Equilibrium Innovation Ecosystems: The Dark Side of Collaborating with Complementors*

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Abstract

The recent years have exhibited a burst in the amount of collaborative activities among firms selling complementary products. This paper aims at providing a rationale for such a large extent of collaboration ties among complementors. To this end, we analyze a game in which the two producers of a certain component have the possibility to form pairwise collaboration ties with each of the two producers of a complementary component. Once ties are formed, each of the four firms decides how much to invest in improving the quality of the match with each possible complementor, under the assumption that a firm with a collaboration link with a complementor puts some weight on the complementor's profit when making investment decisions. Once investment choices have taken place, all firms choose prices for their respective components in a noncooperative manner. In equilibrium, firms end up forming as many collaboration ties as it is possible, although they would all prefer a scenario where collaboration were forbidden. In addition, a social planner would also prefer such a scenario to the one arising in equilibrium. We show that the result that collaboration is inefficient for firms and society does not depend on whether collaboration ties are formed in an exclusive manner: in fact, exclusivity would only worsen the situation.

Key words: Systems Competition, Complementary Products, Interoperability, Collaboration Link, Co-opetition, Exclusivity.

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1 Introduction

The recent decades have witnessed a shift in the competitive paradigm in high-tech industries that is driven to a large extent by the increasing importance of product complementarity. Indeed, cooperation among firms selling complementary products is playing a prominent role in industries such as consumer electronics, semiconductors or telecommunications. More generally, hardware-software industries have exhibited a surge in the extent of cooperation among producers of complementary goods with the aim of improving the interoperability of their respective products (see e.g. Moore 1996, Gawer and Cusumano 2002, Adner 2006, Adner and Kapoor 2010, Gawer and Henderson 2007).[1] Building such *innovation ecosystems* (Adner 2006) with the producers of complementary goods seems to be the key competitive weapon in most high-tech industries, in which the notion of competition has been displaced by that of *co-opetition* (Brandenburger and Nalebuff 1996). A noteworthy feature of collaboration with complementors (i.e., firms selling products that complement each other from the point of view of consumers) is that it is not unusual for firms to collaborate with several complementors that sell substitutes of each other.[2]

A natural question that arises in these settings is whether such extensive collaboration is desirable from the standpoints of firms and consumers. Intuitively, one would be tempted to think that collaboration in improving the interoperability of complementary products is efficient both for the firms involved, and in fact for society as a whole. The purpose of this paper is to show that this need not be the case. We argue that collaboration may result in equilibria in which both firms and society are worse off than when firms do not collaborate with complementors. This holds regardless of whether collaboration ties are exclusive or not, under the assumption that collaboration with a complementor leads a firm to care somewhat about the complementor's profit when deciding on its interoperability investments.

To formally analyze these issues, we consider a game played by two firms X_1 and X_2 that sell components that (perfectly) complement those sold by firms Y_1 and Y_2 (both of which are also engaged in the game). In this mix-and-match setting (Matutes and Regibeau 1988 and Economides 1989), there are four systems that are contemplated by consumers when they make their purchase decisions: X_1Y_1, X_1Y_2, X_2Y_1 and X_2Y_2. The game that we study consists of three stages. In the first stage, each firm decides

[1] The interoperability of the components of which a composite good consists refers to their coherence to work together with each other as a sole system. This is largely related to the absence of conflicts arising from possible incompatibility issues.

[2] To give concrete examples, mobile phone manufacturer Nokia allied first with Intel to develop the MeeGo operating system for smartphones, and later signed an agreement with Microsoft to support the Windows Phone operating system. In addition, the Intel Architecture Lab (IAL) was formed to foster investment in components complementary to Intel's microprocessors by firms that many times competed against each other.

whether to form a (pairwise) collaboration link with each of its possible complementors (collaboration among firms selling substitute components of a system is not allowed). In the second stage, each firm decides how much to invest in improving the interoperability of its component with each of its complementors.[3] It is assumed that a firm that has formed a collaboration link with a complementor puts some weight on the complementor's profit when making investment decisions (e.g., a joint venture is formed). In the third and final stage, each firm decides independently on the price of its component, given past interoperability investments of all the firms involved in the game.

We find in this setting that the (unique) equilibrium collaboration network involves each firm forming (pairwise) collaboration links with its two complementors. If collaboration ties can be formed only in an exclusive manner, then exactly the same forces (subject to the exclusivity restriction) imply that in equilibrium each firm forms a collaboration link with just one of its complementors. In both the exclusive and non-exclusive settings, equilibria exhibit all firms collaborating with at least one complementor, which seems to accord well with the empirical evidence on innovation ecosystems.

Although equilibrium outcomes seem quite intuitive, intuition may conceal the effect of several forces that are working at the same time, and not necessarily in the same direction. Thus, two complementors that form a new collaboration link between them have an incentive to increase their investment in enhancing the interoperability with each other, thus mitigating free riding to some extent. In addition, a new collaboration tie between two complementors induces each to lower its investment in enhancing the interoperability with the complementor's rival. These two effects conform to the intuition that one may have on the impact of a new collaboration link. However, the formation of a new collaboration tie not only affects the incentives of the firms that become collaborators but also affects those of firms not involved in the new collaboration relationship. We show that the strategic reactions elicited by a new collaboration link either make it even more appealing to form the link or are not as negative for collaborators so as to dissuade them from forming the link.[4] Factoring all the incentives, we then have that it is always desirable to form a new collaboration tie with a complementor with which a firm does not have one. This rat race ends when no more ties are possible, and hence each firm collaborates with as many complementors as it can.

In spite of the rat race underlying the unique equilibrium outcome, all firms prefer the situation in which none collaborates with its complementors to that in which all collab-

[3]Greater investment in the interoperability of two components is modeled as an enhancement in the (perceived) quality of the system comprising *both* components (e.g., the investment by X_1 in improving interoperability with component Y_2 is specific to Y_2, and has no effect on the interoperability of components X_1 and Y_1).

[4]Put differently, firms not involved in the new collaboration tie strategically react by weakening or just slightly strengthening the systems in which the firms involved in the new collaboration link do not participate. This is not enough to offset the positive effect of mitigating free-riding that forming a new collaboration tie has.

orate with them. Hence, the equilibrium outcome exhibits the features of a prisonner's dilemma. When a firm collaborates with both complementors, it invests in enhancing interoperability with each as much as it would in the absence of any collaborative tie among firms.[5] The reason is that benefiting one of the complementors comes at the expense of harming the complementor's competitor, so the firm adopts a neutral approach, and the situation is as if no firm collaborated with any other firm (except for the costs of collaboration). Thus, although a firm would benefit from its competitor committing not to collaborate with any complementors, it holds that all of them would be better off if each could make such commitment. Not only do we find that collaboration is excessive from the point of view of firms, but also from that of a social planner that can simply choose how many collaboration links should be formed. Our baseline model assumes that a firm cares when making interoperability decisions about the entire profit made the complementors with which it collaborates. We show that results do not vary (actually, they are strengthened) if collaborating with another firm entails caring only about the profit generated for such a firm by the system in which both firms participate.

Our result that R&D collaboration among complementors results in private and public ineffiencies is in stark contrast with the result that R&D collaboration among firms selling substitute goods may be desirable both for firms and society, as shown in the seminal papers by D'Aspremont and Jacquemin (1988) and Kamien, Muller and Zang (1992). These papers do not consider whether a firm has incentives to collaborate with other firms, a limitation that has been overcome by subsequent work by Bloch (1995) using a coalitions approach, and more recently by Goyal and Moraga-González (2001) using a bilateral link formation approach.[6] Both of these papers show that excessive collaboration may arise in equilibrium. Although we also contend that equilibria displaying collaboration may be inefficient, it is worth noting that the results in Bloch (1995) and Goyal and Moraga-González (2001) are derived for substitute goods, not for complementary goods, as is our focus.

Our paper also contributes to the literature analyzing strategic competition when there exists at least one complementor whose pricing activities interact with those of two firms selling components that constitute substitutes for each other. This literature was pioneered by Economides and Salop (1992) as an extension of early work by Cournot (1838), who analyzed the effect of a merger of two monopolists that produce complemen-

[5] A firm may invest more in enhancing interoperability with each complementor than it would in the absence of any collaborative tie among firms. This happens when collaborating with a complementor entails caring about the profit generated for such a complementor by the system comprising the two products sold by these firms.

[6] See Leahy and Neary (1997) for a generalization of the models in D'Aspremont and Jacquemin (1988) and Kamien, Muller and Zang (1992). See also Bloch (2005) for a comprehensive survey that covers strategic network formation games in settings with R&D activities. Finally, it is worth pointing out that Westbrock (2010) builds on Goyal and Moraga-González (2001) and Goyal and Joshi (2003) so as to analyze how asymmetric R&D networks may be socially efficient if collaboration ties are somewhat costly to establish.

tary goods.

The paper by Economides and Salop (1992) examines the effect of cooperation in prices (i.e., a merger) between the two existing producers of one of the two components of which a system consists. They consider two scenarios, depending on whether or not the two producers of the complementary component are already cooperating in prices. In our work, we do not analyze price cooperation and, in fact, firms always choose prices noncooperatively regardless of the structure of the collaboration network. The network architecture does have an effect on cooperation in R&D activities, though.[7] Our paper is also related to recent work by Casadesus-Masanell, Nalebuff and Yoffie (2008). Their paper provides conditions under which a firm may benefit from having a new competitor enter with a substitute good whenever there exists a complementor for both the firm under consideration and its new competitor. Our framework differs in that it does not focus on the effects of entry on co-opetive settings, as they do, but rather it examines the incentives to form collaboration links and to invest in enhancing interoperability among complementors.

The remainder of the paper is organized as follows. Section 2 introduces the game we consider. Section 3 characterizes the efficiency properties of the unique equilibrium of the game under the assumption that a firm can form any desired number of collaboration links with its complementors. Section 4 examines how results are affected if collaboration ties are assumed to be exclusive. Section 5 shows that results are strengthened in the more realistic case in which collaborating with another firm entails caring just about the profit generated for such a firm by the system in which both firms participate. Section 6 concludes.

2 The model

We define a system as a pair of perfectly complementary goods such as hardware and software. The two perfect complements giving rise to a system are called components X and Y. It is assumed that there are two firms costlessly producing component X, X_1 and X_2, and two firms costlessly producing component Y, Y_1 and Y_2.[8] As a result, there are $n = 4$ systems: X_1Y_1, X_1Y_2, X_2Y_1 and X_2Y_2. System X_iY_j $(i, j = 1, 2)$ can be bought by any consumer at price $p_{i,j} = p_{X_i} + p_{Y_j}$, where p_{X_i} and p_{Y_j} respectively denote the prices at which components X_i and Y_j are sold. Whenever there is no risk of confusion, we will

[7] There is a recent literature on (pure and mixed) bundling by firms that produce two perfectly complementary components in competition with firms that produce just one of these components (see e.g. Denicolò 2000 and Choi 2008). The reason why this stream of research building on Economides and Salop (1992) is not related to our work is that we do not consider bundling, an issue that certainly deserves a separate analysis beyond the scope of our paper.

[8] That production is costless is without loss of generality if the marginal cost of production is constant and the fixed costs of operation are not too large.

write p_{ij} instead of $p_{i,j}$ for system X_iY_j. Also, firms X_1 and X_2 are typically referred to as the complementors of firms Y_1 and Y_2, and vice versa.

