

Ad-valorem platform fees and efficient price discrimination*

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Abstract

Online shopping platforms (such as Amazon and eBay) or payment card networks (such as Visa and MasterCard) serve to facilitate transactions between buyers and sellers. While these platforms typically only incur small per-transaction costs, they often charge sellers a substantial proportional fee plus a small fixed transaction fee. In this paper, we investigate the implications of this use of ad-valorem fees (fees based on value) by a monopoly platform. The platform has to deal with trade in multiple goods with different costs and valuations that it does not observe. We characterize a class of platform demand functions that rationalizes the platform's use of a linear fee schedule (a proportional fee plus fixed transaction fee). We show that allowing the platform to use ad-valorem fees is equivalent to it being allowed to use third-degree price discrimination in the hypothetical situation in which it observes the costs and valuations for each good traded and sets a different optimal fee for each. Surprisingly, we find for this class of demands, allowing the platform to set ad-valorem fees (i.e. price discriminate) generally increases social welfare.

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1 Introduction

There are many platforms that serve to facilitate the transactions between buyers and sellers. Well known examples include online shopping websites (such as eBay and Amazon) and payment card networks (such as Visa and MasterCard). While these platforms typically only incur small (if any) marginal costs per transaction, they typically charge sellers a fee that depends on the value of the transaction (i.e. the sale price), also known as an ad-valorem fee. While they often also include a small fee for each transaction completed, the ad-valorem component accounts for most of their revenue.¹

In this paper, we investigate the implications of this particular fee structure in a setting where a platform has to deal with users trading many different goods, and the implications for welfare of preventing platforms using such ad-valorem fees. Goods are assumed to be heterogenous in their underlying scales (e.g. eBay has goods that are traded for a few dollars and others for thousands of dollars, sometimes within the same broad category of good; and likewise for payment card networks). We do this by assuming the underlying goods being traded have different costs and valuations that vary with a scale parameter. We show how ad-valorem pricing can allow the platform to maximize its profit without having to collect any information about the particular costs or valuations of each good, and for the class of demand functions for which this is true, how the use of ad-valorem pricing can increase welfare compared to the case in which ad-valorem fees are not allowed.

Our findings shed new light on the debate on platform pricing. In recent years, a sizeable body of literature has developed to evaluate whether the structure of fees in the payment card industry (i.e. fees that have been argued to be skewed against retailers and in favor of cardholders) is due to some market failure.² Meanwhile, questions have also been raised about ad-valorem fees: Why do payment card networks set interchange fees (in the case of Visa and MasterCard) or retail fees (in the case of American Express) that depend on transaction values

¹For example, eBay and Amazon typically charge sellers ad-valorem fees of 6 percent to 15 percent, plus a small per-transaction fee which is typically not much more than one dollar. Visa and MasterCard typically set interchange fees which are paid by merchants to card issuers through merchant acquirers with an ad-valorem rate of 0.5 percent to 2.5 percent by broad merchant category, sometimes plus a per-transaction fee of 5 cents to 25 cents.

²See, for instance, Bedre-Defolie and Calvano (2010), Rochet and Tirole (2002, 2003, 2010), Schmalensee (2002) and Wright (2003, 2004, 2011).

rather than being a fixed amount per-transaction? Empirical evidence suggests that the cost components cannot explain the level of ad valorem fees charged.³ The issue is of concern to policymakers in many countries who have recently tried to align payment card fees with costs. While it may be difficult to directly regulate card fee levels, one approach would be to require only fixed per-transaction fees are charged for payment cards.

Shy and Wang (2011) and Wang (2010) consider whether there is a good rationale for the use of proportional fees rather than per-transaction fees by platforms.⁴ For a specific demand specification and considering a single good, they show that when a platform and retailers both have market power, the platform earns higher profit by charging a proportional fee rather than a fixed per-transaction fee. Proportional fees are also shown to increase social welfare, although sellers are worse off. However, their findings, which provide some support for the use of ad-valorem fees, hinge on the presence of double marginalization. Should retailers be competitive, there is no difference between charging a proportional fee and a per-transaction fee in terms of profit or welfare in their setting.

In contrast, our analysis shuts down the double marginalization channel by assuming the sellers are competitive. Nevertheless, we show that the monopoly platform still prefers charging a proportional fee, together with a fixed per-transaction fee, since this allows the platform to maximize its profit when its platform is used to trade different goods with different costs and valuations, and it does not have information about the costs and valuations of each different good traded. Indeed, under a broad class of demand specifications, we find using a proportional plus per-transaction fee allows the platform to achieve the same profit as if it could observe the costs of each good traded and could price discriminate setting a different optimal fee for each. Using this class of demand, we then study the welfare effects of banning ad-valorem pricing. The conditions for social welfare to increase turn out to be the same as whether banning price discrimination improves social welfare within the class of demands considered. This allows us to draw on the substantial literature on monopolistic third-degree price discrimination (e.g. see

³For example, Shy and Wang (2011) report from industry studies that average net fraud losses to card issuers are 0.08 percent for credit cards, 0.05 percent for signature debit cards, and 0.01 percent for PIN debit cards.

⁴Proportional fees are just a special case of ad-valorem fees, which we define as any fee which depends positively on the value of the transaction (i.e. the sale price). Ad-valorem fees are also sometimes called commission fees (see Loertscher and Niedermayer, 2010).

Aguirre *et al.*, 2010 for a recent analysis).

Our setting is quite different from the analysis of optimal fee setting mechanisms in Lortscher and Niedermayer (2010). They establish conditions under which the optimal selling mechanism for trading intermediaries is one in which the intermediary charges a proportional fee and sellers set the price. In their setting the trading intermediaries allow a matched buyer and seller to trade, where each has private information about their valuation for an indivisible good. In our setting, there are multiple sellers of the same good, all with the same valuation of the good (i.e. cost), and we take as given the selling mechanism in which competing sellers set prices and focus on whether a monopoly platform should be allowed to charge ad-valorem fees.

The remainder of this paper is structured as follows. Section 2 introduces the basic model environment. Section 3 characterizes the class of platform demands for which ad-valorem fees are optimal. Section 4 investigates the welfare implications of ad-valorem fees given the class of demands characterized in Section 3. Finally, Section 5 offers some concluding remarks.

2 Model Environment

We consider a trading environment where there are multiple goods which can be traded on a platform. For each such good, a unit mass of buyers want to purchase one unit of the good. There are two or more identical sellers of each good which post a price for the good. Different goods are indexed by c , which can be thought of as a scale parameter, so that different goods can be thought of as having similar demands except they come in different scales. In particular, the per-unit cost of good c to sellers is normalized to c and the value of the good to a buyer on the platform with benefit parameter $b > 0$ is $c(1 + b)$, so the scale parameter multiplies up the cost and the buyer's valuation by the same factor. We assume the lowest and highest values of c are denoted c_L and c_H respectively, with $c_L > 0$. We also assume $1 + b$ is distributed according to some smooth and strictly increasing distribution function H on $[1, 1 + \bar{b}]$, where $\bar{b} > d/c_L$, so it is efficient for some buyers to trade with sellers on the platform, even for the good with the lowest cost. Only buyers know their own b , while H is public information.

This setting captures the idea that the platform enables some types of trades which otherwise would not arise and that the main difference across goods traded on the platform from the point

of view of the platform is their scale (i.e. some are cheap goods and some are expensive). The idea is that, in comparison to scale, potential differences in the shapes of demand functions across the different goods traded is not likely to be of first-order importance.⁵ In Appendix A we provide an alternative setting which generates the same platform demand function below, and so the same results, without requiring demand to take a common form across the different goods. Specifically, we allow there to be an alternative less efficient trading mechanism (e.g. buyers and sellers trade directly) in which case the demand for different goods is allowed to take different forms since it is only the demand for using the platform that is required to be scaled up or down across the different goods traded.

The platform is assumed to incur a small cost d for handling each transaction. In many situations it is reasonable that d is negligible and so can be set to zero. Indeed, some of our welfare analysis will make use of this property for tractability. We assume the platform cannot distinguish anything about which good is being sold (i.e. the scale of the good sold), but only the price at which each good is sold. As a result, a platform's fees can only condition on the price of a transaction (i.e. not on the cost of sellers or valuation of buyers). Since sellers of a particular good are identical and compete in price, competition between sellers for a good c will result in an equilibrium in which sellers who use the platform charge buyers a common price P_c . In general, the platform fee schedule or tariff can be written as $T(P_c)$ for a given price of the transaction P_c , which can be decomposed into a buyer fee schedule $T^b(P_c)$ and seller fee schedule $T^s(P_c)$.⁶ Each fee schedule can consist of two components, a fixed per-transaction fee which is independent of P_c and a variable component which varies with P_c (i.e. the ad-valorem component). When the ad-valorem component is linear in P_c , we call it a proportional fee.

Note that given identical sellers compete for buyers, any fee charged to sellers will be passed through to buyers. The final price faced by buyers will reflect any platform fees, and the buyer treats these the same whether he faces them directly or through sellers.⁷ Due to this neutrality result, we can assume without loss of generality that the platform only charges fees to sellers

⁵If the platform can identify a particular category of goods and set a specific fee schedule for that category, as is often the case, then the analysis in this paper applies to that particular category of goods.

