Information Aggregation in Bargaining

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Abstract

We study information aggregation in a bilateral bargaining model with two-sided incomplete information. In particular, we consider the case in which the size of the common-value pie is stochastic (i.i.d. over time) and players receive private signals on the size of the pie each period. Efficient agreement is a stochastic rule; delay is efficient if the expected size of the today's pie is small and the discount factor is high. Hence information aggregation is crucial for efficiency. We consider a random-proposer bargaining in which the proposer can make an offer of the deal upon agreement, potentially with a message about its own signal. In the case of divisible pie without transfers, an equilibrium that attains efficient agreement rule exists unless the discount factor is in the intermediate range and the accuracy of information is high (except for perfect information). We also show that cheap talk does not expand the attainability of efficient equilibrium. In the case of indivisible pie with transfers, an equilibrium that attains efficient agreement rule exists if the transfer can take continuous values, whereas the attainability is limited if the transfer is discrete.

Keywords: asymmetric information bargaining, information aggregation, common value

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1 Introduction

People exchange information in negotiations. Doing so does not help if players have perfect information about the size of the pie as in Rubinstein (1982). This is also the case as long as the players have common knowledge about the size of the pie today even if the size of the future pie is uncertain as in Merlo and Wilson (1995, 1998). In actual negotiations, however, people do not necessarily agree on how large the exact size of the pie is. If the players have private signal about the size of the pie that they would share, they will benefit from aggregating information across players, especially when the future size or value of the pie is uncertain.

Information aggregation allows the players to collectively make the efficient decision, i.e., to reach an agreement today if and only if the size of the today's pie evaluated with the information is larger than the continuation social value. However, information aggregation may not be straightforward because players' interest can be potentially conflicting. If a player can enjoy better terms of agreement by manipulating information, the player might like to distort the efficient agreement decision for a rent. In other words, a player might have an incentive not to disclose the private signal in order to seek a larger share of the pie at the cost of inefficient agreement decision.

In this paper, we ask to what degree the players can aggregate information in bilateral bargaining, and how efficient the resulting bargaining outcome can be. While information aggregation is studied extensively in the auction and the voting literature, it has attracted less attention in the bargaining literature in spite of its importance. In many bargaining situations, players bargain over a pie having different information about the size of it. Consider, for example, a venture capitalist and an entrepreneur. The common-value profitability of the venture is uncertain, and both parties have private signal about the profitability. They bargain whether to start the venture and how to divide the earnings, or wait. Another example is bargaining between randomly matched buyer and seller, who only have a noisy signal about the size of the surplus from the trade, and will be rematched with other player if they cannot agree. The players have different information about the true size of the surplus, and they will benefit from information aggregation because they gain if they could trade when the surplus is large (and wait for another random match if surplus is small).[1]

To address this issue, we consider a random proposer stochastic bargaining game in which each player has private signal about the size of the pie. The timing of the stage game

[1] Our environment differs from that of Myerson and Satterthwaite (1983) studying bilateral trade with *independent* private values: Our model can be interpreted as bargaining between a buyer and a seller with (perfectly) *correlated* values. Evans (1989) and Vincent (1989) study bargaining with correlated values, assuming that the size of the pie is *persistent*, the incomplete information is one-sided, and only uninformed party makes offers.

is as follows. First, the nature chooses a proposer with equal probability, and a true size of the pie is determined (either high or low with probability a half). Then, each player receives a conditionally independent private signal (also binary) on the size of the pie. The accuracy of signal is measured by the conditional probability that a player receives a high signal given the true size is large. The proposer makes a take-it-or-leave-it offer, which is a function of her private signal, to the responder. The responder chooses whether to accept or reject the offer by inferring the size of the pie using his own signal together with the proposer's offer. The game ends if the responder agrees, but continues to the next period with a new i.i.d. draw of the size of the pie each period.[2]

We also allow players to engage in cheap talk after they receive their signals each period. It is natural to think that people communicate in bargaining with common value, and we show that communication can play a role in our environment. However, we also show that allowing cheap talk does not expand the availability of efficiency, compared with signaling equilibrium.

We begin our analysis by characterizing the efficient agreement rule that maximizes the ex ante social value as a mapping from aggregated private signals $\{h, l\} \times \{h, l\}$ to agreement decisions in a stage $\{Agree, Disagree\}$.[3] This generalizes the efficiency in Merlo and Wilson (1995, 1998) for complete information environment. Efficiency in this context does not imply immediate agreement because of the stochastic nature of the pie. Waiting for a better realization in the next period would be more efficient than agreeing with expectation of a small size of the pie today if the discount factor is high enough. Therefore information aggregation is necessary for efficient delay.

Indeed, there are three possible efficient agreement rules (Proposition 1). For a high discount factor, the players should agree only if they both receive high signals (Agreement Rule 1). For an intermediate discount factor, they should agree unless they both receive low signals (Agreement Rule 2). For a low discount factor, they should agree immediately (Agreement Rule 3). The degree to which efficient information aggregation requires a high discount factor depends on the precision of the signals: the more accurate the signals, the lower the required level of the discount factor for efficient information aggregation. This is because the more accurate signal lowers the likelihood of agreeing when the pie is small.

We then study stationary perfect Bayesian equilibrium of the bargaining game and ask whether an equilibrium exists that attains each efficient agreement rule on the equilibrium path. We initially focus on the case in which the pie is divisible but monetary transfer is

[2] The i.i.d. assumption helps us focus on the issue of information aggregation across players and avoid the complexity due to intertemporal transmission of information.

[3] The i.i.d. assumption implies the irrelevance of history for efficient agreement. We assume transferable utility and thus terms of agreement for players are also irrelevant for efficiency. Note that the efficient agreement rule is based on the information profile at the interim stage but not on the unknown, true size of the pie. An actual efficient agreement could end up with a low social value ex post.

unavailable.

When Agreement Rule 1 is efficient, an equilibrium exists that attains it (Proposition 2). To construct the equilibrium, we adopt *the pessimistic belief* off-the-equilibrium-path, i.e., the responder believes that the proposer's signal is low. With such belief, the responder's tendency to reject an off-path offer is reinforced and thus the proposer's incentive to deviate to such offer is discouraged. A key finding here is that the proposer's bargaining advantage is washed out by this belief; the responder retains a rent by the threat that it can rationally reject a more demanding offer due to the change in belief.[4] As a result, the equilibrium share that can support Rule 1 falls around 1/2; each player in effect "owns" about a half of the pie. The player basically shares interests with the social planner as of information aggregation and agreement decision, and hence behaves in accordance with the efficient agreement rule. Note that cheap talk and signaling work the same in this case; in a cheap talk equilibrium, the proposer offers α with a cheap talk of its own signal, whereas in a signaling equilibrium, the proposer with a high signal offers α but with a low signal offers too demanding share (Corollary 1).

When Agreement Rule 2 is efficient, an equilibrium that attains the rule is not always available. Proposition 3 and 4 characterize the sets of discount factor and signal accuracy for which an associated cheap talk equilibrium and a signaling equilibrium exist, respectively. Consider first a cheap talk equilibrium, in which the proposer demands a share α with a message and the off-path belief is again pessimistic. By contrast to equilibrium for Rule 1, the concern about off-path deviation plays an important role to pin down α; the equilibrium share α needs to be sufficiently high, otherwise the proposer with a low signal would deviate to offer a higher off-path share.[5] But this causes another incentive problem; in cheap talk equilibrium, the proposer with a low signal is tempted to lie. By lying to have a high signal, the proposer induces the responder to accept the offer, even when the signals are both low. If the proposer's share upon agreement is substantially higher than a half, the proposer might prefer today's inefficient agreement to the continuation value.[6] Consequently, an efficient cheap talk equilibrium fails to exist if the discount factor falls below a certain level. On the other hand, in a signaling equilibrium in which the proposer offers different shares for different signals, the incentive condition is relaxed, and therefore an efficient equilibrium

[4]To attain Agreement Rule 1, the responder accepts the offer if and only if the responder expects a large size of the pie, by receiving a high signal and believing the proposer's signal being high. With the pessimistic belief, the responder expects a smaller size of the pie, and would not accept the offer n any case. The proposer with a high signal would not demand more because of this threat.

[5]On the equilibrium path, the proposer with a low signal reveals its information, and therefore the pessimistic belief does not play a role to limit the deviation incentive to make an off-path share.

[6]Because of the random assignment of proposer, the ex ante expected share upon agreement is a half, on the one hand. On the other hand, the proposer's share upon agreement becomes higher than a half after the assignment (due to off-path deviation concern), and therefore the proposer restores a bargaining advantage (effective only within the period).

exists for a larger set of parameter values. Note, however, that the incentive condition cannot be relaxed fully. We find that if the accuracy is high and the discount factor is intermediate, there is no efficient equilibrium.

When Agreement Rule 3 is efficient, agreement should occur immediately and therefore information aggregation is unnecessary for efficiency. We show that an equilibrium that attains the rule fails to exist as the accuracy of information improves (Proposition 5). As the accuracy improves, the proposer with a high signal considers it is more likely that the responder receives the same signal. The proposer would then find it optimal to demand a higher share that is only acceptable by the responder with a high signal, contradicting to pooling equilibrium that is necessary for Agreement Rule 3.

Finally, we consider the case that the pie is not divisible, and players make monetary transfer. We show that the efficient outcome can be achieved for all areas of parameters as long as the monetary transfers can take continuous values. The proposer demands the entire pie by promising the payment that equals the responder's continuation value (with a message of its own signal). The responder is indifferent between accepting and rejecting, so that it can optimally behave according to the efficient agreement rule. The proposer is the residual claimant and thus behaves the same as the planner.

Restricting transfers to discrete values makes it generically impossible to propose a transfer that make the responder exactly indifferent between accepting and rejecting an offer. Thus, the above type of offer can not induce the responder to behave accordingly to an efficient agreement rule if information aggregation is necessary for efficiency. We show that generically no equilibrium that attains Agreement Rule 1 exists.[7] As of Agreement Rule 2, consider the following signaling offers: the proposer with a high signal demands the entire pie in exchange for the continuation value (which the responder always accepts) whereas the proposer with a low signal asks money for letting the responder have the entire pie (which only the responder with a high signal accepts). By choosing appropriate transfers, the responder's payoff is set to the continuation value, and the proposer again becomes the residual claimant. Since this does not hinge on any indifference condition, we can construct an equilibrium that attains Rule 2 as long as the discrete transfer space is sufficiently fine.

The remainder of the paper proceeds as follows. The literature review is given in the next part. Section 2 presents the model and the characterization of efficient agreement rules. In Section 3, we study the case of divisible pie without transfers. In addition to the analysis for fully efficient equilibria, we discuss approximately efficient ones when the

[7] Instead of using the above offer (called, *bid offer*), suppose the proposer makes an *ask offer*, by which the responder receives the entire pie with transfer payment to the proposer. The responder then uses information in making decision of acceptance. However, by using this offer, the proposer's payoff does not depend on information. The proposer has to be indifferent between acceptance and rejection in order to elicit information. With discrete transfer space, this is generically unavailable.

information is almost complete. Section 4 briefly discusses the case of indivisible pie with transfers. Section 5 gives concluding remarks.

1.1 Related Literature

The paper is related to three strands of the literature. The first is the literature on bargaining with incomplete information. Bargaining model with incomplete information has been studied extensively.[8] In this literature, a series of papers analyzes bargaining with one-sided private information and interdependent valuations.[9] Uninformed party makes an offer to the informed party in these studies. By taking a mechanism design approach to the sequential bargaining model, Ausubel and Deneckere (1989) show that every individually rational and incentive compatible bargaining mechanism is implementable if the uninformed player makes all offers, and that the set of implementable equilibria shrinks as the frequency of offers by the informed player increases. There are also a series of papers in which the informed party makes an offer: Reinganum and Wilde (1986), Schweinzer (2010a), and Brooks et. al. (forthcoming) consider a signalling in common value bargaining environment. Two papers close to ours are Schweizer(1989) and Schweinzer (2010b), which are the only papers with two-sided asymmetric information with common value bargaining.[10] In Schweizer (1989) each of two parties has one perfect signal on one of the two dimensional state in one-period pretrial bargaining model. Thus, players have perfect information on one dimension, but not in the other dimension. He shows that outcome cannot be efficient in the refined equilibrium. Schweinzer (2010b) considers an alternating offer sequential bargaining game in which proposal cannot be decreasing over time and offer space is discrete. Our paper differs from the papers in this literature by considering the issue of information aggregation.

The second is the literature on stochastic bargaining. Merlo and Wilson (1995, 1998) consider a complete information bargaining game in which size of the pie is stochastic. Players know the size of the pie at the beginning of each period. They show that efficient delay occurs in such a model when realization of the pie is small. Our model adds two-sided asymmetric information in stochastic bargaining by introducing private signals on the stochastic size of the pie. Our equilibrium converges to their equilibrium as accuracy of signal converges to one.

Finally, the paper is related to the literature on information aggregation in committees. Bond and Eraslan (2010) is one of the closest paper, and they consider a bargaining between

[8] See, e.g., the survey by Kennan and Wilson (1982) and Ausubel et. al. (2002).

[9] See, e.g., Evans (1989), Vincent (1989), Spier (1992), Deneckere and Liang (2006), and Fuchs and Skrzypacz (2010b).