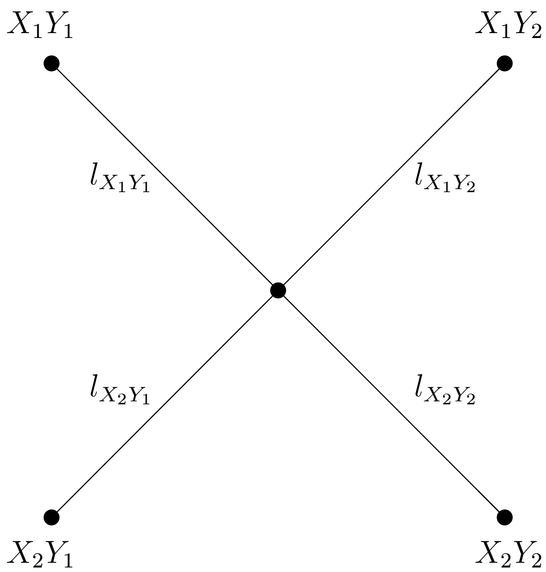
It is assumed that there exists a unit mass of consumers willing to buy at most one system. System X_iY_j is assumed to create a gross utility of $v_{i,j}$ to any consumer (again, we will typically write v_{ij} instead of $v_{i,j}$). The gross utility v_{ij} is largely the outcome of choices by firms X_i and Y_j. More specifically, for some given scalar $v > 0$, we have that $v_{ij} = v + k_i^j + e_j^i$, where k_i^j is firm X_i's R&D investment in improving the quality of the match with firm Y_j's component and e_j^i is firm Y_j's R&D investment in improving the quality of the match with firm X_i's component.[9] Thus, the investment variables k_i^j and e_j^i affect the vertical attributes of system X_iY_j. Given their system-specificity, they can be viewed as investments in improving the interoperability of components X_i and Y_j, although other interpretations are possible and may be more appealing depending on the context.

Besides (possibly) being vertically differentiated, systems are perceived by consumers as being horizontally differentiated in an exogenous manner. To model consumer preferences over horizontally differentiated systems, we follow Chen and Riordan (2007) in using their "spokes" model of nonlocalized differentiation. Thus, each system is represented by a point at the origin of a line of length 1/2, a line which is denoted by $l_{X_iY_j}$ for system X_iY_j $(i, j = 1, 2)$. The other end of a line is called its terminal, and it is assumed that the terminals of all lines meet at a point called the center (see Figure 1). All the existing consumers are uniformly distributed along the four lines. A consumer who is located on line $l_{X_iY_j}$ at distance $d_{X_iY_j} \in [0, 1/2]$ from system X_iY_j must incur a transportation/disutility cost of $td_{X_iY_j}$ when buying X_iY_j, where $t \geq 0$ is a unit transportation cost. The same consumer must incur transportation cost $t(1 - d_{X_iY_j})$ when purchasing any other system (since $l_{X_iY_j} = 1/2$ for all $i, j = 1, 2$). It is assumed that X_iY_j is the preferred system for any consumer on $l_{X_iY_j}$, and any other system has probability $1/(n-1) = 1/3$ of constituting the benchmark against which X_iY_j is to be compared by a consumer on $l_{X_iY_j}$. A system that is not deemed as preferred or as a benchmark for a consumer is assumed to yield no utility to such a consumer. This assumption completes the description of the spokes model we use for modeling the horizontal attributes of systems.[10]

[9]See Goyal, Konovalov and Moraga-González (2008) for another setting with relationship-specific actions.

[10]Note that although in the most general version of the spokes model there are $N \geq n$ systems over which preferences are defined, we have let $N = n$ for the sake of simplicity. This means that we have assumed that there is no uncommercialized system that is possibly the object of desire by (some) consumers.

Figure 1 : The Spoke Model



Given these features of firms and consumers, we study a three-stage game. In the first stage, firms X_i $(i = 1, 2)$ simultaneously form pairwise collaboration links with firms Y_j $(j = 1, 2)$. We let $g_{ij} = \lambda \in (0, 1]$ if a collaboration link between X_i and Y_j is formed and $g_{ij} = 0$ otherwise, with the convention that $g_{ji} = g_{ij}$. In addition, $g_{ij} = \lambda$ implies that both firms X_i and Y_j bear an arbitrarily small cost $\varepsilon > 0$. We denote the network (i.e., the set of collaboration links) by g, that is, $g = \{g_{11}, g_{12}, g_{21}, g_{22}\} \in \{0, \lambda\}^4$. Note that in principle we allow a firm to form more than one collaboration link with its complementors (e.g., it may be possible that $g_{i1} = g_{i2} = \lambda$ for some $i \in \{1, 2\}$). The parameter λ intends to capture how much a firm cares about a collaborator's (net) profit when firms choose investment levels in the second stage. We let $\lambda = 1$ to simplify computations, but insights and results do not vary qualitatively if $\lambda \in (0, 1)$.

In the second stage of the game we consider, we assume that firm X_i chooses k_i^j at the same time as firm Y_j chooses e_j^i $(i, j = 1, 2)$. Investments of k_i^1 and k_i^2 by firm X_i result in an R&D cost equal to $C_{X_i}(k_i^1, k_i^2) = (k_i^1)^2 + (k_i^2)^2$, whereas investments of e_j^1 and e_j^2 by firm Y_j result in an R&D cost equal to $C_{Y_j}(e_j^1, e_j^2) = (e_j^1)^2 + (e_j^2)^2$. It is assumed throughout that $g_{ij} = \lambda$ implies that both firms X_i and Y_j choose their investments in improving their match quality in a (somewhat) cooperative manner. By this, we mean that each also cares to some extent about the investment cost borne by the collaborator and the profit that the collaborator will make in the product market. There are many formal or informal arrangements that may lead a firm that collaborates with a complementor to care about the complementor's payoff when making investment decisions. Reasons range from research alliances (or collusive R&D cartels) to relational capital concerns in ongoing relationships between firms that need each other. (Sometimes, the motives that foster cooperation with a complementor may also preclude a firm from collaborating with the complementor's competitor; this is the reason why we will examine equilibria when arrangements are exclusive in a later section.)

In the third and last stage, prices p_{X_i} and p_{Y_j} are set simultaneously in the standard noncooperative manner, and consumers make their purchase decisions given p_{ij} for $i, j = 1, 2$.

The solution concept is the same as in Goyal and Moraga-González (2001). Thus, for each possible g, we will look for subgame perfect Nash equilibria, which will give equilibrium payoffs given g. In order to solve for the equilibrium network structure in the first stage, we will use the pairwise stability notion proposed by Jackson and Wolinsky (1996). This concept is very weak, and aims at capturing (possibly) complex communication and negotiation activities that would be hard to capture through noncooperative game theory.

Introducing the concept of pairwise stability requires some notation. In particular, we let $g - g_{ij}$ denote the network that results from suppressing the collaboration link between firms X_i and Y_j in network g. We also let $g + g_{ij}$ denote the network that results from adding a collaboration link between firms X_i and Y_j in network g. Denoting the equilibrium payoffs (gross of ε) obtained by firm X_i and Y_j given network g by $\Pi^*_{X_i}(g)$ and $\Pi^*_{Y_j}(g)$, network g would be pairwise stable if the following two conditions held for all $i, j \in \{1, 2\}$: (i) $\Pi^*_{X_i}(g) - \varepsilon \geq \Pi^*_{X_i}(g - g_{ij})$ and $\Pi^*_{Y_j}(g) - \varepsilon \geq \Pi^*_{Y_j}(g - g_{ij})$ for $g_{ij} = \lambda$ and; (ii) $\Pi^*_{X_i}(g + g_{ij}) - \varepsilon \geq \Pi^*_{X_i}(g)$ implies that $\Pi^*_{Y_j}(g + g_{ij}) - \varepsilon < \Pi^*_{Y_j}(g)$. The first condition requires that neither X_i nor Y_j have an incentive to unilaterally break their collaboration relationship (provided it exists). In turn, the second condition requires that, if firms X_i and Y_j are not linked to each other, then a desire by X_i to form a collaboration link with Y_j should not be reciprocal. It is worth noting that the results we derive still hold if the network is required to be pairwise Nash stable, that is, if a firm is allowed to unilaterally break more than one collaboration link at a time.

3 Resolution of the model

3.1 Third stage

As is standard, we solve the last two stages of the game by working backwards. So assume that first-stage and second-stage choices lead to a gross valuation of v_{ij} for system X_iY_j, $i, j = 1, 2$. We first derive the demand functions for each system and then we find out profits attained by each firm as a function of $\{v_{11}, v_{12}, v_{21}, v_{22}\}$. It is assumed throughout that v is large enough so that the market is always fully covered and all firms make positive sales. If collaboration between firms X_i and Y_j drove a system in which none of them participates out of the market, then there would be an additional incentive to form collaboration links. It is in this sense that we make the weakest case for collaboration to take place, and still find that it emerges in equilibrium.

In order to characterize the demand functions of each system, let $l_{X_iY_j} + l_{X_{i'}Y_{j'}} = \{d : d \in l_{X_iY_j} \cup l_{X_{i'}Y_{j'}}\}$ $(i, j, i', j' = 1, 2$, with $i \neq i'$ or $j \neq j'$ or both) denote the set consisting of all the points that belong to either line $l_{X_iY_j}$ or $l_{X_{i'}Y_{j'}}$ or both. In defining $l_{X_iY_j} + l_{X_{i'}Y_{j'}}$, we establish the convention that $i \leq i'$ and $j \leq j'$.[11] To find out the demand for system X_1Y_1, consider a consumer who happens to be on $l_{X_1Y_1} + l_{X_1Y_2}$. This occurs either because X_1Y_1 is her preferred system and X_1Y_2 is the benchmark, or because X_1Y_2 is her preferred system and X_1Y_1 is the benchmark. The consumer will be indifferent between both systems if her distance $d_{11}^{12} \in [0, 1]$ from X_1Y_1 is given by $v_{11} - p_{11} - td_{11}^{12} = v_{12} - p_{12} - t(1 - d_{11}^{12})$,[12] that is, if

$$d_{11}^{12} = \frac{t + v_{11} - v_{12} + p_{12} - p_{11}}{2t}.$$

Because the measure of consumers between the locations of systems X_1Y_1 and X_1Y_2 is $2/n$, we then have that the number of consumers who prefer X_1Y_1 over X_1Y_2 given p_{11} and p_{12} is $2d_{11}^{12}/n$. Similarly, the number of consumer who prefer X_1Y_1 over X_2Y_j $(j = 1, 2)$ can be shown to be $2d_{11}^{2j}/n$, where

$$d_{11}^{2j} = \frac{t + v_{11} - v_{2j} + p_{2j} - p_{11}}{2t}.$$

Conditional upon X_1Y_1 being the preferred system or the benchmark one, we have that X_1Y_2, X_2Y_1 and X_2Y_2 have each probability $1/(n - 1) = 1/3$ of being the system with respect to which X_1Y_1 is to be assessed by consumers. It then follows that demand for X_1Y_1 is

$$Q_{11} = \frac{2(d_{11}^{12} + d_{11}^{21} + d_{11}^{22})}{n(n - 1)}.$$

Simple algebra yields that

$$Q_{11} = \frac{3t + 3v_{11} - v_{12} - v_{21} - v_{22} - 3p_{11} + p_{12} + p_{21} + p_{22}}{12t}.$$

Similar steps lead to the following demand for system X_iY_j $(i, j = 1, 2)$:

$$Q_{ij} = \frac{3t + 3v_{i,j} - v_{3-i,j} - v_{i,3-j} - v_{3-i,3-j} - 3p_{i,j} + p_{3-i,j} + p_{i,3-j} + p_{3-i,3-j}}{12t}.$$

Recalling that $p_{i,j} = p_{X_i} + p_{Y_j}$ and letting $Q_{X_i} \equiv Q_{i1} + Q_{i2}$ denote X_i's demand, we

[11] Observe from the definition of $l_{X_iY_j} + l_{X_{i'}Y_{j'}}$ that $i \neq i'$ or $j \neq j'$ or both, so we cannot have both $i = i'$ and $j = j'$.