⁶All buyers and sellers are assumed to be on the platform (we abstract from participation costs and participation fees).

⁷See Gans and King (2003) for a general statement and proof of neutrality in a platform setting.

(in accordance with the typical practice, e.g. with eBay). Since buyers do not face any fee, the number of transactions on the platform Q_c for a particular good c is then the measure of buyers who obtain non-negative surplus from buying on the platform, i.e. $\Pr(c(1+b) \geq P_c)$, so

$$Q_c(P_c) = 1 - H\left(\frac{P_c}{c}\right). \quad (1)$$

The function Q_c is the market demand function faced by sellers of good c on the platform. We assume Q_c is not too convex. Specifically, let $\lambda = H'/(1-H)$ be the hazard rate of H . We assume throughout that $\lambda'(x) > -\lambda(x)^2$ for all x , so that if the hazard rate is decreasing, it never decreases too fast. This is an even weaker assumption than assuming log-concave demand (which is equivalent to assuming an increasing hazard rate). The corresponding inverse demand for sellers of good c on the platform is $P_c(Q_c) = cH^{-1}(1 - Q_c)$. Define the curvature of inverse demand as the elasticity of the slope of this inverse demand with respect to quantity Q_c (Cowan and Vickers, 2007 and Aguirre *et al.* 2010).

Sellers of good c facing a fee schedule $T(P_c)$ make a profit margin of $P_c - T(P_c) - c$ on each transaction using the platform. The platform makes a profit which is equal to

$$\Pi_c = (T(P_c) - d)Q_c(P_c)$$

for good c . Denote the distribution of c across goods by some arbitrary distribution function G . Integrating over all goods, a platform's profit is

$$\Pi = \int_c \Pi_c dG(c). \quad (2)$$

The platform's problem is to choose the fee schedule $T(P_c)$ to maximize Π .

3 Optimal fee schedules

We are interested in the welfare effects of a restriction on optimal fee schedules that rules out an ad-valorem component. Rather than specifying some particular distributions for G and H , and working out optimal fee schedules, which is in general an intractable problem, we proceed in the

opposite direction. We characterize the class of demand functions (i.e. the class of distributions H) that can rationalize fee schedules that are commonly observed, namely fee schedules that involve a fixed per-transaction fees and proportional fees (i.e. linear fee schedules). These will turn out to be optimal for any distribution of goods, G . We can then proceed to analyze the welfare effects of putting a restriction on the structure of optimal fees (e.g. ruling out a variable component in fees) for this class of demand functions. This approach takes advantage of the fact that the (first-best) optimal transaction fee for each good (i.e. if the platform observed each good) implies a relationship between the price of goods traded and the optimal fee for the same good that can be used to trace out the optimal fee schedule.

Proposition 1 (*Optimal fee schedule*): *The optimal fee structure is linear (involving a proportional fee plus a fixed per-transaction fee) if and only if the distribution of $1 + b$ has a linear inverse hazard rate (including the case the inverse hazard rate is constant). The condition is equivalent to $1 + b$ being distributed according to the generalized Pareto distribution. This condition is also equivalent to the corresponding demand functions faced by sellers on the platform being defined by a class of demands with constant curvature of inverse demand. This includes the case of linear demand, constant-elasticity demand and exponential demand.*

Proof. Consider the hypothetical problem of a platform that observes each good c and can set a separate fee F_c to maximize its profit $\Pi = (F_c - d) Q_c$, where homogenous Bertrand competition implies $P_c = c + F_c$ and so

$$Q_c(F_c) = 1 - H\left(1 + \frac{F_c}{c}\right).$$

The first-order condition to characterize optimality is

$$F_c = d + \frac{c}{\lambda\left(1 + \frac{F_c}{c}\right)}. \quad (3)$$

Let the optimal fee be denoted F_c^* . Note if $F_c = d$, then $d\Pi_c/dF_c = 1 - H(1 + d/c) > 0$ given $\bar{b} > d/c_L$, while if F_c is arbitrarily close to $c\bar{b}$ so that $1 - H(1 + F_c/c)$ is arbitrarily close to 0, then $d\Pi_c/dF_c$ is arbitrarily close to $-(F_c - d)h(1 + F_c/c)/c < 0$ and so is negative. Given our assumption that $\lambda'(x) > -\lambda(x)^2$ for all x , the solution to (3) is therefore uniquely defined

with $d < F_c^* < c\bar{b}$. The second-order condition holds provided $\lambda'(1 + F_c/c) > -c^2/F_c^2$. As will be shown below, the condition $\lambda'(x) > -\lambda(x)^2$ implies $\lambda'(1 + F_c/c) > -c^2/F_c^2$ for the distributions considered. Using that $P_c = c + F_c$, the first-order condition can be rewritten as

$$F_c = d + \frac{P_c - F_c}{\lambda\left(\frac{P_c}{P_c - F_c}\right)}. \quad (4)$$

If the hazard rate function $\lambda(\cdot)$ is such that there is some solution α_0 and α_1 (where these parameters are constants; i.e. they do not depend on P_c) to the following problem

$$\alpha_0 + \alpha_1 P_c = d + \frac{P_c - (\alpha_0 + \alpha_1 P_c)}{\lambda\left(\frac{P_c}{P_c - (\alpha_0 + \alpha_1 P_c)}\right)}, \quad (5)$$

then (3) will hold for each good c regardless of the value of $c > 0$. This condition holds if and only if $1/\lambda(x)$ is a linear function of x , so that either the term $P_c - (\alpha_0 + \alpha_1 P_c)$ cancels or $1/\lambda(x)$ is constant in x .

Without loss of generality, the requirement that the inverse hazard rate be linear is equivalent to requiring

$$\frac{H'(x)}{1 - H(x)} = \frac{\lambda}{1 - \lambda(1 - \sigma)(x - 1)} \quad (6)$$

for some constant parameters λ and σ . The case with a constant hazard rate (and so constant inverse hazard rate λ) arises when $\sigma = 1$. For this special case, solving the differential equation (6) for $H(x)$ implies

$$-\ln(1 - H(x)) = \gamma_0 + \lambda x$$

for some constant γ_0 . Requiring $H(1) = 0$ given that H defines the distribution of $1 + b$ and b is assumed to be non-negative, implies $\gamma_0 = -\lambda$, and the resulting distribution function is

$$H(x) = 1 - e^{-\lambda(x-1)}, \quad (7)$$

which is the left-truncated exponential distribution on $[1, \infty)$, and is a special case of the generalized Pareto distribution below in which $\sigma \rightarrow 1$. The corresponding density function is $h(x) = \lambda e^{-\lambda(x-1)}$, where $\lambda > 0$ is the constant hazard rate. Note the second-order condition

assumed above holds since $\lambda'(x) = 0$ for all x . Demand faced by the platform for good c is given by

$$Q_c(F_c) = e^{-\frac{\lambda F_c}{c}}. \quad (8)$$

The demand faced by sellers on the platform $Q_c(P_c)$ is also exponential. The corresponding inverse demand for sellers is

$$P_c(Q_c) = c - \frac{c}{\lambda} \ln Q_c, \quad (9)$$

which has constant curvature equal to 1.

If instead $\sigma \neq 1$, solving (6) for $H(x)$ implies

$$-\ln(1 - H(x)) = \gamma_0 + \frac{\ln\left(\frac{1}{\lambda} + (\sigma - 1)(x - 1)\right)}{(\sigma - 1)}.$$

Requiring $H(1) = 0$ implies $\gamma_0 = -\ln(\lambda) / (1 - \sigma)$, so

$$H(x) = 1 - (1 + \lambda(\sigma - 1)(x - 1))^{\frac{1}{1-\sigma}} \quad (10)$$

where $\lambda > 0$. This is the generalized Pareto distribution. The corresponding density function is

$$h(x) = \lambda(1 + \lambda(\sigma - 1)(x - 1))^{\frac{\sigma}{1-\sigma}}$$

so that the hazard rate is

$$\lambda(x) = \frac{\lambda}{1 + \lambda(\sigma - 1)(x - 1)}. \quad (11)$$

When $\sigma < 1$ the support of H is $[1, 1 + 1/\lambda(1 - \sigma)]$ and it has increasing hazard, so the second-order condition assumed above holds. Alternatively, when $\sigma > 1$, the support of H is $[1, \infty)$ and it has decreasing hazard. The second-order condition above holds given our assumption that $\lambda'(x) > -\lambda(x)^2$ for all x (this implies $\sigma < 2$ so that $\lambda'(1 + F_c/c) > -c^2/F_c^2$ at the value of F_c solving (3)). Demand faced by the platform for good c is given by

$$Q_c(F_c) = \left(1 + \frac{\lambda(\sigma - 1)F_c}{c}\right)^{\frac{1}{1-\sigma}}. \quad (12)$$

The demand faced by sellers on the platform is given by

$$Q_c(P_c) = \left(1 + \frac{\lambda(\sigma - 1)(P_c - c)}{c}\right)^{\frac{1}{1-\sigma}}. \quad (13)$$

This includes linear demand ($\sigma = 0$) and constant-elasticity demand ($\sigma = 1 + 1/\lambda$).⁸ When $\sigma \rightarrow 1$, it gives the left-truncated exponential distribution above. The corresponding inverse demand for sellers is

$$P_c(Q_c) = c \left(1 - \frac{1}{\lambda(\sigma - 1)}\right) + \frac{c}{\lambda(\sigma - 1)} Q_c^{1-\sigma}, \quad (14)$$

which has constant curvature σ , with $\sigma < 2$.

