The Impact of Financial Market Frictions on Firm Size in a Relational Contracting Model

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Abstract

The present paper gives a new explanation for why growth might be restricted by badly functioning financial markets - based on the premise that firms have to use implicit contracts to motivate their employees. If demand conditions for the firm's output are volatile and it faces a limited liability constraint, i.e., all funds used to compensate its employees must come out of the sale of the output, firms' employment levels may stay inefficiently small. The reason is the enforced variability in compensation, which increases the firm's reneging temptation. However, a well-functioning credit market helps to smooth payments over time and consequently allows for larger firms. Whereas the limited liability constraint and a badly-functioning credit market restricts firm size with regard to employment, it might make it optimal to choose a capacity or capital level that is inefficiently high. Overinvestments thus do not have to be a consequence of agency problems in the owner-manager relationship but can be a mechanism to countervail underemployment driven by the limited liability a firm faces absent a well functioning financial market.

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1 Introduction

Over the last decade, there has been an intense discussion how important the functioning of domestic financial markets is for a country's growth. Moll (2010), Buera, Kaboski, and Shin (2010), and Midrigan and Xu (2010), for example, find that one reason for a misallocation of capital - which is claimed to be a cause for low growth rates in many developing countries - is that financial markets are not sufficiently well developed. The focus of this literature is on the consequences of financial frictions on the financing of investment projects, though. In essence, financial frictions lead to inefficiently low investments, thus hampering the productivity potential of an economy.

The present paper gives a new, complementary explanation for why badly functioning financial markets might be an impediment to growth, namely by restricting a firm's flexibility when handling and compensating their workforce. The argument is based on two premises. First, firms often use implicit contracts to motivate their employees. Especially if law-enforcing institutions are weak - which often goes together with badly working financial markets - arrangements based on trust serve as a substitute for perfectly enforceable, formal contracts between firms and its employees. Second, all funds used to compensate the firm's employees must come out of the sale of its output or be borrowed from outside lenders. Then, if demand conditions for the firm's output are volatile, an employee's compensation will often also have to vary with the company's success. However, the firm's incentives to honor an implicit contracts are lowest in states where it is supposed to pay out most, since the benefits from honoring the contract only depend on expected future profits and not its current realizations. Thus, a larger difference between good and bad states restricts the firm's commitment and keeps employment inefficiently low. This problem is reduced by a well-functioning credit market, which allows to smooth payments over time and average out reneging temptations. Furthermore, a higher stock of (productive) physical assets also relaxes the limited liability constraint, giving rise to overinvestments if sufficient equity financing is available.

The starting point of this paper is the moral hazard principal agent literature, which focuses on unobservable effort choice as a determinant of firm profitability (or productivity). An improved solution to the moral hazard problem will, ceteris paribus, increase productivity. While there exists a large literature, building on Holmström (1979) and Grossman and Hart (1983), focusing on explicit contracts that reward the agent based on verifiable performance measures, there has been an increased interest in implicit contracts as a way to mitigate the moral hazard problem (see, e.g. Bull, 1987, MacLeod and Malcomson, 1989, or Levin, 2003). This literature employs repeated game logic to use observable but unverifiable information and does not rely on explicit, court enforceable, performance contracts to motivate workers. In particular in developing countries with weak institutions it seems reasonable to assume that such implicit incentive contracts play an important role in motivating the work force. In the recent past, within the literature in relational contracts, there have been a number of papers investigating richer dynamics and the effect of stochastic shocks on the efficiency and stability of implicit contracts (see, for example, Li and Matouschek, 2011, or Englmaier and Segal, 2011). Under such conditions, the efficiency of relational contracts can be improved if the contracting parties have access to a financial market to smooth the effects of shocks on their relationship. This paper attempts the, as we believe, natural step to combine this literature with the literature that studies the effects of financial market development on (productivity) growth.

We develop a model where one risk-neutral firm can employ many risk-neutral agents. Neither an agent's effort nor the resulting output are verifiable, and relational contracts must be used to give incentives. In addition, demand conditions for the firm's output vary – they can be either high or low – and only funds that have been earned by selling its output can be used to compensate employees, i.e., the firm faces a limited liability constraint. Especially if markets are volatile and the difference between good and bad outcomes is relatively large, a major part of total compensation is forwarded to agents when demand conditions are good. However, the maximum amount the firm is willing to pay out instead of reneging and shutting down is determined by its expected future profits, independent of the difference between states of the world. Thus, higher payment obligations in the good state increases the firm's reneging temptation, and - at some point - induce an inefficiently low employment level.[1] The firm would therefore rather operate in a more stable environment, even at the cost of a ceteris paribus smaller surplus. Thus, the limited liability constraint induces risk averse-like behavior, even though firm and agents are actually risk neutral.

In a next step, we introduce a credit market where the firm can borrow funds to smooth wage payments over time. The quality of the credit market might be better

[1] Bad contract-enforcing institutions can thus restrict firm size, which is one of the main results in Powell (2011). Different from our paper, he analyzes a market with heterogenous firms and restricts his focus on problems that arise if a firm's expected future profits - which serve as a bond to increase commitment - are too small.

or worse, manifesting in the interest rates demanded for loans and the degree to which the firm's assets can be used as collateral. A good financial environment helps the firm to overcome the restrictions it faces - the impossibility to write formal contracts, as well as its limited liability together with changing demand conditions - and increases implementalbe employment levels and thus firm size.

Subsequently, we analyze the situation when the firm's output level not only depends on employment but also on a physical asset, where the associated capacity level is determined by the firm at the beginning of the game and financed through equity. As a larger capacity increases output and thus the available cash flow in all states of the world, it relaxes the limited liability constraint and allows for higher employment. This gives rise to overinvestments into capacity (in relation to employment), especially when the firm faces a worse credit market but less problems to raise enough equity to fund the initial investment.

Investments that are inefficiently high or low have been analyzed extensively, especially in the corporate finance literature. There, various agency problems between owners and managers of a firm are blamed to spur inefficiencies with regard to corporate investments, causing investment projects and consequently firms to either be inefficiently large or inefficiently small (see Stein, 2003, for an excellent survey). Rationales for overinvestments that have been identified in the literature are empire building (see Jensen, 1986, 1993, or Hart and Moore, 1995), short-termism of managers who focus on activities the market can easily observe (see Stein, 1989, or Bebchuk and Stole, 1993), or managers' overconfidence into their own abilities (see Roll, 1986 or Heaton, 2002). Our model generates overinvestments even though no agency problem between owners and manager exists (the only agency problem present is the non-verifiability of each employee's performance). Moreover, overinvestments are actually *efficiency-enhancing* and help to mitigate the negative impact the firm's limited liability has on enforceable employment. However, overinvestments can be substituted by borrowing and thus decrease with the quality of the credit market. This last point can be related to the observation that firms in developing countries often are either very small or very large. Buera et al. (2011), among others, make this point and claim that less developed countries are particularly unproductive in sectors with larger scale of operation, e.g., manufacturing. Whereas they postulate that this is driven by a misallocation of capital, our model shows that a disproportionately large use of physical assets can be an instrument to countervail underemployment caused by a firm's limited liability combined with a weak financial sector.

