The result of this paper that the growth rate of productivity in telecom increases with the number of firms finds empirical support, despite limited (see, for instance, Boylaud & Nicoletti, 2001; OECD, 2004).³³ It seems that thus far there is no empirical research which tries to identify the impact of trade of knowledge on the relation between productivity growth in telecom (or in similar industry) and the market structure of that industry. Such research can complement the results of this paper.

As a final note, the presented model can find other applications as well. For instance, it can be suitable for analysis of industries where there is a significant amount of R&D partnerships or cross-licensing and where the Shumpeterian arguments seem not to be well applicable. Examples could be the computer software and hardware producing industries, where only several companies capture almost the entire market and the intra-firm R&D seems to be the driver of productivity.

³³For instance, OECD (2004) suggests exactly that the labor productivity growth in ICT industries increases with the number of firms. However, in order to back up this empirical observation OECD (2004) highlights a channel different than the trade for knowledge. This channel emphasizes the role of technological change brought by new firms.

6 References

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7 Technical appendices

7.1 Appendix A.1 - Decentralized equilibrium and a balanced growth path

The optimal rules (15)-(19) result from the j th telecom firm's problem. If written in terms of current value Hamiltonian, the j th telecom firm's problem is given by

$$\begin{aligned} \max_{L_{x_{j,t}}, L_{r_{j,t}}, u_{j,i,t}, u_{i,j,t}} H_j^T = & \left[p_{x_{j,t}} \lambda_{j,t} L_{x_{j,t}} + \sum_{i=1, i \neq j}^N p_{u_{j,i} \lambda_{j,t}} (u_{j,i,t} \lambda_{j,t}) - \dots \right. \\ & \left. \dots - (L_{x_{j,t}} + L_{r_{j,t}}) w_t - \sum_{i=1, i \neq j}^N p_{u_{i,j} \lambda_{i,t}} (u_{i,j,t} \lambda_{i,t}) \right] \\ & + q_{\lambda_{j,t}} \xi \left[\sum_{i=1}^N (u_{i,j,t} \lambda_{i,t})^\alpha \right] \lambda_{j,t}^{1-\alpha} L_{r_{j,t}}. \end{aligned}$$

The telecom firm prefers to sell all her knowledge since it strictly increases the profits, i.e., $u_{j,i,t} = 1$ for any i and t .

The ratiion $\frac{1}{e_{j,t}}$ in (15) and (19) stems from derivating the price of the j th telecom good with respect $L_{x_{j,t}}$ and $\lambda_{j,t}$, correspondingly. Moreover, here and in the rest of the appendices, when relevant, I either put the time in subscript or suppress it.

From (15) and (16), it follows that

$$\frac{p_{x_{j,t}}}{q_{\lambda_{j,t}}} = \frac{g_{\lambda_{j,t}}}{L_{r_{j,t}} b_{j,t}}, \quad (45)$$

where $b_{j,t} = \frac{e_{j,t}-1}{e_{j,t}}$.

Assuming symmetric equilibrium, using the definition of the productivity improvement process (9), and substituting (45) into (19), the following can be obtained

$$g_{q_{\lambda_t}} = r_t - \left\{ g_{\lambda_t} + \frac{L_{x_t}}{L_{r_t}} g_{\lambda_t} \right\}. \quad (46)$$

From (16) it follows that

$$g_{q_{\lambda_t}} = g_{w_t} - \delta g_{N_t} - g_{\lambda_t}.$$

Therefore,

$$[NL_r]_t = \frac{g_{\lambda_t}}{r_t - (g_{w_t} - \delta g_{N_t})} [NL_x]_t. \quad (47)$$

From (15)-(19) and the market clearing condition (23), it follows that

$$\pi = w L_x \Phi(N),$$

where

$$\Phi(N) = \frac{1}{e_t^k - 1} - \frac{g_{\lambda_t}}{r_t - (g_{w_t} - \delta g_{N_t})}, \quad k = C, B.$$

The Φ is a decreasing and convex function of N . First, consider the first term in the square brackets. It can be shown that

$$\frac{\partial e^k}{\partial N} > 0, \frac{\partial^2 e^k}{\partial N^2} < 0, k = C, B. \quad (48)$$

This means that the first term is a decreasing and convex function of the number of firms N . For the second term,

$$\frac{\partial}{\partial N} \frac{g_{\lambda_t}}{r_t - (g_{w_t} - \delta g_{N_t})} = \frac{\frac{NL_x}{L_Y} \left(\frac{\partial}{\partial N} \frac{NL_r}{L_Y} \right) - \frac{NL_r}{L_Y} \left(\frac{\partial}{\partial N} \frac{NL_x}{L_Y} \right)}{\left(\frac{NL_x}{L_Y} \right)^2},$$

where

$$\begin{aligned} \frac{\partial}{\partial N} \frac{NL_x}{L_Y} &= \frac{\partial b}{\partial N} \frac{1}{b} \frac{NL_x}{L_Y}, \\ \frac{\partial}{\partial N} \frac{NL_r}{L_Y} &= \frac{\partial b}{\partial N} \frac{1}{b} \left[\frac{NL_r + L_Y}{L_Y} \right]. \end{aligned}$$

Therefore,

$$\frac{\partial}{\partial N} \frac{g_{\lambda_t}}{r_t - (g_{w_t} - \delta g_{N_t})} = \frac{\partial b}{\partial N} \frac{1 - \sigma}{b^2 \sigma}.$$

It can be shown that

$$\frac{\partial b}{\partial N} > 0, \frac{\partial b}{\partial N^2} < 0. \quad (49)$$

Therefore, the second term is a decreasing and convex function of the number of firms as well. Hence, the Φ is a decreasing and convex function of N .

An alternative proof for $\Phi' < 0$ and $\Phi'' > 0$ uses the labor market clearing condition (24), final and telecom goods production functions (3),(8), and the relation between labor demand in final and telecom goods production. A sufficient condition to observe the desired property of Φ is $b\sigma L \frac{1-\sigma}{1+\mu} < NL_x$, which can be shown to hold from the labor market clearing condition.

The proof of the proposition that the growth rate of productivity in telecom good production is an increasing and concave function of the number of firms requires a bit of algebra. The total labor force employed in telecom goods production (NL_x) can be expressed in terms of constants and the number of firms from the demand for telecom goods bundle (13), demand for a telecom good (14), demand for labor force in final good production (12) and the production function of the final good (3). The total labor force employed in the productivity improvement process (NL_r) can be then expressed in terms of constants and the number of firms through the labor market clearing condition while using the result for NL_x . The resulting expression is

$$NL_r = L - \left(\frac{1 - \sigma + b\sigma}{b\sigma} \right) z b^{\frac{1-\sigma}{1+\mu}} N^{-\frac{1}{\varepsilon-1} \frac{\mu+\sigma}{1+\mu}}, \quad (50)$$

where $z = \left(\frac{\sigma}{1-\sigma} \right)^{\frac{1-\sigma}{1+\mu}} Y^{\frac{1}{1+\mu}} \lambda^{-\frac{\mu+\sigma}{1+\mu}}$ is given. From (12) it can be easily shown that NL_r , and thus g_{λ} , is an increasing and concave function of the number of firms.

7.1.1 Balanced growth path

Under symmetry and on a balanced growth path, the variables in (47) should be time invariant. The return r on asset holdings is time invariant due to the standard Euler equation. The growth rates are time invariant by definition. The total labor force employed in the productivity improvement process $[NL_r]_t$ is time invariant since $NL_r = \frac{1}{\xi}g\lambda$. Therefore, the total labor force employed in telecom goods production $[NL_x]_t$ is time invariant.

It has to be noted that the number of firms on a balanced growth path should be either time invariant or infinite. This immediately follows from the elasticity of substitution between telecom goods perceived by a telecom firm,

$$-e_t = \frac{\partial x_t}{\partial p_{x_t}} \frac{p_{x_t}}{x_t} = \frac{g_{x_t}}{g_{p_{x_t}}}.$$

Since $g_{x_t}, g_{p_{x_t}} = \text{const}$ on the balanced growth path, this elasticity should be constant on as well. According to (20) or (21), it is the case either when $N = \text{const}$ or when $N = \infty$. Moreover, according to (20) or (21), this elasticity is equal to the true elasticity ε when $N = \infty$.

From market clearing condition (22), the constraint (6), production technology of a telecom good (8), and the supply of a telecom good (15), it follows that

$$[NL_x]_t \frac{1 - \sigma}{b_t \sigma} = L_{Y_\tau}. \quad (51)$$

Given that on the balanced growth path e_t should be time invariant, b_t is time invariant by its definition (31). Therefore, given that $[NL_x]_t = NL_x$, the labor force employed out of telecom (L_{Y_t}) is time invariant, on the balanced growth path.

From (47), (51), the labor market clearing condition (24), and the definition of productivity improvement process (9), it follows that

$$r - (g_w - \delta g_N) = D(\xi L - g_\lambda), \quad (52)$$

where

$$D = \frac{b\sigma}{b\sigma + 1 - \sigma}.$$

From the Euler equation, which stems from the household's optimization problem, it follows that

$$r = \theta g_C + \rho, \quad (53)$$

where g_C is the growth rate of consumption.

From the budget constraint $\dot{A} = rA + wL - C$, equilibrium asset holdings $A = VN$, entry condition $\dot{N} = \eta S$, a no arbitrage condition for a telecom firm $rV = \pi + \dot{V}$, $L = \text{const}$, $V = \text{const}$, $r = \text{const}$, and (12), it follows that

$$g_C = g_Y = g_S = g_N = g_w = g_A \equiv g. \quad (54)$$

In turn, the growth rate of final good g can be derived from the final goods

production technology (3) together with the telecom goods production technology (8). The g is given by

$$g = Bg_\lambda, \quad (55)$$

where

$$B = \frac{(\mu + \sigma)(\varepsilon - 1)}{\varepsilon - 1 - \delta(\mu + \sigma)}.$$

Replacing the g_w and g_N in (52) with g from (3), replacing g_C in (53) with the same, and equating (52) and (53) yields

$$g_\lambda = \frac{\xi DL - \rho}{(\theta - 1 + \delta)B + D}.$$

Therefore, on the balanced growth path, the GDP growth is given by

$$g = Bg_\lambda.$$

The labor force allocations are

$$\begin{aligned} NL_r &= \frac{1}{\xi} g_\lambda, \\ NL_x &= \frac{1}{\xi} [(\theta - 1 + \delta) Bg_\lambda + \rho], \\ L_Y &= NL_x \frac{1 - \sigma}{b\sigma} \\ &= L - NL_x - NL_r. \end{aligned}$$

The parameter restrictions are

$$\varepsilon > 1 + \mu + \sigma, \xi DL > \rho.$$

It is worth noting that when the number of firms is infinite, the GDP growth rate is not a function of the number of firms since the perceived elasticity of substitution is equal to the actual one [i.e. the curly brackets in (20) and (21) are zero].

7.2 Appendix A.2 - Comparative statics for the GDP growth rate when there is entry or there are exogenous barriers to entry

The comparative statics can be (and is) done for the general case where δ is not set equal to zero. This allows a short cut when performing similar comparative statics for the case where there is an entry.