[10] There are also paper studying two-sided asymmetric information with private values such as Fudenberg and Tirole (1983) and Chatterjee and Samuelson (1987) where infromation aggregation is not the focus of the studies.

a proposer and a set of voters who collectively decide whether to accept the offer or not in common-value environment. The bargaining is static game, and status quo prevails in case of no agreement. Their focus is on the information aggregation among the voters rather than information aggregation across voters and proposers. Also, we consider different forms of offers in an environment where value of disagreement is endogenously determined by the continuation value of the game. Damiano et. al. (2010) consider information aggregation in collective decision making problem between two players where players may disagree i) because they have different information and/or ii) because they have conflicting interest. Though their paper is close to ours as our players also disagree for the same reasons, our paper consider the problem in bargaining environment in which players endogenously choose an offer.

2 Model

Primitives We consider a two person random-proposer bargaining model in which the value of the pie is a random variable X, which is an i.i.d. draw in each period. Specifically, X takes either values L or H, $0 < L < H$, with probability $1/2$ for each realization. Each party receives a conditionally independent, private signal s_i, $i = 1, 2$, $s_i \in \{h, l\} = S$, with probability $\Pr(s_i = h|X = H) = \Pr(s_i = l|X = L) = q > 1/2$.

When the offer is accepted, the value of the pie materializes. The payoff is the share of the realized value net of monetary transfer. As usual, players discount future payoffs with the common discount factor $\delta \in [0, 1)$.

Timing The bargaining proceeds as follows: at the beginning of a stage, the nature assigns one party as a proposer with probability $1/2$, and then determines the value of the pie X. Each party then receives a private signal conditional on X. The proposer makes an offer to the receiver (details in bargaining protocols will be discussed later), and if it is accepted the bargaining is terminated. Otherwise the bargaining proceeds to the next stage. The proposer can send a message costlessly, although such cheap talk might be unnecessary or irrelevant to achieve efficient agreement in several cases.

Bargaining Protocols Due to asymmetric information, each player evaluates the expected interim pie size by using different information. This immediately implies that the players cannot agree on the monetary value of the pie unless information aggregation is perfect, and therefore offering a share would be different from offering money without dividing the pie. In fact, for signaling and/or eliciting information, having different instruments for offering helps achieve efficient outcomes since the proposer can make an offer by which the responder receives little or no rent, and so the proposer will be interested in maximizing the entire pie. However, having different instruments might be unavailable due to technological or financial constraints. The pie might be indivisible because it might be useless or very costly to manage if divided. The players might be financially constrained so that the limited liability constraints bind. We focus on the following two cases: a divisible pie without transfers and an indivisible pie with transfers. We will see that the analysis for the case with a divisible pie with transfer would directly follow from the two cases in discussing whether efficiency is available in equilibrium.

Accordingly, we consider the following three bargaining offers:

- **Share offer:** the proposer offers $\alpha \in [0, 1]$ share of the pie. If the receiver accepts the offer, the pie is shared accordingly.
- **Ask price offer:** the proposer offers $a \geq 0$. If the receiver accepts the offer, the receiver pays a to the proposer and the receiver obtains the entire pie.
- **Bid price offer:** the proposer offers $b \geq 0$. If the receiver accepts the offer, the proposer pays b to the receiver and the proposer obtains the entire pie.

These offers can be made equivalent under complete information environment. Note that ask and bid price offers are analogous to transactions in financial markets.

Solution Concept As a solution concept we consider stationary symmetric Perfect Bayesian equilibrium in which players' strategies are independent of the past histories and time.

2.1 Efficiency

We first characterize efficient agreement rule with respect to signal realization before moving on to analyzing the bargaining game. Because of the stochastic nature of the size of the pie, forgoing agreement may be preferred to immediate agreement if signal realizations are not good. Thus, efficient agreement rule depends on how large the time discount factor is as well as the precision of the signal and relative size of L and H. Our characterization is a natural extension of the idea of efficient delay in stochastic bargaining by Merlo and Wilson (1995) to the signal realization. We define efficient agreement rule before we characterize it.

Definition 1 *Agreement rule f is a mapping from signal space to agreement or not, i.e. $f : S \times S \rightarrow \{Agreement, No\ agreement\}$.*

Definition 2 *An efficient agreement rule is an agreement rule such that* $f(s,s) = \{Agreement\}$ *iff* $E[x|(s,s)] \geq 2\delta V_f$*, where* V_f *is recursively defined by* $V_f = \frac{1}{2}[\Pr(Agreement|f)E[x|Agreement, f] + \Pr(No\ agreement|f)2\delta V_f]$.

An efficient agreement rule means that players agree if and only if expected size of the pie given the signals are larger than the continuation value. The continuation value is multiplied by two because V is continuation value for each player. Note that we do not need index for each player or period because we assume recognition probability to be equal across players and size of the pie is i.i.d. across periods. To characterize an efficient agreement rule, consider the following three case.

Case 1) δ high: agreement only for $(s_1, s_2) = (h, h)$. This case corresponds to the situation that waiting is not costly (δ is sufficiently high) and agreement would be made only if both signals are h. In this case, the continuation value at the beginning of each period, V_1, is written as

$$V_1 = \frac{1}{2}(\Pr(h,h)E[x|h,h]) + \delta V_1(1 - \Pr(h,h))$$
$$\Leftrightarrow V_1 = \frac{\Pr(h,h)E[x|h,h]}{2(1-(1-\Pr(h,h))\delta)}.$$

If two signals are different (one is h and the other is l), the expected size of the pie is $E[x] = E[x|h,l]$. Hence, the condition that the discount factor should be sufficiently high can be obtained by comparing the expected size of the pie given two different signals, $E[x]$, and the continuation value where agreement is made only if signal is (h,h), i.e.,

$$E[x] \leq 2\delta V_1 = \frac{\delta \Pr(h,h)E[x|h,h]}{1-(1-\Pr(h,h))\delta}$$
$$\Leftrightarrow \delta \geq \frac{E[x]}{\Pr(h,h)E[x|h,h] + (1-\Pr(h,h))E[x]}.$$

For example, in the extreme case that $\delta \approx 1$, the waiting until (h,h) gives larger size of the pie to the players.

Case 2) δ medium: agreement for $(s_1, s_2) = (h,h), (h,l), (l,h)$; This case corresponds to the situation that waiting is relatively costly (δ is not too high) and agreement would be made if at least one signal is h. In this case, the continuation value is written as

$$V_2 = \frac{1}{2}(E[x] - \Pr(l,l)E[x|l,l]) + \delta V_2 \Pr(l,l)$$
$$\Leftrightarrow V_2 = \frac{E[x] - \Pr(l,l)E[x|l,l]}{2(1-\Pr(l,l)\delta)}.$$

Note that efficiency implies that the expected size of the pie given signals (l, l) should be lower than the continuation value of forgoing agreement, i.e.,

$$E[x|l,l] \leq 2\delta V_2 = \frac{\delta(E[x] - \Pr(l,l)E[x|l,l])}{1 - \Pr(l,l)\delta}$$
$$\Leftrightarrow \frac{E[x|l,l]}{E[x]} \leq \delta.$$

Combining this with the Case 1) we obtain agreement in case of $(s_1, s_2) = (h, h), (h, l), (l, h)$ to be efficient if

$$\frac{E[x|l,l]}{E[x]} \leq \delta < \frac{E[x]}{\Pr(h,h)E[x|h,h] + (1 - \Pr(h,h))E[x]}$$

Case 3) δ low: agreement for any profile If discount factor is very low, immediate agreement is always efficient. The associated continuation value, V_3, is $E[x]/2$. The threshold discount factor is obtained by the condition in Case 2) with opposite sign, since $E[x|l,l] \geq 2\delta V_3 \Leftrightarrow \delta \leq E[x|l,l]/E[x]$. This proves the following proposition.

Proposition 1 ***[Agreement Rule 1]*** *An efficient agreement rule is* $f(h,h) = \{Agreement\}, f(h,l) = f(l,h) = f(l,l) = \{No\ Agreement\}$ *for* $\forall\delta \in [\delta_H, 1)$ *where*

$$\delta_H = \frac{E[x]}{\Pr(h,h)E[x|h,h] + (1 - \Pr(h,h))E[x]}.$$

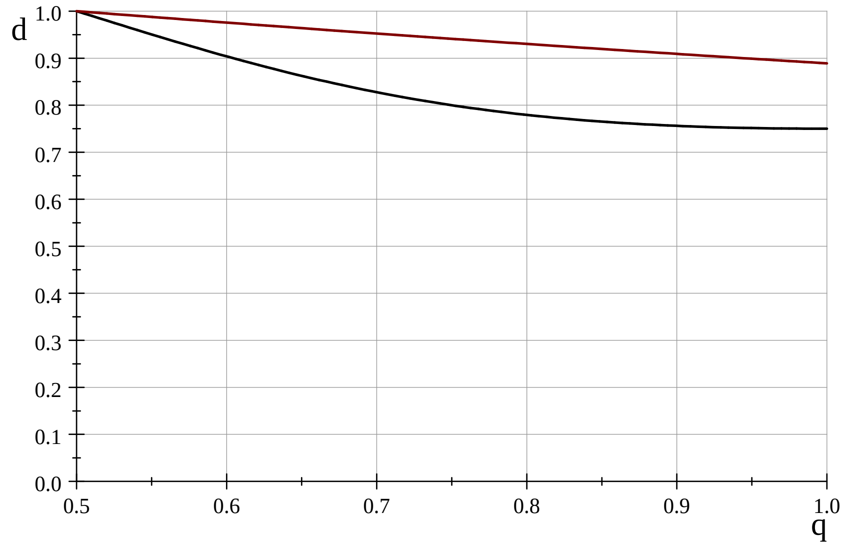
[Agreement Rule 2] *An efficient agreement rule is* $f(h,h) = f(h,l) = f(l,h) = \{Agreement\}, f(l,l) = \{No\ Agreement\}$ *for* $\forall\delta \in [\delta_L, \delta_H)$ *where.*

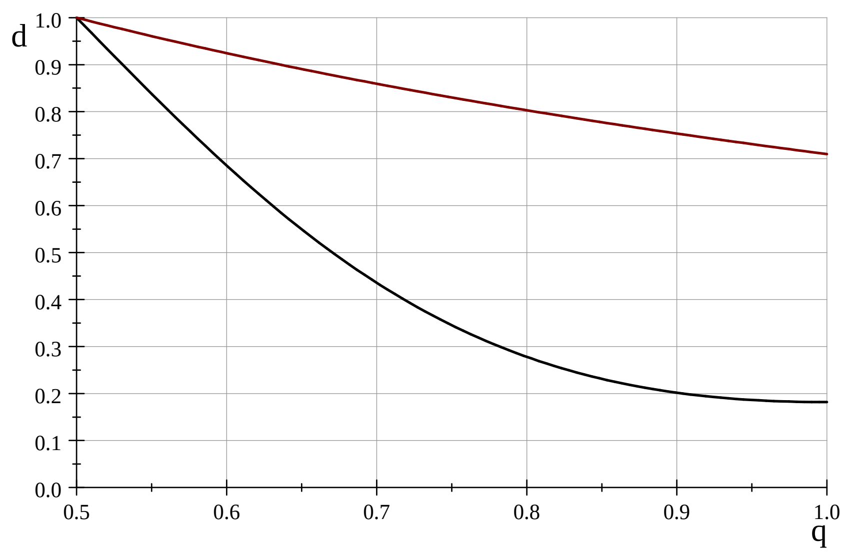
$$\delta_L = \frac{E[x|l,l]}{E[x]}.$$

[Agreement Rule 3] *An efficient agreement rule is* $f(s,s) = \{Agreement\}\forall(s,s) \in S \times S$ *for* $\forall\delta \in [0, \delta_L)$.

Note that thresholds δ_H and δ_L depends on q and L/H. For instance, if q is approaching to 1 and/or L/H to zero, both δ_H and δ_L takes lower values as illustrated in Figure 1. In Figure 1 we plot the thresholds δ_H (red) and δ_L (black) for the case of $L/H = 0.6$ and $L/H = 0.1$.

An important point here is the extent of information aggregation required to achieve efficient agreement rule. For low δ (that is $\delta < \delta_L$) we do not need information aggregation to achieve efficiency because immediate agreement is always preferred. For the first two cases (high and middle δ), information aggregation is necessary to achieve efficiency. The question is whether and how the player can aggregate information in equilibrium.





2.2 Efficiency via Commitment

Suppose that, before the bargaining starts, players can sign a binding contract as of how to "share" the pie in case an agreement occurs. In this case, efficiency is easily attained. Note that there is no sense of roles as a proposer or a responder.

- For a divisible pie, set each player's share to be, say, a half, and leave the decision for agreement to the players. Then it is optimal for each player to sincerely share its private signal (by cheap talk) and to make agreement if it is efficient. This is because players' payoffs are perfectly correlated, and also efficiency and individual payoffs are aligned.

- For an indivisible pie, create a lottery that specifies the probability of receiving the entire pie for each player upon agreement, and leave the decision for agreement to the players. The it is also optimal to make agreement if it is efficient.

The above observation shows that, in our model, information aggregation is not an issue when commitment is possible. Basically, as long as the "share" is determined in advance, there is no conflict between the players regarding when to agree and whether to exchange information. This is due to the pure common value assumption.

However, commitment may not be available for, at least, the following reasons. First, a contract before finalized for agreement might be torn up and one player might propose another, more favorable one to itself. Second, if we interpret our model as bargaining by randomly matched players, commitment for agreement is naturally very costly, since players do not know who they will agree with in advance.

We thus explore the model without the possibility of commitment in the following.

