When Do Markets Tip? A Cognitive Hierarchy Approach*

Tanjim Hossain
University of Toronto

John Morgan
University of California, Berkeley

August 2011

Abstract

The market structure of platform competition is critically important to managers and policy makers. While network effects in these markets predict concentrated industry structures, competitive effects and differentiation suggest the opposite. Standard theory offers little guidance—full rationality leads to multiple equilibria with wildly varying market structures. We relax full rationality in favor of a boundedly rational cognitive hierarchy model. Even small departures from full rationality allow sharp predictions—there is a unique equilibrium in every case. When participants single-home and platforms are vertically differentiated, a single dominant platform emerges. Multi-homing can give rise to a strong-weak market structure: One platform is accessed by all while the other is used as a backup by some agents. Horizontal differentiation, in contrast, leads to fragmentation. Differentiation, rather than competitive effects, mainly determines market structure.

Keywords: Platform competition, tipping, bounded rationality, cognitive hierarchy, vertical and horizontal differentiation

*Tanjim Hossain: Rotman School of Management, University of Toronto (tanjim.hossain@utoronto.ca) and John Morgan: Haas School of Business and Department of Economics, University of California, Berkeley (morgan@haas.berkeley.edu). We thank participants at the 8th Invitational Choice Symposium for useful comments.

1 Introduction

Theorists have long been fascinated by coordination games. Part of this fascination stems from the fact that standard theory offers little guidance as to the outcome—theory predicts that individuals will coordinate but is silent as to what action they will coordinate on. Of course, these limitations are of little practical consequence if one is interested in thought experiments like the famous one proposed by Schelling about strangers trying to meet in New York City. But coordination problems loom large in many high-stakes business settings. Managers and researchers alike stand to benefit from a usable theory that goes beyond the non-predictions of the fully rational framework.

One practical coordination setting of particular importance concerns competition among online platforms, such as Google and Microsoft in search—what we term the competing matchmakers problem. Unlike standard coordination games where players are typically treated symmetrically, the competing matchmakers problem introduces additional complexity owing to the fact that participants may fundamentally differ from one another. For instance, in online auctions, the value of a platform depends not just on how many buyers it attracts, nor how many sellers, but rather on the combination of the two. Moreover, agents of a given type, such as men in an online dating context, care not just about the number of women on the site, but on the number of *other* men also since each additional male represents an additional competitor for a woman's heart. The two-sidedness of the market combined with competitive effects multiply the range of equilibrium possibilities. Indeed, in our baseline model, which nests many of the standard models of platform competition, the main conclusion to be derived from equilibrium under full rationality is that anything can happen: A single platform may dominate the market though the model is silent as to which platform, or the market may be fragmented though again the model is silent as to who gets what share of the market. For empiricists or managers looking to theory as a guide, the full rationality model offers little in the way of help.

But full rationality represents an idealization at best for what motivates the choices of market participants. A growing number of findings in behavioral economics highlight situations where behavior that is inexplicable under full rationality may be readily understood by relaxing this assumption in favor of psychologically plausible alternatives. For instance, behavior in laboratory studies of coordination games is better characterized by models that allow for a range of strategic sophistication among subjects rather than requiring all to be infinitely rational. That is, allowing for hierarchies of cognition leading to differing levels of strategic sophistication provides a much better description of observed outcomes. In these models, non-strategic agents naïvely choose a pre-planned action without analyzing the payoffs. Strategic or sophisticated agents maximize their expected payoff given their beliefs which depend on their cognitive sophistication levels.

We begin with a simple observation: If cognitive hierarchy models are useful in organizing data from coordination games in the lab, perhaps these models might be fruitfully used to offer guidance in more applied settings, such as the study of market shares when there are competing matchmakers. This analysis is the heart of our study.

An important criticism in departing from full rationality is that such models open up a Pandora's box of possibilities where "anything goes" and therefore theory loses much of its predictive power. In our setting, we obtain exactly the opposite conclusion—while nearly any market share outcome is consistent with equilibrium under full rationality, cognitive hierarchy models produce unique equilibrium predictions. In some instances, these predictions coincide with a particular equilibrium under full rationality, in which case our models may be thought of as a kind of behavioral equilibrium refinement. Yet in other settings, the predictions under cognitive hierarchy offer completely novel predictions of behavior. Thus, in addition to offering more precise predictions, behavioral models are, in principle, distinguishable empirically from their fully rational counterparts.

Before proceeding to describe our main findings, a sketch of the model is useful. There are N men and N women choosing between two online dating platforms. Platforms may differ in both the access fees they charge and the efficacy of their matching processes. Both platforms share the common feature that there are benefits from scale—the larger the participant base at a given platform, the better the expected quality of the resulting matches. This effect pushes the market in the direction of concentration. There is, however, a countervailing competitive force. Men may prefer to be on a smaller platform so as to avoid having to compete as intensely with other men for the attention and affections of the women also located on the platform. Likewise for women on the smaller platform. Provided that this competitive force is strong enough, platforms of wildly different sizes can coexist in equilibrium under the fully rational model.

When agents must choose a single platform, as would be the case for a seller of a unique object deciding on which online auction platform to sell, bounded rationality implies that a dominant platform will emerge. All strategic individuals will coordinate on the same platform—regardless of the strength of competitive effects. The particular platform chosen depends on the behavior of non-strategic agents. In the case where these agents are totally uninformed about the details of other platform and hence choose randomly, the unique equilibrium prediction is that strategic agents will coordinate on the *risk dominant platform*, an equilibrium refinement first introduced by Harsanyi and Selten (1988) to select among equilibria in stag hunt type games.

Allowing agents to multi-home (i.e. choose to be on both platforms) adds to the set of equilibrium possibilities under full rationality, but still leads to *herding* under bounded rationality. Again the exact outcome depends on the choices of the naïve agents. Of particular interest is the situation where these agents simply avoid choosing between platforms and instead multi-home. In that case, all strategic agents coordinate on the single platform, but now select the Pareto dominant rather than the risk dominant choice. Here again bounded rationality acts as a kind of equilibrium refinement though importantly the refinement selected depends on the particulars of the institutional setting.

When naïve agents randomize their behavior, equilibrium takes a different form: Relatively unsophisticated strategic agents multi-home while sophisticates opt for the Pareto dominant platform exclusively. This equilibrium shares some of the features of credit card markets. While nearly all US credit card holders have a Visa/MasterCard in their wallet, some also carry a Discover card in addition. But the situation is rarely reversed—few people "single home" using Discover. There is no analogous equilibrium under full rationality. Here the boundedly rational model suggests qualitatively different behavior among strategic players rather than merely selecting a particular equilibrium from the myriad possibilities arising under full rationality.

All of these results suggest that competitive forces alone are not sufficient to prevent a dominant platform from emerging. In every case, one of the platforms is accessed by *all* of the strategic agents (though some may also access a second platform as a kind of backup). While this is a sharp prediction, it is clearly at odds with some market structures arising in real-world online markets. For instance, the market for online dating in the US is highly fragmented.

To better understand this phenomenon, we return to the single homing case but now add horizontal differentiation to the mix. Clearly, this provides an additional force allowing both platforms to share the market. Under full rationality, there is an intuitive equilibrium where each agent chooses his or her (horizontally) preferred platform and the market is split. But there are many other equilibrium possibilities including the emergence of a single dominant platform or a "backwards" equilibrium where every agent chooses her less preferred platform and the market is split. Relaxing full rationality cuts through the clutter. If naïve agents are weakly more likely to choose their horizontally preferred platform then the unique equilibrium corresponds to the intuitive case where every agent chooses her (horizontally) preferred platform and the market is split.

To summarize, even arbitrarily small departures from full rationality dramatically sharpen equilibrium predictions in the competing matchmakers problem. The equilibrium multiplicity endemic to coordination games vanishes. More importantly, bounded rationality models highlight the key structural components determining market share. In particular, when platforms are primarily vertically differentiated, it is always the case that one of the platforms is patronized by all agents (though some of these might also visit the rival under multihoming). This conclusion remains valid regardless of the strength of competitive effects.

When platforms are horizontally differentiated, markets are fragmented, even if competitive effects are small or absent altogether. Thus, the bounded rationality model can rationalize both the dominance of eBay in online auctions as well as the fragmentation of the market for US online dating sites without the need to appeal to sunspots or other mechanisms relying on equilibrium multiplicity.

The remainder of the paper proceeds as follows: We conclude this section by placing our results in the context of the extant literature. Section 2 sketches the model. Section 3 characterizes equilibrium under full and bounded rationality. In section 4, we add multi-homing to the model and explore how this changes choice behavior and market structure. Section 5 adds horizontal differentiation to the model and identifies conditions where platforms coexist. Finally, section 6 concludes. Some of the proofs are discussed in the main body of the paper before the formal propositions are presented. Rest of the proofs are in the appendix.

Related Literature

The literature on platform competition is relatively young (see Katz and Shapiro, 1994 for a survey of the early literature), but has grown in importance with the maturation of the Internet. Typically, this literature takes platform characteristics as given and then studies the equilibrium implications of these characteristics. More recently, Ellison and Fudenberg (2003) and Ellison, Fudenberg, and Möbius (2004) study agents' choice among two competing homogeneous platforms where with fixed characteristics and establish conditions for multiple platforms to coexist in equilibrium. Ambrus and Argenziano (2009) note that consumers must be non-negligible in size for the Ellison, *et al.* mechanism to have force. Our paper highlights that, even when agents are "large," bounded rationality can destroy coexistence.

A separate stand of the literature focuses on endogenous pricing decisions by platforms. Early papers along this line include Caillaud and Jullien (2001, 2003), who examine competing matchmakers who may charge two-part tariffs and different price schedules to the two types of agents. They show only tipping equilibria can be supported by this market. In their model, however, a scale effect is absent. Rochet and Tirole (2003) and Armstrong (2006) study platform competition where platforms are horizontally differentiated and charge only transaction fees. Horizontal differentiation guarantees coexistence. Damiano and Li (2008) show how entry fees among competing platforms can generate coexistence by separating different "types" of consumers. Most of these papers, however, assume single-homing. Our paper allows for multi-homing though at the expense of abstracting away from the optimizing decisions of the platforms. One exception, in addition to the papers already mentioned, is Carrillo and Tan (2006) who analyze consumers' decision to purchase at one or both platforms in a model where platforms bring consumers and complementors together, as is common in video gaming console markets.

The link between behavioral assumptions and coordination outcomes has met with increasing interest. For example, Amaldoss, Bettman, and Payne (2008) show, using laboratory experiments, that behavioral biases by economic agents can, in fact, facilitate coordination. Our approach builds on the suggestion of Ho, Lim, and Camerer (2006) that, by incorporating consumer psychology into choice models, new insights might be gained about market performance. Our specific behavioral model, based on the framework of Camerer, Ho, and Chong (2004), assumes that agents differ in their level of strategic sophistication using the framework of cognitive hierarchy. Nagel (1995) first introduced the cognitive hierarchy framework and showed that vastly outperformed the fully rational model in organizing data from p-beauty contest type games. This model has also proved extremely useful in describing behavior from coordination games. See, for example, Stahl and Wilson (1995) and Costa-Gomes and Crawford (2006) as well as Camerer, Ho, and Chong (2004). More recently, cognitive hierarchy models have been used to examine empirical data in industry settings including technology adoption by Internet service providers (Goldfarb and Yang, 2009), entry in local telephone markets (Goldfarb and Xiao, 2010) and decision-making by movie distributors (Brown, Camerer, and Lovallo, 2009). Our paper contributes to this literature by treating the cognitive hierarchy model as an essential tool in applied modeling in more complex settings.

