Consumer Unawareness and Competitive Strategies

Zhen Liu

Department of Economics, SUNY at Buffalo, zl24@buffalo.edu

Chun (Martin) Qiu

Desautels Faculty of Management, McGill University, chun.qiu@mcgill.ca

Abstract

Many purchase decisions rely on complex information. In reality, some consumers may not be well informed and unaware of their lack of information, a situation termed *consumer unawareness*. In a stylized model of market entry, this paper investigates how consumer unawareness of preference-product match affects firms' strategies in product, pricing, and promotion. It shows that for entry to occur, the level of unawareness must be intermediate. When entry occurs, firms offer differentiated products to avoid head-on competition and use mixed pricing strategies. As the level of unawareness decreases, both the incumbent and entrant become less competitive in pricing. This paper also analyzes how firms strategically promote consumer awareness through advertising. While the promotion of the entrant helps increase its demand after entry, the promotion of the incumbent can serve as either a barrier or an invitation to entry. Fearing the former, the entrant may not enter, or enter with limited promotion; benefiting from the latter, the entrant may enter and free ride on the incumbent's promotion. As a result of the former situation, a relatively high degree of consumer unawareness is maintained despite competition. This paper also identifies the incumbent's brand equity and promotion costs as critical factors that drive firms' strategic choices. Finally, this paper presents economic and managerial implications of the findings.

Keywords: consumer unawareness, bounded rationality, preference match, competitive strategies, market entry

There are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns, the ones we don't know we don't know. —Donald H. Rumsfeld

1. Introduction

Consumers rely on information to make purchase decisions (Bettman, Johnson and Payne 1991). Yet, the information usually involves various aspects of the purchase and consumption, and hence, is complicated (Thorelli and Thorelli 1977). As a result, consumers do not possess all of the necessary information to make a sound purchase (e.g., Tellis and Gaeth 1990). Very often many consumers may do not know their lack of information, a situation termed *consumer unawareness* in this paper.

There are many different types of consumer unawareness given the nature of the information (e.g., Spiegler 2011). [1] This paper focuses on consumer unawareness of the match between consumer personal needs and product features, as consumers may misperceive either the former or the latter. It is frequently observed that some consumers choose a prevailing product when they are unaware of some alternative which provides a better match. For instance, when first-time guitar learners start selecting a guitar, they may not even know there are left-handed guitars available; when parents choose diapers for their newborns, they do not know whether the products will cause any skin irritations, and if so, how to alleviate the symptom (by using hypoallergenic products). The personal care industry (shampoos, shower gels, lotions, body sprays etc) is full of examples of consumer awareness contingent on the introduction of better-matched products. Starting with the era when virtually everyone was using the same soap, the industry gradually developed variations out of basic products to appeal to consumers with different personal characteristics such as gender, age, skin types and hair types (Dyer, Dalzell and Olegario 2004). Meanwhile, some consumers were not ready to appreciate the variations instantly, although they eventually became aware that the variations provide batter matches for many. The most recently launched *Vaseline for Men* by Unilever highlights another example of the unawareness of a better-fitting product for some male consumers.

Such consumer unawareness creates an entry barrier for firms who identified consumers' needs for a new but unaware matched consumption. Yet, overcoming consumer unawareness also means great

---

[1] Some types of consumer unawareness are well recognized in marketing. For example, consumers are often unaware of the relationships of different brands which are closely related. The recently launched Volkswagen's minivan brand Routan was featured as "German Engineering", but few consumers know that it was manufactured in Chrysler's factories and "no more than a Caravan with a Volkswagen badge" (*Automotive Addicts* 2009), but Routan is sold for $10,000 more than Caravan. Also few consumers do not know that many luxury-brand fragrances such as Gucci, Hugo Boss, and D&G are actually manufactured and managed by P&G. In retailing, Consumers are unaware that many private-label products are secretively supplied to retailers by national manufacturers. More than 15% of the store-brand merchandise of Aldi—the biggest discount retailer in Germany—has national-brand twins that are sold at a 15% to 89% higher cost in other stores (Schneider 2005).

business opportunity. For example, in 1968, Estée Lauder launched Clinique as the world's first allergy-tested, dermatologist-driven skin care product line. However, the line did not take off for several years until the company invested millions of dollars in marketing. In 1978, Clinique sales increased to $80 million, almost 30% of Estée Lauder's total revenues that year. (Koehn 2001). In many similar cases of new product diffusion, consumer unawareness of new offerings leads to the obsession with incumbent products. It is not clear how such an environment shapes market entry and competition. While many aspects regarding consumers' lack of information have been extensively studied since the seminal work of Akerlof (1970), not until recently has some issues of consumer unawareness been examined (e.g., Gabaix and Laibson 2006; Li, Peitz and Zhao 2010). This paper intends to explore in depth firms' entry and competitive strategies under consumer unawareness.

Specifically, in a duopoly framework, this paper investigates how consumer unawareness affects market entry and firms' competitive strategies in product and pricing; and in turn, how firms strategically manage consumer unawareness via promotion. In the model, consumers are heterogeneous in preferences and information. Some consumers are aware of one possible product (termed *special product*) serving them better than the prevailing one (termed *generic product*) sold by the incumbent, whereas some are not aware. The entrant intends to offer the special product to those consumers. Yet, the entrant faces the barrier caused by consumer unawareness and the incumbent's pre-emptive moves. We argue that, in equilibrium, the entrant will launch the special product if and only if consumer unawareness is not too low or too high. When entry occurs, both firms use mixed pricing strategies. As the degree of consumer unawareness decreases, the two firms become less competitive in pricing; resulting in an increase in the entrant's profit.

When both firms can promote consumer awareness, the entrant may do so to increase the demand for the special product. However, the incumbent may also promote consumer awareness. Its promotion can serve as either a barrier or an invitation to entry. Fearing the former, the entrant may not enter, or enter with limited promotion; benefiting from the latter, the entrant may enter and free ride on the incumbent's promotion. Thus, competition does not necessarily lead to more promotions of consumer awareness.

We identify two factors that crucially influence the market equilibrium: The incumbent's market establishment, as reflected by its brand equity, and the effectiveness of promotion, as reflected by the unit cost of promotion. When brand equity is low, expecting the entrant to price aggressively, the incumbent may promote consumer awareness to soften the competition without offering the special product. When promotion cost is low, the incumbent is more likely to deter entry through promotion. As a result, a lower cost of promotion makes the entry more difficult.

Conceptually, there is a clear distinction between unawareness and uncertainty (Li, Peitz, and Zhao 2010). Uncertainty, or "known unknowns", implies that a decision maker does not know the exact state at

a moment of time. For example, Jing (2011a) models consumer uncertainty of matched consumption, with the uncertainty per se as common knowledge. Thus, the decision maker is typically associated with full awareness of all possible states in a decision set, including their existence and all related consequences. In comparison, unawareness, or "unknown unknowns", implies that a decision maker is associated with incomplete knowledge of the states in a decision set. Hence, he either ignores the existence of some states or only knows incomplete consequences related to some states. [2]

In addition to contributing to the literature on incumbent's responses to market entry (e.g., Shankar 1997; Narasimhan and Zhang 2000; Shen and Villas-Boas 2010), this paper also contributes to the growing body of literature that studies the relationship between consumers' bounded rationality and firms' marketing strategies. In particular, some studies examine how consumer forgetfulness—"unknown knowns"—influences firms' pricing decisions. For example, Villas-Boas and Villas-Boas (2008) show how consumer forgetfulness regarding their preferences affects the time interval between sales. More recently, Chen, Iyer and Pazgal (2010) have studied the effects of limited memory and categorization on duopoly price competition. In comparison, we investigate the influence of consumer unawareness, or "unknown unknowns," on firms' competitive strategies.

The remainder of the paper is structured as follows. In Section 2, we discuss related literature. In Section 3, we set up the basics. Section 4 analyzes the benchmark model with exogenous consumer unawareness. Section 5 analyzes the extension with consumer unawareness endogenously determined by firms through promotion. We discuss some modeling issues for clarification in Section 6. We conclude in Section 7. A demonstration of unawareness and proofs of intermediate results, lemmas, corollaries, and propositions are in the appendix.

2. Related Literature in Consumer Unawareness

Economic literature dating back to Simon (1955) has discussed the reasons for individual unawareness, yet not until recently did unawareness receive more attention. Dekel, Barton and Rustichini (1998) show that the standard model of information structure cannot formulate unawareness. Modica and Rustichini (1999) consider a modified information partition model to formulate unawareness in an individual decision making setting. Heifetz, Meier and Schipper (2006) andLi (2009) present frameworks that formulate interactive unawareness. Applied work incorporating unawareness is growing in some fields such as industrial organization and finance (for a review, see Spiegler 2011). Yet, most of the studies investigate the unawareness of product features that lead to vertical differentiation, in the form of

---

[2] In Appendix I, we demonstrate the difference between full awareness and unawareness, or partial awareness, in the context of preference-product match.

information shrouding. In contrast, this paper focuses on consumer unawareness of horizontally differentiated products.

Specifically, our paper is closely related to three papers. The first is by Gabaix and Laibson (2006). In their model, firms sell products that are composites of a base good and add-ons. Firms choose not to educate consumers about the price of add-ons when add-ons have close substitutes. While naive consumers are unaware of the price of add-ons and suffer from unexpected high costs, informed consumers exploit the existing low price of the base good and cheap, close substitutes. In contrast, this paper considers horizontally differentiated products and focuses on consumer heterogeneity in both information and preferences.

The second paper by Li, et al. (2010) studies a monopoly firm that has private information about its product's degree of adverse effects. In one case, they allow the representative consumer be aware but uncertain about the adverse effects; in the other case, the consumer is unaware of the adverse effects. They show how mandatory disclosure of private information can affect the total welfare and consumer surplus. Their study complements ours by allowing aware consumers to be Bayesian and by emphasizing the difference between uncertainty and unawareness. By comparison, our study incorporates competition and market entry.

Finally, Kamenica, Mullainathan and Thaler (2011) study how firms can assist consumers when they lack the data on their individual usage of services. While they do not incorporate unawareness, they address a similar issue in that consumers may know too little about their preference types and are unable to find a good match. They find that mandatory information disclosure of the usage data does not always improve consumer welfare, as it can lead to higher prices in equilibrium. Similar to their results, we find reducing unawareness may lead to higher prices and lower consumer welfare. However, we focus on firms' competitive strategies such as product decisions and promotion decisions.

3. Model Setup

There is a cohort of consumers on the market with the total size normalized to one. With $0 < \lambda < 1$, a fraction λ of consumers, termed *special* consumers, demand some particular attribute of the underlying product to meet their consumption needs. All other consumers, termed *general* consumers, do not have such consumption complexity.

A *generic* product (denoted by *G*) matches the needs of general consumers but not those of special consumers. A *special* product (denoted by *S*), a modification of *G*, can satisfy the needs of special consumers as well as those of general consumers. Each consumer needs one unit of the product to satisfy his/her needs, and values it at $v > 0$. Hence, when consumers know their type, a general consumer has a

valuation of v for both G and S,[3] but a special consumer has a valuation of v for S and a lower valuation for G. Such a low valuation can even be negative if special consumers receive disutility from a mismatch. For expositional purposes, we set the valuation under mismatch to zero (results for a general specification can be obtained from the authors). The modeling setting reflects the basic idea of product differentiation where a new product evolves from the base product to satisfy the needs of newly identified consumer segments.

Without loss of generality, the marginal cost of G is normalized to zero, and the marginal cost of S is $c > 0$. Both firms have the same production costs.

Consumer Information Type and Unawareness

We describe consumers' information types. While some special consumers (with a fraction $\lambda - \theta$, $0 < \theta < \lambda$) are aware of their needs for the match, other special consumers (with a fraction θ) are not, and hence, make purchase decisions as general consumers do. Thus, θ measures the degree of consumer unawareness in the market.