Note that (9) and (14) characterize a class of inverse demands which has constant curvature σ (i.e. compare with Cowan and Vickers, 2007, pp.10-11).

Provided the distribution of $1 + b$ belongs to the generalized Pareto distribution above, the platform's optimal fee structure is a simple two-part (i.e. linear) tariff

$$T^*(P_c) = \frac{\lambda d}{1 + \lambda(2 - \sigma)} + \frac{P_c}{1 + \lambda(2 - \sigma)} \quad (15)$$

where $\lambda > 0$ and $\sigma < 2$. This follows by substituting (11) into (4) and solving explicitly for F_c^* . To show (15) is optimal it remains to show that sellers of good c facing this fee schedule and the demand given in (12) will indeed choose a price to sell their goods that implies they are charged F_c^* . Taking into account the fee schedule (15), if the seller of good c sells his unit, he gets a profit of

$$\left(1 - \frac{1}{1 + \lambda(2 - \sigma)}\right) P_c - c - \frac{\lambda d}{1 + \lambda(2 - \sigma)}.$$

Profit is increasing in P_c given $\sigma < 2$ and is non-negative if and only if $P_c \geq c + F_c^*$, where

$$F_c^* = \frac{\lambda d + c}{\lambda(2 - \sigma)}. \quad (16)$$

⁸Note that while the demand faced by the platform is also linear when $\sigma = 0$, it does not take the constant-elasticity form for any σ .

Any seller that sets a higher price will receive zero demand. Any seller that sets a lower price will make a loss. Sellers of good c will therefore price at $c + F_c^*$. Substituting this price into (15) implies $T^*(c + F_c^*) = F_c^*$, which is the optimal fee from (3) given the specified distribution. ■

Proposition 1 characterizes the class of demand specifications that rationalizes the platform's use of a linear fee schedule (a proportional fee plus fixed transaction fee). This turns out to be the broad class of demand functions that has constant inverse demand curvature, which includes the popular cases of linear demand, constant-elasticity demand and exponential demand. As shown in Proposition 1, this class of demands can be derived from the condition that buyers valuation follows a generalized Pareto distribution, so we will also refer to it as the generalized Pareto demand in our following analysis. The optimal fee schedule that emerges from Proposition 1 is given in (15) in the proof. Since the curvature of inverse demand is the constant $\sigma < 2$, the proportional fee is always less than 100% of the corresponding price. The fixed per-transaction fee is only positive if there is a positive cost to the platform of handling each transaction.

4 Welfare analysis

A social planner can achieve maximum welfare if any platform is made to set its fees based on marginal cost, so $F = d$ for all goods. Given the platform's costs are not dependent on the prices charged for different goods, a first-best outcome involves a fee that also does not depend on these prices. This might suggest to policymakers that eliminating ad-valorem fees might be beneficial since it would remove any fee distortion, whereby relative fees across different goods do not reflect the relative costs of the platform in providing its service. Our interest is in whether this idea extends to optimally set fee schedules in our setting. We ask whether simply requiring the monopolist to set the same per-transaction fee for all goods, thereby eliminating any type of ad-valorem fee, will help increase welfare.

Without any restrictions, the monopoly platform chooses a fee schedule to maximize (2). Given generalized Pareto demand, the optimal fee schedule (15) is linear, with a fixed per-transaction fee and a fee that varies in proportion to price (the ad-valorem fee component). If

instead the platform can only choose a single fixed per-transaction fee F across all goods, it will choose F to maximize

$$\Pi = \int_c (F - d) Q_c(F) dG(c), \quad (17)$$

where $Q_c(F) = (1 - H(1 + F/c))$. We assume that $G(c)$ is such that a unique value of F , denoted \hat{F} , maximizes Π . Given generalized Pareto demand, the resulting \hat{F} is between $F_{c_L}^*$ and $F_{c_H}^*$.⁹ Our problem of interest is thus what happens to total welfare in going from the optimal fee schedule (15) which maximizes (2) to the single fee \hat{F} which maximizes (17).

We now show that the solution to this problem can be found by solving a dual problem, which amounts to the welfare effects of third-degree price discrimination. The dual problem involves considering a monopoly firm that sells to distinct and identifiable markets. It sets F_c in each market to maximize profit

$$\Pi_c = (F_c - d) Q_c(F_c). \quad (18)$$

Without any restriction the monopolist will charge a different optimal price F_c in each market to maximize (18). Given the generalized Pareto demand given in Proposition 1, the solution is given already in (16). If third-degree price discrimination is banned, the monopolist will instead choose a uniform price F across all markets to maximize (17). In this dual problem we consider what happens to total welfare in banning third-degree price discrimination.

Proposition 2 (Duality): *Given generalized Pareto demand for goods, the welfare effect of banning ad-valorem platform fees from the optimal fee schedule is identical to the welfare effect of banning third-degree price discrimination in the dual problem in which the platform can observe the different goods (i.e. their costs and valuations) and charge different (optimal) fees to sellers for each good.*

Proof. Consider the optimal fee F_c^* defined in (16), the corresponding price $P_c^* = c + F_c^*$, and the optimal linear fee schedule (15) which has the property $T^*(P_c) = F_c^*$. As shown in

⁹Given that $\sigma < 2$, it can be shown that $d\Pi_c/dF < 0$ if $F > F_c^*$ and $d\Pi_c/dF > 0$ if $F < F_c^*$. Since F_c^* is increasing in c , \hat{F} would never be below $F_{c_L}^*$ since a higher F can increase profit on every good traded on the platform, and likewise it would never be above $F_{c_H}^*$ since a lower F can increase profit on every good traded on the platform.

the proof of Proposition 1, sellers of a good c set the same price P_c^* in equilibrium whether they face the optimal fee schedule $T^*(P)$ or whether they can be identified as selling good c and charged the optimal fee F_c^* . The unrestricted outcomes are therefore the same for the two problems. The restricted outcomes in which ad-valorem fees are banned are also the same in each problem. This follows since the optimal fixed per-transaction fee maximizing (17) does not depend on being able to identify each good. Therefore the two problems are equivalent. ■

Proposition 2 implies that to determine the welfare effects of banning the ad-valorem component of a platform’s optimal linear fee schedule, one can look at the equivalent problem of determining the welfare effects of banning third-degree price discrimination in the situation in which the monopolist sets the price F_c (one price for each market c) to maximize its profit in (18). In this problem Q_c can be interpreted as a standard demand function, F_c the relevant price in each market and c a parameter which shifts demand across different identifiable markets. Note $\partial Q_c/\partial F_c < 0$ and $\partial Q_c/\partial c > 0$ given H is an increasing function, so the demand curve is indeed decreasing in “price” and higher values of c correspond to higher demand curves. Total welfare is $W = \int_c W_c dG(c)$ in which the welfare for each market W_c is written in the standard way

$$W_c = \int_0^{Q_c} (P_c(Q) - c - d) dQ,$$

where recall $P_c(Q)$ is the inverse demand function for goods. This duality result allows us to transform the original welfare problem into a standard problem of whether banning a monopolist from using third-degree price discrimination raises welfare, a problem on which a substantial existing literature exists. We draw on this existing literature but also obtain some new results of our own.

Aguirre *et al.* (2010) focus on the case with two markets which are always covered, and provide quite general conditions to sign the output and welfare effects of price discrimination under non-linear demand. When the curvature of inverse demand function σ is common across markets, as it is in our case, a sufficient condition for total output to increase is that curvature, as measured by σ , is positive and constant in each market. Thus, provided $\sigma > 0$, price discrimination will expand output in our setting with two goods, so that allowing a platform to set ad-valorem fees will increase the number of transactions compared to requiring a fixed per-

transaction fee across the two goods. This includes the case seller demand on the platform is of the constant-elasticity form, so that $\sigma = 1 + 1/\lambda > 1$. Aguirre *et al.* (2010) also show that in the case of two markets that are always covered, when $\sigma > 1$ (as in our constant-elasticity case), price discrimination raises welfare if the discriminatory prices are not far apart. In subsection 4.3 we show this welfare result holds even when discriminatory prices are far apart and when there is a continuum of markets. If instead $\sigma = 0$, then platform demand is linear, and it is well known (e.g. Schmalensee, 1981) that output does not change as a result of price discrimination in this case if both markets are still covered with uniform pricing, and so welfare is lower with price discrimination. However, we are interested in the case in which (i) there may be more than two goods or markets and (ii) not all goods or markets will necessarily be covered because the scale of the different goods traded on the platform is quite different.

To obtain more specific results, we focus on the class of generalized Pareto demand for the full range of σ covered by Proposition 1. We first consider the special cases in which $\sigma = 0$ (linear demand) and $\sigma = 1$ (exponential demand). We then provide numerical results for the full range of σ covering both log-concave demands and log-convex demands (up to $\sigma < 2$). In each case we consider both the standard case from the literature in which there are only two markets corresponding to c_L and c_H , and the more challenging case in which there are a continuum of markets with c uniformly distributed between c_L and c_H . With linear demand (and more generally with log-concave generalized Pareto demand so $\sigma < 1$) we find that welfare will be higher under price discrimination provided there is sufficient dispersion in c . This result arises because under uniform pricing, low- c markets drop out when there is sufficient dispersion in c . With exponential demand, despite the fact markets are always covered, welfare is always found to be higher under price discrimination. The same is true for log-convex generalized Pareto demands more generally (for $1 < \sigma < 2$). The implication is that banning ad-valorem fees will result in lower welfare.

