Mitigating Hold-up Through Complementarities and Refundability

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Abstract: Many valuable composite goods exist only by assembling multiple, monopoly-supplied component goods. Since a monopolist exclusively owns each component, hold-up by the last seller can result. First I design a model to show how sunk cost explains this assembly problem. Then I consider two factors that reduce sunk cost, determining conditions for which each mitigates hold-up. Specifically, I implement varied degrees of complementarities and levels of refundability of the component goods. No known previous research examines the assembly problem in terms of sunk cost or considers refundability, and little research models imperfect complementarities. I show if at least the first component purchased has stand alone value, hold-up is mitigated under any refund level. However, if only the last component purchased has stand alone value, averting hold-up depends on capacity constraints, degree of complementarities, price discrimination, and level of refund. Regardless of the degree of complementarities, full refunds prevent hold-up, while zero and partial refunds do not. Welfare analysis reveals conditions for which a first or second mover advantage exists and when sellers prefer a partial or full refund. This research offers settings when degrees of complementarity and refundability reduce sunk cost, often enough to prevent hold-up. My results also suggest policies to mitigate inefficient outcomes in assembly problems, such as legal requirements on full refunds, regulation on the order in which components must be purchased, and prohibition of price discrimination.

Keywords: anticommons, complementary monopoly, complementarities, refund, hold-up, double marginalization, property rights.

JEL Classification: D4, D70, K11, K19, L1

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1 Introduction

Consider a market structure where the assembly of multiple component goods creates a welfare-improving composite good. The components are complementary: only the combination of these multiple components creates the composite. Also, no substitute exists for any component. Each component is monopoly-supplied, and the monopolists do not collude. I refer to this market for the composite good as a "fragmented market" in which the potential market failure creates the "assembly problem."

Generally, the component and composite goods are bundles of rights, or properties, such as any physical good, creative work, or intangible right to use. Examples of a fragmented market setting include a developer trying to assemble the real property of multiple, distinct land owners; shared owners of a single piece of property (such as heirs), each with veto rights to exclude; or an innovator whose success depends on acquiring multiple licenses from unique patent holders.

The most common terminology used to analyze these fragmented markets is *complementary monopoly* (e.g. Sonnenschein (1968), Machlup and Taber (1960)), *complementary oligopoly* (e.g. Salinger (1989), Parisi and Depoorter (2003), and Dari-Mattiacci and Parisi (2006)), *anticommons property* (e.g. Heller (1998) and Buchanan and Yoon (2000)), and *gridlock* (Heller (2008)). Although the terminology varies by field of study and types of rights considered, the potential market failure is well-known. Because each component is controlled by separate monopolists, each supplier does not internalize the total benefit from the composite good. The fragmentation of rights can result in a total equilibrium price for the composite good higher than even that chosen by a vertically integrated monopolist, yielding suboptimal or no acquisition of the composite good. This inefficient outcome has been referred to as the tragedy of the anticommons, double marginalization, or hold-up.

I seek to understand the underlying mechanism driving the inefficient outcome and how to alter that mechanism so hold-up might be mitigated. In particular, I model this assembly problem in terms of sunk cost. Recognizing how sunk cost drives the inefficient outcome, I then analyze two factors that reduce sunk cost as solutions to overcome hold-up. The first factor, imperfect complementarities of component goods, corresponds to a component's residual benefit from the production technology. Second, refundability, corresponds to an alteration in the contract structure through the return and refund of a previously purchased component. These two factors reveal aspects of the problem not fully characterized in the literature, which itself has not explicitly recognized the role of sunk costs. Using the subgame perfection solution concept, I consider twelve combinations of imperfect complementarities and refundability in a sequential game of two sellers and one representative buyer.

My analysis generalizes a common assumption in models of fragmented markets that component goods are perfectly complementary. In this setting, perfect complementarities means no residual benefit exists in the production technology of any component. Therefore, if components are perfectly complementary, no component has value except in combination with the other required components. If no single component has value in use alone, then once purchased, the full cost of that component is sunk. Knowing the cost of every previously-purchased component is sunk, the last seller rationally responds by pricing without regard to the other components' prices. This results in a total equilibrium price so high that an inefficient quantity of composites – even fewer than sold by a single monopolist – is purchased in equilibrium.

However, monopoly-supplied components are not necessarily perfectly complementary. A land developer desiring to assemble multiple parcels of property might value one of the required parcels even if he is unable to assemble the property via acquisition of all other parcels. Therefore, a natural next question is whether *imperfectly* complementary components overcome the inefficient effects of sunk cost. An imperfectly complementary component has *stand alone value*, or value to the buyer other than in use to create the composite good. Because a component with stand alone value has outside use to the buyer, then the sunk cost to purchasing that component is reduced by that stand alone value. When complementarities are imperfect, the last seller must account for the possibility that potential buyers can use previously purchased components, even without the last seller's good. This fact puts downward pressure on the last seller's optimal price.

In my model, the composite good requires two components. I proceed by varying the degree of complementarity of each of these two components. If at least the first component supplied is imperfectly complementary to the second component supplied, no anticommons tragedy results. Consider the land developer seeking to assemble two distinctly-owned parcels of land, at least one of which can be resold or has use besides creating the composite parcel. As long as the developer purchases a parcel with positive stand alone value first, the last seller has no ability to hold-up the developer. Note that components are purchased in either a predefined or undefined order. This result also suggests that hold-up in fragmented markets can be induced if a regulation requires a particular order of assembling rights. If the order of acquiring rights optimally is chosen by the buyer or optimally is determined by a social planner, hold-up can be mitigated.

On the other hand, the last component purchased might be the only component with stand alone value. In this case, sunk cost remains full, as in the case of perfect complementarities. The last seller still holds up the buyer, and therefore, stand alone value in the last component does nothing to mitigate the anticommons tragedy. This intuitive result is unambiguous in a model where the capacity of the last component is constrained to the exact proportion required to create the composite good. For example, in land or permit acquisitions, capacity for each component is restricted to one unit, where the buyer chooses quantity one (buy) or quantity zero (not buy).

However, the last seller might be capable of supplying units of the component beyond that required to create the composite good. I show hold-up no longer necessarily results if (i) the last seller can supply a quantity beyond that required to create the composite and (ii) the buyer values that excess quantity in use alone. In this case, the degree of stand alone value, precisely defined below, determines whether hold-up results. Pricing incentives change when the last seller supplies not only the required number of units to create the final good, but also additional units for stand alone use. First, if the last component has low stand alone value, full hold-up results. Moreover, if the last seller can price discriminate between the good used alone and the good used to assemble the composite, hold-up results, regardless of the last component's degree of stand alone value. However, if the last component has high stand alone value and sellers cannot price discriminate, the anticommons tragedy is mitigated.

These outcomes result when the buyer must purchase the components in a particular order: the first component has no stand alone value and the last component has positive stand alone value. Therefore, even when a natural order to acquiring rights exists and cannot be changed, and even if the last component has stand alone value, hold-up does not necessarily result; without capacity constraints, high stand alone value in the last component mitigates hold-up, provided the last seller does not price discriminate.

Another factor that reduces sunk cost is a refund for previously purchased components. With a partial or full refund, the buyer has the added option of returning the first component. To my knowledge, no previous research considers refundability. If the first component has imperfect complementarities, the sunk cost from purchasing that component is reduced by that component's stand alone value. Refunds, on the other hand, reduce sunk cost by the amount of the recovered purchase price. While both factors reduce sunk cost, they alter the buyer's payoffs differently. With imperfect complementarities in the first component, the sunk cost can be fully eliminated, depending on the first seller's best response to the last seller's optimal behavior. With refunds, sunk cost is fully eliminated only if the refund is full, regardless of the first seller's optimal price.

From the last seller's perspective, when a positive refund is offered, optimal behavior might require not only ensuring the buyer purchases the last component, but also ensuring the buyer does not return the first component. However, the last seller knows if the first component is returned and the refund is not one hundred percent, the buyer's surplus is negative, because sunk cost remains. Therefore, if the refund is only partial or zero, then the reduction in sunk costs is not enough to give the last seller incentive to account for the effects of a refund. With a zero or partial refund, unless the first component also has stand alone value, full hold-up results with no purchase of the composite good in equilibrium. However, a full refund completely eliminates sunk cost, forcing the last seller to price low enough so the buyer purchases the last component and does not return the first component. Therefore, a full refund ensures that hold-up by the last seller is not privately optimal, guaranteeing a Pareto improving outcome.

My analysis also includes joining the factors of imperfect complementarities and refundability. When the first component has zero stand alone value, which corresponds to the case of perfect complementarities or the case of imperfect complementarities in the last component, I show only a full refund mitigates hold-up (assuming transaction costs are low enough). Recall, stand alone value in the first component mitigates hold-up. Therefore, allowing for refundability does not alter the outcome. However, I provide a welfare analysis, endogenizing the refund level, as well as the order of play, to show how equilibrium payoffs are affected by the level of refund. I show the first seller does not necessarily prefer a full refund. Also, depending on the degree of stand alone value, a first-mover or second-mover advantage exists, suggesting the last seller in fragmented markets does not always receive the highest equilibrium payoff.

My model reveals the role of sunk cost in assembly problems created by anticommons property or complementary monopoly. The results imply situations where complementarities or refundability can and cannot prevent hold-up. One policy response to market power in such settings might include divestiture of an anticompetitive (or potentially anticompetitive) firm. However, fragmenting a market by forcing sell-offs or spin-offs of a vertically-integrated firm into multiple entities might result in a total composite price exceeding the original firm's price, even when it is already the anticompetitive monopoly price. Therefore, the resulting tragedy of the anticommons could yield a decrease in social welfare. Policy prescriptions accounting for complementarities and refundability in such situations might avert a further decrease in efficiency. My results suggest policies, such as legal requirements on full refunds, regulation on order in which components must be purchased, or prohibition of price discrimination, to mitigate inefficient outcomes in assembly problems.

1.1 Related Literature

Many have considered markets for a composite good that cannot exist without assembling multiple, monopoly-supplied components. My contribution rests in motivating the assembly problem through sunk cost, then considering how complementarities and refundability affect sunk cost in a way that might mitigate hold-up. While some bargaining models of hold-up, such as Carmichael and MacLeod (2003) and Gul (2001), include discussion of sunk cost, to my knowledge, no models of the assembly problem explicitly model sunk costs.[1]

The seminal analyses of assembly problems in fragmented markets are due to Cournot (1838) and Ellet (1839). Cournot's examination, most often referred to as complementary monopoly, initially assumes perfectly complementary, monopoly-supplied components with zero costs to produce the components and composite. Assuming simultaneous interaction between monopolists, Cournot concludes that the equilibrium total price is greater than the price charged by a single monopolist. He also shows that as the number of components required to assemble the composite good increases, so does the difference between this equilibrium total price and the vertically integrated price. Sonnenschein (1968) takes insight from Edgeworth (1925) to show how Cournot's theory of complementary monopoly is the dual of his more well known theory of duopoly.

While the complementary monopoly model in Cournot (1838) chooses prices simultaneously, Spengler (1950) considers a similar problem in a sequential setting, *a la* Stackelberg (1934). In both frameworks, the price for the composite increases with the number of required components. In fact, Feinberg and Kamien (2001) show in a game of perfect and complete information an outcome of hold-up is the analog result to double marginalization under a game of imperfect information. Thus, while hold-up results under sequential play, double marginalization results under simultaneous play. In both, the tragedy of the anticommons is the same; the quantity (price) of the composite good purchased in equilibrium is lower (higher) than under vertical integration.

The second seminal work on the assembly problem, Ellet (1839), reaches the same conclusion as Cournot (1838) with respect to total price and welfare. However, Ellet's motivation is that of trade (and tolls, related thereto). Acquiring permits to assemble a composite good motivates many analyses of this market structure. Feinberg and Kamien (2001) and Gardner, Gaston, and Masson (August 2002) consider the assembly of permits purchased from successive monopolists in the form of a toll. In this case, the composite good is a destination or privilege. This perspective provides a clear transition from consumption goods to those providing a "right to use" (such as a permit to traverse real property).

Ellickson (1993) and Fennell (2010) credit the inception of the term *anticommons* to Frank Michelman. Michelman (1982) describes the problem as the converse of a commons problem, calling it a property regime in which everyone has the right to block use and no

[1] Llanes and Trento (2009) incorporate a fixed (sunk) cost in the production of components in their model of optimal patent policy, while in my model, the consumer's sunk cost drives the assembly problem.

one has the exclusive privilege of use.[2]

Heller (1998) brought this assembly problem to greater attention among legal scholars. Buchanan and Yoon (2000) follow up on Heller's account to show the symmetry between the tragedies of the commons and anticommons. Though a less general model than Cournot's, they also show total equilibrium price increases with the number of required components.

Refundability plays an important role in my analysis as the second factor I consider to reduce sunk cost and possibly mitigate hold-up. Buchanan and Yoon (2000) state a full refund is allowed; however, refunds play no role in their analysis. To my knowledge, no research considers refunds in the assembly problem.

Most theoretical and experimental models of complementary monopoly and anticommons property assume perfect complementarities among the component goods (e.g. Cournot (1838), Buchanan and Yoon (2000), Feinberg and Kamien (2001)). I show that relaxing this assumption, allowing for degrees of complementarity, can alter the equilibrium outcome in a Pareto improving way. Several papers, including Feinberg and Kamien (2001), suggest imperfect complementarities should be considered. Parisi and Depoorter (2003) and Parisi, Depoorter, and Schulz (2005) consider imperfect complementarities, but with neither the same motivation nor objective as my model. Cournot (1838) considers the possibility of relaxing the perfect complementarities assumption, but he provides no analysis after concluding the model is too complicated to determine general results. In my consideration of imperfect complementarities and refunds in the assembly problem, I use a sequential model with discrete quantities, continuous price, two sellers, and one buyer, allowing me to derive enlightening results from a straightforward model.