7.2.1 The change of GDP growth rate with $\mu, \sigma, \xi, \rho, \theta, (1 - \alpha)$

With a bit of algebra it can be shown that

$$\frac{\partial g}{\partial B} > 0, \frac{\partial B}{\partial \mu} > 0, \frac{\partial g}{\partial D} > 0, \frac{\partial D}{\partial \sigma} > 0, \frac{\partial B}{\partial \sigma} > 0, \quad (56)$$

and

$$\frac{\partial D}{\partial b} > 0, \frac{\partial b}{\partial e} > 0, \frac{\partial e^k}{\partial \varepsilon} > 0 \quad k = C, B, \frac{\partial D}{\partial \varepsilon} > 0. \quad (57)$$

From (56) and (57), it follows that

$$\frac{\partial g}{\partial \mu} > 0, \frac{\partial g}{\partial \sigma} > 0, \frac{\partial g}{\partial \xi} > 0, \frac{\partial g}{\partial \rho} < 0, \frac{\partial g}{\partial \theta} < 0, \frac{\partial g}{\partial 1 - \alpha} = 0.$$

7.2.2 The change of GDP growth rate with ε

The derivative of B with respect ε is

$$\frac{\partial B}{\partial \varepsilon} = -\delta \left(\frac{\mu + \sigma}{\varepsilon - 1 - \delta(\mu + \sigma)} \right)^2 \leq 0.$$

Therefore, from (56) and (57), it follows that when there are barriers to entry (i.e., $\delta = 0$)

$$\frac{\partial g}{\partial \varepsilon} > 0.$$

It follows also that when there is permanent entry the answer is inconclusive. This is the case since higher ε implies more intensive competition and lower contribution of growth in the number of firms to GDP growth [i.e., the $\frac{\varepsilon}{\varepsilon-1}$ in (4) decreases with ε]. More intensive competition implies a higher growth rate since it reduces the relative price distortions. However, the combined effect is ambiguous.

7.3 Appendix A.3 - The CME and ZP curves in (g_λ, N) space

The CME curve is the growth rate of productivity improvement, i.e.,

$$g_\lambda^{CME} \equiv g_\lambda = \frac{\xi DL - \rho}{(\theta - 1 + \delta)B + D}.$$

The ZP curve is the growth rate of productivity improvement which satisfies the equality in (36), i.e.,

$$g_\lambda^{ZP} \equiv g_\lambda = \frac{\rho}{e^k - 1 - (\theta - 1 + \delta)B}, \quad k = C, B.$$

Both g_λ^{CME} and g_λ^{ZP} are functions of the perceived elasticity of substitution e^k , $k = C, B$. Given the first and second order derivatives of perceived elasticity of substitution with respect number of firms (48) the first and second order derivatives

of g_λ^{ZP} can be easily derived from chain rules. It is convenient drop the competition types since the first and second order derivatives of the perceived elasticity of substitution under both types of competitions have the same sign. It can be shown that

$$\frac{\partial g_\lambda^{ZP}}{\partial e} < 0, \frac{\partial^2 g_\lambda^{ZP}}{\partial e^2} > 0.$$

From the chain rules

$$\frac{\partial g_\lambda^{ZP}}{\partial N} = \frac{\partial e}{\partial N} \frac{\partial g_\lambda^{ZP}}{\partial e}$$

and

$$\frac{\partial^2 g_\lambda^{ZP}}{\partial N^2} = \frac{\partial^2 e}{\partial N^2} \frac{\partial g_\lambda^{ZP}}{\partial e} + \frac{\partial^2 g_\lambda^{ZP}}{\partial e^2} \left(\frac{\partial e}{\partial N} \right)^2,$$

then follows that g_λ^{ZP} is a hyperbola in (g_λ, N) space, i.e.,

$$\frac{\partial g_\lambda^{ZP}}{\partial N} < 0, \frac{\partial^2 g_\lambda^{ZP}}{\partial N^2} > 0.$$

The derivations for *CME* curve require more algebra. For these derivations it necessary to have the first and second order derivatives of b with respect e , D with respect b , and g_λ^{CME} with respect D . It be shown that

$$\begin{aligned} \frac{\partial D}{\partial b} &> 0, \frac{\partial b}{\partial e} > 0, \frac{\partial g_\lambda^{CME}}{\partial D} > 0, \\ \frac{\partial^2 D}{\partial b^2} &< 0, \frac{\partial^2 b}{\partial e^2} < 0, \frac{\partial^2 g_\lambda^{CME}}{\partial D^2} < 0. \end{aligned}$$

From the signs of these derivatives and from the chain rules

$$\begin{aligned} \frac{\partial g_\lambda^{CME}}{\partial N} &= \frac{\partial e}{\partial N} \frac{\partial b}{\partial e} \frac{\partial D}{\partial b} \frac{\partial g_\lambda^{CME}}{\partial D}, \\ \frac{\partial^2 g_\lambda^{CME}}{\partial N^2} &= \frac{\partial^2 D}{\partial N^2} \frac{\partial g_\lambda^{CME}}{\partial D} + \frac{\partial^2 g_\lambda^{CME}}{\partial D^2} \left(\frac{\partial e}{\partial N} \frac{\partial b}{\partial e} \frac{\partial D}{\partial b} \right)^2, \\ \frac{\partial^2 D}{\partial N^2} &= \frac{\partial^2 b}{\partial N^2} \frac{\partial D}{\partial b} + \frac{\partial^2 D}{\partial b^2} \left(\frac{\partial e}{\partial N} \frac{\partial b}{\partial e} \right)^2, \\ \frac{\partial^2 b}{\partial N^2} &= \frac{\partial e^2}{\partial N^2} \frac{\partial b}{\partial e} + \frac{\partial^2 b}{\partial e^2} \left(\frac{\partial e}{\partial N} \right)^2, \end{aligned}$$

it follows that *CME* is a concave function in (g_λ, N) space, i.e.,

$$\frac{\partial g_\lambda^{CME}}{\partial N} > 0, \frac{\partial^2 g_\lambda^{CME}}{\partial N^2} < 0.$$

7.3.1 Stability of equilibrium

There are two conditions which guarantee a stable equilibrium. The first condition is that the entry of firms reduces the profit, equivalently the exit increases the profit. The second condition is that the slope of the rate of return from (53) is steeper in (r, g_λ) space than the same slope of the rate of return from (52). The later condition

implies that when g_λ is smaller than the one which satisfies both (53) and (52), the required rate of return (53) is less than the actual rate of return (52). Therefore, there is an incentive to raise the g_λ up to an equilibrium point where (53) and (52) are equal.

The first condition is automatically satisfied since CME is increasing and ZP is declining with N . The second condition will be satisfied if

$$(\theta - 1)B > -D.$$

This condition automatically holds when $\theta \geq 1$, which is the empirically valid case. Therefore, another parameter restriction is

$$\theta \geq 1.$$

Van de Klundert & Smulders (1997) have the same condition and restriction.

7.4 Appendix A.4 - The GDP growth rate on the balanced growth path when there are exogenous barriers to entry and infinitely many telecom firms compared to the same when there is entry

The GDP growth rate when there is no entry is given by

$$g|_{\dot{N}=0} = B|_{\dot{N}=0} \frac{\xi DL - \rho}{(\theta - 1) B|_{\dot{N}=0} + D},$$

where

$$B|_{\dot{N}=0} = \mu + \sigma.$$

The GDP growth rate when there is entry is given by

$$g|_{\dot{N} \neq 0} = B|_{\dot{N} \neq 0} \frac{\xi DL - \rho}{\theta B|_{\dot{N} \neq 0} + D},$$

where

$$B|_{\dot{N} \neq 0} = \frac{(\mu + \sigma)(\varepsilon - 1)}{\varepsilon - 1 - (\mu + \sigma)}.$$

The comparison of $g|_{\dot{N}=0}$ with $g|_{\dot{N} \neq 0}$ is equivalent to

$$B|_{\dot{N}=0} \frac{\xi DL - \rho}{(\theta - 1) B|_{\dot{N}=0} + D} * B|_{\dot{N} \neq 0} \frac{\xi DL - \rho}{\theta B|_{\dot{N} \neq 0} + D} \Leftrightarrow$$

$$(\varepsilon - 1) * D = \frac{b\sigma}{b\sigma + 1 - \sigma} \leq \sigma$$

Given that $\varepsilon > 1 + \mu + \sigma$, the constant GDP growth rate when there are exogenous barriers to entry and infinitely many telecom firms is higher than the same when there

is entry:

$$g|_{\dot{N}=0} \geq g|_{\dot{N} \neq 0}.$$

7.5 Appendix A.5 - The number of firms when there are endogenous barriers to entry

When there are endogenous barriers to entry, the number of firms can be determined from the intersection of *CME* and *ZP* curves, i.e.,

$$\frac{\xi DL - \rho}{(\theta - 1)B + D} = \frac{\rho}{e^k - 1 - (\theta - 1)B} \quad k = C, B.$$

From this equality it follows that

$$e^k = \frac{\xi L [(\theta - 1)B + 1]}{\xi L - \frac{\rho}{\sigma}} \quad k = C, B.$$

This is an important result. It indicates that the perceived elasticities of substitution between telecom goods are the same under both types of competitions. Moreover, it implies that the growth of productivity in telecom goods production is the same under both types of competitions. Therefore, it implies that the GDP growth rate is the same under both types of competitions.

Under Cournot competition

$$e^C \equiv \frac{\varepsilon}{1 + (\varepsilon - 1)\frac{1}{N}} = \frac{\xi L [(\theta - 1)B + 1]}{\xi L - \frac{\rho}{\sigma}};$$

thus,

$$N^C = (\varepsilon - 1) \frac{\xi L [(\theta - 1)(\sigma + \mu) + 1]}{\varepsilon (\xi L - \frac{\rho}{\sigma}) - \xi L [(\theta - 1)(\sigma + \mu) + 1]}. \quad (58)$$

Under Bertrand competition

$$e^B \equiv \varepsilon - \frac{(\varepsilon - 1)}{N} = \frac{\xi L [(\theta - 1)B + 1]}{\xi L - \frac{\rho}{\sigma}};$$

thus,

$$N^B = (\varepsilon - 1) \frac{\xi L - \frac{\rho}{\sigma}}{\varepsilon (\xi L - \frac{\rho}{\sigma}) - \xi L [(\theta - 1)(\sigma + \mu) + 1]}. \quad (59)$$

The following parameter restrictions should hold

1. $\xi L - \frac{\rho}{\sigma} > 0,$ (60)
2. $\xi L [\varepsilon - (\theta - 1)(\sigma + \mu) - 1] - \varepsilon \frac{\rho}{\sigma} > 0.$

The (60) follows from $\xi DL - \rho > 0$ given that $D < \sigma$. Therefore the only parameter restriction is

$$\xi L [\varepsilon - (\theta - 1)(\sigma + \mu) - 1] - \varepsilon \frac{\rho}{\sigma} > 0.$$

7.5.1 The number of firms under Cournot competition compared to the same under Bertrand competition

The comparison of N^C and N^B is equivalent to

$$\xi L (\theta - 1) (\sigma + \mu) * -\frac{\rho}{\sigma}.$$

Given the parameter restrictions, the left hand side is always greater than the right hand side. Therefore, the number of firms under Cournot competition is higher than the same under Bertrand competition:

$$N^C > N^B.$$

7.6 Appendix A.6 - Comparative statics for the number of telecom firms when there are endogenous barriers to entry

7.6.1 The change of N with $\xi, \varepsilon, \mu, \rho, \theta$

From (58) and (59), it can be shown that

$$\frac{\partial}{\partial \xi} N^k < 0, \frac{\partial}{\partial \varepsilon} N^k < 0, \frac{\partial}{\partial \mu} N^k > 0, \frac{\partial}{\partial \rho} N^k > 0, \frac{\partial}{\partial \theta} N^k > 0, \quad k = C, B. \quad (61)$$

Figure 2 may easily explain this pattern: (1) CME increases with ξ and ZP does not depend on ξ , therefore the number of firms declines; (2) CME increases with ε , while ZP declines with it, therefore the number of firms declines; and (3) CME declines with μ, ρ and θ , while ZP increases, therefore the number of firms increases.