[12] Recall that the set $l_{X_1Y_1} + l_{X_1Y_2}$ has unit (Lebesgue) measure.

have that

$$Q_{X_i}(p_{X_i}, p_{X_{3-i}}) = \frac{3t + v_{i,1} + v_{i,2} - v_{3-i,1} - v_{3-i,2} - 2p_{X_i} + 2p_{X_{3-i}}}{6t}.$$

We have made the arguments of Q_{X_i} explicit to highlight that the volume of sales by firm X_i does not depend on how any complementary product is priced. Under full market coverage, different prices by Y_1 and Y_2 just affect with which component X_i wishes to be matched, but firm X_i's demand solely depends on p_{X_i} and $p_{X_{3-i}}$. One can similarly find out that

$$Q_{Y_j}(p_{Y_j}, p_{Y_{3-j}}) = \frac{3t + v_{1,j} + v_{2,j} - v_{1,3-j} - v_{2,3-j} - 2p_{Y_j} + 2p_{Y_{3-j}}}{6t},$$

where $Q_{Y_j} \equiv Q_{1j} + Q_{2j}$.

Firms X_1 and X_2 choose p_{X_1} and p_{X_2} to maximize $\pi_{X_1}(p_{X_1}, p_{X_2}) \equiv p_{X_1} Q_{X_1}(p_{X_1}, p_{X_2})$ and $\pi_{X_2}(p_{X_2}, p_{X_1}) \equiv p_{X_2} Q_{X_2}(p_{X_2}, p_{X_1})$, respectively. Using the strict concavity of profit functions, we have that the solution to the following system delivers the equilibrium prices for firms X_1 and X_2:

$$3t + v_{11} + v_{12} - v_{21} - v_{22} - 4p_{X_1} + 2p_{X_2} = 0 \tag{1}$$

and

$$3t + v_{21} + v_{22} - v_{11} - v_{12} - 4p_{X_2} + 2p_{X_1} = 0. \tag{2}$$

The system consisting of equations (1) and (2) has the following solution:

$$p^*_{X_i} = \frac{9t + v_{i,1} + v_{i,2} - v_{3-i,1} - v_{3-i,2}}{6}, \; i = 1, 2.$$

Similarly, one can show that

$$p^*_{Y_j} = \frac{9t + v_{1,j} + v_{2,j} - v_{1,3-j} - v_{2,3-j}}{6}, \; j = 1, 2.$$

We then have that the sales of system X_iY_j are

$$Q^*_{ij} = \frac{9t + 5v_{i,j} + v_{3-i,3-j} - 3v_{i,3-j} - 3v_{3-i,j}}{36t}.$$

The profit that system X_iY_j generates for firm X_i $(i, j = 1, 2)$ is $\pi^j_{X_i} \equiv p^*_{X_i} Q^*_{ij}$, so recalling that $v_{ij} = v + k^j_i + e^i_j$, we can write it as a function of second-stage choices:

$$\begin{aligned} \pi^j_{X_i} \;=\; & \frac{1}{216t}(9t + k^j_i + k^{3-j}_i - k^j_{3-i} - k^{3-j}_{3-i} + e^i_j + e^i_{3-j} - e^{3-i}_j - e^{3-i}_{3-j}) \times \\ & (9t + 5k^j_i + k^{3-j}_{3-i} - 3k^j_{3-i} - 3k^{3-j}_i + 5e^i_j + e^{3-i}_{3-j} - 3e^i_{3-j} - 3e^{3-i}_j). \end{aligned}$$

Similarly, the profit that system X_iY_j generates for firm Y_j can be written as follows:

$$\begin{aligned}\pi^i_{Y_j} &= \frac{1}{216t}(9t + k^j_i + k^j_{3-i} - k^{3-j}_i - k^{3-j}_{3-i} + e^i_j + e^{3-i}_j - e^i_{3-j} - e^{3-i}_{3-j}) \times \\ &\quad (9t + 5k^j_i + k^{3-j}_{3-i} - 3k^j_{3-i} - 3k^{3-j}_i + 5e^i_j + e^{3-i}_{3-j} - 3e^i_{3-j} - 3e^{3-i}_j).\end{aligned}$$

Letting $\pi^*_{X_i} \equiv \pi^1_{X_i} + \pi^2_{X_i}$ and $\pi^*_{Y_j} \equiv \pi^1_{Y_j} + \pi^2_{Y_j}$ respectively denote the overall profits made by firms X_i and Y_j, it is easy to show that

$$\pi^*_{X_i} = \frac{(9t + k^1_i + k^2_i - k^1_{3-i} - k^2_{3-i} + e^i_1 + e^i_2 - e^{3-i}_1 - e^{3-i}_2)^2}{108t}, \; i = 1, 2,$$

and

$$\pi^*_{Y_j} = \frac{(9t + k^j_1 + k^j_2 - k^{3-j}_1 - k^{3-j}_2 + e^1_j + e^2_j - e^1_{3-j} - e^2_{3-j})^2}{108t}, \; j = 1, 2.$$

The following is worth noting for $i, j = 1, 2$:

Remark 1 *We have that*

$$\frac{\partial^2 \pi^*_{X_i}}{\partial k^j_i \partial k^{3-j}_i} = \frac{\partial^2 \pi^*_{X_i}}{\partial k^j_i \partial e^i_j} = \frac{\partial^2 \pi^*_{X_i}}{\partial k^j_i \partial e^i_{3-j}} > 0$$

and

$$\frac{\partial^2 \pi^*_{X_i}}{\partial k^j_i \partial k^j_{3-i}} = \frac{\partial^2 \pi^*_{X_i}}{\partial k^j_i \partial k^{3-j}_{3-i}} = \frac{\partial^2 \pi^*_{X_i}}{\partial k^j_i \partial e^{3-i}_j} = \frac{\partial^2 \pi^*_{X_i}}{\partial k^j_i \partial e^{3-i}_{3-j}} < 0.$$

Similarly,

$$\frac{\partial^2 \pi^*_{Y_j}}{\partial e^i_j \partial e^{3-i}_j} = \frac{\partial^2 \pi^*_{Y_j}}{\partial e^i_j \partial k^j_i} = \frac{\partial^2 \pi^*_{Y_j}}{\partial e^i_j \partial k^j_{3-i}} > 0$$

and

$$\frac{\partial^2 \pi^*_{Y_j}}{\partial e^i_j \partial e^i_{3-j}} = \frac{\partial^2 \pi^*_{Y_j}}{\partial e^i_j \partial e^{3-i}_{3-j}} = \frac{\partial^2 \pi^*_{Y_j}}{\partial e^i_j \partial k^{3-j}_i} = \frac{\partial^2 \pi^*_{Y_j}}{\partial e^i_j \partial k^{3-j}_{3-i}} < 0.$$

Essentially, a firm's incentive to invest in enhancing the match quality with any one of its complementors becomes less intense as there is less investment in any of the systems in which the firm participates. This incentive is also weakened as there is more investment in any of the systems in which it does not participate.

Remark 1 will be heavily used in what follows, together with the following one that applies to the cases in which two complementors collaborate with each other:

Remark 2 *For* $i, j = 1, 2$, $\dfrac{\partial^2(\pi^*_{X_i} + \pi^*_{Y_{3-j}})}{\partial k^j_i \partial e^i_j} > 0$, $\dfrac{\partial^2(\pi^*_{X_i} + \pi^*_{Y_{3-j}})}{\partial k^j_i \partial k^{3-j}_{3-i}} = \dfrac{\partial^2(\pi^*_{X_i} + \pi^*_{Y_{3-j}})}{\partial k^j_i \partial e^{3-i}_{3-j}} < 0$ *and*

$$\frac{\partial^2(\pi^*_{X_i} + \pi^*_{Y_{3-j}})}{\partial k^j_i \partial k^{3-j}_i} = \frac{\partial^2(\pi^*_{X_i} + \pi^*_{Y_{3-j}})}{\partial k^j_i \partial e^i_{3-j}} = \frac{\partial^2(\pi^*_{X_i} + \pi^*_{Y_{3-j}})}{\partial k^j_i \partial k^j_{3-i}} = \frac{\partial^2(\pi^*_{X_i} + \pi^*_{Y_{3-j}})}{\partial k^j_i \partial e^{3-i}_j} = 0,$$

whereas $\frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial k_i^{3-j}\partial e^i_{3-j}} > 0,\ \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial k_i^{3-j}\partial k^j_{3-i}} = \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial k_i^{3-j}\partial e_j^{3-i}} < 0$ *and*

$$\frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial k_i^{3-j}\partial k_i^j} = \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial k_i^{3-j}\partial e_j^i} = \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial k_i^{3-j}\partial k_{3-i}^{3-j}} = \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial k_i^{3-j}\partial e_{3-j}^{3-i}} = 0.$$

Similarly, $\frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial e_{3-j}^{3-i}\partial k_{3-i}^{3-j}} > 0,\ \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial e_{3-j}^{3-i}\partial e_j^i} = \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial e_{3-j}^{3-i}\partial k_i^j} < 0$ *and*

$$\frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial e_{3-j}^{3-i}\partial e_{3-j}^i} = \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial e_{3-j}^{3-i}\partial k_i^{3-j}} = \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial e_{3-j}^{3-i}\partial e_j^{3-i}} = \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial e_{3-j}^{3-i}\partial k_{3-i}^j} = 0,$$

whereas $\frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial e_{3-j}^i\partial k_i^{3-j}} > 0,\ \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial e_{3-j}^i\partial e_j^{3-i}} = \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial e_{3-j}^i\partial k_{3-i}^j} < 0$ *and*

$$\frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial e_{3-j}^i\partial e_{3-j}^{3-i}} = \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial e_{3-j}^i\partial k_{3-i}^{3-j}} = \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial e_{3-j}^i\partial e_j^i} = \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_{3-j}})}{\partial e_{3-j}^i\partial k_i^j} = 0.$$

We now have that a firm's incentive to invest in enhancing the match quality with any one of its complementors becomes less intense as there is less investment in the system in which both firms participate. This incentive is also weakened as there is more investment in the system in which neither of them participates.