3 Divisible Pie without Transfers

We first look at the case in which the pie is divisible but transfers are not available, which corresponds to the share offer protocol. In this case the proposer offer the division of the pie $(\alpha, 1-\alpha)$, which gives $\alpha x \geq 0$ to the proposer and $(1-\alpha)x \geq 0$ to the responder, when agreed with the size of the pie x.

3.1 Full Efficiency

We study whether efficiency can be achieved as an equilibrium outcome. We construct an equilibrium for each efficient agreement rule, and find the conditions under which such an equilibrium exists. Note that information aggregation is necessary for Rules 1 and 2, which requires either *signaling* or *cheap talk* by the proposer.[11] Note that cheap talk is unnecessary when signaling successfully conveys information for efficiency. We will see when signaling and cheap talk actually differ.

3.1.1 Equilibrium for Rule 1

Suppose Rule 1 is efficient, or equivalently, $E[x] \leq 2\delta V_1$, where $V_1 = \Pr(h,h)E[x|h,h]/[2(1-(1-\Pr(h,h))\delta)]$. We start our analysis with cheap talk equilibrium in which efficient Agree-

[11] Rule 3 does not require information aggregation because players always agree regardless of the realization of signal.

ment Rule 1 arises. We will later discuss equivalent signaling talk equilibrium, which we can directly construct from cheap talk equilibrium in this particular case.

Cheap Talk Equilibrium We search for an equilibrium in which the proposer offers α with a cheap talk about its signal, and the responder accepts the offer in accordance with Rule 1. We first check the conditions under which the proposer would not lie in message, and the responder behaves accordingly. We then investigate the conditions and some belief system under which the proposer would not make an off-path share offer α'. We show that these conditions pin down the range of α supported in equilibrium.

Responder's On-schedule Incentives Efficiency implies that the responder accepts α if and only if both signals are high. This can be attained in the above form of equilibrium if the following conditions are satisfied;

$$(1-\alpha)E[x|h,h] \geq \delta V_1 \geq (1-\alpha)E[x].$$

The first inequality shows the responder is better off by accepting the offer than rejecting if both signals are high, whereas the second ensures rejection is better if one of the signals is low. Responses off the equilibrium path will be checked below.

Proposer's On-schedule Incentives In equilibrium, the proposer offers the same share with the message of its own signal. There are two kinds of deviations: *on-schedule deviation* by which the proposer pretends to have a different signal, and *off-schedule deviation* by which the proposer makes a share offer that would never arise in equilibrium. First, we investigate the condition that deters the on-schedule deviations:

$$\Pr(h|h)\alpha E[x|h,h] + \Pr(l|h)\delta V_1 \geq \delta V_1 \Leftrightarrow \alpha E[x|h,h] \geq \delta V_1 \tag{IC$_{hl}$}$$

$$\Pr(h|l)\alpha E[x] + \Pr(l|l)\delta V_1 \leq \delta V_1 \Leftrightarrow \alpha E[x] \leq \delta V_1. \tag{IC$_{lh}$}$$

(IC$_{hl}$) says that the proposer with a high signal would not lie, whereas (IC$_{lh}$) deters the other lying.

Off-schedule Incentives We need to check *the off-schedule incentive conditions*, by which the proposer would not make offer $\alpha' \neq \alpha$. To deter such deviations, we consider the *pessimistic* belief system off the equilibrium path: *the responder believes that it is the proposer with a signal l that makes the off-path offer.*[12] Given this belief, the responder's

[12] We adopt this belief because temptation to deviate to an off-path share is weakly larger if the belief is more optimistic, and therefore equilibrium is more likely to fail. In the Appendix, we provide a rationale for the pessimistic belief by arguing D1 criterion by Banks and Sobel (1987).

optimal reaction is to always accept if $(1-\alpha)\, E[x|l,l] \geq \delta V_1$, to accept if it has a high signal and $(1-\alpha)\, E[x|h,l] = (1-\alpha)\, E[x] \geq \delta V_1$, and to reject otherwise.

Lemma 1 *With the pessimistic belief and the on-schedule incentive conditions, the proposer has no incentive to offer* $\alpha' \neq \alpha$.

Proof. There are three cases. (i) for α' such that $\delta V_1 \geq (1-\alpha)E[x|h,l]$, which the responder always rejects. This deviation is effectively the same as in offering α with the message of a low signal, and therefore unprofitable. (ii) for α' such that $(1-\alpha')E[x|h,l] \geq \delta V_1 > (1-\alpha')E[x|l,l]$. The responder with a high signal would accept such α' but with a low signal it would definitely reject. The response is effectively the same as in offering α with the message of a high signal, except that the share is lower. Thus offering α' is clearly less profitable than α with the message of a high signal. (iii) for α' such that $(1-\alpha')E[x|l,l] \geq \delta V_1$, which the responder always accepts. Suppose, hypothetically, the proposer had an option that could implement α without responder's agreement. Under (IC_{lh}), this option is never optimal, since the proposer prefers no agreement if one of the signal is low. Since $\alpha' < \alpha$, deviation to α' is worse than the hypothetical option and therefore is never profitable. ∎

We now establish that if there exists an α that satisfies all the on-schedules IC's, namely,

$$(1-\alpha)E[x|h,h] \geq \delta V_1 \geq (1-\alpha)E[x],$$
$$\alpha E[x|h,h] \geq \delta V_1 \geq \alpha E[x],$$

a corresponding cheap talk equilibrium exists that implements Agreement Rule 1. Combining these inequalities, we have $E[x|h,h] \geq 2\delta V_1 \geq E[x]$, the latter of which is precisely the condition for Rule 1 being efficient. Note that, if α satisfies these conditions, so does $1-\alpha$, and hence the share α that arises in equilibrium forms a closed interval centered at 1/2. Conversely, suppose $E[x|h,h] \geq 2\delta V_1 \geq E[x]$ hold, i.e., Rule 1 is efficient (the former inequality, $E[x|h,h] \geq 2\delta V_1$, is trivial). Then,

$$\frac{1}{2}E[x|h,h] \geq \delta V_1 \geq \frac{1}{2}E[x]$$

and these correspond to the equilibrium conditions for $\alpha = 1/2$. To summarize, we have the following proposition:

Proposition 2 *A cheap talk equilibrium that attains the outcome for Agreement Rule 1 exists iff Rule 1 is efficient. Each share that satisfies responder's and proposer's on-schedule IC can be supported in equilibrium.*

Recall that, if the players can commit a share upon agreement, they voluntarily aggregate information to reach efficient agreement. The above argument shows that, when Rule 1 is efficient, an equilibrium exists in which the proposer's equilibrium offer is $\alpha = 1/2$. In terms of information aggregation, this equilibrium is thus equivalent to the case where they commit to equally share the pie upon agreement.

There are other equilibria in which the proposer's equilibrium offer is $\alpha \neq 1/2$. It is important to note that random proposer bargaining implies the *ex ante* expected share of the pie upon agreement to be just a half, since both players are equally likely to become a proposer. Thus the interim share upon agreement α differs from the ex ante expected one. This is nevertheless not a problem for information aggregation if α is close to $1/2$ and the efficiency condition holds with strict inequality. The reason is that the incentive conditions for information aggregation in the commitment benchmark with $\alpha = 1/2$ do not bind under such condition, and therefore a slight change in the interim share would not impede efficient information aggregation.

Unlike complete information benchmark, the proposer does not intrinsically have the "first-mover" advantage on bargaining share in this efficient equilibrium. In fact, the equilibrium share α lies in a closed interval centered at $1/2$ (the length varies with parameters). This is due to the redundancy of off-schedule ICs. The pessimistic belief enables the responder to act tough against the proposer, to the point where off-schedule offer is never attractive.[13] This characteristic disappears in the cases where Rules 2 and 3 are efficient.

Signaling Equilibrium Now we discuss an equivalent efficient equilibrium with signaling. Consider that the proposer offers $\alpha^h = \alpha$ if it receives a high signal, and, say, $\alpha^l = 1$ otherwise. We maintain the responder's belief being pessimistic for any other share. The associated combination of strategy and belief is essentially the same as above since *offering* α^l *works just as the message that the signal is low.*

Corollary 1 *A signaling equilibrium that attains the outcome for Agreement Rule 1 exists iff Rule 1 is efficient.*

3.1.2 Equilibrium for Rule 2.

In Agreement Rule 2, players reach agreement unless both signals are l. Recall this rule is efficient if $E[x|l,l] \leq 2\delta V_2 < E[x]$ where $V_2 = \{E[x] - \Pr(l,l)E[x|l,l]\}/\{2(1-\Pr(l,l)\delta)\}$.

Cheap Talk Equilibrium We search for an equilibrium in which the proposer offers α with a cheap talk about its signal, and the responder accepts the offer in accordance with

[13] In the Rubinstein bargaining, the first-mover advantage arises since the proposer can make the offer up to which the responder is indifferent between accepting and rejecting.

Rule 2. We check the on- and off-schedule incentive conditions.

Responder's and Proposer's On-schedule Incentives Efficiency implies that the responder should accept α unless both signals are low. Given that the proposer tells the truth, the associated incentive conditions are

$$(1-\alpha)E[x] \geq \delta V_2 \geq (1-\alpha)E[x|l,l].$$

As in the case of Rule 1, the proposer should not lie in message. The associated incentive conditions for not lying are

$$\begin{aligned}
&\Pr(h|h)\alpha E[x|h,h] + \Pr(l|h)\alpha E[x] \geq \Pr(h|h)\alpha E[x|h,h] + \Pr(l|h)\delta V_2 \\
&\Leftrightarrow \alpha E[x] \geq \delta V_2 \\
&\Pr(h|l)\alpha E[x] + \Pr(l|l)\delta V_2 \geq \Pr(h|l)\alpha E[x] + \Pr(l|l)\alpha E[x|l,l] \\
&\Leftrightarrow \delta V_2 \geq \alpha E[x|l,l],
\end{aligned}$$

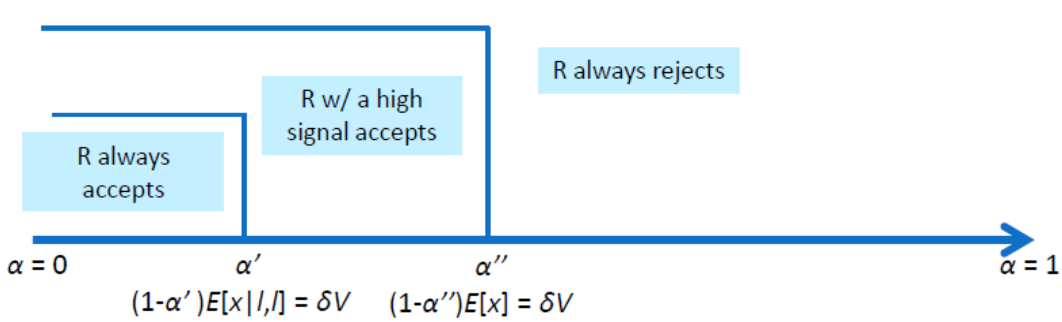
and these are summarized by $\alpha E[x] \geq \delta V_2 \geq \alpha E[x|l,l]$.

These conditions are analogous to those in the previous subsection. However, the off-schedule conditions are essential in this case.

Off-schedule Incentives Suppose the responder's off-path belief is pessimistic: let m denote the message by the proposer. The belief is then

$$\beta(h|\tilde{\alpha}) = \begin{cases} 1 & \tilde{\alpha} = \alpha, \text{ and } m = h, \\ 0 & \text{otherwise.} \end{cases}$$

Given this belief, the responder's off-path best response is to accept if and only if $(1-\tilde{\alpha})E[x|s_r,l] \geq \delta V_2$, where s_r is the responder's private signal. Define α', α'' s.t. $(1-\alpha')E[x|l,l] = \delta V_2$, $(1-\alpha'')E[x] = \delta V_2$ respectively. Then, the responder's off-path best response is depicted as follows:



The off-schedule incentive constraints are such that, given these responses, the proposer must have no incentive to deviate to any $\tilde{\alpha} \neq \alpha$. The following lemma is key to pin down the equilibrium share.

Lemma 2 *By the off-schedule incentive constraints for the proposer with a low signal, the equilibrium share α must satisfy $(1-\alpha)E[x] \leq \delta V_2$.*

Proof. Recall the proposer tells the truth on the equilibrium path and therefore the responder understands the proposer's signal. This implies that the on-path belief for the low-signal proposer is the same as the pessimistic off-path belief. Suppose $(1-\alpha)E[x] > \delta V_2$. Then for the proposer with a low signal, deviation to $\tilde{\alpha}$, slightly larger than α, is profitable since the responder's reaction is the same but the share upon agreement is larger. ■

Unlike the case for Rule 1, the pessimistic belief works poorly to deter off-schedule deviation. Unless the share is sufficiently large, the responder's behavior facing the low-signal proposer remains the same for slightly more demand for the share. This is not a problem in the case for Rule 1 since the proposer with a low signal anticipates no agreement in equilibrium.

