Re-examining the Effects of Switching Costs

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Abstract

In many markets consumers incur costs when switching from one product to another. Recently there has been renewed debate within the literature about whether these switching costs lead to higher prices. This paper builds a theoretical model of dynamic competition and solves it analytically for a wide range of switching costs. We provide a sharp condition on parameters which determines whether long-run prices are higher or lower. The paper also examines short-run variation in prices, and demonstrates that switching costs can even make consumers better off. Finally as a robustness check we allow consumer tastes to be correlated across time.

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1 Introduction

In many markets consumers incur costs if they switch from the product they currently purchase, to another product sold by a different company. These *switching costs* partially lock consumers to their initial supplier, and this creates the well-known trade-off between harvesting and investing. On the one hand a firm might charge a high price and harvest its existing customers, exploiting their reluctance to switch. On the other hand since consumers tend not to switch, there is a strong relationship between current market share and future profitability. A firm might therefore prefer to invest in market share by charging a low price. The conventional wisdom has been that the harvesting effect dominates, such that switching costs lead to higher prices (Farrell and Klemperer 2007). However, recent research challenges this perceived wisdom, arguing that small switching costs lead to lower prices (for example Dubé *et al* 2009, Doganoglu 2010).

In this paper we build a model of dynamic competition between two infinitely-lived firms, and use it to re-examine the effect of switching costs on prices, profits, and consumer surplus. Our approach goes beyond that of other papers in several ways. Firstly we solve our model analytically for a wide range of switching costs. This contrasts with other papers which typically use numerical simulations, or focus only on very small or very large switching costs. Solving the model analytically also leads to sharper and more general conclusions, as well as better intuitions. Secondly other papers only study the long-run (steady state) impact of switching costs, but we also look at the short-run impact. Thirdly much of the literature is focused solely on the question of whether switching costs lead to higher or lower prices. We go further and ask whether switching costs might even lead to such fierce price competition, that they actually improve consumer welfare. Finally we also check the robustness of our results (and those of other recent papers) by allowing consumer tastes to be correlated across time.

Switching costs appear to be quantitatively important across a wide range of industries. For example in the U.S. auto insurance industry, Honka (2010) estimates an average switching cost of \$84, which is around 14% of the average premium. In the U.S. pay-TV market, Shcherbakov (2009) estimates switching costs of \$109 for cable and \$186 for satellite. These amount to about 32% and 52% respectively of what a typical subscriber would spend annually on these services. Evidence of significant switching costs has also been found, amongst others, in markets for health insurance (Handel 2011), domestic gas (Giulietti *et al* 2005), bank loans (Kim *et al* 2003), cell phones and bank deposits (Shy 2002).

Our paper adds to a large theoretical literature on switching costs, which is surveyed in Klemperer (1995) and Farrell and Klemperer (2007). Earlier papers focused on two-period models, in which firms offer 'bargains' to consumers when they are young, and 'rip-offs' when they become old. One potential drawback of these models is that they artificially separate out the investment and harvesting motives into the first and second periods respectively. In reality firms often compete over a long time horizon, and at any moment are trying to both attract new consumers and sell to old ones. For this reason the literature has increasingly focused on infinite-horizon models.

When firms can commit to charge the same price in every future period, von Weizsäcker (1984) shows that switching costs may intensify competition. On the other hand when firms sell homogeneous products and cannot commit to future prices, Farrell and Shapiro (1988) and Padilla (1995) find that switching costs raise prices above marginal cost (i.e. the usual Bertrand outcome). However both papers have the unusual feature that no consumer ever switches in equilibrium. In a similar vein when firms sell differentiated products, Beggs and Klemperer (1992) and To (1996) also find that switching costs lead to unambiguously higher steady state prices. However both papers make the very strong assumption that switching costs are so large that again no consumer ever actually switches.

More recently other papers have argued instead that small switching costs lead to prices which are below those observed in a frictionless market. Doganoglu (2010) presents a model with overlapping generations of consumers, and two firms which each sell a different experience good. When consumers are young they have no information on the characteristics of the two products, and their preferences are determined by advertising hype. When they become old, their preferences are shaped by their experience of the first product that they consumed. Doganoglu shows that starting from zero, a small increase in the switching cost leads to a decrease in the steady state price.[1] Cabral (2008) looks at a different setting where two firms compete for the custom of one infinitely-lived consumer, whose preferences for the two products change independently over time. He also shows that starting from zero, a small switching cost reduces average price. Arie and Grieco (2011) present a model with myopic consumers, and a single

[1] Two related papers are Villas-Boas (2006) and Somaini and Einav (2011). In Villas-Boas's model the distribution of product matches is skewed, and this leads to inertia in consumers' purchase decisions. The skew therefore acts like a switching cost which, depending upon parameters, can lead to higher prices. Somaini and Einav (2011) extend Doganoglu's model to allow for an arbitrary number of firms, and then use it to study the anti-trust implications of mergers in dynamic customer markets.

firm that competes against a non-strategic outside good. A small switching cost again reduces steady state price whenever the firm's market share is sufficiently low.[2] To summarize, these three papers focus on steady state, show that a very small switching cost reduces prices, and then use numerical simulations to show that the same happens for somewhat larger switching costs as well.

In this paper we present a model which is largely inspired by Doganoglu (2010), although we represent consumer preferences using a standard Hotelling line, rather than an experience goods framework. As explained earlier, we go beyond the work of Doganoglu and others in four main ways. Firstly we solve the model analytically, for a wide range of switching costs which are consistent with consumers switching between products in equilibrium. It turns out that comparative statics around a zero switching cost are rather special: for any strictly positive switching cost, steady state price can actually be higher or lower depending upon parameters. We therefore provide a sharp condition on discount rates, which determines whether switching costs are pro- or anti-competitive. After comparing this condition with empirical evidence on rates of time preference, we believe that compared to papers which rely on numerical simulations, our model provides stronger and more general evidence in favor of switching costs being pro-competitive. Our analytical approach is also better because it provides clearer intuitions, and helps explain why switching costs are often pro-competitive when they are relatively small but (as shown in Beggs and Klemperer 1992 and To 1996) not when they are large.

Secondly since dynamic competition is hard to analyze, other papers focus only on the long-run (steady state) impact of switching costs. However we show that in the short-run there is considerable variation in the prices charged by individual firms. On average prices are also higher, so focusing only on steady state may lead to biased conclusions about the pro-competitiveness of switching costs. We then provide a condition on parameters which guarantees that average price is lower in both the short- and long-run. Thirdly an important topic which has nevertheless been neglected in the previous literature, is the question of whether consumers could ever benefit from switching costs. We show that sometimes they do, and provide a condition on rates of time preference which guarantees this.

Finally for tractability, it is standard within the literature to assume that consumer

[2] Also related are Biglaiser *et al* (2010) and Bouckaert *et al* (2010), who explore heterogeneity in switching costs. The papers make different assumptions about the distribution of switching costs, but both show that an increase in switching costs can lead to lower industry profits.

preferences change independently over time. We also check how robust our results are once this assumption is relaxed. In particular we focus only on very small switching costs, but allow preferences to change over time in a very general way. We show that on the one hand independence is not an innocuous assumption: relaxing it may cause switching costs to be anti-competitive. On the other hand it is possible for preferences to be quite strongly correlated over time and yet switching costs still be pro-competitive. We then establish a condition which guarantees that in this more general setting, a small switching cost again leads to lower steady state prices.

Empirical evidence on the effect of switching costs is consistent with the above discussion. Dubé *et al* (2009) look at psychological costs of switching between brands of orange juice and margarine. They estimate switching costs to be on the order of 15-19% of the purchase price. They also estimate that these switching costs reduce market prices for these products by 3-6%. However Viard (2007) finds that number portability led to a 14% reduction in prices charged to firms that had toll-free phone numbers. Since number portability reduces the cost of changing phone providers, this implies that switching costs raise market prices. Intuitively without number portability, a firm that changes its telephone number must advertise this fact to all its potential customers, as well as reprint stationary and business cards. The market for toll-free calls therefore has very large switching costs and so is closer to traditional models like Beggs and Klemperer (1992), but other markets with smaller switching costs are closer to ours and other recent papers.[3]

The rest of the paper proceeds as follows. Section 2 outlines the model, whilst Section 3 proves existence and uniqueness of an equilibrium in affine strategies. We then examine how switching costs affects prices and profits (in Section 4) and consumer surplus (in Section 5). Section 6 then extends the model in two directions. First we allow consumers to live for many periods, and secondly we allow consumer tastes to be correlated across time. Finally we conclude in Section 7 with some directions for future research.

[3]Park's (2011) analysis of the impact of number portability on individual users, also supports this interpretation. Low-volume users are likely to have few contacts to inform of a number change, and therefore have lower switching costs than high-volume users. Since some carriers reacted to number portability by introducing surcharges, low-volume users ended up paying more and high-volume users paying less (page 731). This is consistent with small switching costs being pro-competitive, but larger switching costs being anti-competitive.

2 Model

Time is discrete and there are infinitely many periods, denoted by $t = 1, 2, \ldots$ There are two firms A and B that are located on a Hotelling line at positions $x = 0$ and $x = 1$ respectively. The marginal cost of production is normalized to zero for both firms. Each period a unit mass of new consumers is born, who then live for two periods before exiting the market. Consequently at any moment there are (equal-sized) overlapping generations of 'young' and 'old' consumers in the economy. At the start of period t each consumer is randomly assigned a location x^t on the Hotelling line, which (for old consumers) is independent of their location in the previous period. A consumer with location x^t values product A at $V - x^t$ and product B at $V - (1 - x^t)$.[4] If an 'old' consumer bought from firm i when young but now wants to buy from firm $j \neq i$, she must incur a switching cost $s \in (0, 7/10]$. As explained more fully below, we restrict attention to $s \leq 7/10$ in order to ensure that in equilibrium each firm always has some consumers switching to it and others switching away from it. Consumers and firms are both risk-neutral and have discount factors δ_c and δ_f respectively which lie in $(0, 1)$.

The timing of the model is as follows. In each period t the two firms simultaneously and non-cooperatively choose prices p_A^t and p_B^t, in order to maximize their respective discounted sum of current and future profits. Firms are unable to make any commitments about their future prices. Consumers then observe p_A^t and p_B^t as well as their own personal location x^t. Young consumers buy whichever product maximizes their expected lifetime utility. Old consumers either stay with their initial supplier, or pay the switching cost and buy from the competitor.

Our model is closely related to both To (1996) and Doganoglu (2010). Our set-up differs from To's because he makes the switching cost so large that nobody ever switches; our model is designed in such a way that some switching occurs in every period. Our model differs from Doganoglu's because he also allows consumers to learn about their match for a product by experiencing it. Mathematically the two models are equivalent when Δ (a parameter in his model which captures heterogeneity in consumers' product valuations) equals 2. However Doganoglu only provides comparative statics for steady state price around the point $s = 0$. We study prices both in and out of steady state, as well as consumer surplus. Moreover our analysis is valid for any $s \in (0, 7/10]$, and so is much more general.

[4] Any transport cost parameter would simply scale up all prices in the model, and so without loss of generality is normalized to one.

3 Solving the model

3.1 Consumers

Old consumers Suppose an old consumer bought product A when she was young. When old, she can again buy product A and enjoy a surplus $V - x^t - p_A^t$, or she can switch to product B and get $V - (1 - x^t) - p_B^t - s$. Therefore buying product A again is optimal if and only if $x^t \leq \dot{x}^t = \left(1 + p_B^t - p_A^t + s\right)/2$. Similarly an old consumer who bought product B when she was young, optimally switches to firm A if and only if $x^t \leq \ddot{x}^t = \left(1 + p_B^t - p_A^t - s\right)/2$. We will show that since the switching cost is not too large, in equilibrium $\dot{x}^t, \ddot{x}^t \in (0, 1)$. This means that generically each firm always has both some old consumers switching away from it and others switching towards it.

Young consumers born in period t form (rational) expectations about the prices Ep_A^{t+1} and Ep_B^{t+1} they will face when old (we discuss more fully below how these expectations arise). If a young consumer buys product A she gets an immediate payoff $V - x^t - p_A^t$; when old, she will stay with product A and get $V - x^{t+1} - Ep_A^t$ if x^{t+1} is sufficiently low, otherwise she will switch to product B and get $V - (1 - x^{t+1}) - Ep_B^t - s$. Taking an expectation over all possible values of x^{t+1}, the young consumer can calculate her expected lifetime payoff from buying product A. She can similarly calculate the expected utility from product B, and then buys whichever is best for her. Some simple algebra reveals the following: (Note that all proofs are given in the appendices.)

Lemma 1 *There exists a threshold $\tilde{x}^t$ such that all young consumers with location $x^t \leq \tilde{x}^t$ buy product A, and everyone else buys product B. The threshold satisfies*

$$\tilde{x}^t = \frac{1}{2} + \frac{p_B^t - p_A^t + \delta_c s\left(Ep_B^{t+1} - Ep_A^{t+1}\right)}{2} \tag{1}$$

Young consumers located at $\tilde{x}^t$ expect to get the same lifetime utility from both products and are therefore indifferent between them. People located to the left (right) of $\tilde{x}^t$ have a stronger initial preference for product A (B) and therefore buy it.

3.2 Firms

Each firm has a strategy which specifies a price that should be played, for every time period and for every possible history of the game. The aim of this section is to solve for and characterize that strategy.