Consumer unawareness is persistent in this paper. Consumers do not become automatically aware of the match without using the product for at least a certain period. This assumption follows behavioral studies where over-confidence (Hoch and Deighton 1989; Alba and Hutchinson 2000), or limited cognitive capability and experience (Bloch, Sherrell and Ridgway 1986; Aragones, et al. 2005) can contribute to the persistency of consumer unawareness.

As general consumers value S and G equally at v, whether or not they are unaware of the difference between G and S does not affect their purchase decisions and the subsequent market outcome. Hence, we impose no restrictions on the information types of general consumers.

Asymmetric Duopoly

We consider the game between two asymmetric firms: Firm 1 is a big established firm that already sells G on the market, and firm 2 is a small firm that plots its entry. The two firms differ in three aspects. First, in terms of market establishment, firm 1 enjoys a consumer-based brand equity (Keller 1993), $b \geq 0$, interpreted as the price premium that consumers are willing to pay for the value created by brand name associations and perceptual distortions (Kamakura and Russell 1993). Such a price premium, as Aaker (1996) claimed, "is the best single measure of brand equity." Firm 2, however, is new to the market

---

[3] General consumers' valuation for S depends on the nature of modification, and can be equal to, less than, or greater than v. Specifically, when the modification horizontally differentiates the two products (as in the case of golf clubs for the left-handed), general consumers may have a smaller valuation than v for S. When the modification vertically makes S better (e.g., high quality, more functional), general consumers may have a greater valuation for S. Finally, when general consumers perceive the modification to be irrelevant (as in the case of hypoallergenic products), their valuation remains at v. This paper assumes the last case which is prevalent in many product categories.

and does not have any consumer loyalty. Second, in terms of operations management, firm 1 can manage to produce and sell both G and S by incurring a fixed cost, $k > 0$, which includes all fixed costs incurred such as development costs, supply-chain costs and organizational costs; whereas firm 2 can manage to produce and sell only one product (either G or S). Finally, in terms of business responsiveness, firm 2 moves ahead of firm 1 in both promotion and production; firm 1 does not anticipate firm 2's entry, which is consistent with the literature on small firms (e.g., Acs and Audretsch 1990).

Firm 2 incurs an irreversible entry cost, f. To focus on non-trivial cases, we assume that f is less than the highest profit collectible from all special consumers: $0 < f < \lambda(v-c)$. Table 1 provides a list of notations. Table 2 summarizes the perceived valuation obtained by different types of consumers from the different firms/products.

4. Benchmark Model

We start with the benchmark model where firms take consumer unawareness as exogenous.

Non-preemptive constraint

We first we first identify the condition under which firm 1 offers G only as a monopoly when not threatened by firm 2's entry. Following Table 2, firm 1's monopoly profit by offering G, denoted by π_{1M}^{G}, is

$$\pi_{1M}^{G} = (1 - \lambda + \theta)(v + b);$$

firm 1's monopoly profit by offering both products, denoted by π_{1M}^{GS}, is

$$\pi_{1M}^{GS} = (1 - \lambda + \theta)(v + b) + (\lambda - \theta)(v + b - c) - k.$$

The comparison of π_{1M}^{G} and π_{1M}^{GS} yields the *no non-preemptive constraint* given below, which we assume to hold for the duopoly competition.

$$\theta > \Theta \equiv \lambda - k / (b + v - c). \tag{1}$$

The non-preemptive constraint suggests that there should be a sufficient number of unaware special consumers for firm 1 to offer G only.

Timeline

The game consists of three stages. In stage 1, firm 2 chooses which product to offer, or does not act —in which case the game ends. If firm 2 enters, in stage 2, firm 1 observes firm 2's move and decides

whether to add *S* to its product line. In stage 3, both firms simultaneously set prices to compete as a duopoly. The timeline of the game is depicted as follows.
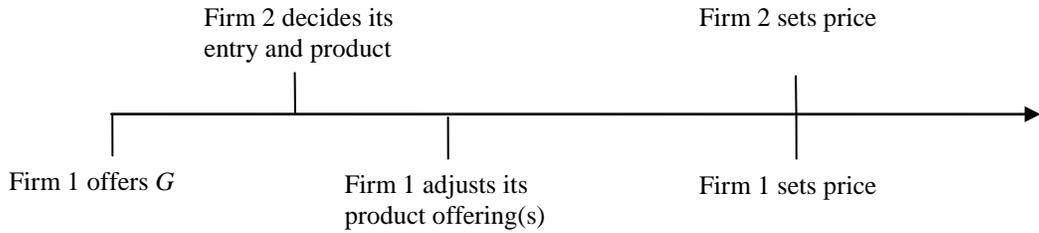
Using backward induction, we begin with the price-competition stage where firms' product choices are given; we then analyze the product-choice stage.

Analysis

In the duopoly competition, there are four subgames in terms of product offerings given as follows:

Subgame 1	Firm 1 sells *G* Firm 2 sells *G*	Subgame 2	Firm 1 sells *G* Firm 2 sells *S*
Subgame 3	Firm 1 sells {*G*, *S*} Firm 2 sells *G*	Subgame 4	Firm 1 sells {*G*, *S*} Firm 2 sells *S*

Whenever the offerings of the two firms overlap, the price competition is a Nash Bertrand competition. When the two firms offer distinct products, only mixed-strategy equilibria in pricing exist. We compute the two firms' equilibrium expected profits following standard procedure (e.g., Shilony 1977; Narasimhan 1988; the derivation is provided in Appendix II), and report the total equilibrium payoffs in Table 3.

_____

Insert Table 3 about here

_____

From Table 3, it is straightforward to see that firm 2's payoff is always –*f* when the two firms' offerings overlap. Hence, it is only possible for firm 2 to enter with *S* in Subgame 2. Nevertheless, to show Subgame 2 to be a SPNE, we also need to show that firm 1 does not want to offer *S* upon firm 2's entry, as in Subgame 4.

Denote firm 1's equilibrium profits in Subgames 2 and 4 as π_{1D}^{G*} and π_{1D}^{GS*}, respectively. By Table 3 they are given by

$$\pi_{1D}^{G*} = (1 - \lambda + \theta)\big[(\lambda - \theta)(v - c) + c + b\big]; \tag{2}$$

$$\pi_{1D}^{GS*} = b + (1 - \lambda + \theta)c - k. \tag{3}$$

Immediately following the above two equations, we have the following lemma.

LEMMA 1. *If* $b < \tilde{b}(\theta)$, $\pi_{1D}^{G*} > \pi_{1D}^{GS*}$, *and vice versa, where*

$$\tilde{b}(\theta) \equiv (1 - \lambda + \theta)(v - c) + \frac{k}{\lambda - \theta}. \tag{4}$$

Lemma 1 highlights the influence of cannibalization on firm 1's choice of product offerings upon firm 2's entry. The intuition is that, when offering the full product line {*G*, *S*}, firm 1 has to decrease its price of *S* to $b + c$ in order to compete with firm 2 on the special consumer segment. As a result, the price of *G* has to fall to $b + c$ to remain attractive to general consumers, who otherwise would purchase *S* instead. Although firm 1 creates a larger consumer base by offering the full product line, when it has small brand equity, i.e., $b < \tilde{b}(\theta)$, the benefit is outweighed by cannibalization as well as the fixed cost of offering multiple products. It is easy to see that when No-Preemption Constraint is satisfied (i.e., $\theta > \Theta$), firm 1 will offer *G* only (see Appendix II A3 for the proof).

We now examine firm 2's profitability upon entry. Firm 2's equilibrium expected profit in Subgame 2, π_{2D}^{S*}, is given by

$$\pi_{2D}^{S*} = (\lambda - \theta)(v - c) - f. \tag{5}$$

Eq. (5) implies that firm 2 needs a sufficient number of aware-special consumers to break even. This condition determines the upper bound of the degree of unawareness at which firm 2 enters, denoted as $\bar{\theta}$:

$$\bar{\theta} \equiv \lambda - f / (v - c). \tag{6}$$

Based on Lemma 1, the equilibrium product-offering strategies are characterized in Proposition 1.

PROPOSITION 1. *In equilibrium, when* $\theta \leq \bar{\theta}$, *firm 1 always sells G, and firm 2 enters to sell S. When* $\theta > \bar{\theta}$, *firm 2 does not enter.*

Proposition 1 suggests that, for firm 2 to enter with *S*, consumer unawareness needs to lie within an intermediate range. Hence, the availability of *S* depends on composition of consumers, as well as on other

aspects such as firm 1's brand equity b and fixed cost k, which determines Θ. It is consistent with the literature which argues that large brand equity gives a firm advantage forming significant entry barriers (Bonanno 1986; Smiley 1988). However, the effect of brand equity is not limited to the preemption considered here. More on this will be discussed in the next section.

When firm 2 enters in equilibrium, Corollaries 1 and 2 respectively describe the relationships of the degree of unawareness and the two firms' average prices and profits.

COROLLARY 1. *In equilibrium where firm 2 enters, when the degree of unawareness decreases, both firms' average prices increase.*

Corollary 1 suggests that when the degree of unawareness decreases, in equilibrium firm 2 is more willing to focus on special consumers and hence, less willing to undercut firm 1 for the general consumers. When firm 2 prices less aggressively, so does firm 1. The result also implies that, if awareness is exogenously increased (e.g., through public campaign or government intervention), the welfare improvement is not Pareto. Specifically, while some unaware-special consumers will gain awareness, and benefit from a better match, other consumers, including aware-special consumers and general consumers, will lose; they now pay a higher price.

COROLLARY 2. *In equilibrium where firm 2 enters, when the degree of unawareness decreases, firm 2's profit increases and firm 1's profit may first increase and then decrease.*
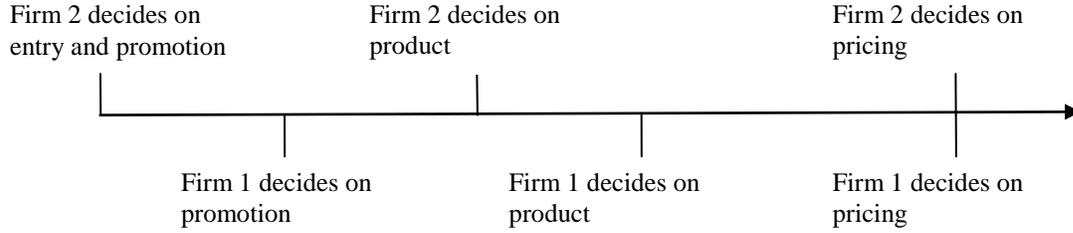
This result follows Equations (2) and (5). As the degree of unawareness decreases, firm 2 obtains a larger consumer base. Consequently, firm 2 can raise prices and earns a larger profit. For firm 1, the decrease of unawareness reduces its consumer base and hurts its profitability (the direct effect); however, firm 1 benefits from the less aggressive pricing of firm 2 (the indirect effect). When the direct effect dominates the indirect effect, firm 2's profit decreases, and vice versa. In the next section, we will revisit this result when firms can promote awareness.

5. Awareness Promotion

In this section, we consider the scenario in which firms can promote consumer awareness. The objective of the promotion is to educate unaware-special consumers about the needs of match for consumption, e.g., the so-called seller induced learning (e.g., Jing 2011b). Hence, the promotion is category-specific rather than brand-specific. As firms cannot *a priori* identify consumer type, promotions are non-targeting.

Timeline of Duopoly Problem with Promotion

Since firm 1 does not intend to offer *S* when acting as a monopoly, it is natural to assume that firm 2 moves ahead of firm 1 in targeting special consumers, and the two firms sequentially make promotional decisions. The timeline of the new game is given below. In stage 1, firm 2 decides whether or not to enter, and if it does, it decides how much promotion is needed. In stage 2, firm 1 decides its promotion. If firm 2 does not enter, the game ends. The rest of the game is identical to the benchmark model.