We also have that the following holds when a firm collaborates with both of its complementors:

Remark 3 *For* $i, j = 1, 2$,

$$\frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_j}+\pi^*_{Y_{3-j}})}{\partial k_i^j\partial e_j^i} > \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_j}+\pi^*_{Y_{3-j}})}{\partial k_i^j\partial k_{3-i}^j} = \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_j}+\pi^*_{Y_{3-j}})}{\partial k_i^j\partial e_j^{3-i}} > 0,$$

and

$$\frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_j}+\pi^*_{Y_{3-j}})}{\partial k_i^j\partial k_{3-i}^{3-j}} = \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_j}+\pi^*_{Y_{3-j}})}{\partial k_i^j\partial e_{3-j}^{3-i}} < \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_j}+\pi^*_{Y_{3-j}})}{\partial k_i^j\partial k_i^{3-j}} = \frac{\partial^2(\pi^*_{X_i}+\pi^*_{Y_j}+\pi^*_{Y_{3-j}})}{\partial k_i^j\partial e_{3-j}^i} < 0.$$

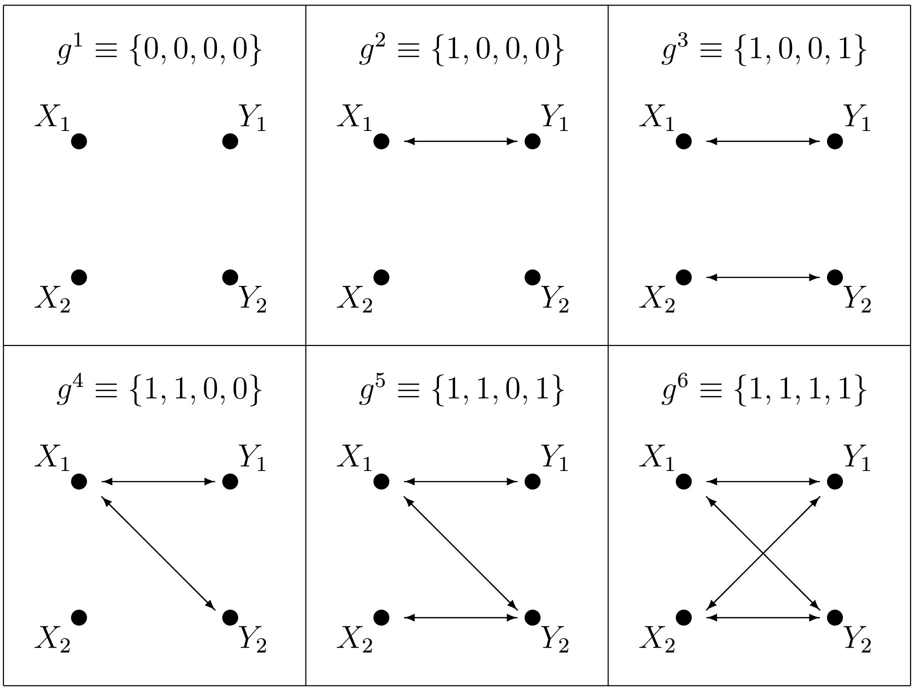
Intuitively, we now have that firm X_i's incentive to invest in enhancing the match quality with Y_j becomes less intense as there is less investment in the systems in which firm Y_j participates (especially in system X_iY_j). This incentive is also weakened as there is more investment in the systems in which Y_j does not participate (especially in system $X_{3-i}Y_{3-j}$).

To build intuition for the subsequent analysis, note that, in principle, collaboration between two complementors has two direct effects. On the one hand, collaborators internalize the positive externality that each imposes on the other, and as a result each invests more in enhancing the quality of the match with the other. We call this the "forget-free-riding" effect. On the other hand, collaboration between two complementors induces each to invest less in enhancing the quality of the match with its complementor's rival. We call this the "harm-my-competitor" effect of collaboration. In the light of Remarks 1, 2 and 3, the "forget-free-riding" and "harm-my-competitor" effects arising from a collaborative link also have several strategic effects whose nature and extent depends on the existing collaboration network. What follows is an analysis of the interaction of these multiple effects for diverse collaborative structures under the assumption that $\lambda = 1$. When $\lambda \in (0,1)$, the existing effects are softened, but do not disappear, so the insights are qualitatively the same (proof available upon request).

3.2 Second and first stages

We now consider the investment subgames for each of the possible network structures arising from the first stage. Up to a relabeling of firms, there are six network structures that should be considered (see Figure 2): $g^1 \equiv \{0,0,0,0\}$, $g^2 \equiv \{1,0,0,0\}$, $g^3 \equiv \{1,0,0,1\}$, $g^4 \equiv \{1,1,0,0\}$, $g^5 \equiv \{1,1,0,1\}$ and $g^6 \equiv \{1,1,1,1\}$. Besides characterizing equilibrium play for each, we also show which one emerges as the unique (pairwise) stable network, thus effectively providing a complete resolution of the network formation game.

Figure 2 : Network Structures



We start by analyzing simple network structures, which means that no firm has more than one collaboration link (i.e., $g = g^1$, or $g = g^2$, or $g = g^3$). We then analyze more complex network structures (i.e., $g = g^4$, or $g = g^5$, or $g = g^6$). At this point, it is useful to recall our behavioral assumptions, so let us define the following functions to this end:

$$\Pi_{X_i}(k_i^1, k_i^2, k_{3-i}^1, k_{3-i}^2, e_j^1, e_j^2, e_{3-j}^1, e_{3-j}^2) \equiv \pi_{X_i}^* - C_{X_i}(k_i^1, k_i^2)$$

and

$$\Pi_{Y_j}(e_j^1, e_j^2, e_{3-j}^1, e_{3-j}^2, k_i^1, k_i^2, k_{3-i}^1, k_{3-i}^2) \equiv \pi_{Y_j}^* - C_{Y_j}(e_j^1, e_j^2).$$

Given network architecture g, we then have that firm X_i $(i = 1, 2)$ chooses $k_i^1 \geq 0$ and $k_i^2 \geq 0$ to maximize $\Pi_{X_i} + \sum_{j=1}^2 g_{ij}\Pi_{Y_j}$, while firm Y_j $(j = 1, 2)$ chooses $e_j^1 \geq 0$ and $e_j^2 \geq 0$ to maximize $\Pi_{Y_j} + \sum_{i=1}^2 g_{ij}\Pi_{X_i}$, where we have suppressed the arguments of the functions to avoid clutter. We also recall that all second-stage choices are made simultaneously. Lastly, we note that we will avoid equilibrium inexistence by making t large enough.[13]

[13] The inexistence problem is already present in Casadesus-Masanell, Nalebuff and Yoffie (2008), who get around it by introducing vertical differentiation.

3.2.1 Simple network structures

We first consider network $g = g^1 \equiv \{0,0,0,0\}$, so firm X_i ($i = 1,2$) maximizes Π_{X_i}, whereas firm Y_j ($j = 1,2$) maximizes Π_{Y_j}. We assume that $t > 1/54$ to make payoff functions strictly concave. We then have that the unique equilibrium is symmetric, and it is characterized by each firm investing $k_i^j(g^1) = e_j^1(g^1) = 1/12$ $(i,j = 1,2)$ in trying to (unilaterally) improve the match with each complementary component. Equilibrium profits for $g = g^1$ are

$$\Pi^*_{X_i}(g^1) = \Pi^*_{Y_j}(g^1) = \frac{54t-1}{72},$$

which are positive for $t > 1/54$.

Recalling that we have assumed that $\lambda = 1$, we now turn to the case in which there is just one collaboration link, i.e., $g = g^2 \equiv \{1,0,0,0\}$. In this case, firms X_1 and Y_1 maximize joint profits $\Pi_{X_1} + \Pi_{Y_1}$. In turn, firm X_2 maximizes Π_{X_2}, whereas firm Y_2 maximizes Π_{Y_2}. If one makes the assumption for $g = g^2$ that $t > 4/54$ to ensure that payoff functions are strictly concave and investment levels are positive, it holds that the unique equilibrium is characterized by the following investments in match quality: $k_1^1(g^2) = e_1^1(g^2) = \dfrac{27t-1}{9(18t-1)}$, $k_1^2(g^2) = e_1^2(g^2) = 0$ and $k_2^1(g^2) = k_2^2(g^2) = e_2^1(g^2) = e_2^2(g^2) = \dfrac{27t-2}{18(18t-1)}$. Equilibrium profits for $g = g^2$ are then

$$\Pi^*_{X_1}(g^2) = \Pi^*_{Y_1}(g^2) = \frac{(27t-1)^3}{81(18t-1)^2}$$

and

$$\Pi^*_{X_2}(g^2) = \Pi^*_{Y_2}(g^2) = \frac{(27t-2)^2(54t-1)}{162(18t-1)^2}.$$

All profits are positive for $t > 2/54$, so our assumption that $t > 4/54$ suffices for making profits positive. It can also be shown that it is sufficient for making equilibrium quantities of each system positive, as required by the full market coverage assumption we have made. We are now in a position to prove the following result.

Lemma 1 *Network $g = g^1$ cannot arise in equilibrium for $t > 4/54$.*

Proof. Noting that $g^2 = g^1 + g_{11}$, it holds that $\Pi^*_{X_1}(g^2) = \Pi^*_{Y_1}(g^2) > \Pi^*_{Y_1}(g^1) = \Pi^*_{X_1}(g^1)$ for $t > 4/54$. For small enough $\varepsilon > 0$, it then follows that $\Pi^*_{X_1}(g^2) - \varepsilon \geq \Pi^*_{X_1}(g^1)$ does not imply that $\Pi^*_{Y_1}(g^2) - \varepsilon < \Pi^*_{Y_1}(g^1)$, and hence $g = g^1$ cannot be a stable network. ■

The result follows because both firms X_1 and Y_1 would benefit from forming a collaboration tie if the network were $g = g^1$. To understand why this happens, note that, relative to the case in which $g = g^1$, there arise several incentives for firms X_1 and Y_1 if

$g = g^2$. The harm-my-competitor effect leads them to decrease k_1^2 and e_1^2.[14] In addition, the forget-free-riding effect leads to higher k_1^1 and e_1^1. Overall, both k_1^1 and e_1^1 end up increasing relative to the case in which $g = g^1$, whereas k_1^2 and e_1^2 are both reduced as much as possible. By Remark 1, the lower k_1^2 has a positive marginal impact on firm X_2's payoff, whereas the lower e_1^2 has a negative impact on firm X_2's marginal payoff. Taking into account that both of these effects cancel out and that system X_1Y_1 is stronger, it follows from Remark 1 that firm X_2 prefers to lower k_2^1. The lower $k_2^1 + e_1^2$ implies that X_2Y_1 is weakened, which coupled with the fact that X_1Y_1 is stronger, induces firm X_2 to reduce k_2^2. Firm X_2 equally benefits from investing in the match with Y_1 or Y_2, so we must have that both k_2^1 and k_2^2 are reduced by the same amount because the strict convexity of R&D costs implies that it is more efficient to spread effort over two complementors rather than just one. Analogous incentives for firm Y_2 imply that both e_2^1 and e_2^2 are lowered by the same amount.

In short, the result that X_1 and Y_1 can mutually benefit from forming a link with each other is largely driven by the forget-free-riding and harm-my-competitor effects, as well as the positive strategic effects that these direct effects have on noncollaborating firms.