2 General framework

Two monopolists, A and B, know a single buyer C finds value in combining component rights a and b to form composite right c.

For $J \in \{A, B\}$ and $j \in \{a, b\}$, seller J chooses price p_j at which to supply quantity $q_j \in \{0, 1\}$ of component j.[3] Let $p_c = p_a + p_b$ be the composite good's total price when components are supplied separately by monopolists A and B. While each seller J offers take-it-or-leave-it price p_j for component j, buyer C chooses to purchase ($q_j = 1$) or not purchase ($q_j = 0$) each component.

[2]Michelman (1982) defines *right* as "others are legally required to leave the object alone save as the owner may permit" and defines privilege as "the owner is legally free to do with the object as he or she wills."

[3]Later I consider $q_a \in \{0, 1\}$ but $q_b \in \{0, 1, 2\}$.

2.1 Information and timing

Players A, B, and C know all potential payoffs and all previous moves taken in the game. The order of play is fixed; seller A always moves first, and buyer C immediately follows each seller. A welfare analysis in Section 6 evaluates how order of play affects the anticommons outcome.

Fixed order of sequential play is not far-fetched. Geographical aspects of the market might naturally result in exogenously determined order. For example, a transportation system composed of multiple links joined by toll booths requires a buyer to purchase a pass at each link in order to proceed towards the destination. The composite good is the overall right to reach the destination. A fisherman might have his sights set on fishing from a lake high in the mountains. Such rights-seeker must obtain not only a permit to fish, but also permits to traverse private and/or government owned property. Buyers and sellers know a fishing license first must be obtained, and the geographical nature of getting to the fishing hole results in the exogenously determined process of obtaining the remaining permits to create the composite good, the right to fish at that lake.

The model rests on the assumption that sellers have the power in trade, where a take-it-or-leave it price is offered, and the buyer only can accept or reject an offer. A model of sellers' take-it-or-leave-it offers allows for examination under the most extreme outcomes of anticommons. This offers the most robust analysis of the role of refunds and stand alone value in mitigating hold-up.

If the buyer had the power of trade, rather than sellers, then trade occurs only if the buyer knows the sellers' valuation of their components. In this case, efficient trade results, and clearly, hold-up cannot occur.

A third mechanism of trade is when neither party has the power in trade and bargaining is possible. Under a bargaining model with fully transparent information and perfect complementarities, an outcome of hold-up depends on the split of cooperative surplus between the seller and the buyer at each stage of bargaining. Even under bargaining, an assumption of rationality results in all players accounting for sunk cost, thereby disregarding the purchase price of component a in the bargain between seller B and buyer C. Even if buyer C captures some of the gains to trade in the bargain with B, if his surplus from his bargain with A yields a negative surplus in sum, then hold-up results. However, if buyer C's bargaining power at both bargaining stages is high enough, hold-up might not result, due only to the buyer's bargaining power, not necessarily other factors such as refunds and stand alone value.

Therefore, a bargaining model is more involved than a model of take-it-or-leave it offers because outcomes depend on the interaction of bargaining power with refunds and/or stand alone value. The goal of this chapter is to show how refunds and stand-alone-value overcome outcomes of anticommons. A sequential model of perfect and complete information with sellers offering take-it-or-leave-it prices provides the best framework with which to highlight the impact of refunds and stand alone value.

2.2 Parameters and assumptions

Next, I explicitly define and distinguish the model's exogenous parameters, refunds and complementarities. Refunds alter the market structure for composite c, while complementarities are inherent to the production technology of a component. My analysis relies on varying the values of these parameters.

Any seller other than the last may offer a refund. Therefore, let $\gamma \in [0, 1]$ be the fraction of purchase price p_a seller A offers as a refund to buyer C. Regardless of whether components are complementary, a refund may be offered.

Definition 1 (refund) ***No refund*** *corresponds to level* $\gamma = 0$*,* ***partial refund*** *corresponds to level* $0 < \gamma < 1$*, while a* ***full refund*** *corresponds to level* $\gamma = 1$*.*

The degree of complementarity is expressed by buyer C's willingness-to-pay for stand alone units of each component. Buyer C's willingness-to-pay for composite c and stand alone units of a and b is $\omega_c > 0$, $\omega_a \geq 0$ and $\omega_b \geq 0$, respectively. I assume buyer C's willingness-to-pay for the composite good is higher than the willingness-to-pay for both component goods separately. This focuses the analysis to that of a fragmented market for composite c.

Assumption 1 (Preference for composite) *Buyer C prefers composite c to components a and b separately:* $\omega_c > (\omega_a + \omega_b) \geq 0$*.*

A component might have value only in use with other components to form the composite good. Suppose component a has value to buyer C only to form composite c. In this case, component a is a perfect complement to component b, even though b may or may not be perfect complements with a.

Definition 2 (perfectly complementary) *The use for composite c is* ***perfectly complementary*** *if all components required to form the composite have no value to the buyer in use alone.*

A component might have value not only in use to create composite c, but also in use alone. For example, although component a is required to form composite c, component a also might have outside value to the buyer. In this case, even if the buyer cannot form composite good c, an outside use for component a exists.

Definition 3 (stand alone value, imperfectly complementary) *For* $j \in \{a, b\}$*, a component has value beyond its use to create the composite, or* ***stand alone value*** (SAV_j)*, if* $\omega_j > 0$*. The use for composite c is* ***imperfectly complementary*** *if at least one component j has stand alone value.*

With two sellers and one representative buyer, the dynamic game of perfect and complete information is composed of four stages, generally outlined in Figure 1.

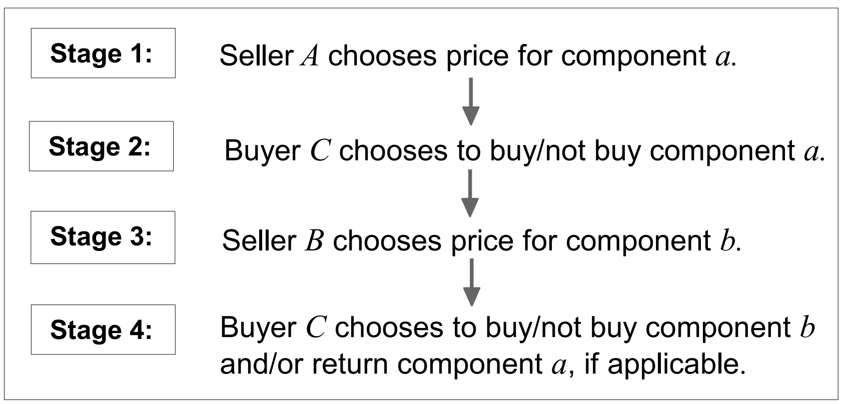

Figure 1: Game Sequence

My analysis uses the subgame perfect equilibrium concept, solving via backward induction. A subgame perfect equilibrium always exists because the game is finite with perfect information. Some games in my model result in multiple equilibria. However, for each game, the outcome is unique with respect to whether an anticommons tragedy results.

I make two equilibrium selection assumptions regarding indifference. These assumptions focus the equilibrium outcomes on those concerning an anticommons tragedy. First, each seller, A and B, incurs a positive cost to supplying any positive quantity of their respective component. These costs could be production costs if components are physical goods, or transaction costs to trading rights. In either case, I call these costs transaction costs.[4] This assumption leaves seller A not indifferent when optimally choosing prices.

[4]For simplicity, I assume fixed costs rather than marginal costs. Given that most of the analysis restricts capacity to exact proportions, fixed and marginal costs of supplying that unit are equivalent. Once I remove the capacity constraint in Section 4.2.1, fixed costs still simplify the analysis; however, I indicate how marginal costs alter the outcome, requiring marginal cost be less than marginal benefit.

Assumption 2 (Transaction Costs) *The cost of supplying any positive quantity of component j is $\epsilon_j > 0$. Furthermore, (i) if either component has positive stand alone value, $\omega_j > 0$, assume $\omega_j > \epsilon_j > 0$ and (ii) if either component has no stand alone value, $\omega_j = 0$, assume $\omega_c - \omega_{\not{j}} > \epsilon_j > 0$ for each $j \in \{a, b\}$.*

Assumption 2 ensures that any outcome in which the composite good is not purchased is due to the anticommons tragedy and not high transaction costs. When transaction costs are positive, each seller must ensure his optimal price yields non-negative payoffs, given the optimal behavior of the other two players. Thus, each seller must account for his positive transaction costs. Parts (i) and (ii) of Assumption 2 provide additional requirements on transaction costs to ensure supplying the component is optimal for a seller.

Assumption 2 part (i) requires that supplying one unit costs less than the buyer's willingness-to-pay for a stand alone unit of that component. This ensures that supplying a component is optimal whenever a component has positive stand alone value. However, when monopolist J's component has no stand alone value, Assumption 2 part (ii) ensures monopolist J optimally supplies his component, j, if the stand alone value of the other component, $\not{j}$, is not too high. This is because the degree of stand alone value of the other component, $\not{j}$, might determine how much surplus supplier J must leave on the table to ensure the buyer purchases his component, j.[5]

Additionally, I assume if the buyer is left indifferent between more than one action, then the indifferent buyer chooses the action that results in the greatest purchased quantity of the component.

Assumption 3 (Indifference Rule) *An indifferent buyer chooses to purchase the maximum possible units of the good.*

In sum, in this two-seller, one-buyer model, composite c is formed from one of the following three structures of components a and b: (i) two perfectly complementary components; (ii) two imperfectly complementary components; or (iii) a hybrid of complementarities. This amounts to four cases:

1. perfect complementarities market for c: $\omega_a = 0$, $\omega_b = 0$ (Baseline case);

2. stand alone value in only component a (SAV_a): $\omega_a > 0$, $\omega_b = 0$;

3. stand alone value in only component b (SAV_b): $\omega_b > 0$, $\omega_a = 0$; and

[5]Proofs of each Lemma, Proposition and Corollary, provided in Section 8: Appendix A, state the exact Assumption 2 conditions required in solving for the equilibrium.

4. stand alone value in both components a and b ($SAV_{a,b}$): $\omega_a > 0$, $\omega_b > 0$.

Each case is analyzed under the three levels of refund (no, partial, and full refunds), leaving a total of twelve cases I consider.

A complete description of the game includes the strategy set for each player. Accounting for refundability, the strategies for each player at each decision node are given by:

$$S_A = p_a \geq 0$$
$$S_B = \{p_{b0} \geq 0, p_{b1} \geq 0\}$$
$$S_C = \{q_a \in \{0,1\}, q_{b0} \in \{0,1\}, q_{b1} \in \{0, 0_{\not{a}}, 1_c, 1_{a,b}, 1_{\not{a},b}\}\}$$

For buyer C's strategy S_C, the first parenthesized term represents action to not buy/buy a; the second parenthesized terms represents action to not buy/buy b if $q_a = 0$; and the third parenthesized terms represents action to not buy/buy b with an option to return a (noted $\not{a}$) when $q_a = 1$. Figure 2 provides a description of the notation used throughout my analysis and in the extensive form games.

Payoffs:
Seller A
Seller B
Buyer C

p_j = price of j = action for J, $j \in \{a, b_0, b_1\}$, $J \in \{A,B\}$
q_j = quantity of j purchased = actions for C, $j \in \{a, b_0, b_1\}$
ε_j = transaction cost for seller J, $j \in \{a,b\}$, $J \in \{A,B\}$
$0 \leq \gamma \leq 1$: fraction of purchase price p_a refunded
δ = additional cost for A (surplus left on table for B) when $\gamma = 1$

$0_{\not{a}}$: don't buy b and return a
$1_{\not{a},b}$: buy b but return a
$1_{a,b}$: buy b; use a and b separately
1_c : buy b; join a and b to create c
$2_{c,b}$: buy b; use one b to create c, use the other b separately
$2_{b,b}$: buy b; use both b separately

SAV_j: component(s) j have positive stand alone value, $j \in \{a,b\}$

Figure 2: Legend for notation

Figure 3 exhibits the most general form of the game. This general form of the game is simpler than it appears. First, notice the game simplifies significantly when the refund is zero. Even with a positive refund, several of buyer C's stage 4 actions are never played in equilibrium. For example, for any game considered in this analysis, by Assumption 1 (preference for the composite good) buyer C never finds it optimal to purchase both components to use separately ($q_{b1} = 1_{a,b}$) over purchasing both components to form composite c ($q_{b1} = 1_c$).