Combining the condition in the lemma and one of the responder's incentive condition, we can pin down the equilibrium share α^*:

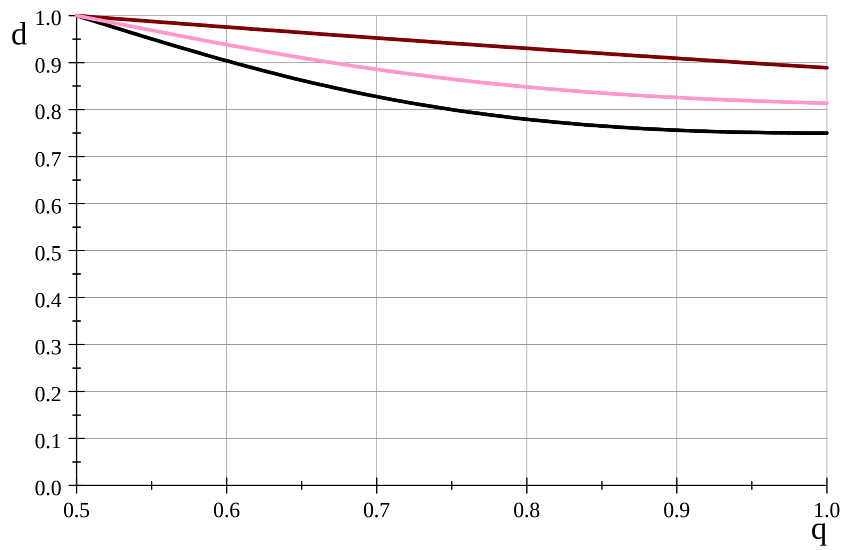
$$(1-\alpha^*)E[x] = \delta V_2.$$

With this α^* and the on-schedule conditions, we can straightforwardly show that the other off-schedule incentive constraints are satisfied (as in Lemma 1). For example, the proposer has no incentive to offer $\tilde{\alpha} \leq \alpha'$ (recall $(1-\alpha')E[x|l,l]$) since the responder's behavior is the same as the proposer offers α^* with the message of a high signal, but the share is much smaller. For the same reason, offering $\tilde{\alpha} \in (\alpha', \alpha'' = \alpha^*)$ is inferior to pretending to have a low signal. Offering $\tilde{\alpha} > \alpha^*$ results in sure rejection. Since $\alpha E[x] \geq \delta V_2$ from on-schedule IC, the proposer only misses profitable agreements by offering $\tilde{\alpha}$.

Combining all the conditions Now the question is whether all the on-schedule conditions holds with α^*, namely,

$$(1-\alpha^*)E[x] \geq \delta V_2 \geq (1-\alpha^*)E[x|l,l]$$
$$\alpha^* E[x] \geq \delta V_2 \geq \alpha^* E[x|l,l].$$

Clearly the responder's on-schedule conditions are satisfied. Note that Rule 2 efficiency implies $E[x] \geq 2\delta V_2$, so that $(\alpha^* + (1-\alpha^*))E[x] \geq 2\delta V_2 \Leftrightarrow \alpha^* E[x] + \delta V_2 \geq 2\delta V_2 \Leftrightarrow$



$\alpha^* E[x] \geq \delta V_2$,thus ($\text{IC}_{hl}$) holds. However, ($\text{IC}_{lh}$), $\delta V_2 \geq \alpha^* E[x|l,l]$, may fail; substituting $\alpha^* = 1 - \delta V_2/E[x]$, we rewrite ($\text{IC}_{lh}$) as

$$\delta V_2 \geq \frac{E[x|l,l]}{1 + E[x|l,l]/E[x]}.$$

This is stronger than $2\delta V_2 \geq E[x|l,l]$, a necessary condition for Rule 2 being efficient. We summarize the above arguments in the following proposition.

Proposition 3 *A cheap talk equilibrium exists that attains the outcome for Agreement Rule 2 iff Rule 2 is efficient and* $\delta V_2 \geq E[x|l,l]/\left(1 + E[x|l,l]/E[x]\right)$*. The equilibrium share* α^* *is uniquely determined by* $(1-\alpha^*)E[x] = \delta V_2$.

Efficiency by the cheap talk equilibrium is not always available. The region of parameter values for $L/H = .6$ in which the equilibrium exists is plotted in Figure 3.1.2 (above the pink curve).

In this cheap talk equilibrium, the proposer does have the first-mover advantage. In fact, the share α^* is so high that the responder with a high signal is just willing to accept if the message is low (i.e., $(1-\alpha^*)E[x] = \delta V_2$). This is because the proposer with a low signal knows that the responder with a high signal accepts any share up to α^* even with the pessimistic belief. Note also that the share is decreasing with δ.

When the discount factor is relatively low, cheap talk does not work to achieve efficiency. The discount factor affects two parts; While the continuation payoff δV_2 increases in δ, the share α^* decreases. This suggests that the proposer with a low signal might have an incentive to lie to induce agreement: the lower the discount factor δ, the less attractive the continuation payoff while the more attractive the agreement even with the worst scenario ($\alpha^* E[x|l,l]$).

The difference between Rule 1 and Rule 2 equilibria comes from the role of off-schedule incentive conditions. Recall that the off-schedule conditions are redundant in Rule 1 equilibrium, whereas in Rule 2 equilibrium, they are essential to determine the share α^*. It is this abusive power granted to the proposer at the interim stage that impedes efficient information aggregation via cheap talk.

Signaling Equilibrium We search for signaling equilibrium which implements Agreement Rule 2. We will show that signaling is more effective than cheap talking for Rule 2 efficiency.

Let α^h and α^l denote proposer's equilibrium offers given it respectively receives h and l. We maintain the pessimistic belief system, under which the following lemma for α^l obtains (the proof is the same as Lemma 2 and the argument that follows).

Lemma 3 *Under the pessimistic belief and the responder's on-schedule conditions,* $\alpha^l = \alpha^*$.

Proof. By the off-schedule incentive conditions for the proposer with a low signal, $(1-\alpha^l)E[x] \leq \delta V_2$ while by the responder's on-schedule incentive condition, $(1-\alpha^l)E[x] \geq \delta V_2$. Therefore $\alpha^l = \alpha^*$. ■

Responder's On-schedule Incentives The conditions for the responder are

$$(1-\alpha^h)E[x] \geq \delta V_2,$$
$$(1-\alpha^l)E[x] \geq \delta V_2 > (1-\alpha^l)E[x|l,l].$$

The first condition says that the responder with a low signal accepts α^h (and so does the responder with a high signal). The conditions in the second line say that only the responder with a high signal accepts α^l. Since $\alpha^l = \alpha^*$, i.e., $(1-\alpha^l)E[x] = \delta V_2$, the latter conditions automatically hold, and for the first condition to hold, we must have $\alpha^h < \alpha^l$. Note that all the incentive conditions for the responder hold if $\alpha^h < \alpha^l$ and $\alpha^l = \alpha^*$.

Lemma 4 *For a signaling equilibrium for Rule 2,* $\alpha^h < \alpha^l$.

Proposer's On-schedule Incentives The associated conditions are the following;

$$\alpha^h E[x|h] \geq \Pr(h|h)\alpha^l E[x|h,h] + \Pr(l|h)\delta V_2, \qquad (\mathrm{IC}_{hl})$$
$$\alpha^h E[x|l] \leq \Pr(h|l)\alpha^l E[x] \qquad + \Pr(l|l)\delta V_2, \qquad (\mathrm{IC}_{lh})$$

where $\alpha^h < \alpha^l$ and $\alpha^l = \alpha^*$. It turns out that the conditions on parameter values under which these two inequalities hold are complicated because of the freedom of α^h. It should be noted here about the difference between signaling and cheap talking. In cheap talk equilibrium, (IC$_{lh}$) may fail even when Rule 2 is efficient whereas (IC$_{hl}$) always holds (it is typically slack). In signaling equilibrium, we have $\alpha^h < \alpha^l$ and $\alpha^l = \alpha^*$ and therefore (IC$_{lh}$) is relaxed. This is why signaling equilibrium can extend the availability of Rule 2 efficiency. We need to be careful about (IC$_{hl}$), might bind for a smaller α^h.

The following technical lemma gives the condition under which there is α^h simultaneously satisfying (IC$_{hl}$) and (IC$_{lh}$).

Lemma 5 *There is α^h for which (IC$_{hl}$) and (IC$_{lh}$) simultaneously hold iff $\delta V_2 \geq A(q)$, where*

$$A(q) = \frac{HL\,(2q-1)\,(H+L)}{q((H+L)^2 + 4HL) - (3H+L)L}.$$

The expression $A(q)$ has the following properties: (i) $A(q)$ is increasing in q, (ii) $E[x] > 2A(q)$ for all q, and (iii) there is a $\tilde{q} \in (.5, 1)$ s.t. $E[x|l,l] \geq 2A(q)$ iff $q \leq \tilde{q}$.

Proof. See Appendix A.1. ■

Although $A(q)$ itself is hard to interpret, we can see that appropriate α^h fails to exist if the accuracy of signal is relatively high (from (iii)). Proposer's signaling thus works better than cheap talking but not perfect for Rule 2 efficiency. Note also that the above lemma does not ensure $\alpha^h < \alpha^l$, which is verified in the next lemma.

Proposer's Off-schedule Incentives Checking the proposer's off-schedule incentives is also more complicated than in the cheap talk equilibrium. We need detailed arguments since α^h has a freedom. However, it suffices to have Rule 2 efficiency and on-schedule IC in order to establish all the off-schedule IC.

Lemma 6 *Suppose $E[x|l,l] \leq 2\delta V_2 < E[x]$ and the proposer's on-schedule IC hold (i.e., $\delta V_2 \geq A(q)$). Then there is $\alpha^h < \alpha^l$ satisfying all the off-schedule incentive conditions.*

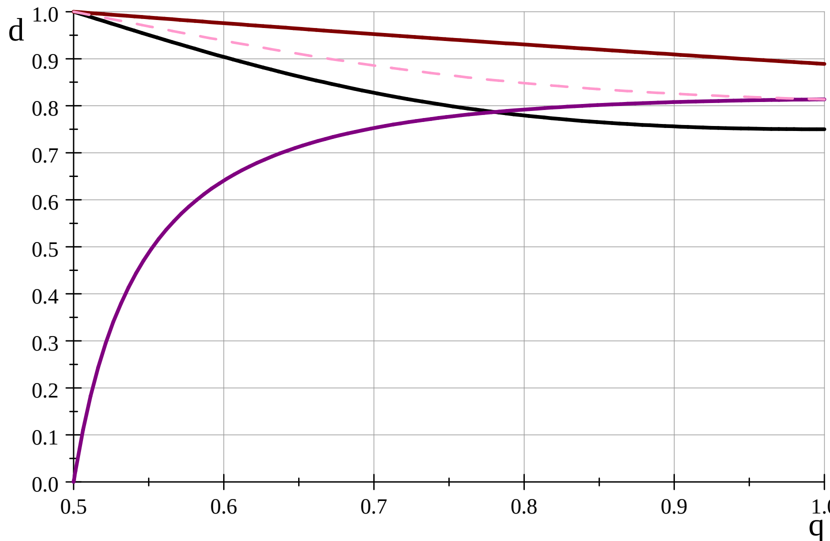
Proof. See Appendix A.2. ■

The case when $2\delta V_2 = E[x]$ (where Rule 1 and 2 are equally efficient) is excluded since the on-schedule ICs indeed imply $\alpha^h = \alpha^l = 1/2$, which contradicts signaling, $\alpha^h \neq \alpha^l$. This is not a big problem since efficient cheap talk equilibrium exists.

With the above lemmas, we can identify the conditions under which an efficient signaling equilibrium for Rule 2 exists.

Proposition 4 *Suppose $E[x|,l,l] \leq 2\delta V_2 < E[x]$. A signaling equilibrium for Rule 2 exists if and only if $\delta V_2 \geq A(q)$.*

We plot the lower bound of δ for which $\delta V_2 \geq A(q)$, i.e., (IC_{hl}) and (IC_{lh}) hold (purple):



The area below the purple curve and above the black curve is where efficiency is not attainable in signaling equilibrium. When Rule 2 is efficient, i) for a lower information accuracy level ($q \approx .78$ or smaller), efficiency is attainable in an equilibrium , while ii) for a higher information accuracy level, it depends on the discount factor; δ must be large enough to achieve efficiency. We can see that the area of δ in which signaling equilibrium exists is approaching to that in which cheap talk equilibrium exists as $q \to 1$. To see why, note that α^h has to be very close to α^l for approximately complete information, otherwise (IC_{hl}) would fail. Thus the equilibrium is converging to the cheap talk equilibrium as $q \to 1$.

It is interesting to see that signaling equilibrium is more powerful to achieve efficient agreement rule than cheap talk equilibrium. To see this, first notice that the offer in cheap talk equilibrium, α^*, and the one by the proposer with a low signal, α^l, are the same. This is due to the abusive power granted to the proposer from the off-schedule incentive constraints. The difference between these two equilibria lies in the on-schedule incentive constraints. Since $\alpha^h < \alpha^l$ in signaling equilibrium, the incentive condition for the proposer with a low signal, (IC_{lh}) is relaxed compared with the one in cheap talk equilibrium. Of course the other on-schedule constraint, (IC_{hl}), arises which is redundant in cheap talk equilibrium. The argument above guarantees that both (IC_{hl}) and (IC_{lh}) hold if α^h lies in a closed interval, so by setting α^h as large as possible (IC_{hl}) is made slack. This is why signaling equilibrium facilitates information aggregation better than cheap talk equilibrium.

3.1.3 Equilibrium for Rule 3

Suppose Rule 3 is efficient, i.e., $E[x|l,l] \geq 2\delta V_3$ where $V_3 = E[x]/2$. We search for a *pooling* equilibrium in which the receiver always accepts the offer by the proposer. Neither signaling nor cheap talk is useful for this equilibrium. Let α be the proposer's offer. As the receiver accepts α regardless of his signal, the following responder's on-schedule incentive conditions should hold;

$$(1-\alpha)E[x|l] \geq \delta V_3.$$

The best offer for the proposer is α s.t. $(1-\alpha)E[x|l] = \delta V_3$. Although we can think of other equilibrium with a smaller pooling share, the associated proposer's incentive conditions are harder to hold. Since we are seeking the possibility of achieving an efficient equilibrium, it is therefore natural to consider the best offer. Hence,[14]

$$\alpha = 1 - \frac{\delta E[x]}{2E[x|l]}.$$

Note that the payoff for the proposer with a low signal $\alpha E[x|l]$ is larger than δV_3, the reservation value.