We know from the previous section that in period t product A is bought by $\tilde{x}^t$ young consumers (defined in Lemma 1) and by $\tilde{x}^{t-1}\dot{x}^t + (1-\tilde{x}^{t-1})\,\ddot{x}^t$ old consumers. Total demand for product A is therefore not just a function of current prices p_A^t and p_B^t, but also depends upon consumers' price expectations and past market share. To reflect this write A's demand as $D_A^t\left(p_A^t, p_B^t, Ep_A^{t+1}, Ep_B^{t+1}, \tilde{x}^{t-1}\right)$. Given any initial history firm A has the following optimization problem:

$$\max_{\{p_A^\tau\}_{\tau=t}^\infty} \sum_{\tau=t}^{\infty} \delta_f^{\tau-t} p_A^\tau D_A^\tau\left(p_A^\tau, p_B^\tau, Ep_A^{\tau+1}, Ep_B^{\tau+1}, \tilde{x}^{\tau-1}\right) \tag{2}$$

subject to i). B's strategy, ii). the process by which expectations $\left\{Ep_A^{\tau+1}, Ep_B^{\tau+1}\right\}_{t=\tau}^{\infty}$ are formed, and iii). subject to equation (1) which specifies how $\tilde{x}^\tau$ evolves over time.

It is natural to simplify the optimization problem (2) by restricting attention to linear Markovian pricing strategies - meaning that each firm's price is a linear function of its stock of old consumers, but does not otherwise depend upon the history of the game.[5] In particular recall that $\tilde{x}^{t-1}$ is a threshold such that in period $t-1$, all young consumers located to the left of $\tilde{x}^{t-1}$ bought from firm A, and all other young consumers bought from firm B. We therefore suppose that firms use the following pricing strategies

$$p_A^t\left(\tilde{x}^{t-1}\right) = J + K\left(\tilde{x}^{t-1} - 1/2\right) \tag{3}$$

$$p_B^t\left(\tilde{x}^{t-1}\right) = J - K\left(\tilde{x}^{t-1} - 1/2\right) \tag{4}$$

The interpretation is that when $\tilde{x}^{t-1} = 1/2$, each firm sold to half of the young consumers born in period $t-1$, so come period t the two firms are symmetric and both charge the same price J. We will prove later that $K > 0$ so if instead $\tilde{x}^{t-1} > 1/2$, firm A sold to more than half of young consumers in period $t-1$, and therefore charges more than B does in period t.

Forward-looking young consumers can use equations (3) and (4) and predict that $Ep_B^{t+1} - Ep_A^{t+1} = -2K\left(\tilde{x}^t - 1/2\right)$. Substituting this into equation (1) the marginal young consumer in period t has location

$$\tilde{x}_t = \frac{1}{2} + \frac{p_B^t - p_A^t}{2\left(1 + K\delta_c s\right)} \tag{5}$$

[5] This is natural because using a simple inductive argument, we can show that the finite-horizon versions of our model have a unique equilibrium, and it involves firms using symmetric linear pricing strategies. Of course given the infinitely-repeated nature of our game, there might for example be other equilibria which exhibit collusive behavior.

It follows that if firms use the pricing strategies (3) and (4) in period t, their demands in that period will be linear in $\tilde{x}^{t-1}$ and their profits quadratic in $\tilde{x}^{t-1}$. This suggests that in period t the net present value to a firm of its current and future profits, will also be quadratic in $\tilde{x}^{t-1}$. Therefore we look for value functions $V_A^t(\tilde{x}^{t-1})$ and $V_B^t(\tilde{x}^{t-1})$ which have the following form:

$$V_A^t\left(\tilde{x}^{t-1}\right) = M + N\left(\tilde{x}^{t-1} - 1/2\right) + R\left(\tilde{x}^{t-1} - 1/2\right)^2 \tag{6}$$
$$V_B^t\left(\tilde{x}^{t-1}\right) = M - N\left(\tilde{x}^{t-1} - 1/2\right) + R\left(\tilde{x}^{t-1} - 1/2\right)^2 \tag{7}$$

A linear Markovian strategy is therefore characterized by the behavioral parameters J, K, M, N and R.

A linear Markov perfect equilibrium (MPE) exists whenever the two firms' strategies are subgame perfect (see Fudenberg and Tirole 1991). Proposition 2 below shows that there is a unique such equilibrium. The proof (in Appendix A.1) proceeds by showing that dynamic optimality imposes just enough conditions to uniquely pin down the five behavioral parameters. A sketch of the proof is as follows. Since product A is bought by $\tilde{x}^t + \tilde{x}^{t-1}\dot{x}^t + (1 - \tilde{x}^{t-1})\ddot{x}^t$ consumers, its demand can be written as

$$D_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) = \frac{1}{2} + \frac{p_B^t - p_A^t}{2(1 + K\delta_c s)} + \frac{1 + p_B^t - p_A^t}{2} + s\left(\tilde{x}^{t-1} - 1/2\right) \tag{8}$$

Firstly A's strategy must be subgame perfect. The principle of optimality says that A will

$$\max_{p_A^t} p_A^t D_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) + \delta_f V_A^{t+1}\left(\tilde{x}^t\right) \tag{9}$$

subject to i). B playing the linear strategy in equation (4) and ii). subject to $\tilde{x}^t$ satisfying equation (5). Subgame perfection requires that the solution to this maximization problem, is precisely the linear strategy given by equation (3). Appendix A.1 shows that this imposes two restrictions on the behavioral parameters. Secondly A's value function must be consistent with the hypothesized pricing strategies. In particular take the righthand side of (9), then use (6) to substitute out for $V_A^{t+1}(\tilde{x}^t)$, (5) to substitute out for $\tilde{x}_t$, and finally (3) and (4) to substitute out for p_A^t and p_B^t. This leaves an expression for A's period-t value, which is only a function of $\tilde{x}^{t-1}$. Consistency then requires that this should equal the value function given in equation (6); Appendix A.1 shows that imposing this leads to three more restrictions on the behavioral parameters. Combining these various restrictions, we can prove:

Proposition 2 *For any $s \in (0, 7/10]$ there is a unique MPE in linear strategies. The behavioral parameter J satisfies*

$$J = \frac{2 + 2K\delta_c s + \delta_f K}{2 + K\delta_c s + \delta_f s} \tag{10}$$

whilst K lies in $[s/3, 3s/8)$ and satisfies the following equation

$$\delta_f K^3 (2 + K\delta_c s) - 3K (2 + K\delta_c s)(1 + K\delta_c s)^2 + 2s (1 + K\delta_c s)^3 = 0 \tag{11}$$

Proposition 2 restricts attention to switching costs that satisfy $s \leq 7/10$. This is because when setting up demand in equation (8), we assumed that generically each firm has both some consumers switching to it and others switching from it. However switching in both directions can only happen if the difference in prices $|p_B^t - p_A^t| = K|2\tilde{x}^{t-1} - 1|$ is not too large. Since K is related to s this means that s cannot be too large either. We show in the appendix that it is sufficient to restrict attention to switching costs that are less than 7/10.[6] To put this into perspective, we show in the next section that although a wide range of prices may be charged by the two firms, in equilibrium no firm will ever charge more than about 1.2. Consequently the equilibrium in Proposition 2 is (at a bare minimum) valid for any switching cost between about 0 and 60% of market prices. Most real-world estimates of switching costs, some of which were summarized in the introduction, comfortably lie in this interval.

Proposition 3 *The economy always converges to a steady state in which firms split the market equally and charge a price J. At any point in time the location of the marginal young consumer satisfies*

$$\tilde{x}^t - 1/2 = -\frac{K}{1 + K\delta_c s}\left(\tilde{x}^{t-1} - 1/2\right) \tag{12}$$

If the firms start off with unequal market shares, over time they will converge to

[6]For example in the special case where $\delta_c = \delta_f = 0$ equation (11) has a unique solution $K = s/3$, and switching occurs in both directions if and only if $s < 3/4$. More generally the relevant solution to equation (11) lies in $(s/3, 3s/8)$, and therefore the critical switching cost will be closer to 7/10. Once this critical threshold is crossed, the analysis becomes substantially more complicated. This is because for some $\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right)$ switching occurs in both directions, whilst for others it only occurs in one direction. Consequently the two firms' demand elasticities are discontinuous in $\tilde{x}^{t-1}$, which makes it technically very challenging to solve for any putative equilibrium, and then check that profits are quasiconcave. Note however that once the switching cost is sufficiently large, no switching occurs in either direction, and the model becomes qualitatively similar to To (1996).

a steady state in which they both sell to exactly half of the consumers. During this convergence process, the position of the marginal young consumer $\tilde{x}^t$ oscillates around 1/2. Consequently the prices set by the two firms also oscillate around J. Oscillation of prices and market shares is common in overlapping generations models, including the papers by To (1996) and Villas-Boas (2006). Intuitively oscillatory behavior arises because in each period, the firm which previously sold to more than half of young consumers, exploits this fact by charging a higher price than its rival. As a result it then sells to fewer than half of the current young consumers.

4 The effect of switching costs on prices

In the long-run the economy converges to a steady state, so we start by examining whether switching costs increase or decrease prices in the long-run. We then repeat this exercise for the short-run, before discussing the effect on profits.

4.1 Steady state

Proposition 4 *The steady state price is decreasing in s around $s = 0$.*

Doganoglu (2010) shows in his model that starting from $s = 0$, steady state price is decreasing in the switching cost. Proposition 4 shows that the same is true in our related model. Doganoglu then uses numerical simulations to show that the steady state price is lower for larger switching costs as well. However we find that away from $s = 0$ the steady state price can actually be higher or lower, depending upon parameters. In particular the discount factors δ_c and δ_f both affect the steady state price and can both take values anywhere on $(0, 1)$. Therefore for each switching cost, there is a whole set of possible steady state prices. In Figure 1 we plot, for each switching cost in $[0, 7/10]$, the infimum and supremum of that set. Note that when $s = 0$ the steady state price is equal to 1. When consumers have switching costs, steady state price can be as much as 8% higher or 17% lower than this, depending upon the specific values assigned to δ_c, δ_f and s. Indeed even for switching costs very close to zero, price may be higher or lower. Therefore the result in Proposition 4 is rather special, and an analytical approach is required to decide whether in general switching costs are more likely to be pro- or anti-competitive.

It is convenient to split up the impact of switching costs on price, into the following four effects, all of which are mentioned in various parts of the literature. These are

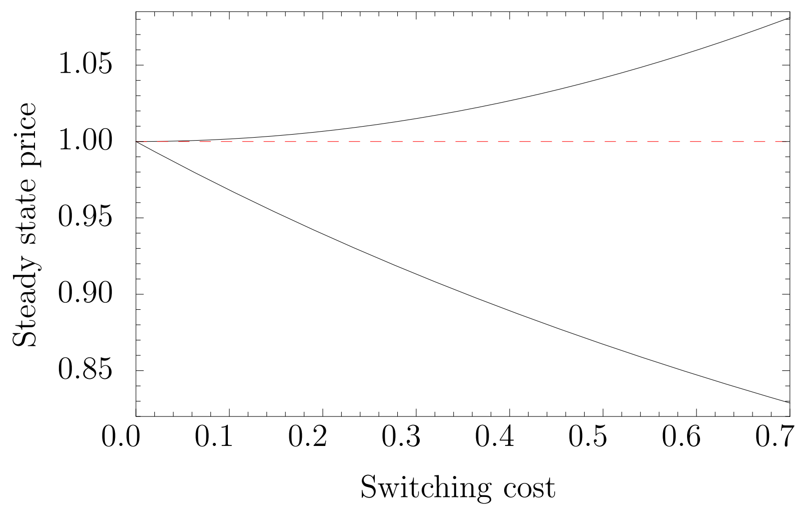

Figure 1: A plot showing the range of possible steady state prices.

the *harvesting*, *poaching*, *investment*, and *consumer price* effects.[7] The first two reflect pricing incentives on old customers. According to the *harvesting effect*, firms should charge a high price and exploit their old customers' reluctance to switch away. However according to the *poaching effect*, firms should charge a low price and poach some of their rival's customers (using the low price to overcome their reluctance to switch). It turns out that since each firm has exactly half of the old customers, *in steady state* the harvesting and poaching effects cancel out.[8] Two assumptions are crucial in this respect. First we assumed that s is small enough to guarantee that switching actually occurs. By contrast in Beggs and Klemperer (1992) and To (1996), the switching cost is so large that nobody ever switches. Poaching is therefore impossible, and both firms just harvest. Second we assumed that preferences change independently over time. In Section 6 we show that if preferences are positively correlated, harvesting dominates poaching, although the amount of upward pressure on steady state price is not necessarily very large.

[7]Different papers use slightly different terminology to describe these four effects. Within the context of two-period models, Fudenberg and Tirole (2000) discuss poaching, whilst Klemperer (1987) describes the other three effects.

[8]This intuition is essentially the same as that given by Arie and Grieco (2011). They argue that a switching cost is like a subsidy to a firm's existing customers but a tax to everybody else. For a myopic monopolist with less (more) than half the market, the tax (subsidy) effect dominates and so the firm reduces (raises) its price. In our problem duopolists have exactly half the market in steady state, and hence the tax and subsidy effects cancel.

Since the harvesting and poaching effects cancel, the steady state price is driven only by pricing incentives on young consumers. It is simple to show that $V_A^{t+1}(\tilde{x}^t)$ and $V_B^{t+1}(\tilde{x}^t)$ are respectively increasing and decreasing in $\tilde{x}^t$. Therefore according to the *investment effect* firms should charge lower prices, as they try to win market share and thereby improve their future profitability. On the other hand if a firm cuts its price, consumers understand that it is only temporary and will be followed by a price increase in the next period (c.f. equations 3 and 4). Young consumers therefore have relatively inelastic demands, and according to the *consumer effect* firms should respond by charging higher prices. Proposition 5 now shows that if firms (consumers) are relatively patient, then the investment (consumer) effect dominates, and switching costs lead to a lower (higher) steady state price.