Finally, there has been some empirical work on platform competition. Much of the empirical work in the area has centered on whether the "efficient" (i.e. better) platform prevails in the market.[1] Indeed, the QWERTY phenomenon—the idea that a vertically inferior platform might prevail owing to path dependence—has been profoundly influential and controversial. The influential study by David (1985) highlights a number of examples including Betamax versus VHS video standards. However, more recent work does not find the QWERTY phenomenon to be that prevalent. For example, Tellis, Yin, and Niraj (2009) use consumer reviews to try to identify the "better" platform and then examine subsequent market share. They conclude that, when a dominant platform emerges, it tends to be of higher quality than its rivals. Moreover, the work of Liebowitz and Margolis (1990 and 1994) casts serious doubt on whether the usual examples of the QWERTY phenomenon, including the QWERTY keyboard itself, are indeed inferior standards or platforms. This is also true of laboratory experiments of platform competition (see Hossain, Minor, and Morgan, 2011). Thus, our paper contributes a theoretical rationale for these findings.

[1] Brown and Morgan (2009) briefly examine this possibility in a competing auctions model and conclude that vertical differentiation in that setting leads to tipping.

2 The Model

Consider a market where there are two competing platforms labeled A and B, serving two types of agents. In terms of exposition, we shall think of these platforms as competing matchmakers and shall refer to the agents as *women* and *men*. There are exactly N of each type of agent. The role of the platform is to match agents of one type with agents of the other, i.e., to match men with women. To perform this service, each platform charges an up-front access fee $p_i > 0$ where $i \in \{A, B\}$.

All agents simultaneously decide which platform to access. For the moment, we assume that only one of the two platforms may be chosen (i.e., no multi-homing) though we relax this assumption later. We also assume that the benefits and fees of the platforms are commonly known and that all agents prefer to participate rather than opting out of choosing either platform.

Payoffs for each agent consist of gross payoffs from the match technology of the platform less the cost of the access fee. Let $u_i(n_{i1}, n_{i2})$ denote the gross payoff from accessing platform i when n_{i1} agents of the same type and n_{i2} agents of the opposite type access the platform. For instance, when n_{i1} women and n_{i2} men access platform i, then each lady enjoys a gross payoff of $u_i(n_{i1}, n_{i2})$ and a net payoff of $u_i(n_{i1}, n_{i2}) - p_i$. Similarly, the payoffs to a man when n_{i1} men and n_{i2} women accessed site i would be identical.

The focus of the model is on the agents' platform choice decisions rather than the strategy of the platforms themselves. Thus, we restrict attention to non–discriminatory pricing schemes where the access fee for men and women is the same. Platforms can also charge non-discriminatory fees for a successful match; however, these can be accounted for in the gross payoff functions. We assume that platforms exhibit standard competition and network effects. Formally,

Assumption 1 (market size effect): Gross payoffs are increasing in the number of agents of the opposite type. For all n_1 and n_2, $u_i(n_1, n_2 + 1) > u_i(n_1, n_2)$.

Assumption 2 (market impact effect): Gross payoffs are decreasing in the number of agents of own type. For all n_1 and $n_2 > 0$, $u_i(n_1, n_2) > u_i(n_1 + 1, n_2)$.

Assumption 3 (positive network externality): Gross payoffs increase when the number of agents of both types on the platform increase equally. For all n_1 and n_2, $u_i(n_1 + 1, n_2 + 1) > u_i(n_1, n_2)$.[2]

Assumption 4: For all $n \in \{1, 2, \ldots, N\}$ and $i \in \{A, B\}$, $u_i(n, 0) = 0$. Moreover, an agent also gets a payoff of 0 from not joining either platform.

We will maintain these assumptions throughout the paper. Assumptions 1 and 2 guarantee that women benefit from a greater choice of men on the platform and suffer from

[2] Our results are unchanged if we recast Assumption 3 as multiplicative. Specifically, it may be replaced by the assumption that, for all $(n_1, n_2) >> 0$, $u_i(kn_1, kn_2) > u_i(n_1, n_2)$ for $k > 1$.

more competing women. (And vice-versa for men.) Assumption 3 guarantees that, all else equal, a larger platform is preferred to a small platform. Assumption 4 says that women are unaffected by competition when there are no men on the platform. We normalize this payoff to zero for simplicity. These assumptions do not provide a complete ranking of the gross payoffs for all possible platform choices by the agents. Indeed, the model is flexible enough to accommodate most models of competing platforms in the extant literature.

Finally, to rule out knife-edge or pathological cases, we restrict attention to generic net payoffs, so that it is not the case that for all $i,j \in \{A, B\}$ and $n_1, n_2 \in \{1, 2, \ldots, N\}$, $u_i(n_1, n_2) - p_i = u_j(n_1, n_2) - p_j$ and assume that access fees are such that agents make positive net payoffs if all of them coordinate on a single platform, i.e., $u_i(N, N) - p_i > 0$.

3 Equilibrium

We now examine equilibria arising in the model under full rationality, restricting attention to pure strategy Nash equilibria. We then relax this assumption, allowing for differences in the strategic sophistication of agents, using the cognitive hierarchy framework proposed by Camerer, Ho, and Chong (2004). As mentioned above, one important justification for this approach is that it has been fruitfully used to analyze laboratory data from coordination games. We provide a detailed description of how the cognitive hierarchy model works and what aspects of bounded rationality it is meant to capture. Our main result is to show that, while a wide array of market share distributions can arise as equilibria under full rationality, adding even a vanishingly small fraction of strategically unsophisticated agents yields a unique prediction—a single dominant platform is selected by all strategic types.

Full Rationality

We first characterize equilibria in the model under the usual assumption of full rationality. Recall that the gender ratio of the market as a whole is 1 to 1.[3] The following lemma shows that in any Nash equilibrium, the gender ratio of agents at each platform is the same as that of the market as a whole. Formally,

Lemma 1 *In any Nash equilibrium, the same number of agents of each type select a given platform.*

While the result is intuitive for the case where both platforms are identical, Lemma 1 shows that, despite asymmetries across platforms, all equilibria remain symmetric in the

[3] Assuming equal numbers of agents facilitates Lemma 1 below, which considerably simplifies the equilibrium characterization. The presence of both tipped and non-tipped equilibria, however, may be readily shown for the case where there are unequal numbers of men and women in the market. The complete description of all interior equilibria becomes cumbersome. Detailed analysis of this case is available from the authors upon request.

sense that the gender ratio is the same across platforms. To see this, suppose more women than men join platform A in equilibrium. This implies that the fee difference $p_B - p_A$ is large enough to offset any gain in payoff a woman located at platform A would enjoy from switching to the platform B that has relatively more men. This, however, implies that a man on platform B would benefit from switching for the same reasons.

The scale effect contained in Assumption 3 implies that these markets are, in a sense, natural monopolies. All else equal, agents benefit from coordination on a single platform. Formally, we say that the market has *tipped* when only one platform is active, i.e., all agents opt for a single platform. When both platforms are active, we say that they *coexist.*

The next proposition shows that tipping is always an equilibrium although it is silent as to which platform will be the "winning" one. To see this, suppose that, in equilibrium, all agents locate on platform i and earn payoffs $u_i(N, N) - p_i > 0$, where the inequality follows by assumption. Now, if an agent deviates to platform j, she earns $u_j(1, 0) - p_j < 0$ since $u_j(1, 0) = 0$ and $p_j > 0$; therefore such a deviation is not profitable. It then follows immediately that

Proposition 1 *Tipping to either platform is an equilibrium. Formally, it is a Nash equilibrium for all agents to select a single platform $i \in \{A, B\}$.*

One might think that something like Assumption 3 is *necessary* for tipping to comprise a Nash equilibrium. This is not the case. Even if platforms exhibited diseconomies of scale, Proposition 1 would still hold owing to Assumption 4 and that fact that coordinating on a single platform yields non-negative surplus. The reason is that, unlike most standard coordination games, deviations by *both* types of agents are needed here to unlock surplus from the inactive platform.

While Assumption 3 is not necessary for tipping, Assumption 2 is required for both platforms to be active in equilibrium (in a pure strategy equilibrium). To see this, define the magnitude of the market impact effect in market i with n agents of each type to be

$$\delta_{i,n} = u_i(n, n) - u_i(n + 1, n).$$

Consider an equilibrium where n agents of each type go to platform A with the remainder going to platform B. The difference in equilibrium utility for agents going to platform A versus those going to platform B is

$$\Delta U_n = u_A(n, n) - u_B(N - n, N - n) - (p_A - p_B).$$

Suppose that $\Delta U_n \geq 0$. Clearly, agents located on A cannot profitably deviate to B since their payoffs are at most $u_B(N - n, N - n) - p_B$ (owing to non-negative market impact effects), which are smaller than the payoffs from locating on platform A since $\Delta U_n > 0$.

Thus, we only need to show that agents located on B have no wish to deviate. Incentive compatibility requires that

$$u_B(N-n, N-n) - p_B \geq u_A(n+1, n) - p_A.$$

Subtracting $u_A(n, n) - p_A$ from both sides of the inequality, we obtain

$$-\Delta U_n \geq -\delta_{A,n}$$

or, equivalently, that market impact effects for platform A must be sufficiently large, i.e., $\delta_{A,n} \geq \Delta U_n$.

The case where $\Delta U_n < 0$ yields the analogous condition that the market impact effects for platform B must be sufficiently large, i.e., $\delta_{B,N-n} \geq -\Delta U_n$. To summarize, we have shown

Proposition 2 *Any market share split is consistent with equilibrium provided market impact effects are sufficiently large.*

Formally, n agents of each type locating on platform A with the remainder choosing platform B comprise a coexisting equilibrium provided that: (1) $\delta_{A,n} \geq \Delta U_n$ when $\Delta U_n \geq 0$ and (2) $\delta_{B,N-n} \geq -\Delta U_n$ when $\Delta U_n < 0$.

A different way to see that market impact effects are necessary for coexisting equilibria to arise is to consider the case where the two platforms are identical. Suppose that platform A enjoys a smaller market share than platform B. In that case, the net payoff to men and women located on A is smaller than that enjoyed by their counterparts on B. What prevents a man on A from profitably deviating is that, were he to switch, the additional competition among men on B would lower the payoffs of men on that platform through the market impact effect. If this effect is large enough to overwhelm the gains from scale offered by B, then such a deviation is not profitable. Essentially, this is the force leading to equilibrium coexistence in the model of Ellison and Fudenberg (2003).

One might worry that coexisting equilibria arising in this model are "knife-edge" in the sense that any small perturbation in agent strategies leads to tipping. This is not the case. Generically, the coexisting equilibria we identify above are *strict Nash equilibria* and hence are robust to small perturbations. Another worry is that coexistence is an artifact of the assumption of exogenous access fees. One might reason that a platform with higher match quality could simply compete Bertrand style in access fees and thereby capture the entire market. The flaw in this intuition is that a platform is only valuable to the extent that it can induce *multilateral* deviations. Regardless of price, it does not pay to switch to a higher quality platform where few other agents are present. In the Appendix, we formalize this intuition and show that coexistence is consistent with equilibrium even when fees are endogenous.

Example 1 To give a concrete example of how the model works, suppose the matching technology is such that when a man joins a platform that has at least as many female participants as male participants (including himself), the market impact effect is relatively small. However, the competition between men become much more acute leading to a larger market impact effect when there are fewer females than males on that platform. A simple gross payoff function based on this matching technology can be described by:

$$u_A(n_1, n_2) = u_B(n_1, n_2) = \begin{cases} \max\left\{100\frac{n_2}{N} - \gamma_1 \frac{n_1 - 1}{N-1}, 0\right\} & \text{if } n_1 \leq n_2 \\ \max\left\{100\frac{n_2}{N} - \gamma_2 \frac{n_1 - 1}{N-1}, 0\right\} & \text{if } n_1 > n_2 \end{cases}$$

where $0 < \gamma_1 < \gamma_2 < \frac{N-1}{N}$. Here, the two different lambda parameters represent the magnitude of the market impact effects. This market satisfies all of the assumptions above. Women gain with an increase in the fraction of men located on a given platform. They lose in proportion to the fraction of women on the same platform, and the effect is more pronounced when women on the platform outnumber men. When $N = 10$, $\gamma_1 = 5$, $\gamma_2 = 60$, $p_A = 2$ and $p_B = 0.01$, there are five coexisting equilibria of this market consisting of equal market shares, a 60-40 split in favor of either platform, and a 70-30 split in favor of either platform. The remaining equilibria consist of tipping to either platform.