2 Model Setup

There is one firm ("principal") which is endowed with an indivisible, physical asset with market value $A > 0$. Furthermore, there is a labor market with N homogenous prospective employees ("agents"). Time is discrete, the time horizon infinite and all players have a discount factor $\delta \in (0,1)$. In every period $t = 1, 2, ...$ - and as long as it still owns the asset - the firm makes an employment offer to $\underline{n}_t \geq 0$ agents (for concreteness, we assume $\underline{n}$ to be continuous). N – the size of the labor market – is sufficiently large for the firm to not be exogenously bounded when choosing $\underline{n}_t$. Agents who received an offer are indexed $i \in [0, \underline{n}_t]$. Each offer consists of a fixed wage $w_{it} \geq 0$ and the promise to make a contingent bonus payment $b_{it} \geq 0$. All agents who received an offer then decide whether to accept it or not, i.e., each of them chooses $d_{it}^A \in \{0, 1\}$. All agents with $d_{it} = 0$, among whom we subsume all of them who rejected an offer as well as those who did not receive one, consume their outside options $\overline{u}$ in the respective period. All agents who chose $d_{it}^A = 1$ and thus received and accepted an offer by the firm subsequently have to make their effort choice. Effort is binary, $e_{it} \in \{0, 1\}$, and associated with private costs $c(e_{it})$, where $c(0) = 0$ and $c(1) = c > 0$. Then, period-t output $y_t = f(n_t)$ is realized, where $n_t \leq \underline{n}_t$ is the number of agents who chose $e = 1$. $f(n)$ is a continous function, strictly increasing and concave

After production, the firm sells the output, generating revenues $\theta_t f(n_t)$. $\theta_t \in \{\theta^l, \theta^h\}$ is a parameter specifying the demand for the firm's output, with $0 < \theta^l < \theta^h$, and is realized after the output has been produced. High demand is observed with probability p, low demand with $1 - p$. These probabilities are independent over time, i.e., there is no persistency in demand conditions. After selling the output, payments w_{it} and b_{it} are actually made. We assume that the firm is wealth constrained and initially not endowed with any liquid financial resources. All funds used to compensate employees must therefore be earned via the sale of its products. However, there exists a credit market where the firm can borrow the loan $L_t \geq 0$ at a per-period interest rate r_B, with $\frac{1}{1+r_B} \leq \delta$. Loans can be collateralized against the asset with market value A. To make the role of a well-functioning credit market more precise, we finally assume that savings are not possible.[2]

At the end of a period, the firm is supposed to pay back loans taken in previous periods (the amount paid back in period t is denoted R_t) and has the option to sell

[2]Alternatively, the savings rate could be set sufficiently small (driven by bad financial markets and a high inflation) to never make saving optimal.

the asset. If it defaults on a collateralized loan, the creditor (whom we do not model further) forces the firm to sell the asset, keeps the amount to be repayed and leaves the rest to the firm. If there is no default, the firm itself decides whether to sell the asset and receive A or whether to run the firm for an additional period. Termination of production is permanent, i.e., once the asset has been sold and liquidated, it is not possible to get it back in any subsequent period. Formally, this choice is characterized by $d_t^P \in \{0,1\}$, where $d_{t+1}^P = 0$ describes that the asset has been sold at the end of period t. Thus, production in t is only possible if $d_t = 1$ at the beginning of period t, where $d_t = d_{t-1}d_{t-1}^P$, and $d_1 \equiv 1$. Note that once $d_t = 0$, $d_{it}^A = 0$ for all agents and all future periods as well.

Information

Each agent's effort choice can be observed by the principal. However, effort and output are not verifiable, implying that that bonus payments have to be made voluntarily. Thus, relational contracts must be used to incentivize agents.

Furthermore, all agents as well as the labor market can detect if the principal reneges on promises made to any employed agent. The reason could be that others are able to observe an agent's effort as well as her compensation, or that turnover that should not occur in equilibrium is detected. This allows the principal to use multilateral relational contracts (Levin, 2002) to incentivize agents. We will explain this in more detail below, just note that it means that if the principals breaks the promise he made to one agent, all others – whether already employed or not – will not cooperate in the future anymore. This increases the commitment power of the principal, who is now going to loose the total future surplus when reneging on just one agent.

Different from output and effort, the realization of the demand parameter is actually verifiable, for example because it reflects the general state of the economy or the specific industry where the firm is active. This assumption is important as it allows loan contracts where the repayment obligation is a function of this state of the world. The revenue generated by selling the firm's output is not verifiable, though (this assumption is without loss of generality as long as the contribution of a single agent to total output cannot be measured).

Payoffs and Strategies

All players are risk neutral and aim to maximize their expected discounted payoff streams in every period t. For agent i, this expected payoff stream equals

$$U_{it} = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left[d_\tau d_{i\tau}^A \left(e_{i\tau} - c(e_{i\tau}) + w_{i\tau} + b_{i\tau} \right) + (1 - d_\tau d_{i\tau}^A) \overline{u} \right],$$

while the firm's payoff in period t equals

$$\Pi_t = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left\{ d_t \left[\left(p\theta^h + (1-p)\theta^l + L_t(\theta_t) - R_t(\theta_t) \right) f(n_t) - \int_0^{\tilde{n}_t} (w_{it} + b_{it})\, di \right] + (1-d_t)(1-\delta)A \right\},$$

where $\tilde{n} \leq \underline{n}_t$ denotes the number of agents who received and accepted an offer. Risk neutrality is also assumed for (not further modelled) creditors.

Since all information is public, the relevant equilibrium concept here is subgame perfect equilibrium (SPE). More precisely, we are looking for the SPE that maximizes the firm's expected profits at the beginning of the game, i.e., that it is able to make a take-it-or-leave-it offer to each agent. We think that this is a natural choice given that the labor market is competitive, which should make it possible for the firm to unilaterally set wages and bonus payments. However, as agents can always walk away and effort and output are not verifiable, several constraints have to be fulfilled and are subsequently derived. Generally, the profit-maximizing SPE is supported by a reversion to the static Nash equilibrium in case the firm failed to pay the promised reward to any employed agent: no bonus payments are made, all agents are thus induced to choose $e_i = 0$, principals offer a wage of zero, leading agents to reject offers and consequently making it optimal for the principal to not make any offer and instead liquidate and consume the asset.

Furthermore, we can – without loss of generality – focus our attention to contracts that are independent of time and thus are able to omit time subscripts t. But contracts will not be completely stationary, since if a loan is taken, this will depend on realized demand conditions. Furthermore, a repayment obligation is only present if a loan has actually been taken. However, because demand conditions are verifiable, induced effort levels as well as optimal employment will be constant over time.

We are also able to omit the index i. Since agents are identical and multilateral relational contracts can – and should – be used, different employees will always be treated the same. Thus, we do not have to differentiate between agents who exert effort and those who do not. Hiring an agent only makes sense if she is supposed to choose $e = 1$ in the respective period, which implies that the number of hired agents coincides with n – the number of agents who actually exert effort.

Furthermore, we make a normalization and set $\overline{u} = 0$. This has no substantial impact on our results but simplifies the analysis. The reason is that a (court-enforceable) fixed wage usually used to compensate agents for their outside utilities can only be credibly promised if it is collateralized against the asset. Finally, the firm's outside option is solely captured by the value of the asset, A.

3 Benchmark Case Without a Credit Market

We start with the assumption that a credit market is not available (alternatively, we could set $r_B = \infty$), which helps to understand the problem associated with the firm's limited liability constraint. The objective of this section is to derive the profit-maximizing employment level n^* (which is constant over time) and the associated wage scheme. Since $\overline{u} = 0$, we can also set $w = 0$. Thus, the wage scheme only consists of bonus payments b which are supposed to be paid if high effort was chosen (if any agent chooses $e = 0$, she obviously does not receive a bonus). The size of the bonus potentially depends on the realization of the state of the world θ and is made at the end of the period. If it has to be state-contingent, the bonus payment is either denoted b^h (when demand conditions are high) or b^l.

As already pointed out, employing an agent only makes sense if this agent is supposed to exert high effort. Then, an employed agent's expected discounted payoff stream equals

$$U = pb^h + (1-p)b^l - c + \delta U$$

Enforcing high effort requires each agent's incentive compatibility (IC) constraint to be satisfied. Given the promised bonus payments b^l and b^h are made, it must be optimal for an agent to actually choose high effort:

(IC)

$$pb^h + (1-p)b^l - c + \delta U \geq 0,$$

and can be simplified to $pb^h + (1-p)b^l - c \geq 0$. Note that we impose the standard assumption that after deviating (which does not occur in equilibrium), the agent is fired. Furthermore, (IC) also captures an agent's individual rationality constraint,

$U \geq 0$. Since we are interested in the profit-maximizing equilibrium, it will be optimal for the firm to set $U = 0$ and thus make (IC) bind.

Furthermore, it must be in the interest of the principal to actually forward b^l or b^h to each agent. If he fails to make a promised payment, the respective relationship is soured and the agent will subsequently not exert high effort anymore. Since we further assume full transparency with respect to the firm's decisions among its employees, every employed agent will find out if the firm reneges on payments meant for any other agent. Thus, the firm can and also should use Multilateral Relational Contracts (Levin, 2002). This implies that a failure to keep promises made to one agent leads to a loss of trust of the whole (current and prospective new)[3] workforce. Therefore, the firm either makes all promised payments or reneges on all of them. In the latter case, it will be optimal to subsequently shut down and consume A, the liquidation value of the asset.