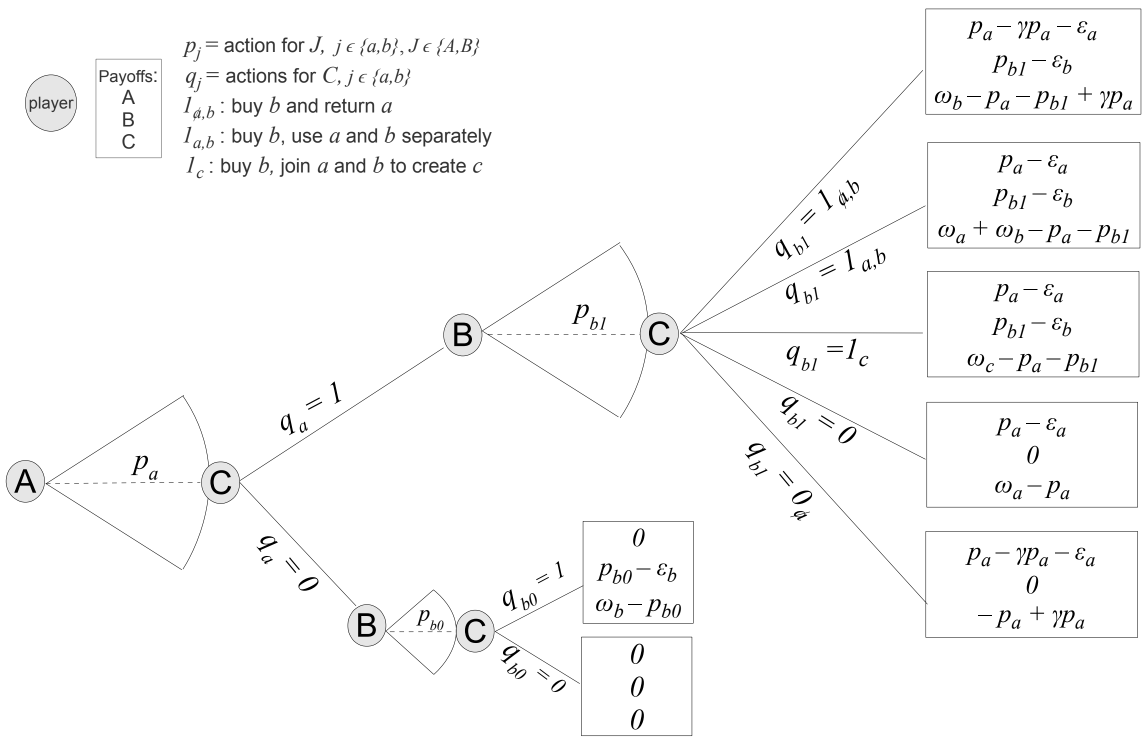

Figure 3: Game of refund $\gamma \in [0, 1]$ with nonnegative SAV_a and SAV_b

Finally, in the general form game, consider the second-smallest subgame along the path $q_a = 0$. In this $q_a = 0$-subgame, seller B chooses p_{b0}, then buyer C chooses q_{b0}. If component b has stand alone value, as long as supplying b has the lowest opportunity cost, seller B prices to sell stand alone units of b at the monopoly price. Buyer C then purchases stand alone units of b at the monopoly price, and seller A receives zero payoff. If component b does not have stand alone value, then by Assumption 2, B prices above zero, resulting in no units of b purchased by buyer C in the subgame equilibrium.

This Nash equilibrium is off the subgame perfect equilibrium path when no anticommons tragedy results and along the equilibrium path when hold-up occurs. However, the focus of the analysis is on averting hold-up. While the equilibrium outcome of each game depends on optimal behavior along the $q_a = 0$ path, regardless of the parameter values, the buyer always compares an equilibrium payoff of zero along the $q_a = 0$ path to the anticipated payoff along the $q_a = 1$ path. Therefore, in referencing the extensive form game tree for each game, focus should remain on the $q_a = 1$ path, where the tragedy ultimately might be averted. Lemma 1 summarizes the equilibrium along this path that is realized only when hold-up results.

Lemma 1 ($q_a = 0$ subgame) *By Assumptions 2 and 3, the Nash equilibrium of the second-smallest subgame along path $q_a = 0$ of the most general form of the game is*

$$\text{(i) if no } SAV_b,\ (p^*_{b0} > \epsilon_b, q^*_{b0} = 0) \text{ and (ii) if } SAV_b \text{ or } SAV_{ab},\ (p^*_{b0} = \omega_b, q^*_{b0} = 1).$$

In equilibrium, if $q_a = 0$, the final good is not purchased, but if component b has stand alone value, stand alone units of b are purchased.

The two most extreme cases of the general game are considered in Proposition 1, Proposition 6, and Proposition 7. The least restrictive case allows for positive refunds and stand alone value in both components. The most restrictive scenario, which serves as my baseline case, assumes no refunds with perfect complements. Feinberg and Kamien (2001) use a toll booth example to show the hold-up outcome in this most restrictive case.

3 Baseline: No refund and perfect complements

For a concrete example, consider the following scenario used throughout the analysis.

> **Example:** Two dilapidated buildings each sit on a quarter-acre parcel of land, a and b. Owners A and B were grandfathered into a city zoning ordinance that prevents the building of new dwellings on any parcel less than a half-acre. Upon sale of either quarter-acre plot, the grandfather clause ends, so a new owner may not construct a new dwelling on either parcel. A land developer desires to combine exactly these two parcels to create a legal-size parcel to build a new home, compliant with the zoning ordinance. No substitute parcels exist. The developer values the combined right to both parcels more than the right to using each property separately. Owner A first offers the developer a take-it-or-leave it price. Upon observing the price, the developer decides whether to purchase parcel a. All parties know the developer has no value for either property (building or land) in use alone. Also, the purchase of parcel a does not include any form of earnest money agreement, options agreement, or other

> contract regarding refunds. Once all parties observe whether the developer purchases parcel a, Owner B offers a take-it-or-leave it price for parcel b. Finally, the developer chooses whether to purchase b.

Assume no refund, $\gamma = 0$. Under perfect complementarities, Buyer C demands each component only when combined to form composite c. The buyer's reservation price for using either component alone is zero, $\omega_j = 0 \; \forall j \in \{a, b\}$.

> **Example continued:** Due to the zoning restrictions, the developer cannot build a new home unless he acquires both parcels of land. Because the developer finds value only in building a new home and has no value for either parcel in use alone, buildings a and b are perfect complements to the developer.

The extensive form game of perfect complements and no refund, given in Figure 6, shows the simplified general game. Without the option to refund component a, Buyer C's stage 4 actions are limited to choosing not to purchase b, to purchase b to combine with a to form c, or to purchase b for use alone. Without stand alone value, clearly b for stand alone use is never chosen.

Proposition 1 (No Refund/Perfect Complements - Tragedy) *Assume seller A offers no refund. By Assumptions 2 and 3, if components a and b are perfect complements, then in equilibrium, buyer C does not purchase either component good. Therefore, in equilibrium, buyer C does not purchase the composite good.*

> **Example continued:** Because parcel a has no use except when combined with parcel b, Owner B, the last seller, knows if the developer purchases parcel a, the purchase price of a is fully sunk. Therefore, b rationally charges the developer's willingness-to-pay for the combined parcels. Given the purchase of parcel a, the developer rationally would purchase parcel b at this price. However, unless the purchase price for parcel a is zero, the developer knows his *ex post* surplus will be negative. If the cost to supplying parcel a is positive, then owner A will never price at zero. Therefore, the developer will never purchase parcels a or b.

In a sequential setting, the anticommons tragedy under perfect complements and no refund amounts to full hold-up. Once the buyer purchases from first seller A, the cost for component a is sunk. The sunk nature of this purchase reveals itself in the final payoffs to buyer C: once buyer C purchases a, for all of buyer C's stage 4 actions, C's payoff is reduced by the purchase price, p_a. This sunk cost created from the transaction between seller A and buyer C yields a positive externality on seller B. With the cost of a sunk, seller B optimally prices not as a monopolist for component b, but as a monopolist for composite good c. This results in negative buyer surplus for C for any $p_a > 0$. However, for positive transaction costs, pricing component a at zero yields a negative payoff to seller A. Therefore, seller A optimally prices above zero. In equilibrium, the total price exceeds buyer C's willingness to pay for composite c, so buyer C does not purchase a or b. Equilibrium surplus is zero for every player.

This discussion clarifies the critical role of sunk costs in causing the anticommons tragedy in a sequential model. I now consider how stand alone value and refund might eliminate the tragedy by reducing sunk costs.

4 Games of no refund and imperfect complements

I now alter the baseline of perfect complements by allowing positive stand alone value in either one or both component goods. None of the players' action sets change, but with changes in the stand alone parameter, payoffs change.

4.1 No refund and SAV_a and SAV_{ab}

Begin by assuming $\omega_a > 0$ and $\omega_b \geq 0$ so *at least* component a has stand alone value to buyer C.

> **Example continued:** Suppose parcel a has stand alone value to the developer. For example, even though the developer cannot use only parcel a to build a new home, he can use the dilapidated building on parcel a as a storage unit, which has value to him, but not as much value as building a new home on parcels a and b combined. Now parcel a, the first parcel offered for sale, has stand alone value to the developer.

Figure 7 provides the merged, extensive form games of no refund with stand alone value in at least component a; the two possible games, SAV_{ab} and SAV_a differ only in the payoffs for actions $q_{b1} = 1_{a,b}$ (purchase b to use with a) and $q_{b0} = 1$ (purchase b for stand alone use only).

Whether the game is one of stand alone value in both components or stand alone value in a alone, Assumption 1 implies buyer C will never choose to purchase both components to use separately (action $q_{b1} = 1_{a,b}$). Therefore, the only difference in the equilibrium of these two games of stand alone value in a is in the stand alone market for component b.

The outcome of both games, with respect to anticommons, is the same. For this reason, I combine the analysis of games of imperfect complementarities in component a.

Proposition 2 (No Refund/SAV_a or SAV_{ab} - No Tragedy) *Assume seller A offers no refund. By Assumptions 2 and 3, if at least first component a has stand alone value, then buyer C purchases the composite good in equilibrium.*

> **Example continued:** Owner B knows once parcel a is purchased, the cost is not fully sunk because parcel a has use to the developer other than in use with parcel b. Therefore, the developer will not be willing to pay the monopoly price for the the assembled property, c, given the purchase of parcel a. Owner B optimally offers a lower price to ensure purchasing b is optimal for the developer. Because b does not hold-up the developer, owner A can price positively and still ensure the developer purchases parcel a.

For both games, the subgame perfect equilibrium along the equilibrium path is

$$(p_a^* = \omega_a, (q_a^* = 1, q_{b1}^* = 1_c), p_{b1}^* = \omega_c - \omega_a).$$

Along the equilibrium path, the composite good is purchased. Only off-the-equilibrium-path equilibria differ between a game of SAV_{ab} and a game of SAV_a.

Proposition 2 offers two implications. First, in equilibrium, positive stand alone value in the first component is enough to prevent full hold-up by the last seller, regardless of whether a refund is offered or the last component has stand alone value. If the first component has stand alone value, the sunk cost to purchasing component a is *reduced* by ω_a, buyer C's stand alone value of a. This reduction in sunk cost via an increase in component a's stand alone value gives the last seller incentive to charge only the composite good monopoly price *less* this change in sunk cost (ω_a). Without incentive to hold-up, enough surplus is left on the table for seller A optimally to price positively and buyer C optimally to purchase both components.

Second, in equilibrium, both sellers earn positive profits, compared to zero profits realized in the perfectly complementary market. Additionally, if component b has stand alone value, then seller B charges $p_{b1}^* = \omega_c - \omega_a$, a higher equilibrium price for a unit of b to be combined with a than could be charged in the stand alone market for component b when buyer C does not purchase component a, $p_{b0}^* = \omega_b < \omega_c - \omega_a$. Therefore, last seller B benefits from the strategic interaction with seller A in the market for composite c, as opposed to being the sole monopolist in the stand alone market for component b.

4.2 No Refund and SAV_b

> **Example continued:** Suppose only parcel b has stand alone value to the developer. For example, even though the developer cannot use parcel b alone to build a new home, he can rent the building on parcel b. However, the total value of renting the building on parcel b is lower than the value of building a home on assembled parcel c.

A game of no refund with stand alone value in component b assumes first component a has no stand alone value ($\omega_a = 0$), last component b has positive stand alone value ($\omega_b > 0$), and seller A offers no refund ($\gamma = 0$). Figure 8 provides the extensive form of the game. Notice that the game's payoffs differ from the perfect complements/no-refund case examined in Section 4 in only two outcomes: the case when buyer C chooses action $q_{b1} = 1_{a,b}$ (purchasing a and b to use alone) and the case when buyer C chooses action $q_{b0} = 1$ (purchasing b to use alone). Therefore, since rational buyer C never chooses $q_{b1} = 1_{a,b}$, equilibrium hold-up results, as in the game of perfect complements with no refund. Stand alone value in component b does not reduce the sunk cost to purchasing a. Optimal behavior yields hold-up, with the total price in equilibrium higher than the buyer's willingness to pay for composite c. Unlike the perfect complements case, since b has stand alone value, a stand alone unit of b is purchased in equilibrium.

Proposition 3 (No Refund/SAV_b - Tragedy) *Assume seller A offers no refund. By Assumptions 2 and 3, if last component b has stand alone value, then buyer C does not purchase the composite good but does purchase a stand alone unit of b in equilibrium.*

> **Example continued:** Owner B knows parcel a has no stand alone value to the developer, so the purchase of parcel a is fully sunk. Owner B optimally prices at the value to the developer for combining both parcels. In equilibrium, this holds up the developer because the total equilibrium price exceeds his willingness-to-pay for both a and b. The developer does not purchase a. Since he values parcel b alone, and since B observes that the developer has not purchased a, owner B offers a price equal to the developer's value for b. The developer therefore purchases parcel b at the price he is willing to pay for using b alone as a rental property.

Some components, such as real or intellectual property, as in the examples I have considered, have a natural capacity constraint of one unit. However, allowing the last supplier to sell more units of b than are required to create composite c can alter the optimal behavior in this game. To understand whether stand alone value in the last component supplied affects hold-up, I relax the capacity constraint on component b.

4.2.1 No price discrimination and relaxed capacity constraint

> **Example continued:** Now suppose Owner B offers for sale a second parcel of similar land having the same stand alone value on the opposite end of owner A's parcel. Either of the b-parcels could be combined with parcel a to form the larger parcel, with the extra b-parcel used alone.

Suppose in stage 4, buyer C has the option to purchase two units of component b, with the quantity of component a supplied held constant at one unit. Therefore, seller B knows if buyer C purchases component a and values b alone, then the two units of component b purchased must be for different uses – one for creating c and one in use alone. If price discrimination were feasible, B would offer different prices for the first and second units of b. If B cannot price discriminate, B must choose a single, per-unit price for component b.

If buyer C purchases component a, then C's set of actions in stage 4 becomes (i) purchase zero units of b ($q_{b1} = 0$); (ii) purchase one unit of b to form composite good c ($q_{b1} = 1_c$); (iii) purchase two units of b: one to form good c and one to use alone ($q_{b1} = 2_{c,b}$); (iv) purchase one unit of b to use alone ($q_{b1} = 1_{a,b}$); and (v) purchase two units of b to use each alone at value ω_b ($q_{b1} = 2_{b,b}$).