Cognitive Hierarchy

The previous analysis relied on the full rationality of market participants. In particular, the choices made by each agent depend on expectations about the choices made by all other agents, which in turn depend on expectations of expectations, and so on. Clearly, this level of sophistication is an idealization at best—some participants are likely to be more naïve and make choices without fully reflecting on the selections of other agents. To capture this idea, we use a model of cognitive hierarchies. Cognitive hierarchy models are meant to capture heterogeneities in the strategic sophistication of participants in the market. Specifically, some fraction of agents are non-strategic. Their choices are determined by rules or heuristics and made irrespective of beliefs about the choices of others. Other agents have limited strategic reasoning. Their expectations are formed based on (flawed) models of the choice behavior of all other agents.

Formally, each agent has a cognitive sophistication level of $l \in \{0, \ldots, \overline{L}\}$ for some finite $\overline{L}$. For simplicity, we assume that the true distribution of the levels of cognitive sophistication is the same for women and men. The total population of agents that are of cognitive sophistication level l is $f(l)$ with $\sum_{l=0}^{\overline{L}} f(l) = 1$. Agents of level $k \geq 1$ assume that all other agents have sophistication levels strictly below k and best respond accordingly. For instance, a level-k woman assumes that all N men and the remaining $N-1$ women are of level $k-1$ or below. Moreover, she perceives that the population fraction of level l is $f(l) / \sum_{t=0}^{k-1} f(t)$ for $l \leq k-1$ and is 0 for $l \geq k$. While these agents are strategic, level-0 agents are non-strategic. They make no inference about the behavior of others around them to determine the correct choice. Instead, they rely on rules or heuristics to guide their choices. In this model with single-homing, there is no obvious choice of heuristics for these non-strategic types. As such, we will be agnostic about their strategy. We will assume that a level-0 agent chooses platform A with probability $\lambda_A \in [0,1]$ and chooses platform B with probability $\lambda_B = 1 - \lambda_A$.

To analyze this game, we introduce a new notation. Let $U_i(\lambda)$ denote the expected gross payoff to an agent from choosing platform i when all other agents independently select this platform with probability λ. That is,

$$U_i(\lambda) = \sum_{s=1}^{N} \sum_{t=0}^{N} \binom{N-1}{s-1} \binom{N}{t} \lambda^{s-1+t} (1-\lambda)^{2N-s-t} u_i(s,t). \quad (1)$$

Clearly, $U_i(\lambda)$ is continuously differentiable, $U_i(\lambda) > U_i(0)$ for all $\lambda \in (0,1]$, and $U_i'(0) > 0$. To make cross-platform comparisons with respect to λ requires some additional structure on payoffs. In particular, we would like payoffs to satisfy the familiar single-crossing condition with respect to λ. It suffices to assume that the payoffs for each platform are single-peaked in λ. Formally,

Assumption 5: If $U_i'\left(\widehat{\lambda}\right) = 0$ then $U_i'(\lambda) < 0$ for all $\lambda > \widehat{\lambda}$.

Assumption 5 guarantees that there is a unique λ^* solving

$$U_i(\lambda^*) - p_i = U_j(1-\lambda^*) - p_j.$$

Moreover, for all $\lambda' > \lambda^*$,

$$U_i(\lambda') - p_i > U_j(1-\lambda') - p_j$$

for $i \in \{A, B\}$, which are the usual single-crossing conditions.

With this notation in hand, let us consider the best responses for each agent. From the perspective of a level-1 agent, all other agents are selecting platforms at random, thus, her expected payoff from choosing platform i is simply $U_i(\lambda_i) - p_i$.[4] Naturally, such an agent chooses platform i over j if and only if

$$U_i(\lambda_i) - p_i > U_j(1-\lambda_i) - p_j \quad (2)$$

Level-1 agents choose platform i provided there is a sufficiently high chance of encountering level-0 agents there. A level-2 agent believes that all other agents go to platform i with probability $\frac{\lambda_i f(0) + f(1)}{f(0)+f(1)} > \lambda_i$ as she believes all other agents are of level 0 or 1. That is, she believes a larger fraction of agents are choosing platform i. The single-crossing property implies that she too prefers platform i to j. (Notice that absent Assumption 5, one might

[4] We ignore the non-generic case where λ_i happens to leave level-1 types indifferent between the two platforms.

encounter the rather implausible situation where an agent who is convinced that i enjoys a higher market share is less likely to choose it compared to an agent who believes that i enjoys a smaller market share.) The same logic obtains for agents with ever higher levels of sophistication. As a consequence, the market will tip to the platform satisfying equation (2). Formally,

Proposition 3 *Under cognitive hierarchy, all agents with sophistication level* $l > 1$ *choose the same platform as level-1 agents.*

Like many models with behavioral types, the choices of level-0 types profoundly influence the decisions of more sophisticated agents, even when level-0 agents are relatively scarce in the population as a whole. Of particular interest is the situation where level-0 agents choose either platform with equal probability, i.e. $\lambda_i = 1/2$. In that case, there is a useful link between cognitive hierarchy and the risk dominance notion of equilibrium selection first introduced by Harsanyi and Selten (1988). Harsanyi and Selten were motivated by the game stag hunt. It is well-known that there are two pure strategy equilibria in stag hunt, one corresponding to the "safe" strategy of hunting hare while the other corresponds to the "risky" strategy of hunting stag. Of course, in equilibrium, neither strategy is truly risky in that the behavior of the other agent is known with certainty. Yet, in a real sense, hunting stag is riskier—an agent's payoff could be lower if the co-agent unexpectedly took an action. Harsanyi and Selten sought to capture this notion of stag as a riskier strategy through the risk dominance equilibrium refinement. Specifically, given two pure equilibria, E and E' of a bi-matrix game, equilibrium E is said to be *risk dominant* if the expected payoff to each agent is higher under E than under E' given random (equiprobable) play on the part of the co-agent. So long as the downside of hunting stag is sufficiently large, hunting hare is the risk dominant equilibrium in the game. The same holds true in our setting and hence:

Remark 1 *Suppose that* $\lambda_i = \frac{1}{2}$ *and* $f(0) \to 0$, *then the unique equilibrium under cognitive hierarchy converges to the risk dominant equilibrium.*

While the cognitive hierarchy corresponds to risk-dominance under the specific assumption of equiprobable choice behavior by level-0 agents, the model predicts herding—all more sophisticated agents will mimic the choices of level-1 agents—regardless of the particular specification of level-0 behavior. Indeed, this herding phenomenon is quite robust.

The Camerer-Ho-Chong specification of cognitive hierarchy is one of several in the literature; however, all of these specifications share the herding effect. For instance, in the Nagel-Stahl-Wilson specification, a level-l agent believes that all other agents have cognitive sophistication level of $l - 1$. It is straightforward to show that the behavior of level 1 agents is unchanged under this specification. Naturally, all other cognitive types will choose the same platform as level-1 agents.

The herding result is also robust to relaxing the assumption that the gross payoffs treat men and women symmetrically. So long as the expected payoff maximizing platform is the same for both types of agents, the cognitive hierarchy model will again predict a unique equilibrium where all agents will herd on the choice of the level-1 agents. Likewise, the result straightforwardly extends to the case where there are more than two competing platforms.

4 Multi-Homing

The previous section follows much of the literature on platform competition by focusing on the case where agents can only choose a single platform. In practice, however, there are many circumstances where such an assumption is patently unrealistic. For instance, if one were interested in applying the model to study credit card markets, assuming that merchants only accept a single type of card or that consumers only have one card in their wallets is clearly at odds with reality. One reason for restricting attention to the single-homing case is tractability. As we saw, equilibrium multiplicity was a serious problem in the fully rational model even when agents were restricted to choosing a single platform. The analysis only grows more complex with the additional option of multi-homing. A second reason for such a restriction is that the single-homing assumption might be innocuous—the analysis may be fundamentally unchanged despite the added complexity.

In this section, we amend the model to allow for multi-homing. Formally, each agent's choice set consists of $\{A, B, AB\}$ where AB denotes multi-homing. We show that, in the fully rational case, this additional option is not innocuous—the set of coexisting equilibria change when multi-homing is permitted. This added complexity, however, does not change the simplicity of the cognitive hierarchy approach. There remains a unique equilibrium; however the character of the equilibrium does change. In particular, even when level-0 agents choose each available option with equal probability, it is no longer the case that the risk dominant platform prevails in the market. Indeed, the addition of multi-homing tends to favor the "better" platform in the sense of Pareto dominance. Thus, the assumption of single-homing is far from innocuous regardless of the assumed level of rationality.

Assumptions 1-4 imply that one of the platforms will be Pareto dominant—payoffs for all participants are maximized when everyone chooses this platform exclusively. Let platform i denote the Pareto dominant platform and note that this implies that $u_i(N, N) - p_i > u_j(N, N) - p_j$.

Amending the model to allow for multi-homing requires more than merely adding this option to the choice sets of each agent. It also requires some specification of how the matching process works (and hence payoffs are generated) when agents choose to multi-home. We assume that agents follow a lexicographic rule: First, they go to the better (Pareto dominant) platform and enjoy payoffs from whoever else is at that platform. That is, an agent enjoys payoffs $u_i(n_{i1}, n_{i2}) - p_i$. Next, they go to the worse platform and enjoy payoffs from any *new* individuals of the opposite type they encounter. Of course, they still suffer costs from competition associated with *all* individuals of the same type visiting the worse platform. That is, an agent who multi-homes enjoys incremental payoffs of $u_j\left(n_{j1}, n_{j2}^E\right) - p_j$ where $n_{j2}^E = N - n_{i2}$. Thus, the net payoff for a multi-homing agent is

$$u_i(n_{i1}, n_{i2}) + u_j\left(n_{j1}, n_{j2}^E\right) - p_A - p_B$$

This type of rule is intuitive in a dating market context. It makes sense that a woman will first search for matches on the better dating platform, collecting contact information for the attractive men located there. Having obtained this information, she then visits the less attractive dating platform. Obviously, the only additional value such a visit provides is the contact information for *new* attractive men not already encountered on the better platform. Of course, she faces competition from all of the women located at each platform regardless of duplication. While this rule seems intuitive, it is not essential to our main result (Proposition 6)—that under cognitive hierarchy the better platform prevails. In particular, the same result would still hold if we assumed that all agents visited the worse platform before the better platform.

Full Rationality

Tipping to either platform remains an equilibrium even when we add the option of multi-homing. To see, this, suppose women all choose platform $i \in \{A, B\}$ exclusively, then men have no incentive to join platform j or to multi-home since there is no benefit to visiting platform which is devoid of women. The same is, of course true of women when men join platform i exclusively.

Likewise, under some parameter values, it remains an equilibrium for n agents of each type join platform A and the remaining $N - n$ agents of each type join platform B, the analog to coexisting equilibria under single homing. Proposition 4 formalizes this.