4.1 Log-concave demand

When $\sigma = 0$, (10) collapses to the uniform distribution on $[1, 1 + 1/\lambda]$ with the corresponding density function $h(x) = \lambda$ and $\lambda > 0$. The platform faces linear demand for good c given by

$$Q_c(F_c) = \left(1 - \frac{\lambda F_c}{c}\right). \quad (19)$$

Demand functions are linear with equal horizontal intercepts but different vertical intercepts (choke prices). At any price, higher demands (corresponding to higher values of c) have lower elasticities. The results for linear demand then follow fairly direct from the existing literature.

4.1.1 Two markets

We know from the existing literature that in the case with two markets where demands are different and both markets are served under uniform pricing, that welfare is lower with price discrimination (output is the same but price discrimination distorts the relative prices). On the other hand, when demands are sufficiently different, the monopolist will want to drop the low demand market and as a result set the monopoly price for the high demand market. Since this is the same price that it would set under price discrimination, welfare is unambiguously lower under uniform pricing in this case, given the low demand market would have been served under price discrimination. It is straightforward to verify that the condition for this to arise is $c_H > 3c_L$ when $d = 0$. Thus, provided the scale of goods differs enough across the two markets and the low demand market is still served under price discrimination, welfare is higher with price discrimination.

Figure 1 (in Appendix B) illustrates when c_L is normalized so $c_L = 1$ and $c_H = kc_L$, and k varies from 1 to 10. The demand parameter $\lambda = 4.5$ implies that the optimal ad-valorem fee is 10 percent, although this is just a normalization. Three values of d are considered: $d = 0, 0.1$, and 0.2 , which show that the welfare results continue to hold for positive d . Note that for $d = 0.2$, the low demand market on the platform has very little demand under price discrimination, which is why price discrimination has only a very small positive welfare effect when the low demand market is dropped under uniform pricing.

Clearly, the same conclusion holds for any demand specification in which the low demand

market will be dropped with uniform prices when demand differences are large enough. E.g. the conclusion holds for the more general case in which the market is not always covered since demand is log-concave (i.e. generalized Pareto with $\sigma < 1$). This is shown in figure 2, which confirms the same qualitative findings as the linear demand two market case.

4.1.2 Continuum of markets

Corresponding to (19), inverse demand faced by the platform in market c is

$$F_c(Q_c) = \frac{c(1 - Q_c)}{\lambda}.$$

Set $d = 0$ (the monopolist has zero cost) and $c \in U[c_L, c_H]$ with $c_L > 0$, $c_H = kc_L$ and $k > 1$. Then the problem is stated in exactly the same form as the third-degree price discrimination problem analyzed by Malueg and Schwartz (1994), other than we allow inverse demand to be multiplied by a constant positive parameter and we allow that the uniform distribution on c be no longer centered at unity.¹⁰ It turns out what matters for Malueg and Schwartz's results is the ratio of the highest to lowest value of c in the support of the distribution, i.e. k . Therefore reinterpreting the relevant part of their Proposition 1 to our setting, it implies that for large enough dispersion $k > k_0$, some markets are dropped under uniform pricing; in this range the ratio of welfare under price discrimination to welfare under uniform pricing increases monotonically with dispersion, and exceeds 1 when dispersion is sufficiently large.

To calculate these points precisely define $k_0 > 1$ which solves $1 + 2 \ln k_0 = k_0$, so $k_0 \simeq 3.513$. Then the point at which dispersion is sufficiently large for welfare to increase under price discrimination arises when¹¹

$$k > \frac{3k_0 - \sqrt{3k_0(4 - k_0)}}{k_0 - 4 + \sqrt{3k_0(4 - k_0)}} \simeq 4.651.$$

Thus, provided there is sufficient dispersion in costs, welfare is unambiguously higher with price

¹⁰Their specification can be obtained by setting $\lambda = 1$, $c = a$, $c_L = 1 - x$ and $c_H = 1 + x$.

¹¹There is a typo in Malueg and Schwartz's stated formula for this cutoff (in their footnote 17) which does not generate the approximate numerical value they state in the footnote. However, their stated numerical value corresponds to ours which we derived directly with our specification. I.e. if their cutoff is denoted x_e and ours is denoted k_e , then it can be checked that $k_e = (1 + x_e) / (1 - x_e)$.

discrimination. Figure 3 illustrates for a continuum of values of c between $c_L = 1$ and $c_H = kc_L$, with k again varying between 1 and 10. Figure 4 shows the same qualitative results hold for the more general case with a continuum of markets (for any $\sigma < 1$, so that markets are not always covered due to demand being log-concave).

4.2 Exponential demand

When $\sigma = 1$, the demand faced by the platform is given by (8), with corresponding inverse demand

$$F_c(Q_c) = -\frac{c \ln Q_c}{\lambda},$$

where $\lambda > 0$ is the constant hazard rate. We assume $d = 0$ for tractability.

4.2.1 Two markets

Suppose there are only two markets, with c taking the values c_L and c_H . Define $k = c_H/c_L > 1$. For a particular market c

$$W_c = F_c e^{-\lambda \frac{F_c}{c}} + \frac{ce^{-\lambda \frac{F_c}{c}}}{\lambda} \quad (20)$$

so that $W_c(F_c^*) = 2ce^{-1}/\lambda$ under price discrimination since $F_c^* = c/\lambda$. Therefore, welfare across both markets under price discrimination is $W_{PD} = 2c_L(1+k)e^{-1}/\lambda$. Now consider welfare without price discrimination. The monopolist will set the uniform price F to maximize

$$\Pi = F \left(e^{-\frac{\lambda F}{c_L}} + e^{-\frac{\lambda F}{c_H}} \right).$$

The optimal uniform price \widehat{F} solves the first-order condition

$$e^{-\frac{\lambda \widehat{F}}{c_L}} \left(1 - \frac{\lambda \widehat{F}}{c_L} \right) + e^{-\frac{\lambda \widehat{F}}{c_H}} \left(1 - \frac{\lambda \widehat{F}}{c_H} \right) = 0.$$

The solution can be written as $\widehat{F} = -\rho c_L/\lambda$, where ρ solves $(1 + \rho)e^\rho + (1 + \rho/k)e^{\rho/k} = 0$. Note ρ is just a function of k .

Welfare under uniform pricing is

$$\begin{aligned} W_U &= \widehat{F} \left(e^{-\frac{\lambda \widehat{F}}{c_L}} + e^{-\frac{\lambda \widehat{F}}{c_H}} \right) + c_L \left(\frac{e^{-\frac{\lambda \widehat{F}}{c_L}}}{\lambda} \right) + c_H \left(\frac{e^{-\frac{\lambda \widehat{F}}{c_H}}}{\lambda} \right) \\ &= \frac{c_L}{\lambda} \left((k - \rho) e^{\frac{\rho}{k}} - (\rho - 1) e^\rho \right). \end{aligned}$$

Therefore

$$W_{PD} - W_U = \frac{c_L}{\lambda} \left(2(1 + k) e^{-1} + (\rho - 1) e^\rho - (k - \rho) e^{\frac{\rho}{k}} \right).$$

Since ρ is just a function of k , and the term in brackets in $W_{PD} - W_U$ is just a function of ρ and k , the sign of $W_{PD} - W_U$ just depends on k . It is then straightforward to verify $W_{PD} - W_U > 0$ for all $k > 1$.

The limit case as $k \rightarrow \infty$ is particularly instructive. In the limit as $k \rightarrow \infty$, $\rho \rightarrow -k$.¹² Then $W_{PD} - W_U \rightarrow 2c_L e^{-1}/\lambda > 0$. This implies as $k \rightarrow \infty$, $\widehat{F} \rightarrow kc_L/\lambda = c_H/\lambda = F_{c_H}^*$. In other words, for large k , the uniform price converges to the price that the monopolist would set to the high cost market under price discrimination. Indeed, this is approximately true even for moderately large k . For example, if $k = 10$ then $z = -10$ to one decimal place and if $k = 20$ then $\rho = -20$ to five decimal places. Surprisingly, even though with exponential demand there is always positive demand for the monopolist from both markets (at any prices), when faced with having to set a uniform price, the monopolist acts as though it effectively drops the low demand market and sets its price focusing almost purely on the high demand market. Thus, welfare is unambiguously higher with price discrimination.

Figure 5 illustrates. Note we set $\lambda = 9$ (which implies the same normalization to a 10 percent ad-valorem fee as before). The figure shows that the uniform price is indeed very close to the optimal price for the high demand market when k is moderately large (even with $k > 5$). The figure suggests this property may hold even for lower k when $d > 0$.