Firm 2 decides on entry and promotion
Firm 2 decides on product
Firm 2 decides on pricing

Firm 1 decides on promotion
Firm 1 decides on product
Firm 1 decides on pricing

Promotional Technology

Using the advertising response function developed by Butters (1977), we model the percentage change of the degree of unawareness as

$$\frac{\mu - \mu'}{\mu} = 1 - e^{-A/\alpha},$$ (7)

where μ and μ' are the pre-promotional and post-promotional degrees of unawareness, respectively, $\mu' < \mu$; hence, $(\mu-\mu')/\mu$ is equivalent to the probability that an unaware special consumer becomes aware. *A* is a firm's total promotional efforts measured in the monetary unit; α is the unit cost of promotional packet that guarantees a "conversion" of an unaware special consumer. Each one of A/α promotional packets is randomly and independently sent to all consumers with equal probability. When the number of consumers is large, the probability with which an unaware special consumer receives at least one packet is $1 - e^{-A/\alpha}$.

ASSUMPTION. *The unit cost of promotion, α, is sufficiently small such that* $\alpha < v - c$.

We rewrite Eq. (7) in terms of the promotional cost to reduce the degree of unawareness from μ to μ':

$$A(\mu' \mid \mu) = \alpha(\ln \mu - \ln \mu').$$ (8)

Note that $A' < 0$ and $A'' > 0$, suggesting the marginal return of promotion is decreasing.

Preliminary Analysis

It is easy to verify that the non-preemptive constraint specified in the benchmark model still guarantees that firm 1 offers G only in the monopoly market. [4] Nevertheless, the non-preemptive constraint no longer guarantees that firm 1 offers G only in the duopoly, since it can alter the unawareness through promotions prior to its product decision when facing entry.

We denote the initial degree of unawareness prior to any promotion as θ, the degree of unawareness *ex post* firm 2's promotion as θ', and the degree of unawareness *ex post* firm 1's promotion as θ''. If firm 1 does not promote, $\theta'' = \theta'$; if firm 2 does not promote, $\theta' = \theta$.

In the stage of price competition, we denote firm 1's equilibrium profits in Subgames 2 and 4 as $\pi_{1D}^{G*}(\theta'')$ and $\pi_{1D}^{GS*}(\theta'')$, respectively. In Table 3 they are given as

$$\pi_{1D}^{G*}(\theta'') = \left(1 - \lambda + \theta''\right)\left((\lambda - \theta'')(v - c) + c + b\right); \tag{9}$$

$$\pi_{1D}^{GS*}(\theta'') = b + (1 - \lambda + \theta'')c - k . \tag{10}$$

Replace θ with θ'' in Eq. (4) shown in Lemma 1. Now we obtain the same result regarding the market equilibrium *ex post* promotion: Firm 1 offers G if $b < \tilde{b}(\theta'')$ and $\{G, S\}$ if $b > \tilde{b}(\theta'')$, where

$$\tilde{b}(\theta'') = (1 - \lambda + \theta'')(v - c) + \frac{k}{\lambda - \theta''} .$$

Since $\tilde{b}(\theta'')$ is a increasing function of θ'', its minimum (reached at $\theta'' = 0$) is

$$b^* \equiv (1 - \lambda)(v - c) + k / \lambda. \tag{11}$$

Thus, for any $b \leq b^*$, the inequality $b < \tilde{b}(\theta'')$ is automatically satisfied, and firm 1 offers G upon firm 2's entry with S. For $b \geq b^*$, firm 1 may consider offering either G or $\{G, S\}$, depending on the two firms' sequential promotional strategies. We analyze the two cases separately.

Low Brand Equity ($b \leq b^*$)

When $b \leq b^*$, in the product offering stage, firm 1 offers G whenever firm 2 enters with S, irrespective of the degree of unawareness. Back to the promotion stage, firm 1 maximizes its profit

---

[4] It is because firm 1 does not promote when it offers G only, or when it offers both G and S. To see why, consider when firm 1 promotes θ to $\theta' < \theta$, if it offers G only, its profit is $(1 - \lambda + \theta')(v + b) - A(\theta' \mid \theta)$, which increases in the degree of unawareness. If it offers both G and S, firm 1's profit is $(1 + \lambda - \theta')(v + b) + (\lambda - \theta')(v + b - c) - k - A(\theta' \mid \theta)$. Note the profit margin of S, $v+b-c$, is lower than that of G, $v + b$. Again, firm 1 does not gain through promotion.

$\pi_{1D}^{G*}(\theta") - A(\theta"\,|\,\theta')$ by choosing $\theta" \in (0,\ \theta']$. According to Corollary 2, $\pi_{1D}^{G*}(\theta")$ may decrease or increase with the degree of unawareness; thus, firm 1 may or may not promote consumer awareness. Finally, given the anticipated response of firm 1, $\theta"(\theta')$, firm 2 maximizes its profit $(\lambda - \theta')(v - c) - f - A(\theta'\,|\,\theta)$ by choosing $\theta' \in (0,\ \theta]$. The following proposition characterizes the market equilibrium.

PROPOSITION 2**.** *When* $b \le b^*$ *, the market equilibrium is summarized as follows.*

Condition 1	Condition 2	Firm 1's decisions	Firm 2's decisions
$\theta"^* \le \min\{\theta, \bar{\theta}\}$	$(\theta"^* - \theta'^*)(v - c) \le A(\theta"^*\,|\,\theta)$	*Promote; G*	*Not promote; S*
$\theta'^* \le \min\{\theta, \tilde{\theta}\}$	$(\theta"^* - \theta'^*)(v - c) > A(\theta"^*\,|\,\theta)$	*Not promote; G*	*Promote; S*
$\theta \le \min\{\theta'^*, \theta"^*, \bar{\theta}\}$	-	*Not promote; G*	*Not promote; S*
All other conditions	-	*Not promote; G*	*Not enter*

where $\theta'^* \equiv \alpha/(v - c)$, $\theta"^* = \dfrac{(b - \hat{b}) + \sqrt{(b - \hat{b})^2 + 8\alpha(v - c)}}{4(v - c)}$, $\hat{b} \equiv v - 2c - 2\lambda(v - c)$, *and*

$$\tilde{\theta} \equiv \frac{\alpha}{v - c}\exp\left[\frac{\lambda(v - c) - f}{\alpha} - 1\right]. \qquad (12)$$

Proposition 2 implies that the entry and promotion behavior is complex. Upon firm 2's entry, three types of promotion are possible: either firm promotes, or none promotes. The most interesting case is that firm 1 promotes and firm 2 free rides on the effort.[5] This happens when the optimal degree of unawareness of firm 1 is lower than that of firm 2's, or when $\theta"^* \le \theta'^*$; it may also happen when $\theta"^* > \theta'^*$, as firm 2 finds it not optimal to promote unawareness below $\theta"^*$.

Following Proposition 2, we report the following results on promotion.

COROLLARY 3. *The following properties of the equilibrium are observed:*

1. *In the case where firm 2 enters without promotion and firm 1 promotes, the initial degree of unawareness can be as high as* λ *.*

---

[5] Although firm 1 does not produce *S,* it may promote awareness while promoting *G,* by showing that *G* is not suitable for special consumers. For example, the instruction of a sunscreen may state that the product is only intended for non-sensitive skins.

2. *In the case where firm 2 promotes, the upper bound of the pro-entry unawareness, $\tilde{\theta}$, is higher than that without promotion, $\overline{\theta}$. In addition, $\tilde{\theta}$ decreases as the cost of promotion, α, decreases.*

3. *In the case where firm 2 enters without promotion and firm 1 promotes, it is necessary that the brand equity of firm 1 is low.*

4. *If $b + \alpha / \lambda > v - 2c$, firm 1 does not promote.*

The first two points show that promotion helps firm 2 to enter under even higher degree of unawareness than in the benchmark model. It also implies that a more effective promotion technology lowers entry barriers. Point 3 demonstrates that, although firm 2 may free ride on firm 1's promotion, it happens only if firm 1's brand equity is sufficiently low. This is because only then does firm 1 expect firm 2 to act aggressively in pricing, and is more willing to promote to soften the competition. Point 4 shows that firm 1 does not promote when brand equity or the cost of promotion is sufficiently large.

Next, we analyze the market equilibrium when firm 1's brand equity $b > b^*$.

High Brand Equity ($b > b^*$)

It is not certain whether firm 1 should offer *G* only or {*G, S*} when $b > b^*$. First, we establish the condition under which firm 1 accommodates entry by selling *G* only, and firm 2 is profitable by selling *S*. The result follows from Proposition 1 and Lemma 1.

LEMMA 2. *When $b > b^*$, firm 2 is profitable if, and only if, $\theta'' \in (\underline{\theta}, \ \overline{\theta}]$, where*

$$\underline{\theta} = \max\{\lambda - \frac{-(b - v + c) + \sqrt{\Delta}}{2(v - c)}, \ 0\}, \tag{13}$$

where $\Delta = (b - v + c)^2 + 4k(v - c)$.

Going back to the stage where firm 1 makes promotion decisions, given the degree of unawareness it faces, θ', firm 1 can decide how it wants the consumer unawareness to be: It can set $\theta'' \in (\underline{\theta}, \ \overline{\theta}]$ to accommodate entry, or set $\theta'' \notin (\underline{\theta}, \ \overline{\theta}]$ to deter entry. In the first case, it earns a profit of

$$\pi_{1D}^G(\theta'' | \theta') = (1 - \lambda + \theta'')[(\lambda - \theta'')(v - c) + c + b] - A(\theta'' | \theta'). \tag{14}$$

In the second case, firm 1 remains a monopoly and has two product choices:

1. To offer *G* only at a price equal to *v+b* with a profit of

$$\pi_{1M}^G(\theta'' | \theta') = (1 - \lambda + \theta'')(v + b) - A(\theta'' | \theta'); \tag{15}$$

2. To offer the full product line {*G, S*} with both prices equal to *v+b* and a profit of

$$\pi_{1M}^{GS}(\theta''\,|\,\theta') = v + b - (\lambda - \theta'')c - k - A(\theta''\,|\,\theta'). \tag{16}$$

Correspondingly, define $\pi_{1D}^{G*}(\theta')$, $\pi_{1M}^{G*}(\theta')$ and $\pi_{1M}^{GS*}(\theta')$ as firm 1's optimal profits subject to the constrained range of θ'', for the functions given in Eq. (14), (15) and (16), respectively. Note that all three optimal profits are functions of θ'. We then define

$$\theta_0 = \{\theta' \le \overline{\theta}\,|\,\pi_{1D}^{G*}(\theta') = \pi_{1M}^{GS*}(\theta')\}. \tag{17}$$

In Appendix II A8, we derive properties of the above optimal profit functions. In Appendix II A9, we show that if θ_0 exists, then $\theta_0 > \underline{\theta}$. Based on the comparison of the optimal profits, the following lemma characterizes firm 1's equilibrium response in the subgame where firm 2 enters.

LEMMA 3.

If $\theta' \in (\theta_0, \overline{\theta}]$, firm 1 accommodates firm 2's entry. It offers G without promotion.

If $\theta' \le \theta_0$, firm 1 deters firm 2's entry by offering {G, S}. In particular, if $\theta' \in (\underline{\theta}, \theta_0]$ firm 1 over-promotes such that $\theta'' = \underline{\theta}$.

If $\theta' > \overline{\theta}$, firm 1 does not promote and offers either G only or {G, S}.

Figure 1 depicts firm 1's decisions described above. The red segment indicates the range in which firm 1 accommodates entry. In view of this, firm 2 will never promote θ' below θ_0.

_____

Insert Figure 1 about here

_____

Back to firm 2's promotional decision, the market equilibrium of the full game depends on the existence of θ_0 and its value if it exists, which is characterized by the following proposition.