Proposition 5 *For any* δ_c *and* $s \in (0, 7/10]$*, there exists a unique threshold* $\widetilde{\delta_f} \in (\delta_c s/2, 3\delta_c s/5)$ *such that in the long-run switching costs are pro-competitive if* $\delta_f > \widetilde{\delta_f}$ *and anti-competitive otherwise.*

Proposition 5 confirms analytically that price can be either higher or lower depending upon parameters. However it also suggests that in most real-world markets switching costs will be pro-competitive. This is because even if consumers are twice as patient as firms - that is even if δ_c is as high as $2\delta_f$ - it will still be the case that $\delta_f \geq \widetilde{\delta_f}$. Introspection suggests that in most cases consumers will not be more patient than firms. In addition experimental evidence from Denmark places consumer discount rates somewhere between 10% (Andersen *et al* 2008) and 28% (Harrison *et al* 2002). These discount rates are lower than many other studies and in line with or exceeding market interest rates. Moreover suppose that we interpret time periods in our model as years. Then based on these studies, firms would need to have discount rates in excess of 120% before the condition $\delta_c \leq 2\delta_f$ failed to hold. Therefore Proposition 5 suggests that in reality, the investment effect will outweigh the consumer effect, and steady state price will therefore be lower. The interpretation is that firms cut their price as a defensive measure, to prevent their rival from stealing valuable market share.

To understand why the investment effect usually dominates, recall from Section 3.1 that old consumers definitely buy product A if $x^{t+1} \leq (1-s)/2$ and definitely buy product B if $x^{t+1} \geq (1+s)/2$. We also know that if old consumers are in the "lock-in region" $x^{t+1} \in [(1-s)/2, (1+s)/2]$, they stay with their initial supplier. Consider the *investment effect*. If firm i captures a few extra young consumers, they are valuable in the next period if i). they buy product i when old and ii). if, but for buying i when young, they would buy $j \neq i$ when old. Equivalently these extra young consumers are valuable if and only if they lie in the lock-in region in the next period. The probability of actually being in the lock-in region is s. Moreover the value created for the firm by these additional young consumers is J since this is what they will contribute to future revenue. Therefore if a firm acquires a few extra young consumers, the direct effect on future profits is $J\delta_f s$. However at the same time there is an indirect effect because the rival firm will become more aggressive in the next period, and reduce its price in proportion to K. As a result we can show that $dV_A/dx = -K + Js$.

Now compare this with the *consumer effect*. According to equation (5) the marginal young consumer has location

$$\tilde{x}_t = \frac{1}{2} + \frac{p_B^t - p_A^t}{2 + 2K\delta_c s}$$

where the term $2K\delta_c s$ in the denominator measures the consumer effect. Intuitively if firm i slightly reduces its price it will increase its market share. Using equations (3) and (4) young consumers can infer that in the following period, i's price will be higher and j ($\neq i$)'s price will be lower. In particular $p_i^{t+1} - p_j^{t+1}$ increases in proportion to $2K$. However a young consumer who buys product i only incurs an expected future loss of $2K\delta_c s$. Intuitively only when a consumer finds herself in the lock-in region, does her initial decision to choose i over j actually *cause* her to pay the extra $2K$.[9] Since the probability of ending up in the lock-in region is only s, the consumer effect is only on the order of $2K\delta_c s$.

Summarizing the two previous paragraphs, the investment effect is on the order of $\delta_f [-K + Js]$ whilst the consumer effect is on the order of $2K\delta_c s$. This has three implications. Firstly it explains why according to Proposition 5, switching costs are usually pro-competitive. Since J is close to 1 but $K < 3s/8$, the investment effect usually dominates the consumer effect. This means that the benefit to a firm from locking in some extra young consumers, is greater than the expected loss incurred by those consumers. As a result switching costs usually lead to fiercer competition and a

[9] For example suppose hypothetically that the young consumer knows that when she becomes old, her location will satisfy $x^{t+1} \notin [(1-s)/2, (1+s)/2]$. She therefore knows that her initial purchase decision will have no effect on her subsequent one, and moreover that she is equally likely to buy either of the two products. Hence her future payoff from locking in to i or j is the same. (Of course an infinitesimally small increase in the relative future price of good i is bad news if the consumer turns out to really like product i in the following period, and is good news if she ends up really liking product j, but this is immaterial ex ante.)

lower steady state price. Secondly it explains why according to Proposition 4, starting from $s = 0$ a small switching cost always reduces the steady state price. This is because the investment effect is first-order in s, whilst the consumer effect $2K\delta_c s$ is only second-order because K is of the same order as s. Hence around $s = 0$ the investment effect always dominates. Thirdly it also explains why simulations in other papers often find that the steady-state price is U-shaped in the switching cost. Intuitively the investment effect is roughly linear in s, but the consumer effect $2K\delta_c s$ is more than linear because K is also related to s. This means that the investment effect dominates for small switching costs, and the consumer effect dominates for larger switching costs. As a result the steady state price follows a U-shape. Moreover we can prove the following:

Proposition 6 *If $\delta_c \leq \delta_f$ and $s \leq 1/2$ then the steady state price is monotonically decreasing in s.*

Proposition 6 shows that for a wide range of parameters, switching costs monotonically reduce the steady state price. Consequently only when $s \in (1/2, 7/10]$ might the consumer effect become strong enough to reverse this pattern.

Finally, why are switching costs often pro-competitive when they are relatively small, but (as shown by Beggs and Klemperer 1992 and To 1996) not when they are very large? As discussed earlier, one reason is that when switching costs are very large, firms are unable to poach from their rival. Consequently they focus more on harvesting, which is a force for higher prices. A second related reason is that when switching costs are very large, the link between a firm's market share and its price will become stronger. Equivalently, the consumer effect will be stronger than in our model, and is likely to dominate the investment effect. This again explains why very large switching costs lead to a higher steady state price. It is worth noting again, however, that all our results are valid (at a minimum) for any switching cost between about 0 and 60% of the steady state price. Therefore a very large switching cost is required to overturn our results.

4.2 Outside of steady state

In the short-run the market will not be in steady state, unless the firms happened to start off with equal shares of young consumers from the previous period. Instead the larger firm focuses more on harvesting and charges a higher price, whilst its rival focuses more on poaching and charges a lower price. Moreover this short-run price variation can be quantitatively very important. A simple calculation shows that depending upon parameters, one firm might charge as much as 33% more than its rival.

We say that switching costs are pro-competitive in the short-run if they reduce the average (transaction) price. Since there are two units of consumers in every period, the average price paid by consumers at time t is:

$$\frac{p_A^t D_A^t + p_B^t \left(2 - D_A^t\right)}{2} = J + \left(\tilde{x}^{t-1} - 1/2\right)^2 K \left(s - K \frac{2 + K\delta_c s}{1 + K\delta_c s}\right) \tag{13}$$

Although the two firms' prices are symmetric around J, the larger firm is able to both charge a higher price *and* sell to more than half of the market. This means that average price is higher in the short-run - and this is clear from equation (13) because the final term is positive whenever $\tilde{x}^{t-1} \neq 1/2$. One implication is that since $|\tilde{x}^{t-1} - 1/2|$ decreases over time, switching costs are more likely to be pro-competitive in markets that are more mature. A related implication is that even if switching costs lead to lower prices in the long-run (that is if $J < 1$), the same need not be true in the short- to medium-run. Nevertheless:

Proposition 7 *Provided that $\delta_c \leq \delta_f$ and $\delta_f \gtrapprox 0.14$, switching costs always lead to a lower average price.*

Proposition 7 shows that under relatively mild conditions, switching costs are also pro-competitive outside of steady state. However unlike for the steady state price, it is not sufficient to merely ensure that δ_f/δ_c is large enough to make the investment effect dominate the consumer effect. It is also necessary to ensure that δ_f is large enough in absolute terms, to counteract the upward pressure on price caused by the dominant firm's harvesting.

In light of Proposition 7 it is natural to ask whether a switching cost could cause *both firms* to charge a lower price. This will clearly depend upon how mature the market is, so for simplicity we look at the most extreme case possible. The highest price one could ever observe in the market is $J + K/2$, which occurs when $\tilde{x}^{t-1} \in \{0, 1\}$. This is the price charged by a firm which was previously a monopolist (and so has everybody locked into it) but whose market has just been opened up to competition.

Proposition 8 *Start with $s = 0$ and introduce a small switching cost. The highest observable price $J + K/2$ decreases if and only if $\delta_f > 1/2$.*

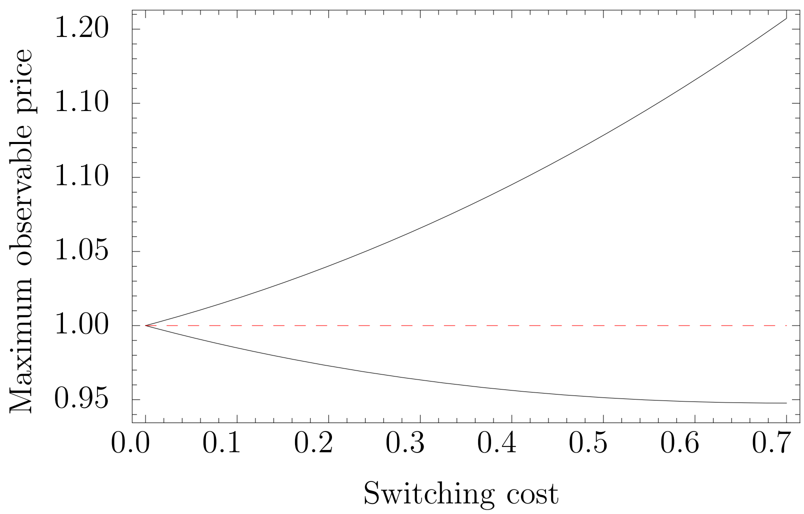

Figure 2: A plot showing the range of prices charged by an incumbent monopolist.

Surprisingly even an incumbent monopolist facing a brand new entrant, may react to a switching cost by cutting its price. The result is surprising because out of everybody, a (recent) monopolist has the strongest incentive to harvest its (very large) customer base. Nevertheless if the firm cares enough about the future, it will follow the entrant and cut its price as a defensive measure to avoid losing too much market share. One would expect that as the switching cost grows, the incumbent's power over its old customers grows and therefore the harvesting effect should start to dominate. This is shown by Figure 2, which plots the range of prices which an incumbent monopolist might charge. As usual there is a whole set of possible prices, depending upon the values assigned to δ_c and δ_f; Figure 2 plots the infimum and supremum of that set. Clearly as s increases, the distribution of prices tends to shift up. However even for very high switching costs, there are combinations of δ_c and δ_f such that the incumbent's price is below the frictionless benchmark 1. Therefore even very large switching costs may cause both firms in the market to charge a lower price.

To summarize, we have proved analytically that switching costs are often pro-competitive both inside and outside steady state. Strikingly we have also shown that under certain conditions, even an incumbent monopolist facing a brand new entrant, may decide to reduce its price when consumers have switching costs.

4.3 Profits

Since there is a close connection between prices and profits, the previous two sections suggest that firms are likely to be made worse off by switching costs. Firstly *in steady state* each firm charges a price J and sells to one unit of consumers in every period. Therefore switching costs reduce long-run profits if and only if they reduce long-run price. Proposition 5 then implies that switching costs are bad for firms except when consumers are especially patient. Secondly *outside of steady state*, total industry profit in any single period is equal to $p_A^t D_A^t + p_B^t D_B^t$. The latter is proportional to the average price charged in period t, which we defined earlier in equation (13). Therefore Proposition 7 says that unless firms are particularly impatient, industry profit will be lower in every period. Of course if firms start off with unequal market shares, the larger firm may still benefit from switching costs. However analogous to Proposition 8, we can show that provided $\delta_f > 1/2$, a small switching cost reduces every firm's (discounted sum of) profits. In particular even a recent monopolist can be harmed by the introduction of a small switching cost.

5 The effect of switching costs on consumer welfare

There is at least a theoretical possibility that switching costs could lead to such fierce price competition, that they actually benefit consumers. Although this is an interesting (and from a policy perspective) important question, it has not been discussed in the previous literature. This section therefore seeks to fill this gap. To do this, we will focus on steady state consumer welfare.

In steady state both firms charge the same price, so equation (5) shows that *young consumers* buy from A if $x^t \leq 1/2$ and buy from B if $x^t > 1/2$. However these are exactly the same choices that they would make in a market without switching costs. Therefore when consumers are young, they benefit from switching costs if and only if the equilibrium price is lower. This is obviously not true for *old consumers*, because some of them incur the switching cost (a direct loss), whilst others of them keep buying an inferior product to avoid paying the switching cost (an indirect loss). In principle old consumers could still benefit from switching costs, if the steady state price falls enough to compensate them for these other losses. However the following lemma shows that this never happens:

Lemma 9 *Switching costs always make old consumers worse off.*

Lemma 9 therefore suggests that in most interesting cases there will be a trade-off, with switching costs benefitting consumers when they are young through lower prices, but harming them when they are old. The overall effect of switching costs on consumers will therefore depend upon how the payoffs of young and old consumers are weighted. A natural measure of consumer surplus is the ex ante lifetime expected utility of a young consumer who is about to enter the market (i.e. weights of 1 and δ_c on young and old consumption respectively). Using this measure of consumer surplus, we find that:

Proposition 10 *For any δ_f and s, there exists a threshold $\widetilde{\delta_c} > 0$ such that switching costs raise consumer surplus if and only if $\delta_c < \widetilde{\delta_c}$.*

Proposition 10 is intuitive. When δ_c is very low, a consumer's lifetime utility is mainly affected by how well off she is when young. Moreover Proposition 5 says that for sufficiently small δ_c the steady state price must be lower with switching costs, and therefore that consumers are better off when young. Two things change as δ_c increases. Firstly the consumer cares more about her utility when she is older, and this is less favorable for switching costs. Secondly we can show that the steady state price increases in δ_c, therefore consumers become worse off in both periods of their life. We now provide two examples to illustrate Proposition 10:

Example 11 *Starting from $s = 0$, a small switching cost increases consumer surplus if and only if*

$$\delta_c < \tilde{\delta}_c = \frac{2\delta_f}{3 - 2\delta_f}$$

Example 11 shows that a small switching cost benefits consumers under a wide range of circumstances. For instance when $\delta_f \geq 1/2$, a sufficient condition for consumers to benefit is that they are weakly less patient than firms. As a second example, Figure 3 plots the critical threshold $\tilde{\delta}_c$ for the case where $s = 1/2$. This switching cost benefits consumers whenever (δ_c, δ_f) lies in the shaded area. Therefore provided consumers discount the future a little more than firms, they are again better off. The reason is that although the losses associated with switching costs are higher when s is larger, so are the gains from paying a lower price (c.f. Proposition 6). Numerical simulations show a similar pattern for other values of the switching cost as well. In summary the base model suggests that switching costs often lead to fiercer competition, and this harms retailers but benefits consumers.