¹²To see this, suppose not so that ρ is bounded from below by some finite $-r$ (with $r > 1$). Then there is no real number solution for ρ since this requires $r = -\ln(1/(r-1))$ but the right hand side is only positive for $r > 2$, in which case the left hand side is strictly larger. On the other hand if $\rho = -rk$ then the solution for ρ requires $(1 - rk)e^{-rk} + (1 - r)e^{-r} = 0$ but the left hand-side is negative for all $k > 1$ including in the limit as $k \rightarrow \infty$ unless $r \rightarrow 1$. Therefore, it must be that as $k \rightarrow \infty$, then $\rho \rightarrow -k$.

4.2.2 Continuum of markets

Normalize $c_L = 1$ and $c_H = k > 1$ so $c \in U[1, k]$ and the density function is $g(c) = 1/(k-1)$.

Then total welfare is

$$\begin{aligned} W_{PD} &= \int_1^k \left(\frac{W_c(F_c^*)}{k-1} \right) dc \\ &= \left(\frac{1+k}{\lambda} \right) e^{-1} \end{aligned}$$

under price discrimination. Now consider welfare without price discrimination. The monopolist will set the uniform price F to maximize

$$\Pi = \int_1^k \left(\frac{F e^{-\frac{\lambda F}{c}}}{k-1} \right) dc.$$

The optimal uniform price $\hat{F} = \rho/\lambda$, where

$$\int_1^k \left(\frac{e^{-\frac{\rho}{c}}}{k-1} \right) \left(1 - \frac{\rho}{c} \right) dc = 0 \quad (21)$$

(21) can be rewritten

$$Ei\left(1, \rho\right) - Ei\left(1, \frac{\rho}{k}\right) = \frac{e^{-\rho} - k e^{-\frac{\rho}{k}}}{2\rho}, \quad (22)$$

where

$$Ei(1, y) = \int_1^\infty \frac{e^{-xy}}{x} dx.$$

Now

$$W_U = \int_1^k \left(\frac{F e^{-\frac{\lambda F}{c}}}{k-1} \right) dc + \int_1^k \left(\frac{c e^{-\frac{\lambda F}{c}}}{\lambda(k-1)} \right) dc$$

so

$$\begin{aligned} W_{PD} - W_U &= \left(\frac{1+k}{\lambda} \right) e^{-1} - \int_1^k \left(\frac{F e^{-\frac{\lambda F}{c}}}{k-1} \right) dc - \int_1^k \left(\frac{c e^{-\frac{\lambda F}{c}}}{\lambda(k-1)} \right) dc \\ &= \frac{4(k^2-1)e^{-1} - \rho(k e^{-\frac{\rho}{k}} - e^{-\rho}) - 2(k^2 e^{-\frac{\rho}{k}} - e^{-\rho})}{4\lambda(k-1)}, \end{aligned} \quad (23)$$

where we have used (22) to get (23). The solution to (21) just depends on k and since the sign of (23) just depends on ρ and k , the sign of (23) just depends on k . Evaluating (23), it can be confirmed that it is always positive for $1 < k \leq 5$.

Now consider the expression for larger k . Given $k > 1$, the expression in (23) is increasing in ρ . For all $k > 5$, the solution to (21) satisfies $\rho > rk$, where $r = 0.61$. Thus, evaluating (23) with $\rho = rk$ gives a lower bound for the change in welfare from allowing price discrimination. Then

$$W_{PD} - W_U > \frac{k^2 Z}{4\lambda(k-1)}$$

where

$$Z = 4 \left(1 - \frac{1}{k^2}\right) e^{-1} - r \left(e^{-r} - \frac{e^{-rk}}{k}\right) - 2 \left(e^{-r} - \frac{e^{-rk}}{k^2}\right),$$

which is positive for all $k > 5$, so this shows that (23) is also positive for all large k , including in the limit as $k \rightarrow \infty$. Figure 6 illustrates.

4.3 Log-convex demand

So far we have considered generalized Pareto demands which are log-concave ($\sigma < 1$) or exponential ($\sigma = 1$). In this section we consider the case where $\sigma > 1$ so that demand is log-convex. We assume $d = 0$ for tractability.

Aguirre *et al.* (2010) show that in the case of two markets that are always covered, when $\sigma > 1$ price discrimination raises welfare if the discriminatory prices are not far apart. Figure 7 shows the welfare effects of price discrimination allowing for a wider range of demands and so discriminatory prices. Price discrimination always increases welfare. Figure 8 shows the same results for a continuum of markets. Welfare is again always higher with price discrimination. Given markets are always covered when demand is log-convex, the rationale for this is the same as the exponential case.

5 Concluding remarks

In this paper, we investigated a puzzle and possible policy concern: for platforms which only incur small per-transaction costs, why do they charge sellers ad-valorem fees (fees based on

value)? Our analysis suggested that price discrimination provides a major explanation. A platform has to deal with trade in multiple goods with different costs and valuations that it does not observe. Under the class of generalized Pareto demand functions, we showed the use of a linear fee schedule (a proportional fee plus fixed transaction fee) is optimal for the platform and allows it to achieve the same profit that it would obtain under third-degree price discrimination if it could observe the costs and valuations for each good traded and set a different optimal fee for each. We also found this linear fee schedule generally increases social welfare for this class of demands when compared to not allowing the platform to use ad-valorem fees, provided there is enough heterogeneity in the scale of goods traded. Therefore, caution should be taken when policymakers consider restricting platforms from using such ad-valorem pricing.

Our framework can also address some related policy questions which we are currently working on incorporating. For instance, it can address whether a regulator that wants to cap platform fees would rather do this by requiring the platform to only set a single per-transaction fee which is at or below the cap or by allowing the platform to set ad-valorem fees provided the weighted average fees implied by the fee schedule are at or below the cap.¹³ This can also be framed as a Ramsey problem. Suppose the platform incurs fixed cost for its operation in addition to per-transaction costs. A regulator may want the platform to set fees to just recover both costs. The policy question is whether the regulator should allow the use of ad valorem fees in doing so?

Similarly, our analysis can usefully be applied to the large literature on specific vs. ad-valorem taxation. The existing literature addresses situations in which one type of tax raises a fixed amount of revenue more efficiently than another. None of the literature seems to cover the case that we are addressing in which the good being taxed varies in scale (so in cost and valuation). Thus, suppose we interpret the platform as the tax authority and suppose, as seems reasonable, the tax authority cannot identify the actual goods traded but only their reported prices. Imposing a specific per-unit tax will impose a higher distortion on low value goods and cause some not to be traded at all. In contrast, an ad-valorem tax does not suffer from this problem since low value goods are taxed less. Our current findings suggest that under the class of demands considered, a taxing authority that wants to maximize tax revenue for a category of

¹³This is an issue that has arisen in the regulation of interchange fees in Australia and the United States.

goods which varies significantly in scale can use a pure ad-valorem tax to collect more revenue than a specific tax, and at the same time achieve higher social welfare. It remains to extend the finding to a situation where the taxing authority wants to collect a fixed amount of revenue and wants to minimize any efficiency loss from doing so. We are working on this extension which should provide a new rationale for why an ad-valorem tax may be better than a specific tax, namely that it would not shut down the trade of low value goods.

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Appendix A. Alternative trading mechanism

In this appendix we consider an alternative setting which gives rise to the same demand specification (1) and for which all of the results of the paper continue to hold. Suppose buyers can buy from sellers through the platform or through an alternative trading mechanism. For example, the platform may be eBay or Amazon, and the alternative may be to purchase the good from the seller's own website or shop (possibly locally). Alternatively, the platform may be Visa or MasterCard, and the alternative may be to purchase the good using cash. Different goods are indexed by c , as before, which is the scale parameter. For good c , sellers all incur a unit cost c to provide the good and buyers all value the good at $v_c > c$ which is sufficiently large such that all buyers want to buy the good (either on the platform or off the platform). Note buyers valuation for each different good need not be scaled by c any more. There are assumed to be sufficient identical sellers so that there is homogenous Bertrand competition both on and off the platform.

The platform is more efficient in the sense that buyers purchasing through the platform can avoid an inconvenience or loss that is expected when purchasing through the alternative means, a loss which is equal to bc , where $b \geq 0$. For the examples of payment card platforms such as those offered by Visa and MasterCard, this could be the inconvenience of having to use cash for payment (i.e. so it is assumed to be proportional to the amount of cash used, which equals cost c given Bertrand competition amongst cash sellers). For the examples of eBay and Amazon, this could be the additional costs of distributing locally (i.e. the costs of storage at a retailer) if trade is not through the platform, with these additional costs assumed to be proportional to the cost of the good itself so that the retail price buying directly is $(1 + b)c$.