PROPOSITION 3. *When $b > b^*$, firm 1 offers G without promotion. Firm 2 enters only if θ_0 exists and $\theta > \theta_0$. When the necessary conditions are satisfied, under additional conditions, firm 2 enters with S and promotes differently as specified below:*

Conditions	Firm 2's promotion decisions
$\theta'^* \in (\theta_0, \theta)$ and $\theta \le \tilde{\theta}$	Promote to $\theta' = \theta'^*$
$(\lambda - \theta_0)(v - c) - f - A(\theta_0\,|\,\theta) \ge 0$ and $\theta'^* \le \theta_0$	Promote to $\theta' = \theta_0 + \varepsilon$

$\theta \le \min\{\theta^{r^*}, \bar{\theta}\}$	*Does not promote*

Proposition 3 shows that with promotion, firm 2 may enter when the original consumer unawareness, θ, is within the range of $(\theta_0, \tilde{\theta}]$. The effect of promotion is two-fold. It enables entry for high θs, as shown in Corollary 3. It also restricts entry for low θs, because the lower bound of pro-entry θ is θ_0, which is greater than $\underline{\theta}$, the minimum degree of θ'' for which firm 1 accommodates entry. This is due to the strategic effect of promotion: when unawareness is relatively low, firm 1 can over-promote to deter the entry for a high profit. Proposition 3 also shows when and why firm 2 under-promotes consumer awareness. If firm 2 promotes too much, firm 1 free rides on firm 2's promotional efforts by offering both products. Therefore, an equilibrium may prevail where no firms promote or only firm 2 under-promotes. As a result, a relatively high degree of consumer unawareness is maintained despite competition.

COROLLARY 4. *The critical threshold θ_0 increases when the unit cost of promotion, α, decreases.*

Corollary 4 suggests that a more effective promotion technology does not necessarily result in more awareness promotion, nor does it necessarily induce market entry and competition. While a high cost of promotion may *discourage* firm 2's entry, as it cannot afford a large promotion to lift the demand, a low cost of promotion may allow firm 1 to *deter* firm 2's entry, as firm 1 can easily manage consumer unawareness. In the extreme, when the promotion cost is zero, firm 2 may never enter.

Discussion: Established firms often create entry barriers by using heavy advertising to differentiate their own products from new products, namely, generating high brand equity (see Smiley (1988) for empirical evidence and Bonanno (1986) for theoretical argument). Our analysis demonstrates subtle effects of brand equity: In the benchmark model, no entry deterrence is associated with firm 1's brand equity b; in the model with promotions, firm 1 with high brand equity may deter entry, while firm 1 with low brand equity may invite entry.

The effect of brand equity can be understood as follows. First, note that we impose the non-preemptive constraint such that for firm 1 the cost of offering multiple products outweighs the gain under a monopoly. If the non-preemptive constraint does not hold, then firm 1 always offers both *G* and *S*, and firm 2's entry is unprofitable as a result of price competition of homogeneous products. Other things equal, the non-preemptive constraint becomes more restrictive when brand equity increases. Hence, a sufficiently high brand equity will cause firm 1 to offer *S* and preempt the market of special consumers.

Second, when the non-preemptive constraint holds, the effect of brand equity depends on the interaction of price competition, intra-brand cannibalization, as well as promotion. In the benchmark model, although offering both products helps firm 1 to deter entry, it is too costly due to intra-brand cannibalization between G and S, in addition to the fixed cost. Thus, firm 1 does not deter entry. In the model with promotion, if firm 2 promotes awareness, then more special consumers are only interested in S. As a direct effect (by holding firm 2's pricing) firm 1's profit decreases when it offers G only. Therefore, at the present level of unawareness, firm 1 may switch to offer both products. The actual result critically depends on the level of brand equity. When brand equity is high, it is more likely that the benefit of adding more consumer base outweighs the cost of cannibalization and the fixed cost. Thus, entry deterrence is more likely. When brand equity is too low, then not only is entry accommodated, firm 1 may promote and essentially invite entry. It is because, as an indirect effect, promoting awareness softens the competition.

To sum up, the strategic advantage of high brand equity and the strategic disadvantage of low brand equity are both amplified when unawareness can be reduced by promotions. Moreover, it can be argued that the advertising related to brand equity is more persuasive and the advertising related to awareness is more informational. Our findings thus offer some insight into the interaction between the two types of advertising.

The above analysis also illustrates a phenomenon that we term "pioneer victim": If the first mover (firm 2) fails to recognize the strategic effect of promotion and brand equity in our context, it can fall victim to its pioneer marketing activities. Such activities shape the market environment to benefit firm 2 in the short term, but prompt firm 1's entry, and may eventually hurt firm 2 in the long term. If firm 2 cannot quickly build consumer loyalty or other competitive advantages before firm 1's entry, what it has done on the market environment will favor only firm 1. In fact, knowing that firm 2 is myopic, firm 1 may prefer to act more slowly than firm 2 to free ride on its marketing efforts later on. Hence, our study provides an alternative explanation to why pioneer firms eventually concede their leading places to late-but big-comers, as documented in empirical studies (Golder and Tellis 1993).

6. General Discussion

There are several issues in the model that we would like to clarify here. First, we model consumer unawareness specifically on preference-product match, which should not be confused with unawareness on brand or product. Indeed, in our model, after the entry of firm 2, unaware consumers know that product S is available.

Second, traditional rational models often incorporate consumer bias by attributing consumers with imperfect information. In such cases, uninformed consumers are nevertheless fully aware, as they are

attributed with a full set of possible states. In addition, advertising or promotions can only modify their beliefs by Bayesian update. We deviate by allowing consumers to have an incomplete set of states such that advertising can add new states in consumer's posterior belief. Researchers may wonder how our approach differs from the traditional approach, or how our results can be rationalized via modified information (Spiegler 2011).

To address this issue, consider replacing all unaware consumers with uninformed Bayesian consumers. In the simplest case, the uninformed know that they are of the type of general consumers with probability q, $0 < q < 1$, or of the type of special consumers with probability $1–q$. Their expected valuations are given below in comparison with these of unaware consumers as in the original model.

Consumer type	Special consumers		Uninformed Bayesian
	Match-aware	Match-unaware	
Segment size	$\lambda - \theta$	θ	θ
Perceived valuation			
Purchase G from firm 1	0	$v+b$	$q(v+b)$
Purchase S from firm 1	$v+b$	$v+b$	$v+b$
Purchase G from firm 2	0	v	qv
Purchase S from firm 2	v	v	v

It is easy to see that, relative to unaware consumers, uninformed consumers have a lower value for G and an identical value for S. For the former to be close to the unaware consumers' value for G, q would have to be close to 1. It implies that the proportion of special-consumer type among uninformed consumers, $1-q$, must be very small. If this is true, then the model with promotion is not interesting, because firm 2 has little interest in promoting information of match to uninformed consumers, regardless of firm 1's strategic reaction. Conversely speaking, for firm 2 to have sufficient incentive to promote the information of match, the proportion of special-consumer type must be sufficiently large. Then, the ex-ante valuation of G, $q(v+b)$, is much lower than that of unaware consumers as well as that of general consumers. This change may considerably affect firms' equilibrium pricing strategies derived in the benchmark model. For example, for firm 1 there will be a trade-off between attracting only general consumers and attracting both general consumers and uninformed special consumers.

In conclusion, we doubt that any traditional rational model can replicate the predictions of the models presented in this paper by simply replacing unaware consumers with uninformed ones. The main reason is that belief update in traditional models is more "consistent" than in our model.

Finally, additional distinction emerges if we extend the model by allowing consumers to learn useful information. Plausibly, it is more difficult for unaware consumers to learn aspects that they are unaware of, as they simply do not perceive these uncertainties. Because of this, in order to induce learning, effective advertising would be costly. In traditional models, uninformed consumers want to learn because they are driven by the perceived uncertainties. Hence, less costly advertising may still be effective.

7. Conclusion, Limitation and Managerial Implications

While sellers must make consumers *aware* of a product in order to sell it, very often it is also true that, to have consumers stay with existing products, sellers must keep them *unaware* of the benefits of better alternatives. This paper explores how firms compete in product offerings, pricing, and promotion under consumer unawareness about preference-product match. The analysis shows that it requires a moderate degree of unawareness to entice market entry. When firms compete after entry, a low degree of unawareness makes their pricing less competitive, and the entrant more profitable; but the incumbent does not always gain. When firms can promote consumer awareness, the entrant may strategically under-promote the awareness, and, essentially, suppress the potential demand for its products. Conversely, the incumbent may strategically promote to deter entry or soften competition. Together with the initial degree of unawareness, the establishment of the incumbent and the effectiveness of promotion technology are also critical in determining the strategic choices of both firms. These findings suggest that, given a proper market environment, consumer unawareness affects firms' competitive strategies profoundly.

Managerially, consumer unawareness precludes a suitable product—which offers consumers a better match—from being consumed. Opportunities arise when firms identify the existence of such a match, launch new products that align with the match, and promote consumer awareness of the products (Epstein 2000). However, anecdotal observations suggest that the process is not always as smooth as it sounds when factors such as competition, cannibalization, logistics, and culture are taken into account. For example, when Budweiser launched Bud Light in 1982 with lower alcohol and calorie content to cater to the 25- to 44-year-old professionals, the rapid growth of Bud Light sales was largely at the expense of its regular Budweiser brand sales rather than the competing Miller Lite (Munching 1998). In another example, Research in Motion (RIM) customizes its Blackberry smart phones with the AZERTY keyboard to cater to French-speaking users in Europe, in addition to the regular QWERTY keyboard for Anglophone users. Nevertheless, as a Canadian company whose headquarters is only 300 miles from Canada's French-speaking province of Quebec, RIM does not make its predominant Francophone consumers in Quebec aware of the AZERTY keyboard (not to mention even offering such a choice). These anecdotes highlight the importance of consumer unawareness in marketing activities.

In response to the challenges brought up by consumer unawareness, our research provides important insights for managers who consider entering a new market or launching a new product with a prerequisite of educating consumers about their unknown needs. As Hoch and Deighton (1989) suggest, market leaders usually avoid educating their consumers with new product knowledge. Yet, we find that for small firms, it might also be less desirable to fully promote consumer awareness, as doing so may entice big firms to free ride on their promotional efforts by entering the segment afterwards, resulting in a head-on competition. Conversely, for large firms, it is not always a bad situation if it cannot move before myopic small firms, as the promotion of consumer awareness can be converted into a second-mover advantage.

Our study is limited in that consumer learning is only promotion-based. We are aware that consumer awareness can be triggered in a market where consumers interact with each other, or when they search for general information. Subsequently, unaware consumers can learn about their type, and which product is suitable for their type. Further analyses are needed to explore the different tools to trigger consumer awareness, and how consumers dynamically respond to those tools.

Another important direction for future research is to examine how consumers distinguish a true match from an artificial one imposed via firms' marketing efforts. An entrant firm may claim a questionable matched consumption (e.g., should toothpaste be made gender-specific). How shall the incumbent respond? Shall the incumbent accept the claim and does what we analyze in the paper? Or shall it try directly disproving the claim and maintaining consumer confidence in its product? What are the driving factors?

Finally, this paper offers several theoretical findings, which can be empirically tested. We expect that future research will develop practical measures of consumer unawareness on the market level, and test its impact on firms' entry and competitive strategies.