7.6.2 The change of N with σ

Under Cournot competition

$$\frac{\partial}{\partial \sigma} N^C = (\varepsilon - 1) \frac{\xi L (\theta - 1) \varepsilon \left(\xi L - \frac{\rho}{\sigma} \right) - \xi L [(\theta - 1) (\sigma + \mu) + 1] \varepsilon \frac{\rho}{\sigma^2}}{\left(\varepsilon \left(\xi L - \frac{\rho}{\sigma} \right) - \xi L [(\theta - 1) (\sigma + \mu) + 1] \right)^2} * 0$$

\Rightarrow

$$\xi L (\theta - 1) \frac{\varepsilon \left(\xi L - \frac{\rho}{\sigma} \right)}{\xi L [(\theta - 1) (\sigma + \mu) + 1]} - \frac{\varepsilon \rho}{\sigma^2} * 0.$$

From parameter restriction $\frac{\varepsilon \left(\xi L - \frac{\rho}{\sigma} \right)}{\xi L [(\theta - 1) (\sigma + \mu) + 1]} > 1$ and having in mind that $\frac{\left(\xi L - \frac{\rho}{\sigma} \right)}{\xi L [(\theta - 1) (\sigma + \mu) + 1]} < 1$ the following is true

$$\xi L (\theta - 1) - \frac{\rho}{\sigma^2} > \xi L (\theta - 1) \frac{\varepsilon \left(\xi L - \frac{\rho}{\sigma} \right)}{\xi L [(\theta - 1) (\sigma + \mu) + 1]} - \frac{\varepsilon \rho}{\sigma^2} > \xi L (\theta - 1) - \frac{\varepsilon \rho}{\sigma^2};$$

thus,

$$\begin{aligned}\xi L(\theta - 1) - \frac{\varepsilon \rho}{\sigma^2} &> 0 \Rightarrow \frac{\partial}{\partial \sigma} N^C > 0, \\ \xi L(\theta - 1) - \frac{\rho}{\sigma^2} &< 0 \Rightarrow \frac{\partial}{\partial \sigma} N^C < 0.\end{aligned}$$

Under Bertrand competition

$$\begin{aligned}\frac{\partial}{\partial \sigma} N^B &= (\varepsilon - 1) \frac{\xi L(\theta - 1) \left(\xi L - \frac{\rho}{\sigma} \right) - \frac{\rho}{\sigma^2} \xi L [(\theta - 1)(\sigma + \mu) + 1]}{\left(\varepsilon \left(\xi L - \frac{\rho}{\sigma} \right) - \xi L [(\theta - 1)(\sigma + \mu) + 1] \right)^2} * 0 \\ \Rightarrow \\ \xi L(\theta - 1) \left(\xi L - \frac{\rho}{\sigma} \right) - \frac{\rho}{\sigma^2} \xi L [(\theta - 1)(\sigma + \mu) + 1] &* 0 \\ \Leftrightarrow \\ \xi L(\theta - 1) \varepsilon \left(\xi L - \frac{\rho}{\sigma} \right) - \xi L [(\theta - 1)(\sigma + \mu) + 1] \frac{\varepsilon \rho}{\sigma^2} &* 0.\end{aligned}$$

Therefore, similar to Cournot competition

$$\xi L(\theta - 1) - \frac{\rho}{\sigma^2} > \xi L(\theta - 1) \frac{\varepsilon \left(\xi L - \frac{\rho}{\sigma} \right)}{\xi L [(\theta - 1)(\sigma + \mu) + 1]} - \frac{\varepsilon \rho}{\sigma^2} > \xi L(\theta - 1) - \frac{\varepsilon \rho}{\sigma^2};$$

thus,

$$\begin{aligned}\xi L(\theta - 1) - \frac{\varepsilon \rho}{\sigma^2} &> 0 \Rightarrow \frac{\partial}{\partial \sigma} N^B > 0, \\ \xi L(\theta - 1) - \frac{\rho}{\sigma^2} &< 0 \Rightarrow \frac{\partial}{\partial \sigma} N^B < 0.\end{aligned}$$

Figure 2 may easily explain this pattern also. Both CME and ZP increase with σ , therefore the change in the number of firms depends on the relative change of CME and ZP , which is a function of model parameters.

7.7 Appendix A.7 - Comparative statics for the GDP growth when there are endogenous barriers to entry

It has been shown that the elasticities of substitution perceived by the telecom good producers are the same under both types of competition. They are equal to

$$e = e^B = e^C = \frac{\xi L [(\theta - 1)(\sigma + \mu) + 1]}{\xi L - \frac{\rho}{\sigma}}.$$

7.7.1 The change of GDP growth rate with the scale of direct network externalities (μ)

$$\frac{\partial g}{\partial \mu} = \frac{\partial B}{\partial \mu} \frac{\partial g}{\partial B} + \frac{\partial e}{\partial \mu} \frac{\partial b}{\partial e} \frac{\partial D}{\partial b} \frac{\partial g}{\partial D} > 0$$

Since both terms are positive, the change of GDP growth rate with the scale of direct

network externalities is positive:

$$\frac{\partial g}{\partial \mu} > 0.$$

7.7.2 The change of GDP growth rate with the efficiency of productivity improvements in telecom production (ξ)

$$\frac{\partial g}{\partial \xi} = g \frac{DL}{\xi DL - \rho} + \frac{\partial e}{\partial \xi} \frac{\partial b}{\partial e} \frac{\partial D}{\partial b} \frac{\partial g}{\partial D},$$

where

$$\begin{aligned} \frac{\partial e}{\partial \xi} &= -\frac{\rho}{\xi \sigma} \frac{e}{\xi L - \frac{\rho}{\sigma}}, \quad \frac{\partial b}{\partial e} = \frac{1}{e^2}, \\ \frac{\partial D}{\partial b} &= D \frac{(1 - \sigma)}{b(b\sigma + 1 - \sigma)}, \quad \frac{\partial g}{\partial D} = g \frac{1}{\xi DL - \rho} \frac{\xi L(\theta - 1)B + \rho}{(\theta - 1)B + D}. \end{aligned}$$

Substituting these expressions and B into $\frac{\partial g}{\partial \xi}$ yields

$$\frac{\partial g}{\partial \xi} = g \frac{D}{\xi DL - \rho} \left[L - \frac{\rho}{\xi \sigma} \frac{\xi L(\theta - 1)(\sigma + \mu) + \rho}{\xi L(\theta - 1)(\sigma + \mu) + \frac{\rho}{\sigma}} (1 - \sigma) \sigma b \frac{D}{(\theta - 1)(\sigma + \mu) + D} \right].$$

From

$$\frac{\xi L(\theta - 1)(\sigma + \mu) + \rho}{\xi L(\theta - 1)(\sigma + \mu) + \frac{\rho}{\sigma}}; (1 - \sigma); \sigma; b; \frac{D}{(\theta - 1)(\sigma + \mu) + D} \leq 1,$$

and

$$D \leq \sigma, \xi DL - \rho \geq 0 \Rightarrow \xi \sigma L - \rho \geq 0,$$

it follows that the GDP growth rate increases with the efficiency in productivity improvement,

$$\frac{\partial g}{\partial \xi} \geq 0.$$

7.8 Appendix A.8 - When ε is a choice variable

In this section I show that the telecom good producers prefer to enter a market where the elasticity of substitution between telecom goods ε is low. This means that if the telecom firms had a chance to set ε at time zero and after they know the prices for telecom goods for any ε , they would set it as low as possible. Given that the costs of a telecom firm are not a function of ε , it is sufficient to discuss the effect of ε on the revenue of the j th telecom firm, under different types of competition.

Under Cournot competition the revenue of the j th telecom firm R_j^C can be obtained by substituting the inverse demand function instead of the price of the j th telecom good:

$$R_j^C = P_X \left(\sum_{i=1}^N x_i^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}} x_j^{\frac{\varepsilon-1}{\varepsilon}}.$$

The derivative of R_j^C with respect to ε is

$$P_X \left(\sum_{i=1}^N x_i^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{1}{\varepsilon-1}} x_j^{\frac{\varepsilon-1}{\varepsilon}} \left[\frac{1}{\varepsilon^2} \ln x_j + \left(-\frac{\ln \left(\sum_{i=1}^N x_i^{\frac{\varepsilon-1}{\varepsilon}} \right)}{(\varepsilon-1)^2} + \frac{1}{\varepsilon^2} \frac{\sum_{i=1}^N x_i^{\frac{\varepsilon-1}{\varepsilon}} \ln x_i}{\sum_{i=1}^N x_i^{\frac{\varepsilon-1}{\varepsilon}}} \right) \right].$$

Under symmetry this expression reduces to

$$-P_X N^{\frac{1}{\varepsilon-1}} x \frac{\ln N}{(\varepsilon-1)^2},$$

which is less than zero when the number of firms is greater than one.

Under Bertrand competition the revenue of the j th telecom firm R_j^B can be obtained by substituting the demand function instead of the production of the j th telecom good:

$$R_j^B = \sigma Y \left(\frac{p_{x_j}}{P_X} \right)^{1-\varepsilon}$$

The derivative of R_j^B with respect to ε is

$$-\sigma Y \left(\frac{p_{x_j}}{P_X} \right)^{1-\varepsilon} \ln \left(\frac{p_{x_j}}{P_X} \right).$$

Under symmetry this expression reduces to

$$-\sigma Y N \frac{1}{\varepsilon-1} \ln N,$$

which is less than zero when number of the firms is greater than one, given that $\varepsilon > 1 + \sigma + \mu$.

Therefore, the revenue reduces with higher elasticity of substitution, or lower product differentiation, and the telecom firms have an incentive to enter a market where ε is lower or try to set a low ε .