Proposition 4 *When agents can multi-home, tipping to either platform is an equilibrium. Furthermore, any market share split is consistent with equilibrium provided market impact effects are sufficiently large–even though none of the agents multi-homes.*

Formally, there exists an equilibrium where all agents choose platform $i \in \{A, B\}$. There exists an equilibrium where n agents of each type choose platform $i \in \{A, B\}$ with the remainder choosing platform j provided that:

$$\delta_{A,n} \geq u_A(n, n) - p_A \geq 0 \text{ and } \delta_{B,N-n} \geq u_B(N - n, N - n) - p_B \geq 0 \tag{3}$$

Since the multi-homing option is not exercised for the coexisting equilibria characterized in Proposition 4, we can examine how multi-homing affects the chance that platforms coexist. Define $\bar{\delta}_{A,n}^{MH} \equiv u_A(n,n) - p_A$ to be the critical threshold for market impact effects on platform A to sustain coexistence in an equilibrium where n agents choose platform A under multi-homing. Under single-homing, the relevant critical threshold is $\bar{\delta}_{A,n}^{SH} \equiv \Delta U_n$. The critical thresholds for the market impact effects on platform B are analogous. Now, since $u_B(N-n, N-n) - p_B \geq 0$, it follows immediately that $\bar{\delta}_{A,n}^{MH} \geq \bar{\delta}_{A,n}^{SH}$ and similarly $\bar{\delta}_{B,n}^{MH} \geq \bar{\delta}_{B,n}^{SH}$; that is, market impact effects must be larger to sustain coexistence under multi-homing than under single-homing. Thus, the option to multi-home undermines the prospects of equilibrium coexistence (for the class of equilibria where the multi-homing option is not exercised). This is intuitive in that multi-homing offers an additional possibility for deviation from equilibrium, namely co-locating on both platforms. The required conditions to rule such deviations out are, accordingly, more stringent.

Of course, Proposition 4 only considers the set of equilibria in which the option to multi-home is not exercised in equilibrium. Equilibrium coexistence might also arise when it is an equilibrium for one or both types of agents to multi-home. We study these equilibria next.

One can easily rule out the possibility that all agents multi-home. To see this, notice that, since all women are on both platforms, there is no incremental benefit to men from visiting the worse platform. Moreover, such visits are costly. Hence, men can profitably deviate by single-homing at the better platform and likewise for women.

Similarly, it can never be an equilibrium for all men to choose platform i and some men to multi-home. Under this circumstance, all women would choose to visit platform i exclusively and hence the multi-homing men derive no benefit from also accessing platform j. (An identical argument rules out the case where all women visit platform i and some multi-home.)

There are, however, coexisting equilibria where some agents of each type exclusively use each of the platforms while others multi-home. for instance, some men exclusively use platform A, others exclusively use platform B, while the remainder multi-home and symmetrically for the women. Since some men find it optimal to multi-home, one may wonder why it is not profitable for a man currently using a single platform to deviate by multi-homing. What prevents this is the market impact effect—by adding a second platform, competition among men on this platform is increased—which deters such deviations. Proposition 5 formalizes the exact conditions where the market impact effects can sustain this type of coexistence.

Proposition 5 *Provided market impact effects are sufficiently large, coexistence where some agents multi-home is an equilibrium.*

Formally, suppose that i is the Pareto dominant platform and that

$$\begin{aligned}
\delta_{i,n_i} + \delta_{j,n_j} &\geq p_i - p_j + u_j(n_j, n_j) - u_i(n_i + 1, n_i) \geq 0 \\
\gamma_{j,n_j} &\geq p_j - u_j(n_j + 1, N - n_i) \geq 0 \\
\delta_{i,n_i} &\geq p_i - u_i(n_i + 1, n_i) + u_j(n_j, n_j) - u_j(n_j, N - n_i) \geq 0
\end{aligned}$$

where $\gamma_{j,n_j} \equiv u_j(n_j, N - n_i) - u_j(n_j + 1, N - n_i)$. *Then it is a Nash equilibrium for* $N - n_j$ *agents of each type locate only on platform* i, $N - n_i$ *agents of each type locate only on platform* j, *and* $n_i + n_j - N$ *agents of each type multi-home.*

Proposition 5 reveals that the multi-homing behavior seen in practice in credit card markets is consistent with a coexisting equilibrium under full rationality. Moreover, it is essential that not all individuals on the same side of the market make the same choices. Some consumers will use Visa/MasterCard exclusively while others will also carry Discover card. Likewise, not all merchants will accept both cards. One counterfactual aspect of the equilibrium is that it requires that some merchants and some consumers use/accept Discover card exclusively. While the exclusive acceptance of Discover was, at one time, the policy of both Sears and Sam's Club, this is no longer the case. Thus, a coexisting equilibrium is capable of rationalizing some but not all behavior with respect to multi-homing. Perhaps more importantly, such equilibria are ruled out (by assumption) by limiting attention to the single-homing case.

Taken together, Propositions 4 and 5 point out that equilibrium offers little guidance as to what market structures emerge with platform competition under full rationality. Indeed, if anything, the picture is even more muddled than under single homing with full rationality. For instance, one can easily choose parameter values such that the addition of multi-homing merely expands the (already considerable) set of equilibria that previously arose under single-homing.

Cognitive Hierarchy

We saw that relaxing the assumption of full rationality in favor of the arguably more realistic cognitive hierarchy formulation substantially clarified predictions about market structure under single-homing regardless of the assumptions made about the behavior of level-0 agents. Multi-homing introduces additional possibilities for modeling the choices made by these individuals. Now the probabilistic mix is multi-dimensional rather than single dimensional. Assuming single peakedness (Assumption 5) guaranteed that the problem of best responses for level 1 and higher agents was well-behaved thus facilitating full characterization under single-homing. The situation is more nuanced in the multi-homing case. Thus, rather than characterizing equilibria under arbitrary choices of level-0 agents, we temporarily restrict attention to circumstances where these choices are in pure strategies. Later, we relax this assumption to allow for symmetric randomization behavior by these agents; that is, level-0 agents (stochastically) choose either of the platforms with equal probability and otherwise multi-home.

Pure Strategy Choices by Level-0 Agents

First, consider the case where level-0 agents avoid choosing between competing platforms; they simply multi-home. We claim that all strategically sophisticated agents choose the better platform. We first sketch the idea of the proof and then present a formal statement of the result and proof. When level-0 agents multi-home, which platform should level-1 agents choose? Since they view all agents as being level-0, then they believe that all agents will be present on both platforms. There is, effectively, no risk associated with choosing either platform and, as a consequence, level-1 agents select the better (i.e. Pareto dominant) platform. A level-2 agent believes that all agents are level-1 or level-0 and hence believes that all agents will be present on the Pareto dominant platform. As a consequence, such agents are best served by mimicking the choices of the level-1 agents. The same holds of all agents with higher levels of strategic sophistication. Formally, we may conclude:

Proposition 6 *Suppose that all level-0 agents multi-home. Under cognitive hierarchy, all agents with sophistication level* $l \geq 1$ *choose the Pareto dominant platform.*

Proposition 6 reinforces the notion that, by allowing for some degree of bounded rationality, market impact effects are not enough to sustain equilibrium coexistence—one of the platforms will enjoy 100% market share of sophisticated agents while the rival platform gets 0% market share. Moreover, it sharpens the prediction as to the identity of the winning platform. In particular, it suggests that the QWERTY phenomenon—the possibility of agents getting locked in to the inferior platform—does not arise. That is, the possibility of being locked in to an inferior platform is not a product of bounded rationality, rather it is an artifact of the hyper-sophistication engendered by assuming full rationality. It is perhaps for this reason that examples of this type of lock-in are rare.

Next, consider the case where all level-0 agents choose platform i exclusively. Clearly level-1 agents will follow suit. There is no gain to accessing platform j either exclusively or through multi-homing since no agents are believed to be present on the platform. The same logic applies to all agents with higher levels of sophistication. Thus, we have shown that

Proposition 7 *Suppose that all level-0 agents choose platform* i*. Then, under cognitive hierarchy, the market tips to platform* i*—all agents utilize this platform exclusively.*

Propositions 6 and 7 highlight several key properties of bounded rationality and platform competition. First, the "herding" effect where all agents of higher levels of sophistication mimic the choices of level-1 agents is a robust feature of the model. Second, despite the option to multi-home, all agents of higher levels of sophistication opt for a single platform. Third, and most importantly, even in the presence of multi-homing, a single dominant platform emerges as the equilibrium market structure.

Stochastic Level-0 Agent Choices

One may worry, however, that the tendency toward tipping is purely an artifact of our restriction to pure strategy behavior on the part of level-0 agents. We now partially relax this assumption to allow for non-deterministic behavior on the part of level-0 agents. Specifically, we assume that level-0 agents choose to access platform i exclusively with the same probability that they choose to access platform j. With remaining probability, level-0 agents multi-home. Notice that the conditions given in Proposition 6 are a special case of this specification of level-0 behavior.

Before proceeding with the analysis, we need to introduce some additional notation to account for stochastic choices on the part of other agents. As usual, let i be the Pareto dominant platform. Suppose that an agent of a given level of rationality believes that all other agents select platform i (exclusively) with probability λ_i, select platform j with probability λ_j, and multi-home with the remaining probability $1-\lambda_i-\lambda_j$. In that case, her payoff from multi-homing when exactly $s-1$ agents of the same type choose i, r multi-home, and t of the opposite type choose platform i (either exclusively or through multi-homing) is simply $\left(u_i\left(s+r,t\right)+u_j\left(N-s+1,N-t\right)\right)$. The chance of this event is

$$pr\left(s,r,t\right)=\binom{N-1}{s-1}\binom{N-s}{r}\binom{N}{t}\lambda_i^{s-1}\left(1-\lambda_i-\lambda_j\right)^r\lambda_j^{N-s-r}\left(1-\lambda_j\right)^t\lambda_j^{N-t}.$$

Expecting over all possible events yields the expected utility from multi-homing,

$$U_{mh}\left(\lambda_i,\lambda_j\right)=\sum_{s=1}^{N}\sum_{r=0}^{N-s}\sum_{t=0}^{N}pr\left(s,r,t\right)\left(u_i\left(s+r,t\right)+u_j\left(N-s+1,N-t\right)\right).$$

When an agent chooses platform i exclusively, on the other hand, she gets payoff from all other agents who join platform i, exclusively or not. That is, she believes that an agent will locate on platform i with probability $1-\lambda_j$. Therefore, her expected payoff from joining platform i exclusively is

$$\begin{aligned}U_{sh,i}\left(\lambda_i,\lambda_j\right) &= \sum_{s=1}^{N}\sum_{t=0}^{N}\binom{N-1}{s-1}\binom{N}{t}\left(1-\lambda_j\right)^{s-1}\lambda_j^{N-s}\left(1-\lambda_j\right)^t\lambda_j^{N-t}u_i\left(s,t\right)\\ &= \sum_{s=1}^{N}\sum_{t=0}^{N}\binom{N-1}{s-1}\binom{N}{t}\left(1-\lambda_j\right)^{s+t-1}\lambda_j^{2N-s-t}u_i\left(s,t\right)\end{aligned}$$

While the delineation of λ_i and λ_j is needed in determining the payoffs under multi-homing, it is not strictly necessary under single homing. Indeed, $U_{sh,i}\left(\lambda_i,\lambda_j\right)=U_i\left(1-\lambda_j\right)$ as defined in equation (1). Similarly, when an agent chooses platform j exclusively, she earns

$$U_{sh,j}(\lambda_i,\lambda_j)=\sum_{s=1}^{N}\sum_{t=0}^{N}\binom{N-1}{s-1}\binom{N}{t}(1-\lambda_i)^{s+t-1}\lambda_i^{2N-s-t}u_j(s,t)=U_j(1-\lambda_i).$$

Each of these functions is well-defined and continuously differentiable in λ_i and λ_j. The case where $\lambda_i=\lambda_j=0$ corresponds to the situation where all other agents are perceived to multi-home. As we saw in the proof of Proposition 6, an agent's best response was to select the better platform exclusively given these beliefs. That is,

$$U_{sh,i}(0,0)-p_i>U_{mh}(0,0)-p_A-p_B \tag{4}$$

For multi-homing to be a viable best response to symmetric choices by level-0 agents, we assume that

$$U_{mh}\left(\frac{1}{2},\frac{1}{2}\right)-p_A-p_B>U_{sh,i}\left(\frac{1}{2},\frac{1}{2}\right)-p_i,U_{sh,j}\left(\frac{1}{2},\frac{1}{2}\right)-p_j \tag{5}$$

This assumption merely guarantees that, if all other agents locate on a single platform exclusively, with equal probability for each platform, then the benefits of encountering all of the agents of the opposite type exceed the costs of multi-homing.