One can easily show buyer C never chooses actions (iv) or (v). However, under certain conditions, buyer C might choose to purchase two units of b: one to form c and the other to use b alone. If buyer C does use one unit of b alone, he gets value ω_b for that unit. Therefore, if C purchases two units, he will never agree to pay more than the value of b used alone. As a result, seller B knows he cannot charge a price greater than C's willingness-to-pay for b alone.

However, selling two units of b at a per-unit price no higher than ω_b might not yield higher payoffs to B than selling just one unit of B at a price higher than ω_b. Seller B knows C would purchase one unit of b to use with a at a price no higher than ω_c. In that case, B could price above ω_b and no higher than ω_c. Then, seller B earns profit for selling only one unit of b, but at a price higher than ω_b. Therefore, seller B prices to sell either two units of b or one unit of b, depending on the relationship between ω_b and ω_c.

Lemma 2 formalizes this pricing incentive by analyzing the Nash equilibrium of the second smallest subgame in the full game. This Lemma highlights the importance of the degree of stand alone value in affecting optimal behavior. The following definition provides a measure of the degree of stand alone value:

Definition 4 (high (low) stand alone value) *For $j \in \{a, b\}$, component j has **high stand alone value** to buyer C if $2\omega_j > \omega_c$. Otherwise, component j has **low stand alone value**.*

Thus, if a component's stand alone value is more than half of the value of the composite good, then that component has high stand alone value. Otherwise, that component has low stand alone value. If both components, a and b, have high stand alone value, then $2\omega_a + 2\omega_b > \omega_c + \omega_c \Leftrightarrow \omega_a + \omega_b > \omega_c$, which contradicts Assumption 1. Therefore, at most one component can have high stand alone value. If both components have low stand alone value, Assumption 1 is satisfied.

If component b has high stand alone value, seller B optimally prices to sell two units, even though he cannot price above ω_b in doing so. However, if component b has low stand alone value, B optimally charges a higher price and sells only one unit of b. It is not optimal to price lower to sell two units when the stand alone value, and thus the feasible price, of the units is low.

Lemma 2 (No Refund/SAV_b unconstrained capacity: B's pricing incentives) *Assume component b has stand alone value. Consider the second smallest subgame along branch $q_a = 1$ in a game of no refund with stand alone value in last component b. Assume C may purchase up to two units of component b, and seller B cannot price discriminate. By Assumptions 2 and 3, (i) seller B optimally prices at $p_{b1}^* = \omega_b$ to sell $q_{b1}^* = 2_{c,b}$ if and only if last component b has high stand alone value and (ii) seller B optimally prices at the monopoly price for the composite good, $p_{b1}^* = \omega_c$, to sell $q_{b1}^* = 1_c$ if and only if last component b has low stand alone value.*

Figure 4 illustrates how the demand for component b is affected by the relationship between the stand alone value of component b and the reservation value of composite c. The demand for a component b with high stand alone value, ω_{b-high} in the graph, is more elastic than demand for a component b with low stand alone value, ω_{b-low} for the following reasons. By Assumption 1, since buyer C values the composite good to using either component alone, then regardless of the stand alone value in b, the buyer is willing to pay up to ω_c for the first unit of b. For the second unit of b, buyer C only will pay up to his stand alone value for the component, either ω_{b-low} or ω_{b-high}. In the standard monopoly model, the monopoly price under more price elastic demand is lower than the monopoly price under a less price elastic demand. This is exactly the result shown in Lemma 2; by Assumption 1, with high stand alone value in b, seller B's optimal price (ω_b) is lower than the monopoly price for composite c (ω_c), which is B's optimal price when b has low stand alone value. Using B's optimal pricing strategy given in Lemma 2, Proposition 4 provides conditions for which buyer C purchases composite c, yielding no anticommons tragedy, and conditions for which B holds up C, resulting in an anticommons tragedy.

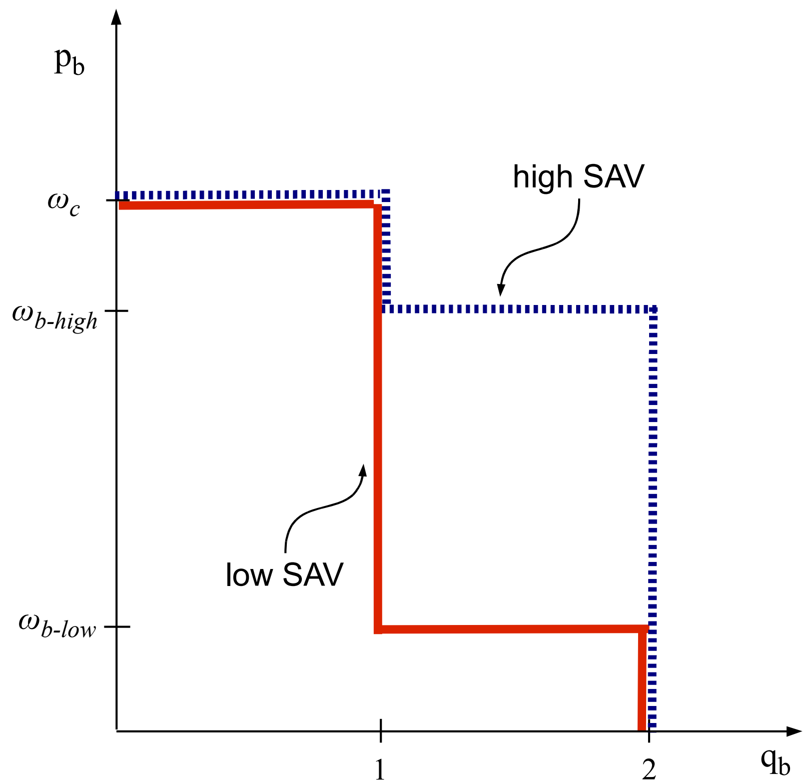

Figure 4: Demands for component b by degree of stand alone value

Proposition 4 (No Refund/High(Low) SAV_b - No Tragedy (Tragedy)) *Assume seller A offers no refund and only last component b has stand alone value. Assume buyer C may purchase up to two units of component b, and seller B does not price discriminate. By Assumptions 2 and 3, in equilibrium, (i) if last component b has high stand alone value, then the buyer purchases the composite good and (ii) if last component b has low stand alone value, then the buyer does not purchase the composite good, but does purchase stand alone units of b.*

Compared to the baseline of perfect complements, if buyer C purchases component a, the sunk cost still equals the purchase price of component a, p_a. However, with the option of selling more than one unit of b, the last seller can increase profits by reducing the price to sell two units of b. Thus, when buyer C highly values component b, seller B has an incentive not to hold-up. Therefore, stand alone value in the last component b can be enough to mitigate the anticommons tragedy when capacity is not constrained to the exact proportion required to create the composite good.

4.2.2 Price discrimination and relaxed capacity constraint

Once the last seller offers to sell more than one unit of component b, it is natural to suppose seller B will price discriminate. If buyer C purchases more units of a component than required to form the composite good, then the seller knows the buyer intends to use those additional units for an alternate, less valuable use. By Assumption 1, seller B knows buyer C places value $\omega_c > \omega_b$ for any units of b purchased to create composite c. Therefore, buyer C is willing to pay up to ω_c for these units. The buyer will pay no more than reservation price ω_b for any additional units of b purchased. Thus, seller B might first degree price discriminate by charging a price $p_{b1c} \leq \omega_c$ for any units of b used to create c and price $p_{b1b} \leq \omega_b$ for any units of b not used to create composite c. Clearly, B prices to capture all buyer surplus through perfect price discrimination because reservation prices are known. This provides no incentive for A to price so that buyer C purchases component a. Therefore, the composite is not purchased in equilibrium.

Proposition 5 (No Refund/SAV_b: Price Discrimination - Tragedy) *Assume seller A offers no refund and component b has stand alone value. Assume buyer C has the option to purchase up to two units of component b. By Assumptions 1, 2, and 3, if seller B perfectly price discriminates, then in equilibrium, buyer C does not purchase composite c but does purchase two stand alone units of b.*

Thus, hold-up results if the last seller price discriminates. This inefficient outcome depends on an assumption of perfect and complete information. In real world examples of assembly problems, information can be transparent due to close relationships between parties. In such cases, sellers can determine a buyer's willingness-to-pay, so allowing for price discrimination can result in the anticommons tragedy.

Price discrimination by seller B imposes a negative externality on seller A because it leaves A without an optimal pricing strategy that gives buyer C incentive to purchase component a. With or without price discrimination, seller B sells two units of component b. However, with price discrimination, the composite good is not purchased in equilibrium. Since buyer C values stand alone units of b, seller B is indifferent to price discriminating or not. However, when B price discriminates, seller A is worse off, and the anticommons tragedy results. Recall, if component b has high stand alone value and B does not price discriminate, hold-up is averted. Therefore, when b has high stand alone value, banning price discrimination yields a weak Pareto improvement.

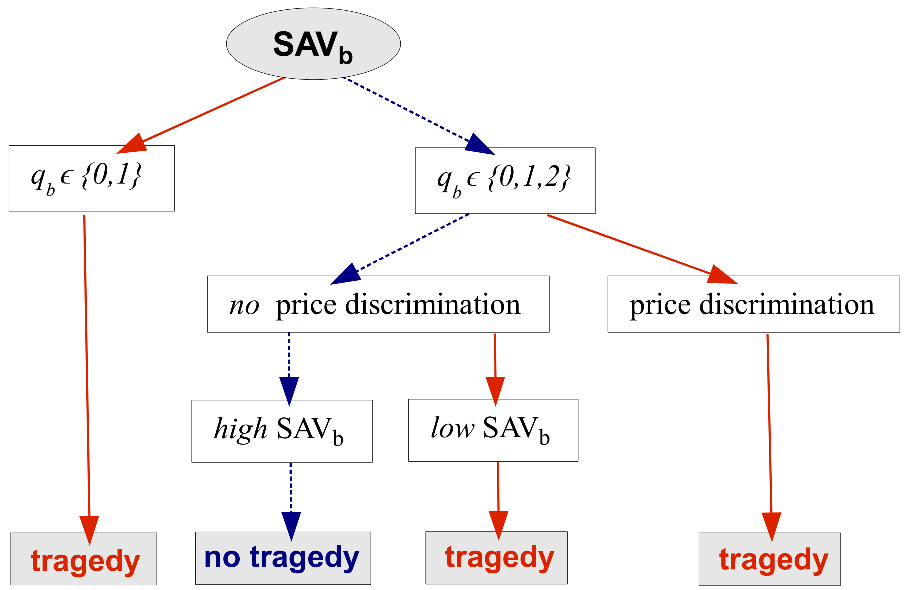

Figure 5: Results: Games of stand alone value in only component b

Figure 5 summarizes the results from games with stand alone value in component b. Whenever A does not offer a refund and b has stand alone value, then the anticommons tragedy is mitigated if and only if (1) C has the option to purchase more than one unit of b, *and* (2) b has high stand alone value, *and* (3) B does not engage in price discrimination. If C cannot purchase more than one unit of b, or if b has low stand alone value, or if B price discriminates, then B prices to capture all of the surplus from buyer C, leaving seller A unable to profitably sell any units of a. In such cases, C does not purchase a and does purchase stand alone units of b, resulting in an anticommons tragedy.

5 Games of refunds

I now incorporate positive refundability, $\gamma \in (0, 1]$. I consider partial refund, $\gamma \in (0, 1)$, and full refund, $\gamma = 1$.

> **Example continued:** Now, assume a contract between owner A and the developer includes an earnest money option for the purchase of parcel a. Rather than outright purchasing parcel a, the developer and owner A agree upon a

> value of earnest money to be paid to seller a, enough to signal the developer's intent to purchase, but not too much to dissuade the developer from participating. Only one unit of parcel b is available for purchase. Under the agreement, if the developer does not purchase parcel b, some portion of the earnest money for parcel a is refunded and parcel a is not sold.

As shown above, B's ability to hold-up the buyer derives from the sunk cost of purchasing component a. A full refund completely eliminates this sunk cost, while a partial refund reduces it. Figure 10 again provides the extensive form of the general game, allowing for all degrees of complementarity and refunds. Notice how refunds reduce sunk cost, either fully or partially, when C chooses to return a.

Consider the perfect complements case. Since component a does not have stand alone value, C's payoff will be negative in the event he does not purchase component b. When seller A offers a partial refund, full recovery of the purchase price for a does not occur. However, a full refund makes buyer C no worse off when purchasing a. Therefore, a partial refund, which leaves the buyer without a potential for positive surplus, is not enough to change seller B's incentive to hold-up. On the other hand, with a threat of full refund, B must price low enough to ensure C has incentive to purchase both components. In fact, with sunk costs fully eliminated, B's best response directly depends on seller A's optimal behavior. A captures (nearly) all surplus, ω_c, leaving just enough surplus on the table to ensure an indifferent seller B optimally prices to sell component b.

Let $\delta > 0$ be this added surplus seller A leaves on the table for B to optimally price so C chooses $q_b = 1_c$ and to ensure B is not indifferent. Therefore, this smallest element of surplus acts as a cost to seller A to ensure buyer C purchase component a in equilibrium.

Proposition 6, below, summarizes the partial refund's weakness as a mechanism for mitigating hold-up. If buyer C purchases the first component but not the last, then the buyer is better off *ex post* with even partial recovery of the sunk cost. However, this partial recovery is not enough to alter the optimal behavior of seller B, so *ex ante*, the buyer has no incentive to purchase the first component anyway. This result holds regardless of the degree of complementarities. Therefore, under cases of complementarities and no refund where hold-up results, a partial refund will not avert an anticommons tragedy. Since partial refunds do nothing to alter the optimal behavior of B, when at least component a has stand alone value, full hold-up is mitigated irrespective of a partial refund. Even so, I show below that seller A might prefer a partial refund to a full refund. Therefore, a partial refund might not affect the anticommons outcome, but it can affect the equilibrium welfare.