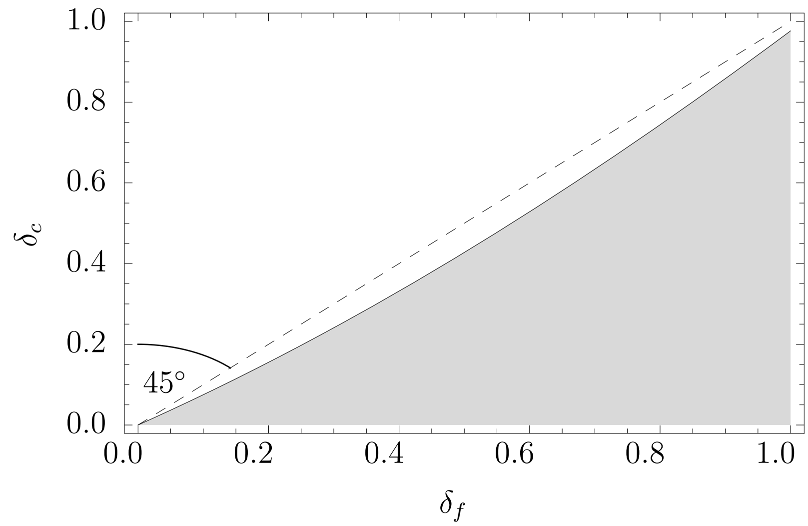

Figure 3: Switching costs improve consumer welfare whenever δ_c and δ_f lie in the shaded region.

6 Discussion

We now investigate how our results might change if consumers live for more than two periods, and have preferences which are correlated over time. In both cases a firm's demand is not a linear function of its past market share, and consequently there no longer exists an equilibrium in linear strategies. The difficulty of finding a Markovian equilibrium in this more general setting is well-known within the literature, and Dutta and Sundaram (1998) provide a good discussion of this topic.[10] For this reason we do not explore issues of equilibrium existence. Instead we take the following approach. When $s = 0$ there are no payoff-relevant state variables, so a MPE does trivially exist, and it involves the two firms playing the (static) Hotelling equilibrium in each period. Assuming that a (continuous) MPE also exists in the neighborhood of $s = 0$, we can derive first order conditions and use them to study comparative statics in this neighborhood. The results are then compared with the equivalent comparative statics exercise which we performed in the base model.

[10]Existence results are available in games where the transition between states is stochastic (see for example Duggan 2011). However in our case market share evolves deterministically over time, and so these results are not applicable.

6.1 Longer-lived consumers

We start by relaxing the assumption that consumers live for two periods. In particular we now assume that in each period, any given consumer 'dies' and exits the market with probability $\rho \in (0,1)$, whereupon they are replaced with a brand new young consumer who has no prior product attachment. We continue to assume that firms are infinitely-lived, and that consumer preferences change independently over time (the latter is relaxed in the following section).

Proposition 12 *Starting from $s = 0$, a small increase in the switching cost causes steady state price to change by*

$$-\frac{2\delta_f}{3}(1-\rho)$$

and causes expected lifetime consumer surplus to increase whenever $\delta_c < 4\delta_f/3$.

Starting from a zero switching cost, the investment effect is again first-order whilst the consumer effect is only second-order. Therefore as in the earlier model, a small switching cost is always pro-competitive. The resulting price drop is less pronounced, however, when ρ is larger. Intuitively a high ρ means that each firm's existing customers are unlikely to survive till the next period. This weakens the link between current market share and future profitability, which reduces the incentive for firms to invest in market share. As a comparison in our earlier model, half of all consumers (namely the old ones) exited the market each period, and a small switching cost also reduced the steady state price by $\delta_f/3$. Proposition 12 also shows that small switching costs are again beneficial for consumers, provided they are not too patient relative to firms. In fact the condition on discount factors $\delta_c < 4\delta_f/3$ is weaker than the equivalent condition in Example 11.

How is the balance between the investment and consumer effects likely to change, as the switching cost increases? As before, consider a thought experiment in which firm i reduces p_i^t slightly and acquires some extra consumers. We expect that during the following periods, firm i will charge a higher price than firm j, but that over time the firms' market shares will converge back to 1/2 and therefore the gap in prices will shrink accordingly. We also expect that when ρ is smaller, the link between market share and price will be stronger, and so the price gap (between i and j) will ceteris paribus be larger. There will also be a lock-in region, such that an old consumer who finds herself located within it, will stay with her supplier from the previous period. The initial reduction in i's price will again have both direct and indirect effects. Firstly the reductions in j's future prices will indirectly reduce i's profits in each future period. Secondly the direct effect will also carry over into in each future period. In particular of those consumers who buy product i in period t, a fraction of them will survive and be located in the lock-in region for Y consecutive periods, where $Y = 1, 2, \ldots$ Their initial decision to buy product i in period t therefore causes these consumers to also buy i in each of these Y consecutive periods. In each of those Y periods, they directly increase firm i's profits by an amount equal to the price that they pay (which in our base model, is J). Similarly in each of those Y periods, the consumers are made worse off by having to pay an amount proportional to the difference in the two firms' prices (which in our base model, is $2K$). We can then derive expressions for the consumer and investment effects by summing up these expressions over each possible value of Y. Note however that as explained above, the gap in prices will decay over time. This suggests that unless ρ is particularly small, the investment effect is even more likely to dominate the consumer effect, suggesting that relatively small switching costs remain pro-competitive.

6.2 Correlated preferences

We now relax the assumption that consumer preferences evolve independently over time. There is lots of evidence that a consumer's valuation for any given product is often serially correlated across time. Moreover Dubé *et al* (2010) have shown that after controlling for switching costs, this persistence in preferences offers an additional explanation for why consumers exhibit inertia in their brand choices. As a simplification we will again restrict attention to the case where each consumer lives for two periods.

There are many ways in which independence could be relaxed. One possible approach is to let a consumer's location on the Hotelling line be the same in both periods - in other words let preferences be perfectly correlated. Another approach is to have a mix of independence and perfect correlation: some consumers keep their old location, whilst others get a new location which is independent of their old one. The latter approach is used by both Klemperer (1987) in a two-period model, and by Viard (2005) in an infinite-horizon model. However a pure-strategy equilibrium fails to exist under either approach when the switching cost is small.[11]

[11] To illustrate, suppose that tastes are perfectly correlated and that $\delta_c = \delta_f = 0$. Assume that in period $t-1$ all young consumers with $x^{t-1} \leq 1/2$ bought product A and all those with $x^{t-1} > 1/2$ bought product B. If the firms use symmetric pricing strategies, no old consumer will switch in period t because $p_A^t = p_B^t$. It is simple to show that if a pure strategy equilibrium exists, each firm charges a

To avoid problems of non-existence of equilibrium, we model correlation in a smoother and more general way. In particular young consumers born in period t are randomly assigned a location on the Hotelling line. When a young consumer with location x^t becomes old, she is assigned a new location x^{t+1} which is drawn using a conditional density $f\left(x^{t+1} \middle| x^t\right)$. This conditional density is assumed to be continuous, atomless and strictly positive for all $(x^t, x^{t+1}) \in [0,1]^2$. It is also assumed to be i). symmetric over time, which means that $f\left(x^{t+1} \middle| x^t\right) = f\left(x^t \middle| x^{t+1}\right)$, and ii). radially symmetric, which says that $f\left(y \middle| z\right) = f\left(1-y \middle| 1-z\right)$ for all $y, z \in [0,1]$.

Proposition 13 *Starting from $s = 0$, a small switching cost reduces average price if and only if*

$$\frac{1}{2}\frac{\partial \Pr\left(X \geq 1/2 | Y = 1/2\right)}{\partial Y} + \frac{\delta_c}{2}\frac{\partial \Pr\left(X \geq 1/2 | Y = 1/2\right)}{\partial Y} - \delta_f \frac{f\left(1/2|1/2\right)}{3} < 0 \qquad (14)$$

To interpret Proposition 13, when preferences are independent $\Pr\left(X \geq 1/2|Y\right) = 1/2$, and so the inequality (14) definitely holds. When instead preferences are positively correlated, we expect that a consumer who is more attached to product B in one period, is also more likely to prefer B over A in another period. Equivalently we expect that $\partial \Pr\left(X \geq 1/2|Y\right)/\partial Y \geq 0$, in which case (14) might not be satisfied. Before commenting further on this, we provide some brief intuition behind expression (14).

The first term in (14) is a combination of the harvesting and poaching effects, whilst the second term is the consumer effect, and the third term is the investment effect. The *harvesting* and *poaching effects* are therefore now (weakly) positive, and the intuition is as follows. As explained earlier, firms can exploit their own old consumers with a high price, or poach some of their rival's customers with a low price. With independent preferences half of old consumers are locked in to the 'wrong' firm, and the incentives to exploit and poach cancel. When instead preferences are positively correlated, fewer marginal consumers are locked in to the 'wrong' firm. This makes it easier to harvest and harder to poach, so the former effect now dominates.

The second term of (14) is the *consumer effect* and it too is now positive. As reported earlier, the standard explanation is that young consumers are less responsive to price cuts, because they expect a price rise to follow in the next period. Starting

price $\bar{p} = 2$ and earns profit $\bar{\pi} = 2$. However it is also simple to show that whenever $s \lessapprox 1/3$, A can do better by 'deviating' and reducing its price to $(6-s)/4$. This allows it to poach some of B's old customers, and earn a profit $(6-s)^2/16$. Therefore a pure strategy equilibrium does not exist for a wide range of switching costs.

from $s = 0$, we showed that this effect is only second-order when consumer tastes evolve independently over time; for similar reasons it is also second-order even when tastes are correlated across time. Instead the positive consumer effect in (14) is caused by a quite different mechanism which, to our knowledge, has not previously been mentioned in the literature. It arises due to expected changes in future preferences. For example suppose that firm A reduces p_A^t and tries to attract some young consumers located slightly to the right of $x^t = 1/2$. Since preferences are positively correlated, these young consumers expect to prefer product B in the next period. This makes them more reluctant to buy A now, which causes demand to become less elastic.

The final term of (14) is the *investment effect*. As in the base model, firms compete for the marginal young consumer who is located at $x^t = 1/2$. As argued previously, this marginal consumer is valuable in the future if she turns out to be located in the lock-in region. Starting from $s = 0$, a small increase in the switching cost changes her probability of being in the lock-in region by $f(1/2|1/2)$. Since preferences are correlated $f(1/2|1/2)$ may exceed the unconditional density, and therefore the investment effect may be stronger than in our earlier model.

To summarize when consumer preferences are correlated over time, the first two terms of inequality (14) are positive, and therefore even very small switching costs are not necessarily pro-competitive. Note however that only the behavior of $f\left(x^{t+1}\middle|\, x^t\right)$ around the point $x^t = 1/2$, is relevant for whether inequality (14) holds. Correlation on the other hand is a global concept, which summarizes the behavior of $f\left(x^{t+1}\middle|\, x^t\right)$ for all x^t. This immediately implies that the amount of correlation has *no* direct bearing on whether switching costs are pro- or anti-competitive. What matters instead is whether $\partial \Pr\left(X \geq 1/2 | Y = 1/2\right)/\partial Y$ is large or small. As an example, suppose that if a young consumer is almost indifferent about which product to buy, she is also equally likely to prefer A or B when she becomes old. Then $\partial \Pr\left(X \geq 1/2 | Y = 1/2\right)/\partial Y$ is zero and switching costs are definitely pro-competitive, even if in the wider population there is a strong positive correlation between x^t and x^{t+1}. Therefore although our earlier assumption of independence is not innocuous, it can be substantially relaxed without changing the conclusion that small switching costs are pro-competitive. Furthermore, whenever consumer tastes are correlated, fewer old consumers need to actually incur the switching cost because they are already locked into the 'correct' firm. This means that conditional on switching costs being pro-competitive, there is again a good chance that they improve consumer welfare.

7 Conclusion

There has recently been renewed interest in the question of whether switching costs lead to higher or lower prices. Traditionally they were perceived as being anti-competitive, but this conclusion was based on models in which no consumers actually switched in equilibrium. Some more recent papers have focused on the polar case of very small switching costs, and come to the opposite conclusion. In this paper we have presented a tractable model of dynamic competition, and solved it for a very wide (and empirically valid) range of switching costs. In the long-run switching costs can be either pro- or anti-competitive, depending upon how patient firms are relative to consumers. However using evidence on rates of time preference, we concluded that in most markets switching costs are likely to lead to lower prices. Intuitively this is because firms cut prices as a defensive measure, to prevent their rival from stealing valuable market share. We then used the model to address some other issues which have been neglected by the previous literature. We showed that short-run prices can be extremely heterogeneous, and that focusing on steady state may lead to biased conclusions about the pro-competitiveness of switching costs. We also examined the wider effects of switching costs, on for example consumer welfare. Switching costs often act as a way of transferring surplus from old to young consumers. When consumers are relatively impatient, this trade-off is favorable and consumer welfare is increased. Finally we investigated how our conclusions might change when, amongst other things, consumer tastes are correlated over time. On the one hand even starting from zero, small switching costs need no longer be pro-competitive. On the other hand independence of preferences can be substantially relaxed, without significantly affecting any of our earlier conclusions.