7.9 Appendix S.1 - Social optimum

Denote

$$a_1 = \frac{\varepsilon}{\varepsilon-1}(\sigma + \mu), a_2 = \sigma + \mu, a_3 = 1 - \sigma.$$

From (3) it follows that

$$Y = N^{a_1} \lambda^{a_2} L_x^{a_2} L_Y^{a_3}.$$

The Social Planner's problem then is

$$\max U = \int_0^{\infty} \frac{C_t^{1-\theta}}{1-\theta} e^{-\rho t} dt$$

s.t.

$$C + S = Y \quad (62)$$

$$Y = N^{a_1} \lambda^{a_2} L_x^{a_2} L_Y^{a_3} \quad (63)$$

$$L_Y = L - N(L_x + L_r) \quad (64)$$

$$\dot{N} = \eta S \quad (65)$$

$$\dot{\lambda} = \xi N \lambda L_r \quad (66)$$

The problem, if written in terms of current value Hamiltonian, is given by

$$\max_{\{C, S, L_x, L_r\}} H_S = \frac{C^{1-\theta}}{1-\theta} + q_Y [Y - C - S] + q_N \eta S + q_\lambda \xi N \lambda L_r, \quad (67)$$

where the q_N and q_λ are the shadow values of increasing the number of firms and improving the productivity, respectively. The first order conditions and the state evolution laws are

$$[C] : \quad C^{-\theta} = q_Y, \quad (68)$$

$$[S] : \quad q_Y = \eta q_N, \quad (69)$$

$$[L_x] : \quad N L_x = \frac{a_2}{a_3} L_Y, \quad (70)$$

$$[L_r] : \quad q_Y a_3 \frac{Y}{L_Y} = q_\lambda \xi \lambda, \quad (71)$$

$$[N] : \quad \dot{q}_N = \rho q_N - \frac{\partial H_S}{\partial N}, \quad (72)$$

$$[\lambda] : \quad \dot{q}_\lambda = \rho q_\lambda - \frac{\partial H_S}{\partial \lambda}. \quad (73)$$

From (64) and (70), it follows that

$$N L_x = \frac{a_2}{a_2 + a_3} [L - N L_r] \quad (74)$$

The equations (72) and (73) are given by

$$g_{q_N} = \rho - \left\{ \frac{q_\lambda}{q_N} \xi \lambda L_r + \frac{q_Y}{q_N} \left[a_1 \frac{Y}{N} - a_3 \frac{Y}{N} \left(\frac{L - L_Y}{L_Y} \right) \right] \right\}, \quad (75)$$

$$g_{q_\lambda} = \rho - \left(\frac{q_Y}{q_\lambda} a_2 \frac{Y}{\lambda} + \xi N L_r \right). \quad (76)$$

From (71), (76), and (70), it follows that

$$g_{q\lambda} = \rho - \left(\frac{a_2}{a_2 + a_3} \xi L + \frac{a_3}{a_2 + a_3} \xi N L_r \right).$$

From (66), (71) and (68), it follows that

$$-\theta g + g_Y - g_{L_Y} = \rho - \frac{a_2}{a_2 + a_3} (\xi L - g_\lambda). \quad (77)$$

A bit of algebra is required in order to show that the economy grows at constant rates in the social optimum. Using the (75), (70), (71), (74) (76), log-differentiated (70), entry condition (7), productivity improvement process (9), and the production function of the final goods (3), it can be shown that

$$g_{L_Y} = -\frac{a_2 \xi}{a_2 + a_3 - 1} L + \frac{a_2}{a_3} \xi L_Y + \frac{a_1 - a_2}{a_2 + a_3 - 1} \eta \frac{C}{N}. \quad (78)$$

Given that $\frac{a_2}{a_3} \xi$ and $\frac{a_1 - a_2}{a_2 + a_3 - 1} \eta$ are positive and C increases with Y , which (in a period) increases with L_Y , the growth rate of L_Y increases with L_Y . Therefore, if L_Y grows, the optimal rule (70) will not hold after sometime, which is a contradiction. Consequently, the labor allocation in the final goods production is constant over time in the social optimum. This, from (70) and (74), implies that other labor allocations are also constant and the economy always grows at constant rates.

Given that the labor allocations are constant, from (62) and (77), it follows that

$$(\theta - 1)g + \rho = \frac{a_2}{a_2 + a_3} (\xi L - g_\lambda). \quad (79)$$

In its turn, from (63), (65), and (62), it follows that

$$g = \frac{a_2}{1 - (a_1 - a_2)} g_\lambda, \quad (80)$$

where $1 - (a_1 - a_2) = \frac{\varepsilon - 1 - (\sigma + \mu)}{\varepsilon - 1}$ should be greater than zero. Denote

$$\begin{aligned} D^S &= \frac{a_2}{a_2 + a_3} \equiv \frac{\sigma + \mu}{1 + \mu}, \\ B^S &= \frac{a_2}{1 - (a_1 - a_2)} \equiv \frac{(\varepsilon - 1)(\sigma + \mu)}{\varepsilon - 1 - (\sigma + \mu)}. \end{aligned}$$

From (79) and (80), it follows that

$$g_\lambda = \frac{\xi D^S L - \rho}{(\theta - 1)B^S + D^S},$$

and

$$g = B^S g_\lambda.$$

Denote these growth rates by g^S and g_λ^S respectively. In addition to $\varepsilon > 1 + \sigma + \mu$, a parameter restriction is $\xi D^S L - \rho > 0$. The optimal conditions (68) and (69) suggest that in the social optimum there is permanent entry.

7.9.1 Socially optimal labor allocations

$$\begin{aligned} NL_r^S &= \frac{1}{\xi} g_\lambda^S, \\ NL_x^S &= \frac{\sigma + \mu}{1 + \mu} [L - NL_r^S], \\ L_Y^S &= \frac{1 - \sigma}{\sigma + \mu} NL_x^S. \end{aligned}$$

7.9.2 When ε is a choice variable

In (67) only Y is a function of ε . Therefore, the sign of the derivative of Y with respect to ε is sufficient in order to infer the choice of the Social Planner,

$$\frac{\partial Y}{\partial \varepsilon} = \frac{\partial a_1}{\partial \varepsilon} \frac{\partial N^{a_1}}{\partial a_1} \frac{\partial Y}{\partial N^{a_1}},$$

where

$$\frac{\partial a_1}{\partial \varepsilon} = -\frac{\sigma + \mu}{(\varepsilon - 1)^2}, \quad \frac{\partial N^{a_1}}{\partial a_1} = N^{a_1} \ln N, \quad \frac{\partial Y}{\partial N^{a_1}} = \frac{Y}{N^{a_1}}.$$

Therefore, when the number of firms is greater or equal 1, the derivative has a negative sign:

$$\frac{\partial Y}{\partial \varepsilon} < 0.$$

Since the sign is negative, the Social Planner would prefer lower ε . This is evident also from the formulae of Y . The growth rate of Y increases with lower ε .

7.10 Appendix S.2 - Comparative statics for the socially optimal GDP growth rate

7.10.1 The change of GDP growth rate with $\mu, \sigma, \varepsilon, \xi, \rho, \theta, (1 - \alpha)$

It can be shown that

$$\frac{\partial g^S}{\partial B^S} > 0, \quad \frac{\partial B^S}{\partial \mu} > 0, \quad \frac{\partial g^S}{\partial D^S} > 0, \quad \frac{\partial D^S}{\partial \mu} > 0, \quad \frac{\partial D^S}{\partial \sigma} > 0, \quad \frac{\partial B^S}{\partial \sigma} > 0, \quad \frac{\partial B^S}{\partial \varepsilon} < 0 \quad (81)$$

From (81) it can be shown that

$$\frac{\partial g^S}{\partial \mu} > 0, \quad \frac{\partial g^S}{\partial \sigma} > 0, \quad \frac{\partial g^S}{\partial \varepsilon} < 0, \quad \frac{\partial g^S}{\partial \xi} > 0, \quad \frac{\partial g^S}{\partial \rho} < 0, \quad \frac{\partial g^S}{\partial \theta} < 0, \quad \frac{\partial g^S}{\partial 1 - \alpha} = 0.$$

7.11 Appendix S.3 - The socially optimal growth rate of productivity in telecom goods production compared to the decentralized equilibrium one

The growth rates of the productivity in telecom goods production in the social optimum and in the decentralized equilibrium are

$$g_\lambda^S = \frac{\xi D^S L - \rho}{(\theta - 1)B^S + D^S}, g_\lambda = \frac{\xi DL - \rho}{(\theta - 1 + \delta)B + D},$$

respectively. Comparison of these growth rates is equivalent to comparison of the following expression with zero:

$$\xi L [(\theta - 1)B^S D - (\theta - 1 + \delta)BD^S] + \rho [(\theta - 1 + \delta)B + D - (\theta - 1)B^S - D^S]. \quad (82)$$

7.11.1 In the case where there is an entry in the decentralized equilibrium

Since in this case $B = B^S$, the comparison of (82) with zero is equivalent to

$$B [D - D^S] (\xi L \theta + \rho) - B (\xi DL - \rho) * 0.$$

The parameter restriction $\xi DL - \rho > 0$ and

$$D \leq \sigma \leq \frac{\sigma + \mu}{1 + \mu} = D^S$$

imply that the socially optimal growth rate is higher than the decentralized equilibrium one:

$$g_\lambda \leq g_\lambda^S.$$

7.11.2 In the case where there is no entry in the decentralized equilibrium

It can be shown that $\frac{\partial D}{\partial N} > 0$ and $\frac{\partial g_\lambda}{\partial D} > 0$; therefore, it is sufficient to take $N = \infty$ if interested in $g_\lambda \leq g_\lambda^S$. According to (82), when $N = \infty$, there is no entry in the decentralized equilibrium, and $g_\lambda \leq g_\lambda^S$, the following holds

$$\begin{aligned} & \xi L (\theta - 1) \left[\frac{(\mu + \sigma)(\varepsilon - 1)}{\varepsilon - 1 - (\mu + \sigma)} \frac{\varepsilon - 1}{\varepsilon - \sigma} \sigma - (\sigma + \mu) \frac{\sigma + \mu}{1 + \mu} \right] \\ & + \rho \left((\theta - 1) \left(-\frac{(\mu + \sigma)^2}{\varepsilon - 1 - (\mu + \sigma)} \right) + \frac{\varepsilon - 1}{\varepsilon - \sigma} \sigma - \frac{\sigma + \mu}{1 + \mu} \right) \\ & \leq 0. \end{aligned}$$

Given that $\frac{\sigma+\mu}{1+\mu} \geq \sigma$, a sufficient condition for $g_\lambda \leq g_\lambda^S$ is

$$\begin{aligned} & \xi L(\theta - 1)\sigma(\mu + \sigma) \left[\frac{(\varepsilon - 1)^2 - (\varepsilon - \sigma)(\varepsilon - 1 - (\mu + \sigma))}{(\varepsilon - 1 - (\mu + \sigma))(\varepsilon - \sigma)} \right] \\ & + \rho \left((\theta - 1) \left(-\frac{(\mu + \sigma)^2}{\varepsilon - 1 - (\mu + \sigma)} \right) + \frac{\varepsilon - 1}{\varepsilon - \sigma} \sigma - \frac{\sigma + \mu}{1 + \mu} \right) \\ & \leq 0. \end{aligned}$$

The second bracket is always negative; therefore, for $g_\lambda \leq g_\lambda^S$ it is sufficient to have

$$(\varepsilon - 1)^2 - (\varepsilon - \sigma)(\varepsilon - 1 - (\mu + \sigma)) \leq 0,$$

or equivalently

$$\varepsilon \geq \frac{1 - \sigma(1 + \mu + \sigma)}{1 - \mu - 2\sigma}.$$

The last ratio increases with σ ; therefore, it would be sufficient if the inequality holds when $\sigma \approx 1$. When $\sigma \approx 1$, this ratio is equal to unity, and under the parameter restriction $\varepsilon > 1 + \mu + \sigma$, the inequality always holds. Therefore, the socially optimal growth rate of the productivity in telecom goods production is always higher than the one in the decentralized equilibrium:

$$g_\lambda \leq g_\lambda^S.$$

7.12 Appendix S.4 - The socially optimal labor allocations compared to the ones of the decentralized equilibrium

7.12.1 In the case where there is an entry in the decentralized equilibrium

$$NL_r = \frac{g_\lambda}{\xi} \leq \frac{g_\lambda^S}{\xi} = NL_r^S.$$

From (47) and (74) follows that

$$\begin{aligned} NL_x &= \frac{1}{\xi} [\theta g + \rho], \\ NL_x^S &= \frac{1}{\xi} [(\theta - 1)g^S + \rho]. \end{aligned}$$

Therefore, the comparison of NL_x and NL_x^S is equivalent to the comparison of

$$\theta g * (\theta - 1)g^S.$$

When θ is close to 1, $NL_x > NL_x^S$. However, when θ is sufficiently high, $NL_x < NL_x^S$ since $g_\lambda < g_\lambda^S$.