Suppose as in the main text, without loss of generality, platform fees are only on the seller side. Thus, when a transaction takes place, a buyer of good c with (benefit) parameter b and facing a price for good c on the platform of P_c receives consumer surplus of $v_c - P_c$ using the platform and $v_c - (1 + b)c$ using the alternative. We assume the parameter $b \geq 0$ varies across buyers. In the case of a card payment network, this captures the idea that different buyers face different costs of using the alternative to cards, say cash. In the examples of eBay and Amazon, this captures the idea that the additional costs of local supply differ for buyers located

in different cities compared to buying on the common platform.¹⁴ The assumptions on $1 + b$ and other parameters then follow as in Section 2. The number of transactions on the platform Q_c for a particular good c is the measure of buyers whose surplus from using the platform is greater than using the alternative, i.e.

$$\Pr \left(1 + b \geq \frac{P_c}{c} \right),$$

which gives rise to the demand function (1) as in the benchmark model. Facing the same demand function as in the benchmark model, the rest of the analysis follows as before. Note that when consumers have the option to purchase off the platform, welfare in each market W_c needs to be adjusted by adding the constant \overline{W}_c , where \overline{W}_c is a constant term (i.e. that does not depend on the fees charged by the platform). Formally, welfare from good c can be written as

$$\begin{aligned} \widehat{W}_c &= \int_1^{1+\frac{F_c}{c}} (v_c - cx) dH(x) + \int_{1+\frac{F_c}{c}}^{1+\bar{b}} (v_c - c - d) dH(x) \\ &= \int_1^{1+\bar{b}} (v_c - cx) dH(x) + \int_{1+\frac{F_c}{c}}^{1+\bar{b}} (cx - c - d) dH(x) \\ &= \overline{W}_c + W_c, \end{aligned}$$

since

$$\int_{1+\frac{F_c}{c}}^{1+\bar{b}} (cx - c - d) dH(x) = \int_0^{Q_c} (P_c(Q) - c - d) dQ$$

and $Q_c = 1 - H(1 + F_c/c)$.

¹⁴For example, suppose different cities involve different costs of storage of good c . Buyers located at location b face an effective price of $(1 + b)c$, where $b \geq 0$. In contrast, the price on the platform which is available to everyone regardless of their location is determined simply by the cost c and the platform fee.

Appendix B. Figures

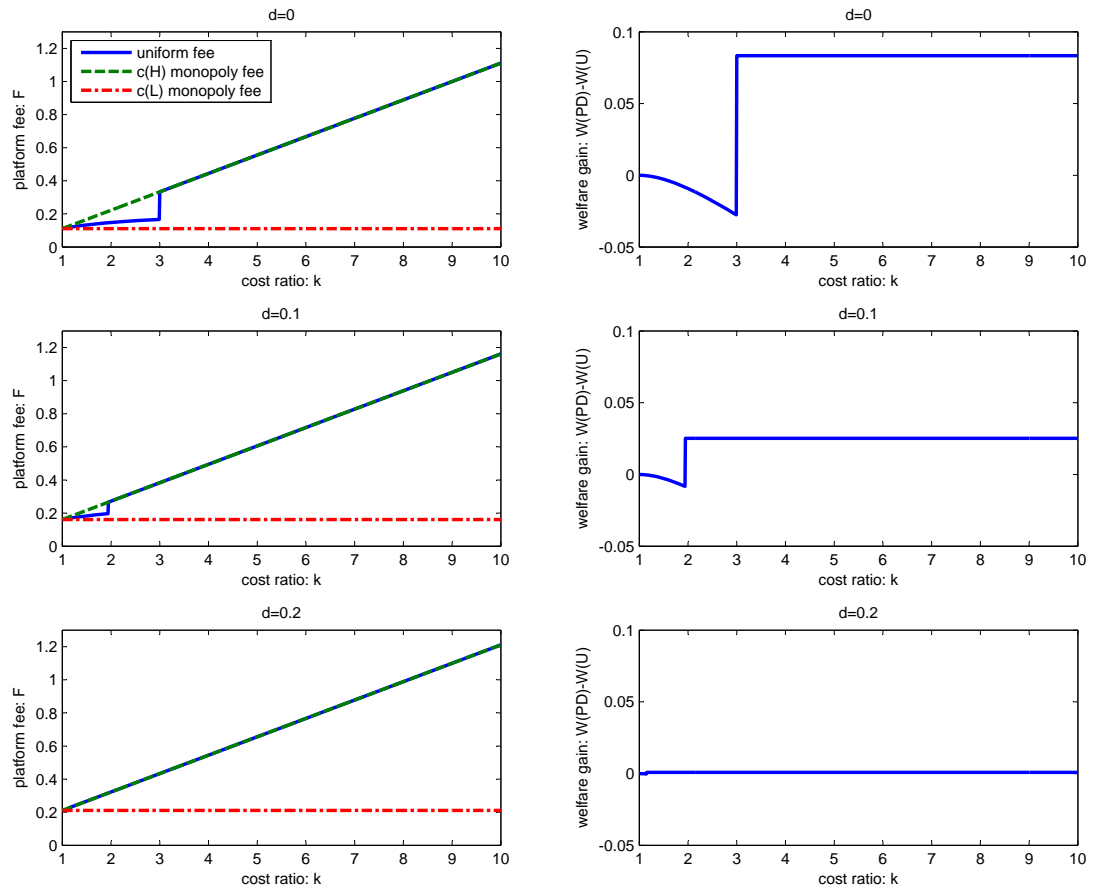


Figure 1: Two markets with linear demand

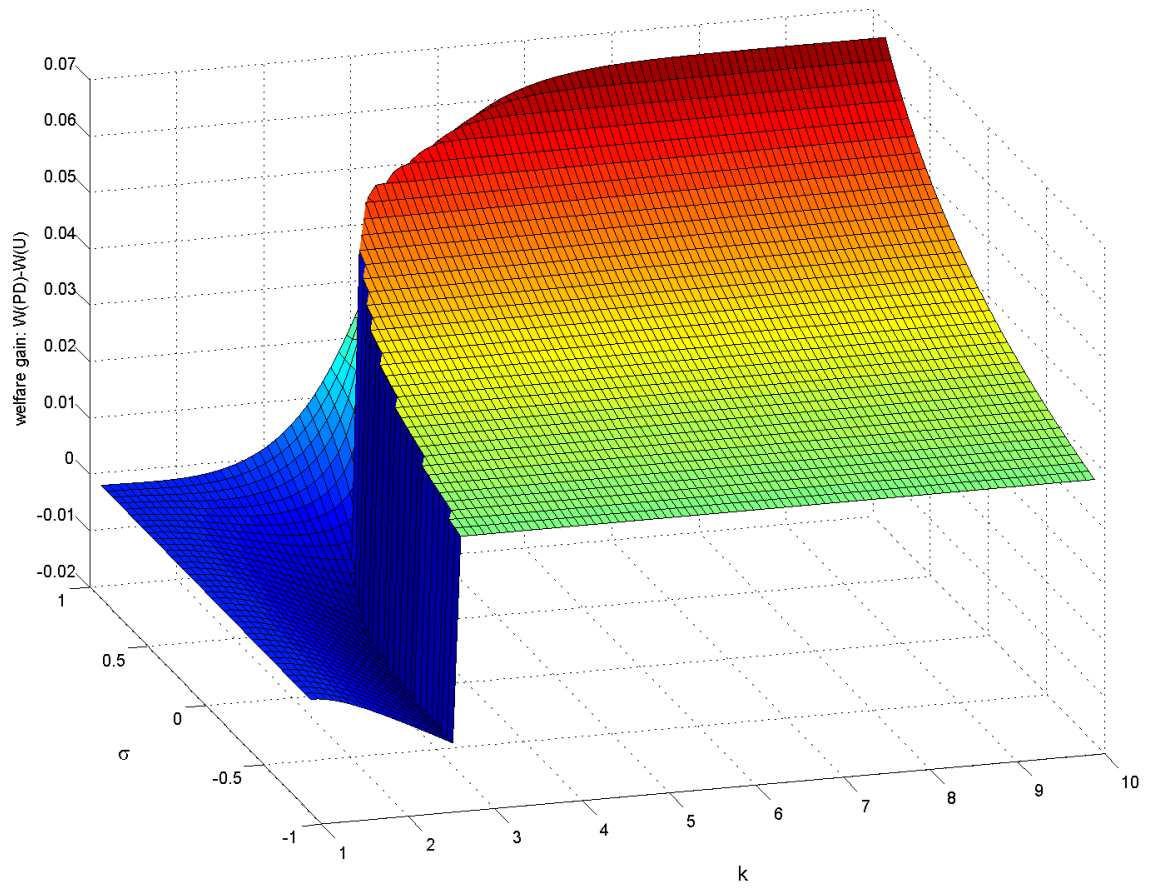


Figure 2: Two markets with log-concave demand ($\sigma < 1$)