Throughout the paper we have assumed that the switching cost is exogenously given. However in practice manufacturers can often choose to make their products more or less compatible with those of their rivals. Retailers can also, at some cost to themselves, make it more difficult for their customers to cancel subscriptions or move to another provider. Therefore an interesting way to extend the current model, would be to allow each firm to influence how easily its customers can switch to its rival. Our existing results already show that profits are usually maximized when consumers are able to switch costlessly. However we conjecture that at least sometimes, firms may end up playing a Prisoner's dilemma. In particular it seems plausible that each firm might benefit from making it slightly more difficult for its existing customers to switch. However once both firms realize this and set a positive switching cost, price competition is intensified and they end up earning lower profits. A natural implication is that firms might try to 'collude' and establish industry standards that make it easier for consumers to change providers. We hope to think more about this in future work.

A Appendix

A.1 Proofs for Section 3

Proof of Lemma 1. As explained in the text, the expected lifetime payoff from buying product A in period t is:

$$V-x^t-p_A^t+\delta_c\left[\int_0^{\dot{x}^{t+1}}\left(V-y-Ep_A^{t+1}\right)dy+\int_{\dot{x}^{t+1}}^{1}\left(V-(1-y)-Ep_B^{t+1}-s\right)dy\right] \quad (15)$$

where $\dot{x}^{t+1}=\left(1+Ep_B^{t+1}-Ep_A^{t+1}+s\right)/2$ is the location of the old consumer who will be just indifferent about switching from A to B in period $t+1$. After adding and subtracting $\int_{\dot{x}^{t+1}}^{1}\left(V-y-Ep_A^{t+1}\right)dy$ (15) can be rewritten as

$$V-x^t-p_A^t+\delta_c\left[V-\frac{1}{2}-Ep_A^{t+1}+\int_{\dot{x}^{t+1}}^{1}\left[-(1-2y)+Ep_A^{t+1}-Ep_B^{t+1}-s\right]dy\right] \quad (16)$$

Similarly the expected lifetime payoff from buying product B in period t is

$$V-\left(1-x^t\right)-p_B^t+\delta_c\left[V-\frac{1}{2}-Ep_B^{t+1}+\int_0^{\ddot{x}^{t+1}}\left[(1-2y)+Ep_B^{t+1}-Ep_A^{t+1}-s\right]dy\right] \quad (17)$$

where $\ddot{x}^{t+1}=\left(1+Ep_B^{t+1}-Ep_A^{t+1}-s\right)/2$ is the location of the old consumer who will be just indifferent between switching from B to A in period $t+1$. The difference between (16) and (17) is clearly decreasing in x^t. Therefore provided $|p_B^t-p_A^t|$ is not too large, there exists an $\tilde{x}^t\in(0,1)$ such that (16) and (17) are equal when evaluated at $x^t=\tilde{x}^t$. (This also means that (16) exceeds (17) when $x^t<\tilde{x}^t$, whilst the opposite is true when $x^t>\tilde{x}^t$.) The final step is to therefore to equate (16) and (17), and then substitute in $x^t=\tilde{x}^t$. After some algebraic manipulations we find that $\tilde{x}^t$ satisfies equation (1). ■

A.1.1 Proof of Proposition 2

Proof of Proposition 2. Lemma 14 below derives expressions for J,M,N,R as a function of K, which must hold in any equilibrium. Lemma 15 then shows there is a unique K consistent with our problem, and shows that it lies on $[s/3,3s/8]$. ■

Lemma 14 *K satisfies equation (11) and in addition*

$$J = \frac{2 + 2K\delta_c s + \delta_f K}{2 + K\delta_c s + \delta_f s} \tag{18}$$

$$M = \frac{J}{1-\delta_f} \tag{19}$$

$$N = \frac{2s\left(1 + K\delta_c s\right) - K\left(2 + K\delta_c s\right)}{2 + K\delta_c s + \delta_f s} \tag{20}$$

$$R = \frac{K^2}{2}\left(\frac{2 + K\delta_c s}{1 + K\delta_c s}\right) \tag{21}$$

Proof of Lemma 14. Let $\pi_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) = p_A^t D_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right)$ be flow profit in period t. Firm A chooses p_A^t to maximize $\pi_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) + \delta_f V_A^{t+1}\left(\tilde{x}^t\right)$. Take $\pi_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) + \delta_f V_A^{t+1}\left(\tilde{x}^t\right)$ and use equations (5) and (6) to substitute out for $V_A^{t+1}\left(\tilde{x}^t\right)$. Then maximize with respect to p_A^t to get a first order condition

$$D_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) - \frac{p_A^t}{2}\frac{2 + K\delta_c s}{1 + K\delta_c s} - \frac{\delta_f N}{2\left(1 + K\delta_c s\right)} - \frac{\delta_f R\left(p_B^t - p_A^t\right)}{2\left(1 + K\delta_c s\right)^2} = 0 \tag{22}$$

Substitute out p_A^t and p_B^t using equations (3) and (4), collect terms, and then rewrite (22) in the form $\alpha_1 + \alpha_2\left(\tilde{x}^{t-1} - \frac{1}{2}\right) = 0$. Setting $\alpha_1 = \alpha_2 = 0$ gives the following conditions[12]

$$1 - \frac{J}{2}\frac{2 + K\delta_c s}{1 + K\delta_c s} - \frac{\delta_f N}{2\left(1 + K\delta_c s\right)} = 0 \tag{23}$$

$$s - \frac{3K}{2}\frac{2 + K\delta_c s}{1 + K\delta_c s} + \frac{\delta_f R K}{\left(1 + K\delta_c s\right)^2} = 0 \tag{24}$$

To find an expression for A's period-t valuation, take $\pi_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) + \delta_f V_A^{t+1}\left(\tilde{x}^t\right)$ and again use equations (5) and (6) to substitute out for $V_A^{t+1}\left(\tilde{x}^t\right)$. Then use equations (3) and (4) to eliminate p_A^t and p_B^t. After collecting terms, A's period-t valuation can we expressed in the form $\alpha_3 + \alpha_4\left(\tilde{x}^{t-1} - \frac{1}{2}\right) + \alpha_5\left(\tilde{x}^{t-1} - \frac{1}{2}\right)^2$. Since we assumed in equation (6) that this value equals $M + N\left(\tilde{x}^{t-1} - \frac{1}{2}\right) + R\left(\tilde{x}^{t-1} - \frac{1}{2}\right)^2$, we can equate coefficients

[12] We can also prove that $\pi_A^t\left(p_A^t, p_B^t, \tilde{x}^{t-1}\right) + \delta_f V_A^{t+1}\left(\tilde{x}^t\right)$ is quasiconcave in p_A^t. Details are available upon request.

and get three equations

$$\alpha_3 = J + \delta_f M = M \tag{25}$$

$$\alpha_4 = Js - JK\frac{2 + K\delta_c s}{1 + K\delta_c s} + K - \frac{\delta_f KN}{1 + K\delta_c s} = N \tag{26}$$

$$\alpha_5 = Ks - K^2\frac{2 + K\delta_c s}{1 + K\delta_c s} + \frac{\delta_f RK^2}{(1 + K\delta_c s)^2} = R \tag{27}$$

Since $s > 0$ equation (24) implies that $K \neq 0$. Therefore rewrite equation (24) as

$$R = \frac{3\,(2 + K\delta_c s)\,(1 + K\delta_c s)}{2\delta_f} - \frac{s\,(1 + K\delta_c s)^2}{\delta_f K} \tag{28}$$

and then substitute this into equation (27) and rearrange to find $\phi(K) = 0$ where

$$\phi(K) = \delta_f K^3 (2 + K\delta_c s) - 3K(2 + K\delta_c s)(1 + K\delta_c s)^2 + 2s(1 + K\delta_c s)^3 \tag{29}$$

Setting $\phi(K) = 0$ gives equation (11) in the text. Substituting equation (28) into the lefthand side of (27) gives the expression for R in equation (21). To get the expressions for J and N in (18) and (20), jointly solve equations (23) and (26). ■

Lemma 15 *Equation (11) has a unique solution consistent with our problem, and it lies in* $[s/3, 3s/8]$.

Proof of Lemma 15. The demand expression (8) is valid if and only if $|K| \leq 1 - s$. To see this, note firstly that (8) is only well-defined if $1 + K\delta_c s \neq 0$, which is satisfied provided $|K| \leq 1 - s$. Secondly (8) assumes that each firm sells to a positive mass of young consumers. This requires that $\tilde{x}^t \in (0, 1)$ which, using equation (5), is equivalent to $|K| < 1 + K\delta_c s$. This is again satisfied provided $|K| \leq 1 - s$. Thirdly (8) assumes that generically (i.e. whenever $\tilde{x}^{t-1} \notin \{0, 1\}$) each firm has old consumers both switching to and away from it. Using Section 3.1 this requires $\ddot{x} > 0$ and $\dot{x} < 1$. The condition $\ddot{x} > 0$ is equivalent to $1 - s > 2K(\tilde{x}^{t-1} - 1/2)$, so a necessary and sufficient condition for this to hold for any $\tilde{x}^{t-1} \in (0, 1)$, is that $|K| \leq 1 - s$. A similar argument applies to the condition $\dot{x} < 1$.

Aim: in light of the above, we first show that equation (11) has exactly one solution on the interval $[-(1 - s), 1 - s]$, and that it lies in $[s/3, 3s/8)$. Note then that $K \leq 1 - s$ hold, because by assumption $s \leq 7/10$.

Step 1: Show that equation (11) has exactly one solution on the interval $[0, 1 - s]$.

Step 1a. Show that $\frac{\partial \phi(K)}{\partial K} < 0$ for all $K \in [0, 1-s]$. Using equation (29):

$$\frac{1}{2}\frac{\partial \phi(K)}{\partial K} = K^2\delta_f(3+2K\delta_c s) + 3(1+K\delta_c s)\left[-1 - 4K\delta_c s - 2(K\delta_c s)^2 + \delta_c s^2 + K\delta_c^2 s^3\right]$$

Since we are considering $K \geq 0$, $3 + 2K\delta_c s \leq 3(1+K\delta_c s)$ and therefore

$$\frac{1}{2}\frac{\partial \phi(K)}{\partial K} \leq 3(1+K\delta_c s)\left[K^2\delta_f - 1 - 4K\delta_c s - 2(K\delta_c s)^2 + \delta_c s^2 + K\delta_c^2 s^3\right]$$

Notice that $-4K\delta_c s + K\delta_c^2 s^3 < 0$ since $\delta_c s^2 < 4$, and that $K^2\delta_f - 1 + \delta_c s^2 \leq (1-s)^2 - 1 + s^2 = 2s(s-1) < 1$. Therefore $\frac{\partial \phi(K)}{\partial K} < 0$ for all $K \in [0, 1-s]$.

Step 1b. Note that $\phi(0) = 2s > 0$. Now prove that $\phi(1-s) < 0$. Substituting $K = 1-s$ into $\phi(K)$ and then simplifying, we find that:

$$\phi(1-s) = \delta_f(1-s)^3[2+\delta_c s(1-s)] - [1+\delta_c s(1-s)]^2\left(6 - 8s + 3\delta_c s - 8\delta_c s^2 + 5\delta_c s^3\right)$$

Clearly $[2+\delta_c s(1-s)]/[1+\delta_c s(1-s)]^2 \leq 2$ because we know that $[2+X]/[1+X]^2$ decreases in X for any $X \geq 0$. Therefore to show that $\phi(1-s) < 0$, it is sufficient to prove that

$$2(1-s)^3 < 6 - 8s + 3\delta_c s - 8\delta_c s^2 + 5\delta_c s^3 \qquad (30)$$

First if $s \in [0, 3/5]$ then the righthand side of (30) increases in δ_c. Therefore if (30) holds for $\delta_c = 0$ it will also hold for any $\delta_c \in [0,1]$. When $\delta_c = 0$ (30) becomes $2(1-s)^3 < 6 - 8s$, which is easily shown to hold for any $s \in [0, 3/5]$. Second if instead $s \in [3/5, 7/10]$ then the righthand side of (30) decreases in δ_c. Therefore if (30) holds for $\delta_c = 1$ it will also hold for any $\delta_c \in [0,1]$. When $\delta_c = 1$ (30) becomes $0 < 4 + s - 14s^2 + 7s^3$ which is easily seen to hold for any $s \in [3/5, 7/10]$. Therefore $\phi(1-s) < 0$ for any $s \in [0, 7/10]$.

Step 1c. Combining steps 1a and 1b, it is clear that there is a unique $K \in [0, 1-s]$ which solves $\phi(K) = 0$.

Step 2: Show that the solution on $[0, 1-s]$ to $\phi(K) = 0$, actually lies on $[s/3, 3s/8)$.

Step 2a. Show that $\phi(s/3) > 0$. The terms $-3K(2+K\delta_c s)(1+K\delta_c s)^2 + 2s(1+K\delta_c s)^3$ become $(1+K\delta_c s)^2 K\delta_c s^2\big|_{K=s/3} > 0$. Also $\delta_f K^3(2+K\delta_c s) > 0$ since $K > 0$.