7.12.2 In the case where there is no entry in decentralized equilibrium

From (47) and (74), it follows that

$$\begin{aligned} NL_x &= \frac{1}{\xi} [(\theta - 1)g + \rho], \\ NL_x^S &= \frac{1}{\xi} [(\theta - 1)g^S + \rho]. \end{aligned}$$

Given that

$$g \leq g^S,$$

it follows that

$$NL_x \leq NL_x^S.$$

In turn, given that $g_\lambda \leq g_\lambda^S$, the following holds

$$NL_r \leq NL_r^S. \quad (83)$$

When there are exogenous barriers to entry, the growth rate of productivity in telecom goods production g_λ and the GDP growth rate g increase with the number of firms N and with the toughness of competition. Therefore, NL_x and NL_r increase with N and toughness of competition. This means that as N increases and/or the competition becomes tougher, NL_x tends to NL_x^S , and NL_r tends to NL_r^S .

The proof for the inequality (83) in the transition period of the decentralized equilibrium is based on three observations. First, during the transition the number of firms increases. Second, the balanced growth path follows the transition. Third, NL_r is an increasing function of the number of firms. Therefore, it is sufficient to compare the balanced growth path NL_r to NL_r^S .

7.13 Appendix S.5 - Socially optimal GDP growth rate compared to the one of decentralized equilibrium

Since $\frac{\partial g}{\partial B} > 0$, $\frac{\partial g}{\partial D} > 0$ and

$$\begin{aligned} B &= \frac{(\mu + \sigma)(\varepsilon - 1)}{\varepsilon - 1 - \delta(\mu + \sigma)} \leq \frac{(\sigma + \mu)(\varepsilon - 1)}{\varepsilon - 1 - (\sigma + \mu)} = B^S, \\ D^{CE} &= \frac{b\sigma}{b\sigma + 1 - \sigma} \leq \sigma \leq \frac{\sigma + \mu}{1 + \mu} = D^S, \\ \theta - 1 + \delta &\geq \theta - 1, \end{aligned}$$

the socially optimal GDP growth rate is higher than the one of the decentralized equilibrium,

$$g < g^S.$$

The inequality is strict since even when $\varepsilon = \infty$ and $\mu = 0$ ($\Rightarrow D = D^S$), δ should be equal to 1 in order to have $B = B^S$. However, in that case the denominator of the growth rate from the decentralized equilibrium increases.

7.14 Appendix DE.1 - Policies that deliver socially optimal allocations as a decentralized equilibrium outcome

The socially optimal outcome differs from the decentralized equilibrium outcome in three ways. First, the resource allocations are different. Second, there is permanent entry in the social optimum. Third, the returns on productivity improvements do not decline with entry of firms in social optimum. The policy which can deliver the first best allocations in the decentralized equilibrium should take into account all these aspects.

In this section I derive two policies which deliver the socially optimal allocations as a decentralized equilibrium outcome. The policies subsidize telecom goods production, productivity improvements and guarantee that the profit of any telecom firm is never negative. Other policies may subsidize the demand for telecom goods instead of subsidizing the production.

Everything else being the same, the j th telecom good producer's problem under these policies is

$$\max V_t = \int_t^{\infty} \{p_{x_{j,\tau}} x_{j,\tau} - (L_{x_{j,\tau}} (1 - \tau_{L_x}) + L_{r_{j,\tau}} (1 - \tau_{L_r})) w_{\tau} \quad (84)$$

$$\dots - \sum_{i=1, i \neq j}^N p_{u_{i,j} \lambda_{i,\tau}} (u_{i,j,\tau} \lambda_{i,\tau}) + \sum_{i=1, i \neq j}^N p_{u_{j,i} \lambda_{j,\tau}} (u_{j,i,\tau} \lambda_{j,\tau})$$

$$\dots + \tau_{\lambda} p_{\lambda_j} \lambda_{j,\tau} + T_{\pi,j,\tau} \} e^{-\int_t^{\tau} r_s ds} d\tau$$

s.t.

$$x_{j,\tau} = \lambda_{j,\tau} L_{x_{j,\tau}} \quad (85)$$

$$\dot{\lambda}_{j,\tau} = \xi \left(\sum_{i=1}^N (u_{i,j,\tau} \lambda_{i,\tau})^{\alpha} \right) \lambda_{j,\tau}^{1-\alpha} L_{r_{j,\tau}} \quad (86)$$

where the policy is given by $(\tau_{L_x}, \tau_{L_r}, \tau_{\lambda}, T_{\pi,j})$. τ_{L_x} and τ_{L_r} can fix the resource misallocations. $\tau_{\lambda} p_{\lambda_j} \lambda_{j,\tau}$ can cancel the decline in the rate of return on productivity improvements due to firm entry. $T_{\pi,j}$ is a lump-sum transfer which guarantees non-negative profits, therefore, permanent entry. This policy is financed by lump-sum tax G imposed on the household.

7.14.1 Balanced growth path under symmetric equilibrium

Let a symmetric equilibrium hold. The optimal rules can be derived as in the decentralized equilibrium. Compared to the decentralized equilibrium scenario, the only market clearing condition which changes is the budget constraint. Here it is given by

$$\dot{A} = rA + wL - C - G,$$

where G is the lump-sum tax, which finances the policy, i.e.,

$$G = N \left[\frac{w}{N} (\tau_{L_x} N L_x + \tau_{L_r} N L_r) + \tau_{\lambda} p_{\lambda} \lambda + T_{\pi} \right].$$

When there is permanent entry, which G is supposed to guarantee, G should grow at rate of N on the balanced growth path. This can be assumed to hold and verified later.

With derivations similar to those for market equilibrium, it can be shown that

$$g^{DE} = Bg_\lambda^{DE},$$

where g^{DE} is the GDP growth rate and g_λ^{DE} is the growth rate of productivity, which is given by

$$g_\lambda^{DE} = \frac{D_{DE}\xi L - \rho}{(\theta - 1)B + B - \alpha\tilde{t}_\lambda - \frac{g^{(1-\tau_{L_r})}}{g_\lambda^{DE}} + D_{DE}}, \quad (87)$$

where

$$D_{DE} = \left[\frac{1 - T_{L_r}}{1 - T_{L_x}} \left(\frac{1 - \sigma}{b\sigma} (1 - T_{L_x}) + 1 \right) \right]^{-1},$$

$$\tilde{t}_\lambda = \frac{\tau_\lambda}{N}.$$

Moreover, similarly it can be shown that

$$\xi NL_r = g_\lambda^{DE},$$

$$L_r = \frac{g_\lambda^{DE}}{r - \{g_w + g^{(1-\tau_{L_r})} - g_N + \alpha g_\lambda^{DE} \tilde{t}_\lambda\}} \frac{1 - \tau_{L_x}}{1 - \tau_{L_r}} L_x, \quad (88)$$

$$NL_x (1 - T_{L_x}) \frac{1 - \sigma}{b\sigma} = L_Y. \quad (89)$$

7.14.2 Policy 1 - Equal subsidies to telecom goods production and to investment for productivity improvement

The policy should make sure that

$$g_\lambda^{DE} = g_\lambda^S,$$

$$NL_x = NL_x^S \Leftrightarrow D_{DE} = D^S \text{ and } g_\lambda^{DE} = g_\lambda^S,$$

and

$$T_\pi : \pi = \text{const} > 0.$$

Therefore, let

$$\tau_{L_x} = \tau_{L_r}$$

and

$$\alpha\tilde{t}_\lambda = B - \frac{g^{(1-\tau_{L_r})}}{g_\lambda^{DE}}.$$

From $D_{DE} = D^S$ it follows that

$$\tau_{L_x} = \tau_{L_r} = 1 - b \frac{\sigma}{\sigma + \mu}.$$

Therefore, when $N = \infty$ or $N = \text{const}$, $g_{(1-\tau_{Lr})} = 0$. Moreover, T_π is such that $\pi = \text{const} > 0$, where

$$\begin{aligned}\pi &= b \frac{\sigma}{\sigma + \mu} \frac{w}{N} (NL_x^S) \left\{ \frac{1}{e-1} - \frac{NL_r^S}{NL_x^S} \left[1 - \alpha \left(B - \frac{g_{(1-\tau_{Lr})}^S}{g_\lambda^S} \right) \right] \right\} + T_\pi, \\ &\equiv \pi_1 + T_\pi.\end{aligned}$$

Let

$$T_\pi = \nu - \pi_1,$$

where ν is determined from (75) given that in both cases the returns r should be the same

$$\begin{aligned}\nu &= (a_1 - a_2) \left(\frac{Y}{N} \right)^S \\ &= \frac{\theta}{\eta} (g^S + \rho).\end{aligned}$$

Note that when without any policy interventions there is permanent entry, T_π can be set to zero.

Under this policy,

$$\begin{aligned}\tau_{L_x} &= \tau_{L_r} = 1 - b \frac{\sigma}{\sigma + \mu}, \\ \alpha \tilde{t}_\lambda &= B - \frac{g_{(1-\tau_{Lr})}^S}{g_\lambda^{DE}}, \\ T_\pi &: \pi = \nu > 0,\end{aligned}$$

the GDP growth rates and the growth rates of productivity in telecom goods production in the decentralized equilibrium and the social optimum coincide, i.e.,

$$\begin{aligned}g^{DE} &= g^S, \\ g_\lambda^{DE} &= g_\lambda^S.\end{aligned}$$

Therefore,

$$NL_r^{DE} = NL_r^S.$$

Moreover,

$$\begin{aligned}NL_x^{DE} &= \frac{1}{\xi} [(\theta - 1) B g_\lambda^{DE} + \rho] \\ &= \frac{1}{\xi} [(\theta - 1) B g_\lambda^S + \rho] \\ &= NL_x^S.\end{aligned}$$

From $NL_r^{DE} = NL_r^S$ and $NL_x^{DE} = NL_x^S$ it follows that

$$L_Y^{DE} = L_Y^S.$$

Therefore, under this policy the growth rates and the labor allocations are the same in decentralized equilibrium and social optimum.