We conclude this subsection by analyzing what happens if each firm has one, and only one, collaboration link, i.e., $g = g^3 \equiv \{1, 0, 0, 1\}$. Then firms X_i and Y_j $(i, j = 1, 2; i = j)$ maximize their joint payoff, $\Pi_{X_i} + \Pi_{Y_j}$. Under the assumption that $t > 1/54$, all payoff functions are strictly concave, and the unique equilibrium is symmetric, and it is characterized by the following investments in match quality: $k_1^1(g^3) = k_2^2(g^3) = e_1^1(g^3) = e_2^2(g^3) = 1/6$ and $k_1^2(g^3) = k_2^1(g^3) = e_1^2(g^3) = e_2^1(g^3) = 0$. Equilibrium profits for $g = g^3$ are

$$\Pi^*_{X_i}(g^3) = \Pi^*_{Y_j}(g^3) = \frac{27t - 1}{36},$$

which are positive for $t > 2/54$. However, we make the somewhat stronger condition that $t > 12/54$ in order to make quantity sold of each system positive in equilibrium. We then have all the elements to rule out $g = g^2$ as an equilibrium outcome.

Lemma 2 *Network $g = g^2$ cannot arise in equilibrium for $t > 12/54$.*

Proof. Noting that $g^3 = g^2 + g_{22}$, it holds that $\Pi^*_{X_2}(g^3) = \Pi^*_{Y_2}(g^3) > \Pi^*_{Y_2}(g^2) = \Pi^*_{X_2}(g^2)$ for $t > 12/54$. For small enough $\varepsilon > 0$, it then follows that $\Pi^*_{X_2}(g^3) - \varepsilon \geq \Pi^*_{X_2}(g^2)$ does not imply that $\Pi^*_{Y_2}(g^3) - \varepsilon < \Pi^*_{Y_2}(g^2)$, and hence $g = g^2$ cannot be a stable network. ■

Starting from $g = g^2$, let us consider the incentive for firms X_2 and Y_2 to form a tie, an incentive that is somewhat similar to the one that firms X_1 and Y_1 to form a link

[14] Notice by Remark 1 that the decrease in k_1^2 creates an incentive to decrease k_1^1, whereas the lower e_1^2 stimulates higher k_1^1. Because k_1^2 and e_1^2 have the same impact on firm X_1's marginal payoff, both effects offset each other, and hence there is no incentive for X_1 to change k_1^1. A similar argument explains why firm Y_1 has no incentive to change e_1^1.

starting from $g = g^1$. The harm-my-competitor leads them to decrease k_2^1 and e_2^1, whereas the forget-free-riding effect leads to higher k_2^2 and e_2^2. Overall, both k_2^2 and e_2^2 end up increasing relative to the case in which $g = g^2$, whereas k_2^1 and e_2^1 are both reduced as much as possible. We now turn to how firms X_1 and Y_1 react. In choosing k_1^1, firm X_1 does not care about system X_2Y_1 because of its collaboration with firm Y_1 (by Remark 2), so the fact that system X_2Y_2 becomes stronger induces it to lower k_1^1 relative to the case in which $g = g^2$. In addition, the fact that system X_1Y_2 is weakened creates a pressure towards lowering k_1^2, but there arises a tension to increase k_1^2 as X_2Y_1 becomes weaker (by Remark 2). Because both effects cancel out, k_1^2 remains at the same level as when $g = g^2$. A closely similar analysis explains why firm Y_1 reduces e_1^1 and leaves e_1^2 unchanged.

In short, g^2 is not a stable network because firms X_2 and Y_2 would mutually benefit from forming a link. This incentive to form a link arises because of the forget-free-riding and harm-my-competitor effects, as well as the the positive strategic effect of their collaboration. Thus, firms X_1 and Y_1 reduce their investment in each other, and leave unchanged the investments in enhancing the match with the complementors with which they do not collaborate.

3.2.2 Complex network structures

We now deal with network structures in which at least one firm has more than one collaboration link. We start by analyzing $g = g^4 \equiv \{1, 1, 0, 0\}$. Firm X_1 maximizes $\Pi_{X_1} + \Pi_{Y_1} + \Pi_{Y_2}$, whereas firm X_2 maximizes Π_{X_2}. In turn, firm Y_j ($j = 1, 2$) maximizes $\Pi_{X_1} + \Pi_{Y_j}$.

Given that we focus throughout on large enough t, the unique equilibrium in this case happens to be noninterior.[15] In particular, we assume that $t > 5/54$ to ensure that payoffs are strictly concave as well as the non-negativity of equilibrium profits, investment levels and quantities sold. Then we have that $k_1^1(g^4) = k_1^2(g^4) = \dfrac{54t - 1}{36(18t - 1)}$, $k_2^1(g^4) = k_2^2(g^4) = \dfrac{54t - 5}{36(18t - 1)}$, $e_1^2(g^4) = e_2^2(g^4) = 0$ and $e_1^1(g^4) = e_2^1(g^4) = \dfrac{27t - 1}{9(18t - 1)}$. As for equilibrium profits for $g = g^4$, they are

$$\Pi^*_{X_1}(g^4) = \frac{(54t - 1)^3}{648(18t - 1)^2},$$

$$\Pi^*_{X_2}(g^4) = \frac{(54t - 5)^2(54t - 1)}{648(18t - 1)^2}$$

[15] It can be shown when $g = g^4$ that there exist smaller values of t for which a noninterior equilibrium exists (proof available on request). However, the equilibrium payoffs cannot be compared with those of some network structures for which an equilibrium does not exist if t is not small enough. This is the reason why we always let t be large enough for our purposes of comparison among all the possible network structures.

and

$$\Pi^*_{Y_j}(g^4) = \frac{78732t^3 - 11664t^2 + 459t - 4}{324(18t-1)^2},\ j = 1, 2.$$

To get some understanding of what is going on, let us take the case in which $g = g^2$ as a benchmark of the current situation. Relative to $g = g^2$, k_1^2 and e_2^1 increase (owing to the forget-free-riding effect), which leads to slightly higher e_2^2 by Remark 2. In addition, relative to $g = g^2$, k_1^1 and e_2^2 decrease because of the harm-my-competitor effect, an effect that also creates some pressure towards slightly lower e_2^1 in the light of Remark 2. So, overall, we have that k_1^2 and e_2^1 increase and k_1^1 and e_2^2 decrease relative to the case in which $g = g^2$.

After analyzing how collaboration affects those directly involved, we now turn to analyzing the competitive reaction when going from network $g = g^2$ to $g = g^4$. By Remark 1, the lower k_1^1 has a positive marginal impact on firm X_2's payoff, whereas the lower e_2^2 has a negative impact on firm X_2's marginal payoff. Both of these effects cancel out, so the fact that system X_1Y_2 is stronger implies by Remark 1 that firm X_2 prefers to lower k_2^2. The decrease in $k_2^2 + e_2^2$, together with the stronger X_1Y_2, induces firm X_2 to reduce k_2^1 too (as per Remark 1). The change in firm Y_1's behavior is more complex, though. Firm Y_1 does not care about system X_1Y_2 becoming stronger because it collaborates with firm X_1. However, the lower k_1^1 creates a tension to decrease e_1^1, which is exacerbated after taking into account that firm Y_1 cares about firm X_1's profit. The lower e_2^2 creates pressure to increase e_1^1, though, a pressure accentuated because Y_1 cares about firm X_1's profit. Because the tensions to increase and decrease e_1^1 offset each other, e_1^1 ends up not changing relative to $g = g^2$. Furthermore, the fact that firm Y_1 cares about firm X_1's profit implies that it does not care about systems X_1Y_1 and X_2Y_2 becoming weaker or stronger when it comes to choosing e_1^2. Because system X_1Y_2 is stronger, there arises a tension to lower e_1^2 relative to the case in which $g = g^2$. As a result, e_1^2 is kept at its minimum possible level, namely zero.

We analyze now the cases in which $g = g^5 \equiv \{1, 1, 0, 1\}$. Firm X_1 maximizes $\Pi_{X_1} + \Pi_{Y_1} + \Pi_{Y_2}$, whereas firm X_2 maximizes $\Pi_{X_2} + \Pi_{Y_2}$. In turn, firm Y_1 maximizes $\Pi_{X_1} + \Pi_{Y_1}$, whereas firm Y_2 maximizes $\Pi_{X_1} + \Pi_{X_2} + \Pi_{Y_2}$. In this case, the unique equilibrium that exists with all firms active is noninterior, and it requires that t be sufficiently large.[16] More precisely, we let $t > 6/54$, which guarantees that payoffs are strictly concave and that equilibrium profits, investment levels and quantities sold are all non-negative. Solving for an equilibrium then yields $k_1^1(g^5) = e_2^2(g^5) = \frac{27t-2}{18(18t-1)}$, $k_1^2(g^5) = e_2^1(g^5) = \frac{3t}{2(18t-1)}$, $k_2^1(g^5) = e_1^2(g^5) = 0$ and $k_2^2(g^5) = e_1^1(g^5) = \frac{1}{6}$. As for profits in equilibrium for $g = g^5$,

[16]Equilibrium inexistence for t that is not large enough also happens when $g = g^6$. The proof is available upon request.

they are

$$\Pi^*_{X_1}(g^5) = \Pi^*_{Y_2}(g^5) = \frac{39366t^3 - 3645t^2 + 108t - 2}{162(18t-1)^2}$$

and

$$\Pi^*_{X_2}(g^5) = \Pi^*_{Y_1}(g^5) = \frac{3(8748t^3 - 1620t^2 + 84t - 1)}{(18t-1)^2}.$$

We then have enough elements to discard $g = g^3$ as an equilibrium outcome.

Lemma 3 *Network $g = g^3$ cannot arise in equilibrium for $t > 12/54$.*

Proof. Noting that $g^5 = g^3 + g_{12}$, it holds that $\Pi^*_{X_1}(g^5) = \Pi^*_{Y_2}(g^5) > \Pi^*_{Y_2}(g^3) = \Pi^*_{X_1}(g^3)$ for $t > 12/54$. For small enough $\varepsilon > 0$, it then follows that $\Pi^*_{X_1}(g^5) - 2\varepsilon \geq \Pi^*_{X_1}(g^3) - \varepsilon$ does not imply that $\Pi^*_{Y_2}(g^5) - 2\varepsilon < \Pi^*_{Y_2}(g^3) - \varepsilon$, and hence $g = g^3$ cannot be a stable network. ■

We now provide an intuitive explanation so as to understand what changes when firms X_1 and Y_2 form an extra link in $g = g^3$ in order to give rise to network configuration $g = g^5$. As usual, the forget-free-riding effect increases k_1^2 and e_2^1, but no incentive to vary k_1^1 or e_2^2 arises as a result of these changes.[17] In turn, the harm-my-competitor effect decreases k_1^1 and e_2^2, and again there arises no incentive to vary k_1^2 or e_2^1. Consequently, k_1^2 and e_2^1 are higher and k_1^1 and e_2^2 are lower under $g = g^3$ than under $g = g^5$. The lower k_1^1 induces firm Y_1 to decrease e_1^1, but the fact that system X_2Y_2 is weaker induces it to increase e_1^1.[18] Because both effects cancel out, e_1^1 finally does not change relative to $g = g^3$. In addition, the fact that system X_1Y_2 is stronger creates a tension to lower e_1^2 below its level under $g = g^3$, namely zero, so firm Y_1 chooses not to vary e_1^2 either. Using a similar argument for firm X_2 explains why k_2^1 and k_2^2 remain at the same level as in the case in which $g = g^3$.