Proposition 6 (Partial Refund) *Assume seller A offers only a partial refund $\gamma \in (0, 1)$. Composite c is purchased in equilibrium if at least component a has stand alone value.*

(Therefore, composite c is not purchased in equilibrium if components a and b are perfect complements or if only component b has stand alone value.) If component b has stand alone value, the stand alone unit of b is purchased in equilibrium.

Though partial refunds do nothing to mitigate the anticommons tragedy, full refunds do. Proposition 7 shows conditions for each case of complementarities that ensure hold-up is mitigated when seller A offers a full refund. The result relies on seller A leaving just enough surplus on the table, δ, to induce seller B to sell component b. As long as this additional transaction cost to seller A, δ, is low enough, hold-up is mitigated.

Proposition 7 (Full Refund) *Assume seller A offers full refund, $\gamma = 1$. (i) If components are perfect complements or if only component a has stand alone value, the composite good is purchased in equilibrium as long as $\omega_c - \epsilon_a > \delta > \epsilon_b$. (ii) If at least component b has stand alone value, the composite good is purchased in equilibrium as long as long as $\omega_c - \omega_b - \epsilon_a > \delta$ holds.*

With the option to return component a for a full refund, seller A must price to ensure seller B does not price so component a is returned: A wants B to price so b is purchased and C keeps a. When component b does not have stand alone value, assuming transaction costs are not too high, the condition ensuring these optimal actions is weak.

However, the option to return component a for a full refund increases buyer C's strategy set in a way that could benefit B. In the case where at least component b has stand alone value, if A prices "too high," C's optimal action may be to return a and consume b alone. In this case, to ensure seller B prices so buyer C will not return a, the stand alone value of component b (plus a small δ) must be left as surplus for seller B. Therefore, as shown in Table 1, when at least component b has stand alone value, a refund option can yield seller B significantly nonzero ($\omega_b + \delta$) equilibrium payoffs.

6 Welfare analysis

Using the results from above, outlined in Table 1, I consider how welfare is affected by degree of complementarity, refunds and the order of play. Endogenizing the choice of refund and endogenizing the order of play offer additional insight regarding optimal welfare.

6.1 Endogenous refund

Suppose seller A chooses a refund level in stage 0, and component a has stand alone value to buyer C. If the marginal gain to offering a full refund outweighs the marginal cost to offering a full refund, then seller A optimally chooses a full refund. This means C's stand alone value of a cannot be too similar to C's value for composite c, or in the case of stand alone value in only component a, B's transaction costs are not too high. Corollary 1 summarizes this result.

Corollary 1 (A chooses full refund) *In stage 0, let refund level $\gamma \in [0, 1]$ be chosen by seller A. A full refund $\gamma^* = 1$ maximizes seller A's payoffs in the cases where (i) component a has stand alone value to buyer C and $\omega_c - \omega_a > \delta > \epsilon_b$ and (ii) both components a and b have stand alone value to buyer C and $\omega_c - (\omega_a + \omega_b) > \delta$.*

Thus, when only component a has stand alone value to C, if a's stand alone value is too high, it might be optimal for seller A to relinquish payoffs to seller B by offering no or partial refund. A full refund is not necessarily optimal for the first seller.

As a matter of efficiency and ensuring no hold-up, a social planner should exogenously set a full refund in the case where the order of acquiring rights is fixed and either (i) only the last component required to complete the assembly of rights has stand alone value, or (ii) no components have stand alone value. This ensures full hold-up does not result. However, as a matter of wealth distribution, in the case where at least the first component has stand alone value, an exogenously set full refund could transfer surplus from the first seller to the last seller, if component a has high enough stand alone value or if B's transaction costs are high enough. Notice when only component b has stand alone value or no component has stand alone value, seller A always chooses a full refund because equilibrium payoffs are zero, otherwise.

6.2 Endogenous order

Next, relax the assumption of exogenously determined order of play. Below, the preferred order of play is considered in the cases when (i) the buyer chooses the order of play and (ii) sellers choose order of play.

6.2.1 Buyer chooses order

Buyer C's equilibrium surplus is zero in every case considered, above. Recall, by Assumption 1, buyer C participates in the market for composite c because C values the composite good more than using the component goods alone. Therefore, buyer C optimally chooses the order of play that yields no hold-up in equilibrium. Regardless of the level of refund, buyer C is guaranteed no hold-up as long as a component with stand alone value is purchased first. On the other hand, if a full refund is offered, the order in which buyer C purchases the components does not affect his ability to successfully create composite c in equilibrium.

6.2.2 Sellers choose order

A seller's equilibrium payoffs may be higher under a particular order of play. In this case, I consider when seller A has a first-mover advantage and when seller B has a second-mover advantage. Under no or partial refund, the degree of component a's stand alone value determines whether seller A has a first-mover advantage or seller B has a second-mover advantage.

Corollary 2 (First (Second) mover advantage) *Assume seller A offers no or partial refund. (i) If first component a has high (low) stand alone value, then* $p_a^* > (<) p_{b1}^*$. *(ii) If* $\epsilon_a = \epsilon_b$ *then* $\pi_A^* > (<) \pi_B^*$.

> **Example continued:** The developer's realized value in parcel a alone is not only enough to ensure Owner B does not hold-up, but if using a as a storage unit is not too valuable to the developer (use as a storage unit has low stand alone value), then in equilibrium Owner B could receive higher payoff than owner A. If owner A's building is not too damaged and the developer highly values a's building as a storage unit, then in equilibrium a could realize higher payoff than Owner B.

Regardless of whether b has stand alone value and no refund is offered, if component a has high stand alone value to buyer C, seller A's equilibrium price is higher than B's equilibrium price. A sufficient condition for a first-seller advantage is that transaction costs are equivalent and a has high stand alone value. A necessary condition for a first seller advantage is A's costs not be too high, compared to the stand alone value for a, or a's stand alone value be high enough, compared to A's transaction costs.

By similar reasoning and assumptions, under low stand alone value in component a, seller B may have a second-mover advantage. Therefore, even if C values use of a alone and does not value use of b alone, seller B still may earn higher equilibrium payoffs than seller A. Though stand alone value in a is sufficient to prevent hold-up by seller B, the degree of stand alone value determines which seller achieves higher payoffs in equilibrium. This result shows, for example, in a private takings situation, the homeowner waiting the longest to sell does not necessarily earn as high of payoffs than the first seller.

Corollary 3 shows that under a full refund, seller B could have a second-mover advantage. For stand alone value in at least component b, if that stand alone value is high, then seller B's equilibrium payoffs are higher than seller A's equilibrium payoffs.

Corollary 3 (Full refund second-mover advantage) *Assume seller A offers a full refund. If at least last component b has high stand alone value and $\delta > \epsilon_b$ (or $\delta - \epsilon_b$ is not too negative), then $\pi^*_B > \pi^*_A$.*

7 Summary of results and conclusion

Table 1 adds to the results given in Figure 5 and summarizes the equilibrium outcomes and equilibrium payoffs from the eight cases of complementarities and refundability, as well as the three cases relaxing capacity.

First, regardless of degree of complementarity, a full refund ensures the composite good is purchased in equilibrium. Therefore, a fragmented market of perfect complementarities does not result in hold-up if full refunds are allowed. Although full hold-up does not always result under partial refund, comparison of partial refund to no refund reveals the partial refund does nothing to mitigate the anticommons tragedy; the cases of no tragedy under partial refund are due to the stand alone value of component a. Tragedy does not result under no refund with at least stand alone value in the first component. When the first component does not have stand alone value, with a partial or no refund, tragedy results.

A comparison of equilibrium payoffs reveals, as expected in a game of completely transparent information, in equilibrium, the sellers fully capture the buyer's value of the composite good. Under the assumption an indifferent buyer purchases, then the buyer always is left with no surplus in equilibrium, but in cases of no tragedy, the composite good successfully is assembled.

7.1 Conclusion

Building on the general framework of complementary monopoly and anticommons property, this research first models the assembly problem in terms of sunk cost. Motivating the potential inefficiency in these fragmented markets through sunk cost reveals that solutions to hold-up can rely on factors that reduce those sunk costs. I show the effects of imperfect complementarities and refundability, two factors that reduce sunk cost, in mitigating hold-up. These two factors are relevant in real world examples of anticommons property and complementary monopoly. My analysis offers settings where stand alone value and a refund option can alter potential hold-up.

The order of purchase also affects the incentives for hold-up. Since hold-up is prevented when at least the first component purchased has stand alone value, a social planner should take notice at any required ordering to obtaining rights of use, efficiently determining order or allowing buyers to determine order based on their stand alone values for components.

When only the last component has stand alone value, hold-up can be mitigated if capacity is not constrained to only the proportion required to create the composite good. Allowing the buyer to purchase more units of the last component than required for assembly alters optimal behavior. However, in this case, the level of that last component's stand alone value relative to the composite good's value determines whether the last seller continues to have incentive to hold-up the buyer. This result implies that even though it appears stand alone value in the last component only heightens the last seller's incentive to hold-up, in fact, if additional stand alone units of the last component are supplied, the last seller may no longer have incentive to cause an anticommons tragedy.

Additionally, perfect price discrimination in these market settings results in an inefficient outcome. Because the last seller is indifferent to price discriminating, a ban on price discrimination is Pareto improving when the first component has high stand alone value.

Finally, full refunds mitigate the anticommons tragedy in my model, while a partial refund is no better than no refund for overcoming the anticommons tragedy. My welfare analysis reveals conditions for which a first seller or a last seller advantage exists; if a seller's component has stand alone value, that seller does not necessarily earn higher equilibrium payoff than his rival. Therefore, stand alone value in a rival's component could be a positive externality for the other seller.

Since the literature has not modeled refundability and a little research exists regarding imperfect complementarities, my analysis builds on the current research of anticommons property and complementary monopoly. Furthermore, the results suggest policies, such as legal requirements on full refunds, regulation on order in which components must be purchased, or legal prohibition of price discrimination, to mitigate inefficient outcomes in assembly problems.

Extensions to my research include a continuous, sequential market model of the assembly problem. I have investigated the continuous model under perfect complementarities with repeated, sequential play to gain insight into how repetition affects the incentive to hold-up; I hope to incorporate complementarities and refundability into such model. Finally, the model presented in this research lends itself to experimental testing in the laboratory; current research includes designing this experimental model. Also, I have considered a setting in which to empirically examine hold-up using naturally-occurring data, and I continue to determine the appropriate model and data requirements.

8 Appendix A: Proofs

Proof of Lemma 1. Assume $q_a = 0$.

(i) Let $\omega_b = 0$. By Assumption 3, the indifference rule, buyer C chooses $q_{b0} = 1$ as long as $p_{b0} \leq 0$. Observing buyer C's optimal strategy, seller B chooses to solve

$$\max_{p_{b0}} \{p_{b0} - \epsilon_b, 0\} \text{ s.t. } p_{b0} \leq 0.$$

Therefore, the Nash equilibria are $\{p_{b0}^* > \epsilon_b, q_{b0}^* = 0\}$.

(ii) Let $\omega_b > 0$. By Assumption 3, the indifference rule, buyer C chooses $q_{b0} = 1$ as long as $p_{b0} \leq \omega_b$. Observing buyer C's optimal strategy, seller B chooses to solve

$$\max_{p_{b0}} \{p_{b0} - \epsilon_b, 0\} \text{ s.t. } p_{b0} \leq \omega_b.$$

Therefore, assuming transaction costs are low enough (Assumption 2(i) ($\epsilon_b < \omega_b$)), the Nash equilibrium is $\{p_{b0}^* = \omega_b, q_{b0}^* = 1\}$. ■

Proof of Proposition 1. *(No Refund/Perfect Complements - Tragedy)*

Assume $\gamma = 0$, $\omega_a = 0$, and $\omega_b = 0$. In stage 4 and then stage 3:

(i) if $q_a = 0$, then by Lemma 1, the Nash equilibrium of the second smallest subgame is $\{p_{b0}^* > \epsilon_b, q_{b0}^* = 0\}$.

(ii) if $q_a = 1$, then in stage 4, by Assumption 1, buyer C never chooses $q_{b1} = 1_{a,b}$. Buyer C chooses $q_{b1} = 1_c$ if and only if

$$\pi_C(q_{b1} = 1_c) \geq \pi_C(q_{b1} = 0) \Leftrightarrow \omega_c - p_a - p_b \geq -p_a.$$

Therefore, by Assumption 3, the indifference rule, as long as $p_{b1} \leq \omega_c$, buyer C chooses $q_{b1} = 1_c$. In stage 3, seller B solves

$$\max_{p_{b1}} \{p_{b1} - \epsilon_b, 0\} \text{ s.t. } p_{b1} \leq \omega_c.$$

Therefore, assuming transaction costs are low enough (by Assumption 2(ii), ($\omega_c > \omega_c - \omega_a > \epsilon_b$)), seller B chooses $p_{b1}^* = \omega_c$.

In the third smallest subgame at stage 2, buyer C, knowing the optimal actions of seller B in stage 3, chooses $q_a \in \{0, 1\}$. By Assumption 3, the indifference rule, buyer C chooses to purchase component a as long as

$$\pi_C(q_a = 1) = \omega_c - p_a - p_{b1}^* = -p_a \geq \pi_C(q_a = 0) = 0.$$

This is possible only when $p_a \leq 0$. Therefore, buyer C chooses $q_a^* = 1$ if $p_a \leq 0$ and chooses $q_a^* = 0$ otherwise.