Again we consider the pessimistic off-path belief. Formally,

$$\beta(h|\tilde{\alpha}, s_r = l) = \begin{cases} 2q(1-q) & \text{if } \tilde{\alpha} = \alpha \\ 0 & \text{otherwise} \end{cases}, \quad \beta(h|\tilde{\alpha}, s_r = h) = \begin{cases} q^2 + (1-q)^2 & \text{if } \tilde{\alpha} = \alpha \\ 0 & \text{otherwise} \end{cases}.$$

The responder's reaction to an off-path offer $\tilde{\alpha}$ would be to always accept if $\tilde{\alpha} \leq \alpha'$ s.t. $(1-\alpha')\,E[x|l,l] = \delta V_3$, to accept if $\tilde{\alpha} \in (\alpha', \alpha'']$, where $(1-\alpha'')\,E[x] = \delta V_3$ and $s_r = h$, and to reject if $\tilde{\alpha} > \alpha''$.

Proposer's Off-schedule Incentives We first argue that the only relevant off-schedule deviation is to offer α'' s.t. $(1-\alpha'')\,E[x] = \delta V_3$ among all the off-path offers. First, it is easy to see that offer $\tilde{\alpha} \leq \alpha'$ is unprofitable since the responder's reaction is the same as on-path but the share is smaller ($\alpha' < \alpha$). For $\tilde{\alpha} > \alpha''$, note that $(1-\alpha)E[x|l] = \delta V_3$ and Rule 3 efficiency imply

$$\alpha E[x|l] = E[x|l] - \delta V_3 > E[x|l,l] - \delta V_3 \geq 2\delta V_3 - \delta V_3 = \delta V_3$$

and therefore the payoff of the proposer with a low signal is higher than the continuation value. It is then also unprofitable for the proposer to offer $\tilde{\alpha} > \alpha''$, which the responder always rejects. Among $\tilde{\alpha}$ in $(\alpha', \alpha'']$ which the responder with a high signal accepts, $\tilde{\alpha} = \alpha''$

[14]Note that in the region where Rule 3 is efficient, $E[x|l,l]/E[x] \geq \delta$. This ensures that the best offer is always nonnegative.

gives the proposer the largest share and is therefore the most profitable deviation. Note that $(1-\alpha'')E[x|h,l] = \delta V_3 \Leftrightarrow \alpha'' = 1-\delta/2$.

To deter deviation to α'', the following conditions must hold:

$$\alpha E[x|h] \geq \Pr(h|h)\alpha'' E[x|h,h] + \Pr(l|h)\delta V_3 \tag{IC$_h$}$$

$$\alpha E[x|l] \geq \Pr(h|l)\alpha'' E[x|h,l] + \Pr(l|l)\delta V_3. \tag{IC$_l$}$$

The second constraint (IC$_l$) condition is indeed equivalent to Rule 3 efficiency:

$$\begin{aligned}
&\left(1-\frac{\delta V_3}{E[x|l]}\right) E[x|l] \geq \Pr(h|l)E[x]\left(1-\frac{\delta V_3}{E[x]}\right) + \Pr(l|l)\delta V_3 \\
&\Leftrightarrow \delta V_3 \left(-\Pr(h|l) + 1 + \Pr(l|l)\right) \leq -\Pr(h|l)E[x] + E[x|l] \\
&\Leftrightarrow 2\delta V_3 \Pr(l|l) \leq \Pr(l|l)E[x|l,l] \Leftrightarrow 2\delta V_3 \leq E[x|l,l]
\end{aligned}$$

The first condition (IC$_h$) does not always hold even when Rule 3 is efficient. The following technical lemma gives the requirement for (IC$_h$):

Lemma 7 *(IC$_h$) holds iff $\delta V_3 \leq B(q)$ where*

$$B(q) = \frac{E[x]\Pr(l|h)}{2\Pr(l|h) - E[x|h]\left(1/E[x|l] - 1/E[x]\right)},$$

with the following properties:

1. $2B(1) = 0$, *and* $2B(1/2) = E[x] = E[x|l,l]|_{q=1/2}$
2. *If* $L > 0$, *then there is a* $\tilde{q}$ *s.t.* $2B(q) > E[x|l,l]$ *for* $q \in (1/2, \tilde{q})$, *and* $2B(q) < E[x|l,l]$ *for* $q > \tilde{q}$.
3. *If* $L = 0$, $2B(q) \geq E[x|l,l]$ *for all* q *(equality holds for* $q = 1/2$, 1*).*

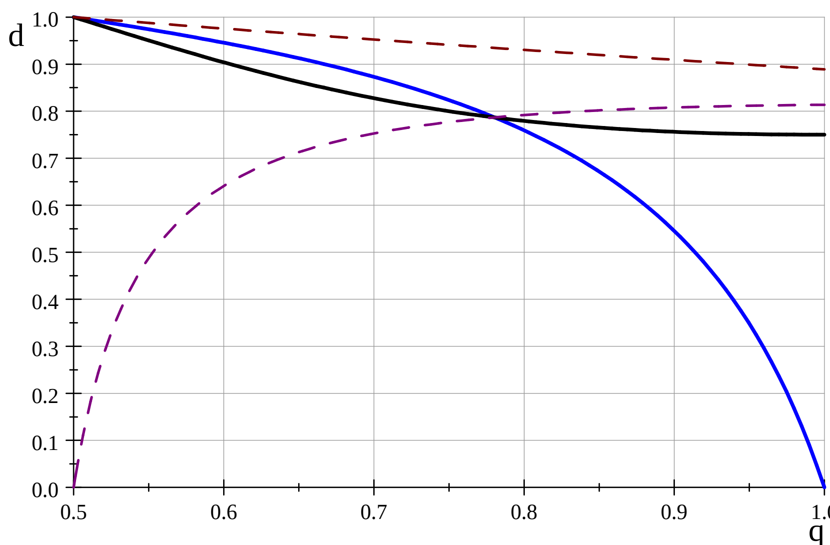
Proof. See Appendix A.3. ■

The condition $\delta V_3 \leq B(q)$ and the properties of $B(q)$ provide information about when Rule 3 is efficient but (IC$_h$) fails (see also the diagram below). If $L = 0$, it never fails since $2B(q) \geq E[x|l,l] \geq 2\delta V_3$ always holds under Rule 3 efficiency. However, if $L > 0$, there is a $\tilde{q}$ s.t. $q > \tilde{q} \Rightarrow 2B(q) < E[x|l,l]$ and hence $2B(q) < 2\delta V_3 \leq E[x|l,l]$ can arise.

The properties of $B(q)$ implies that (IC$_h$) would be harder to hold as δ is higher, and the threshold of δ is decreasing as q increases. Intuitively, if the signal accuracy is very high, the proposer with an h would expect that it is very likely that the responder has the same signal and accepts a deviation offer $\alpha' > \alpha$. This implies that the deviation to an inefficient offer α' can be profitable. To deter such deviation, basically the difference between $\alpha' - \alpha = \delta(E[x]/E[x|l] - 1)$ must be small enough, and this is why δ is necessarily small (this also makes forgoing to the next period more costly as well, which also contributes to (IC_h) being satisfied). Moreover, the lemma confirms that, if q is above a certain level, the threshold is below the one for Rule 3 being efficient. This immediately implies that a fully efficient equilibrium for Rule 3 fails to exist if the accuracy of information is relatively high, and if the discount factor is not too low.

Proposition 5 *Suppose Rule 3 is efficient, i.e.,* $E[x|,l,l] \geq 2\delta V_3$*. An equilibrium for Rule 3 exists if* $\delta V_3 \leq B(q)$*.*

We plot the diagram for $L/H = .6$, in which $\delta V_3 \leq B(q)$ holds in the area below the blue curve.



Efficiency is not always attainable. The incentive condition (IC_h) says that the proposer with an h is likely to deviate if the discount factor is not so low and the information accuracy is high. Basically the proposer's benefit to use signal-sensitive offers is large when the accuracy is very high, and therefore deviation temptation from pooling offer is too high to sustain the efficient pooling equilibrium.

Consider the perfect information case, $q = 1$, in which efficiency obtains by separating offers conditional on the signals. Notice that this efficient equilibrium cannot be achieved in the limit of efficient equilibria as $q \to 1$. Instead, a sequence of separating equilibria can be made arbitrarily close to the efficient equilibrium by the approximate efficiency result in the next subsection.

3.2 Approximate Efficiency

We have shown that an efficient equilibrium sometimes fails to exist for Rules 2 and 3. In this section, we consider if the efficient outcomes can be attained in the limit as information becomes arbitrarily close to perfect, i.e., $q \to 1$.

Equilibrium For Rule 2 Consider a pooling equilibrium in which the responder rejects the equilibrium offer if it receives a low signal. The ex ante value from this agreement rule is

$$V_p = \frac{1}{2}(\frac{1}{2}E[x|h] + \frac{1}{2}2\delta V_p) = \frac{E[x|h]}{2(2-\delta)},$$

since agreement occurs if and only if the responder's signal is h. Let α denote the pooling offer, and assume the pessimistic belief for off-path offers. The responder's on-path incentive condition is

$$(1-\alpha)E[x|h] \geq \delta V_p \geq (1-\alpha)E[x|l],$$

while its off-path best response is just as before. Analogous to the construction for an equilibrium for Agreement Rule 3, it would be natural to think about the best offer for the proposer since otherwise it is harder to sustain the equilibrium. We thus take α s.t.

$$(1-\alpha)E[x|h] = \delta V_p = \frac{\delta E[x|h]}{2(2-\delta)} \Leftrightarrow \alpha = 1 - \frac{\delta}{2(2-\delta)}.$$

Given α as above, an offer $\alpha' > \alpha$ is always rejected. The most profitable deviation for the proposer is then α'' s.t.

$$(1-\alpha'')E[x|l,l] = \delta V_p \Leftrightarrow \alpha'' = 1 - \frac{\delta}{2(2-\delta)} \frac{E[x|h]}{E[x|l,l]}$$

which the responder always accepts. Thus the incentive conditions are

$$\Pr(h|h)\alpha E[x|h,h] + \Pr(l|h)\delta V_p \geq \alpha'' E[x|h], \qquad (\text{IC}_h)$$

$$\Pr(h|l)\alpha E[x] + \Pr(l|l)\delta V_p \geq \alpha'' E[x|l]. \qquad (\text{IC}_l)$$

Consider the situation in which Rule 2 is efficient but efficient equilibrium is not available. We then ask the following question; *given δ and L/H, as the accuracy of information is approaching to $q = 1$, does the approximately efficient equilibrium exist in this region?* The answer is indeed affirmative. To show that, we just have to verify the two ICs to hold as $q \to 1$. Since (IC_h) trivially holds with strict inequality at $q = 1$, by continuity it must hold for q sufficiently close to 1. (IC$_l$) holds at $q = 1$ if

$$\begin{aligned}
\delta V_p \geq \alpha'' E[x|l] &\Leftrightarrow \delta \frac{E[x|h]}{2(2-\delta)} \geq \left(1 - \frac{\delta}{2(2-\delta)} \frac{E[x|h]}{E[x|l,l]}\right) E[x|l] \\
&\Leftrightarrow \delta \frac{E[x|h]}{2(2-\delta)} \left(1 + \frac{E[x|l]}{E[x|l,l]}\right) \geq E[x|l] \\
&\Leftrightarrow \delta E[x|h] \left(1 + \frac{E[x|l]}{E[x|l,l]}\right) \geq 2(2-\delta) E[x|l] \\
&\Leftrightarrow \delta \left\{ E[x|h] \left(1 + \frac{E[x|l]}{E[x|l,l]}\right) + 2E[x|l] \right\} \geq 4E[x|l] \\
&\Leftrightarrow \delta \geq \frac{4E[x|l]}{E[x|h]\left(1 + \frac{E[x|l]}{E[x|l,l]}\right) + 2E[x|l]} = \frac{4L}{2H + 2L} = \left.\frac{E[x|l,l]}{E[x]}\right|_{q=1}
\end{aligned}$$

Hence this condition at the limit is the same as the lower bound restriction for δ for which Rule 2 is efficient, namely $\delta \geq E[x|l,l]/E[x]$ at $q = 1$. Since $E[x|l,l]/E[x]$ is decreasing in q and (IC$_l$) condition is continuous in q, we can conclude that (IC$_l$) holds for q sufficiently close to 1 (indeed, a tedious calculation shows that (IC$_l$) is redundant if $\delta \geq E[x|l,l]/E[x]$ for all q).

Equilibrium For Rule 3 Consider a separating equilibrium in which the proposer offers $\alpha^h > \alpha^l$, and the responder accepts α^h if it also has an h while accepting a^l always. The ex ante value from this agreement rule is

$$V_s = \frac{1}{2}(E[x] - \Pr(h,l)(E[x] - 2\delta V_s)) = \frac{1 - \Pr(h,l)}{1 - \delta \Pr(h,l)} \frac{E[x]}{2}.$$

The responder's incentive conditions are

$$\begin{aligned}
(1 - \alpha^h)E[x|h,h] \geq \delta V_s \geq (1 - \alpha^h)E[x], \\
(1 - \alpha^l)E[x|l,l] \geq \delta V_s.
\end{aligned}$$

Again we assume the pessimistic belief for off-path offers.