The two dynamic enforcement (DE) constraints, one for b^l and one for b^h equal

(DEl)

$$nb^l \leq \delta\Pi - \overline{\Pi} = \frac{\delta}{1-\delta}\left(p(\theta^h f(n) - nb^h) + (1-p)(\theta^l f(n) - nb^l)\right) - A$$

(DEh)

$$nb^h \leq \delta\Pi - \overline{\Pi} = \frac{\delta}{1-\delta}\left(p(\theta^h f(n) - nb^h) + (1-p)(\theta^l f(n) - nb^l)\right) - A$$

where Π and $\overline{\Pi} = A$ are the respective on and off equilibrium expected discounted payoff streams. Note that we slightly deviate from standard models by assuming that if the principal reneges, he can still shut down and consume the asset in the same period instead of waiting for one period. This has no real impact on our results but makes the characterization of the optimal loan contract easier (see below). However, this makes our previously made assumption necessary that shutting down and liquidating the asset is a permanent decision. Given both (DE) constraints, the firm's individual rationality constraint, $\Pi \geq \overline{\Pi}$, is satisfied as well.

Since the right hand sides of (DEl) and (DEh) are identical, only one of them has to be considered, depending on whether b^l or b^h is larger. Thus, the principal would always weakly prefer to have $b^h = b^l$. However, the resulting payment has to be sufficiently large to incentivize each agent and at the same time must not violate the firm's limited

[3]Even if prospective new agents cannot observe the principal's actions, the fact that they received an employment offer can serve as a similar signal.

liability (LL) constraints. The latter state that bonus payments in every period cannot exceed respective revenues, or

(LLl)

$$nb^l \leq \theta^l f(n)$$

(LLh)

$$nb^h \leq \theta^h f(n)$$

Before characterizing the optimal employment level subject to all constraints, we derive some thresholds for n. First of all, note that – given production takes place – first-best employment is characterized by

$$f(n^{FB})' \left(p\theta^h + (1-p)\theta^l\right) - c = 0.$$

To avoid trivial solutions, we make

Assumption 1: $f(n^{FB}) \left(p\theta^h + (1-p)\theta^l\right) - n^{FB}c > \frac{1-\delta}{\delta}A.$

Assumption 1 implies that – given first best employment is feasible – running the firm is strictly better than not running it and consuming the asset instead. First best employment could for example always be enforced if an agent's effort were verifiable. Note that the limited liability constraints would not harm in this case, at least if long-term contracts were possible.

In a next step, we derive another benchmark where effort is not verifiable but where the principal is not subject to a limited liability constraint. This gives

Lemma 1: *Assume effort is not verifiable but that the limited liability constraints are not in force. Then, any employment level n with all agents exerting positive effort is enforceable if and only if*

$$nc \leq \delta f(n) \left(p\theta^h + (1-p)\theta^l\right) - (1-\delta)A. \tag{1}$$

Proof: As there is no limited liability constraint, we can set $b^h = b^l \equiv b$. Using the binding (IC) constraint further gives $b = c$. Plugging this into the principal's (DE) constraint, which now equals $nb \leq \delta f(n) \left(p\theta^h + (1-p)\theta^l\right) - (1-\delta)A$, provides the necessity of (1). Sufficiency immediately follows: Assume (1) is satisfied and set $b \equiv c$.

Q.E.D.

It follows from Assumption 1 and Lemma 1 that if n^{FB} satisfies (1) and if $\theta^l f(n^{FB}) \geq n^{FB}c$, efficient employment can and will be chosen by the principal, also if the limited liability constraints are in force.

Now, define $\overline{n}$ as the employment level where (LLl) just becomes binding, i.e.,

$$\theta^l f(\overline{n}) = \overline{n}c.$$

Due to the concavity of $f(n)$, (LLl) will be slack for all employment levels below $\overline{n}$. Then, it is also possible and optimal to set $b^h = b^l = c$, as this payment structure makes it most likely that (DE) constraints are satisfied for a given employment level $n \leq \overline{n}$.

Furthermore, (LLl) binds for all $n > \overline{n}$. This implies that for any employment level larger than $\overline{n}$, b^h must be higher than b^l, and thus that (DEl) is automatically satisfied given (DEh). To keep the analysis interesting and to focus on potential problems induced by the firm's limited liability, we impose two further assumptions.

Assumption 2: $\overline{n}$ can be enforced, i.e., $c\overline{n} < \delta f(\overline{n}) \left(p\theta^h + (1-p)\theta^l\right) - (1-\delta)A$.

Assumption 3: $\overline{n} < n^{FB}$

If either one of Assumptions 2 or 3 were not satisfied, equilibrium employment would have non-binding limited liability constraints and bonus payments that are identical in both states of the world.

However, given Assumptions 2 and 3, we can establish the following Lemma.

Lemma 2: *Given Assumptions 2 and 3 are satisfied, equilibrium employment n^* satisfies $n^* \geq \overline{n}$. Furthermore,*

$$b^l = \theta^l \frac{f(n^*)}{n^*} \leq b^h$$

Proof: $n^* \geq \overline{n}$ immediately follows from Assumptions 2 and 3: An employment level $\overline{n}$ is enforceable (Assumption 2) and gives strictly higher profits than any lower

n (Assumption 3, together with the strict concavity of $f(n)$). Furthermore, recall that Assumption 2 also implies that $\delta\Pi(\overline{n}) > A$, and thus that it is optimal to produce instead of consuming the asset value A.

Concerning the second part, assume that $b^l < \theta^l \frac{f(n^*)}{n^*}$. Since $n \geq \overline{n}$, we know that $b^h > b^l$. Furthermore, the binding (IC) constraint implies that $b^h = \frac{c-(1-p)b^l}{p}$. Increasing b^l by a positive but small amount ε does not violate (LLl). It further does not violate (DEl), which is slack for $b^l < b^h$, but relaxes (DEh). If $n^* < n^{FB}$, employment can then be increased, which consequently raises profits. If $n^* = n^{FB}$, b^h can be adjusted accordingly to keep profits constant and without violating (IC).

Q.E.D.

Furthermore, we can state that (LLh), the limited liability constraint in the high state, will not bind.

Lemma 3: *Given Assumptions 2 and 3 are satisfied, (LLh) will not bind in equilibrium, i.e.* $\theta^h f(n^*) > n^* b^h$.

Proof: Assume (LLh) binds, i.e., $\theta^h f(n^*) = n^* b^h$. Since (LLl) binds as well, this would mean that $\Pi(n^*) = 0$. But then, n^* cannot be enforced, and choosing the enforceable employment level $\overline{n}$ would strictly increase profits (Assumption 2 and 3).

Q.E.D.

Concluding, (IC) and (LLl) constraints will bind in equilibrium, whereas (LLh) and (DEl) are automatically satisfied. Thus, (DEh) will ultimately determine whether n^{FB} can be enforced or not. Substituting $b^h = \frac{c-(1-p)b^l}{p}$ and $b^l = \theta^l \frac{f(n)}{n}$ into (DEh) gives

(ICDEh)

$$nc \leq \frac{\delta p^2}{1-\delta+\delta p}\theta^h f(n) + (1-p)\theta^l f(n) - \frac{p(1-\delta)}{1-\delta+\delta p}A \qquad (2)$$

(ICDEh) might or might not bind in equilibrium. However, note that the likelihood that it binds not only depends on the discounted expected total surplus - as would be the case in standard relational contracts model without limited liability on the principal's side. Proposition 1 below shows that for a given employment level and surplus, and an arbitrary discount factor δ, (DEh) is violated if p is sufficiently small.

Proposition 1: *Fix $n > \overline{n}$, θ^l, c, as well as the total per-period surplus (given positive effort), $s = f(n)\left(p\theta^h + (1-p)\theta^l\right) - nc$. Then, there always exists a $\underline{p}$ such that for $p < \underline{p}$, (ICDEh) is violated.*

Proof: See Appendix

Increasing the spread between θ^h and θ^l and adjusting p to keep the surplus fixed, (ICDEh) will be violated at some point. Note that this result is driven by the limited liability constraint. Without (LLl), the enforceability of an employment level n would – for a given discount factor δ – only depend on the future surplus, independent of the exact specification of p and θ.