Finally, the analysis is greatly simplified if we extend the notion of Pareto dominance to situations where platforms enjoy less than 100% market share. Specifically, we say that platform i is *super dominant* if, for a given market share, payoffs are higher on platform i than on platform j. For instance, were j to enjoy 60% market share, then payoffs to those on platform j would be lower than to agents on platform i when i enjoys this same market share. Formally, we assume that, for all λ,λ'

$$U_{sh,i}(\lambda,\lambda')-p_i>U_{sh,j}(\lambda',\lambda)-p_j.$$

Obviously, super dominance implies Pareto dominance.

As for the single-homing case, we require some additional structure to ensure that the expected payoff functions are well-behaved. Specifically, parallel to Assumption 5, we assume that $U_{mh}(\lambda,\lambda)$ is single-peaked in λ. Moreover, we assume that relative attractiveness of multi-homing over single-homing at platform i is decreasing in the probability of an agent choosing platform i and is increasing in the probability of an agent choosing platform j. Note that Assumption 5 already implies that $U_{sh,i}(\lambda_i,\lambda_j)$ is single-peaked in λ_j. Formally,

Assumption 6: If $U'_{mh}\left(\widehat{\lambda},\widehat{\lambda}\right)=0$ then $U'_{mh}(\lambda,\lambda)<0$ for all $\lambda>\widehat{\lambda}$. Moreover, $U_{mh}(\lambda_i,\lambda_j)-U_{sh,i}(\lambda_i,\lambda_j)$ is decreasing in λ_i and increasing in λ_j.

With these assumptions, we can now analyze the behavior of level-1 agents. Let $\lambda_i=\lambda_j=\tilde{\lambda}$ denote the choice probabilities of level-0 agents. Clearly, if $\tilde{\lambda}$ is small, then the best response for a level-1 agent is to single-home, exclusively choosing the super dominant platform. This follows from continuity and the inequality in equation (4). Similarly, if $\tilde{\lambda}$ is close to 50%, then the best response for a level-1 agents is to multi-home, which follows from continuity and the inequality in equation (5). Thus, there exists for intermediate probability, $\tilde{\lambda} = \lambda^*$, where level-1 agents are exactly indifferent between single and multi-homing. Clearly, level-1 agents multi-home if and only if $\tilde{\lambda} \geq \lambda^*$.[5]

When $\tilde{\lambda} < \lambda^*$, level-1 agents choose the super dominant platform. Naturally, this makes this platform more attractive for higher level agents, and we obtain the familiar herding result—all more sophisticated agents mimic the behavior of level-1 agents and choose the super dominant platform exclusively.

Of greater interest is the case where $\tilde{\lambda} \geq \lambda^*$. Here, level-1 agents choose to multi-home and thus, from the perspective of a level-2 agent, the fraction of other agents choosing to be exclusively on platform i or j falls to $\lambda' < \lambda^*$. As a consequence, multi-homing is now less attractive. Eventually, there exists a level-$\underline{k}$ agent for whom λ' has fallen sufficiently that it is now below the critical threshold, λ^*. This agent then chooses to visit the super-dominant platform exclusively and, as usual, all more sophisticated agents follow suit.

While the above sketches the essence of the proof, it omits a number of technical details needed to ensure that the intuitive behavior described above is, indeed, optimal. Proposition 8 presents a formal statement of the result. The detailed proof is relegated to the appendix.

Proposition 8 *Suppose that i is super dominant and Assumptions 1-6 hold. Then, under cognitive hierarchy:*

If level-0 agents single-home on each platform with probability $\tilde{\lambda} < \lambda^*$, all strategically sophisticated agents choose the super-dominant platform.

If level-0 agents single-home on each platform with probability $\tilde{\lambda} \geq \lambda^*$, then there exists $\infty > \underline{k} > 1$ such that all agents of sophistication levels $\{1, 2, ..., \underline{k} - 1\}$ multi-home while more sophisticated agents choose the super-dominant platform.

Proposition 8 highlights that the addition of multi-homing offers the possibility of a much richer set of choice behavior in equilibrium under cognitive hierarchy. While it remains the case that bounded rationality leads to unique predictions that entail herding behavior where more sophisticated agents mimic the choices of less sophisticated agents, it is no longer the case that there is a single, dominant platform selected by sophisticated agents. When the fraction of level-0 agents who single-home is high enough, relatively less sophisticated strategic agents respond by multi-homing while sophisticates choose the better platform exclusively. This behavior is qualitatively consistent with what one sees in the credit card

[5] We assume that, when there is a tie between single and multi-homing, level-1 agents choose to multi-home. The particular tie-breaking rule is inessential to the result.

market—some people carry Visa/MasterCard and Discover in their wallet while others use Visa/MasterCard exclusively. Likewise for merchants—Discover cards are not universally accepted while Visa/MasterCards are. It is also unlike *any* equilibrium under full rationality. Thus, in principle, the distinction between the two models is empirically testable.

5 Horizontal Differentiation

While models with full rationality offered little in the way of predictions about market structure, bounded rationality models offered more precise predictions. Specifically, regardless of the size of market impact effects, vertical differentiation, or single versus multi-homing, a ubiquitous platform always arose in equilibrium. Under single-homing, this implied that there was a single, dominant platform selected by all strategic agents. Under multi-homing, both platforms might coexist, but one of the platforms would be "universal" in the sense that all sophisticated agents chose it either exclusively or through multi-homing. While this matches many platform competition situations where there is a single big agent, in other situations the market is more fragmented. In this section, we enrich the model to account for differences in individual preferences across platforms, i.e. to permit horizontal as well as vertical differentiation across platforms.

Up until now, we have assumed that the payoffs for all individuals of a given type choosing a given platform were the same. Thus, men might view the platforms Match.com and eHarmony as different, all men felt the same way about each platform. Clearly, this is an unrealistic assumption. One key dimension along which Match and eHarmony differ is whether the user browses to find the right match versus whether the site provides the user with a short list of suitable matches. A user visiting Match.com is free to browse the profiles of all others signed up to the site and decide who to contact. Browsing, however, is not permitted on eHarmony. Instead, the user receives a list of a small set of potential matches based on compatibility algorithms at the website. Some users prefer the do it yourself approach of Match while others prefer the top-down approach of eHarmony.

To model this, we suppose that each agent has a horizontally preferred platform. By choosing the preferred platform, the agent receives a discount of $\theta > 0$ off of the access fee. Suppose platform i is a given man's preferred platform where $n_1 - 1$ other men and n_2 women has joined and the remaining men and women have joined his non-preferred platform j. Then his payoff from joining platforms i and j will be $u_i(n_1, n_2) - p_i + \theta$ and $u_j(N - n_1 + 1, N - n_2) - p_j$, respectively. The model is uninteresting when the discount is so large as to induce the agent to go to his preferred platform even when he or she is alone on the platform. Thus, we assume that if platform i is the preferred platform then

$$\theta < u_j(N, N) - p_j + p_i \tag{6}$$

Suppose $\widetilde{n_A}$ male and $\widetilde{n_A}$ female agents have a preference for platform A and $\widetilde{n_B} = N - \widetilde{n_A}$ agents of each type have a preference for platform B. To examine the pure effect of horizontal differentiation, we revert to the case where only single-homing is allowed.

Full Rationality

We do not characterize all equilibria for this model. However, we show that both tipping and coexistence occur in equilibrium. Importantly, adding horizontal differentiation admits a new possibility—for generic parameter values, it may be that neither platform is Pareto dominant when tipped. Specifically, horizontal differentiation must be sufficiently unimportant for a Pareto dominant platform to exist. Formally, a Pareto dominant platform i exists if and only if

$$\theta \leq u_i(N, N) - u_j(N, N) - p_i + p_j \tag{7}$$

for some i. It may be readily verified that the inequality given in equation (7) is more stringent than that given in equation (6); thus it may be that tipping to either platform is not Pareto dominant. Regardless of whether the inequality in equation (7) holds, tipping to either platform remains an equilibrium. If all agents are located on platform i, even an agent whose preferred platform is j cannot benefit from unilaterally switching to platform j given the upper bound on θ as specified in equation (6). Thus, we have shown

Proposition 9 *Under horizontal differentiation, tipping to either platform is a Nash equilibrium.*

Under horizontal differentiation, coexisting equilibria continue to exist. The most intuitive of these is one where each agent goes to her (horizontally) preferred platform; however, there are many other classes of equilibria where platforms coexist. For instance, we identify conditions where every agent chooses to go to her non-preferred platform. A mix of the two is also possible. As usual, the key to equilibrium coexistence in the rational model is the size of market impact effects. For the intuitive equilibrium, the magnitude of these effects required for coexistence is reduced by the discount θ from an agent choosing her preferred platform. The following proposition derives formal conditions on market impact effects for equilibrium coexistence to arise. Note, however, that adding horizontal differentiation complicates the equilibrium multiplicity already present under the baseline model; thus, Proposition 10 is not a complete characterization of all equilibria in which coexistence occurs. Formally,

Proposition 10 *Platform coexistence is consistent with equilibrium under horizontal differentiation provided that market impact effects are large enough. Specifically,*

I. All agents joining their preferred platforms is a coexisting equilibrium if (1) $\delta_{A,\widetilde{n_A}} + \theta \geq \Delta U_{\widetilde{n_A}}$ *when* $\Delta U_{\widetilde{n_A}} \geq 0$ *and (2)* $\delta_{B,\widetilde{n_B}} + \theta \geq -\Delta U_{\widetilde{n_A}}$ *when* $\Delta U_{\widetilde{n_A}} < 0$.

II. All agents joining their non-preferred platform is a coexisting equilibrium if $\delta_{B,\widetilde{n_A}} - \theta \geq -\Delta U_{\widetilde{n_B}}$ *and* $\delta_{A,\widetilde{n_B}} - \theta \geq \Delta U_{\widetilde{n_B}}$*.*

III. Moreover, $\widetilde{n_A} - m$ *pairs of male and female agents choosing their preferred platform A,* m *pairs of male and female agents choosing their non-preferred platform B and* $\widetilde{n_B}$ *pairs of male and female agents choosing their preferred platform B for some* $m \in \{1, 2, \ldots, \widetilde{n_A} - 1\}$ *is an equilibrium if* $\delta_{A,\widetilde{n_A}-m} - \theta \geq \Delta U_{\widetilde{n_A}-m}$ *and* $\delta_{B,\widetilde{n_B}+m} + \theta \geq -\Delta U_{\widetilde{n_A}-m}$*.*

We can illustrate the existence of multiple coexisting equilibria under horizontal differentiation using Example 1 with the additional assumptions that $\widetilde{n_A} = \widetilde{n_B} = 5$ and $\theta = 10$. Then equal market shares for both platforms and 60-40 and 70-30 splits in favor of either platform constitute coexisting equilibria. Within these market share splits, any combination of agents choosing their preferred or non-preferred platforms constitute an equilibrium. Moreover, a 80-20 splits in favor of either platform where two pairs of men and women choose their preferred platform and all other agents choose the other platform (which is the preferred platform for five men and five women located there) is an equilibrium. In this example, the possible set of coexisting equilibria under horizontal differentiation is larger than that of the benchmark model when agents are rational. However, this depends on the size of θ. If $\theta = 100$, all agents choosing their preferred platforms is the only coexisting equilibrium.

To summarize, adding horizontal differentiation to the single-homing model under full rationality does little to clarify predictions about market structures. Moreover, depending on the type of equilibrium, market impact effects and horizontal differentiation can interact in peculiar ways. In a coexisting equilibrium where agents choose their preferred platform, horizontal differentiation aids in sustaining coexistence whereas in an equilibrium where agents choose non-preferred platform, market impact effects must be especially strong to overcome horizontal differentiation. Regardless, equilibrium coexistence is by no means assured—tipping remains an equilibrium.

Cognitive Hierarchy

Once again we relax the full rationality assumption. Our main result in this section is to show that the cognitive hierarchy model predicts a unique outcome—provided horizontal differentiation is sufficiently important, each strategic agent chooses her preferred platform and hence both platforms coexist in equilibrium.