Table 1 Some Notations

Symbol	Meaning
λ	The fraction of special consumers in the population
θ	The fraction of unaware special consumers in the population
k	Firm 1's fixed operations costs by introducing S
b	Firm 1's brand equity, measured as additional willingness to pay for its products.
f	Firm 2's lump-sum entry cost
v	Consumer's valuation when perceiving to have a matched consumption.
a	Unit promotional cost of a promotional packet
A	Total cost of promotion
Θ	$\Theta \equiv \lambda - k/(b+v-c)$, the lower bound of θ for firm 1 not to preempt entry by offering special product
$\bar{\theta}$	$\bar{\theta} \equiv \lambda - f/(v-c)$, the upper bound of θ for firm 2 to be profitable to enter when there is no promotion.
θ''	The fraction of unaware special consumers in the population after both firms finish promotion
θ'	The fraction of unaware special consumers in the population after firm 1 finishes promotion
$\tilde{\theta}$	$\tilde{\theta} \equiv \dfrac{\alpha}{v-c}\exp\left[\dfrac{\lambda(v-c)-f}{\alpha}-1\right]$, the upper bound of θ'' for firm 2 to be profitable to enter when it fully promotes
$\underline{\theta}$	$\underline{\theta} \equiv \max\{\lambda - \dfrac{-(b-v+c)+\sqrt{\Delta}}{2(v-c)},0\}$, the lower bound of θ'' for firm 2 to be profitable to enter
θ_0	$\theta_0 \equiv \{\theta' \le \bar{\theta} \mid \pi_{1D}^{G*}(\theta') = \pi_{1M}^{GS*}(\theta')\}$, the lower bound of θ' for firm 1 to accommodate entry
b^*	$b^* \equiv (1-\lambda)(v-c)+k/\lambda$, the highest level of b such that firm 1 accommodates entry for any level of θ''
$\hat{b}$	$\hat{b} \equiv v - 2c - 2\lambda(v-c)$, for notation convenience

Table 2 Consumer Type, Segment Size, and Perceived Valuation

| Consumer type	General consumers	Special consumers	
		Match-aware	Match-unaware
Segment size	$1-\lambda$	$\lambda-\theta$	θ
Perceived valuation			
Purchase G* from firm 1	$v+b$	0	$v+b$
Purchase S from firm 1	$v+b$	$v+b$	$v+b$
Purchase G from firm 2	v	0	v
Purchase S from firm 2	v	v	v

*G stands for the generic product and S stands for the special product.

Table 3   The Two Firms' Expected Payoffs

	Firm 2 sells G	Firm 2 sells S
Firm 1 sells G	$(1-\lambda+\theta)b,\ -f$	$(1-\lambda+\theta)(\overline{x}+b),\ (\lambda-\theta)(v-c)-f$
Firm 1 sells (G, S)	$b+(\lambda-\theta)(v-c)-k,\ -f$	$b+(1-\lambda+\theta)c-k,\ -f$

* $\overline{x} = (\lambda - \theta)(v - c) + c$.

Figure 1 Firm 1's Promotion and Product Decisions

$\pi_{1,M}^{GS*}(\theta')$

$\pi_{1,D}^{G*}(\theta')$

$\pi_{1,M}^{G*}(\theta')$

$\underline{\theta}$ θ_0 $\bar{\theta}$ θ'
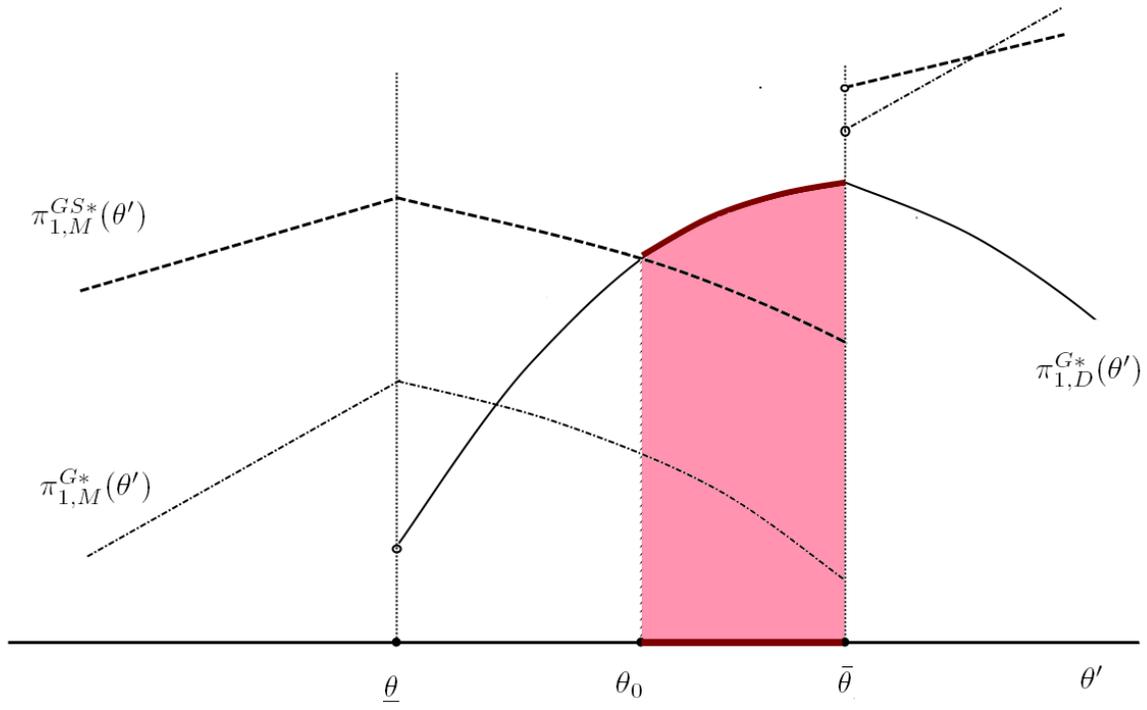
References

Aaker, D. A. 1996. Measuring Brand Equity across Products and Markets. *California Management Review*. 38(Spring). 102-120.

Acs, Z., D. Audretsch. 1990. *Innovation and Small Firms* MIT Press.

Akerlof, G. A. 1970. Market for Lemons--Quality Uncertainty and Market Mechanism. *Quarterly Journal of Economics*. 84(3). 488-500.

Alba, J. W., J. W. Hutchinson. 2000. Knowledge Calibration: What Consumers Know and What They Think They Know. *The Journal of Consumer Research*. 27(2). 123-156.

Aragones, E., I. Gilboa, A. Postlewaite, D Schmeidler. 2005. Fact-Free Learning. *American Economic Review*. 95(5). 1355-1368.

AutomotiveAddicts. 2009. Major Fail: Vw Badged Chrysler Caravan. *www.AutomotiveAddicts.com*. Available at http://www.automotiveaddicts.com/3773/major-fail-vw-badged-chrysler-caravan-no-more-vw-routan.

Bettman, J. R., E. Johnson, J. W. Payne. 1991. Consumer Decision Making in *Handbooks of Consumer Theory and Research.* R. Kassarjian, eds. Prentice-Hall.

Bloch, P. H., D. L. Sherrell, N. M. Ridgway. 1986. Consumer Search: An Extended Framework. *The Journal of Consumer Research*. 13(1). 119-126.

Bonanno, G. 1986. Advertising, Perceived Quality and Strategic Entry Deterrence and Accommodation. *Metroeconomica*. 88(3). 257-280.

Butters, G. R. 1977. Equilibrium Distributions of Sales and Advertising Prices. *The Review of Economic Studies*. 44(3). 465-491.

Chen, Y., G. Iyer, A. Pazgal. 2010. Limited Memory, Categorization and Competition. *Marketing Science*. 29 (4). 650-670.

Dekel, E., L. L. Barton, A. Rustichini. 1998. Standard State-Space Models Preclude Unawareness. *Econometrica*. 66(1). 159-173.

Dyer, D., F. Dalzell, R. Olegario. 2004. *Rising Tide: Lessons from 165 Years of Brand Building at Procter and Gamble* Harvard Business School Press, Boston.

Epstein, R. 2000. *Estee Lauder: Beauty Business Success* Franklin Watts, NY.

Gabaix, X., D. Laibson. 2006. Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets. *Quarterly Journal of Economics*. 121(2). 505-540.

Golder, P. N., G. J. Tellis. 1993. Pioneer Advantage: Marketing Logic or Marketing Legend? *Journal of Marketing Research*. 30(2). 158-170.

Heifetz, A., M. Meier, B. C. Schipper. 2006. Interactive Unawareness. *Journal of Economic Theory*. 130(1). 78-94. .

Hoch, S. J., J. Deighton. 1989. Managing What Consumers Learn from Experience. *The Journal of Marketing*. 53(2). 1-20.

Jing, B. 2011a. Seller Honesty and Product Line Pricing. *Quantitative Marketing and Economics*. forthcoming.

Jing, B. 2011b. Exogenous Learning, Seller-Induced Learning, and Marketing of Durable Goods. *Management Science*. forthcoming.

Kamakura, W. A., G. J. Russell. 1993. Measuring Brand Value with Scanner Data. *International Journal of Research in Marketing*. 10(1). 9-22.

Kamenica, E., S. Mullainathan, R. H. Thaler. 2011. Helping Consumers Know Themselves. *American Economic Review,* 101(3). 417-422 .

Keller, K. L. 1993. Conceptualizing, Measuring, and Managing Customer-Based Brand Equity. *Journal of Marketing*. 57(1). 1-22.

Koehn, N. F. 2001. Estee Lauder and the Market for Prestige Cosmetics. *Harvard Business School Case Study*. Prod. #: 801362-PDF-ENG

Li, J. 2009. Information Structures with Unawareness. *Journal of Economic Theory*. 144(3). 977-993.

Li, S., M. Peitz, X. Zhao. 2010. Worried About Adverse Product Effects? Information Disclosure and Consumer Awareness. *Working paper, Toulouse School of Economics*.

Modica, S., A. Rustichini. 1999. Unawareness and Partitional Information Structures. *Games and Economic Behavior*. 27. 265-298.

Munching, P. V. 1998. *Beer Blast: The Inside Story of the Brewing Industry's Bizarre Battles for Your Money* Crown Business Crown Publishing Group, New York.

Narasimhan, C. 1988. Competitive Promotional Strategies. *Journal of Business*. 61(4). 427-449.

Narasimhan, C., Z. J. Zhang. 2000. Market Entry Strategy under Firm Heterogeneity and Asymmetric Payoffs. *Marketing Science*. 19(4). 313-327.

Schneider, M. 2005. *Aldi: What Brand Is It? 100 Aldi Products and Their Prominent Manufacturers (in German)* West Publishing, Munich Germany.

Shankar, V. 1997. Pioneers' Marketing Mix Reactions to Entry in Different Competitive Game Structures: Theoretical Analysis and Empirical Illustration. *Marketing Science*. 16(3). 271-293.

Shen, Q., J. M. Villas-Boas. 2010. Strategic Entry before Demand Takes Off. *Management Science*. 56(8). 1259-1271.

Shilony, Y. 1977. Mixed Pricing in Oligopoly. *Journal of Economic Theory*. 14(2). 373-388.

Simon, H. 1955. A Behavioral Model of Rational Choice. *Quarterly Journal of Economics*. 69(1). 99-118.

Smiley, R. 1988. Empirical Evidence on Strategic Entry Deterrence. *International Journal of Industrial Organization*. 6(2). 167-180.

Spiegler, R. 2011. *Bounded Rationality and Industrial Organization.* Oxford University Press, New York.

Tellis, G. J., G. J. Gaeth. 1990. Best Value, Price-Seeking, and Price Aversion: The Impact of Information and Learning on Consumer Choices. *Journal of Marketing*. 54(2). 34-45.

Thorelli, H. B., S. V. Thorelli. 1977. *Consumer Information Systems and Consumer Policy.* Ballinger Publishing Company, Cambridge, MA.

Villas-Boas, S. B., J. M. Villas-Boas. 2008. Learning, Forgetting, and Sales. *Management Science*. 54(11). 1951-1960.