For example, first-best employment might be enforceable without limited liability constraints, requiring $n^{FB}c \leq \delta f(n^{FB})\left(p\theta^h + (1-p)\theta^l\right) - (1-\delta)A$ to be satisfied. However, in the limit with $p \to 0$ and an appropriate increase of θ^h to keep the surplus constant, the enforceable employment level even then approaches $\overline{n}$.

The problem addressed here is similar to one without limited liability constraint but with asymmetric information concerning the agent's effort. Then, a relational contract can only be based on the resulting output. Furthermore, the maximum enforceable bonus is characterized by the expected future surplus. Thus, only one dynamic enforcement constraint matters, namely the one for the highest bonus payment. A lower spread of bonus payments thus improves enforceability, whereas a higher spread increases incentives. Fong and Li (2010) show that in this case, the situation might be improved if the agent is not able to observe the output in every period, but a signal generated by an information garbling process that smoothes necessary payments over time. In our setting, smoothing the bonus payments over time would also be optimal, but is restricted by limited liability constraints. Below, we show how taking loans can and will be used for smoothing purposes, even if this is costly.

For concreteness, we further characterize the employment level that induces a binding (DEh) constraint. Thus, $\hat{n}$ is

$$\hat{n}c = \frac{p^2\delta}{1-\delta+p\delta}\theta^h f(\hat{n}) + (1-p)\theta^l f(\hat{n}) - \frac{p(1-\delta)}{1-\delta+p\delta}A. \tag{3}$$

Due to the concavity of $f(n)$, $\hat{n}$ exists, and (ICDEh) is satisfied for $n \leq \hat{n}$, and violated for $n > \hat{n}$. Thus, if $\hat{n} < n^{FB}$, only an inefficiently low employment level can be enforced. Again, note that the driving force here is the limited liability constraint, and inefficiently low employment might also be the outcome if first-best employment could be enforced absent limited liability. To further strenghten this point, we can establish

Lemma 4: *Fix θ^l, c, as well as the total per-period surplus (given positive effort), $s = f(n)\left(p\theta^h + (1-p)\theta^l\right) - nc$. For $\hat{n} > \overline{n}$, $\hat{n}$ is strictly increasing in p.*

Proof: Follows from the fact that the right hand side of (2) is increasing in p, given the surplus is held constant (see the proof to Proposition 1).

Q.E.D.

If θ^h is very high, but the prospects of it being realized are low, enforceable employment is smaller than with a lower θ^h but higher probability p. We should thus observe that industries that ceteris paribus are more volatile have lower employment levels than industries where earnings are more balanced.

Concluding, this section established the potential enforcement problem induced by the limited liability constraint. Bonus payments must vary across states, whereas the maximum power of incentives is determined by the expected future surplus. This implies that the highest equilibrium bonus level determines whether high effort is enforceable or not. If the likelihood of the good state is too low, the necessary bonus to imply effort becomes so high that it is not optimal for the principal to honor his promises anymore, consequently leading to inefficiently low employment.

Taking a credit market might now help to smooth bonus payments over time and consequently relax the limited liability constraints - however at the costs of higher repayment obligations. This will be shown in the following.

4 Credit Market

Now, assume there is a perfect credit market where the principal can borrow at the rate r_B, with $\frac{1}{1+r_B} \leq \delta$. Any loan can be collateralized against the asset necessary for production. The previous analysis then remains true for employment levels $n \leq \hat{n}$. Since taking a loan is costly, the principal will try to avoid it. Only if the profits of increasing employment above $\hat{n}$ outweigh its costs, taking a loan will be optimal.

Two aspects become important now. First of all, we assume that the state of the world is verifiable. Then, repayment can and optimally will be contingent on the realization of θ. More precisely, the firm will prefer credit contracts that require repayment only in high states. If repayment was also due in the low state, this would further reduce feasible bonus payments in this case, making necessary wage payments in the good state even higher (or the firm would have to increase the loan, i.e., take an additional loan to repay the old – but borrowing is costly). However, taking a loan is only optimal if the (DEh) constraint already binds, and a further increase of wages would lead to a violation.

Furthermore, the principal will only be able to borrow if the loan is collateralized. We assume that it can be pledged against the asset with market value A (if a further collateral was available, for example private property of the principal, this could improve the situation). If defaulting on the loan would just resulted in making it impossible for the principal to enter the credit market in any subsequent period, the respective dynamic enforcement constraint would not be relaxed. Thus, the collateralization of the loan generally is not an option, but a necessary feature of any debt contract.

Concerning the timing, we make the following assumption. As above, a reneging of the principal will make it possible to liquidate the asset in the same period (accompanied by the impossibility of buying it back in the following period). Thus, a default on repayment of the loan will also give the creditor immediate access to the asset. Then, the creditor will sell the asset necessary for production, keep the respective amount and give the rest to the firm which is not able to produce anymore from then on.

Concluding, taking a loan helps to relax the limited liability constraint in the low state without having a too severe negative impact on (DEh). The reason is that although payments made by the principal increase in the high state, he can only partially renege on them; formally, repayment obligations to creditors equivalently reduce left and right hand side of the (DEh) constraint.

Now, assume a loan L is taken (and paid out to the agents who exerted effort) in state l. If repayment only occurs in the following period, the repaid amount must equal $(1 + r_B)L$ in expectation. Since repayment should be shifted to the high state where the limited liability constraint does not bind (we will further address the issue of (LLh) below), the creditor will receive $R(\theta^h) = \frac{(1+r_B)L}{p}$ and $R(\theta^l) = 0$.

Then, we can recursively define the principal's payoffs,

$$\begin{aligned}
\Pi^{hh}(n) =& f(n)\theta^h - nb^h + \delta\left(p\Pi^{hh} + (1-p)\Pi^{hl}\right) \\
\Pi^{lh}(n) =& f(n)\theta^h - nb^h - \frac{1+r_B}{p}L + \delta\left(p\Pi^{hh} + (1-p)\Pi^{hl}\right) \\
\Pi^{hl}(n) =& \Pi^{ll} \equiv \Pi^l = f(n)\theta^l - nb^l + \delta\left(p\Pi^{lh} + (1-p)\Pi^l\right)
\end{aligned}$$

Π^{hh} is the payoff if the current state is high and if the state in the previous period was high as well. Then, there are no repayment obligations. If the state is high today but was low in the previous period, the loan has to be paid back. As the firm has no repayment obligations in state l, the payoff there is independent of the state in the previous period.

This gives us the following constraints. First of all, repayment obligations must not exceed the value of the collateral, i.e., a collateral debt (CD) constraint has to be satisfied:

$$A \geq \frac{(1+r_B)L}{p}.$$

Furthermore, an agent's (IC) constraint must hold. This is the same as before with the only difference that now she does not only receive b^l but her share of the loan, $\frac{L}{n}$, in the low state. Thus,

(IC)

$$pb^h + (1-p)(b^l + \frac{L}{n}) - c = 0,$$

where we already take into agent that (IC) optimally binds and thus agents get no rent in equilibrium. The firm's (DE) constraint in the low state equals

(DEl)

$$-nb^l + \delta\left(p\Pi^{lh} + (1-p)\Pi^l\right) - A \geq 0$$

The loan L does not enter (DEl) since it is directly paid out to the agents (however, the repayment obligation is part of Π^{lh}). Note that given (DEl) is satisfied, it is optimal to forward the loan to the agents once it has been taken as long as$-b^l + L - L + \delta\left(p\Pi^{lh} + (1-p)\Pi^{ll}\right) \geq A + L - \delta\frac{(1+r_B)L}{p}$. Since $1 - \delta\frac{(1+r_B)L}{p} \leq 0$, this condition is implied by (DEl).