In short, adding a link between X_1 and Y_2 in network $g = g^3$ has no strategic effect, so the result is all driven by the positive direct effects of collaboration.

Not only can $g = g^3$ be discarded as an equilibrium outcome, but also $g = g^4$ can be ruled out.

Lemma 4 *Network $g = g^4$ cannot arise in equilibrium for $t > 6/54$.*

Proof. Noting that $g^5 = g^4 + g_{22}$, it holds that $\Pi^*_{X_2}(g^5) > \Pi^*_{X_2}(g^4)$ and $\Pi^*_{Y_2}(g^5) > \Pi^*_{Y_2}(g^4)$ for $t > 6/54$. For small enough $\varepsilon > 0$, it then follows that $\Pi^*_{X_2}(g^5) - \varepsilon \geq \Pi^*_{X_2}(g^4)$

[17] Note by Remark 2 that $\dfrac{\partial^2(\pi^*_{X_1} + \pi^*_{Y_1})}{\partial k_1^1 \partial k_1^2} = \dfrac{\partial^2(\pi^*_{X_1} + \pi^*_{Y_1})}{\partial k_1^1 \partial e_2^1} = 0$ and $\dfrac{\partial^2(\pi^*_{Y_2} + \pi^*_{X_1})}{\partial e_2^2 \partial e_2^1} = \dfrac{\partial^2(\pi^*_{Y_2} + \pi^*_{X_1})}{\partial e_2^2 \partial k_1^2} = 0$.

[18] Note that the fact that X_1Y_2 is weaker is not viewed as something that stimulates higher e_1^1 because firm Y_1 also cares about X_1's profit, and firm X_1 would like to lower e_1^1 if X_1Y_2 became weaker for some reason.

does not imply that $\Pi^*_{Y_2}(g^5) - 2\varepsilon < \Pi^*_{Y_2}(g^4) - \varepsilon$, and hence $g = g^4$ cannot be a stable network. ■

We now explain why firms X_2 and Y_2 have an incentive to form a link in $g = g^4$ in order to give rise to network $g = g^5$. On the one hand, the forget-free-riding effect leads to higher k_2^2 and e_2^2, which creates a pressure towards slightly increasing k_2^1 even though e_2^1 remains invariant. On the other hand, the harm-my-competitor effect leads to lower k_2^1 and e_2^1, which results in slightly lower k_2^2 even though e_2^2 remains invariant. Overall, k_2^2 and e_2^2 increase, whereas k_2^1 and e_2^1 decrease. In the light of Remark 3, the facts that system X_2Y_2 becomes stronger and X_2Y_1 becomes weaker induce firm X_1 to lower k_1^1 relative to $g = g^4$ (despite the reduction in e_2^1 weakens system X_1Y_2, which creates an incentive to increase k_1^1). In addition, firm X_1 increases k_1^2 because of the stronger X_2Y_2 and weaker X_2Y_1, despite the reduction in e_2^1 creates some incentive to lower lower k_1^2. In turn, firm Y_1 has an incentive to lower e_1^1 because of the reduction in k_1^1 and the stronger system X_2Y_2, as stems from Remark 2. However, firm Y_1 does not care about system X_2Y_2 when choosing how to vary e_1^2, but the lower k_1^2 and e_2^1 induce it to reduce e_1^2, as does the lower k_2^1. Because e_1^2 cannot be diminished below zero, it remains unchanged at zero.

In short, the forget-free-riding and harm-my-competitor effects create an incentive for firms X_2 and Y_2 to form a new link starting from network $g = g^4$. In addition, their respective competitors react by weakening system X_1Y_1 and strengthening X_1Y_2, so the strategic effect is positive for firm X_2 and has an ambiguous sign for firm Y_2 (recall that Y_2 cares about X_1's profit). Overall, however, firms X_2 and Y_2 mutually benefit from forming a link, and hence $g = g^4$ cannot be stable.

We deal now with the final network that needs to be considered, namely $g = g^6 \equiv \{1, 1, 1, 1\}$. Firm X_i $(i = 1, 2)$ maximizes $\Pi_{X_i} + \Pi_{Y_1} + \Pi_{Y_2}$, whereas firm Y_j maximizes $\Pi_{X_1} + \Pi_{X_2} + \Pi_{Y_j}$. Under the assumption that $t > 1/54$ (which ensures payoff concavity and non-negativity of the relevant variables), the unique equilibrium is symmetric and involves the following investment levels: $k_i^j(g^6) = e_j^i(g^6) = 1/12$ $(i, j = 1, 2)$. Equilibrium profits for $g = g^6$ are

$$\Pi^*_{X_i}(g^6) = \Pi^*_{Y_j}(g^6) = \frac{54t - 1}{72},$$

which are positive for $t > 1/54$. Using these profits, we can rule out $g = g^5$ as an equilibrium network.

Lemma 5 *Network $g = g^5$ cannot arise in equilibrium for $t > 6/54$.*

Proof. Noting that $g^6 = g^5 + g_{21}$, it holds that $\Pi^*_{X_2}(g^6) > \Pi^*_{X_2}(g^5)$ and $\Pi^*_{Y_1}(g^6) > \Pi^*_{Y_1}(g^5)$ for $t > 6/54$. For small enough $\varepsilon > 0$, it then follows that $\Pi^*_{X_2}(g^6) - 2\varepsilon \geq \Pi^*_{X_2}(g^5) - \varepsilon$ does not imply that $\Pi^*_{Y_1}(g^6) - 2\varepsilon < \Pi^*_{Y_1}(g^5) - \varepsilon$, and hence $g = g^5$ cannot be a stable network. ■

We provide the intuition for why firms X_2 and Y_1 would like to be linked to each other starting from $g = g^5$. On the one hand, the forget-free-riding effect results in higher k_2^1 and e_1^2, and no tension arises to vary either k_2^2 or e_1^1. On the other, the harm-my-competitor effect results in lower k_2^2 and e_1^1, and no tension arises to vary either k_2^1 or e_1^2. Hence, we have that k_2^1 and e_1^2 increase relative to $g = g^5$, but k_2^2 and e_1^1 decrease. Based on Remark 3, the facts that system X_2Y_1 becomes stronger and X_2Y_2 becomes weaker induce firm X_1 to increase k_1^1 relative to $g = g^5$ (despite the reduction in e_1^1 weakens system X_1Y_1, which creates an incentive to lower k_1^1). In addition, firm X_1 lowers k_1^2 because of the stronger X_2Y_1 and weaker X_2Y_2, despite the reduction in e_1^1 creates some incentive to raise k_1^2. One can similarly explain why e_2^1 is reduced, but e_2^2 turns out to increase.

The fact that $\Pi_{X_i}^*(g^6) - 2\varepsilon > \Pi_{X_i}^*(g^5) - \varepsilon$ for $i = 1, 2$ and $\Pi_{Y_j}^*(g^6) - 2\varepsilon > \Pi_{Y_j}^*(g^5) - \varepsilon$ for $j = 1, 2$ implies that $g = g^6$ is an equilibrium network for $t > 6/54$. This result, together with all the above Lemmata, leads to our first result.

Proposition 1 *For $t > 12/54$, the unique equilibrium network structure is the complete network, namely $g^* = \{1, 1, 1, 1\}$. In equilibrium, firm X_i chooses to invest $k_i^j(g^*) = 1/12$ in improving the quality of its match with complementor Y_j, whereas firm Y_j chooses to invest $e_j^i(g^*) = 1/12$ in improving the quality of its match with complementor X_i $(i, j = 1, 2)$. Each firm earns a payoff of $(54t - 1)/72 - 2\varepsilon$.*

We recall at this point that we have used the notion of pairwise stability as our solution concept for the strategic network formation game we consider. In the context of our game, the main drawback of this solution concept has to do with the possibility that a firm with several links may want to sever more than one link at a time.[19] However, this criticism does not apply to the game under consideration. Indeed, the fact that $\Pi_{X_2}^*(g^6) - 2\varepsilon > \Pi_{X_2}^*(g^4)$ for small $\varepsilon > 0$ implies that the complete network is stable even if the pairwise stability solution concept is augmented to allow for the deletion of several links at a time (the complete network is then said to be pairwise Nash stable).

We study from now on the efficiency properties of the complete network and the competitive play it implies. Taking into account that forming a collaboration link costs $\varepsilon > 0$ and $k_i^j(g) = e_j^i(g) = 1/12$ $(i, j = 1, 2)$ for both $g = g^1$ and $g = g^*$, we have just proved the following result about the desirability of the equilibrium outcome.

Proposition 2 *For $t > 12/54$, the unique equilibrium network $g^* = \{1, 1, 1, 1\}$ is socially suboptimal and results in a payoff for each firm smaller than that achieved when $g = \{0, 0, 0, 0\}$.*

Hence, firms (and society) would do better if they could commit not to collaborate, since collaboration is costly and it cannot improve upon the case in which each firm acts

[19] See Jackson (2008, pp. 156 and 371-376) for a thorough discussion of the virtues and limitations of pairwise stability as a solution concept.

uncoordinatedly. This result follows because the positive marginal impact on firm Y_j's profit of an increase in k_i^j is completely offset by the negative marginal impact on firm Y_{3-j}'s profit. So the fact that both of firm X_i's collaborators compete with each other implies that firm X_i's choice of k_i^j does not depend on how such choice affects both of its collaborators. In consequence, firm X_i behaves in the same way as when it does not collaborate with either of them. An analogous reasoning explains why firm Y_j's choice of e_j^i is the same both for $g = g^1$ and for $g = g^*$. The existence of opportunity costs of forming collaboration links then finally provides the explanation why firms and society would be better off without collaboration.

4 Exclusive collaboration

We now study what happens if firms can only form exclusive collaboration links. This may be due to explicit or implicit contracting requirements, or to highly competitive conditions that preclude several complementors from being willing to collaborate with the same firm. In the light of our previous analysis, it is clear that the stable network that arises when collaboration is exclusive is $g = g^3$. Under $g = g^3$, there is no investment in improving the match with the complementor with which a firm does not collaborate. As we showed earlier, this is not an outcome that arises owing to the exclusivity requirement, a noteworthy feature of the outcome when $g = g^3$ that fits quite well with such a requirement (although it does depend on λ equaling 1). In addition, the facts that $\sum_{j=1}^2 k_i^j(g^3) = \sum_{j=1}^2 k_i^j(g^1)$ and $\sum_{i=1}^2 e_j^i(g^3) = \sum_{i=1}^2 e_j^i(g^1)$ imply that there is as much as total investment by each firm in an equilibrium under exclusivity as when $g = g^1$. However, all investment is now concentrated on the complementor with which a firm collaborates, which is worse from the viewpoint of R&D costs because of their strict convexity. Gross profits in the product market are the same both under $g = g^1$ and $g = g^3$, since $\sum_{j=1}^2 k_i^j(g^3) = \sum_{j=1}^2 k_i^j(g^1)$ and $\sum_{i=1}^2 e_j^i(g^3) = \sum_{i=1}^2 e_j^i(g^1)$. As a result, it follows that $\Pi_{X_i}^*(g^3) = \Pi_{Y_j}^*(g^3) < \Pi_{X_i}^*(g^1) = \Pi_{Y_j}^*(g^1)$ for $i, j = 1, 2$, and each firm would be better off if collaboration was forbidden or impossible.