In stage 1, seller A chooses price p_a to solve

$$\max_{p_a} \{p_a - \epsilon_a, 0\} \text{ s.t. } p_a \leq 0.$$

Because $p_a = 0$ yields negative payoff, seller A optimally chooses $p_a^* > \epsilon_a$. By Assumption 2(ii) ($\omega_c > \epsilon_b$), ensuring transaction costs are low enough, the subgame perfect equilibria are

$$\{p_a^* > \epsilon_a, (q_a^* = 0, q_{b1}^* = 1_c, q_{b0}^* = 0), (p_{b1}^* = \omega_c, p_{b0}^* > \epsilon_b)\}.$$

Along the equilibrium path, $\{p_a^* > \epsilon_a, (q_a^* = 0, q_{b0}^* = 0), p_{b0}^* > \epsilon_b\}$. Therefore, neither the composite good nor either component good is purchased in equilibrium. ■

Proof of Proposition 2. *(No Refund/SAV_a/SAV_{ab} - No Tragedy)*

Assume $\gamma = 0$, $\omega_a > 0$, and $\omega_b \geq 0$. In stage 4 and then stage 3:

(i) if $q_a = 0$, then by Lemma 1, the Nash equilibrium of the second smallest subgame is $\{p_{b0}^* > \epsilon_b, q_{b0}^* = 0\}$ if $\omega_b = 0$ and $\{p_{b0}^* = \omega_b, q_{b0}^* = 1\}$ if $\omega_b > \epsilon_b$.

(ii) if $q_a = 1$, then in stage 4, by Assumption 1, buyer C never chooses $q_{b1} = 1_{a,b}$ for any $\omega_b \geq 0$. Buyer C chooses $q_{b1} = 1_c$ if and only if

$$\pi_C(q_{b1} = 1_c) \geq \pi_C(q_{b1} = 0) \Leftrightarrow \omega_c - p_a - p_b \geq \omega_a - p_a.$$

Therefore, by Assumption 3, the indifference rule, as long as $p_{b1} \leq \omega_c - \omega_a$, buyer C chooses $q_{b1} = 1_c$. In stage 3, seller B solves

$$\max_{p_{b1}} \{p_{b1} - \epsilon_b, 0\} \text{ s.t. } p_{b1} \leq \omega_c - \omega_a.$$

Therefore, seller B chooses $p_{b1}^* = \omega_c - \omega_a$.

Notice, assuming $\omega_b > \epsilon_b$, which holds by Assumption 2(i), then $\omega_b > 0$. Therefore, $\omega_c - \omega_a > \omega_b > \epsilon_b$, by Assumption 1. Otherwise, $\omega_b > 0$ holds by Assumption 2(ii) ($\omega_c - \omega_a > \epsilon_b$).

In the third smallest subgame at stage 2, buyer C, knowing the optimal actions of seller B in stage 3, chooses $q_a \in \{0, 1\}$. By Assumption 3, the indifference rule, buyer C chooses to purchase component a as long as

$$\pi_C(q_a = 1) = \omega_c - p_a - p_{b1}^* = \omega_a - p_a \geq \pi_C(q_a = 0) = 0.$$

Therefore, buyer C chooses $q_a^* = 1$ if $p_a \leq \omega_a$ and chooses $q_a^* = 0$ otherwise.

In stage 1, seller A chooses p_a to solve

$$\max_{p_a} \{p_a - \epsilon_a, 0\} \text{ s.t. } p_a \leq \omega_a.$$

By Assumption 2(i) ($\omega_a > \epsilon_a$), then $p_a^* = \omega_a$. Therefore, assuming $\omega_c - \omega_a > \epsilon_b$ and $\omega_a > \epsilon_a$, the subgame perfect Nash equilibria are:

$$\begin{aligned} &\{p_a^* = \omega_a, (q_a^* = 1, q_{b0}^* = 1, q_{b1}^* = 1_c), (p_{b0}^* = \omega_b, q_{b1}^* = \omega_c - \omega_a)\} \quad \text{if } SAV_{ab} \\ \text{and } &\{p_a^* = \omega_a, (q_a^* = 1, q_{b0}^* = 0, q_{b1}^* = 1_c), (p_{b0}^* > 0, q_{b1}^* = \omega_c - \omega_a)\} \quad \text{if } SAV_a. \end{aligned}$$

Notice, only off the equilibrium path do equilibria differ between the two games. For both games, along the equilibrium path,

$$\{p_a^* = \omega_a, (q_a^* = 1, q_{b1}^* = 1_c), q_{b1}^* = \omega_c - \omega_a\}.$$

Thus, the composite good is purchased in equilibrium. ■

Proof of Proposition 3. *(No Refund/SAV_b)*

Assume $\gamma = 0$, $\omega_b > 0$, and $\omega_a = 0$. As explained in the text, a the game of no refund with stand alone value in only component b yields the same off-the-equilibrium path equilibria and equilibrium outcome of hold-up as the game of no refund with perfect complements. The only difference is the equilibria along the equilibrium path; since $\omega_b > 0$, then by Lemma 1, stand alone units of b are purchased in equilibrium. Therefore, the proof of Proposition 3 is the same as the proof of Proposition 1, except in the case where $q_a = 0$.

In stage 4 and then stage 3:

(i) if $q_a = 0$, then by Lemma 1, the Nash equilibrium of the second smallest subgame is $\{p_{b0}^* = \omega_b, q_{b0}^* = 1\}$.

(ii) if $q_a = 1$, the proof is identical to the proof of Proposition 1 for $q_a = 1$. Assuming $\omega_c > \epsilon_b$, the subgame perfect equilibria are

$$\{p_a^* > \epsilon_a, (q_a^* = 0, q_{b1}^* = 1_c, q_{b0}^* = 1), (p_{b1}^* = \omega_c, p_{b1}^* = \omega_b)\}.$$

Along the equilibrium path, $\{p_a^* > \epsilon_a, (q_a^* = 0, q_{b0}^* = 1), p_{b1}^* = \omega_b\}$. Therefore, the composite good is not purchased in equilibrium, but stand alone units of b are purchased in equilibrium.

■

Proof of Lemma 2. *(B's pricing incentives under multiple quantities)*

Recall, $q_{b1} = 2_{c,b}$ means buyer C purchases two units of b, one to create composite c and one to use alone. Assume $\gamma = 0$, $\omega_b > 0$, and $\omega_a = 0$. Buyer C purchases two units of b as long as

$$\begin{aligned} \pi_C(q_{b1} = 2_{c,b}) &\geq \pi_C(q_{b1} = 1_c) \Leftrightarrow \\ (\omega_c - p_a - p_{b1}) + (\omega_b - p_{b1}) &\geq (\omega_c - p_a - p_{b1}) \Leftrightarrow \\ p_{b1} &\leq \omega_b \end{aligned} \tag{1}$$

and

$$\begin{aligned} \pi_C(q_{b1} = 2_{c,b}) &\geq \pi_C(q_{b1} = 0) \Leftrightarrow \\ (\omega_c - p_a - p_{b1}) + (\omega_b - p_{b1}) &\geq (-p_a) \Leftrightarrow \\ 2p_{b1} &\leq \omega_c + \omega_b. \end{aligned} \tag{2}$$

By Assumption 1, $\omega_c > \omega_b \Rightarrow \frac{\omega_c + \omega_b}{2} > \omega_b$. Therefore, condition (1), $p_{b1} \leq \omega_b$, is a necessary and sufficient condition to ensure buyer C purchases two units of b: one to create c and one to use alone.

Buyer C purchases one unit of b as long as

$$\begin{aligned} \pi_C(q_{b1} = 1_c) &> \pi_C(q_{b1} = 2_{c,b}) \Leftrightarrow \\ \omega_c - p_a - p_{b1} &> (\omega_c - p_a - p_{b1}) + (\omega_b - p_{b1}) \Leftrightarrow \\ \omega_b &< p_{b1} \end{aligned} \tag{3}$$

and

$$\begin{aligned} \pi_C(q_{b1} = 1_c) &\geq \pi_C(q_{b1} = 0) \Leftrightarrow \\ \omega_c - p_a - p_{b1} &\geq -p_a \Leftrightarrow \\ p_{b1} &\leq \omega_c. \end{aligned} \tag{4}$$

Therefore, by conditions (3) and (4), as long as $\omega_b < p_b \leq \omega_c$, buyer C purchases one unit of b to create composite c.

The highest price at which seller B sells two units is $p_{b1} = \omega_b$, yielding payoff $\pi_B(q_{b1} = 2_{c,b}) = 2\omega_b - \epsilon_b$. The highest price B sells one unit is $p_{b1} = \omega_c$, yielding payoff $\pi_B(q_{b1} = 1_c) = \omega_c - \epsilon_b$. Seller B chooses price $p_{b1} = \{\omega_c, \omega_b\}$ that yields the highest payoff. This price depends on the stand alone value of component b.

Recall, component b has *low stand alone value* if $\omega_c \geq 2\omega_b$ and *high stand alone value* if $\omega_c < 2\omega_b$. Therefore, by Assumption 2(i) ($\omega_c > \omega_b > \epsilon_b$), seller B prices to sell *two* units whenever b has *high stand alone value*:

$$\pi_B(q_{b1} = 2_{c,b}) > \pi_B(q_{b1} = 1_c) \quad \Leftrightarrow \quad 2\omega_b - \epsilon_b > \omega_c - \epsilon_b.$$[6]

Similarly, by Assumption 2(i) ($2\omega_b > \omega_b > \epsilon_b$), seller B prices to sell one unit whenever b has *low stand alone value*:

$$\pi_B(q_{b1} = 1_c) > \pi_B(q_{b1} = 2_{c,b}) \quad \Leftrightarrow \quad \omega_c - \epsilon_b > 2\omega_b - \epsilon_b.$$

■

Proof of Proposition 4. *(No Refund/High SAV_b - No Tragedy, Low SAV_b - Tragedy)*

Assume $\gamma = 0$, $\omega_b > 0$, and $\omega_a = 0$.

part (i): In stage 4 and then stage 3:

(a) if $q_a = 0$, then by Lemma 1, the Nash equilibrium of the second smallest subgame is $\{p^*_{b0} = \omega_b, q^*_{b0} = 2\}$.

(b) if $q_a = 1$, then using the result and proof of Lemma 2, $p^*_{b1} = \omega_b$ and buyer C purchases $q_{b1} = 2_{c,b}$, using one b to create c and one to use alone.

In stage 2, buyer C faces profit of $\pi_C(q_a = 0) = 0$ and $\pi_C(q_a = 1) = \omega_c - \omega_b - p_a$. Therefore, as long as $p_a \leq \omega_c - \omega_b$, buyer C purchases $q_a = 1$.

In stage 1, seller A faces $\pi_A(p_a > \omega_c - \omega_b) = 0$ and $\pi_A(p_a \leq \omega_c - \omega_b) \leq \omega_c - \omega_b - \epsilon_a$. Therefore, by Assumption 2(ii) ($\omega_c - \omega_b > \epsilon_a$), to maximize profit, seller A chooses $p^*_a = \omega_c - \omega_b$.

The subgame perfect equilibrium, assuming $\omega_c > \epsilon_b$ and $\omega_c - \omega_b > \epsilon_a$ is:

$$\{p^*_a = \omega_c - \omega_b, (q^*_a = 1, q^*_{b1} = 2_{c,b}, q^*_{b0} = 1), (p^*_{b1} = \omega_b, p^*_{b0} = \omega_b)\}.$$

Along the equilibrium path, $\{p^*_a = \omega_c - \omega_b, (q^*_a = 1, q^*_{b1} = 2_{c,b}), (p^*_{b1} = \omega_b)\}$. Therefore, if b has *high stand alone value*, the composite good is purchased in equilibrium.

part (ii): (a) if $q_a = 0$, then by Lemma 1, the Nash equilibrium of the second smallest subgame is $\{p^*_{b0} = \omega_b, q^*_{b0} = 1\}$.

[6] If transaction costs are marginal rather than fixed, then the result depends on high stand alone value in b *along with* the marginal benefit to supplying the second unit greater than the marginal cost to supplying the second unit. I leave the result in terms of fixed costs to keep focus on stand alone value.

(b) if $q_a = 1$, then in stage 4, buyer C purchases $q_{b1} = 1_c$ as long as $\omega_b \leq p_{b1} \leq \omega_c$. Buyer C purchases $q_{b1} = 2_{c,b}$ as long as $p_{b1} \leq \omega_b$ and $2p_{b1} \leq \omega_c + \omega_b$. Since $\frac{\omega_c+\omega_b}{2} > \omega_b$, buyer C purchases $q_{b1} = 2_{c,b}$ as long as $p_{b1} \leq \omega_b$.

Lemma 2 shows that when component b has *low stand alone value*, seller B prices at the monopoly price for composite good c. Therefore, by Assumption 2(i) ($\omega_c > \omega_b > \epsilon_b$), seller B chooses $p_b^* = \omega_c$.

The remainder of the proof follows the proof of Proposition 1 (and hence, the proof of Proposition 3).

In stage 2, buyer C faces profit $\pi_C(q_a = 0) = 0$ and $\pi_C(q_a = 1) = -p_a$. As long as seller A prices at $p_a \leq 0$, then buyer C purchases component a. However, in stage 1, by the assumption of transaction costs, at price at $p_a \leq 0$, seller A's surplus is negative. Therefore, seller A chooses $p_a^* > 0$ to maximize surplus.

Assuming b has low stand alone value, the subgame perfect equilibria, assuming $2\omega_b > \epsilon_b$, are:

$$\{p_a^* > 0, (q_a^* = 0, q_{b1}^* = 1_c, q_{b0}^* = 2), (p_{b1}^* = \omega_c, p_{b0}^* = \omega_b)\}.$$

Along the equilibrium path, component a is not purchased. Therefore, if b has low stand alone value, the composite good is not purchased in equilibrium, but stand alone units of b (two, in this case) are purchased in equilibrium. ■

Proof of Proposition 5. *(No Refund/SAV$_b$: Price Discrimination - Tragedy)*

Assume $\gamma = 0$, $\omega_b > 0$, and $\omega_a = 0$. Let p_{b1c} be the price B chooses, corresponding to the purchase of b to use to form c. Let p_{b1b} be the price B chooses, corresponding to the purchase of b to use alone. In stage 4 and then stage 3:

(i) if $q_a = 0$, then by a similar result to Lemma 1 (but extending $q_{b0} \in \{0, 1, 2\}$), the Nash equilibrium of the second smallest subgame is $\{p_{b0}^* = \omega_b, q_{b0}^* = 2\}$.