Appendix I

To demonstrate the concept of unawareness in the context we study here, we can use the example of acoustic guitars. The match between a consumer (guitar player) and a product (guitar) depends on multiple aspects of consumer preferences. For the purpose of demonstration, we only consider two aspects. Firstly, consumers differ in whether they are right-handed or left-handed. (Some consumers may be able to play both ways, but we ignore this case.) We denote this aspect by a set $H = \{r, l\}$, where H, for *Handedness*, denotes this dimension of differentiation in general, and r or l denotes whether a consumer is right-handed or left-handed. Secondly, consumers differ in whether they are particularly interested in playing solo or not. We denote this aspect by another set $S = \{s, -s\}$, where S, for *Style*, denotes this dimension of differentiation in general, and s or $-s$, denotes whether a consumer is particularly interested in playing solo or not. For a complete set, or the space, of all possible preferences types, we define

$$T \equiv H \times S = \{(r,s),(r,\neg s),(l,s),(l,\neg s)\}.$$

If a buyer has full awareness but does not know her exact type, she must use set T for her purchase decision. For example, a regular acoustic guitar is designed for right-handed players and not for solo players, because for most solo players, an additional design called a cutaway (an indentation around the base of the guitar neck for easy access to high notes) is desired. The possible value of a regular acoustic guitar, depending on the type of consumer, is:

Type	Valuation
(r, s)	7
(r, -s)	10
(l, s)	2
(l, -s)	2

Meanwhile, an acoustic guitar designed for a right-handed solo player can have the following valuation:

Type	Valuation
(r, s)	10
(r, -s)	7
(l, s)	2
(l, -s)	2

Now consider a buyer who is aware of the first but not the second aspect, due to various reasons such as limited experience or influences of popular singers. The space of all possible types he/she considers is set $T^u = H$ In the case where he/she unknowingly follows the majority's opinion on the second aspect, his/her subjective valuations of a regular acoustic guitar conditional on his/her type are below. They are identical to the valuations of (r, -s) and (l, -s) in the full awareness case.

Type	Valuation
(r)	10
(l)	2

Though not quite likely, in the case when he/she unknowingly follows the minority's opinion on the second aspect, his/her subjective valuations of a regular acoustic guitar conditional on his/her type are below. They are identical to the valuations of (l, s) and (l, -s) in the full awareness case.

Type	Valuation
(r)	7
(l)	2

Finally, please note that whether awareness is full or not, consumers can be uncertain about their types. To account for that, standard approaches using prior beliefs may apply, and then consumers evaluate the products by computing expected utility.

Appendix II

A1. The derivation of equilibria in the price-competition stage.

In Stage 3, the equilibrium prices and profits are as follows.

i. If both firms offer the generic product, then $p_1^G = b$, $p_2^G = 0$, $\pi_1 = (1 - \lambda + \theta)b$, $\pi_2 = 0$.

ii. If firm 1 offers both products and firm 2 offers the generic product, then $p_1^G = b$, $p_1^S = v + b$, $p_2^G = 0$, and $\pi_1 = b + (\lambda - \theta)(v - c)$, $\pi_2 = 0$.

iii. If firm 1 offers both products and firm 2 offers the special product, then $p_1^G = b + c$, $p_1^S = b + c$, $p_2^S = c$, and $\pi_1 = b + (1 - \lambda + \theta)c$, $\pi_2 = 0$.

iv. If firm 1 offers the generic product and firm 2 offers the special product, there is no pure strategy equilibrium. In a mixed-strategy equilibrium, the pricing rules are as follows. Define $\bar{x} = (\lambda - \theta)(v - c) + c$. Firm 1's price distribution has support over the interval $[\bar{x} + b, v + b]$, and the cumulative distribution function is

$$
prob\{p_1^G \leq x\} =
\begin{cases}
0 & if \quad x < \bar{x} + b \\
\frac{1}{1 - \lambda + \theta} - \frac{(\lambda - \theta)(v - c)}{(1 - \lambda + s\theta)(x - b - c)} & if \quad x \in [\bar{x} + b, v + b) \\
1 & if \quad x \geq v + b
\end{cases}
\tag{1}
$$

Firm 2's price distribution has support over the interval $[\bar{x}, v]$, and the cumulative distribution function is

$$
prob\{p_2^S \leq y\} =
\begin{cases}
0 & if \quad y < \bar{x} \\
1 - \frac{\bar{x} + b}{y + b} & if \quad y \in [\bar{x}, v) \\
1 & if \quad y \geq v
\end{cases}
\tag{2}
$$

The profits are $\pi_1 = (1 - \lambda + \theta)(\bar{x} + b)$ and $\pi_2 = (\lambda - \theta)(v - c)$.

1

Proof. **The proof of points $i - iii$.**

When both firms offer G, they compete in the Bertrand fashion: One firm's best response to each other's price is a lower price until at least one of them breaks even. Hence, firm 2 sets a price equal to the marginal cost and firm 1 sets a price equal to the marginal cost plus the brand loyalty. It explains point i.

When firm 1 offers both product types and firm 2 offers G, they still compete on G in the Bertrand fashion. But firm 1 sets the price of S at $v+b$, which is the highest price that aware special consumers are willing to pay, because any of the lower prices will not increase firm 1's profit. Firm 1's profit is therefore $\pi_1 = (1-\lambda+\theta)b + (\lambda-\theta)(v+b-c) = b + (\lambda-\theta)(v-c)$. Firm 2's profit is zero. This explains point ii.

When firm 1 offers both product types and firm 2 offers S, they compete on S in the Bertrand fashion. Firm 2 charges a price of S equal to the marginal cost. Firm 1 sets the price of S equal to the marginal cost plus the brand loyalty. For general consumers and unaware special consumers, G and S are perfect substitutes, therefore any price of G higher than the price of S will not increase firm 1's profit, and any lower price will decrease firm 1's profit. Hence, the price of S is $c+b$. Firm 1's profit is $\pi_1 = (1-\lambda+\theta)(c+b) + (\lambda-\theta)b = b + (1-\lambda+\theta)c$. Firm 2's profit is zero. This explains point iii.

The proof of point iv.

The proof takes three steps. In step 1 we identify the best reply function of each firm in the price competition, then show pure-strategy equilibria do not exist. In step 2 we construct a strategy profile that is a candidate of mixed-strategy equilibrium. In step 3 we show that such a strategy profile is a Nash equilibrium.

STEP 1. Given $p_2^S \geq c$, firm 1 sets the highest price such that general consumers prefer the generic product over the special product, namely

$$p_1^G = BR(p_2^S) = p_2^S + b - \epsilon > 0,$$

where $\epsilon > 0$. Note firm 1 has no incentives to undercut firm 2 for special consumers, because by assumption these consumers have zero valuations about the generic product.

Given $p_1^G \geq 0$, firm 2 may set a price that just undercuts firm 1 and earn $p_1^G - b - \epsilon' - c > 0$, where $\epsilon' > 0$, or set a price equal to v and earn $(v-c)(\lambda - \theta)$. For firm 2 to be better off in the first case, it must be that $p_1^G - b - c - \epsilon' \geq (v-c)(\lambda - \theta)$ or $p_1^G > \bar{x} + b$. The best

reply of firm 2 is

$$p_2^S = BR(p_1^G) = \begin{cases} p_1^G - b - \epsilon' & \text{if } p_1^G > \bar{x} + b \\ v & \text{if } p_1^G \leq \bar{x} + b \end{cases}.$$

Given these best reply functions, we show that no pure-strategy equilibria exist. For any $p_1^G = x > \bar{x} + b$, the best reply of firm 2 is $x - b - \epsilon'$, with respect to which the best reply of firm 1 is $x - b - \epsilon' + b - \epsilon < x - \epsilon'$. Thus, firms undercut each other until $x = \bar{x} + b$. When that happens, firm 2's best reply becomes v. In turn, firm 1's best reply is $v + b - \epsilon > \bar{x} + b$. The cycle renews. Therefore, in the price competition, no pure-strategy equilibrium exists.

STEP 2. We construct a mixed-strategy equilibrium. In this equilibrium, firm 1 randomizes its price over the interval of $[\bar{x} + b, v + b]$, and firm 2 randomizes its price over the interval of $[\bar{x}, v]$. Note that when $p_2^S = v$, independent of firm 1's price, firm 2 always earns a constant profit of $(v - c)(\lambda - \theta)$; when $p_2^S < v$, firm 2 earns a profit of

$$\text{prob}(p_1^G - b > p_2^S)(p_2^S - c) + \text{prob}(p_1^G - b < p_2^S)(p_2^S - c)(\lambda - \theta)$$
$$+ \text{prob}(p_1^G - b = p_2^S)(p_2^S - c)[(\lambda - \theta) + \tfrac{1}{2}(1 - \lambda + \theta)] \quad .$$

We assume when common consumers are indifferent between two firms, they buy from either one with the same probability. In addition, assume in equilibrium $\text{prob}(p_1^g - b = p_2^s) = 0$. For a mixed strategy to hold, the two profits must equal, or

$$\text{prob}(p_1^G - b > p_2^S)(p_2^S - c) + \text{prob}(p_1^G - b < p_2^S)(p_2^S - c)(\lambda - \theta) = (v - c)(\lambda - \theta).$$

Replacing p_2^S by $x - b$, we solve the probability distribution that defines firm 1's strategy:

$$\text{prob}(p_1^G \leq x) = \begin{cases} 0 & \text{if} & x < \bar{x} + b \\ \frac{1}{1 - \lambda + \theta} - \frac{(\lambda - \theta)(v - c)}{(1 - \lambda + \theta)(x - b - c)} & \text{if} & x \in [\bar{x} + b, v + b) \\ 1 & \text{if} & x \geq v + b \end{cases} \quad .$$

Note that $p_1^G = v + b$ is not a mass point.

Consider firm 2's strategy. Note for any $p_2^S > \bar{x}$, by setting $p_1^G = \bar{x} + b$, firm 1 earns a profit equal to $(1 - \lambda + \theta)(\bar{x} + b)$. When firm 1 uses $p_1^G > \bar{x} + b$, it expects to earn a profit equal to

$$\text{prob}(p_2^S = p_1^G - b)(1 - \lambda + \theta)\tfrac{1}{2}p_1^G + \text{prob}(p_2^S > p_1^G\text{-}b)(1 - \lambda + \theta)p_1^G + \text{prob}(p_2^S < p_1^G - b)0.$$

Again assume that in equilibrium $\text{prob}(p_2^S = p_1^G - b) = 0$, for all p_2^S used by firm 2. For a mixed strategy to hold, the two profits must equal, or

$$\text{prob}(p_2^S > p_1^G\text{-b})(1 - \lambda + \theta)p_1^G = (1 - \lambda + \theta)(\bar{x} + b).$$

Replacing $p_1^G - b$ by y, we can solve the probability that defines firm 2's strategy:

$$\text{prob}(p_2^S \leq y) = \begin{cases} 0 & \text{if} & y < \bar{x} \\ 1 - \frac{\bar{x}+b}{y+b} & \text{if} & y \in [\bar{x}, v) \\ 1 & \text{if} & y \geq v \end{cases}.$$

Note that $p_2^S = v$ is a mass point with $\text{prob}(p_2^S = v) = \frac{\bar{x}+b}{v+b}$.

STEP 3. We need to show this mixed strategy profile is a Nash Equilibrium. In other words, we need to show each firm's strategy is optimal given the other firm's strategy. Fixing the mixed strategy of firm 1, we define an arbitrary mixed strategy of firm 2 by a cumulative density function F_2 over $[c, v]$. The expected profit of firm 2 is

$$\begin{aligned} & \int_c^v [\text{prob}(p_1^G - b > p_2^S)(p_2^S - c) + \text{prob}(p_1^G - b < p_2^S)(p_2^S - c)(\lambda - \theta)]dF_2(p_2^S) \\ = & \int_c^{\bar{x}}(p_2^S - c)dF_2(p_2^S) + \int_{\bar{x}}^v (\lambda - \theta)(v - c)dF_2(p_2^S) \\ \leq & (\bar{x} - c)[F_2(\bar{x}) - 0] + (\lambda - \theta)(v - c)[1 - F_2(\bar{x})] \\ = & (\lambda - \theta)(v - c) \end{aligned}$$

The first equality follows that $\text{prob}(p_1^G \geq \bar{x} + b) = 1$ and firm 2 always earns a constant profit of $(v - c)(\lambda - \theta)$ when firm 1's strategy is fixed and $p_2^S \geq \bar{x}$. The last equality follows $\bar{x} = (v - c)(\lambda - \theta) + c$. This shows that firm 1's strategy is optimal given firm 2's strategy.