The proposer's off-schedule incentive condition implies $(1 - \alpha^l)E[x|l,l] \leq \delta V_s$, otherwise the proposer can deviate to offer α' slightly higher than α^l but it would still be accepted. Hence

$$\alpha^l = 1 - \frac{\delta V_s}{E[x|l,l]}.$$

We consider the case in which the responder receives no rent, i.e.,

$$(1 - \alpha^h)E[x|h,h] = \delta V_s \Leftrightarrow \alpha^h = 1 - \frac{\delta V_s}{E[x|h.h]}.$$

Consider the on-schedule incentive conditions:

$$\Pr(h|h)\alpha^h E[x|h,h] + \Pr(l|h)\delta V_s \geq \alpha^l E[x|h]$$
$$\Pr(h|l)\alpha^h E[x] + \Pr(l|l)\delta V_s \leq \alpha^l E[x|l]$$

The first IC is redundant for q sufficiently large, since $q \to 1$ implies

$$\begin{aligned}\alpha^h E[x|h,h] &= (1 - \frac{\delta V_s}{E[x|h.h]})E[x|h,h] \\ &= H - \delta\frac{H+L}{4} \\ &> H - \delta\frac{H+L}{4L}H = (1 - \frac{\delta V_s}{E[x|l,l]})E[x|h] = \alpha^l E[x|h]\end{aligned}$$

The second one is

$$\delta\frac{H+L}{4} \leq L - \delta\frac{H+L}{4} = \alpha^l E[x|l] \Leftrightarrow \delta \leq \frac{L}{(H+L)/2} = \frac{E[x|l,l]}{E[x]}$$

An argument analogous to Approximate Efficiency for Rule 2 shows that as q goes to 1, an approximately efficient equilibrium exists.

4 Indivisible Pie with Transfers

We consider the case in which the pie is indivisible and player can make monetary transfers. In this case, the players bargain over who obtains the pie and how much monetary transfer the other party has to make. The party who receives the entire pie have strong incentive to aggregate information, while the other party only cares about a larger transfer (and not the size of the pie itself per se). This suggests information aggregation for Agreement Rules 1 and 2 is harder to obtain.

In the following, we argue that the possibility of efficient information aggregation depends on how fine the transfer can take. Actually, if transfers can take continuum of values, an equilibrium exists that attains the efficient agreement rule for each of the Agreement Rules 1 and 2. This result is driven by the fact that the party who receives money can elicit information about its signal for free if the monetary transfer upon agreement is equal to the continuation value upon rejection. The result is not robust if transfer can only take discrete values, generically violating the indifference condition. In fact, it is not possible to attain Agreement Rule 1 in equilibrium. However, by considering a signaling offer, efficient outcome can still be supported in equilibrium for Agreement Rule 2.

4.1 Full Efficiency with Continuous Transfers

In the case of continuous transfers, full efficiency can be attained for all three agreement rules. Consider the following offer: the proposer pays to the responder the exact amount of the discounted continuation payoff δV in order to obtain the entire pie (bid price offer). The responder is then indifferent between accepting and rejecting no matter what signal he receives, because the signal is irrelevant for the current payoff (transfer) and for the future payoff. The responder would then release its information for free and could behave in accordance with the efficient agreement rule.[15] Given this behavior of the responder, the proposer can be considered as the residual claimant, and makes offers so that the efficiency is attained.

4.2 (In)Efficiency with Discrete Transfers

In this subsection, we consider whether efficient outcome could be attained in the case where transfer can only take discrete values (such as one cent or one yen). The fact that the transfer can only take discrete value implies that proposer cannot make offer such as $b = \delta V$ generically. Denote the set of discrete values transfers can take as $T = \{0, ..., \bar{t}\}$. For notational convenience, we use notation of x_+ for a monetary amount that is the minimum of all the amounts larger than x, i.e., $x_+ = \min_{t \in T}\{t | t \geq x\}$, and similarly $x_- = \max_{t \in T}\{t | t \leq x\}$.

Equilibrium For Rule 1 We show that generically an equilibrium for Rule 1 does not exist. Recall that in an equilibrium for Rule 1, the proposer with a high signal makes an offer that only the responder with a high signal accepts, while the offer by the proposer with a low signal must be always rejected. This shows that the proposer with a high signal needs to make an offer that screens the responder. However, offering a menu of contracts to screen the responder is unnecessary in this case, since screening is completed by acceptance or rejection. Therefore, the proposer with a high signal make either an ask price offer or a bid price offer for screening, and the proposer with a low signal make an offer that is definitely rejected.

First, consider the proposer with a high signal makes an ask offer a. The incentive constraints of the proposer are

$$\Pr(l|h)\delta V_1 + \Pr(h|h)a \geq \delta V_1 \Leftrightarrow a \geq \delta V_1$$
$$\delta V_1 \geq \Pr(l|l)\delta V_1 + \Pr(h|l)a \Leftrightarrow \delta V_1 \geq a$$

[15] For Agreement Rule 2, the responder should agree if at least one of the signals is high. To achieve this consequence, the proposer needs to offer with a cheap talk of its signal.

These conditions cannot be met generically if transfer is discrete. Similarly, if the proposer with a high signal makes a bid offer b, the incentive constraints are

$$\begin{aligned} \Pr(l|h)\delta V_1 + \Pr(h|h)\left(E[x|h,h]-b\right) \geq \delta V_1 &\Leftrightarrow E[x|h,h]-b \geq \delta V_1 \\ \delta V_1 \geq \Pr(l|l)\delta V_1 + \Pr(h|l)\left(E[x|h,h]-b\right) &\Leftrightarrow \delta V_1 \geq E[x|h,h]-b. \end{aligned}$$

Again, these constraints cannot be satisfied generically. Hence, Rule 1 outcome cannot be sustained in equilibrium if transfer is discrete.

Equilibrium For Rule 2 Let us consider the following signaling strategy: the proposer makes a bid offer b if her signal is h, and an ask offer a if her signal is l. The responder accepts the offer if (i) it is a bid offer $\tilde{b}$ and $\tilde{b} \geq \delta V_2$ or (ii) the offer is ask offer $\tilde{a}$ and $E[x|\text{updated belief and its own signal}] - \tilde{a} \geq \delta V_2$, and rejects otherwise. We adopt the pessimistic belief for off-path offers as before, i.e.,

$$\beta(h|\text{offer}) = \begin{cases} 1 & \text{offer is } a, \\ 0 & \text{otherwise (including offer is } b). \end{cases}$$

We show that this is an equilibrium for an appropriate choice of a and b.

The on-schedule incentive conditions for the responder to sustain Agreement Rule 2 are

$$\begin{aligned} b &\geq \delta V_2 \\ E[x]-a &\geq \delta V_2 > E[x|l,l]-a, \end{aligned}$$

and those for the proposers are

$$\begin{aligned} \Pr(l|h)E[x] + \Pr(h|h)E[x|h,h] - b &\geq \Pr(h|h)a + \Pr(l|h)\delta V_2 \\ \Pr(h|l)a + \Pr(l|l)\delta V_2 &\geq \Pr(l|l)E[x|l,l] + \Pr(h|l)E[x] - b. \end{aligned}$$

Suppose that the proposer chooses ask and bid offers as $a = [E[x]-\delta V_2]_-$ and $b = [\delta V_2]_+$. Clearly these satisfy the responder's on-schedule IC. Also, it is easy to check that the proposer's off-schedule IC holds with these on-path offers.

Intuitively, the proposer receives almost all rent if the discrete transfer space is sufficiently fine, and therefore it will behave accordingly to the efficient agreement rule. For a formal argument, denote $\varepsilon_a = E[x] - \delta V_2 - a > 0$, and $\varepsilon_b = \delta V_2 - b > 0$, and

$$\delta_L^+ = \frac{E[x|l,l] + (\Pr(h|l)\varepsilon_a + \varepsilon_b)/\Pr(l|l)}{E[x] + \Pr(l,l)(\Pr(h|l)\varepsilon_a + \varepsilon_b)/\Pr(l|l)}.$$

Note that $\delta_L^+ > \delta_L = E[x|l,l]/E[x]$, the lower bound of the discount factor for Rule 2. The former incentive constraint is actually redundant:

$$\Pr(l|h)E[x] + \Pr(h|h)E[x|h,h] - [\delta V_2]_+ \geq \Pr(h|h)[E[x] - \delta V_2]_- + \Pr(l|h)\delta V_2$$

$$\Leftrightarrow \delta < \frac{E[x] + \frac{\Pr(h|h)}{\Pr(l|h)}(E[x] - E[x|l,l]) + \left[\frac{\Pr(h|h)}{\Pr(l|h)}\varepsilon_a + \varepsilon_b\right]}{E[x] + \frac{\Pr(l,l)}{\Pr(l|h)}\left(E[x] - E[x|l,l]\right) + \Pr(l,l)\left[\frac{\Pr(h|h)}{\Pr(l|h)}\varepsilon_a + \varepsilon_b\right]}, \text{ (note, RHS>1)}$$

The latter one is rewritten as

$$\Pr(h|l)a + \Pr(l|l)\delta V_2 \geq \Pr(l|l)E[x|l,l] + \Pr(h|l)E[x] - b$$

$$\Leftrightarrow \frac{E[x|l,l] + (\Pr(h|l)\varepsilon_a + \varepsilon_b)/\Pr(l|l)}{E[x] + \Pr(l,l)(\Pr(h|l)\varepsilon_a + \varepsilon_b)/\Pr(l|l)} \leq \delta \Leftrightarrow \delta_L^+ \leq \delta$$

Thus, all the constraints are satisfied if $\delta_L^+ \leq \delta$ where $\delta_L^+ \to \delta_L$ as discreteness vanishes (ε_a and ε_b goes to zero).

Equilibrium For Rule 3 For Rule 3, information aggregation is unnecessary and therefore the concern about the indifference condition for eliciting information is vacuous. In fact, we can easily modify the equilibrium for Rule 3 with continuous transfer: a pooling equilibrium in which the proposer always propose $b = [\delta V_3]_+$, and the responder accept if $b \geq \delta V_3$ and reject otherwise regardless of the signal. It is straightforward to see that this is an equilibrium.

5 Concluding Remarks

We have studied how and when dispersed information can be aggregated for efficient agreement in bilateral bargaining with common value. For a divisible pie without transfers, we find that efficient information aggregation is available in an equilibrium if the discount factor is relatively large. We also find that too accurate (but not perfect) information disturbs attainment of fully efficient equilibrium because the proposer would be able to squeeze rent with distorting efficient agreement. For an indivisible pie without transfers, the attainability of efficient information aggregation depends on the property of transfer space. Generally speaking, the party who receives money, not the indivisible pie does not care about the information on the pie, and hence has no incentive to signal information at a cost. We find that this problem is serious if the discount factor is very large.

We have focused on the availability of efficient information aggregation in equilibrium. We have already seen multiplicity of efficient equilibria (see Proposition 2), and other inefficient equilibria (see the approximately efficient equilibria studied in the end of Section 3). We actually discuss plausibility of the pessimistic beliefs in efficient equilibrium with a standard stability argument in the Appendix, and show that an efficient equilibrium with information aggregation passes the D1 criterion. For better predictions, we need further study on other equilibria and the equilibrium selection issues.

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A Proofs

A.1 Proof of Lemma 5

Define, from the IC's,

$$\alpha_1 = \frac{\Pr(h|h)\alpha^l E[x|h,h] + \Pr(l|h)\delta V_2}{E[x|h]}, \quad \alpha_2 = \frac{\Pr(h|l)\alpha^l E[x] + \Pr(l|l)\delta V_2}{E[x|l]}.$$

That is, α_1 is the lowest α^h satisfying (IC$_{hl}$) and α_2 is the highest α^h satisfying (IC$_{lh}$). To assure there is $\alpha^h \in [\alpha_1, \alpha_2]$, we need $\alpha_1 \leq \alpha_2$. The condition $\alpha_1 \leq \alpha_2$ can be rewritten as

$$\begin{aligned}
&\frac{\Pr(h|h)\alpha^l E[x|h,h] + \Pr(l|h)\delta V}{E[x|h]} \leq \frac{\Pr(h|l)\alpha^l E[x] + \Pr(l|l)\delta V}{E[x|l]} \\
&\Leftrightarrow [\Pr(h|h)(1 - \frac{\delta V}{E[x]})E[x|h,h] + \Pr(l|h)\delta V]E[x|l] \leq [\Pr(h|l)(1 - \frac{\delta V}{E[x]})E[x] + \Pr(l|l)\delta V]E[x|h] \\
&\Leftrightarrow \Pr(h|h)E[x|h,h]E[x|l] - \Pr(h|l)E[x]E[x|h] \\
&\leq \delta V[\{\Pr(h|h)\frac{E[x|h,h]}{E[x]} - \Pr(l|h)\}E[x|l] + \{\Pr(l|l) - \Pr(h|l)\}E[x|h]] \\
&= \delta V[\{\Pr(h|h) - \Pr(l|h)\}[E[x|l] + E[x|h]] + \Pr(h|h)(\frac{E[x|h,h]}{E[x]} - 1)] \\
&\Leftrightarrow \delta V \geq \frac{\Pr(h|h)E[x|h,h]E[x|l] - \Pr(h|l)E[x]E[x|h]}{\{\Pr(h|h)\frac{E[x|h,h]}{E[x]} - \Pr(l|h)\}E[x|l] + \{\Pr(l|l) - \Pr(h|l)\}E[x|h]}
\end{aligned}$$

Define the RHS of the last inequality as $A(q)$. Since δV is increasing in δ, $\delta V \geq A(q)$ defines a lower bound of δ for which $\alpha_1 \leq \alpha_2$ holds.[16]

The numerator of $A(q)$ is

$$\begin{aligned}
&\Pr(h|h)E[x|h,h]E[x|l] - \Pr(h|l)E[x]E[x|h] \\
&= \left(q^2 H + (1-q)^2 L\right)(qL + (1-q)H) - q(1-q)(H+L)(qH + (1-q)L) \\
&= \left(q^2(H+L) + (1-2q)L\right)(-q(H-L) + H) - q(1-q)(H+L)(q(H-L) + L) \\
&= (H+L)(H-L)(-q^3 - q^2(1-q)) + (H+L)(q^2 H - q(1-q)L) + (1-2q)L(-q(H-L) + H) \\
&= -q^2(H+L)(H-L) + q^2(H+L)^2 - qL(H+L) + (1-2q)L(-q(H-L) + H) \\
&= 2Lq^2(H+L) - qL(H+L) + (1-2q)L(-q(H-L) + H) \\
&= q(2q-1)L(H+L) + (1-2q)L(-q(H-L) + H) \\
&= (2q-1)L(2qH - H) \\
&= HL(2q-1)^2
\end{aligned}$$

[16] It is straightforward to show that $\delta V \geq A(q)$ is equivalent to $\delta \geq \frac{2A(q)}{E[x] - \Pr(l,l)(E[x|l,l] - 2A(q))}$.