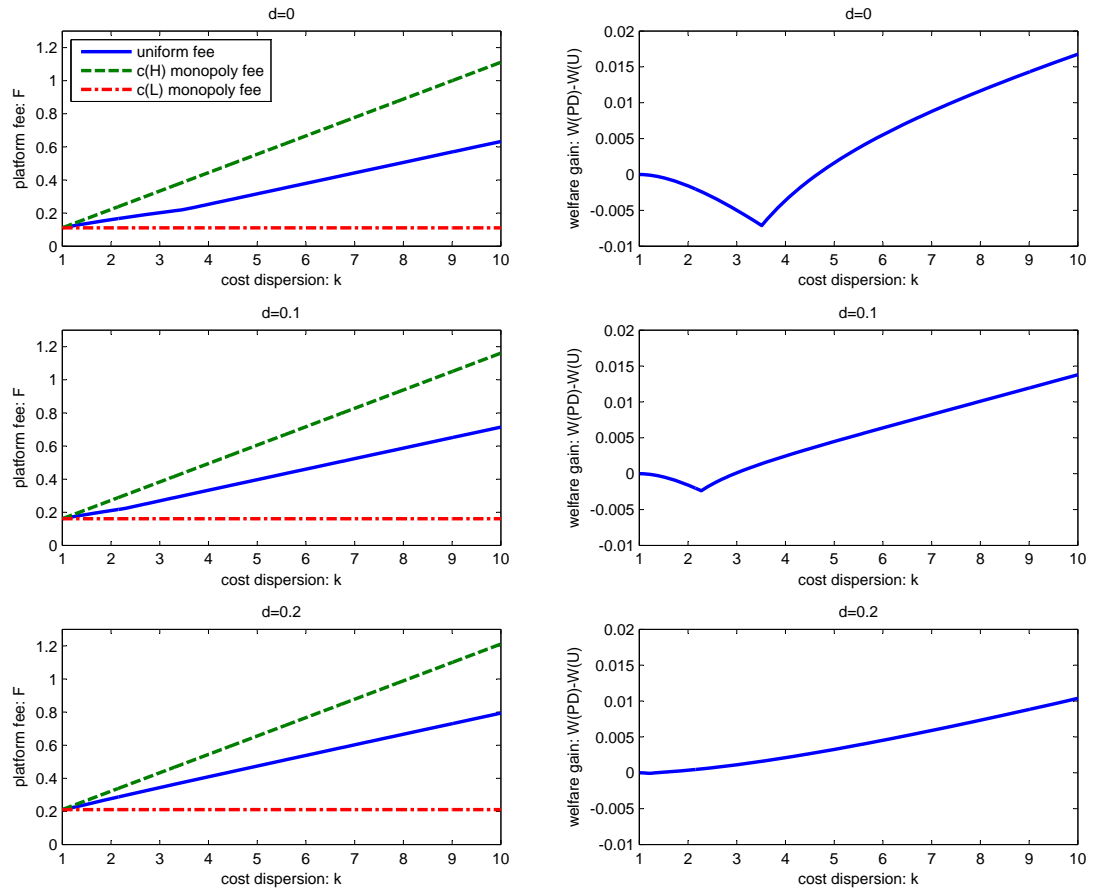


Figure 3: Continuum of markets with linear demand

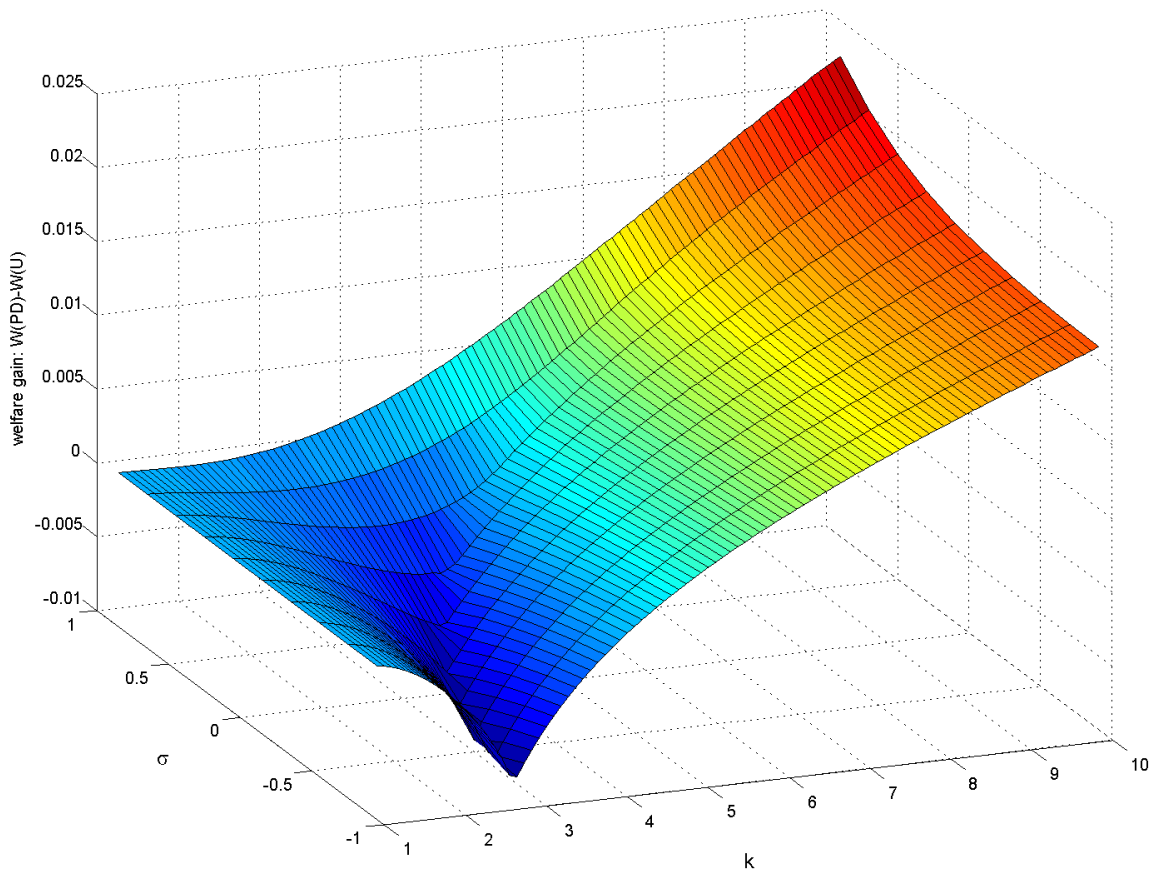


Figure 4: Continuum of markets with log-concave demand ($\sigma < 1$)

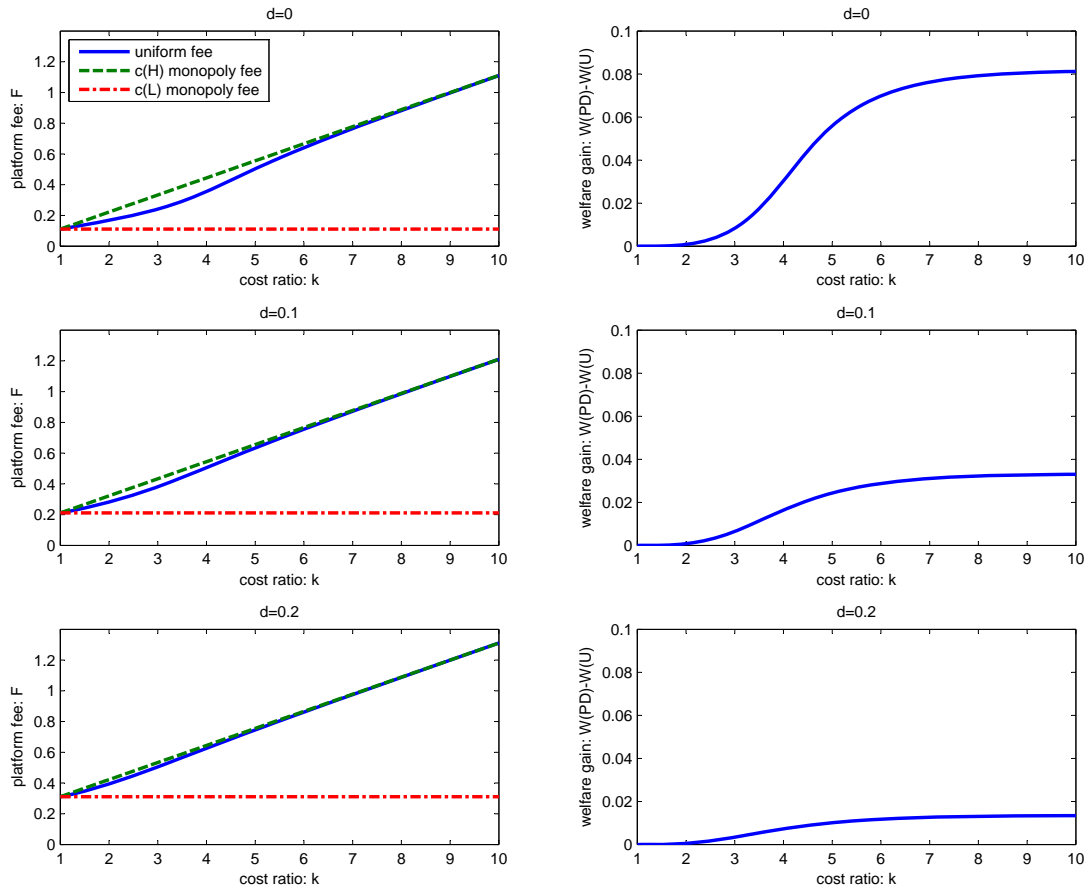


Figure 5: Two markets with exponential demand ($\sigma = 1$)