While we were agnostic about the behavior of level-0 agents when horizontal differentiation was absent, here we place some (mild) additional structure on their choices: We assume that level-0 agents are weakly more likely to choose their preferred platform than their non–preferred platform. This rules out the bizarre case where being horizontally preferred reduces the chance that it is selected by a non-strategic agent.

We also require that the discount for horizontal differentiation be sufficiently large. Again, this is intuitive. Our baseline model is, in effect a special case of the horizontal differentiation model where the discount θ from choosing the preferred platform is zero. As we showed, in that case a single, dominant platform is chosen by all strategic agents. By continuity, if θ is small, this continues to be the case. Formally, the condition we require is that

Assumption 7: $\theta > U_j\left(1 - \min\left\{\frac{1}{2}, \frac{\widetilde{n}_i}{N}\right\}\right) - U_i\left(\min\left\{\frac{1}{2}, \frac{\widetilde{n}_i}{N}\right\}\right) - (p_i - p_j)$ for $i \in \{A, B\}$. Assumption 7 is fairly weak. Among other things, it merely ensures that when the choices of all other agents are random, it is better for an agent to choose her preferred platform over the non-preferred platform. With this assumption, we are now in a position to state our main result of this section:

Proposition 11 *When horizontal differentiation is sufficiently large, platforms coexist under cognitive hierarchy.*

Formally, suppose level-0 agents weakly choose their preferred platform and Assumptions 1-5, 7 hold. Then strategically sophisticated agents choose their preferred platform in the unique equilibrium.

We sketch the proof below, but relegate the formal proof to an appendix. When level-1 agents are determining which platform to select, they anticipate that level-0 agents are weakly more likely to choose their preferred platform. Notice that, even when level-0 agents are selecting randomly, Assumption 7 implies that level-1 agents optimally select their preferred platform. Likewise, when level-0 agents are always selecting their preferred platform, level-1 agents find it optimal to do so as well (since this is a Nash equilibrium under full rationality). Assumption 5 guarantees that, for any convex combination of these two extremes, it remains optimal for level-1 agents to choose their preferred platform. Level-2 agents likewise face a convex combination of random choice and selection based on preferred platforms and respond identically to level-1 agents. And so on for more sophisticated agents.

Comparing Propositions 3 and 11 reveals striking differences in market structure under bounded rationality. When horizontal differentiation is only a secondary consideration, there is a strong tendency toward industry concentration—all strategic agents choose the same platform—regardless of market impact effects. However, once horizontal differentiation becomes an important consideration, the industry tends to remain fragmented regardless of the magnitude of positive network externalities. Thus, the cognitive hierarchy model is capable of rationalizing the vast difference in the market structure of online auctions (extremely concentrated) and online dating markets (extremely fragmented). While the technology used by platforms in both of these markets is similar, idiosyncratic match characteristics (horizontal differentiation) are much more important in selecting a date or a life partner than they are in selecting a Beanie Baby or a new golf club. Differences in the market structure for video game consoles (fragmented) versus office software and high definition optical disc format (concentrated) can also be explained along the same lines.

6 Conclusion

While models of bounded rationality have been strongly embraced in interpreting data from laboratory experiments, their acceptance in applied settings has been much more limited. A compelling objection against their use is that the very flexibility that makes these models attractive for organizing lab data undermines their ability to make sharp predictions. For instance, quantal response equilibrium is a commonly used solution concept for analyzing experimental data, but, as shown by Haile, Hortacsu, and Kosenok (2008), its use is clearly problematic in applied settings as it can rationalize any set of data.

We showed that, in the setting of platform competition, the situation is exactly reversed. The standard, fully rational model can justify a wide range of market structures owing to the combination of complementarity and competitive effects in our setting. In contrast, the boundedly rational cognitive hierarchy model yields unique predictions. Moreover, by varying key features of the platform competition setting, such as the ability to multi-home or the degree to which the platforms are horizontally differentiated, we can identify which structural features lead to industry concentration versus those that lead to fragmentation. In particular, competition among agents of the same type, such as sellers on an online auction platform, does little to prevent the emergence of a dominant platform. Horizontal differentiation, however, leads to fragmentation even if the degree of differentiation is relatively modest.

It is, however, worth noting that our cognitive hierarchy model shares a defect common to many models of bounded rationality—the choice behavior of non-strategic players is a free variable and, even when these types are a vanishingly small fraction of the population, their choices play a critical role in the resulting decisions of strategic players. The situation is analogous to that of behavioral types in the reputation literature (see, e.g. Kreps et al., 1982). Despite this, several key qualitative features of industry structure, notably the emergence of a single platform accessed by *all* strategic types absent horizontal differentiation, occur regardless of the assumed behavior of naïve types.

Saying more requires judgment about the motives of non-strategic types. One interpretation is that these types are completely uninformed about the particulars of each platform and hence choose at random. In the single homing model, we showed that this connected the cognitive hierarchy model to a much older equilibrium refinements literature—choice behavior of strategic types corresponds to a risk dominant equilibrium. Thus, one (modest) contribution of the paper is to provide a behavioral micro-foundation for this refinement. But the predictions under bounded rationality do not always coincide with risk dominance. Allowing for multi-homing does not change the identity of the risk dominant platform but substantially changes the behavior of strategic types. They now respond with a combination of multi-homing and exclusively choosing the Pareto dominant platform.

Compared to theory offerings, the empirical literature on platform competition is relatively sparse. Certainly the complexity of these models combined with the resulting equilibrium multiplicity is not helpful in this regard. Perhaps our most important contribution is to show how allowing for bounded rationality gives rise to clear, testable predictions about how the structural features of platform competition translate into resulting market share performance. While our results are consistent with several features of these markets, an important next step is to carefully examine these predictions empirically. This remains for future research.

References

[1] Amaldoss, Wilfred, James R. Bettman, John W. Payne (2008): "Biased but Efficient: An Investigation of Coordination Facilitated by Asymmetric Dominance," *Marketing Science*, 27(5), 903-921.

[2] Ambrus, Attila and Rossella Argenziano (2009): "Asymmetric Networks in Two-sided markets," *American Economic Journal: Microeconomics*, 1(1), 17-52.

[3] Armstrong, Mark (2006): "Competition in Two-Sided Markets," *RAND Journal of Economics*, 37(3), 668-691.

[4] Brown, Alexander L., Colin F. Camerer, and Dan Lovallo (2009): "To Review or Not to Review? Limited Strategic Thinking at the Movie Box Office," Working Paper, Texas A&M University.

[5] Brown, Jennifer, and John Morgan (2009): "How much is a Dollar Worth? Tipping versus Equilibrium Coexistence on Competing Online Auction Sites," *Journal of Political Economy*, 117(4), 668-700.

[6] Caillaud, Bernard and Bruno Jullien (2001): "Competing Cybermediaries," *European Economic Review*, 45, 797-808.

[7] Caillaud, Bernard and Bruno Jullien (2003): "Chicken and Egg: Competition among Intermediation Service Providers." *RAND Journal of Economics*, 34(2), 521-552.

[8] Camerer, Colin F., Teck-Hua Ho, and Juin-Kuan Chong (2004): "A Cognitive Hierarchy Model of Games," *Quarterly Journal of Economics*, 119(3), 861-898.

[9] Carrillo, Juan and Guofu Tan (2006): "Platform Competition: The Role of Multihoming and Complementors," Working Paper, University of Southern California.

[10] Costa-Gomes, Miguel A., and Vincent P. Crawford (2006): "Cognition and Behavior in Two-Person Guessing Games: An Experimental Study," *American Economic Review*, 96(5), 1737-1768.

[11] Damiano, Ettore and Li, Hao (2008): "Competing Matchmaking," *Journal of the European Economic Association*, 6(4), 789-818.

[12] David, Paul A. (1985): "Clio and the Economics of QWERTY," *American Economic Review*, 77(2): 332-337.

[13] Ellison, Glenn, and Drew Fudenberg (2003): "Knife-Edge or Plateau: When Do Market Models Tip?," *Quarterly Journal of Economics*, 118(4), 1249-1278.

[14] Ellison, Glenn, Drew Fudenberg, and Markus Möbius (2004): "Competing Auctions," *Journal of the European Economic Association*, 2(1), 30-66.

[15] Goldfarb, Avi and Mo Xiao (2010): "Who Thinks About the Competition? Managerial Ability and Strategic Entry in US Local Telephone Markets," *American Economic Review*, Forthcoming.

[16] Goldfarb, Avi and Botao Yang (2009): "Are All Managers Created Equal?" *Journal of Marketing Research*, 46(5), 612-622.

[17] Haile, Philip A., Ali Hortacsu, and Grigory Kosenok (2008): "On the Empirical Content of Quantal Response Equilibrium," *American Economic Review*, 98(1), 180-200.

[18] Harsanyi, John C., and Reinhard Selten (1988): *A General Theory of Equilibrium Selection in Games*, MIT Press.

[19] Ho, Teck H., Noah Lim, and Colin F. Camerer (2006): "Modeling the Psychology of Consumer and Firm Behavior with Behavioral Economics," *Journal of Marketing Research*, 43(3), 307-331.

[20] Hossain, Tanjim, Dylan Minor, and John Morgan (2011): "Competing Matchmakers: An Experimental Analysis," *Management Science*, Forthcoming.

[21] Katz, Michael L. and Carl Shapiro (1994): "Systems Competition and Network Effects," *Journal of Economic Perspectives*, 8 (2), 93-115.

[22] Kreps, David, Paul Milgrom, John Roberts, and Robert Wilson (1982): "Rational Cooperation in the Finitely Repeated Prisoners' Dilemma," *Journal of Economic Theory*, 27(2), 245-252.

[23] Liebowitz, S. J. and Stephen E. Margolis (1990): "The Fable of the Keys," *Journal of Law and Economics*, 33(1): 1-25.

[24] Liebowitz, S. J. and Stephen E. Margolis (1994): "Network Externality: An Uncommon Tragedy," *Journal of Economic Perspectives*, 8(2): 133-150.

[25] Nagel, Rosemarie (1995): "Unravelling in Guessing Games: An Experimental Study," *American Economic Review*, 85(5), 1313-1326.

[26] Rochet, Jean-Charles and Jean Tirole (2003): "Platform Competition in Two-Sided Markets," *Journal of the European Economic Association*, 1(4), 990-1029.

[27] Stahl, Dale O., and Paul Wilson (1995): "On Players' Models of Other Players: Theory and Experimental Evidence," *Games and Economic Behavior*, 10(1), 213-254.

[28] Tellis, Gerard J., Yiding Yin, and Rakesh Niraj (2009): "Does Quality Win? Network Effects versus Quality in High-Tech Markets," *Journal of Marketing Research*, 46(2), 135-149.