In the high state of the world, the principal has to repay the loan. The fact that the loan is pledge makes it impossible to default on it, even if the principal reneges on compensating the agents. Thus, the (DEh) constraint in states with a repayment obligation equals $-nb^h - \frac{1+r_B}{p}L + \delta\left(p\Pi^{hh} + (1-p)\Pi^{hl}\right) \geq A - \frac{(1+r_B)}{p}L$. As $-\frac{1+r_B}{p}L$ cancels out, the (DEh) constraint is independent of whether the principal has to repay a loan in the respective period and equals

(DEh)

$$-nb^h + \delta\left(p\Pi^{hh} + (1-p)\Pi^l\right) - A \geq 0$$

Finally, we have the limited liability constraints

(LLl)

$$\frac{f(n)}{n}\theta^l \geq b^l$$

(LLlh)

$$f(n)\theta^h \geq nb^h + \frac{1+r_B}{p}L$$

(LLhh)

$$\frac{f(n)}{n}\theta^h \geq b^h$$

Obviously, we can omit (LLhh) given (LLlh). However, in the following we will further assume that (LLlh) does not bind. This is done for simplicity and unlikely to have a big impact on results. To see this, assume that taking a loan is optimal. Furthermore, assume (LLlh) binds and is the condition determining L. Then, the loan can be increased if repayment not only occurs if the state is high in the first subsequent period, but also if it is high in the second subsequent period as well. However, this would further increase costs, as interest would have to be paid for two periods. Furthermore, this mechanism could be extended to an arbitrary number of subsequent high-demand periods where repayments have to occur. But then, the resulting total limited liability constraint would never bind in equilibrium, since this would imply that profits are zero, which is always dominated by an equilibrium with employment $\overline{n}$ (see Lemma 3). For these reasons, we make the simplifying

Assumption 4: LLlh with potential repayment obligations only in the first subsequent period does not bind in equilibrium

Given Assumption 4, we are safe to assume that repayment can only occur in the first period after the loan has been taken. Finally, we need a constraint stating that a loan must be non-negative (recall that savings are excluded by assumption), i.e.,

(NN)

$$L \geq 0$$

Taken together, we can establish

Proposition 2: *Taking a loan is optimal if and only if the condition*

$$p\theta^h f(\hat{n})' + (1-p)\theta^l f(\hat{n})' - c > (\delta(1+r_B)-1)\left(c-(1-p)\theta^l f(\hat{n})'\right) \tag{4}$$

is satisfied, where $\hat{n}$ *is defined by* (3) *above and denotes the employment level that just makes (ICDEh) binding if no loan is taken.*

Proof: See Appendix

Thus, if condition 4 is satisfied and a loan is taken, equilibrium employment will either be determined by

$$\left[pf(n)'\theta^h + (1-p)f(n)'\theta^l - c\right] + (\delta(1+r_B)-1)\left[(1-p)f(n)'\theta^l - c\right] = 0$$

or – if either (DEl) or (CD) binds – by the respective constraint. In any case, we can state

Proposition 3: *Equilibrium employment is decreasing in* r_B*.*

Proof: See Appendix

The intuition underlying Proposition 3 is straightforward. Optimal employment is aimed at balancing marginal benefits with marginal costs. If a loan is taken and marginal costs increase, equilibrium employment must go down due to the concavity of the production function. If either (DEl) or (CD) binds, marginal benefits of a higher employment are higher than the marginal costs. However, larger interest rates also tighten the constraints, forcing the firm to employ even less agents.

5 Endogenous Asset Value

In this section, we take a closer look at the role of the asset and allow for different investment levels. Until this point, the asset's role in our setting is threefold. It is necessary for production, determines the firm's outside, and can be used as a collateral.

Among these previously made assumptions, the discreteness of the asset's role in the production process can be questioned: If it is used, the production function is determined by $\theta f(n)$. If not, nothing can be produced. In the following, we thus assume that the initial investment, i.e., the value A, can differ and has an impact on output.

More precisely, we adjust the production function to $f(n, A)$, with $f_1 > 0$, $f_2 > 0$, $f_{11} < 0$, $f_{22} < 0$ and $f_{12} \geq 0$. Furthermore, the asset can solely be financed through equity, given to the firm by outsiders who require a return of at least $\frac{1}{\delta}$.[4] There is no further exogenous bound on the capital being raised. These outsiders are passive investors who are also not willing to later lend money to the firm to pay its wage obligations. The costs of investments into the asset are kA, with $k \geq 1$. We furthermore make the assumption that only the whole asset – and not part of it – can be liquidated. Liquidation remains to be a permanent decision.

The total expected ex-ante surplus in this case – before the decision on the level of A – equals $S = \frac{1}{1-\delta}\left[f(n, A)\left(p\theta^h + (1-p)\theta^l\right) - nc\right] - kA$, where we make the simplifying assumption that there is no delay between the point in time when the investment is made and when production can start.

First-best values, n^{FB} and A^{FB}, are thus characterized by

$$f_1\left(p\theta^h + (1-p)\theta^l\right) - c = 0$$

and

$$f_2\left(p\theta^h + (1-p)\theta^l\right) - (1-\delta)k = 0.$$

In addition, the second-order condition must be satisfied, i.e., $\left(p\theta^h + (1-p)\theta^l\right)\begin{pmatrix} f_{11} & f_{12} \\ f_{12} & f_{22} \end{pmatrix}$ must be negative definite, which further requires $f_{11}f_{22} - f_{12}^2 > 0$ and what we subsequently assume.

[4] Even if the manager of the firm has sufficient own funds in the beginning, he might not be able to credibly commit to use it for wages.

Furthermore, we make

Assumption 5: $\left[f(n^{FB}, A^{FB})\left(p\theta^h + (1-p)\theta^l\right) - n^{FB}c\right] - (1-\delta)kA^{FB} > 0$

Assumption 5 states that the firm will be able to raise enough capital for efficient employment and capacity levels.

In the following, we show that even though capacity investments (i.e., the choice regarding the asset's value) are not restricted, the value of A might be chosen inefficiently if first best employment cannot be enforced – even in relation to the enforceable employment level. To see that, we first assume that there is no limited liability constraint (implying that borrowing is not optimal), and only incentive compatibility as well as dynamic enforcement constraints must be satisfied. Then, any employment level can be enforced if and only if

$$nc \leq \delta f(n, A)\left(p\theta^h + (1-p)\theta^l\right) - (1-\delta)A \tag{5}$$

This gives us

Proposition 4: *If* (5) *does not bind,* $n^* = n^{FB}$ *and* $A^* = A^{FB}$*. Otherwise,* n^* *is inefficiently low. If* $k = \frac{1}{\delta}$*,* A^* *is set efficiently* ***in relation*** *to equilibrium employment* n^**. For* $k > \frac{1}{\delta}$*,* A^* *is set inefficiently high and for* $k < \frac{1}{\delta}$*,* A^* *is set inefficiently low,* ***in relation*** *to* n^**.*

Proof: The firm's problem is to $\max\limits_{e,A} \frac{1}{1-\delta}\left[f(n, A)\left(p\theta^h + (1-p)\theta^l\right) - nc\right] - kA$, s.t. (5).

This gives the Lagrange function $LG = \frac{1}{1-\delta}\left[f(n, A)\left(p\theta^h + (1-p)\theta^l\right) - nc\right] - kA + \lambda\left(\delta f(n, A)\left(p\theta^h + (1-p)\theta^l\right) - nc - (1-\delta)A\right)$, with first-order conditions

$\frac{\partial LG}{\partial n} = \frac{1}{1-\delta}\left[f_1\left(p\theta^h + (1-p)\theta^l\right) - c\right] + \lambda\left[\delta f_1\left(p\theta^h + (1-p)\theta^l\right) - c\right] = 0$

$\frac{\partial LG}{\partial A} = \frac{1}{1-\delta} f_2\left(p\theta^h + (1-p)\theta^l\right) - k + \lambda\left[\delta f_2\left(p\theta^h + (1-p)\theta^l\right) - (1-\delta)\right] = 0$

If (5) binds, we have $\lambda > 0$, and $\frac{\partial LG}{\partial A} = 0$ becomes

$f_2\left(p\theta^h + (1-p)\theta^l\right) - (1-\delta)k + \delta(1-\delta)\lambda\left[f_2\left(p\theta^h + (1-p)\theta^l\right) - \frac{(1-\delta)}{\delta}\right] = 0$, which, together with $f_{22} < 0$, proves the proposition.

Q.E.D.