The denominator of $A(q)$ is

$$\{\Pr(h|h)\frac{E[x|h,h]}{E[x]} - \Pr(l|h)\}E[x|l] + \{\Pr(l|l) - \Pr(h|l)\}E[x|h]$$
$$= ((q^2 H + (1-q)^2 L)\,\frac{2}{H+L} - 2q(1-q))(qL + (1-q)H)$$
$$+ ((q^2 + (1-q)^2 - 2q(1-q))(qH + (1-q)L)$$
$$= ((q^2(H+L) + (1-2q)L)\,\frac{2}{H+L} - 2q(1-q))(q(L-H)+H) + (2q-1)^2(q(H-L)+L)$$
$$= (4q^2 - 4q + 1 - (2q-1) + 2(1-2q)\frac{L}{H+L})(q(L-H)+H) + (2q-1)^2(q(H-L)+L)$$
$$= ((2q-1)^2 - (2q-1)(\frac{2L}{H+L} - 1))(q(L-H)+H) + (2q-1)^2(q(H-L)+L)$$
$$= (2q-1)^2(H+L) - (2q-1)(\frac{2L}{H+L} - 1))(q(L-H)+H)$$
$$= \frac{2q-1}{H+L}\left[(2q-1)(H+L)^2 - (2L-(H+L))(q(L-H)+H)\right]$$
$$= \frac{2q-1}{H+L}\left[(2q-1)(H+L)^2 + (H-L)(q(L-H)+H)\right]$$
$$= \frac{2q-1}{H+L}\left[H^2 q + L^2 q - 3HL - L^2 + 6HLq\right]$$
$$= \frac{2q-1}{H+L}[q((H+L)^2 + 4HL) - (3H+L)L]$$

This shows

$$A(q) = \frac{HL\,(2q-1)\,(H+L)}{q((H+L)^2 + 4HL) - (3H+L)L}$$

We show that $A(q)$ is increasing in q. In fact, the derivative of $A(q)$ w.r.t. q is

$$\frac{\partial}{\partial q}\left(\frac{HL\,(2q-1)\,(H+L)}{q((H+L)^2+4HL)-(3H+L)L}\right)$$
$$= \frac{2HL(H+L)(q((H+L)^2+4HL)-(3H+L)L) - HL(2q-1)(H+L)((H+L)^2+4HL)}{(q((H+L)^2+4HL)-(3H+L)L)^2}$$

while the denominator is

$$2HL(H+L)(q((H+L)^2+4HL)-(3H+L)L) - HL(2q-1)(H+L)((H+L)^2+4HL)$$
$$= 2HL(H+L)(q((H+L)^2+4HL)-(3H+L)L) - 2HL(H+L)q((H+L)^2+4HL)$$
$$+ HL(H+L)((H+L)^2+4HL)$$
$$= 2HL(H+L)(-(3H+L)L) + HL(H+L)((H+L)^2+4HL)$$
$$= HL(H+L)(-2(3H+L)L + (H+L)^2 + 4HL)$$
$$= HL(H+L)(H^2 - L^2) > 0$$

Hence $A(q)$ is increasing.

Recall $E[x]$ is constant and $E[x|l,l]$ is decreasing in q, and that $E[x|l,l] = E[x]$ for $q = 1/2$. We can also see that

$$A(1) = \frac{(H+L)HL}{H(H+3L)} = \frac{(H+L)L}{(H+3L)}, \quad A(1/2) = 0$$

and

$$\begin{aligned} \frac{(H+L)L}{(H+3L)} &= \frac{H+L}{4}\frac{4L}{(H+3L)} = \left.\frac{E[x]}{2}\right|_{q=1} \times \frac{4L}{(H+3L)} < \left.\frac{E[x]}{2}\right|_{q=1}, \\ \frac{(H+L)L}{(H+3L)} &= \frac{L}{2}\frac{2(H+L)}{(H+3L)} = \left.\frac{E[x|l,l]}{2}\right|_{q=1} \times \frac{2(H+L)}{(H+3L)} > \left.\frac{E[x|l,l]}{2}\right|_{q=1} \end{aligned}$$

This shows that $E[x] > 2A(q)$ for all q, and there is a threshold $\tilde{q} \in (0,1)$ s.t. $E[x|l,l] > 2A(q)$ for $q < \tilde{q}$ and $E[x|l,l] < 2A(q)$ for $q > \tilde{q}$.

A.2 Proof of Lemma 6

Consider α^h satisfying all the on-schedule IC. We first show that $\alpha^h < \alpha^l$ if $E[x] > 2\delta V_2$. It suffices to show α_1 in Lemma 5 is smaller than α^l. Indeed

$$\begin{aligned} \alpha_1 - \alpha^l &= \frac{\Pr(h|h)\alpha^l E[x|h,h] + \Pr(l|h)\delta V_2}{E[x|h]} - (1 - \frac{\delta V_2}{E[x]}) \\ &= (\frac{\Pr(h|h)E[x|h,h]}{E[x|h]} - 1)(1 - \frac{\delta V_2}{E[x]}) + \frac{\Pr(l|h)\delta V_2}{E[x|h]} \\ &= (\frac{-\Pr(l|h)E[x]}{E[x|h]})(1 - \frac{\delta V_2}{E[x]}) + \frac{\Pr(l|h)\delta V_2}{E[x|h]} = \frac{\Pr(l|h)(-E[x] + 2\delta V_2)}{E[x|h]} < 0. \end{aligned}$$

Now we check the off-schedule incentive conditions.

(i) For $\tilde{\alpha} > \alpha^l = \alpha^*$, the responder with the pessimistic belief would always reject, and the proposer's resulting payoff would be δV_2. To deter this deviation,

$$\alpha^h E[x|h] \geq \delta V_2$$
$$\Pr(h|l)\alpha^l E[x] + \Pr(l|l)\delta V_2 \geq \delta V_2 \Leftrightarrow \alpha^l E[x] \geq \delta V_2$$

should hold. The former condition is redundant given the latter one and (IC_{hl}), since

$$\begin{aligned} \alpha^h E[x|h] &\geq \Pr(h|h)\alpha^l E[x|h,h] + \Pr(l|h)\delta V_2, \\ &\geq \Pr(h|h)\alpha^l E[x] \qquad + \Pr(l|h)\delta V_2 \geq \delta V_2. \end{aligned}$$

The latter condition can be rewritten as

$$\alpha^l E[x] \geq \delta V_2 \Leftrightarrow (1 - \frac{\delta V_2}{E[x]})E[x] \geq \delta V_2 \Leftrightarrow E[x] \geq 2\delta V_2$$

which holds if Rule 2 is efficient.

(ii) For $\tilde{\alpha} \in (\alpha', \alpha^*)$, where $(1-\alpha')E[x|l,l] = \delta V_2$, the responder accepts only if its own signal is high. This deviation is nonsense since the same acceptance rule applies for α^*. As long as the on-schedule IC hold, this deviation is never profitable.

(iii) For $\tilde{\alpha} \leq \alpha'$, the responder always accepts. There are two cases. First suppose $\alpha^h \geq \alpha'$. For this case, this deviation is nonsense since the same acceptance rule applies for a^h. On the other hand, suppose $\alpha^h < \alpha'$. Then the proposer with a high signal would be better off by offering α' instead of α^h. To deter such deviation, we thus need $\alpha^h \geq \alpha'$, which is equivalent to

$$\alpha^h \geq 1 - \frac{\delta V_2}{E[x|l,l]}.$$

Recall that the upper bound of α^h is either α_2 in Lemma 5, or $\alpha^l = 1 - \delta V_2/E[x]$. We show this inequality by establishing $\alpha_2 \geq 1 - \delta V_2/E[x|l]$, which implies $\alpha_2 > 1 - \delta V_2/E[x|l,l] = \alpha'$.[17] Indeed

$$\begin{aligned}
\alpha_2 - (1 - \frac{\delta V_2}{E[x|l]}) &= \frac{\Pr(h|l)\alpha^l E[x] + \Pr(l|l)\delta V_2}{E[x|l]} - (1 - \frac{\delta V_2}{E[x|l]}) \\
&= \frac{\Pr(h|l)(E[x] - \delta V_2) + \Pr(l|l)\delta V_2}{E[x|l]} - (1 - \frac{\delta V_2}{E[x|l]}) \\
&= \frac{\Pr(h|l)E[x] - E[x|l] + 2\Pr(l|l)\delta V_2}{E[x|l]} = \frac{\Pr(l|l)(-E[x|l,l] + 2\delta V_2)}{E[x|l]} \geq 0.
\end{aligned}$$

A.3 Proof of Lemma 7

Rearranging (IC_h), we have

$$\begin{aligned}
&(1 - \delta V_3/E[x|l])\, E[x|h] \geq (1 - \delta/2)\Pr(h|h)E[x|h,h] + \Pr(l|h)\delta V_3 \\
&\Leftrightarrow \delta V_3 \left(\Pr(h|h)E[x|h,h]/E[x] - E[x|h]/E[x|l] - \Pr(l|h)\right) \geq \Pr(h|h)E[x|h,h] - E[x|h] \\
&\Leftrightarrow \delta V_3 \left(\frac{E[x|h] - \Pr(l|h)E[x]}{E[x]} - \frac{E[x|h]}{E[x|l]} - \Pr(l|h) \right) \geq -\Pr(l|h)E[x] \\
&\Leftrightarrow \delta V_3 \leq \frac{\Pr(l|h)E[x]}{2\Pr(l|h) - E[x|h](1/E[x] - 1/E[x|l])}
\end{aligned}$$

[17] Recall that $\alpha_1 < \alpha^l$ if $E[x] > 2\delta V_2$, and that $\alpha_1 \leq \alpha_2$ if both on-schedule ICs hold. By definition, $\alpha' < \alpha^l$. This implies that if $\alpha' < \alpha_2$, two intervals $[\alpha_1, \alpha_2]$ and $[\alpha', \alpha^l]$ must overlap, and we can find α^h satisfying $\alpha_1 \leq \alpha^h < \alpha^l$, and $\alpha' \leq \alpha^h$.