A Proofs

Proof of Lemma 1

Proof. Suppose, in an equilibrium, s women and t men enter platform i. Without loss of any generality, we assume $s > t$. Since women in platform i have no incentive to move to platform j,

$$\begin{aligned} u_i(s,t) - p_i &\geq u_j(N-s+1, N-t) - p_j \\ \Rightarrow p_j - p_i &\geq u_j(N-s+1, N-t) - u_i(s,t). \end{aligned}$$

The assumption of $s > t$ implies

$$u_i(s,t) \leq u_i(t+1,t) < u_i(t+1,s)$$

and

$$u_j(N-(s-1), N-t) \geq u_j(N-t, N-t) > u_j(N-t, N-s).$$

Therefore,

$$\begin{aligned} u_j(N-s+1, N-t) - u_i(s,t) &> u_j(N-t, N-s) - u_i(t+1,s) \\ \Longrightarrow p_j - p_i &> u_j(N-t, N-s) - u_i(t+1,s) \\ \Longrightarrow u_i(t+1,s) - p_i &> u_j(N-t, N-s) - p_j. \end{aligned}$$

However, this implies that men in platform j will have incentives to move to platform i. Therefore, if s women and t men entering platform i is an equilibrium, then $s = t$. ■

Proof of Proposition 3

Proof. Suppose we draw $U_A(\lambda) - p_A$ and $U_B(1-\lambda) - p_B$ on the same graph for $\lambda \in [0,1]$. Given the market size and positive network externalities effects,

$$U_i(1) - p_i = u_i(N,N) - p_i > U_j(0) - p_j = u_j(1,0) - p_j$$

for $i, j \in \{A, B\}$. If both U_A and U_B are increasing functions of the probability of an agent choosing that platform, then that immediately implies single-crossing of the two curves. Otherwise, $U_B(\lambda) - p_B$ and $U_A(1-\lambda) - p_A$ will intersect at most twice given Assumption 5. However, if they intersect twice then $U_i(1) - p_i$ must be smaller than $U_j(0) - p_j$ with $i \neq j$ for at least one i, which is impossible. This implies that $U_A(\lambda) - p_A$ and $U_B(1-\lambda) - p_B$ intersect exactly once and there is a unique λ^* such that $U_A(\lambda^*) - p_B = U_B(1-\lambda^*) - p_B$. Moreover, $U_A(\lambda) - p_A < U_B(1-\lambda) - p_B$ for all $\lambda < \lambda^*$ and $U_A(\lambda) - p_A > U_B(1-\lambda) - p_B$ for all $\lambda > \lambda^*$.

Now we analyze the equilibria under the cognitive hierarchy model. A level-0 agent chooses to join platform i with probability λ_i. As a level-1 agent assumes that all other agents are of level-0, her expected payoff from joining platforms A and B are $U_A(\lambda_A) - p_A$ and $U_B(1-\lambda_A) - p_B$, respectively. First suppose $\lambda_A < \lambda^*$. Then, all level-1 agents will choose to go to platform B. A level-2 agent believes that any of the other agents is of level-0 with probability $\frac{f(0)}{f(0)+f(1)}$ and of level-1 with probability $\frac{f(1)}{f(0)+f(1)}$. Moreover, the agent believes that a level-0 agent chooses platform B with probability $1-\lambda_A$ and a level-1 agent chooses platform B with probability 1. The expected payoff of a level-2 agent from platform A and B are $U_A\left(\frac{\lambda_A f(0)}{f(0)+f(1)}\right) - p_A$ and $U_B\left(\frac{(1-\lambda_A)f(0)+f(1)}{f(0)+f(1)}\right) - p_B$, respectively. As $\frac{\lambda_A f(0)}{f(0)+f(1)} < \lambda_A < \lambda^*$, a level-2 agent will choose platform B. It can easily be shown that, a level-l agent believes that another agent chooses platform B with probability $1 - \frac{\lambda_A f(0)}{\sum_{k=0}^{l-1} f(k)}$ for all $l \geq 1$. As a result, her best response is to join platform B. Similar logic shows that if $\lambda_A > \lambda^*$, then all level-l agents will choose platform A for $l \geq 1$. ■

Proof of Proposition 4

Proof. The proof that tipping is an equilibrium is analogous to the argument in Proposition 1.

To establish conditions where coexisting equilibria exist, suppose for some $n \in \{1, 2, \ldots, N-1\}$,

$$u_A(n,n) \geq p_A \geq u_A(n+1,n) \quad (8)$$

$$u_B(N-n, N-n) \geq p_B \geq u_B(N-n+1, N-n). \quad (9)$$

Note that, for each equation, one of the inequalities will be strict because of the market impact effects. Then n players of each type choosing platform A and $N-n$ players of each type choosing platform B is an equilibrium. Under these platform choices, all agents make non-negative payoff. If a female agent on platform B also joins platform A, she will have access to n new male agents while competing with n other female agents and paying an access fee of p_A. However, as $u_A(n+1,n) \leq p_A$, she will have no incentive to multi-home. She will also have no incentive to choose platform A exclusively. Similarly, as $u_B(n+1,n) \leq p_B$, an agent on platform A will have no incentive to switch to platform B or multi-home. Subtracting $u_A(n,n)$ from the inequalities in equation (8) and $u_B(N-n,N-n)$ from the inequalities in equation (9) yields the inequalities in equation (3). ■

Proof of Proposition 5

Proof. To ensure that the proposed equilibrium exists, the following conditions need to be satisfied. An agent who is single-homing on platform i will not deviate to single-home on platform j if

$$u_i(n_i,n_i) - p_i \geq u_j(n_j+1,n_j) - p_j \tag{10}$$

and will not multi-home if

$$u_i(n_i,n_i) - p_i \geq u_i(n_i,n_i) - p_i + u_j(n_j+1,N-n_i) - p_j. \tag{11}$$

On the other hand, an agent single-homing on platform j will not single-home on the platform i and will not choose to multi-home if

$$u_j(n_j,n_j) - p_j \geq u_i(n_i+1,n_i) - p_i \tag{12}$$

and

$$u_j(n_j,n_j) - p_j \geq u_i(n_i+1,n_i) - p_i + u_j(n_j,N-n_i) - p_j, \tag{13}$$

respectively. Finally, an agent who chooses to multi-home in this equilibrium will not deviate by choosing just one of the platforms if

$$u_i(n_i,n_i) - p_i + u_j(n_j,N-n_i) - p_j \geq u_j(n_j,n_j) - p_j \tag{14}$$

and

$$u_i(n_i,n_i) - p_i + u_j(n_j,N-n_i) - p_j \geq u_i(n_i,n_i) - p_i. \tag{15}$$

We next rearrange and simplify these six equilibrium conditions. Equations (10) and (12) together imply

$$u_i(n_i+1,n_i) - u_j(n_j,n_j) \leq p_i - p_j \leq u_i(n_i,n_i) - u_j(n_j+1,n_j).$$

Equations (11) and (15) lead to

$$u_j(n_j, N - n_i) \geq p_j \geq u_j(n_j + 1, N - n_i)$$

and the two remaining equations suggest that

$$u_i(n_i, n_i) + u_j(n_j, N - n_i) - u_j(n_j, n_j) \geq p_i \geq u_i(n_i + 1, n_i) + u_j(n_j, N - n_i) - u_j(n_j, n_j).$$

Writing these expressions in terms of market impact effects yields the set of inequalities in the statement of the proposition. ■

Proof of Proposition 6

Proof. Suppose $U_i(N, N) - p_i > U_j(N, N) - p_j$ and all level-0 agents join both platforms i and j. A level-1 agent assumes that all other agents are of level 0. Hence, she believes that all other agents join both platforms. Given that belief, if she joins only platform j, her net payoff is $U_j(N, N) - p_j$ and her expected payoff if she joins only platform i is $U_i(N, N) - p_i$. If she joins both platforms then she does not gain any benefit from joining platform j as she meets all the agents of the opposite type already at the Pareto dominant platform i. Her net payoff from multi-homing, thus, is $U_i(N, N) - p_A - p_B$. Hence, all level-1 agents will choose to go to platform i. A level-2 agent believes that any of the other agents is of level 0 with probability $\frac{f(0)}{f(0)+f(1)}$ and of level 1 with probability $\frac{f(1)}{f(0)+f(1)}$. Moreover, she believes that all other agents join platform i and level-0 agents join platform j in addition to joining platform i. Hence, her optimal action is to join only platform i. Similar arguments show that all agents with a higher level of cognitive ability will choose to join only platform i. In the unique equilibrium, a level-0 agent joins both platform and a level-l agent joins only platform i for all $l \geq 1$. ■

Proof of Proposition 8

Proof. Since i is the super-dominant platform, any agent with sophistication level of 1 or higher will never choose single-homing on platform j over single-homing on platform i. Moreover, given Assumptions 5 and 6, both U_{mh} and $U_{sh,i}(\lambda, \lambda)$ are single-peaked in λ. Note that $U_{mh}(0, 0) - p_A - p_B < U_{sh,i}(0, 0) - p_i$ and $U_{mh}\left(\frac{1}{2}, \frac{1}{2}\right) - p_A - p_B > U_{sh,i}\left(\frac{1}{2}, \frac{1}{2}\right) - p_i$. Using similar logic to those in the proof of Proposition 3, one can show that there is exactly one λ^* such that $U_{mh}(\lambda^*, \lambda^*) - p_A - p_B = U_{sh,i}(\lambda^*, \lambda^*) - p_i$ and $U_{mh}(\lambda^*, \lambda^*) - p_A - p_B < U_{sh,i}(\lambda^*, \lambda^*) - p_i$ for $\lambda < \lambda^*$ and $U_{mh}(\lambda^*, \lambda^*) - p_A - p_B > U_{sh,i}(\lambda^*, \lambda^*) - p_i$ for $\lambda > \lambda^*$.

Now we analyze the best responses of sophisticated agents given level-0 agents' behavior. First, consider the case that $\widetilde{\lambda} < \lambda^*$; that is, relatively few level-0 agents choose a platform exclusively. Then it is optimal for a level-1 agent to choose only platform i as $U_{mh}\left(\widetilde{\lambda}, \widetilde{\lambda}\right) -$

$p_A - p_B < U_{sh,i}\left(\widetilde{\lambda}, \widetilde{\lambda}\right) - p_i$. A level-2 agent then believes that other agents choose platforms i and j exclusively with probabilities $\frac{\widetilde{\lambda} f(0)+f(1)}{f(0)+f(1)}$ and $\frac{\widetilde{\lambda} f(0)}{f(0)+f(1)}$, respectively and chooses to multi-home with probability $\frac{\left(1-2\widetilde{\lambda}\right) f(0)}{f(0)+f(1)}$. That is, according to her beliefs, more agents join platform i exclusively and fewer agents join platform j exclusively compared to the beliefs of level-1 agents. Given Assumption 6, she gets strictly higher payoff by single-homing on platform i than multi-homing and will choose platform i exclusively in any equilibrium. Similarly, one can show that all level-l agents for $l \geq 1$ will choose platform i when $\widetilde{\lambda} < \lambda^*$.

Next suppose $\widetilde{\lambda} \geq \lambda^*$. Then, it is optimal for level-1 agents to multi-home. A level-2 agent believes that all other agents are of level 0 or 1 and will choose platforms A or B exclusively with probability $\frac{\widetilde{\lambda} f(0)}{f(0)+f(1)}$ each and will multi-home with probability $\frac{\left(1-2\widetilde{\lambda}\right) f(0)+f(1)}{f(0)+f(1)}$. If $\frac{\widetilde{\lambda} f(0)}{f(0)+f(1)} > \lambda^*$ then the level-2 agent will multi-home. Otherwise, she will choose platform i exclusively. In general, suppose $\underline{k} > 1$ is such that $\frac{\widetilde{\lambda} f(0)}{\sum_{k=0}^{\underline{k}-2} f(k)} \geq \lambda^* > \frac{\widetilde{\lambda} f(0)}{\sum_{k=0}^{\underline{k}-1} f(k)}$. Then agents of level l will multi-home for $l < \underline{k}$ and will choose platform i exclusively for $l \geq \underline{k}$ in the unique equilibrium. ■

Proof of Proposition 10

Proof. Suppose all agents choose to join their preferred platform. That is, $\widetilde{n_A}$ pairs of males and females join platform A and $\widetilde{n_B}$ pairs of males and females join platform B. If

$$\Delta U_{\widetilde{n_A}} = u_A\left(\widetilde{n_A}, \widetilde{n_A}\right) - p_A - u_B\left(\widetilde{n_B}, \widetilde{n_B}\right) + p_B \geq 0$$

then, given the benefit from choosing one's own preferred platform (θ) and the market impact effects, an agent located on platform A will have no incentive to join platform B instead. Now, if $\delta_{A,\widetilde{n_A}} + \theta \geq \Delta U_{\widetilde{n_A}}$ then

$$\begin{aligned} u_A\left(\widetilde{n_A}, \widetilde{n_A}\right) - u_A\left(\widetilde{n_A}+1, \widetilde{n_A}\right) + \theta &\geq u_A\left(\widetilde{n_A}, \widetilde{n_A}\right) - p_A - u_B\left(\widetilde{n_B}, \widetilde{n_B}\right) + p_B \\ \Longrightarrow u_B\left(\widetilde{n_B}, \widetilde{n_B}\right) - p_B + \theta &\geq u_A\left(\widetilde{n_A}+1, \widetilde{n_A}\right) - p_A. \end{aligned}$$

In that case, an agent locating on platform B will have no incentive to join platform A instead. Similarly, if $\Delta U_{\widetilde{n_A}} < 0$ then $\delta_{B,\widetilde{n_B}} + \theta \geq -\Delta U_{\widetilde{n_A}}$ ensures that none of the agents will have an incentive to deviate from the strategy of choosing her preferred platform.