Step 2b. Show that $\phi(3s/8) < 0$. Letting $Y = 3\delta_c s^2/8$,

$$\begin{aligned}\phi\left(\frac{3s}{8}\right) &= \left(\frac{27}{512}\right)\delta_f s^3(2+Y) - \frac{9s}{8}(2+Y)(1+Y)^2 + 2s(1+Y)^3 \\ &= \left(\frac{27}{512}\right)\delta_f s^3(2+Y) - \frac{13s}{500}(2+Y)(1+Y)^2 - \frac{1099s}{1000}(2+Y)(1+Y)^2 + 2s(1+Y)^3\end{aligned}$$

The first two terms are negative provided that $27s^2/512 < 13/500$, and this holds because $s < 7/10$. The final two terms are negative if and only if $Y < 198/901 \approx 0.21$, and this holds because $Y \leq (3/8)(7/10)^2 = 0.18375$. Therefore $\phi(3s/8) < 0$.
Step 2c. Since $\phi(K)$ strictly decreases on $[0, 1-s]$, and $\phi(s/3) > 0$ but $\phi(3s/8) < 0$, the solution to $\phi(K) = 0$ must lie on $[s/3, 3s/8)$.
Step 3. Now show there is no solution to $\phi(K) = 0$ for $K \in [-(1-s), 0]$.
Step 3a. Using Step 1a

$$\frac{1}{6}\frac{\partial^2\phi(K)}{\partial K^2} = 2K\delta_f(1+K\delta_c s) + \delta_c s\left(-5 - 12K\delta_c s - 6(K\delta_c s)^2 + 2\delta_c s^2 + 2K\delta_c^2 s^3\right)$$

Since $K \in [-(1-s), 0]$, the first term $2K\delta_f(1+K\delta_c s)$ is negative. To show the remainder is also negative, it is sufficient to show that $-5 - 12K\delta_c s + 2\delta_c s^2 < 0$. The latter is toughest to satisfy when K is very negative and δ_c is large, so substitute in $K = -(1-s)$ and $\delta_c = 1$. It it is then sufficient to prove that

$$-5 + 12s(1-s) + 2s^2 < 0 \tag{31}$$

The lefthand side of (31) is concave in s and maximized when $s = 3/5$. Since by direct computation (31) holds when $s = 3/5$, it also holds for any other $s \in [0, 7/10]$. Therefore $\partial^2\phi(K)/\partial K^2 < 0$ ($\phi(K)$ is concave) for all $K \in [-(1-s), 0]$.
Step 3b. Show that $\phi(-(1-s)) > 0$. Rewrite $\phi(K)$ as

$$\begin{aligned}&-K(2+K\delta_c s)\left[3(1+K\delta_c s)^2 - \delta_f K^2\right] + 2s(1+K\delta_c s)^3 \\ =\ &(1-s)(2-\delta_c s(1-s))\left[3(1-\delta_c s(1-s))^2 - \delta_f^2(1-s)^2\right] + 2s(1-\delta_c s(1-s))\end{aligned}$$

which by inspection in positive. We also showed in Step 1b that $\phi(0) > 0$. Therefore since $\phi(K)$ is concave on $[-(1-s), 0]$ and positive at the boundaries of that set, it must be true that $\phi(K) > 0 \forall K \in [-(1-s), 0]$, hence there is no root to $\phi(K)$ on that interval. ■

A.1.2 Remaining proof in Section 3

Proof of Proposition 3. To get equation (12), substitute the expressions for p_A^t and p_B^t (equations 3 and 4) into the expression for $\tilde{x}^t$ (equation 5). Proposition 2 says that $K \in [s/3, 3s/8)$ therefore $K < 1 + K\delta_c s$. Equation (12) therefore says that $\tilde{x}^t \to 1/2$ as $t \to \infty$; using equations (3) and (4), this also implies that $p_A^t, p_B^t \to J$ as $t \to \infty$. ■

A.2 Intermediate proofs

Lemmas 16 and 17 show that the steady state price J is increasing in δ_c and decreasing in δ_f. These results are used to prove Propositions 5 and 10.

Lemma 16 *J is strictly decreasing in δ_f.*

Proof. *Step 1.* Totally differentiate the expression for J given in equation (18). J decreases in δ_f if and only if $\partial K/\partial \delta_f < \alpha_1/\alpha_2$ where

$$\alpha_1 = 2s\left(1 + K\delta_c s\right) - K\left(2 + K\delta_c s\right), \quad \alpha_2 = 2\left[\left(2\delta_c s + \delta_f\right)\left(1 + \frac{s}{2}\delta_f\right) - \delta_c s\right]$$

Step 2. Firstly the derivative of α_1 with respect to K is $2\delta_c s^2 - 2 - K\delta_c s$, which is negative because $\delta_c s^2 < 1$. Secondly by inspection α_2 is increasing in δ_c, δ_f and s. Proposition 2 says that $K < 3s/8$, so:

$$\frac{\alpha_1}{\alpha_2} > \frac{\alpha_1|_{K=3s/8}}{\alpha_2|_{s=7/10,\delta_c=\delta_f=1}} = \frac{s\left(\frac{5}{4} + \frac{39}{64}\delta_c s^2\right)}{5.08} = \frac{s}{5.08}\left(\frac{5}{4} + \frac{39}{64}\delta_c s^2\right)$$

Step 3. Obtain an expression for $\partial K/\partial \delta_f$. Totally differentiate equation (29) with respect to δ_f and then rearrange to get $\partial K/\partial \delta_f = \alpha_3/\alpha_4$ where

$$\begin{aligned}
\alpha_3 &= K^3\left(2 + K\delta_c s\right) \\
\alpha_4 &= 3\left(1 + K\delta_c s\right)^2\left[2\left(1 - \delta_c s^2\right) + K\delta_c s\right] + 3K\delta_c s\left(1 + K\delta_c s\right)\left(5 + 3K\delta_c s\right) - \alpha_5 \\
\alpha_5 &= \delta_f K^2\left(6 + 4K\delta_c s\right)
\end{aligned}$$

Step 4. α_3 increases in K therefore $\alpha_3 < \alpha_3|_{K=3s/8} = \frac{27s^3}{512}\left(2 + \frac{3\delta_c s^2}{8}\right)$.
Step 5. $\alpha_4 + \alpha_5$ increases in K. Proposition 2 says that $K \geq s/3$ so:

$$\alpha_4 + \alpha_5 \geq \left(\alpha_4 + \alpha_5\right)|_{K=s/3} = \left(1 + \frac{\delta_c s^2}{3}\right)\left[6 + 2\delta_c s^2 - \frac{2}{3}\left(\delta_c s^2\right)^2\right]$$

By inspection $(\alpha_4+\alpha_5)|_{K=s/3}$ increases in s, so $\alpha_4+\alpha_5 \geq (\alpha_4+\alpha_5)|_{K=s/3,s=0}=6$.
Step 6. α_5 increases in K, δ_c, δ_f and s. Therefore $\alpha_5 \leq \alpha_5|_{K=3s/8,s=7/10,\delta_c=\delta_f=1} < 1/2$. Consequently $\alpha_4 \geq 6-1/2 = 11/2$.
Step 7. Combining steps 4-6

$$\frac{\alpha_3}{\alpha_4} \leq \frac{2}{11}\frac{27s^3}{512}\left(2+\frac{3\delta_c s^2}{8}\right) = \frac{27s^3}{2816}\left(2+\frac{3\delta_c s^2}{8}\right)$$

and then simple calculation shows that (since $s \leq 7/10$) the expression for α_3/α_4 given in Step 7 is less than the expression for α_1/α_2 given in Step 2, as required. ■

Lemma 17 *J is strictly increasing in δ_c.*

Proof. *Step 1.* Totally differentiate the expression for J given in equation (18). J increases in δ_c if and only if $\beta_1+\beta_2(\partial K/\partial\delta_c) > 0$ where

$$\beta_1 = Ks\left(2+2\delta_f s - K\delta_f\right), \quad \beta_2 = 2\delta_f + (\delta_f)^2 s + 2\delta_c s + 2\delta_c\delta_f s^2$$

Clearly $\beta_1, \beta_2 > 0$ so it is sufficient to show that $\partial K/\partial\delta_c > 0$.
Step 2. Totally differentiate the expression for K given in equation (29) with respect to δ_c and then rearrange to get $\partial K/\partial\delta_c = \beta_3/\alpha_4$ where

$$\beta_3 = \delta_f K^4 s + 3Ks\left(1+K\delta_c s\right)^2(2s-K) - 6K^2 s\left(1+K\delta_c s\right)\left(2+K\delta_c s\right)$$

and where α_4 is exactly the same as in the proof of Lemma 16. We know that $\alpha_4 \geq 11/2$ so $\partial K/\partial\delta_c > 0$ if and only if $\beta_3 > 0$. The latter holds provided that

$$\begin{aligned} 3Ks\left(1+K\delta_c s\right)^2(2s-K) &> 6K^2 s\left(1+K\delta_c s\right)\left(2+K\delta_c s\right) \\ 2s\left(1+K\delta_c s\right) &> 5K\left(1+3K\delta_c s/5\right) \end{aligned}$$

which is true because $K \in [s/3, 3s/8)$. ■

A.3 Proofs for Sections 4 and 5

Proof of Proposition 4. Totally differentiate equation (29) with respect to s:

$$\begin{aligned}0 \;=\;& 3\delta_f K^2 (2+K\delta_c s)\frac{\partial K}{\partial s} + \delta_c\delta_f K^3\left[K + s\frac{\partial K}{\partial s}\right] - 3\frac{\partial K}{\partial s}(2+K\delta_c s)(1+K\delta_c s)^2 \\ & -3K\delta_c\left[K+s\frac{\partial K}{\partial s}\right](1+K\delta_c s)^2 - 6K\delta_c(2+K\delta_c s)(1+K\delta_c s)\left[K+s\frac{\partial K}{\partial s}\right] \\ & +2(1+K\delta_c s)^3 + 6s\delta_c(1+K\delta_c s)^2\left[K+s\frac{\partial K}{\partial s}\right]\end{aligned}$$

If $s = 0$ then $K = 0$ as well, so the above simplifies to $\partial K/\partial s|_{s=0} = 1/3$. Then totally differentiate the expression for J in equation (18) with respect to s, and impose $s = K = 0$. This gives $\partial J/\partial s|_{s=0} = (\delta_f/2)(-1 + \partial K/\partial s|_{s=0}) = -\delta_f/3 < 0$. ■

Proof of Proposition 5. From equation (10) steady state price is 1 when $s = 0$. First, using equation (10) $J < 1$ if and only if

$$K\delta_c s - \delta_f(s-K) < 0 \tag{32}$$

which is harder to satisfy as K increases. We also know from Proposition 2 that $K < 3s/8$. Therefore if inequality (32) holds when evaluated at $K = 3s/8$, it always holds. Substituting $K = 3s/8$ into (32), we get a condition $\delta_f > (3\delta_c s)/5$. Second, $J > 1$ if and only if

$$K\delta_c s - \delta_f(s-K) > 0 \tag{33}$$

which is easier to satisfy as K increases. We again know from Proposition 2 that $K \geq s/3$. Therefore if inequality (33) holds when evaluated at $K = s/3$, it always holds. Substituting $K = s/3$ into (33), we get a condition $\delta_f < (\delta_c s)/2$.

Therefore steady state price is definitely lower if $\delta_f > (3\delta_c s)/5$ and definitely higher if $\delta_f < (\delta_c s)/2$. Lemma 16 says that the steady state price strictly decreases in δ_f. Therefore there exists a unique threshold between $(\delta_c s)/2$ and $(3\delta_c s)/5$ such that $J = 1$ when $\delta_f = \widetilde{\delta_f}$. ■

Proof of Proposition 6. *Step 1.* Totally differentiate the expression for J in equation (18) with respect to s. J decreases in s if and only if $\partial K/\partial s < A/B$ where

$$A = -2K\delta_c + 2\delta_f + K^2\delta_c\delta_f + K(\delta_f)^2, \quad B = 2\delta_c s + 2\delta_c\delta_f s^2 + 2\delta_f + (\delta_f)^2 s$$

By inspection A and B are respectively decreasing and increasing in δ_c. Therefore J

definitely decreases in s whenever $\partial K/\partial s < (A/B)_{\delta_c=\delta_f} = \bar{A}/\bar{B}$, where

$$\bar{A} = -2K + 2 + K^2\delta_f + K\delta_f, \qquad \bar{B} = 2s + 2\delta_f s^2 + 2 + \delta_f s$$

Step 2. Firstly the derivative of $\bar{A}$ with respect to K is $-2 + 2K\delta_f + \delta_f$ which is clearly negative. Then note that $K < 3/16$, since $K < 3s/8$ and $s \leq 1/2$. Secondly $\bar{B}$ is increasing in s, therefore

$$\frac{\bar{A}}{\bar{B}} > \frac{\bar{A}\big|_{K=3/16}}{\bar{B}\big|_{s=1/2}} = \frac{\frac{13}{8} + \frac{57}{256}\delta_f}{3 + \delta_f} = \frac{416 + 57\delta_f}{256\,(3 + \delta_f)} > \left.\frac{416 + 57\delta_f}{256\,(3 + \delta_f)}\right|_{\delta_f=1} > \frac{23}{50}$$

where the second-last inequality follows because the third-last expression is decreasing in δ_f, and where the final inequality follows from direct computation.
Step 3. Totally differentiate the expression containing K in equation (29) with respect to s. We find that $\partial K/\partial s = C/D$ where

$$\begin{aligned} C &= K^4\delta_c\delta_f - 3K^2\delta_c\,(1 + K\delta_c s)^2 - 6K^2\delta_c\,(1 + K\delta_c s)\,(2 + K\delta_c s) \\ &\quad +2\,(1 + K\delta_c s)^3 + 6K\delta_c s\,(1 + K\delta_c s)^2 \\ D &= -3K^2\delta_f\,(2 + K\delta_c s) - K^3\delta_c\delta_f s + 3\,(1 + K\delta_c s)^2\,(2 + K\delta_c s) \\ &\quad +3K\delta_c s\,(1 + K\delta_c s)^2 + 6K\delta_c s\,(1 + K\delta_c s)\,(2 + K\delta_c s) - 6\delta_c s^2\,(1 + K\delta_c s)^2 \end{aligned}$$