However, we cannot make general statements concerning the absolute value of A^* compared to A^{FB}. Since labor and capital are assumed to be complements, though, an inefficiently low employment level induced by the binding (5) constraint induces the firm to having a lower absolute size. This is consistent with Powell (2011) who shows that contracting frictions restrict the firm size of less productive firms, i.e., those for whom the enforceability constraint is binding.

Thus, an inefficiently high relative level of capacity might be chosen in order to relax (5). If the limited liability constraints bind, this further induces the firm to increase its capacity in relation to equilibrium employment: Since a higher value of A increases output, the funds generated by selling this output will be higher in both states of the world, relaxing (LL) constraints. Thus, it might be optimal to have a capacity levels associated with marginal investment costs higher than its marginal returns.

Proposition 5: *If (LLl) binds, (CD) and (DEl) do not, and* $k + \frac{(1-p)}{\delta p} f_2 \theta^l > \frac{1}{\delta}$*, the value of* A *will generally be inefficiently high in relation to equilibrium employment* n^**. In this case, a higher interest rate* r_B *further increases* A

Proof: See Appendix

6 Conclusion

This paper showed that the negative impact of badly functioning credit market is not just necessarily driven by making initial capital investments more difficult. Instead, the need to use relational contracts to motivate employees in addition to a limited liability constraint the firm faces might restrict firm size. Then, borrowing and overinvestments into physical assets is used to smooth payments made to compensate employees across differing demand conditions, and allow for a larger emplyoment levels. However, this tool will be worth less if borrowing is rather costly.

As next steps, it could be worthwhile to create dynamics by relaxing the assumption that the state of the world of the demand condition is verifiable. In this case, employment will not be constant over time but vary, dependent on past realizations of demand states. Taking savings into account would enable us to address an issue raised by Banerjee and Moll (2009): Why do distortions that restrict growth remain? Since assets can also be accumulated instead of acquired on markets, high returns on these assets should give incentives to accumulate them. Allowing for savings would relax – but never fully solve – the problem in our setup.

Appendix – Omitted Proofs

Proof to Proposition 1: As the surplus is supposed to be fixed as well as θ^l and n, a decrease in p has to be compensatedby an appropriate increase in θ^h. More precisely, taking the total derivative of s, $ds = f(n)\left(dp\theta^h + pd\theta^h - dp\theta^l\right) = 0$, implies $\frac{d\theta^h}{dp} = -\frac{(\theta^h-\theta^l)}{p}$.

Take an arbitrary high-state probability $\overline{p}$ where (ICDEh) is satisfied (however, the latter is not crucial for the argument – if (ICDEh) does not hold for $\overline{p}$, it is also violated for any $p \leq \overline{p}$. This follows from the fact that the right hand side of (2) is increasing in p, which we show below in this proof). For any probability $p^* < \overline{p}$, always counterbalanced by an increase of θ^h that keeps the surplus constant, the right hand side of (ICDEh) equals

$$\frac{\delta}{1-\delta+p^*\delta}\left(p^*\right)^2\left(\theta^h + d\theta^h\right)f(n) + (1-p^*)\theta^l f(n) - \frac{p^*(1-\delta)}{1-\delta+p^*\delta}A$$

$$= \frac{\delta}{1-\delta+p^*\delta}\left(p^*\right)^2\left(\theta^h - (\theta^h-\theta^l)\int_{p^*}^{\overline{p}}\frac{1}{p}dp\right)f(n) + (1-p^*)\theta^l f(n) - \frac{p^*(1-\delta)}{1-\delta+p^*\delta}A$$

$$= \frac{\delta}{1-\delta+p^*\delta}\left(p^*\right)^2\left(\theta^h - (\theta^h-\theta^l)\ln\overline{p} + (\theta^h-\theta^l)\ln p^* + C\right)f(n) + (1-p^*)\theta^l f(n) - \frac{p^*(1-\delta)}{1-\delta+p^*\delta}A$$

For $p^* \to 0$, this becomes $\frac{\delta}{1-\delta(1-p^*)}(\theta^h-\theta^l)\frac{\ln p^*}{\frac{1}{(p^*)^2}}f(n) + \theta^l f(n) = \frac{\delta}{1-\delta}(\theta^h-\theta^l)\frac{(p^*)^2}{-2}f(n) + \theta^l f(n) = \theta^l f(n)$.

Since $n > \overline{n}$ and thus $\theta^l f(n) < nc$, an employment level $n > \overline{n}$ will eventually not be enforceable anymore.

Finally, we have to show that the right hand side of (2) is increasing in p.

$$\frac{d}{dp} = \delta\frac{2p(1-\delta+p\delta)-p^2\delta}{(1-\delta+p\delta)^2}\theta^h f(n) + \frac{p^2\delta}{1-\delta+p\delta}\frac{d\theta^h}{dp}f(n) - \theta^l f(n) - (1-\delta)\frac{(1-\delta)}{(1-\delta+p\delta)^2}A$$

$$= \frac{(1-\delta)}{(1-\delta+p\delta)^2}\left[\delta\left(\theta^h f(n)p + (1-p)\theta^l f(n)\right) - (1-\delta)A - \theta^l f(n)\right].$$

Since $\theta^l f(n) < cn$, this expression is positive as long as n is enforceable if the limited liability constraint is absent. However, if n is not enforceable without the principal's limited liability constraint, (ICDEh) is violated anyway.

Proof to Proposition 2: Note that condition 4 also implies that $n^{FB} > \hat{n}$. If the right hand side of (4) is positive[5], $p\theta^h f(\hat{n})' + (1-p)\theta^l f(\hat{n})' - c$ must be positive as well

[5] Which is always the case if (LLl), $c \geq \theta^l\frac{f(n)}{n}$, binds. Due to the concavity of $f(n)$, $\frac{f(n)}{n} > f(n)'$.

to have the condition satisfied. If $c-(1-p)\theta^l f(\hat{n})' < 0$ and the right hand side is thus negative, the left hand side is positive in any case.

Generally, the firm solves the following problem (where – due to Assumption 4 – we omit the limited liability constraint for the high state):

$$\max_{n,b^h,b^l,L} p\Pi^{hh}(n)+(1-p)\Pi^l(n) = \frac{1}{1-\delta}\left[p\left(f(n)\theta^h - nb^h\right) + (1-p)\left(f(n)\theta^l - nb^l - \delta(1+r_B)L\right)\right]$$

s.t.

(IC) $pb^h + (1-p)(b^l + \frac{L}{n}) - c \geq 0$

(DEl) $-nb^l + \delta\left(p\Pi^{lh} + (1-p)\Pi^l\right) - A \geq 0$

(DEh) $-nb^h + \delta\left(p\Pi^{hh} + (1-p)\Pi^l\right) - A \geq 0$

(LLl) $f(n)\theta^l \geq nb^l$

(CD) $A \geq \frac{(1+r_B)L}{p}$

(NN) $L \geq 0$

As already pointed out, (IC) has to bind. If it does not, b^h can be slightly reduced without violating any other constraint, thereby increasing expected profits. Furthermore, we use $\Pi^{hl} = \Pi^{hh} - \frac{1+r_B}{p}L$ to rewrite (DEl), which becomes $-nb^l + \delta\left(p\Pi^{hh} - (1+r_B)L + (1-p)\Pi^l\right)$ $A \geq 0$, and use $b^h = \frac{c-(1-p)(b^l+\frac{L}{n})}{p}$. Then, we have the Lagrangian

$LG = \frac{1}{1-\delta}\left[pf(n)\theta^h + (1-p)f(n)\theta^l - nc + (1-p)L\left(1-\delta(1+r_B)\right)\right]$

$+\lambda_{DEl}\left[-nb^l - \delta(1+r_B)L + \frac{\delta}{1-\delta}\left[pf(n)\theta^h - nc + (1-p)f(n)\theta^l + (1-p)L\left(1-\delta(1+r_B)\right)\right] - A\right]$

$+\lambda_{DEh}\left[-n\frac{c-(1-p)(b^l+\frac{L}{n})}{p} + \frac{\delta}{1-\delta}\left[pf(n)\theta^h - nc + (1-p)f(n)\theta^l + (1-p)L\left(1-\delta(1+r_B)\right)\right] - A\right]$