It remains to be verified that this policy satisfies the balanced growth path. It can be shown that

$$G = N \left[\nu + NL_r^S \frac{w}{N} \right].$$

Therefore, G is proportional to N and the policy satisfies the balanced growth path.

7.14.3 Policy 2 - Without subsidies proportional to the knowledge for telecom goods production

This policy does not transfer payments proportional to the knowledge. In this case τ_{L_x} and τ_{L_r} will not be equal. This policy (1) corrects the ratio of labor force employed in the productivity improvement process and in telecom goods production; (2) corrects the ratio of labor force employed in final and telecom goods productions; and (3) if needed transfers lump-sum payments in order to guarantee permanent entry. Therefore,

$$\begin{aligned} (i) \quad & \tau_\lambda = 0, \\ (ii) \quad & \frac{1 - \tau_{L_x}}{1 - \tau_{L_r}} = \frac{g_\lambda}{r - g_w} \frac{r - \{g_w + g_{(1-\tau_{L_r})} - g_N\}}{g_\lambda} \Rightarrow NL_r = \frac{g_\lambda}{r - g_w} NL_x, \\ (iii) \quad & D_{DE} = D^S \Leftrightarrow \frac{b\sigma}{b\sigma + (1 - \sigma)(1 - \tau_{L_x})} \frac{1 - \tau_{L_x}}{1 - \tau_{L_r}} = \frac{\sigma + \mu}{1 + \mu}, \\ (iv) \quad & T_\pi : \pi = \nu > 0 \Rightarrow B = B^S. \end{aligned}$$

From (i) it follows that $\tilde{t}_\lambda = 0$. From (ii) it will follow that $\frac{NL_r}{NL_x}$ is the socially optimal one once the growth rate of productivity in telecom good production is the same as the socially optimal one. From (iii) it will follow that once $\frac{NL_r}{NL_x}$ is equal to the socially optimal one, $\frac{L_Y}{NL_x}$ is equal to the socially optimal one as well. From (ii), (iii), and (iv) it will follow that the growth rate of productivity improvement is the socially optimal one.

The subsidy rates τ_{L_x} and τ_{L_r} can be solved from (ii) and (iii), i.e.,

$$\begin{aligned} 1 - \tau_{L_x} &= \frac{b\sigma}{1 - \sigma} \left(\frac{1 + \mu}{\sigma + \mu} \frac{r - \{g_w + g_{(1-\tau_{L_r})} - g_N\}}{g_\lambda} - 1 \right), \\ 1 - \tau_{L_r} &= (1 - \tau_{L_x}) \frac{r - g_w}{r - \{g_w + g_{(1-\tau_{L_r})} - g_N\}}. \end{aligned}$$

For this policy to be consistent with the balanced growth path, the combination of τ_{L_x} , τ_{L_r} and T_π should be constant on it, or

$$G = N [w(\tau_{L_x} L_x + \tau_{L_r} L_r) + T_\pi],$$

should grow at the rate of N . This is verified at the end of this section. Under this policy the relation between NL_x and L_Y is

$$NL_x (1 - \tau_{L_x}) \frac{1 - \sigma}{b\sigma} = L_Y.$$

From this expression it follows that

$$1 - \tau_{L_x} = b \frac{\sigma}{\sigma + \mu},$$

and

$$1 - \tau_{L_r} = b \frac{\sigma}{\sigma + \mu} \frac{r - g_w}{r - \{g_w + g_{(1-\tau_{L_r})} - g_N\}}.$$

From labor market clearing and (88) it follows that

$$\begin{aligned} L &= \left[\frac{1 - \sigma}{b\sigma} (1 - \tau_{L_x}) + 1 \right] \times \\ &\quad \times \frac{r - g_w}{g_\lambda} NL_r + NL_r. \end{aligned}$$

From (86) it follows that

$$g_\lambda = \xi NL_r.$$

Therefore,

$$r - g_w = D_{DE}(\xi L - g_\lambda). \quad (90)$$

From (3), (11), (54), given that the policy satisfies the balanced growth path,

$$g^{DE} = Bg_\lambda^{DE},$$

and

$$r = \theta Bg_\lambda^{DE} + \rho. \quad (91)$$

By equating (90) and (91) the following holds:

$$g_\lambda^{DE} = \frac{D_{DE}\xi L - \rho}{(\theta - 1)B + D_{DE}} = g_\lambda^S.$$

Therefore,

$$NL_r = NL_r^S.$$

Moreover,

$$\begin{aligned} NL_x &= \frac{1}{\xi} [(\theta - 1)Bg_\lambda^{DE} + \rho], \\ &= \frac{1}{\xi} [(\theta - 1)Bg_\lambda^S + \rho], \\ &= NL_x^S, \end{aligned}$$

and

$$NL_r = NL_r^S; NL_x = NL_x^S \Rightarrow L_Y = L_Y^S.$$

In a similar way as in the previous case it can be shown that the lump-sum tax that financed this policy is proportional to the number of firms and grows at a constant rate.

7.15 Appendix DE.2 - Second best policies

7.15.1 A policy without one of the two components that fix the resource misallocations is not optimal

There are three targets for any policy. The policy should fix two labor force misallocations and the GDP growth rate. It should set

$$\begin{aligned}
 1. & NL_r^{DE} = NL_r^S \Leftrightarrow g_\lambda^{DE} = g_\lambda^S, \\
 2. & NL_x^{DE} = NL_x^S \Leftrightarrow \\
 & \left\{ \left[(\theta - 1 + \delta) B^{DE} - \alpha \tilde{t}_\lambda - \frac{g(1-T_{L_r})}{g_\lambda^{DE}} \right] g_\lambda^{DE} + \rho \right\} \frac{1 - \tau_{L_r}}{1 - \tau_{L_x}} = (\theta - 1) B^S g_\lambda^S + \rho, \\
 3. & g^{DE} = g^S \Leftrightarrow B^{DE} g_\lambda^{DE} = B^S g_\lambda^S.
 \end{aligned}$$

7.15.1.1 A policy only with τ_{L_r} (unconditional on the presence of permanent entry)

Let this policy correct the labor force allocation to productivity improvement. Therefore, this policy sets g_λ^{DE} equal to g_λ^S , where the last is a given constant. Therefore, the following holds:

$$D_{DE} = \frac{g_\lambda^S (\theta - 1 + \delta) B^{DE} + \rho}{\xi L - g_\lambda^S},$$

or alternatively

$$1 - T_{L_r} = \frac{b\sigma}{b\sigma + 1 - \sigma} \frac{\xi L - g_\lambda^S}{g_\lambda^S (\theta - 1 + \delta) B^{DE} + \rho}.$$

What remains to be shown is that this policy would fail to fix the labor force allocation to telecom goods production (i.e., $NL_x^{DE} = NL_x^S$). Given that $NL_r^{DE} = NL_r^S$ a necessary and sufficient condition for the equality of NL_x^{DE} and NL_x^S is

$$\frac{L_Y^{DE}}{NL_x^{DE}} = \frac{L_Y^S}{NL_x^S},$$

where

$$\begin{aligned}
 \frac{L_Y^{DE}}{NL_x^{DE}} &= \frac{1 - \sigma}{b\sigma}, \\
 \frac{L_Y^S}{NL_x^S} &= \frac{1 - \sigma}{\sigma + \mu}.
 \end{aligned}$$

Therefore, it should be the case that

$$b = \frac{\sigma + \mu}{\sigma} \geq 1.$$

This cannot be the case given that by definition $b < 1$. Note that if the policy is not able to correct this ratio it does not matter whether it first sets the equality between NL_r^{DE} and NL_r^S or NL_x^{DE} and NL_x^S . Let it first set the equality between NL_x^{DE} and

NL_x^S , given that the ratio $\frac{L_Y^{DE}}{NL_x^{DE}}$ is not equal to $\frac{L_Y^S}{NL_x^S}$, NL_r^{DE} would not be equal to NL_r^S ,

$$\begin{aligned} L &= L_Y^{DE} + NL_x^{DE} + NL_r^{DE} \Rightarrow \\ NL_r^{DE} &= L - \left(\frac{L_Y^{DE}}{NL_x^{DE}} + 1 \right) NL_x^S \neq NL_r^S. \end{aligned}$$

7.15.1.2 A policy only with τ_{L_r} and τ_λ (unconditional on the presence of permanent entry)

This policy would still fail to fix the labor force allocation in telecom goods production (or in the productivity improvement process) due to the same logic as for the previous policy. It would not be able to equate the ratios $\frac{L_Y^{DE}}{NL_x^{DE}} = \frac{L_Y^S}{NL_x^S}$.

7.15.1.3 A policy only with τ_{L_x} (unconditional on the presence of permanent entry)

Let again this policy correct the labor force allocation to productivity improvement. Therefore, this policy sets the g_λ^{DE} equal to g_λ^S , where the last is a given constant. Consequently, the following holds

$$1 - \tau_{L_x} = \frac{1}{\frac{\xi L - g_\lambda^S}{g_\lambda^S(\theta - 1 + \delta)B^{DE} + \rho} - \frac{1 - \sigma}{b\sigma}}.$$

What remains to be shown is that this policy would fail to fix the labor force allocation to telecom goods production. Again this can be shown in terms of the ratios

$$\frac{L_Y^{DE}}{NL_x^{DE}} = \frac{L_Y^S}{NL_x^S}, \quad (92)$$

where in this case

$$\begin{aligned} \frac{L_Y^{DE}}{NL_x^{DE}} &= \frac{1 - \sigma}{b\sigma} (1 - T_{L_x}), \\ \frac{L_Y^S}{NL_x^S} &= \frac{1 - \sigma}{\sigma + \mu}. \end{aligned}$$

From (92) it follows that

$$\left(\frac{\sigma + \mu}{1 + \mu} \xi L - \rho \right) \left[(\theta - 1 + \delta) B^{DE} + \frac{b\sigma}{1 + \mu} \right] = \left(\frac{b\sigma}{1 + \mu} \xi L - \rho \right) \left[(\theta - 1) B^S + \frac{\sigma + \mu}{1 + \mu} \right],$$

or

$$\begin{cases} \frac{\sigma + \mu}{1 + \mu} \left[(\theta - 1 + \delta) B^{DE} + \frac{b\sigma}{1 + \mu} \right] - \frac{b\sigma}{1 + \mu} \left[(\theta - 1) B^S + \frac{\sigma + \mu}{1 + \mu} \right] = 0 \\ \left[(\theta - 1 + \delta) B^{DE} + \frac{b\sigma}{1 + \mu} \right] - \left[(\theta - 1) B^S + \frac{\sigma + \mu}{1 + \mu} \right] = 0 \end{cases}.$$

This would hold either when $b = \frac{\sigma + \mu}{\sigma} > 1$ or under some parameter restrictions, which do not have to hold in general.