Not only would firms be better off by forbidding collaboration, but also consumers as a whole would get a higher surplus. Thus, the price of a component is the same under $g = g^1$ and $g = g^3$, and the extra gross utility attained by some consumers is exactly offset by the lower gross utility attained by the others. (This happens because, on the aggregate, consumers just care about total investment, which is the same under $g = g^1$ and $g = g^3$.) However, transportation costs increase for consumers as a whole under $g = g^3$ because some systems are less appealing in their vertical attributes, and hence are bought less than under $g = g^1$. In other words, the vertical differences among some systems that arise under $g = g^3$ steal consumption away from systems that are preferred from a horizontal standpoint, thus generating some disutility that does not arise under

$g = g^1$. As a result, consumer welfare is greater under $g = g^1$ than $g = g^3$, as are (net) profits made by firms. We then have the following result.

Proposition 3 *Let* $t > 12/54$ *and suppose that collaborating with a complementor precludes a firm from collaborating with the complementor's competitor. Then the unique (up to a relabeling of firms) equilibrium network* $g^{**} = \{1, 0, 0, 1\}$ *is socially suboptimal and results in a payoff for each firm smaller than that achieved when* $g = \{0, 0, 0, 0\}$.

Proof. Since forming a link costs $\varepsilon > 0$ and $\Pi^*_{X_i}(g^3) = \Pi^*_{Y_j}(g^3) < \Pi^*_{X_i}(g^1) = \Pi^*_{Y_j}(g^1)$ for $i, j = 1, 2$, it suffices to show that consumer welfare under $g = g^3$ is smaller than under $g = g^1$. Both for $g = g^1$ and $g = g^3$, it holds that $p^*_{X_1} = p^*_{X_2} = p^*_{Y_1} = p^*_{Y_2} = 3t/2$, so $p^*_{11} = p^*_{12} = p^*_{21} = p^*_{21} = 3t$. In addition, the number of consumers purchasing system X_iY_j $(i, j = 1, 2)$ under $g = g^1$ is $Q^*_{ij}(g^1) = 1/4$. However, the number of consumers purchasing systems X_1Y_1 and X_2Y_2 under $g = g^3$ is $Q^*_{11}(g^3) = Q^*_{22}(g^3) = 1/4 + 1/(18t)$, whereas the number of consumers purchasing systems X_1Y_2 and X_2Y_1 under $g = g^3$ is $Q^*_{12}(g^3) = Q^*_{21}(g^3) = 1/4 - 1/(18t)$. Taking into account that line $l_{X_iY_j}$ $(i, j = 1, 2)$ has a length of $1/2$ and that there exists a unit mass of consumers uniformly spread all over the four existing lines, the aggregate consumer surplus under $g = g^1$ is

$$CS(g^1) = 4\left[\frac{1}{2}\left(v + \frac{1}{12} + \frac{1}{12} - 3t - t\int_0^{1/2} zdz\right)\right],$$

while the aggregate consumer surplus under $g = g^3$ is

$$CS(g^3) = 2\left[\frac{1}{2}\left(v + \frac{1}{6} + \frac{1}{6} - 3t - t\int_0^{1/2+1/(9t)} zdz\right)\right] + 2\left[\frac{1}{2}\left(v - 3t - t\int_0^{1/2-1/(9t)} zdz\right)\right].$$

Because $CS(g^1) - CS(g^3) = 1/(81t) > 0$, the desired result follows. ■

5 Extensions

We now assume that collaborating with another firm entails caring about the profit generated for such a firm by the system in which both participate, a feature that is probably more realistic. Thus, if firm X_i collaborates with Y_j, but not with Y_{3-j}, then firm X_i chooses k_i^j and k_i^{3-j} to maximize $\pi^*_{X_i} + \lambda(\pi^j_{X_i} - (e^i_j)^2) - C_{X_i}(k^j_i, k^{3-j}_i)$. In turn, if firm Y_j collaborates with both X_i and X_{3-i}, then firm Y_j chooses e^i_j and e^{3-i}_j to maximize $\pi^*_{Y_j} + \lambda(\pi^i_{Y_j} - (k^i_j)^2) + \lambda(\pi^{3-i}_{Y_j} - (k^{3-i}_j)^2) - C_{Y_j}(e^i_j, e^{3-i}_j)$. As before, we will let $\lambda = 1$ to simplify computations, even though results hold for any $\lambda \in (0, 1]$.

Our main result in this context coincides with the one derived before.

Proposition 4 *For $t > 11/36$, the unique equilibrium network structure is the complete network, namely $g^* = \{1,1,1,1\}$. In equilibrium, firm X_i chooses to invest $k_i^j(g^*) = 1/8$ in improving the quality of its match with complementor Y_j, whereas firm Y_j chooses to invest $e_j^i(g^*) = 1/8$ in improving the quality of its match with complementor X_i ($i,j = 1,2$). Each firm earns a payoff of $(24t-1)/32$, which is smaller than the payoff that each could achieve if all of them could commit not to collaborating.*

Proof. See Appendix. ■

Relative to Proposition 1, the fact that a firm choosing its investment levels does not fully internalize the harm they provoke on its complementors implies that there is more investment and hence more inefficiencies (from the firms' viewpoints). It is worth noting that these results hold even if forming a collaboration link is not costly, so inefficiencies are much severe in this setting.

To conclude, we show that results under exclusivity do not change when a firm that collaborates with a complementor does not care about its entire profit but rather the profit that it makes from the system in which both firms participate.

Proposition 5 *Let $t > 11/36$ and suppose that collaborating with a complementor precludes a firm from collaborating with the complementor's competitor. Then the unique (up to a relabeling of firms) equilibrium network $g^{**} = \{1,0,0,1\}$ is socially suboptimal and results in a payoff for each firm smaller than that achieved when $g = \{0,0,0,0\}$.*

Proof. Both for $g = g^1$ and $g = g^3$, it holds that $p^*_{X_1} = p^*_{X_2} = p^*_{Y_1} = p^*_{Y_2} = 3t/2$, so $p^*_{11} = p^*_{12} = p^*_{21} = p^*_{21} = 3t$. In addition, the number of consumers purchasing system X_iY_j ($i,j = 1,2$) under $g = g^1$ is $Q^*_{ij}(g^1) = 1/4$, so consumer surplus is

$$CS(g^1) = 4\left[\frac{1}{2}\left(v + \frac{1}{12} + \frac{1}{12} - 3t - t\int_0^{1/2} zdz\right)\right].$$

Under network $g = g^3$, we have that $Q^*_{11} = Q^*_{22} = \frac{1}{4} + \frac{5(4-t)}{4(77t-2)}$ and $Q^*_{12} = Q^*_{21} = \frac{1}{4} - \frac{5(4-t)}{4(77t-2)}$. Suppose first that $t \leq 4$, so that $Q^*_{11} = Q^*_{22} \geq 1/4$ and $Q^*_{12} = Q^*_{21} \leq 1/4$. Then consumer surplus is

$$\begin{aligned} CS(g^3) &= 2\left[\frac{1}{2}\left(v + \frac{15t}{72t-2} + \frac{15t}{72t-2} - 3t - t\int_0^{\frac{1}{2}+\frac{5(4-t)}{2(77t-2)}} zdz\right)\right] + \\ &\quad 2\left[\frac{1}{2}\left(v - 3t - t\int_0^{\frac{1}{2}-\frac{5(4-t)}{2(77t-2)}} zdz\right)\right], \end{aligned}$$

so it holds that

$$CS(g^3) - CS(g^1) = \frac{16 + 112t - 31172t^2 + 235119t^3 - 2700t^4}{12(36t-1)(77t-2)^2} > 0.$$

However, joint profits under network g^1 are greater than under g^3, and the difference equals

$$\frac{3t(1296t^2 - 147t + 1)}{(36t-1)^2} - \frac{54t-1}{18} = \frac{1 - 72t - 2754t^2}{18(36t-1)^2}.$$

Because this difference exceeds $CS(g^3) - CS(g^1)$, we have that the equilibrium network under exclusivity is socially suboptimal. When $t > 4$, we have that

$$\begin{aligned} CS'(g^3) &= 2\left[\frac{1}{2}\left(v + \frac{15t}{72t-2} + \frac{15t}{72t-2} - 3t - t\int_0^{\frac{1}{2}-\frac{5(4-t)}{2(77t-2)}} zdz\right)\right] + \\ & \quad 2\left[\frac{1}{2}\left(v - 3t - t\int_0^{\frac{1}{2}+\frac{5(4-t)}{2(77t-2)}} zdz\right)\right], \end{aligned}$$

so the fact that $CS'(g^3) = CS(g^3)$ implies that the above proof goes through directly. ■

6 Conclusion

The locus of strategic interaction in many high-tech industries has broadened from the traditional competitive approach based on value capture towards one in which cooperative aspects with regards to value creation also play a critical role, as Brandenburger and Nalebuff (1996) emphasize. Not surprisingly, such "co-opetitive" settings display rich innovation ecosystems in which complementors collaborate with each other in R&D actitivities. This paper has shown that such rich innovation ecosystems may be an equilibrium phenomenon with disturbing properties for their members. In particular, we have shown that they may be an inefficient outcome for competing firms that can collaborate with complementors. They may also be inefficient for society. These results hold under a variety of scenarios (e.g., regardless of whether or not collaboration exhibits exclusive features).

In this paper, we have abstracted away from dynamics to clarify our points, but there are many issues that have to do with dynamic variables. For example, collaboration may refer to the timing at which complementary products are brought to the market. Exploring this kind of issues seems promising enough to warrant further work on this completely unexplored area.

Appendix

Proof of Proposition 4. Let $g = g^1$ and assume that $t > 1/54$ so that payoff functions are strictly concave and (equilibrium) investment levels, profits and quantities sold of each system are nonnegative. Then we know that $k_1^1(g^1) = 1/12$, $k_1^2(g^1) = 1/12$, $e_1^1(g^1) = 1/12$, $e_1^2(g^1) = 1/12$, $k_2^1(g^1) = 1/12$, $k_2^2(g^1) = 1/12$, $e_2^1(g^1) = 1/12$ and $e_2^2(g^1) = 1/12$. In addition,

$$\Pi^*_{X_i}(g^1) = \Pi^*_{Y_j}(g^1) = \frac{54t-1}{72} \text{ for all } i, j = 1, 2.$$