Likewise, we define an arbitrary mixed strategy of firm 1 by a cumulative density function F_1 over $[0, v + b]$. Fixing the mixed strategy of firm 2, the expected profit of firm 1 is

$$\begin{aligned} & \int_0^{v+b} \text{prob}(p_2^S > p_1^G - b)(1 - \lambda + \theta)p_1^G dF_1(p_1^G) \\ = & \int_0^{\bar{x}+b}(1 - \lambda + \theta)p_1^G dF_1(p_1^G) + \int_{\bar{x}+b}^{v+b}(1 - \lambda + \theta)(\bar{x} + b)dF_1(p_1^G) \\ \leq & (1 - \lambda + \theta)(\bar{x} + b)[F_1(\bar{x} + b) - 0] + (1 - \lambda + \theta)(\bar{x} + b)[1 - F_2(\bar{x} + b)] \\ = & (1 - \lambda + \theta)(\bar{x} + b) \end{aligned}$$

The first equality follows that $\text{prob}\{p_2^S \geq \bar{x}\} = 1$ and firm 1 always earns a constant profit of $(1 - \lambda + \theta)(\bar{x} + b)$ when firm 2's strategy is fixed and $p_2^S \geq \bar{x}$. This shows that firm 1's

strategy is optimal given firm 2's strategy. In conclusion, the strategy profile is a Nash equilibrium. $\square$

A2. The proof of Lemma 1.

The proof is straightforward and is skipped.

A3. The proof of Proposition 1.

Proof. Note that if $\theta > \Theta = \lambda - \frac{k}{v+b-c}$, then $b < \frac{k}{\lambda-\theta} - v + c < \frac{k}{\lambda-\theta} + (1-\lambda+\theta)(v-c) = \tilde{b}(\theta)$. Then we can apply Lemma 1. $\square$

A4. The proof of Corollaries 1 and 2.

Proof. Following the equilibrium pricing strategy derived under point vi in A1, the average price of firm 1 is

$$
\begin{aligned}
E[p_1^G] &= \int_0^{v+b} x d\mathrm{prob}\{p_1^G \leq x\} \\
&= \int_{\bar{x}+b}^{v+b} x d\left[\frac{1}{1-\lambda+\theta} - \frac{(\lambda-\theta)(v-c)}{(1-\lambda+\theta)(x-b-c)}\right] \\
&= \int_{\bar{x}+b}^{v+b} x \left(\frac{(\lambda-\theta)(v-c)}{(1-\lambda+\theta)}\right)\frac{1}{(x-b-c)^2}dx \\
&= (b+c) - \frac{(\lambda-\theta)(v-c)}{(1-\lambda+\theta)}\ln(\lambda-\theta)
\end{aligned}
\tag{3}
$$

Define $k \equiv \lambda - \theta$, the comparative static is

$$
\frac{\partial E[p_1^G]}{\partial \theta} = \frac{\partial \left(\frac{k(v-c)}{(1-k)}\ln k\right)}{\partial k} = (v-c)\frac{1-k+\ln k}{(1-k)^2} < 0.
\tag{4}
$$

The last inequality is because that $0 < k < 1$.

Likewise, the average price of firm 1 is

$$
\begin{aligned}
E[p_2^S] &= \int_0^v y d\mathrm{prob}\{p_2^G \leq y\} \\
&= \frac{\bar{x}+b}{v+b}v + \int_{\bar{x}}^v y d\left[1 - \frac{\bar{x}+b}{y+b}\right] \\
&= \frac{\bar{x}+b}{v+b}v + \int_{\bar{x}}^v y \frac{\bar{x}+b}{(y+b)^2}dy \\
&= \bar{x} + (\bar{x} + b)\ln\frac{v+b}{\bar{x}+b}
\end{aligned}
\tag{5}
$$

Because $\bar{x} < v$, $\bar{x} + (\bar{x} + b) \ln \frac{v+b}{\bar{x}+b}$ increases in $\bar{x}$. Since $\bar{x}$ decreases in θ, the comparative static is

$$\frac{\partial E[p_2^S]}{\partial \theta} < 0. \tag{6}$$

As to the equilibrium profits, $\pi_{2D}^{S*} = (\lambda - \theta)(v - c) - f$ so it decreases in θ. For π_{1D}^{G*}, notice that $\frac{d\pi_1^{G*}}{d\theta} = 2c+b-v+2(\lambda-\theta)(v-c)$, so it increases in θ only when $2c+b-v+2(\lambda-\theta)(v-c) > 0$. $\qquad\square$

A5. The proof of Proposition 2.

Proof. First we interpret θ''^*, θ'^*, and $\tilde{\theta}$. When firm 1 promotes, solving the first order condition, the θ'' that maximizes $\pi_1^{G*}(\theta'') - A(\theta''|\theta')$ is given by θ''^*. Similarly, if firm 2 promotes, the θ' that maximizes $\pi_{2D}^{S*} = (\lambda - \theta')(v - c) - f - A(\theta'|\theta)$ is θ'^*. Since firm 1's profit without promotion is positive, its profit at the optimal level is also positive. However, firm 2's optimal profit may be negative. So for it to be non-negative, we need

$$(\lambda - \frac{\alpha}{v-c})(v - c) - f - \alpha[\ln \theta - \ln(\frac{\alpha}{v-c})] \geq 0.$$

The highest level of θ under which firm 2 earns a non-negative profit is determined by the above inequality, as

$$\tilde{\theta} = \frac{a}{v-c} \exp[\frac{\lambda(v-c)-f}{\alpha} - 1].$$

Whenever $\theta''^* \leq \theta$, promoting the level of unawareness to θ''^* is viable for firm 1. Also, when $\theta''^* \leq \bar{\theta}$, firm 2 can earn a non-negative profit at θ''^*. But firm 2 may instead promote $\theta' = \theta'^*$. For this not to happen, the benefit from promoting by itself must be less or equal to the benefit from free riding on firm 1's promotion, or

$$(\lambda - \theta'^*)(v - c) - A(\theta'^*|\theta) \leq (\lambda - \theta''^*)(v - c),$$

namely $(\theta'' * -\theta'^*)(v - c) - A(\theta'^*|\theta) \leq 0$.

When $\theta'^* < \theta \leq \tilde{\theta}$, promoting the level of unawareness to θ'^* is viable for firm 2. For firm 2 not want to free ride on firm 1's promotion, the benefit from promoting by itself must be greater, or

$$(\lambda - \theta'^*)(v - c) - A(\theta'^*|\theta) > (\lambda - \theta''^*)(v - c),$$

namely $(\theta'' * -\theta'^*)(v-c) - A(\theta'^*|\theta) > 0$.

When $\theta \leq \min\{\theta'^*, \theta''^*, \bar{\theta}\}$, promotion is not viable for either firm, but firm 2's profit is non-negative, so it enters without promotion.

Finally, in other cases, firm 2 does not enter. Firm 1 stays unchanged. $\square$

A6. The proof of Corollary 3.

Proof. Point 1 is straightforward because there is no restriction on the upside of θ.

Point 2. By Proposition 2, firm 2 promotes when $\theta > \frac{\alpha}{v-c}$ and $\theta \leq \tilde{\theta}$. It implies $\tilde{\theta} > \frac{\alpha}{v-c}$ is a necessary condition for firm 2 to promote. For $\tilde{\theta} = \frac{a}{v-c}\exp[\frac{\lambda(v-c)-f}{\alpha} - 1] > \frac{\alpha}{v-c}$, it must be true that $\frac{\lambda(v-c)-f}{\alpha} - 1 > 0$. Then, $\tilde{\theta} = \frac{a}{v-c}\exp[\frac{\lambda(v-c)-f}{\alpha} - 1] > \frac{a}{v-c}(\frac{\lambda(v-c)-f}{\alpha}) = \bar{\theta}$, because $x - 1 > \ln x$ when $x > 1$. The result follows. Second, the partial derivative $\frac{\partial \tilde{\theta}}{\partial \alpha} = \frac{a}{v-c}\exp[\frac{\lambda(v-c)-f}{\alpha} - 1](1 - \frac{\lambda(v-c)-f}{\alpha})$. It is negative by the same assumption.

Point 3. If $b > v - 2c - \alpha/\lambda$, then it can be shown that $b - \hat{b} > 2\lambda(v-c) - \alpha/\lambda$ and $\frac{d(\pi_1^{G*}(\theta'') - A(\theta''|\theta'))}{d\theta''} > 0$. When the latter is true, firm 1's profit decreases when it lowers the level of unawareness. Thus, firm 1 does not promote for sure.

Point 4. By Proposition 2, for firm 1 to promote in equilibrium, it is necessary that $(\theta''^* - \theta'^*)(v-c) \leq A(\theta'^*|\theta)$. Expand the inequality, we have

$$\sqrt{(b-\hat{b})^2 + 8\alpha(v-c)} \leq 4\alpha(1 + \ln\theta - \ln\theta'^*) - (b-\hat{b}). \tag{7}$$

For it to hold, $4\alpha(1 + \ln\theta - \ln\theta'^*) - (b-\hat{b}) > 0$, or

$$b - v + 2c + 2\lambda(v-c) < 4\alpha(1 + \ln\theta - \ln\theta'^*).$$

Also Inequality 7 implies that

$$b + c + 2\lambda(v-c) < 2\alpha(1 + \ln\theta - \ln\theta'^*)^2.$$

Thus, only small value of b satisfies the inequalities. $\square$

A7. The proof of Lemma 2.

Proof. By Lemma 4, in the price-competition stage, $\pi_1^{G*}(\theta'') > \pi_1^{GS*}(\theta'')$ if, and only if, $b < (1 - \lambda + \theta'')(v - c) + \frac{k}{\lambda - \theta''}$. For notational convenience, define $\beta = \lambda - \theta''$, and $b < (1 - \lambda + \theta'')(v - c) + \frac{k}{\lambda - \theta''}$ is equivalent to $b + (1 - \beta)c - k < (1 - \beta)[\beta(v - c) + c + b]$, or

$$(v - c)\beta^2 + (b - v + c)\beta - k < 0.$$

For the inequality to hold, it must be true that $\beta \in (\beta_1, \beta_2)$, where $\beta_1 = \frac{-(b-v+c)-\sqrt{\Delta}}{2(v-c)}$ and $\beta_2 = \frac{-(b-v+c)+\sqrt{\Delta}}{2(v-c)}$, and $\Delta = (b - v + c)^2 + 4k(v - c)$. Note $\beta_2 > 0$ and $\beta_1 < 0$. Because $\beta > 0$ by definition, the actual range of β is $(0, \beta_2)$. As $\beta = \lambda - \theta$, the constraint of θ'' is $\theta'' > \underline{\theta} = \max\{0, \lambda - \frac{-(b-v+c)+\sqrt{\Delta}}{2(v-c)}\}$. The proof of the upper bound $\bar{\theta}$ follows the same argument of Proposition 1. $\square$

A8. The derivation of firm 1's optimal profits under various assumptions on market structure and product offering.