7.15.2 Subsidies to entry

On the balanced growth path, the GDP growth rate is equal to the socially optimal GDP growth rate if and only if there is permanent entry, conditional on $g_\lambda^{DE} = g_\lambda^S$. This is the case since

$$\begin{aligned} g^{DE} &= B^{DE} g_\lambda^{DE} \\ &= B^{DE} g_\lambda^S, \end{aligned}$$

and

$$B^{DE} = B^S,$$

if there is permanent entry.

The idea behind these subsidies is to have constant profits (thus a balanced growth path for any N in the decentralized equilibrium) and to equalize B^{DE} and B^S .

7.15.3 Second best policies that deliver higher social welfare than a policy in the spirit of the Telecommunications Act of 1996, when there cannot be permanent entry

The policies derived in this section fix the resource misallocations, in the case where there cannot be permanent entry. With a socially optimal resource allocation the economy grows faster. Therefore, these policies deliver higher social welfare than those which do not fix the resource misallocations.

In order to fix the resource misallocations the policy should subsidize both telecom goods production and the investments for productivity improvement. Under this policy

$$\begin{aligned} g_\lambda^{DE} &= \frac{\xi D_{DE} L - \rho}{(\theta - 1) B_{\dot{N}=0} - \alpha \tilde{t}_\lambda + D_{DE}} \\ &= \frac{\xi D^S L - \rho}{(\theta - 1) B^S + D^S} = g_\lambda^S, \end{aligned}$$

where

$$B_{\dot{N}=0} = \sigma + \mu,$$

and

$$\begin{aligned} NL_x^{DE} &= \frac{1}{\xi} \left([(\theta - 1) B_{\dot{N}=0} - \alpha \tilde{t}_\lambda] g_\lambda + \rho \right) \frac{1 - \tau_{Lx}}{1 - \tau_{Lr}} \\ &= \frac{1}{\xi} \left((\theta - 1) B^S g_\lambda^S + \rho \right) = NL_x^S. \end{aligned}$$

7.15.3.1 Policy 1 - With transfers proportional to the knowledge for production of telecom goods

Let

$$\begin{aligned}\alpha\tilde{t}_\lambda &= (\theta - 1) (B^S - B_{\dot{N}=0}), \\ \tau_{L_x} &= \tau_{L_r}, \\ \tau_{L_x} &= \tau_{L_r} = 1 - \frac{b\sigma}{\sigma + \mu}.\end{aligned}$$

The subsidy rates $\alpha\tilde{t}_\lambda$, τ_{L_x} , and τ_{L_r} are constant on the balanced growth path. Therefore, this policy satisfies balanced growth path. Moreover, under this policy the labor force allocations are the socially optimal ones. Therefore, the growth rate of productivity or equivalently the GDP growth rate are higher than in the decentralized equilibrium, and

$$\begin{aligned}g_\lambda^{DE} &= g_\lambda^S, \\ NL_r^{DE} &= NL_r^S, \\ NL_x^{DE} &= NL_x^S, \\ L_Y^{DE} &= L_Y^S.\end{aligned}$$

7.15.3.2 Policy 2 - Without any transfers proportional to the knowledge for production of telecom goods

Let again

$$\begin{aligned}g_\lambda^{DE} &= g_\lambda^S, \\ NL_x^{DE} &= NL_x^S.\end{aligned}$$

These two conditions are equivalent to

$$\begin{aligned}\frac{\left[\frac{1-T_{L_r}}{1-T_{L_x}} \left(\frac{1-\sigma}{b\sigma} (1 - T_{L_x}) + 1\right)\right]^{-1} \xi L - \rho}{(\theta - 1) B_{\dot{N}=0} + \left[\frac{1-T_{L_r}}{1-T_{L_x}} \left(\frac{1-\sigma}{b\sigma} (1 - T_{L_x}) + 1\right)\right]^{-1}} &= g_\lambda^S, \\ \frac{1 - \tau_{L_x}}{1 - \tau_{L_r}} &= \frac{(\theta - 1) B^S g_\lambda^S + \rho}{(\theta - 1) B_{\dot{N}=0} g_\lambda^S + \rho}.\end{aligned}$$

These two are equivalent to

$$\frac{1 - \sigma}{b\sigma} (1 - \tau_{L_x}) + 1 = \frac{\xi L - g_\lambda^S}{(\theta - 1) B_{\dot{N}=0} g_\lambda^S + \rho} \frac{1 - \tau_{L_x}}{1 - \tau_{L_r}}, \quad (93)$$

$$\frac{1 - \tau_{L_x}}{1 - \tau_{L_r}} = \frac{(\theta - 1) B^S g_\lambda^S + \rho}{(\theta - 1) B_{\dot{N}=0} g_\lambda^S + \rho}, \quad (94)$$

respectively. Given that the relation between NL_x and L_Y is given by (89), the policy that guarantees that these conditions hold is

$$1 - \tau_{L_x} = b \frac{\sigma}{\sigma + \mu}; 1 - \tau_{L_r} = (1 - \tau_{L_x}) \frac{(\theta - 1) B_{\dot{N}=0} g_\lambda^S + \rho}{(\theta - 1) B^S g_\lambda^S + \rho}.$$

Therefore, this policy satisfies the balanced growth path since $\tau_{L_x}, \tau_{L_r} = \text{const}$ on it. Moreover, from (93) and (94), it follows that in order to achieve the desired outcome, the policy cannot set τ_{L_r} to zero when the $\alpha \tilde{t}_\lambda$ is set to zero.

7.16 Appendix NT.1 - Decentralized equilibrium and balanced growth path without trade of knowledge

The derivation of the decentralized equilibrium GDP growth rate when there is no trade of knowledge can be done analogously to the one when there is a trade. The results are the same as those of van de Klundert & Smulders (1997), except there are no spillovers, which van de Klundert and Smulders consider. The decentralized equilibrium GDP growth rate in this case is given by

$$g^{NT} = B^{NT} g_\lambda^{NT},$$

where

$$g_\lambda^{NT} = \frac{\xi D^{NT} \frac{L}{N^{NT}} - \rho}{(\theta - 1 + \delta) B^{NT} + D^{NT}}.$$

The definitions of D^{NT} and B^{NT} are the same as the definitions of D and B .

When there is no trade of knowledge, the productivity improvement process is given by

$$\dot{\lambda} = \xi \lambda L_r.$$

Therefore, when the total labor supply is fixed, there cannot be an entry of telecom firms on a balanced growth path with a positive growth in productivity. Similar to van de Klundert & Smulders (1997), I consider the case when there is no entry, i.e., $\delta = 0$ and $g_\lambda^{NT} > 0$.

The parameter restrictions in this case are

$$\xi D^{NT} \frac{L}{N^{NT}} > \rho, \varepsilon > 1 + \sigma + \mu.$$

These restrictions nest those that should hold for the case where there is a trade of knowledge.

7.17 Appendix NT.2 - The number of firms in case there is no trade of knowledge compared with the number of firms in the case where there is trade

In the case where there is trade of knowledge and there are no barriers to entry, the number of firms on a balanced growth path is always higher than the same number in the case where there is no trade, again on the balanced growth path. This holds

because in the first case the number of firms is infinite and in the second case it is a finite number given that there should be endogenous barriers to entry.

A zero profit condition is needed in order to compare the number of firms when there are endogenous barriers to entry and trade of knowledge to the number of firms when there is no trade. The profit of a telecom good producer when there is no trade is derived analogously to the one derived for the case when there is a trade and is given by

$$\pi^{NT} = w^{NT} L_x^{NT} \left[\frac{1}{e^{NT} - 1} - \frac{g_\lambda^{NT}}{(\theta - 1) B^{NT} g_\lambda^{NT} + \rho} \right],$$

where $B^{NT} = B$ given that in both cases where there are barriers for entry.

It turns out that the analytical solution for the number of firms in the case where there is no trade is cumbersome. Therefore, instead of directly comparing the number of firms I compare the perceived elasticities of substitution under which the profit is zero. Given that the perceived elasticities of substitution are strictly monotonically increasing with the number of firms, comparing them is sufficient for comparing the number of firms. After substituting the expression for g_λ^{NT} into π^{NT} , the following holds when $\pi^{NT} = 0$:

$$e^{NT} = \frac{\xi \sigma \frac{L}{N^{NT}} [(\theta - 1)B + 1]}{\xi \sigma \frac{L}{N^{NT}} - \rho}.$$

The same type of expression can be derived for the case there is a trade of knowledge,

$$e = \frac{\xi \sigma L [(\theta - 1)B + 1]}{\xi \sigma L - \rho}.$$

Denote

$$e(x) = \frac{\xi \sigma x [(\theta - 1)B + 1]}{\xi \sigma x - \rho}.$$

Since

$$\frac{\partial}{\partial x} e(x) = \frac{-\xi \sigma [(\theta - 1)B + 1] \rho}{(\xi \sigma x - \rho)^2} < 0$$

and $N > 1$, the following holds

$$e^{NT} > e.$$

Since $\frac{\partial}{\partial N} e^{C,B} > 0$

$$N^{NT} > N.$$

Therefore, on a balanced growth path, if there are endogenous barriers to entry in the case when there is trade of knowledge, the number of firms is higher when there is no trade.

7.18 Appendix NT.3 - The constant GDP growth rate when there is a trade of knowledge compared to the same when there is no trade

As derived earlier, the GDP growth rates in these cases are given by

$$\begin{array}{l}
\text{With trade} \quad g = Bg_\lambda, \quad g_\lambda = \frac{\xi DL - \rho}{(\theta - 1 + \delta)B + D}, \quad B = \frac{(\varepsilon - 1)(\sigma + \mu)}{\varepsilon - 1 - \delta(\sigma + \mu)}, \quad D = \frac{b\sigma}{b\sigma + 1 - \sigma}; \\
\text{Without trade} \quad g^{NT} = B^{NT}g_\lambda^{NT}, \quad g_\lambda^{NT} = \frac{\xi D^{NT} \frac{L}{N^{NT}} - \rho}{(\theta - 1)B^{NT} + D^{NT}}, \quad B^{NT} = \sigma + \mu, \quad D^{NT} = \frac{b^{NT}\sigma}{b^{NT}\sigma + 1 - \sigma}.
\end{array}$$

7.18.1 When there are exogenous barriers to entry and the number of firms is the same in both cases

When $\dot{N} = 0$, B is equal to B^{NT} . Moreover, when $N = N^{NT}$, D is equal to D^{NT} . Therefore, the GDP growth rate when there is a trade is always higher than that when there is no trade,

$$g > g^{NT}.$$

7.18.2 When the number of firms is sufficiently high in both cases

The GDP growth increases with B , and D is an increasing and concave function of the number of firms. Therefore, when both N and N^{NT} are high, though not equal, the D and D^{NT} are quite close to each other. Therefore, also in this case the GDP growth rate when there is a trade is higher than that when there is no trade:

$$g > g^{NT}.$$

7.18.3 When there are endogenous barriers to entry

It turns out that it is not that easy to compare the growth rates in this case. The major issue is that the formulae for the number of firms in the case there is no trade of knowledge is very cumbersome. In this section I derive a sufficient condition under which the GDP growth rate in the case there is no trade of knowledge is lower than that when there is trade.