We now consider $g = g^2$ under the assumption that $t > 6/54$ so that payoff functions are strictly concave and (equilibrium) investment levels, profits and quantities sold of each system are nonnegative. Firm X_1 chooses k_1^1 and k_1^2 to maximize $\pi^*_{X_1} + \pi^1_{X_1} - (e_1^1)^2 - C_{X_1}(k_1^1, k_1^2)$, whereas firm X_2 chooses k_2^1 and k_2^2 to maximize $\pi^*_{X_2} - C_{X_2}(k_2^1, k_2^2)$. In addition, firm Y_1 chooses e_1^1 and e_1^2 to maximize $\pi^*_{Y_1} + \pi^1_{Y_1} - (k_1^1)^2 - C_{Y_1}(e_1^1, e_1^2)$, whereas firm Y_2 chooses e_2^1 and e_2^2 to maximize $\pi^*_{Y_2} - C_{Y_2}(e_2^1, e_2^2)$. Then we have that $k_1^1(g^2) = \dfrac{9t(180t-7)}{2+648t(12t-1)}$, $k_1^2(g^2) = 0$, $e_1^1(g^2) = \dfrac{9t(180t-7)}{2+648t(12t-1)}$, $e_1^2(g^2) = 0$, $k_2^1(g^2) = \dfrac{72t(9t-1)}{2+648t(12t-1)}$, $k_2^2(g^2) = \dfrac{72t(9t-1)}{2+648t(12t-1)}$, $e_2^1(g^2) = \dfrac{72t(9t-1)}{2+648t(12t-1)}$ and $e_2^2(g^2) = \dfrac{27t-1-\lambda}{6(54t-2-\lambda)}$. Also,

$$\Pi^*_{X_1}(g^2) = \frac{3t\{4+243t[8t(67+18t(432t-73))-9]\}}{(2+648t(12t-1))^2},$$

$$\Pi^*_{Y_1}(g^2) = \frac{3t\{4+243t[8t(67+18t(432t-73))-9]\}}{(2+648t(12t-1))^2}$$

$$\Pi^*_{X_2}(g^2) = \frac{10368t^2(54t-1)(9t-1)^2}{(2+648t(12t-1))^2}$$

and

$$\Pi^*_{Y_2}(g^2) = \frac{10368t^2(54t-1)(9t-1)^2}{(2+648t(12t-1))^2}.$$

We now let $g = g^3$ and assume that $t > 11/36$ so that payoff functions are strictly concave and (equilibrium) investment levels, profits and quantities sold of each system are nonnegative, as usual. Firm X_1 chooses k_1^1 and k_1^2 to maximize $\pi^*_{X_1} + \pi^1_{X_1} - (e_1^1)^2 - C_{X_1}(k_1^1, k_1^2)$, whereas firm X_2 chooses k_2^1 and k_2^2 to maximize $\pi^*_{X_2} + \pi^2_{X_2} - (e_2^2)^2 - C_{X_2}(k_2^1, k_2^2)$. In addition, firm Y_1 chooses e_1^1 and e_1^2 to maximize $\pi^*_{Y_1} + \pi^1_{Y_1} - (k_1^1)^2 - C_{Y_1}(e_1^1, e_1^2)$, whereas firm Y_2 chooses e_2^1 and e_2^2 to maximize $\pi^*_{Y_2} + \pi^2_{Y_2} - (k_2^2)^2 - C_{Y_2}(e_2^1, e_2^2)$. Then $k_1^1(g^3) = \dfrac{15t}{72t-2}$, $k_1^2(g^3) = 0$, $e_1^1(g^3) = \dfrac{15t}{72t-2}$, $e_1^2(g^3) = 0$, $k_2^1(g^3) = 0$, $k_2^2(g^3) = \dfrac{15t}{72t-2}$, $e_2^1(g^3) =$

0 and $e_2^2(g^3) = \dfrac{15t}{72t-2}$. In addition, we have that

$$\Pi^*_{X_i}(g^3) = \Pi^*_{Y_j}(g^3) = \frac{3t(1296t^2 - 147t + 1)}{4(36t-1)^2} \text{ for } i, j = 1, 2.$$

We turn now to $g = g^4$ and assume that $t > 13/108$ so that payoff functions are strictly concave and (equilibrium) investment levels, profits and quantities sold of each system are nonnegative. Firm X_1 chooses k_1^1 and k_1^2 to maximize $\pi^*_{X_1} + \pi^1_{X_1} - (e_1^1)^2 + \pi^2_{X_1} - (e_2^1)^2 - C_{X_1}(k_1^1, k_1^2)$, whereas firm X_2 chooses k_2^1 and k_2^2 to maximize $\pi^*_{X_2} - C_{X_2}(k_2^1, k_2^2)$. In addition, firm Y_1 chooses e_1^1 and e_1^2 to maximize $\pi^*_{Y_1} + \pi^1_{Y_1} - (k_1^1)^2 - C_{Y_1}(e_1^1, e_1^2)$, whereas firm Y_2 chooses e_2^1 and e_2^2 to maximize $\pi^*_{Y_2} + \pi^1_{Y_2} - (k_1^2)^2 - C_{Y_2}(e_2^1, e_2^2)$. It then holds that $k_1^1(g^4) = \dfrac{3(36t-1)}{8(108t-7)}$, $k_1^2(g^4) = \dfrac{3(36t-1)}{8(108t-7)}$, $e_1^1(g^4) = \dfrac{540t-17}{24(108t-7)}$, $e_1^2(g^4) = 0$, $k_2^1(g^4) = \dfrac{108t-13}{12(108t-7)}$, $k_2^2(g^4) = \dfrac{108t-13}{12(108t-7)}$, $e_2^1(g^4) = \dfrac{540t-17}{24(108t-7)}$ and $e_2^2(g^4) = 0$. This results in the following profits:

$$\Pi^*_{X_1}(g^4) = \frac{3[16t(14 + 27t(216t-13)) - 3]}{32(108t-7)^2},$$

$$\Pi^*_{X_2}(g^4) = \frac{(54t-1)(108t-13)^2}{72(108t-7)^2}$$

and

$$\Pi^*_{Y_j}(g^4) = \frac{(108t-1)(46656t^2 - 8316t + 289)}{576(108t-7)^2} \text{ for } j = 1, 2.$$

Let $g = g^5$ and assume that $t > (71 + \sqrt{409})/432$ so that payoff functions are strictly concave and (equilibrium) investment levels, profits and quantities sold of each system are nonnegative. In this case, firm X_1 chooses k_1^1 and k_1^2 to maximize $\pi^*_{X_1} + \pi^1_{X_1} - (e_1^1)^2 + \pi^2_{X_1} - (e_2^1)^2 - C_{X_1}(k_1^1, k_1^2)$, whereas firm X_2 chooses k_2^1 and k_2^2 to maximize $\pi^*_{X_2} + \pi^2_{X_2} - (e_2^2)^2 - C_{X_2}(k_2^1, k_2^2)$. In addition, firm Y_1 chooses e_1^1 and e_1^2 to maximize $\pi^*_{Y_1} + \pi^1_{Y_1} - (k_1^1)^2 - C_{Y_1}(e_1^1, e_1^2)$, whereas firm Y_2 chooses e_2^1 and e_2^2 to maximize $\pi^*_{Y_2} + \pi^1_{Y_2} - (k_1^2)^2 + \pi^2_{Y_2} - (k_2^2)^2 - C_{Y_2}(e_2^1, e_2^2)$. it holds that $k_1^1(g^5) = \dfrac{5832t^2 - 675t - 1}{24(1944t^2 - 261t + 4)}$, $k_1^2(g^5) = \dfrac{27t(216t-25)}{24(1944t^2 - 261t + 4)}$, $e_1^1(g^5) = \dfrac{9720t^2 - 1143t + 1}{24(1944t^2 - 261t + 4)}$, $e_1^2(g^5) = 0$, $k_2^1(g^5) = 0$, $k_2^2(g^5) = \dfrac{9720t^2 - 1143t + 1}{24(1944t^2 - 261t + 4)}$, $e_2^1(g^5) = \dfrac{27t(216t-25)}{24(1944t^2 - 261t + 4)}$ and $e_2^2(g^5) = \dfrac{5832t^2 - 675t - 1}{24(1944t^2 - 261t + 4)}$. All this results in the following profits:

$$\Pi^*_{X_1}(g^5) = \Pi^*_{Y_2}(g^5) = \frac{27t\{31 + 54t[24t(871 + 216t(216t-55)) - 341]\} - 1}{576(4 + 9t(216t-29))^2}$$

and

$$\Pi^*_{X_2}(g^5) = \Pi^*_{Y_1}(g^5) = \frac{9t\{5297 + 9t[5184t(196 + 27t(144t - 55)) - 50809]\} - 1}{576(4 + 9t(216t - 29))^2}.$$

Finally, let $g = g^6$ and assume that $t > 5/108$ so that payoff functions are strictly concave and (equilibrium) investment levels, profits and quantities sold of each system are nonnegative. Then $k_1^1(g^6) = 1/8$, $k_1^2(g^6) = 1/8$, $e_1^1(g^6) = 1/8$, $e_1^2(g^6) = 1/8$, $k_2^1(g^6) = 1/8$, $k_2^2(g^6) = 1/8$, $e_2^1(g^6) = 1/8$ and $e_2^2(g^6) = 1/8$. In addition,

$$\Pi^*_{X_i}(g^6) = \Pi^*_{Y_j}(g^6) = \frac{24t - 1}{32} \text{ for all } i, j = 1, 2.$$

We now show that the unique equilibrium network is $g = g^6$ if it holds that $t > 11/36$. Note first that, since $\Pi^*_{X_1}(g^2) = \Pi^*_{Y_1}(g^2) > \Pi^*_{Y_1}(g^1) = \Pi^*_{X_1}(g^1)$, it follows that $\Pi^*_{X_1}(g^2) \geq \Pi^*_{X_1}(g^1)$ does not imply that $\Pi^*_{Y_1}(g^2) < \Pi^*_{Y_1}(g^1)$, and hence $g = g^1$ cannot be a stable network. In addition, the fact that $\Pi^*_{X_2}(g^3) = \Pi^*_{Y_2}(g^3) > \Pi^*_{Y_2}(g^2) = \Pi^*_{X_2}(g^2)$ yields that $\Pi^*_{X_2}(g^3) \geq \Pi^*_{X_2}(g^2)$ does not imply that $\Pi^*_{Y_2}(g^3) < \Pi^*_{Y_2}(g^2)$, and hence $g = g^2$ cannot be a stable network either. Because $\Pi^*_{X_1}(g^5) = \Pi^*_{Y_2}(g^5) > \Pi^*_{Y_2}(g^3) = \Pi^*_{X_1}(g^3)$, it also follows that $\Pi^*_{X_1}(g^5) \geq \Pi^*_{X_1}(g^3)$ does not imply that $\Pi^*_{Y_2}(g^5) < \Pi^*_{Y_2}(g^3)$, which shows that $g = g^3$ cannot be a stable network. To show that the same applies to network $g = g^4$, note that it holds that $\Pi^*_{X_2}(g^5) > \Pi^*_{X_2}(g^4)$ and $\Pi^*_{Y_2}(g^5) > \Pi^*_{Y_2}(g^4)$ so it follows that $\Pi^*_{X_2}(g^5) \geq \Pi^*_{X_2}(g^4)$ does not imply that $\Pi^*_{Y_2}(g^5) < \Pi^*_{Y_2}(g^4)$, which shows that $g = g^4$ cannot be a stable network. Because it also holds that $\Pi^*_{X_2}(g^6) > \Pi^*_{X_2}(g^5)$ and $\Pi^*_{Y_1}(g^6) > \Pi^*_{Y_1}(g^5)$, we have that $\Pi^*_{X_2}(g^6) \geq \Pi^*_{X_2}(g^5)$ does not imply that $\Pi^*_{Y_1}(g^6) < \Pi^*_{Y_1}(g^5)$, and hence $g = g^5$ cannot be a stable network. The fact that $\Pi^*_{X_i}(g^6) > \Pi^*_{X_i}(g^5)$ for $i = 1, 2$ and $\Pi^*_{Y_j}(g^6) > \Pi^*_{Y_j}(g^5)$ for $j = 1, 2$ implies that $g = g^6$ is an equilibrium network for $t > 11/36$. ■

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