Define

$$
\begin{aligned}
B(q) &= \frac{\Pr(l|h)E[x]}{2\Pr(l|h) - E[x|h]\left(1/E[x] - 1/E[x|l]\right)} \\
&= \frac{q(1-q)(H+L)}{4q(1-q) - (qH + (1-q)L))(2/(H+L) - 1/(qL + (1-q)H))}
\end{aligned}
$$

It is easy to see

$$
\begin{aligned}
B(1) &= \frac{0}{0 - H(2/(H+L) - 1/(2L))} = 0 \\
B(1/2) &= \frac{\frac{1}{2}E[x]}{1 - E[x] \times 0} = \frac{E[x]}{2} = \left.\frac{E[x|l,l]}{2}\right|_{q=1/2}.
\end{aligned}
$$

By definition,

$$
\begin{aligned}
&2B(q) - E[x|l,l] \\
&= \frac{2\Pr(l|h)E[x]}{2\Pr(l|h) - E[x|h]\left(1/E[x] - 1/E[x|l]\right)} - E[x|l,l] \\
&= \frac{2E[x]\Pr(l|h) - E[x|l,l][2\Pr(l|h) - E[x|h]\left(1/E[x] - 1/E[x|l]\right)]}{2\Pr(l|h) - E[x|h]\left(1/E[x] - 1/E[x|l]\right)} \\
&= \frac{2E[x]^2\Pr(l|h)E[x|l] - E[x|l,l][2E[x]\Pr(l|h)E[x|l] - E[x|h]\left(E[x|l] - E[x]\right)]}{2E[x]\Pr(l|h)E[x|l] - E[x|h]\left(E[x|l] - E[x]\right)}
\end{aligned}
$$

Note that the denominator is positive for all q. The numerator is

$$
\begin{aligned}
&2E[x]^2\Pr(l|h)E[x|l] - E[x|l,l][2E[x]\Pr(l|h)E[x|l] - E[x|h]\left(E[x|l] - E[x]\right)] \\
&= 2E[x]\Pr(l|h)E[x|l][E[x] - E[x|l,l]] - E[x|l,l]E[x|h]\left(E[x] - E[x|l]\right)] \\
&= (H+L)2q(1-q)(qL + (1-q)H)[\frac{H+L}{2} - \frac{q^2L + (1-q)^2H}{q^2 + (1-q)^2}] \\
&- \frac{q^2L + (1-q)^2H}{q^2 + (1-q)^2}(qH + (1-q)L)[\frac{H+L}{2} - (qL + (1-q)H)] \\
&= (H+L)\frac{q(1-q)}{q^2 + (1-q)^2}(qL + (1-q)H)[(q^2 - (1-q)^2)(H-L)] \\
&- \frac{q^2L + (1-q)^2H}{q^2 + (1-q)^2}(qH + (1-q)L)\frac{(2q-1)(H-L)}{2} \\
&= \frac{(2q-1)(H-L)}{2(q^2 + (1-q)^2)}[2(H+L)q(1-q)(qL + (1-q)H) - (q^2L + (1-q)^2H)(qH + (1-q)L)]
\end{aligned}
$$

The fraction in the last line is positive for $q > 1/2$. If $q = 1/2$, the expression in the square bracket is

$$
2(H+L)\frac{1}{4}\frac{H+L}{2} - \frac{H+L}{4}\frac{H+L}{2} = \frac{(H+L)^2}{8} > 0
$$

Also, the expression can be rewritten as

$$
\begin{aligned}
&2(H+L)q(1-q)(qL+(1-q)H)-(q^2L+(1-q)^2H)(qH+(1-q)L)\\
&=H^2q^3-2H^2q^2+H^2q-5HLq^2+5HLq-HL-L^2q^3+L^2q^2\\
&=H^2(q^3-2q^2+q)+HL(-5q^2+5q)+L^2(-q^3+q^2)-HL\\
&=H^2q(1-q)^2+5HLq(1-q)+L^2q^2(1-q)-HL\\
&=((1-q)H^2+qL^2+5HL)q(1-q)-HL
\end{aligned}
$$

Since $((1-q)H^2+qL^2)$ and $q(1-q)$ are decreasing for $q \in (1/2,1)$, the expression is monotone decreasing in q and goes to $-HL$. Therefore, as long as $L > 0$, there is $\tilde{q} \in (1/2,1)$ s.t. $2B(q) < E[x|l,l]$ iff $q > \tilde{q}$.

B Plausibility of Off-Path Beliefs

We examine whether the off-path beliefs of the above equilibria are reasonable in line with D1 criterion of Banks and Sobel (1987). Two remarks are noted. First, the game is not a standard signaling game in which each player can move only once. However, the information in our model is not persistent and so we can still apply the refinement argument within a period (at least for stationary equilibrium). Second, we are dealing with two-sided incomplete information with correlation. We need to consider a belief system conditional on the responder's type. To incorporate correlation, we adopt the following "two-stage" beliefs.

Let $\beta(\alpha)$ denote the belief that the proposer has signal h given offer α, *ignoring the responder's signal* (think of it as an outsider's belief). Suppose α is offered with some probability. By Bayes' rule

$$
\beta(\alpha)=\frac{\Pr(\alpha|s_p=h)\Pr(s_p=h)}{\Pr(\alpha|s_p=h)\Pr(s_p=h)+\Pr(\alpha|s_p=l)\Pr(s_p=l)}=\frac{\Pr(\alpha|h)}{\Pr(\alpha|h)+\Pr(\alpha|l)}.
$$

For example, with a pooling offer α, i.e., $\Pr(\alpha|h)=\Pr(\alpha|l)=1$, hence $\beta(\alpha)=.5$. With a separating offer α^h,α^l, i.e., $\Pr(\alpha^h|h)=\Pr(\alpha^l|l)=1$, hence $\beta(\alpha^h)=1$ and $\beta(\alpha^l)=0$. Note that

$$
\begin{aligned}
\Pr(s_p=h,\alpha|s_r=h)&=\Pr(\alpha|s_p=h,s_r=h)\Pr(s_p=h|s_r=h)\ \text{(Bayes' rule)}\\
&=\Pr(\alpha|s_p=h)\Pr(s_p=h|s_r=h)\ \text{(the choice of }\alpha\text{ only depends on }s_p)
\end{aligned}
$$

Responder's belief $\beta(h|\alpha, s_r)$ can be induced by $\beta(\alpha)$:

$$\begin{aligned}
\beta(s_p = h|\alpha, s_r = h) &= \frac{\Pr(h|\alpha, s_r = h)}{\Pr(h|\alpha, s_r = h) + \Pr(l|\alpha, s_r = h)} \\
&= \frac{\Pr(h, \alpha|s_r = h)}{\Pr(h, \alpha|s_r = h) + \Pr(l, \alpha|s_r = h)} \text{ (Bayes' rule + eliminating } \Pr(\alpha|s_r = h)) \\
&= \frac{\Pr(h|s_r = h) \Pr(\alpha|s_p = h)}{\Pr(h|s_r = h) \Pr(\alpha|s_p = h) + \Pr(l|s_r = h) \Pr(\alpha|s_p = l)} \text{ (see the above note)} \\
&= \frac{\Pr(h|h)\beta(\alpha)}{\Pr(h|h)\beta(\alpha) + \Pr(l|h)(1 - \beta(\alpha))} \\
\beta(s_p = h|\alpha, s_r = l) &= \frac{\Pr(h|l)\beta(\alpha)}{\Pr(h|l)\beta(\alpha) + \Pr(l|l)(1 - \beta(\alpha))}
\end{aligned}$$

This is consistent with, say, the on-path belief of pooling, $\beta(h|\alpha, h) = q^2 + (1-q)^2$. We apply the above formulas for off-path beliefs given an arbitrary $\beta(\alpha)$. For notational simplicity, we denote β for $\beta(\alpha)$, $\beta^h = \beta(s_p = h|\alpha, s_r = h)$, and $\beta^l = \beta(s_p = h|\alpha, s_r = l)$.

Now we study the plausibility of pessimistic belief $\beta = 0$ off-the-equilibrium paths. A version of D1 criterion for our model can be stated as follows (see Fudenberg and Tirole, 1991, p.452): *for an off-path α, consider $\beta \in [0, 1]$ for which the proposer with an h is weakly better off than the associated equilibrium payoff, given the best responses by the responder for β. If, given β, the proposer with an l is always strictly better off than the associated equilibrium payoff, and if there is β' for which only the proposer with an l can be strictly better off, then the belief should set $\beta = 0$.*

A straightforward algebra shows that $\beta^h \geq \beta^l$ for all $\beta \in [0, 1]$, where equality holds at $\beta = 0, 1$. There are three possible best responses by the responder given α, β and V: if

$$(1 - \alpha) \left[\beta^l E[x] + (1 - \beta^l)E[x|l, l]\right] \geq \delta V$$

the responder always accepts, if

$$(1 - \alpha) \left[\beta^l E[x] + (1 - \beta^l)E[x|l, l]\right] < \delta V \leq (1 - \alpha) \left[\beta^h E[x|h, h] + (1 - \beta^h)E[x]\right] \geq \delta V$$

only the responder with an h accepts, and if

$$(1 - \alpha) \left[\beta^h E[x|h, h] + (1 - \beta^h)E[x]\right] < \delta V$$

the responder always rejects.

To support $\beta = 0$ under the criterion, we just need to show that the opposite ($\beta = 1$ is only plausible) does not hold.

Efficient equilibrium for Rule 1. Consider the case where $(1-\alpha^h)E[x|h,h] \geq \delta V$ binds. Here for $\alpha > \alpha^h$, the responder cannot accept even for the best scenario, $\beta = 1$. For $\alpha < \alpha^h$, consider the highest α' s.t. the responder's best response of always accepting can arise;

$$(1-\alpha')E[x] \geq \delta V$$

with the best scenario $\beta = 1$. For such α', the payoff of the proposer with an h would be

$$\begin{aligned}
\alpha' E[x|h] &= \alpha' \Pr(h|h)E[x|h,h] + \alpha' \Pr(l|h)E[x] \\
&\leq \alpha' \Pr(h|h)E[x|h,h] + \Pr(l|h)(E[x] - \delta V) \text{ (by } (1-\alpha')E[x] \geq \delta V) \\
&\leq \alpha' \Pr(h|h)E[x|h,h] + \Pr(l|h)\delta V \text{ (by } 2\delta V \geq E[x] \text{ for Rule 1)} \\
&< \alpha^h \Pr(h|h)E[x|h,h] + \Pr(l|h)\delta V
\end{aligned}$$

Thus there is no off-path α by which the proposer with an h can be better off than the equilibrium payoff.

Consider the case where $(1-\alpha^h)E[x|h,h] \geq \delta V$ does not bind. In this case we have seen that in an equilibrium IC_{lh} binds. Now for $\alpha > \alpha^h$ (but $\alpha \leq \bar{\alpha}^h$ s.t. $(1-\bar{\alpha}^h)E[x|h,h] = \delta V$), if β is high enough, the responder with an h would accept. However, given such belief and the best response, the proposer with either signal would be better off than the equilibrium payoff (by $\alpha > \alpha^h$ for h, and by the binding IC_{lh} for l). The same discussion applies for $\alpha < \alpha^h$. Thus the belief here is also arbitrary under D1 criterion.

Efficient equilibrium for Rule 2. We show that *the equilibrium passes the D1 test iff IC_{lh} is binding.*

Since α^h is always accepted on the equilibrium path, $\alpha < \alpha^h$ would never be better for the proposer with an h. Note that $(1-\alpha^l)E[x] = \delta V_2$, implying that $\alpha' > \alpha^l$ cannot be accepted by the responder with an l even with the best scenario $\beta = 1$. Consider α' s.t. $\alpha^l < \alpha' \leq \bar{\alpha}$ where $(1-\bar{\alpha})E[x|h,h] = \delta V$, which can be accepted if β is sufficiently high. Obviously this α' is strictly preferred by the proposer with an l, as far as it is accepted by the responder with an h and this implies $\beta = 0$ for α' is plausible under D1. Consider α' s.t. $\alpha^h < \alpha' < \alpha^l$. If we can find some α' and β s.t. α' is always accepted but is not preferred by the proposer with an l, we must have $\beta = 1$ under D1. For α' s.t. $(1-\alpha')E[x] \geq \delta V$, the responder always accepts if $\beta = 1$, hence for $\alpha^h < \alpha' < \alpha^l$ the offer is always accepted with the best scenario. Consider an equilibrium s.t. IC_{lh} binds, namely

$$\alpha^h E[x|l] = \Pr(h|l)\alpha^l E[x] + \Pr(l|l)\delta V_2.$$

Given this equilibrium, the proposer would strictly prefer α' s.t. $\alpha^h < \alpha' < \alpha^l$, regardless of its type. Hence D1 criterion does not regulate off-path belief for such α'. On the other hand, for an equilibrium in which IC_{lh} is slack, there is an α'' close to α^h s.t. the proposer with an h would better off while one with an l would not. Hence the equilibrium in which IC_{lh} binds only survives the D1 test.

Efficient equilibrium for Rule 3. Indeed, *some efficient Rule 3 equilibrium fails to pass the D1 test.* If $\alpha' > \alpha$ (equilibrium) is always accepted by the responder, the proposer wishes to deviate regardless of its signal. Consider the case where $\alpha' > \alpha$ is accepted only by the responder with an h. We check whether the proposer with an h would strictly prefer α' than equilibrium α while the proposer with an l would not.

Consider the best offer α', i.e.,

$$(1-\alpha')E[x|h,h] = \delta V \Leftrightarrow \alpha' = 1 - \frac{\delta V}{E[x|h,h]}.$$

An offer higher than α' would surely be rejected even though the responder has signal h. Offer α' is strictly preferred by the proposer with an h iff (recalling $\alpha = 1 - \delta V/E[x|l]$)

$$\begin{aligned}
&\alpha E[x|h] \le \alpha' \Pr(h|h)E[x|h,h] + \Pr(l|h)\delta V \\
&\Leftrightarrow (1-\delta V/E[x|l])\, E[x|h] \le \Pr(h|h)E[x|h,h] + (2\Pr(l|h)-1)\delta V \\
&\Leftrightarrow \Pr(l|h)E[x] \le (E[x|h]/E[x|l] + 2\Pr(l|h) - 1)\, \delta V \\
&\Leftrightarrow \delta V \ge \frac{\Pr(l|h)E[x]}{2\Pr(l|h) + E[x|h](1/E[x|l] - 1/E[x|h])}.
\end{aligned}$$

Otherwise, the proposer with an h would never prefer deviating α' which is accepted only by the responder with an h, and therefore the pessimistic belief can be maintained under the D1 test.

Recall IC_h for this equilibrium:

$$\delta V \le \frac{\Pr(l|h)E[x]}{2\Pr(l|h) + E[x|h](1/E[x|l] - 1/E[x])}$$

Since $-1/E[x] < -1/E[x|h]$, there is some parameter region where the above two inequalities are satisfied. Note also that, if q is close to 1, the proposer with an l is nearly certain that offer α' would be rejected and thus would not prefer α' (we could proceed to the tedious details for this argument). This implies that the region where efficient Rule 3 equilibrium exists and passes the D1 test is smaller than the one identified earlier. But if

$$\delta V \le \frac{\Pr(l|h)E[x]}{2\Pr(l|h) + E[x|h](1/E[x|l] - 1/E[x|h])}$$

such an equilibrium exists