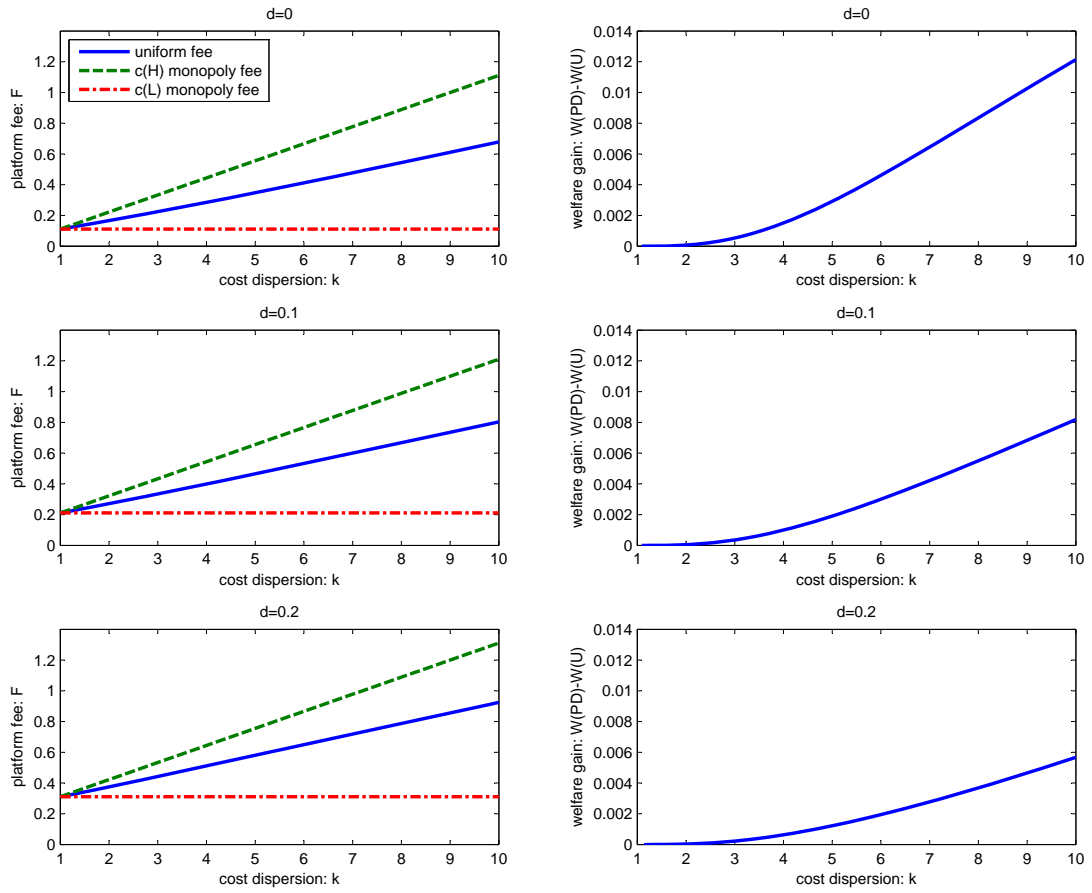


Figure 6: Continuum of markets with exponential demand ($\sigma = 1$)

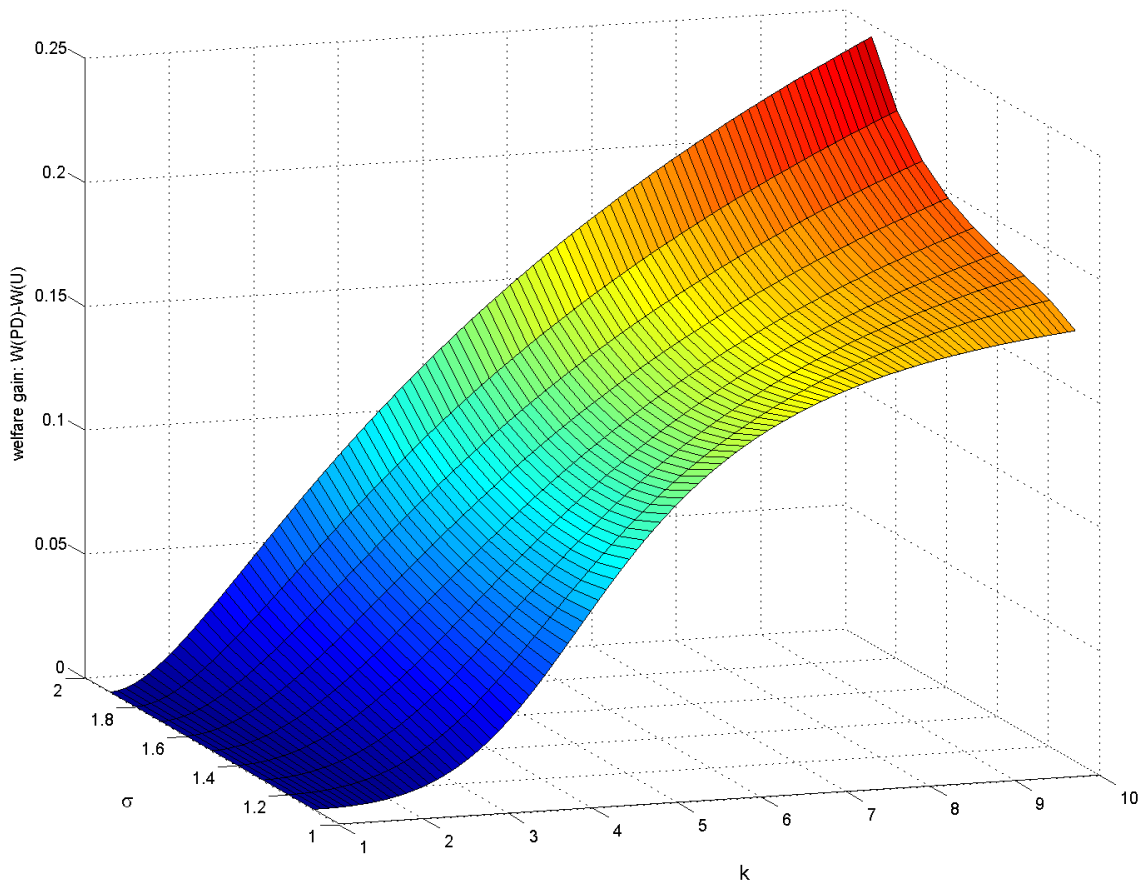


Figure 7: Two markets with log-convex demand ($1 < \sigma < 2$)

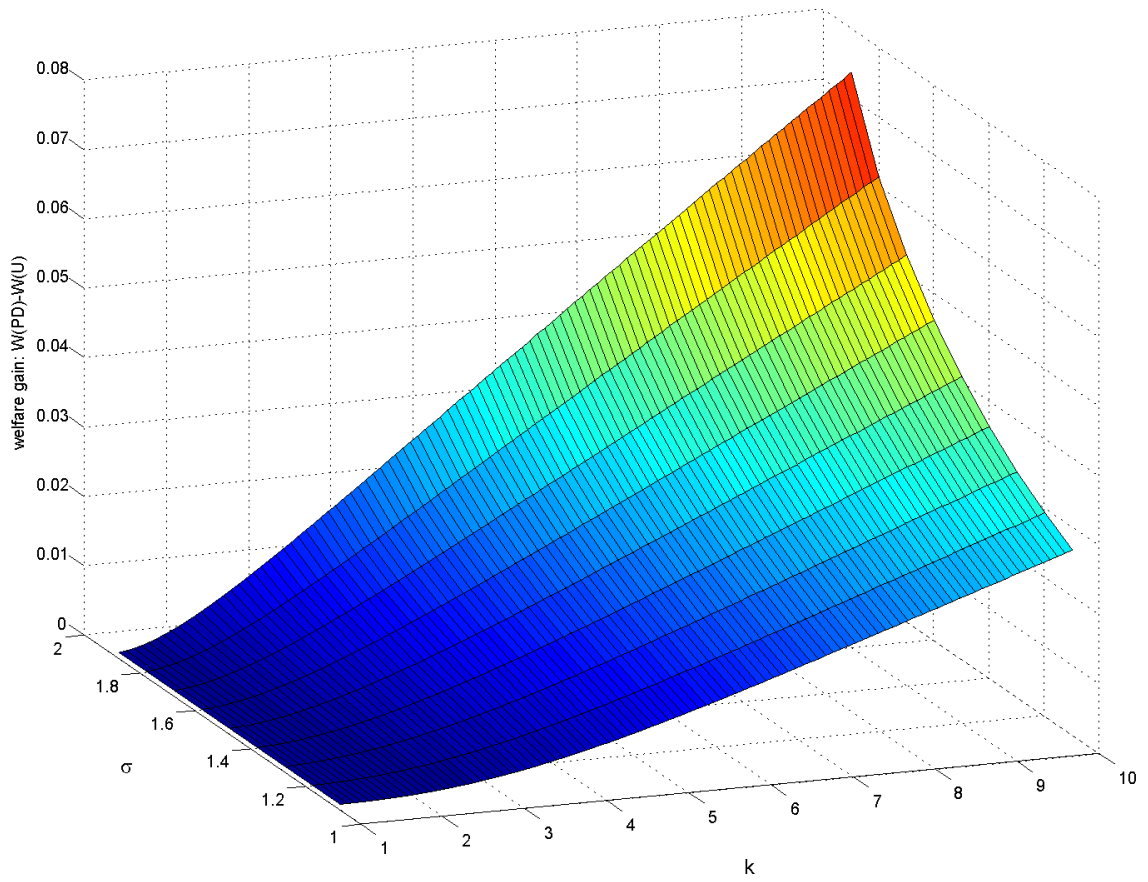


Figure 8: Continuum of markets with log-convex demand ($1 < \sigma < 2$)