Using step 2, to prove the Proposition it is sufficient to show that $50C - 23D < 0$.
Step 4. We must now show that $50C - 23D$ increases in δ_c. The proof is lengthy and is therefore available on request. Since $50C - 23D$ increases in δ_c, the condition $50C - 23D < 0$ is least likely to hold when $\delta_c = \delta_f$. Therefore substitute $\delta_c = \delta_f = \delta$ into the expressions for C and D:

$$\begin{aligned} C &= K^4\delta^2 - 3K^2\delta\,(1 + K\delta s)^2 - 6K^2\delta\,(1 + K\delta s)\,(2 + K\delta s) \\ &\quad +2\,(1 + K\delta s)^3 + 6K\delta s\,(1 + K\delta s)^2 \\ D &= 3\delta\,(2 + K\delta s)\left[2Ks\,(1 + K\delta s) - K^2\right] + \left[3K\delta s\,(1 + K\delta s)^2 - K^3\delta^2 s\right] \\ &\quad +3\,(1 + K\delta s)^2\left(2 + K\delta s - 2\delta s^2\right) \end{aligned}$$

Step 5. Place an upper bound on C. The derivative of C with respect to K, can be simplified and written in the form

$$4K^3\delta^2{+}6K\delta^2 s\left[(2s-4K)+K\delta s\,(2s-3K)\right]{+}6\delta\,(1+K\delta s)\left[(2s-5K)+K\delta s\,(2s-3K)\right]$$

which is positive because $K < 3s/8$. Therefore $C < C|_{K=3s/8}$. After simplifications we find that

$$C|_{K=3s/8} = 2 + \frac{153}{64}\delta s^2 + \frac{5265}{4096}\delta^2 s^4 + \frac{999}{4096}\delta^3 s^6$$

which is increasing in both δ and s. Therefore $C < C|_{K=3s/8,s=1/2,\delta=1} = 2.6818$.
Step 6. Place a lower bound on D. By inspection D comprises three terms, each of which is positive and increasing in K, therefore $D \geq D|_{K=s/3}$. After some algebra, we find that

$$D|_{K=s/3} = 6 + \delta s^2\left(\frac{10}{3} - \frac{4\delta s^2}{27} - \frac{2\delta^2 s^4}{9}\right) \geq 6$$

Step 10. Steps 5 and 6 imply that $50C - 23D < 50\,(2.6818) - 23\,(6) < 0$ as required. ■

Proof of Proposition 7. Note that $(2+X)\,/\,(1+X)$ decreases in X, and that $K\delta_c s < 147/800$ because $K < 3s/8$ and $s \leq 7/10$. This implies that $(2+K\delta_c s)\,/\,(1+K\delta_c s) > 1.84$. Therefore using equation (13), the average price can never exceed $J{+}K\,(s-1.84K)\,/4$. Lemma 17 shows that J increases in δ_c and therefore provided $\delta_c \leq \delta_f$, $J \leq J|_{\delta_c=\delta_f}$. Therefore the average price is less than

$$J|_{\delta_c=\delta_f} + \frac{K}{4}\,(s-1.84K) = \frac{2+2K\delta_f s+\delta_f K}{2+K\delta_f s+\delta_f s} + \frac{K}{4}\,(s-1.84K) \tag{34}$$

It is simple to show that the first term of (34) is increasing in K, whilst the second term is decreasing in K since its derivative is proportional to $s - 3.68K$ and $K \geq s/3$. Therefore evaluating the first term at $K = 3s/8$ and the second term at $K = s/3$, the average price is always less than:

$$\frac{2+3\delta_f s^2/4+3\delta_f s/8}{2+3\delta_f s^2/8+\delta_f s} + \frac{29}{900}s^2$$

Therefore a sufficient condition for the average price to always be below 1, is that

$$\frac{2+3\delta_f s^2/4+3\delta_f s/8}{2+3\delta_f s^2/8+\delta_f s} + \frac{29}{900}s^2 < 1 \iff s\left[\delta_f\frac{3s-5}{16+3\delta_f s^2+8\delta_f s} + \frac{29}{900}s\right] < 0$$

The term in square brackets is increasing is s so this condition is less likely to hold when s is large. Substituting in $s = 7/10$, we find that the condition holds provided that $\delta_f > 0.132$. ■

Proof of Proposition 8. This follows simply from information provided in the earlier proof of Proposition 4. ■

Proof of Lemma 9. In steady state, old consumers have expected surplus of

$$V - J - \frac{1}{4} + \frac{s^2}{4} - \frac{s}{2} \tag{35}$$

This is because consumers who previously bought product A will i). stay with A and earn $V - J - x^t$ if their location satisfies $x^t \leq (1+s)/2$ or ii). switch to B and earn $V - J - (1 - x^t) - s$ otherwise. Integrating over all possible values of x^t, gives (35). Similarly one can show that consumers who previously bought product B also have expected surplus equal to (35).

When $s = 0$ consumer surplus is therefore $V - 5/4$ (because $J = 1$ when $s = 0$). So switching costs make old consumer worse off if and only if

$$\frac{s^2}{4} - \frac{s}{2} < J - 1 \tag{36}$$

Using the expression for J in equation (10), we know that

$$J - 1 = \frac{K\delta_c s + \delta_f (K - s)}{2 + K\delta_c s + \delta_f s} > \frac{\delta_f (K - s)}{2 + K\delta_c s + \delta_f s} > \frac{-\frac{2}{3}\delta_f s}{2 + K\delta_c s + \delta_f s} > \frac{-\frac{2}{3}\delta_f s}{2 + \delta_f s} > \frac{-2s}{3(2+s)}$$

therefore in order to prove (36) it is sufficient to prove that

$$\frac{s^2}{4} - \frac{s}{2} < \frac{-2s}{3(2+s)}$$

which is easily shown to hold for any $s \in (0, 7/10]$. ■

Proof of Proposition 10. The proof follows from arguments in the text, and Lemma 17 which shows that J increases in δ_c. ■

A.4 Proof of Proposition 12

The result in Proposition 12 holds for *any* δ_c. However to simplify the exposition, we will only prove the result for the special case of $\delta_c = 0$.

Lemma 18 *Firm A's period-t demand is*

$$D_A^t\left(p_A^t, p_B^t, D_A^{t-1}\right) = 1 + p_B^t - p_A^t + s\left(1-\rho\right)\left(D_A^{t-1} - 1\right) \tag{37}$$

Proof. Firstly there are 2ρ young consumers. Since $\delta_c = 0$, these young consumers get utility $V - p_A^t - x^t$ from product A and utility $V - p_B^t - (1 - x^t)$ from product B. Therefore they buy A if and only if $x^t \leq \left(1 + p_B^t - p_A^t\right)/2$. Secondly firm A sold to D_A^{t-1} consumers in the previous period, but only a fraction $1-\rho$ survive to the next period, and they buy A if and only if $x^t \leq \left(1 + p_B^t - p_A^t + s\right)/2$. Thirdly B sold to $2 - D_A^{t-1}$ consumers in the previous period, of which only a fraction $1-\rho$ survive, and they only switch to firm A if $x^t \leq \left(1 + p_B^t - p_A^t - s\right)/2$. Summing these three sources of demand and then simplifying, gives equation (37). ■

Proof of the Proposition. The payoff-relevant state variable for period t is D_A^{t-1}. We look for a symmetric MPE which is continuous in s around $s = 0$, and where the steady state has $D_A^{t-1} = 1$ and both firms charging the same price.

Firm A chooses p_A^t to maximize $p_A^t D_A^t(\cdot) + \delta_f V_A^{t+1}\left(D_A^t(\cdot)\right)$. Differentiating with respect to p_A^t and noting that $\partial D_A^t(\cdot)/\partial p_A^t = -1$, gives a *F.O.C.*:

$$D_A^t(\cdot) - p_A^t - \delta_f \frac{dV_A^{t+1}\left(D_A^t\right)}{dD_A^t} = 0 \tag{38}$$

First letting $\bar{p}$ denote the steady state price, in steady state the *F.O.C.* simplifies to $\bar{p} = 1 - \delta_f\left[dV_A^{t+1}(1)/dD_A^t\right]$. Therefore:

$$\left.\frac{\partial\bar{p}}{\partial s}\right|_{s=0} = -\delta_f \left.\frac{\partial\left(dV_A^{t+1}(1)/dD_A^t\right)}{\partial s}\right|_{s=0} \tag{39}$$

Second substitute the expression for $D_A^t(\cdot)$ in equation (37) into the *F.O.C.* (38), and then totally differentiate with respect to D_A^{t-1} at the steady state, to get:

$$\frac{dp_B^t(1)}{dD_A^{t-1}} - 2\frac{dp_A^t(1)}{dD_A^{t-1}} + s\left(1-\rho\right) - \delta_f \frac{d^2V_A^{t+1}(1)}{d\left(D_A^t\right)^2}\left[\frac{dp_B^t(1)}{dD_A^{t-1}} - \frac{dp_A^t(1)}{dD_A^{t-1}} + s\left(1-\rho\right)\right] = 0 \tag{40}$$

We look for a symmetric equilibrium in which $p_A^t\left(D_A^{t-1}\right) = p_B^t\left(2 - D_A^{t-1}\right)$, and therefore $dp_A^t(1)/dD_A^{t-1} = -dp_B^t(1)/dD_A^{t-1}$. Also note that when $s = 0$, $p_A^t = 1$ and $V_A^{t+1} = 1/\left(1-\delta_f\right)$ which are both invariant to changes in D_A^{t-1} and D_A^t respectively. Therefore differentiating equation (40) with respect to s around $s = 0$ gives:

$$\left.\frac{\partial\left(dp_A^t/dD_A^{t-1}\right)}{\partial s}\right|_{s=0} = \frac{1-\rho}{3} \tag{41}$$

Thirdly by the principle of optimality

$$V_A^t\left(D_A^{t-1}\right) = \max_{p_A^t} p_A^t D_A^t(\cdot) + \delta_f V_A^{t+1}\left(D_A^t(\cdot)\right)$$

Differentiating this equation at the steady state, and using the envelope theorem:

$$\frac{dV_A^t(1)}{dD_A^{t-1}} = \left[\bar{p} + \delta_f \frac{dV_A^{t+1}(1)}{dD_A^t}\right]\frac{dp_B^t(1)}{dD_A^{t-1}} + s(1-\rho)\left[\bar{p} + \delta_f \frac{dV_A^{t+1}(1)}{dD_A^t}\right]$$

Using A's F.O.C. and noting that $dp_A^t(1)/dD_A^{t-1} = -dp_B^t(1)/dD_A^{t-1}$, this can be re-expressed as:

$$\frac{dV_A^t(1)}{dD_A^{t-1}} = -\frac{dp_A^t(1)}{dD_A^{t-1}} + s(1-\rho)\left[\bar{p} + \delta_f \frac{dV_A^{t+1}(1)}{dD_A^t}\right]$$

Differentiating this with respect to s around the point $s = 0$ and then combining with equations (41) and (39), gives the required expression for $\partial\bar{p}/\partial s|_{s=0}$ given in the text.

Consumer surplus The impact of a small increase in s (starting from $s = 0$) on a consumer's lifetime expected consumer surplus is

$$\frac{2\delta_f}{3}(1-\rho) + \sum_{\tau=1}^{\infty}\left[\delta_c(1-\rho)\right]^{\tau}\left[-\frac{1}{2} + \frac{2\delta_f}{3}(1-\rho)\right] \tag{42}$$

because the consumer pays $2\delta_f(1-\rho)/3$ less in each period that she stays in the market, but in every period except the first will pay the switching cost with probability 1/2. (42) is easily shown to be positive when the condition given in the proposition holds. ■

A.5 Proof of Proposition 13

To simplify the notation, we use Δ^t as a shorthand for $p_B^t - p_A^t$, and also Δ_e^{t+1} as a shorthand for $Ep_B^{t+1} - Ep_A^{t+1}$. We start with some preliminary lemmas.

Lemma 19 *Suppose that in period $t-1$ all young consumers with $x^{t-1} \leq \tilde{x}^{t-1}$ bought from A and all others bought from B. Then demand for product A in period t is:*

$$D_A^t = \int_0^{\tilde{x}^{t-1}} F\left(\frac{1+\Delta^t+s}{2}\middle| z\right) dz + \int_{\tilde{x}^{t-1}}^1 F\left(\frac{1+\Delta^t-s}{2}\middle| z\right) dz + \tilde{x}^t \tag{43}$$

where $\Delta^t = p_B^t - p_A^t$, and where $\tilde{x}^t$ is implicitly defined by the equation

$$1 - 2\tilde{x}^t + \Delta^t + \delta_c \left[s + \int_{\dot{x}^{t+1}}^1 f\left(z|\tilde{x}^t\right)\left(2z - 1 - \Delta_e^{t+1} - s\right) dz\right] \\ - \delta_c \left[\int_{\ddot{x}^{t+1}}^1 f\left(z|\tilde{x}^t\right)\left(2z - 1 - \Delta_e^{t+1} + s\right) dz\right] = 0 \tag{44}$$

and where $\dot{x}^{t+1}$ and $\ddot{x}^{t+1}$ are as defined earlier, namely $\dot{x}^{t+1} = \left(1+\Delta_e^{t+1}+s\right)/2$ and $\ddot{x}^{t+1} = \left(1+\Delta_e^{t+1}-s\right)/2$.

Proof. The first two terms of (43) are demand from old consumers. In the previous period A sold to all young consumers with $x^{t-1} \leq \tilde{x}^{t-1}$; as shown earlier in the paper, they will buy A in period t if and only if $x^t \leq \left(1+p_B^t - p_A^t + s\right)/2$. In the previous period B sold to all young consumers with $x^{t-1} \geq \tilde{x}^{t-1}$; as shown earlier in the paper, they will switch to A in period t if and only if $x^t \leq \left(1+p_B^t - p_A^t - s\right)/2$.