(ii) if $q_a = 1$, in stage 4, by Assumption 3, buyer C purchases $q_{b1} = 1_c$ as long as $p_{b1c} \leq \omega_c$ and $p_{b1b} \geq \omega_b$. Buyer C purchases $q_{b1} = 2_{c,b}$ as long as $p_{b1b} \leq \omega_b$ and $p_{b1c} + p_{b1b} \leq \omega_c + \omega_b$.

Therefore, by Assumption 1 and Assumption 2(i) ($\omega_c > \omega_b > \epsilon_b$) in stage 3, seller B chooses $p_{b11}^* = \omega_c$ for the first unit of b, since seller B knows buyer C prefers to use this unit with the one-unit of a purchased in stage 2. For the second unit of b purchased by buyer C, seller B can price up to buyer C's reservation value of using b alone. Therefore, by Assumption 2(i) ($\omega_b > \epsilon_b$), $p_{b12}^* = \omega_b$.

Based on these optimal pricing decisions by seller B, buyer C's payoffs for any number of units of b purchased is $-p_a$. Therefore, in stage 2, by Assumption 3, the indifference rule, buyer C purchases $q_a = 1$ only if $p_a \leq 0$. However, $p_a \leq 0$ is suboptimal for seller A given transaction costs are positive. Therefore, A chooses $p_a^* > 0$ to maximize surplus.

The subgame perfect equilibria, assuming $\omega_b > \epsilon_b$, are:

$$\{p_a^* > 0, (q_a^* = 0, q_{b1}^* = 1_c, q_{b0}^* = 2), (p_{b11}^* = \omega_c, p_{b12}^* = \omega_b, p_{b0}^* = \omega_b)\}.$$

Along the equilibrium path, component a is not purchased. Therefore, in equilibrium, the composite good is not purchased, but stand alone units of b (two in this case) are purchased. ■

Proofs of Proposition 6 and Proposition 7. *(Partial and Full Refunds)*

Each proof of full refund builds from the proofs of partial refund. Therefore, I complete the proofs for partial and full refunds in the following order: (I.a) and (I.b): perfect complements; (II.a) and (II.b): SAV_b; (III.a) and (III.b): SAV_a; and (IV.a) and (IV.b): SAV_{ab}.

(I.a) Perfect complements and partial refund: Assume $\omega_b = 0$, $\omega_a = 0$ and $0 < \gamma < 1$. In stage 4 and then stage 3:

(i) if $q_a = 0$, then by Lemma 1, the Nash equilibria of the second smallest subgame is $\{p_{b0}^* > \epsilon_b, q_{b0}^* = 0\}$.

(ii) if $q_a = 1$, then in stage 4, buyer C never chooses $q_{b1} = 0$ or $q_{b1} = 1_{a,b}$. By Assumption 3, the indifference rule, buyer C chooses $q_{b1} = 1_c$ as long as

$$\omega_c - \gamma p_a \geq p_{b1} \quad \text{and} \quad \omega_c - \gamma p_a > 0.$$

Buyer C chooses $q_{b1} = 1_{\not{a},b}$ as long as

$$p_{b1} \leq 0 \quad \text{and} \quad \omega_c - \gamma p_a < 0.$$

In stage 3, buyer B maximizes payoffs given the optimal behavior of buyer C in stage 4. Seller B faces

$$\pi_B(q_{b1} = 1_c) = p_{b1} - \epsilon_b = \omega_c - \gamma p_a - \epsilon_b,$$

$$\pi_B(q_{b1} = 0_{\not{a}}) = 0, \quad \text{and}$$

$$\pi_B(q_{b1} = 1_{\not{a},b}) = p_{b1} = \epsilon_b = -\epsilon_b.$$

Therefore, seller B maximizes payoffs choosing

$$p_{b1}^* = \begin{cases} \omega_c - \gamma p_a & \text{if } \omega_c - \gamma p_a - \epsilon_b > 0 \\ 0 & \text{if } \omega_c - \gamma p_a - \epsilon_b \leq 0 \end{cases}$$

In stage 2, buyer C faces the following payoffs:

$$\text{if } q_a = 0\text{: } \pi_C = 0$$

and

$$\text{if } q_a = 1\text{: } \begin{cases} \pi_C(q_{b1} = 1_c) = \omega_c - p_a - p_{b1}^* = \omega_c - p_a - \omega_c + \gamma p_a = p_a(\gamma - 1) \\ \quad \text{if } \omega_c - \gamma p_a - \epsilon_b > 0 \quad \text{and} \\ \pi_C(q_a = 1, q_{b1} = 0_{\not{c}}) = p_a(\gamma - 1) \\ \quad \text{if } \omega_c - \gamma p_a - \epsilon_b \leq 0. \end{cases}$$

Therefore, as long as $p_a(\gamma - 1) \geq 0$, then buyer C chooses $q_a = 1$. Under a partial refund, $p_a(\gamma - 1) \geq 0$ only for $p_a \leq 0$.

In stage 1, seller A solves

$$\max_{p_a} \{p_a - \epsilon_a, 0\} \quad \text{s.t} \quad p_a \leq 0.$$

Therefore, since there exists no positive price for which seller A achieves positive payoffs, seller A chooses $p_a^* > \epsilon_a$.

The subgame perfect equilibria for the case of partial refund and perfect complementarities are:

$$\text{SPNE} = \begin{cases} p_a^* > \epsilon_a \\ (q_a^* = 0, q_{b0}^* = 0, q_{b1}^* = 1_c) & \text{if } p_{b1}^* \leq \omega_c - \gamma p_a^* \text{ and } \gamma p_a^* < \omega_c. \\ (q_a^* = 0, q_{b0}^* = 0, q_{b1}^* = 0) & \text{if } p_{b1}^* = 0 \text{ and } \omega_c < \gamma p_a^* \\ (p_{b0}^* > \epsilon_b, p_{b1}^* = \omega_c - \gamma p_a^*) & \text{if } \gamma p_a^* < \omega_c - \epsilon_b \\ (p_{b0}^* > \epsilon_b, p_{b1}^* = 0) & \text{if } \gamma p_a^* > \omega_c - \epsilon_b \end{cases}$$

Along the equilibrium path,

$$\{p_a^* > \epsilon_a, q_{b0}^* = 0, p_{b0}^* > \epsilon_b\}.$$

In equilibrium, nether the composite nor either component is purchased. Equilibrium payoffs are $\pi_A^* = 0, \pi_C^* = 0, \pi_B^* = 0$.

(I.b) Perfect complements and full refund: Now assume $\gamma = 1$. The proof follows that from (I.a) up to stage 2. At stage 2, recall as long as $p_a(\gamma - 1) \geq 0$ then buyer C chooses $q_a = 1$. Under full refund, this condition holds for all $p_a \geq 0$. In stage 1, seller A

chooses $p_a \geq 0$; however, seller A must price to ensure seller B prices to sell b so buyer C chooses $q_{b1} = 1_c$ and not $q_{b1} = 1_{\not{a},b}$. If seller A prices higher than $\omega_c - \epsilon_b$ then seller B's profits are negative. Therefore, per seller B's optimal stage 3 behavior, seller A chooses $p_a < \omega_c - \epsilon_b$. Let $\delta > 0$ be the added surplus seller A leaves on the table for seller B to optimally price so C chooses $q_a = 1_c$ and to ensure seller B is not indifferent. Then seller A chooses $p_a^* = \omega_c - \delta - \epsilon_b$ as long as $\omega_c - \delta - \epsilon_b - \epsilon_a > 0$.

Therefore, the subgame perfect Nash equilibrium under a full refund and perfect complements, assuming $\delta > 0$ and $\omega_c - \delta - \epsilon_b - \epsilon_a > 0$ is:

$$\{p_a^* = \omega_c - \delta - \epsilon_b, (q_a^* = 1, q_{b0}^* = 0, q_{b1}^* = 1_c), (p_{b0}^* > \epsilon_b, p_{b1}^* = \epsilon_b + \delta)\}.$$

Therefore, in equilibrium, buyer C purchases the composite good. Equilibrium payoffs are:

$$\pi_A^* = \omega_c - \epsilon_b - \delta - \epsilon_a, \quad \pi_C^* = 0, \quad \pi_B^* = \delta.$$

Therefore, in equilibrium, when no component has stand alone value, for any refund less than full (zero or partial), buyer C does not purchase the composite good, and for a full refund, buyer C purchases the composite good.

(II.a) ***SAV_b* and partial refund:** Assume $\omega_b > 0$, $\omega_a = 0$, and $0 < \gamma < 1$. In stage 4 and then stage 3:

(i) if $q_a = 0$, then by Lemma 1, the Nash equilibria of the second smallest subgame is $\{p_{b0}^* = \omega_b, q_{b0}^* = 1\}$.

(ii) if $q_a = 1$, then by Assumption 3, the indifference rule, in stage 4, buyer C purchases $q_b = 1$ and keeps component a as long as $p_b \leq \omega_c - \gamma p_a$ and $\omega_c - \omega_b > \gamma p_a$. Buyer C purchases $q_b = 1$ and returns component a whenever $p_b \leq \omega_b$ and $\omega_c - \omega_b < \gamma p_a$.

In stage 3, seller B chooses p_b to maximize surplus. Let p_{bb_1} and p_{bb_2} be two prices seller B chooses to offer, determined in the following way. The highest price seller B can offer when $\omega_c - \omega_b > \gamma p_a$ is $p_{bb_1} = \omega_c - \gamma p_a$. The highest price seller B can offer when $\omega_c - \omega_b < \gamma p_a$ is $p_{bb_2} = \omega_b$. Seller B chooses $p_{bb_1} = \omega_c - \gamma p_a$ whenever $\omega_c - \omega_b > \gamma p_a$, yielding $\pi_B(q_b^{keep_a}) = \omega_c - \gamma p_a - \epsilon_b$. Seller B chooses $p_{bb_2} = \omega_b$ whenever $\omega_c - \omega_b < \gamma p_a$, yielding $\pi_B(q_b^{ret_a}) = \omega_b - \epsilon_b$. Seller B's optimal price $p_{b1}^* \in \{p_{bb_1}, p_{bb_2}\}$ depends on the price chosen by seller A.

Under a partial refund assumption, in stage 2, buyer C chooses $q_a = \{0, 1\}$ to maximize surplus. Buyer C faces $\pi_C(q_a = 1) = \gamma p_a - p_a$ and $\pi_C(q_a = 0) = 0$. By the indifference rule, as long as $\gamma p_a - p_a = p_a(\gamma - 1) \geq 0$, then buyer C chooses $q_a = 1$. Clearly, unless $\gamma = 1$, this inequality does not hold. Therefore, for any $\gamma \in [0, 1)$, when component b has stand alone value, regardless of the price chosen by seller A, buyer C never chooses to purchase component a, resulting in hold-up. The subgame perfect Nash equilibria in the case of positive SAV_b and partial refund are:

$$\text{SPNE} = \begin{cases} p_a^* > \epsilon_a \\ (q_a^* = 0, q_{b0}^* = 1, q_{b1}^* = 1_c) & \text{if } p_{b1}^* \leq \omega_c - \gamma p_a^* \text{ and } \gamma p_a^* < \omega_c - \omega_b. \\ (q_a^* = 0, q_{b0}^* = 0, q_{b1}^* = 1_{\not{a}b}) & \text{if } p_{b1}^* \leq \omega_b \text{ and} \omega_c - \omega_b < \gamma p_a^*) \\ (p_{b0}^* = \omega_b, p_{b1}^* = \omega_c - \gamma p_a^*) & \text{if } \gamma p_a^* < \omega_c - \omega_b \\ (p_{b0}^* = \omega_b, p_{b1}^* = \omega_b) & \text{if } \gamma p_a^* > \omega_c - \omega_b \end{cases}$$

In equilibrium, composite good c is not purchased, but stand alone units of b are purchased. Equilibrium payoffs are

$$\pi_A^* = 0, \quad \pi_C^* = 0, \quad \pi_B^* = \omega_b - \epsilon_b.$$

(II.b) ***SAV_b*** **and full refund:** Now assume $\gamma = 1$. Under a full refund assumption, if seller A offers a full refund, then in stage 2, when buyer C chooses $q_a = \{0, 1\}$, by the indifference rule, buyer C chooses $q_a = 1$ for any $p_a \geq 0$. In order for seller A to maximize profits, seller A chooses p_a in such a way that seller B chooses p_{b_1} (buyer C keeps a) rather than p_{b_2}. Therefore, seller A chooses $p_a^* < \omega_c - \omega_b$ to ensure seller B's payoff is highest when buyer C keeps component a. Assuming p_a^* is δ smaller than $p_a^* = \omega_c - \omega_b$, then seller B chooses $p_{b1} = \omega_c - p_a^* = \omega_b + \delta$. Seller A must leave just more than ω_b on the table to give seller B incentive to price to sell $q_{b1} = 1_c$ rather than $q_{b1} = 1_{\not{a},b}$.

In equilibrium, assuming $\omega_c - \omega_b - \epsilon_a > \delta > 0$,

$$\{p_a^* = \omega_c - \omega_b - \delta, (q_a^* = 1, q_{b0}^* = 1, q_{b1}^* = 1_c), (p_{b0}^* = \omega_b, p_{b1}^* = \omega_b + \delta)\}$$

Composite good c is purchased in equilibrium. Equilibrium payoffs are:

$$\pi_A^* = \omega_c - \epsilon_b - \delta - \epsilon_a, \quad \pi_C^* = 0, \quad \pi_B^* = \delta.$$

Therefore, in equilibrium, when only component b has stand alone value, for any refund less than full (zero or partial), buyer C does not purchase the composite good, and for a full refund, buyer C purchases the composite good.