Claim 1. *When Inequality (1) holds and $b > b^*$, the profit of firm 1 when it offers G in the duopoly competition, or $\pi_{1,D}(\theta'') \equiv (1 - \lambda + \theta'')[(\lambda - \theta'')(v - c) + c + b]$, increases in θ''.*

Proof. We want to show that $\frac{\partial \pi_{1,D}(\theta'')}{\partial \theta''} > 0$, or $b > E \equiv v - 2c - 2(\lambda - \theta'')(v - c)$. Given Inequality (1) and $b > b^*$, we have that $b > F \equiv \frac{1}{\theta}(2\lambda - \theta - \lambda^2)(v - c)$. It suffices to show that $F > E$. Note that $E = [1 - 2(\lambda - \theta'')](v - c) - c$, which means it suffices to show $\frac{1}{\theta}(2\lambda - \theta - \lambda^2) > 1 - 2(\lambda - \theta'')$, or $2\theta\theta'' + (1 - 2\lambda)\theta + \lambda^2 - 2\lambda < 0$. Note that by definition $\theta'' \leq \theta$, so it suffice to show $H(\theta) \equiv 2\theta^2 + (1 - 2\lambda)\theta + \lambda^2 - 2\lambda < 0$. Finally, note that $\theta \in (0, \lambda)$, which means that $H(\theta) < \max\{H(0), H(\lambda)\} = \max\{\lambda^2 - 2\lambda, \lambda^2 - \lambda\} < 0$, because $0 < \lambda < 1$. $\square$

Claim 2. *Assume $\pi_{1,D}^G(\theta'' = \bar{\theta})$, $\pi_{1,M}^G(\theta'' = \underline{\theta})$, and $\pi_{1,M}^{GS}(\theta'' = \underline{\theta})$ are non-negative. The*

optimal profits of firm 1 under a duopoly is

$$\pi_{1,D}^{G*}(\theta') = \begin{cases} \text{Irrelevant} & \theta' \leq \underline{\theta} \\ \pi_{1,D}^{G}(\theta'' = \theta'), & \theta' \in (\underline{\theta}, \bar{\theta}] \\ \pi_{1,D}^{G}(\theta'' = \bar{\theta}), & \theta' > \bar{\theta} \end{cases} . \tag{8}$$

The optimal profits of firm 1 under a monopoly with offering g only is

$$\pi_{1,M}^{G*}(\theta') = \begin{cases} \pi_{1,M}^{G}(\theta'' = \theta'), & \theta' \notin (\underline{\theta}, \bar{\theta}] \\ \pi_{1,M}^{G}(\theta'' = \underline{\theta}), & \theta' \in (\underline{\theta}, \bar{\theta}] \end{cases} . \tag{9}$$

and under a monopoly with offering both products is

$$\pi_{1,M}^{GS*}(\theta') = \begin{cases} \pi_{1,M}^{GS}(\theta'' = \theta'), & \theta' \notin (\underline{\theta}, \bar{\theta}] \\ \pi_{1,M}^{GS}(\theta'' = \underline{\theta}), & \theta' \in (\underline{\theta}, \bar{\theta}] \end{cases} . \tag{10}$$

Proof. By Claim 1, $\pi_{1,D}^{G}(\theta''|\theta') = (1-\lambda+\theta'')[(\lambda-\theta'')(v-c)+c+b] - A(\theta''|\theta')$ is maximized when $\theta'' = \theta'$ for $\theta' \in (\underline{\theta}, \bar{\theta}]$; If $\theta' \leq \underline{\theta}$, it is irrelevant because promotion can only increase awareness and a duopoly will not exist. If $\theta' > \bar{\theta}$, then the optimal $\theta'' = \bar{\theta}$ to maintain a duopoly at the lowest promotional cost.

For $\pi_{1,M}^{G}(\theta''|\theta') = (1 - \lambda + \theta'')(v+b) - A(\theta''|\theta')$, again, the profit excluding the promotion expense is an increasing function of θ'', so the optimal $\theta'' = \theta'$ for $\theta' \notin (\underline{\theta}, \bar{\theta}]$. Otherwise the optimal $\theta'' = \underline{\theta}$ for $\theta' \in (\underline{\theta}, \bar{\theta}]$ to maintain a monopoly at the lowest promotional cost.

For $\pi_{1,M}^{GS}(\theta''|\theta') = b + v - (\lambda - \theta'')c - k - A(\theta''|\theta')$, again, the profit excluding the promotion expense is an increasing function of θ'', so the optimal $\theta'' = \theta'$ for $\theta' \notin (\underline{\theta}, \bar{\theta}]$. Otherwise the optimal $\theta'' = \underline{\theta}$ for $\theta' \in (\underline{\theta}, \bar{\theta}]$ to maintain a monopoly at the lowest promotional cost. $\square$

Claim 3. *The following facts hold.*

i. When $\theta' \leq \bar{\theta}$, $\pi_{1,M}^{GS*}(\theta') > \pi_{1,M}^{G*}(\theta')$

ii. When $\theta' > \bar{\theta}$, $\pi_{1,M}^{G*}(\theta') > \pi_{1,D}^{G*}(\theta')$

Proof. **The proof of point** *i*. When $\theta' \leq \underline{\theta}$, $\pi_{1,M}^{GS*}(\theta') > \pi_{1,M}^{G*}(\theta')$ is equivalent to $\pi_{1,M}^{GS*}(\theta'' = \theta') > \pi_{1,M}^{G*}(\theta'' = \theta')$, which is true when $k < (\lambda - \theta')(v + b - c)$. For the latter hold, we

use the fact that $\underline{\theta}$ is the solution of $k = (\lambda - \underline{\theta})[b - (1 - \lambda + \underline{\theta})(v - c)]$. Then $k \leq (\lambda - \theta')[b - (1 - \lambda + \theta')(v - c)] < (\lambda - \theta')(v + b - c)$ for any $\theta' \leq \underline{\theta}$.

When $\theta' \in (\underline{\theta}, \bar{\theta}]$, $\pi_{1,M}^{GS*}(\theta') > \pi_{1,M}^{G*}(\theta')$ is equivalent to $\pi_{1,M}^{GS*}(\theta'' = \underline{\theta}) > \pi_{1,M}^{G*}(\theta'' = \underline{\theta})$, which is true when $k < (\lambda - \underline{\theta})(v + b - c)$. Again, we can use the fact that $k = (\lambda - \underline{\theta})[b - (1 - \lambda + \underline{\theta})(v - c)] < (\lambda - \underline{\theta})(v + b - c)$.

The proof of point *ii.* When $\theta' > \bar{\theta}$, $\pi_{1,M}^{G*}(\theta') = \pi_{1,M}^{G}(\theta'' = \theta')$ and $\pi_{1,D}^{G*}(\theta') = \pi_{1,D}^{G}(\theta'' = \bar{\theta})$. Then $\pi_{1,M}^{G}(\theta'' = \theta') > \pi_{1,D}^{G}(\theta'' = \bar{\theta})$, or $(1 - \lambda + \theta')(v + b) > (1 - \lambda + \bar{\theta})[(\lambda - \bar{\theta})(v - c) + c + b] - A(\bar{\theta}|\theta')$, because $v + b > (\lambda - \bar{\theta})(v - c) + c + b$ and $\theta' > \bar{\theta}$ □

A9. The proof of Lemma 3.

Proof. First we show that if θ_0 exists, then $\theta_0 > \underline{\theta}$, by studying the property of optimal profit curves . By Claims 2 and 3 in Appendix A8, we see that when $\theta' \leq \bar{\theta}$, under a monopoly, firm 1 prefers offering both products than offering G. Hence, when $\theta' \leq \bar{\theta}$ the optimal profit under a monopoly is

$$\pi_{1,M}^{GS*}(\theta') = \begin{cases} \pi_{1,M}^{GS}(\theta'' = \theta'), & \theta' \notin (\underline{\theta}, \bar{\theta}] \\ \pi_{1,M}^{GS}(\theta'' = \underline{\theta}), & \theta' \in (\underline{\theta}, \bar{\theta}] \end{cases}.$$

Note that when $\theta' \leq \bar{\theta}$, $\pi_{1,M}^{GS*}$ is continuous and decreases with θ' when $\theta' \in (\underline{\theta}, \bar{\theta}]$, because

$$\pi_{1,M}^{GS}(\theta'' = \underline{\theta}) = b + v - (\lambda - \underline{\theta})c - k - \alpha \ln(\frac{\theta'}{\underline{\theta}}). \tag{11}$$

Also, when $\theta' = \underline{\theta}$, it can be shown that $\pi_{1,D}^{G}(\theta'' = \theta') < \pi_{1,M}^{G}(\theta'' = \theta') < \pi_{1,M}^{GS*}(\theta')$. By continuity of $\pi_{1,D}^{G}(\theta'' = \theta')$ and $\pi_{1,M}^{GS*}(\theta')$, within the range of $\theta' \in (\underline{\theta}, \bar{\theta}]$, the increasing curve $\pi_{1,D}^{G*}(\theta')$ and the decreasing curve of $\pi_{1,M}^{GS*}(\theta')$ must cross each other at most once at $\theta_0 \in (\underline{\theta}, \bar{\theta}]$. Then whenever θ_0 exists, $\theta_0 > \underline{\theta}$.

Therefore, when $\theta' \leq \theta_0$, the optimal profit under duopoly, $\pi_{1,D}^{G*}(\theta')$, is lower than or equal to the optimal profit under monopoly when firm 1 offers both products, $\pi_{1,M}^{GS*}(\theta')$. Hence, firm 1 does not accommodate the entry. It also ensures that $\theta'' = \underline{\theta}$ if $\theta' > \underline{\theta}$. When $\theta' \in (\theta_0, \bar{\theta}]$, $\pi_{1,D}^{G*}(\theta') > \pi_{1,M}^{GS*}(\theta')$, firm 1 accommodates the entry and does not promote. When $\theta' > \bar{\theta}$, by Claim 2, firm 1 does not accommodate the entry and does not promote.

Firm 1's product choice depends on the comparison between $\pi_{1,M}^{GS*}(\theta')$ and $\pi_{1,M}^{G*}(\theta')$. In particular, if $k > (\lambda - \theta')(v + b - c)$, then firm 1 offers G only. $\qquad\square$

A10. The proof of Proposition 3.

Proof. The proof is based on Lemma 3. First consider when θ_0 exists.

If $\theta \in (\theta_0, \tilde{\theta}]$, when firm 2 sets $\theta' > \theta_0$, firm 1 will accommodate the entry. In particular, following Proposition 2, if $\theta_0 < \theta'^* < \theta$, then firm 2's optimal profit is achieved at the interior optimal solution. If $\theta'^* \leq \theta_0 < \theta$, then firm 2's optimal profit cannot be achieved at the interior solution. Because firm 2's profit decreases in θ' when $\theta'^* < \theta'$, firm 2's solution is to set $\theta' = \theta_0 + \epsilon$, as long as the resulting profit is non-negative. If $\theta \leq \min\{\theta'^*, \bar{\theta}\}$, then firm 2's solution is to set $\theta' = \theta$. Otherwise, firm 2 does not enter.

If $\theta \leq \theta_0$, then whether or not firm 2 promotes, the degree of unawareness faced by firm 1 is $\theta' \leq \theta_0$, upon which firm 1 will deter entry. Hence, firm 2 does not promote. On the other hand, for firm 2 to be profitable via entry, by Proposition 2, the maximum original degree of unawareness is $\tilde{\theta}$. Therefore, if $\theta > \tilde{\theta}$, firm 2 does not promote.

Second, if θ_0 does not exist, then the curve of the optimal profit under duopoly, $\pi_{1,D}^{G*}(\theta')$, does not cross the curve of the optimal profit under monopoly when firm 1 offers both products, $\pi_{1,M}^{GS*}(\theta')$. In other words, $\pi_{1,D}^{G*}(\theta') < \pi_{1,M}^{GS*}(\theta')$. Firm 1 will always deter firm 2's entry. In equilibrium, firm 2 does not enter. $\qquad\square$

A11. The proof of Corollary 4.

Proof. By definition, θ_0 is the solution of $\pi_{1,D}^{G*}(\theta') = \pi_{1,M}^{GS*}(\theta')$, subject to $\theta' \leq \bar{\theta}$. By Lemma 6, when it exists, $\theta_0 \in (\underline{\theta}, \bar{\theta}]$. In this range, $\pi_{1,D}^{G}(\theta'' = \theta')$ is an increasing function of θ' and does not change when α decreases. Also by Equation 11, $\pi_{1,M}^{GS*}$ is a decreasing function of θ'. It also shifts up when α decreases, because

$$\frac{\partial \pi_{1,M}^{GS}(\theta'' = \underline{\theta})}{\partial \alpha} = -\ln(\frac{\theta'}{\underline{\theta}}) < 0.$$

This implies that the intersection of two curves must increase when α decreases. $\qquad\square$