Since $B = B^{NT}$, $N^{NT} > N$, and when the number of firms is the same $g > g^{NT}$ holds, if g^{NT} declines with the number of firms, the inequality $g > g^{NT}$ should always hold. Therefore, a sufficient condition is g^{NT} declines with the number of firms [this is the case considered in van de Klundert & Smulders (1997)].

It is sufficient to focus on the growth rate of productivity in telecom goods production since the GDP growth rate is proportional to that growth rate.

7.18.3.1. The change of g_λ^{NT} with N^{NT}

$$\frac{\partial g_\lambda^{NT}}{\partial N^{NT}} = \frac{\left(\frac{\partial D^{NT}}{\partial N^{NT}} - \frac{D^{NT}}{N^{NT}} \right) \xi \frac{L}{N^{NT}} \left[(\theta - 1)B^{NT} + D^{NT} \right] - \frac{\partial D^{NT}}{\partial N^{NT}} \left(\xi D^{NT} \frac{L}{N^{NT}} - \rho \right)}{\left[(\theta - 1)B^{NT} + D^{NT} \right]^2}.$$

Therefore,

$$\begin{aligned}
\frac{\partial g_\lambda^{NT}}{\partial N^{NT}} < 0 &\Leftrightarrow \\
\left(\frac{\partial D^{NT}}{\partial N^{NT}} - \frac{D^{NT}}{N^{NT}} \right) \xi \frac{L}{N^{NT}} \left[(\theta - 1)B^{NT} + D^{NT} \right] - \frac{\partial D^{NT}}{\partial N^{NT}} \left(\xi D^{NT} \frac{L}{N^{NT}} - \rho \right) &< 0.
\end{aligned}$$

The second term should be always positive under the parameter restrictions; hence, a sufficient condition for the negative relation is

$$\frac{\partial D^{NT}}{\partial N^{NT}} - \frac{D^{NT}}{N^{NT}} < 0.$$

The derivative of D^{NT} with respect N^{NT} is

$$\frac{\partial D^{NT}}{\partial N^{NT}} = (D^{NT})^2 (1 - \sigma) \frac{1}{(e^{NT} - 1)^2} \frac{\partial e^{NT}}{\partial N^{NT}}.$$

Under Cournot competition, the last term is equal to

$$\frac{\partial e_C^{NT}}{\partial N^{NT}} = (e_C^{NT})^2 \frac{1}{(N^{NT})^2} \frac{\varepsilon - 1}{\varepsilon}.$$

Therefore, under Cournot competition

$$\frac{\partial D^{NT}}{\partial N^{NT}} = \frac{D^{NT}}{N^{NT}} \sigma \frac{(1 - \sigma)}{b^{NT} \sigma + 1 - \sigma} \left(\frac{e_C^{NT}}{e_C^{NT} - 1} \right) \frac{1}{N^{NT}} \frac{\varepsilon - 1}{\varepsilon}.$$

The sufficient condition to have a negative relation between growth rate of productivity improvement and number of firms then is given by

$$\sigma \frac{(1 - \sigma)}{b^{NT} \sigma + 1 - \sigma} \left(\frac{e_C^{NT}}{e_C^{NT} - 1} \right) \frac{1}{N^{NT}} \frac{\varepsilon - 1}{\varepsilon} - 1 < 0.$$

Since

$$\frac{1 - \sigma}{b^{NT} \sigma + 1 - \sigma} < 1,$$

in order for this inequality to hold, it is sufficient to have

$$\frac{e_C^{NT}}{e_C^{NT} - 1} < \frac{1}{\sigma} N^{NT} \frac{\varepsilon}{\varepsilon - 1}.$$

This is equivalent to having

$$\frac{\varepsilon}{(\varepsilon - 1) \left(\frac{N^{NT} - 1}{N^{NT}} \right)} < \frac{1}{\sigma} N^{NT} \frac{\varepsilon}{\varepsilon - 1},$$

or

$$N^{NT}|_C > 1 + \sigma. \quad (95)$$

Therefore, under Cournot competition, when the number of firms is greater than $1 + \sigma$, the GDP growth rate decreases with the number of firms. Under Bertrand competition, following the same steps as under Cournot competition, it can be shown that the sufficient condition is the same.

Given that the share of telecom goods consumption tends to be relatively low (for instance, in the US it is about 0.03), having N^{NT} more than 1 plus that share seems reasonable. Under (95) the growth rate of GDP in the case there is no trade

of knowledge and there are endogenous barriers to entry is certainly lower than the one in the case where there is trade.

To proceed further and to derive a sufficient condition which is related to model parameters only, I use the result that $N^{NT} > N$. From this result it follows that it is sufficient to have $N \geq 1 + \sigma$ in order to observe the negative relation.

Under Cournot competition a sufficient condition is

$$\theta \geq 1 + \frac{\sigma}{\sigma + \mu}.$$

Under Bertrand competition a sufficient condition is

$$\theta \geq 1 + \frac{\sigma}{\sigma + \mu} \frac{\varepsilon - 1}{1 + \sigma}.$$

Given that $\theta > 1$ is the empirically valid case and it is largely argued that ε is relatively small for telecom goods (see Taylor, 1980; Trotter, 1996), these parameter restrictions seem to be reasonable.

7.19 Appendix DA.1 - Labor force allocations

The following tables use all the available data on the number of employees in telecom from the International Telecommunication Union (ITU) database and total labor force from the OECD.STAT database. These tables suggest that the share of employees in telecom (in percent) has not varied significantly in the US, UK, France, and Germany.

Similar results can be found from the EU KLEMS database while using the share of total hours worked, instead of number of employees. The only issue with the EU KLEMS data is the 2-digit disaggregation level, where telecom is together with postal services.

Similar to Vourvachaki (2009), 1995 is selected as a cut-off point for comparison between means of shares. The year 1990 is selected as another cut-off point in order to have more or less comparable number of observations given that the ITU data do not suggest any peculiarities at any point of time.

Table 1 - US

	1975-2004	1975-1990	1990-2004	1975-1995	1995-2004
Share of employees in telecom (in percent)					
Average	0.009	0.009	0.008	0.009	0.008
St. Deviation	0.001	0.001	0.001	0.001	0.001
Annual growth rate of the share					
Average	-0.013	-0.028	-0.000	-0.018	-0.005
St. Deviation	0.045	0.036	0.050	0.039	0.056

Table 2 - UK

	1975-2002	1975-1990	1990-2002	1975-1995	1995-2002
Share of employees in telecom (in percent)					
Average	0.008	0.009	0.007	0.009	0.007
St. Deviation	0.001	0.000	0.001	0.001	0.001
Annual growth rate of the share					
Average	-0.003	-0.002	-0.005	-0.024	0.043
St. Deviation	0.007	0.026	0.094	0.051	0.078

Table 3 - Germany

	1975-2007	1975-1990	1990-2007	1975-1995	1995-2007
Share of employees in telecom (in percent)					
Average	0.0061	0.0074	0.0062	0.0072	0.0061
St. Deviation	0.0007	0.0004	0.0003	0.0006	0.0002
Annual growth rate of the share					
Average	-0.0071	0.0061	-0.0175	-0.0062	-0.0084
St. Deviation	0.0401	0.0198	0.0486	0.0451	0.0331

Table 4 - France

	1977-2007	1977-1990	1990-2007	1977-1995	1995-2007
Share of employees in telecom (in percent)					
Average	0.0061	0.0072	0.0063	0.0071	0.0061
St. Deviation	0.0007	0.0003	0.0007	0.0003	0.0007
Annual growth rate of the share					
Average	-0.0090	0.0041	-0.0206	0.0056	-0.0344
St. Deviation	0.0353	0.0266	0.0387	0.0318	0.0259

The second column of the Table 1 suggests that the share of employees has decreased significantly in the US, in the period between 1975 and 1990. Table 4 offers similar observations for France for the period between 1995 and 2007. However, the first columns of both tables suggest that, on average, over the entire time-span of the available data there was no significant change.

7.20 Appendix T.1 - Transition dynamics

The transition dynamics of the model can be described by the following system of equations

$$\begin{aligned} \frac{\dot{w}}{w} - \frac{\dot{N}}{N} &= \eta \frac{w}{N} \frac{b\sigma}{b\sigma + 1 - \sigma} \left(L - \frac{1}{\xi} \frac{\dot{\lambda}}{\lambda} \right) \left[\frac{1}{e-1} - \frac{b\sigma + 1 - \sigma}{b\sigma} \frac{\frac{1}{\xi} \frac{\dot{\lambda}}{\lambda}}{\left(L - \frac{1}{\xi} \frac{\dot{\lambda}}{\lambda} \right)} \right] \\ &\quad - \frac{b\sigma}{b\sigma + 1 - \sigma} \left(\xi L - \frac{\dot{\lambda}}{\lambda} \right), \\ \frac{\dot{C}}{C} &= \frac{1}{\theta} \left(\eta \frac{w}{N} \frac{b\sigma}{b\sigma + 1 - \sigma} \left(L - \frac{1}{\xi} \frac{\dot{\lambda}}{\lambda} \right) \left[\frac{1}{e-1} - \frac{b\sigma + 1 - \sigma}{b\sigma} \frac{\frac{1}{\xi} \frac{\dot{\lambda}}{\lambda}}{\left(L - \frac{1}{\xi} \frac{\dot{\lambda}}{\lambda} \right)} \right] - \rho \right), \end{aligned}$$

$$\frac{w}{N} \frac{1}{b\sigma} = N^{\frac{\sigma + \mu + 1 - \varepsilon}{\varepsilon - 1}} \lambda^{\sigma + \mu} \left(\frac{1 - \sigma}{b\sigma} \right)^{1 - \sigma} \left[\frac{b\sigma}{b\sigma + 1 - \sigma} \left(L - \frac{1}{\xi} \frac{\dot{\lambda}}{\lambda} \right) \right]^{\mu},$$

$$b = \frac{e - 1}{e},$$

$$e = f_i(N; \varepsilon), i = C, B,$$

$$\frac{\dot{N}}{N} = \eta \left[\frac{w}{N} \frac{b\sigma}{b\sigma + 1 - \sigma} \left(L - \frac{1}{\xi} \frac{\dot{\lambda}}{\lambda} \right) \frac{1}{b} - \frac{C}{N} \right].$$

This system of equations can be written in terms of variables which do not grow on the balanced growth path. Moreover, at the same time, it can be reduced to a system of three interrelated and non-linear differential equations.

Due to non-linearity and inter-relations, it is very hard to provide full description of the transition dynamics of this model. There is only limited evidence, based on numerical simulations, that the economy converges to a balanced growth path.

For instance, the economy converges to a balanced growth path for parameter values ($\sigma = 0.05$; $\mu = 0.002$; $\varepsilon = 2.3$; $\theta = 4$; $\rho = 0.0242$; $\xi = 0.9$; $\eta = 1$; $L = 1$), which were selected such that (1) to be close to the share of telecom good consumption implied by the EU KLEMS data; (2) to be close to the suggestion of Roller & Waverman (2001) on the average contribution of telecom to economic growth in the US; and (3) to have low elasticity of substitution between telecom goods which is in accordance with the suggestions of numerous empirical papers that try to measure that elasticity.