The last term of (43) is demand from young consumers. Let us define $W_A^{t+1} = V - E\left(x^{t+1}\middle| x^t\right) - Ep_A^{t+1}$. Then using the same arguments as when proving Lemma 1, a young consumer in period t with location x^t has expected payoffs from buying A and B given by the following:

$$V - x^t - p_A^t + \delta_c\left[W_A^{t+1} + \int_{\dot{x}^{t+1}}^1 f\left(z|x^t\right)\left(2z-1-\Delta_e^{t+1}-s\right) dz\right] \tag{45}$$

$$V - \left(1-x^t\right) - p_B^t + \delta_c\left[W_A^{t+1} - s + \int_{\ddot{x}^{t+1}}^1 f\left(z|x^t\right)\left(2z-1-\Delta_e^{t+1}+s\right) dz\right] \tag{46}$$

Now (45) minus (46) is strictly decreasing in x^t when $s=0$ (and therefore by continuity, when s is sufficiently close to zero). Hence there exists an $\tilde{x}^t$ such that all consumers in period t with location $x^t \leq \tilde{x}^t$ buy A, and all others buy B. Substituting $x^t = \tilde{x}^t$ into (45) and (46) and then equating them, gives equation (44) in the lemma. ■

Lemma 20 *Recall the definition of the marginal young consumer $\tilde{x}^t$ in Lemma 19. In* *steady state i). when $s = 0$, $d\tilde{x}^t/dp_A^t = -1/2$, and ii). the following holds:*

$$\left.\frac{\partial\left(d\tilde{x}^t/dp_A^t\right)}{\partial s}\right|_{s=0} = \frac{\delta_c}{2}\frac{\partial \Pr\left(z \geq 1/2 | x^t = 1/2\right)}{\partial x^t} \tag{47}$$

Proof. Totally differentiate (44) with respect to p_A^t at the steady state, to get:

$$\frac{d\tilde{x}^t}{dp_A^t} = -\frac{1}{\bar{\gamma}}$$

where $\bar{\gamma}$ is defined to be

$$\begin{aligned}\bar{\gamma} &= 2 - \delta_c \int_{\frac{1+s}{2}}^{1} \frac{\partial f\left(x^{t+1}|x^t = 1/2\right)}{\partial x^t}\left(2z - 1 - s\right) dz \\ &+ \delta_c \int_{\frac{1-s}{2}}^{1} \frac{\partial f\left(x^{t+1}|x^t = 1/2\right)}{\partial x^t}\left(2z - 1 + s\right) dz - \delta_c \int_{\frac{1-s}{2}}^{\frac{1+s}{2}} f\left(z|x^t = 1/2\right)\left(\frac{d\Delta_e^{t+1}\left(1/2\right)}{d\tilde{x}^t}\right) dz\end{aligned} \tag{48}$$

To prove part i). note that $\bar{\gamma} = 2$ when $s = 0$. To prove part ii). note that when $s = 0$, $\Delta_e^{t+1}\left(1/2\right) = 0$ which is invariant to changes in s. Therefore after differentiating (48) with respect to s around $s = 0$, we find that:

$$\begin{aligned}\left.\frac{1}{\delta_c}\frac{\partial\bar{\gamma}}{\partial s}\right|_{s=0} &= 2\int_{1/2}^{1} \frac{\partial f\left(z|x^t = 1/2\right)}{\partial x^t} dz = 2\frac{\partial \Pr\left(z \geq 1/2|x^t = 1/2\right)}{\partial x^t} \\ \Longrightarrow \left.\frac{\partial\left(d\tilde{x}^t/dp_A^t\right)}{\partial s}\right|_{s=0} &= \frac{1}{\left(\bar{\gamma}|_{s=0}\right)^2}\left.\frac{\partial\bar{\gamma}}{\partial s}\right|_{s=0} = \frac{\delta_c}{2}\frac{\partial \Pr\left(z \geq 1/2|x^t = 1/2\right)}{\partial x^t}\end{aligned}$$

■

Lemma 21 *In steady state i). when $s = 0$, $\partial D_A^t/\partial p_A^t = -1$ and ii). the following holds*

$$\left.\frac{\partial\left(dD_A^t/dp_A^t\right)}{\partial s}\right|_{s=0} = \frac{1+\delta_c}{2}\frac{\partial \Pr\left(z \geq 1/2|x^t = 1/2\right)}{\partial x^t}$$

Proof. Differentiate the demand expression in equation (43) with respect to p_A^t at the steady state. Using $\bar{\gamma}$ defined in Lemma 20 this gives:

$$\frac{dD_A^t}{dp_A^t} = -\frac{1}{\bar{\gamma}} - \frac{1}{2}\int_0^{1/2} f\left(\left.\frac{1+s}{2}\right| z\right) dz - \frac{1}{2}\int_{1/2}^{1} f\left(\left.\frac{1-s}{2}\right| z\right) dz \tag{49}$$

Now differentiate equation (49) with respect to s around $s=0$:

$$\left.\frac{\partial\left(dD_A^t/dp_A^t\right)}{\partial s}\right|_{s=0}=\frac{\delta_c}{2}\frac{\partial\Pr\left(x^{t+1}\geq 1/2|x^t=1/2\right)}{\partial x^t}+\frac{\int_{1/2}^{1}f'\left(\frac{1}{2}\middle|z\right)dz-\int_0^{1/2}f'\left(\frac{1}{2}\middle|z\right)dz}{4}$$

To get equation (47), first note that $\int_0^{1/2}f'\left(\frac{1}{2}\middle|z\right)dz=-\int_{1/2}^{1}f'\left(\frac{1}{2}\middle|z\right)dz$ by radial symmetry. Then note that by assumption $f\left(1/2|z\right)=f\left(z|1/2\right)$, therefore:

$$\int_{1/2}^{1}f'\left(\frac{1}{2}\middle|z\right)dz\equiv\int_{1/2}^{1}\frac{\partial f\left(y=\frac{1}{2}\middle|z\right)}{\partial y}dz=\int_{1/2}^{1}\frac{\partial f\left(z\middle|y=\frac{1}{2}\right)}{\partial y}dz=\frac{\partial\Pr\left(z\geq\frac{1}{2}|y=\frac{1}{2}\right)}{\partial y}$$

■

Lemma 22 *In steady state:*

$$\left.\frac{\partial D_A^t/\partial\tilde{x}^{t-1}}{\partial s}\right|_{s=0}=f\left(\frac{1}{2}\middle|\frac{1}{2}\right)\tag{50}$$

Proof. Differentiate the demand expression (43) with respect to $\tilde{x}^{t-1}$:

$$\frac{\partial D_A^t}{\partial\tilde{x}^{t-1}}=F\left(\frac{1+\Delta^t+s}{2}\middle|\tilde{x}^{t-1}\right)-F\left(\frac{1+\Delta^t-s}{2}\middle|\tilde{x}^{t-1}\right)\tag{51}$$

Substitute in $\tilde{x}^{t-1}=1/2$ and $\Delta^t=0$, then differentiate the resulting expression with respect to s and impose $s=0$. ■

Lemma 23 *In steady state the derivative of* $\frac{\partial\left(dD_A^t/dp_A^t\right)}{\partial\tilde{x}^{t-1}}$ *with respect to* s *around* $s=0$ *is zero.*

Proof. Differentiate equation (51) with respect to p_A^t to get

$$\frac{\partial\left(dD_A^t/dp_A^t\right)}{\partial\tilde{x}^{t-1}}=\frac{1}{2}\left[-f\left(\frac{1+\Delta^t+s}{2}\middle|\tilde{x}^{t-1}\right)+f\left(\frac{1+\Delta^t-s}{2}\middle|\tilde{x}^{t-1}\right)\right]$$

Then substitute in $\tilde{x}^{t-1}=1/2$ and $\Delta^t=0$. Now differentiate with respect to s, and note that radial symmetry implies $f'\left(1/2|1/2\right)=0$. ■

Now for the main proof.

Proof of Proposition 13. In this problem the payoff-relevant state variable in period t is $\tilde{x}^{t-1}$. We again look for a symmetric MPE which is continuous around $s=0$, and where the steady state has $\tilde{x}^{t-1}=1/2$ and both firms charging the same price.

Firm A chooses p^t_A to maximize $p^t_A D^t_A + \delta_f V^{t+1}_A(\tilde{x}^t)$, which gives a $F.O.C.$

$$D^t_A + p^t_A \frac{dD^t_A}{dp^t_A} + \delta_f \frac{dV^{t+1}_A(\tilde{x}^t)}{d\tilde{x}^t}\frac{d\tilde{x}^t}{dp^t_A} = 0 \tag{52}$$

Firstly Impose steady state on the $F.O.C.$, and then differentiate it with respect to s around $s = 0$. In doing this, note i). that in steady state $D^t_A = 1$, ii). the properties of dD^t_A/dp^t_A given in Lemma 21, iii). that when $s = 0$ firm A charges a price of 1 and has value $V^{t+1}_A = 1/(1-\delta_f)$ which is not a function of $\tilde{x}^t$, and iv). that $d\tilde{x}^t/dp^t_A = -1/2$ when $s = 0$ (Lemma 20). Letting $\bar{p}$ be the steady state price, we find that:

$$\left.\frac{\partial \bar{p}}{\partial s}\right|_{s=0} = \frac{1+\delta_c}{2}\frac{\partial \Pr\left(x^{t+1} \geq 1/2 | x^t = 1/2\right)}{\partial x^t} - \frac{\delta_f}{2}\left[\left.\frac{\partial\left(dV^{t+1}_A(1/2)/d\tilde{x}^t\right)}{\partial s}\right|_{s=0}\right] \tag{53}$$

Secondly differentiate the $F.O.C.$ in equation (52) with respect to $\tilde{x}^{t-1}$ to get:

$$\frac{d(F.O.C.)}{dp^t_A}\frac{dp^t_A}{d\tilde{x}^{t-1}} + \frac{d(F.O.C.)}{dp^t_B}\frac{dp^t_B}{d\tilde{x}^{t-1}} + \left[\frac{\partial D^t_A}{\partial \tilde{x}^{t-1}} + p^t_A \frac{\partial\left(dD^t_A/dp^t_A\right)}{\partial \tilde{x}^{t-1}}\right] = 0$$

The aim now is to differentiate this equation with respect to s around $s = 0$. To do this note that when $s = 0$, i). $F.O.C. = 1 + p^t_B - 2p^t_A$, ii). p^t_A and p^t_B both equal 1 and so are not a function of $\tilde{x}^{t-1}$, and iii). that according to Lemma 23, both $\partial\left(dD^t_A/dp^t_A\right)/\partial\tilde{x}^{t-1}$and its derivative with respect to s, are zero. Also use Lemma 22 which gives an expression for the derivative of $\partial D^t_A/\partial\tilde{x}^{t-1}$ with respect to s. Then

$$-2\left.\frac{\partial\left(dp^t_A/d\tilde{x}^{t-1}\right)}{\partial s}\right|_{s=0} + \left.\frac{\partial\left(dp^t_B/d\tilde{x}^{t-1}\right)}{\partial s}\right|_{s=0} + f\left(\frac{1}{2}\middle|\frac{1}{2}\right) = 0$$

We look for a symmetric equilibrium in which $p^t_A(\tilde{x}^{t-1}) = p^t_B(1-\tilde{x}^{t-1})$ therefore $dp^t_A/d\tilde{x}^{t-1}\left(\frac{1}{2}\right) = -dp^t_B/d\tilde{x}^{t-1}\left(\frac{1}{2}\right)$. So differentiating the above equation with respect to s at $s = 0$, gives:

$$\left.\frac{\partial\left(dp^t_A\left(\frac{1}{2}\right)/d\tilde{x}^{t-1}\right)}{\partial s}\right|_{s=0} = \frac{f\left(\frac{1}{2}\middle|\frac{1}{2}\right)}{3} \tag{54}$$

Thirdly by the principle of optimality

$$V^t_A\left(\tilde{x}^{t-1}\right) = \max_{p^t_A} p^t_A D^t_A(\cdot) + \delta_f V^{t+1}_A\left(\tilde{x}^t\right)$$

Totally differentiating this equation with respect to $\tilde{x}^{t-1}$and using the envelope theorem, gives:

$$\frac{dV_A^t}{d\tilde{x}^{t-1}} = \left[p_A^t \frac{dD_A^t}{dp_B^t} + \delta_f \frac{dV_A^{t+1}}{d\tilde{x}^t} \frac{d\tilde{x}^t}{dp_B^t}\right] \frac{dp_B^t}{d\tilde{x}^{t-1}} + p_A^t \frac{\partial D_A^t}{\partial \tilde{x}^{t-1}}$$

Since $D_A^t(\cdot)$ and $\tilde{x}^t$ only depend upon current prices through Δ^t, $dD_A^t/dp_B^t = -dD_A^t/dp_A^t$ and $d\tilde{x}^t/dp_B^t = -d\tilde{x}^t/dp_A^t$. Therefore using A's $F.O.C.$ (52), the above becomes:

$$\frac{dV_A^t}{d\tilde{x}^{t-1}} = D_A^t \frac{dp_B^t}{d\tilde{x}^{t-1}} + p_A^t \frac{\partial D_A^t}{\partial \tilde{x}^{t-1}} \tag{55}$$

Next impose a steady state on (55) and substitute in $dp_B^t/d\tilde{x}^{t-1}\left(\frac{1}{2}\right) = -dp_A^t/d\tilde{x}^{t-1}\left(\frac{1}{2}\right)$. Then differentiate the resulting equation with respect to s at $s = 0$. Finally combine this last equation with equations (54) and (53), to get the required expression for $\partial\bar{p}/\partial s|_{s=0}$. ■

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