$+\lambda_{LLl}\left[f(n)\theta^l - nb^l\right] + \lambda_{CD}\left[A - \frac{(1+r_B)L}{p}\right] + \lambda_{NN}L$, giving first order conditions

$\frac{dLG}{dn} = \frac{1}{1-\delta}\left[pf(n)'\theta^h + (1-p)f(n)'\theta^l - c\right] + \lambda_{DEl}\left[-b^l + \frac{\delta}{1-\delta}\left[pf(n)'\theta^h + (1-p)f(n)'\theta^l - c\right]\right]$

$+\lambda_{DEh}\left[-\frac{c-(1-p)b^l}{p} + \frac{\delta}{1-\delta}\left[pf(n)'\theta^h + (1-p)f(n)'\theta^l - c\right]\right] + \lambda_{LLl}\left[f(n)'\theta^l - b^l\right] = 0$

$\frac{dLG}{db^l} = -\lambda_{DEl} + \lambda_{DEh}\frac{(1-p)}{p} - \lambda_{LLl} = 0$

$\frac{dLG}{dL} = -\frac{1}{1-\delta}(1-p)\left(\delta(1+r_B)-1\right) - \lambda_{DEl}\left[\delta(1+r_B) + \frac{\delta}{1-\delta}(1-p)\left(\delta(1+r_B)-1\right)\right]$

$+\lambda_{DEh}\left[\frac{(1-p)}{p} - \frac{\delta}{1-\delta}(1-p)\left(\delta(1+r_B)-1\right)\right] - \lambda_{CD}\frac{(1+r_B)}{p} + \lambda_{NN} = 0$

$\frac{dLG}{db^l} = 0$ gives that if $\lambda_{LLl} > 0$, λ_{DEh} has to be strictly positive as well. However, that does not imply that if (LLl) binds, the same is true for (DEh). As long as (DEh) does not bind, a binding (LLl) is not costly for the firm. Then, the bound on b^l can be offset by an appropriate increase of b^h.

$\frac{dLG}{dL} = 0$ shows the following. If $\lambda_{DEl} = \lambda_{DEh} = 0$, λ_{NN} must be positive, giving $L = 0$ (note that this also implies that $\lambda_{CD} = 0$). Plugging this into $\frac{dLG}{dn} = 0$ (and noting that in this case, $\lambda_{LLl} = 0$ as well) gives

$\frac{1}{1-\delta}\left[pf(n)'\theta^h + (1-p)f(n)'\theta^l - c\right] = 0$, and $n^* = n^{FB}$.

Now, assume that $\lambda_{NN} = 0$. Then, λ_{DEh} must be positive (note that without $\frac{1}{p} - \frac{\delta}{1-\delta}(\delta(1+r_B)-1) > 0$, taking a loan cannot be optimal). This also implies that if (DEh) binds without a loan, the same is always true for $L > 0$. Thus, taking a loan will only be useful if it helps to increase employment above $\hat{n}$.

Then, we have three possibilities. Either (DEl) binds, or (CD), or none of them (both will generally not bind at the same time).

If none of them binds, we have $\lambda_{LLl} = \lambda_{DEh}\frac{(1-p)}{p}$ and $\lambda_{DEh}\left[\frac{1}{p} - \frac{\delta}{1-\delta}(\delta(1+r_B)-1)\right] = \frac{1}{1-\delta}(\delta(1+r_B)-1)$ to plug into $\frac{dLG}{dn} = 0$

This gives $\frac{1}{1-\delta}\left[pf(n)'\theta^h + (1-p)f(n)'\theta^l - c\right](1+\delta\lambda_{DEh}) + \lambda_{DEh}\frac{1}{p}\left[(1-p)f(n)'\theta^l - c\right] = 0$ or

$$\left[pf(n)'\theta^h + (1-p)f(n)'\theta^l - c\right] + (\delta(1+r_B)-1)\left[(1-p)f(n)'\theta^l - c\right] = 0. \quad (6)$$

Thus, 4 is satisfied.

If (DEl) or (CD) binds, we thus must have $\left[pf(n)'\theta^h + (1-p)f(n)'\theta^l - c\right] - (\delta(1+r_B)-1)\left[c - (1-p\right.$ 0: If no constraint binds, this is satisfied as an equality. Due to the concavity of $f(n)$, a binding constraint will imply a lower employment than if this constraint does not bind.

Thus, a positive loan is associated with

$$\left[pf(n)'\theta^h + (1-p)f(n)'\theta^l - c\right] + (\delta(1+r_B)-1)\left[(1-p)f(n)'\theta^l - c\right] \geq 0,$$

with a strict inequality if either (DEl) or (CD) binds.

Proof to Proposition 3: If no loan is taken, which is always the case if 4 is not satisfied, i.e., either if we already have $n^{FB} \leq \hat{n}$ or if taking a loan is too expensive at $\hat{n}$, equilibrium employment is $n^* = \min\{n^{FB}, \hat{n}\}$, and a higher r_B has no impact on n^*.

If a loan is taken, equilibrium employment is either determined by

1. The condition $\left(p\theta^h f(n)' + (1-p)\theta^l f(n)' - c\right) = (\delta(1+r_B)-1)\left(c - (1-p)\theta^l f(n)'\right)$
2. The binding (DEl) constraint $-nb^l + \delta\left(p\Pi^{lh} + (1-p)\Pi^l\right) - A = 0$, or
3. The binding (CD) constraint $A = \frac{(1+r_B)L}{p}$,

depending on what determines a lower value of n.

In the first case, $\frac{dn}{d(1+r_B)} = \frac{\delta\left(c-(1-p)\theta^l f(n)'\right)}{\left(p\theta^h f(n)''+(1-p)\theta^l f(n)''\right)+(\delta(1+r_B)-1)(1-p)\theta^l f(n)''} < 0$.

In the second case, $\frac{dn}{d(1+r_B)} = -\frac{\frac{\partial(DEl)}{\partial(1+r_B)}}{\frac{\partial(DEl)}{\partial n}}$. $\frac{\partial(DEl)}{\partial n}$ has to be negative since otherwise, a higher employment would relax (DEl). Thus, $\text{sgn}\frac{dn}{d(1+r_B)} = \text{sgn}\frac{\partial(DEl)}{\partial(1+r_B)}$. We can simplify (DEl) by nothing that (DEh) binds as well if a strictly positive loan is taken. Taking (DEl), $-nb^l + \delta\left(p\Pi^{lh} + (1-p)\Pi^l\right) - A = 0$ and (DEh), $-nb^h + \delta\left(p\Pi^{hh} + (1-p)\Pi^l\right) - A = 0$, as well as $\Pi^{hl} = \Pi^{hh} - \frac{1+r_B}{p}L$, we have $nb^l + \delta(1+r_B)L = nb^h$. Furthermore, using $b^h = \frac{c-(1-p)(b^l+\frac{L}{n})}{p}$ (binding (IC)) and $b^l = \frac{f(n)}{n}\theta^l$ (binding (LLl)) gives $\frac{nc}{p} - f(n)\theta^l\frac{1}{p} - \frac{(1-p)}{p}L - \delta(1+r_B)L = 0$. Thus, $\frac{\partial(DEl)}{\partial(1+r_B)} = -\frac{(1-p)}{p}\frac{\partial L}{\partial(1+r_B)} - \delta L - \delta(1+r_B)\frac{\partial L}{\partial(1+r_B)}$.

Furthermore, from binding (IC) and (DEh), we get

$L = \frac{(1-\delta+p\delta)\left(nc-(1-p)f(n)\theta^l\right)-\delta p^2 f(n)\theta^h+p(1-\delta)A}{(1-p)(1-\delta+\delta p(1-\delta(1+r_B)))}$ (> 0 as (DEh) binds). Thus,

$\frac{\partial L}{\partial(1+r_B)} = \frac{(1-\delta+p\delta)\left(nc-(1-p)f(n)\theta^l\right)-\delta p^2 f(n)\theta^h+p(1-\delta)A}{(1-p)(1-\delta+\delta p(1-\delta(1+r_B)))^2}\delta^2 p = \frac{\delta^2 p}{1-\delta+\delta p(1-\delta(1+r_B))}L > 0$, completing the proof that $\frac{\partial(DEl)}{\partial(1+r_B)} < 0$.