Now suppose all agents join their non-preferred platforms. That is, $\widetilde{n_B}$ pairs of males and females join platform A and $\widetilde{n_A}$ pairs of males and females join platform B. An agent on platform A receives a net payoff of $u_A\left(\widetilde{n_B}, \widetilde{n_B}\right) - p_A$. If she decided to join her preferred platform B instead, she can earn a net payoff of $u_B\left(\widetilde{n_A}+1, \widetilde{n_A}\right) - p_B + \theta$. Suppose $\delta_{B,\widetilde{n_A}} - \theta \geq -\Delta U_{\widetilde{n_B}}$. In that case,

$$\begin{aligned} u_B\left(\widetilde{n_A}, \widetilde{n_A}\right) - u_B\left(\widetilde{n_A}+1, \widetilde{n_A}\right) - \theta &\geq -u_A\left(\widetilde{n_B}, \widetilde{n_B}\right) + p_A + u_B\left(\widetilde{n_A}, \widetilde{n_A}\right) - p_B \\ \Rightarrow u_A\left(\widetilde{n_B}, \widetilde{n_B}\right) - p_A &\geq u_B\left(\widetilde{n_A}+1, \widetilde{n_A}\right) - p_B + \theta. \end{aligned}$$

Therefore, an agent located on platform A will have no incentive to locate on her preferred platform B instead. Similarly, agents locating on platform B will have no incentive to locate on platform A if $\delta_{A,\widetilde{n_B}} - \theta \geq \Delta U_{\widetilde{n_B}}$.

Finally, suppose $\widetilde{n_A} - m$ pairs of male and female agents choose their preferred platform A, m pairs of male and female agents choose their non-preferred platform B and $\widetilde{n_B}$ pairs of male and female agents choose their preferred platform B for some $m \in \{1, 2, \ldots, \widetilde{n_A} - 1\}$. Now, if $\delta_{A,\widetilde{n_A}-m} - \theta \geq \Delta U_{\widetilde{n_A}-m}$ then

$$\begin{aligned} -u_A\left(\widetilde{n_A} - m + 1, \widetilde{n_A} - m\right) - \theta &\geq -u_B\left(\widetilde{n_B} + m, \widetilde{n_B} + m\right) - p_A + p_B \\ \Rightarrow u_B\left(\widetilde{n_B} + m, \widetilde{n_B} + m\right) - p_B &\geq u_A\left(\widetilde{n_A} - m + 1, \widetilde{n_A} - m\right) - p_A + \theta. \end{aligned}$$

In that case, an agent who is located on her platform B will have no incentive to join platform A instead no matter whether her preferred platform is A or B. If $\delta_{B,\widetilde{n_B}+m} + \theta \geq -\Delta U_{\widetilde{n_A}-m}$ then

$$\begin{aligned} -u_B\left(\widetilde{n_B} + m + 1, \widetilde{n_B} + m\right) + \theta &\geq -u_A\left(\widetilde{n_A} - m, \widetilde{n_A} - m\right) + p_A - p_B \\ \Longrightarrow u_A\left(\widetilde{n_A} - m, \widetilde{n_A} - m\right) - p_A + \theta &\geq u_B\left(\widetilde{n_B} + m + 1, \widetilde{n_B} + m\right) - p_B. \end{aligned}$$

Therefore, an agent locating on her preferred platform A will have no incentive to switch to platform B. Note that this condition is trivially satisfied when $\Delta U_{\widetilde{n_A}-m} \geq 0$. ■

Proof of Proposition 11

Proof. Suppose each level-0 agent chooses her preferred platform with probability $\widetilde{\lambda} \geq \frac{1}{2}$. Given the bound on θ stipulated by equation (6), $U_i(0) - p_i + \theta < U_j(1) - p_j$ for $i \in \{A, B\}$. Moreover, $U_i(1) - p_i + \theta > U_j(0) - p_j$. Assumption 5 implies single-crossing of $U_i(\lambda) - p_i + \theta$ and $U_j(1-\lambda) - p_j$ for $\lambda \in [0,1]$, $i \in \{A, B\}$ and $j \neq i$. Assumption 7 then implies that for all $\lambda > \min\left\{\frac{1}{2}, \frac{\widetilde{n_i}}{N}\right\}$, $U_i(\lambda) - p_i + \theta > U_j(1-\lambda) - p_j$. Consider a level-1 agent who prefers platform i. She believes that all agents are of level 0 and each of them chooses platform i with probability $\widetilde{\lambda}\frac{\widetilde{n_i}}{N} + \left(1 - \widetilde{\lambda}\right)\frac{\widetilde{n_j}}{N}$. If $\widetilde{n_i} \geq \widetilde{n_j}$ then $\frac{\widetilde{n_i}}{N} \geq \widetilde{\lambda}\frac{\widetilde{n_i}}{N} + \left(1 - \widetilde{\lambda}\right)\frac{\widetilde{n_j}}{N} \geq \frac{1}{2}$ and if $\widetilde{n_i} < \widetilde{n_j}$ then $\frac{1}{2} \geq \widetilde{\lambda}\frac{\widetilde{n_i}}{N} + \left(1 - \widetilde{\lambda}\right)\frac{\widetilde{n_j}}{N} \geq \frac{\widetilde{n_i}}{N}$. Therefore, a level-1 agent who prefers platform i will choose platform i. A level-2 agents believe that level-0 agents choose platform i with probability $\widetilde{\lambda}\frac{\widetilde{n_i}}{N} + \left(1 - \widetilde{\lambda}\right)\frac{\widetilde{n_j}}{N}$ and level-1 agents choose their preferred platforms. That is, she believes that an agent is likely to choose platform i with probability $\frac{\left(\widetilde{\lambda}\frac{\widetilde{n_i}}{N} + \left(1-\widetilde{\lambda}\right)\frac{\widetilde{n_j}}{N}\right)f(0) + \frac{\widetilde{n_i}}{N}f(1)}{f(0)+f(1)}$. Of course, if $\widetilde{n_i} \geq \widetilde{n_j}$ then $\frac{\widetilde{n_i}}{N} \geq \frac{\left(\widetilde{\lambda}\frac{\widetilde{n_i}}{N} + \left(1-\widetilde{\lambda}\right)\frac{\widetilde{n_j}}{N}\right)f(0) + \frac{\widetilde{n_i}}{N}f(1)}{f(0)+f(1)} \geq \frac{1}{2}$ and $\frac{1}{2} \geq \frac{\left(\widetilde{\lambda}\frac{\widetilde{n_i}}{N} + \left(1-\widetilde{\lambda}\right)\frac{\widetilde{n_j}}{N}\right)f(0) + \frac{\widetilde{n_i}}{N}f(1)}{f(0)+f(1)} \geq \frac{\widetilde{n_i}}{N}$ otherwise. Therefore, a level-2 agent who prefers platform i will choose platform i. In general, an agent of sophistication level l for $l > 0$, whose preferred platform is platform i, believes that her expected net payoffs from joining platforms i and j are $U_i(\lambda) - p_i + \theta$ and $U_j(1-\lambda) - p_j$, respectively for some $\lambda \in \left[\frac{1}{2}, \frac{\widetilde{n_i}}{N}\right]$ if $\widetilde{n_i} \geq \widetilde{n_j}$ and for some $\lambda \in \left[\frac{\widetilde{n_i}}{N}, \frac{1}{2}\right]$ otherwise. Thus, all agents with sophistication level $l > 0$ will choose their preferred platform in the unique equilibrium. ■

B Endogenizing Access Fees

While the model treats access fees as exogenous, in this section we show that coexistence is consistent with equilibrium even when platforms choose fees optimally. Specifically, suppose that platforms simultaneously choose access fees prior to agents deciding on which platform to locate. As is the case in the rest of the model, platforms charge the same access fee to male and female agents. The following proposition shows that the key condition for coexistence is that the magnitude of the market impact effects must be sufficiently large. Formally,

Proposition 12 *Suppose that market impact effects are such that, for some $n \in \{1, ..., N-1\}$*

$$\begin{aligned} \delta_{i,n} &\geq \frac{N-n}{n} u_i(n+1, n) \\ \delta_{j,N-n} &\geq \frac{n}{N-n} u_j(N-n+1, N-n) \end{aligned}$$

Then it is a coexisting equilibrium for n agents of each type to choose platform i with the remainder choosing platform j where i charges $p_i = u_i(n, n)$ and j charges $p_j = u_j(N-n, N-n)$.

Proof. Consider the following proposed equilibrium. First, platforms i and j choose access fees $p_i = u_i(n, n)$ and $p_j = u_j(N-n, N-n)$. Then, agents 1 to n of each type, for some $n \in \{1, 2, \ldots, N-1\}$, follow the following strategy: choose platform i if

$$u_i(n, n) - p_i \geq u_j(N-n+1, N-n) - p_j \text{ and } u_i(n, n) \geq p_i,$$

choose platform j otherwise as long as $u_j(N-n+1, N-n) \geq p_j$ and else choose neither platform. Similarly, agents $n+1$ to N of each type choose platform j if

$$u_j(N-n, N-n) - p_j \geq u_i(n+1, n) - p_i \text{ and } u_j(N-n, N-n) \geq p_j.$$

Then, first n pairs of male and female agents join platform i because they get zero net payoff from platform i and negative net payoff from platform j. The remaining agents join platform j because they get zero net payoff from that platform and negative net payoff from platform i. Now, platforms i and j will have no incentive to change their pricing in the first stage if they cannot raise profit by choosing different access fees. Take platform i: to attract agents who would choose platform j otherwise (agents 1 to n of each type), it needs to charge an access fee of $u_i(n+1,n)$ or lower. In that case, all agents will choose platform i. This is not profitable if

$$\begin{aligned} nu_i(n,n) &\geq Nu_i(n+1,n) \\ \Longrightarrow n\left(u_i(n,n)-u_i(n+1,n)\right) &\geq (N-n)\,u_i(n+1,n) \\ \Longrightarrow n\delta_{i,n} &\geq (N-n)\,u_i(n+1,n) \\ \Longrightarrow \delta_{i,n} &\geq \frac{N-n}{n}u_i(n+1,n). \end{aligned}$$

Similarly, platform j will not try to attract agents otherwise choosing platform i by reducing p_j if

$$\begin{aligned} (N-n)\,u_j(N-n,Nn) &\geq Nu_j(N-n+1,N-n) \\ \Longrightarrow \delta_{j,n} &\geq \frac{n}{N-n}u_j(N-n+1,N-n). \end{aligned}$$

Thus, the proposed strategies constitute a subgame perfect coexisting equilibrium where platforms choose profit maximizing access fees. ■

While Proposition 12 specifies conditions on market impact effects where coexistence can occur in equilibrium, one may worry about whether such conditions can ever be satisfied. To allay this concern, notice that the market in Example 1 supports three coexisting equilibria when platform choose the access fees. Five pairs of men and women joining each platform with $p_A = p_B = 47.78$ is an equilibrium. Moreover, 4 pairs of men and women joining platform i and 6 pairs of men and women joining platform j with $p_i = 38.33$ and $p_j = 57.22$ are equilibria for $i \in \{A, B\}$. Hence, unequal market shares are also consistent with optimal fee choice by platforms.