If (CD) binds, n^* is determined by $A - \frac{(1+r_B)L}{p} = 0$. Due to the same reasons as above, $\text{sgn}\frac{dn}{d(1+r_B)} = \text{sgn}\frac{\partial(CD)}{\partial(1+r_B)}$. This gives us $\frac{\partial(CD)}{\partial(1+r_B)} = -\frac{L}{p} - \frac{(1+r_B)}{p}\frac{\partial L}{\partial(1+r_B)} < 0$.

Addition to Proposition 5: Absence of Credit Market:

$LG = \frac{1}{1-\delta}\left[pf(n,A)\theta^h + (1-p)f(n,A)\theta^l - nc\right] - kA$

$+\lambda_{DEl}\left[-nb^l + \frac{\delta}{1-\delta}\left[pf(n,A)\theta^h - nc + (1-p)f(n,A)\theta^l\right] - A\right]$

$+\lambda_{DEh}\left[-n\frac{c-(1-p)b^l}{p} + \frac{\delta}{1-\delta}\left[pf(n,A)\theta^h - nc + (1-p)f(n,A)\theta^l\right] - A\right] + \lambda_{LLl}\left[f(n,A)\theta^l - nb^l\right]$

$\frac{\partial LG}{\partial b^l} = -\lambda_{DEl} + \lambda_{DEh}\frac{(1-p)}{p} - \lambda_{LLl} = 0$

$\frac{\partial LG}{\partial n} = \frac{1}{1-\delta}\left[pf_1\theta^h + (1-p)f_2\theta^l - c\right]$

$+\lambda_{DEl}\left[-b^l + \frac{\delta}{1-\delta}\left[pf_1\theta^h - c + (1-p)f_1\theta^l\right]\right]$

$+\lambda_{DEh}\left[-\frac{c-(1-p)b^l}{p} + \frac{\delta}{1-\delta}\left[pf_1\theta^h - c + (1-p)f_1\theta^l\right]\right] + \lambda_{LLl}\left[f_1\theta^l - b^l\right]$

$\frac{\partial LG}{\partial A} = \frac{1}{1-\delta}\left[pf_2\theta^h + (1-p)f_2\theta^l\right] - k$

$+\lambda_{DEl}\left[\frac{\delta}{1-\delta}\left[pf_2\theta^h + (1-p)f_2\theta^l\right] - 1\right]$

$+\lambda_{DEh}\left[\frac{\delta}{1-\delta}\left[pf_2\theta^h + (1-p)f_2\theta^l\right] - 1\right] + \lambda_{LLl}f_2\theta^l = 0$

Slack (DEl), binding (DEh) and (LLl):

$\lambda_{DEh}\frac{(1-p)}{p} - \lambda_{LLl} = 0$

$\frac{1}{1-\delta}\left[pf_1\theta^h + (1-p)f_1\theta^l - c\right] + \lambda_{DEh}\left[\frac{\delta}{1-\delta}\left[pf_1\theta^h - c + (1-p)f_1\theta^l\right] - c\right] + \lambda_{DEh}\frac{(1-p)}{p}f_1\theta^l = 0$

$\frac{\partial LG}{\partial A} = \left[pf_2\theta^h + (1-p)f_2\theta^l\right] - (1-\delta)k + \lambda_{DEh}\left[\delta\left[pf_2\theta^h + (1-p)f_2\theta^l\right] - (1-\delta)\right] + \lambda_{DEh}(1-\delta)\frac{(1-p)}{p}f_2\theta^l = 0$

Proof to Proposition 5: The problem we want to solve is identical to the general case above, just that we also have to consider that A is endogenous as well. This gives the Lagrange function

$LG = \frac{1}{1-\delta}\left[pf(n,A)\theta^h + (1-p)f(n,A)\theta^l - nc + (1-p)L\left(1-\delta(1+r_B)\right)\right] - kA$
$+\lambda_{DEl}\left[-nb^l - \delta(1+r_B)L + \frac{\delta}{1-\delta}\left[pf(n,A)\theta^h - nc + (1-p)f(n,A)\theta^l + (1-p)L\left(1-\delta(1+r_B)\right)\right] -\right.$
$+\lambda_{DEh}\left[-n\frac{c-(1-p)(b^l+\frac{L}{n})}{p} + \frac{\delta}{1-\delta}\left[pf(n,A)\theta^h - nc + (1-p)f(n,A)\theta^l + (1-p)L\left(1-\delta(1+r_B)\right)\right] - A\right]$
$+\lambda_{LLl}\left[f(n,A)\theta^l - nb^l\right] + \lambda_{CD}\left[A - \frac{(1+r_B)L}{p}\right] + \lambda_{NN}L$, with first-order conditions
$\frac{\partial LG}{\partial n} = \frac{1}{1-\delta}\left[pf_1\theta^h + (1-p)f_1\theta^l - c\right] + \lambda_{DEl}\left[-b^l + \frac{\delta}{1-\delta}\left[pf_1\theta^h + (1-p)f_1\theta^l - c\right]\right]$
$+\lambda_{DEh}\left[-\frac{c-(1-p)b^l}{p} + \frac{\delta}{1-\delta}\left[pf_1\theta^h + (1-p)f_1\theta^l - c\right]\right] + \lambda_{LLl}\left[f_1\theta^l - b^l\right] = 0$
$\frac{\partial LG}{\partial b^l} = -\lambda_{DEl} + \lambda_{DEh}\frac{(1-p)}{p} - \lambda_{LLl} = 0$
$\frac{\partial LG}{\partial L} = -\frac{1}{1-\delta}(1-p)\left(\delta(1+r_B)-1\right) - \lambda_{DEl}\left[\delta(1+r_B) + \frac{\delta}{1-\delta}(1-p)\left(\delta(1+r_B)-1\right)\right]$
$+\lambda_{DEh}\left[\frac{(1-p)}{p} - \frac{\delta}{1-\delta}(1-p)\left(\delta(1+r_B)-1\right)\right] - \lambda_{CD}\frac{(1+r_B)}{p} + \lambda_{NN} = 0$
$\frac{\partial LG}{\partial A} = pf_2\theta^h + (1-p)f_2\theta^l - (1-\delta)k + \delta\left[pf_2\theta^h + (1-p)f_2\theta^l - \frac{(1-\delta)}{\delta}\right](\lambda_{DEl} + \lambda_{DEh}) +$
$(1-\delta)\lambda_{LLl}f_2\theta^l + (1-\delta)\lambda_{CD} = 0$

As before – when the asset value was given exogenously – taking a loan can only be optimal if (DEh) binds.

Thus, let us assume that (DEh) and (LLl) bind.

If (DEl) and (CD) are slack, equilibrium employment is determined by $\left[pf_1\theta^h + (1-p)f_1\theta^l - c\right] + (\delta(1+r_B)-1)\left[(1-p)f_1\theta^l - c\right] = 0$. Using $\lambda_{LLl} = \lambda_{DEh}\frac{(1-p)}{p}$ $\lambda_{DEh} = \frac{1}{1-\delta}\frac{(\delta(1+r_B)-1)}{\left[\frac{1}{p} - \frac{\delta}{1-\delta}(\delta(1+r_B)-1)\right]}$ gives equilibrium capacity

$$\left(pf_2\theta^h + (1-p)f_2\theta^l - (1-\delta)k\right) + p\left(\delta(1+r_B)-1\right)\delta\left(k - \frac{1}{\delta} + \frac{(1-p)}{\delta p}f_2\theta^l\right) = 0.$$

Thus, equilibrium investments are inefficiently high in relation to employment if $k + \frac{(1-p)}{\delta p}f_2\theta^l > \frac{1}{\delta}$. Furthermore, $\frac{dA}{d(1+r_B)} = -\frac{\delta^2\left(k-\frac{1}{\delta}+\frac{(1-p)}{\delta p}f_2\theta^l\right)}{\left(pf_{22}\theta^h+(1-p)f_{22}\theta^l\right)\frac{1}{p}+(\delta(1+r_B)-1)\left(\frac{(1-p)}{p}f_{22}\theta^l\right)}$, where the denominator is negative.

If (CD) and (DEl) bind, the asset choice is also determined by whether a higher asset relaxes the relevant constraints, which might or might not be the case.

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