(III.a) ***SAV_a*** **and partial refund:** Assume $\omega_a > 0$, $\omega_b = 0$, and $0 < \gamma < 1$. In stage 4 and then stage 3:

(i) if $q_a = 0$, then by Lemma 1, the Nash equilibria of the second smallest subgame is $\{p^*_{b0} > \epsilon_b, q^*_{b0} = 0\}$.

(ii) if $q_a = 1$, in stage 4, buyer C chooses $q_{b1} = 1_c$ as long as $p_{b1} \leq \omega_c - \max\{\omega_a, \gamma p_a\}$ and $\gamma p_a < \omega_c$. In stage 3, seller B chooses $p_{b1} > \epsilon_b$ to maximize profit. To sell one unit of b, $p_{b1} \in \{0, \omega_c - \max\{\omega_a, \gamma p_a\}\}$. Since $\pi_B(p_{b1} = 0) = -\epsilon_b$, seller B would always choose to sell zero units of b than price at zero. Therefore, as long as $\omega_c - \max\{\omega_a, \gamma p_a\} - \epsilon_b > 0$ then seller B chooses $p^*_b = \omega_c - \max\{\omega_a, \gamma p_a\}$ so buyer C purchases b to form c.

In stage 2, by the indifference rule, buyer C chooses $q_a = 1$ as long as $p_{b1} = \omega_c - \max\{\omega_a, \gamma p_a\}$ and $\pi_C(q_a = 1, q_{b1} = 1_c) = \max\{\omega_a, \gamma p_a\} - p_a \geq 0$.

In stage 1, seller A chooses p_a to maximize payoffs. Seller A prices to sell a as long as $p^*_a > \epsilon_a$. If seller A chooses $p_a = \omega_a$ then buyer C purchases and keeps a. However, since buyer C values composite c more than component a, seller A can price higher than ω_a and still sell a. Seller A chooses p_a so buyer C chooses $q_a = 1$ and seller B chooses p_{b1} so that $q_{b1} = 1_c$. This means p^*_a must satisfy (1) $\max\{\omega_a, \gamma p^*_a\} \geq p^*_a$; (2) $\omega_c - \max\{\omega_a, \gamma p^*_a\} - \epsilon_b > 0$; and (3) $\gamma p_a \leq \omega_c$.

In the case of partial refund, seller A must choose p_a so $\omega_a = \max\{\omega_a, \gamma p^*_a\}$; otherwise, there exists no p_a such that condition (1) is satisfied. The only p_a for which condition (1) holds is $p_a = \omega_a$. Assuming ϵ_b is small enough, for $p_a = \omega_a$, condition (2) is satisfied. Condition (3) is satisfied since $\omega_a < \omega_c$. Therefore, assuming ϵ_a is small enough, A maximizes payoffs by choosing $p^*_a = \omega_a$.

Under partial refund, by Assumption 2(i) and (ii), ($\omega_a > \epsilon_a$ and $\omega_c - \omega_a > \epsilon_b$), the subgame perfect equilibrium under a partial refund when only component a has stand alone value is

$$\{p^*_a = \omega_a, (q^*_a = 1, q^*_{b0} = 0, q^*_{b1} = 1_c), (p^*_{b0} > 0, p^*_{b1} = \omega_c - \omega_a)\}.$$

The composite good is purchased in equilibrium under partial refund, and the equilibrium payoffs are

$$\pi^*_A = \omega_a - \epsilon_a,\ \pi^*_C = 0,\ \pi^*_B = \omega_c - \omega_a - \epsilon_b.$$

(III.b) SAV_a **and full refund:** Now assume $\gamma = 1$. In the case of full refund, condition (1), above, is satisfied for any p_a. Therefore, it is possible for seller A to price higher than ω_a, but condition (2), and hence condition (3), are satisfied if and only if $p_a < \omega_c - \epsilon_b$. By Assumption 2(ii) where $\omega_c - \omega_a > \epsilon_b$, then for $\delta < \omega_c - \epsilon_b$, seller A maximizes payoff by choosing $p^*_a = \omega_c - \epsilon_b - \delta$.

Now, $\omega_c - \epsilon_b - \delta > \epsilon_a$ implies $\delta < \omega_c - \epsilon_b$. Under full refund, by Assumption 2(ii)

$(\omega_c - \epsilon_b > \omega_a)$ and assuming $\omega_c - \epsilon_b - \epsilon_a > \delta > 0$, the subgame perfect equilibrium is

$$\{p_a^* = \omega_c - \epsilon_b - \delta, (q_a^* = 1, q_{b0}^* = 0, q_{b1}^* = 1_c), (p_{b0}^* > \epsilon_b, p_{b1}^* = \epsilon_b + \delta)\}.$$

The composite good is purchased in equilibrium, and the equilibrium payoffs are

$$\pi_A^* = \omega_c - \epsilon_b - \delta - \epsilon_a,\ \pi_C^* = 0,\ \pi_B^* = \delta.$$

Therefore, in equilibrium, when only component a has stand alone value, for any refund (zero, partial or full), buyer C purchases the composite good.

(IV.a) SAV_{ab} **and partial refund:** Assume $\omega_a > 0$, $\omega_b > 0$, and $0 < \gamma < 1$. When both components have stand alone value, the proof for a partial refund is similar to that when only component a has stand alone value. The stand alone value in a is enough to overcome hold-up by seller B, and the partial refund only affects equilibrium payoffs. The subgame perfect Nash equilibrium along the equilibrium path is the same as the case of SAV_a:

$$\{p_a^* = \omega_a, (q_a^* = 1, q_{b0}^* = 1, q_{b1}^* = 1_c), (p_{b0}^* = \omega_b, p_{b1}^* = \omega_c - \omega_a)\}$$

Composite c is purchased in equilibrium, and equilibrium payoffs are

$$\pi_A^* = \omega_a - \epsilon_a, \quad \pi_C^* = 0, \quad \pi_B^* = \omega_c - \omega_a - \epsilon_b.$$

(IV.b) SAV_{ab} **and full refund:** Now assume $\gamma = 1$. When both components have stand alone value and a full refund is offered, the proof is similar to that of stand alone value in only component b. Seller A must account for the fact that stand alone value in b provides leverage to seller B that prevents seller A from taking all of buyer C's surplus. Seller A must leave enough surplus on the table for seller B to have incentive to price to sell b so buyer C does not return a $(q_{b1} = 1_c)$ rather than C purchase b to use alone and return a (i.e does not choose $q_{b1} = 1_{\not{a},b}$). In equilibrium, assuming $\omega_c - \omega_b - \epsilon_a > \delta > 0$,

$$\{p_a^* = \omega_c - \omega_b - \delta, (q_a^* = 1, q_{b0}^* = 1, q_{b1}^* = 1c), (p_{b0}^* = \omega_b, p_{b1}^* = \omega_b + \delta)\}$$

Equilibrium payoffs are:

$$\pi_A^* = \omega_c - \epsilon_b - \delta - \epsilon_a, \quad \pi_C^* = 0, \quad \pi_B^* = \delta$$

and the composite good is purchased in equilibrium. ∎

Therefore, in equilibrium, when both components a and b have stand alone value, for any refund (zero, partial or full), buyer C purchases the composite good.

Proof of Corollary 1.

Seller A chooses a full refund if the equilibrium payoff to seller A under a zero or partial refund is lower than the equilibrium payoff to seller A under a full refund.

(i) As long as $\omega_c - \omega_a > \delta$ then

$$\pi_A(SAV_a, \gamma = 1) = \omega_c - \delta - \epsilon_a \quad > \quad \pi_A(SAV_a, \gamma < 1) = \omega_a - \epsilon_a.$$

Since $\delta > \epsilon_b$ when component a has stand alone value, then either seller B's transaction costs cannot be too high or component a's stand alone value cannot be too high.

(ii) As long as $\omega_c - (\omega_a + \omega_b) > \delta$ then

$$\pi_A(SAV_{ab}, \gamma = 1) = \omega_c - \omega_b - \delta - \epsilon_a \quad > \quad \pi_A(SAV_a, \gamma < 1) = \omega_a - \epsilon_a.$$

Therefore, either the net value of the composite must be greater than the amount of surplus seller A must leave on the table for seller B, or the values of components a and b cannot be too high.

For (i) and (ii), if the gains from offering a full refund are greater than the surplus foregone in order to ensure B prices to sell b with a, then seller A prefers a full refund. ■

Proof of Corollary 2.

The equilibrium outcome of both a game of SAV_a and game of SAV_{ab} yields equilibrium surplus

$$\pi_A^* = \omega_a - \epsilon_a \quad \text{and} \quad \pi_B^* = \omega_c - \omega_a - \epsilon_b.$$

By definition of *high (low) stand alone value* in component a, $\omega_c < (>) 2\omega_a$. Assuming transaction costs are equivalent, $\epsilon_a = \epsilon_b = \epsilon$, then

$$\begin{aligned}
\omega_c &< (>) \; 2\omega_a && \Leftrightarrow \\
\omega_c - \omega_a &< (>) \; \omega_a && \Leftrightarrow \\
p_{b1}^* &< (>) \; p_a^* && \Leftrightarrow \\
\pi_B^* + \epsilon_b &< (>) \; \pi_A^* + \epsilon_a && \Leftrightarrow \\
\pi_B^* &< (>) \; \pi_A^*
\end{aligned}$$

which proves the claim. ■

Proof of Corollary 3.

If seller A offers a full refund and at least component b has stand alone value, then seller B realizes equilibrium payoffs

$$\pi_B^* = \omega_b + \delta - \epsilon_b$$

while seller A realizes equilibrium payoffs

$$\pi_A^* = \omega_c - \omega_b - \delta - \epsilon_a.$$

Therefore,

$$\begin{aligned} \pi_B^* &> \pi_A^* &\Leftrightarrow \\ \omega_b + \delta - \epsilon_b &> \omega_c - \omega_b - \delta - \epsilon_a &\Leftrightarrow \\ (\delta + \epsilon_a) + (\delta - \epsilon_b) &> \omega_c - 2\omega_b. \end{aligned}$$

Assuming $\delta > \epsilon_b$, the first term above is positive. Assuming high stand alone value in b, the second term is negative. Therefore, the result holds. ■

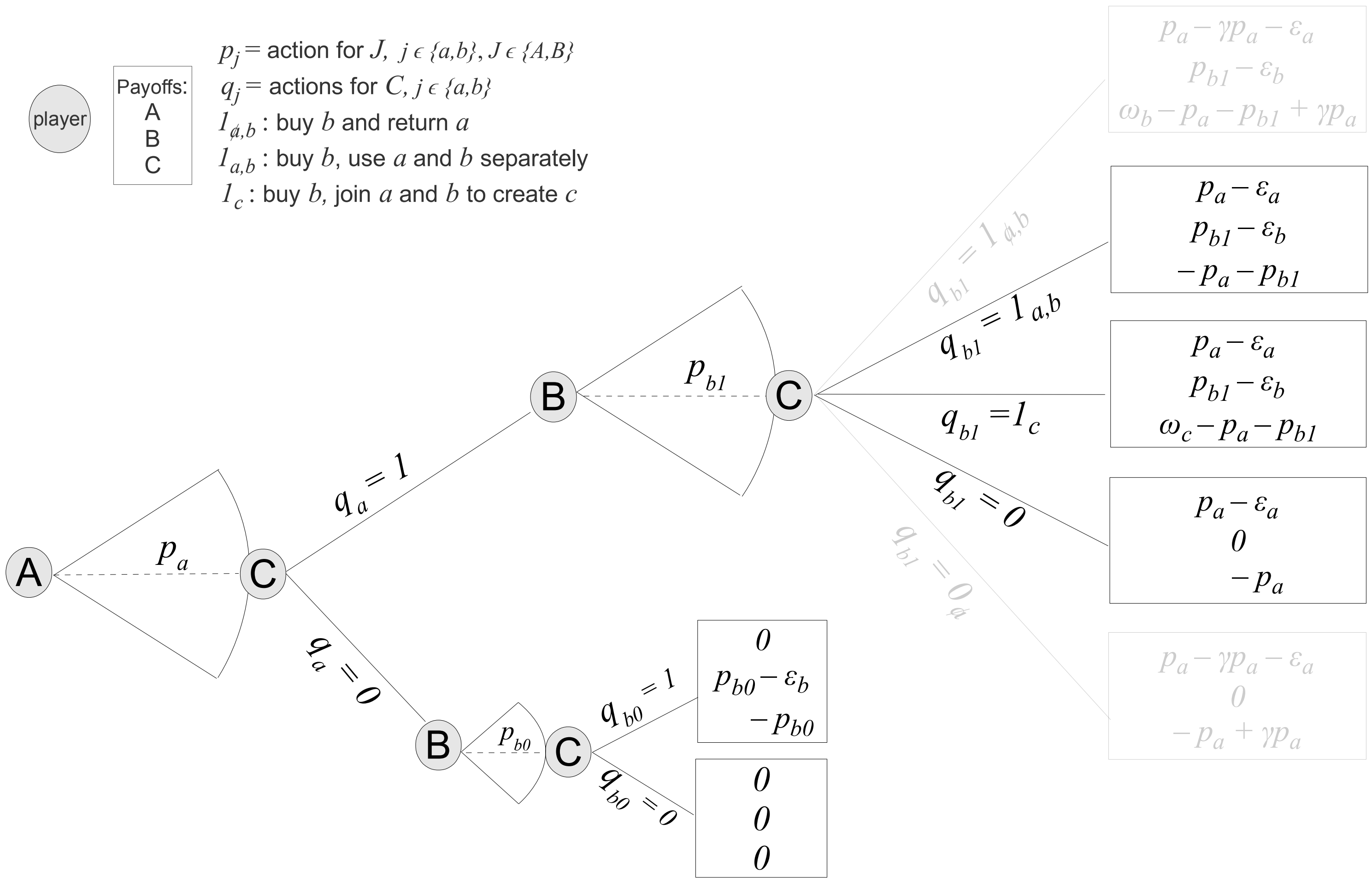

Figure 6: Game of no refund with perfect complements

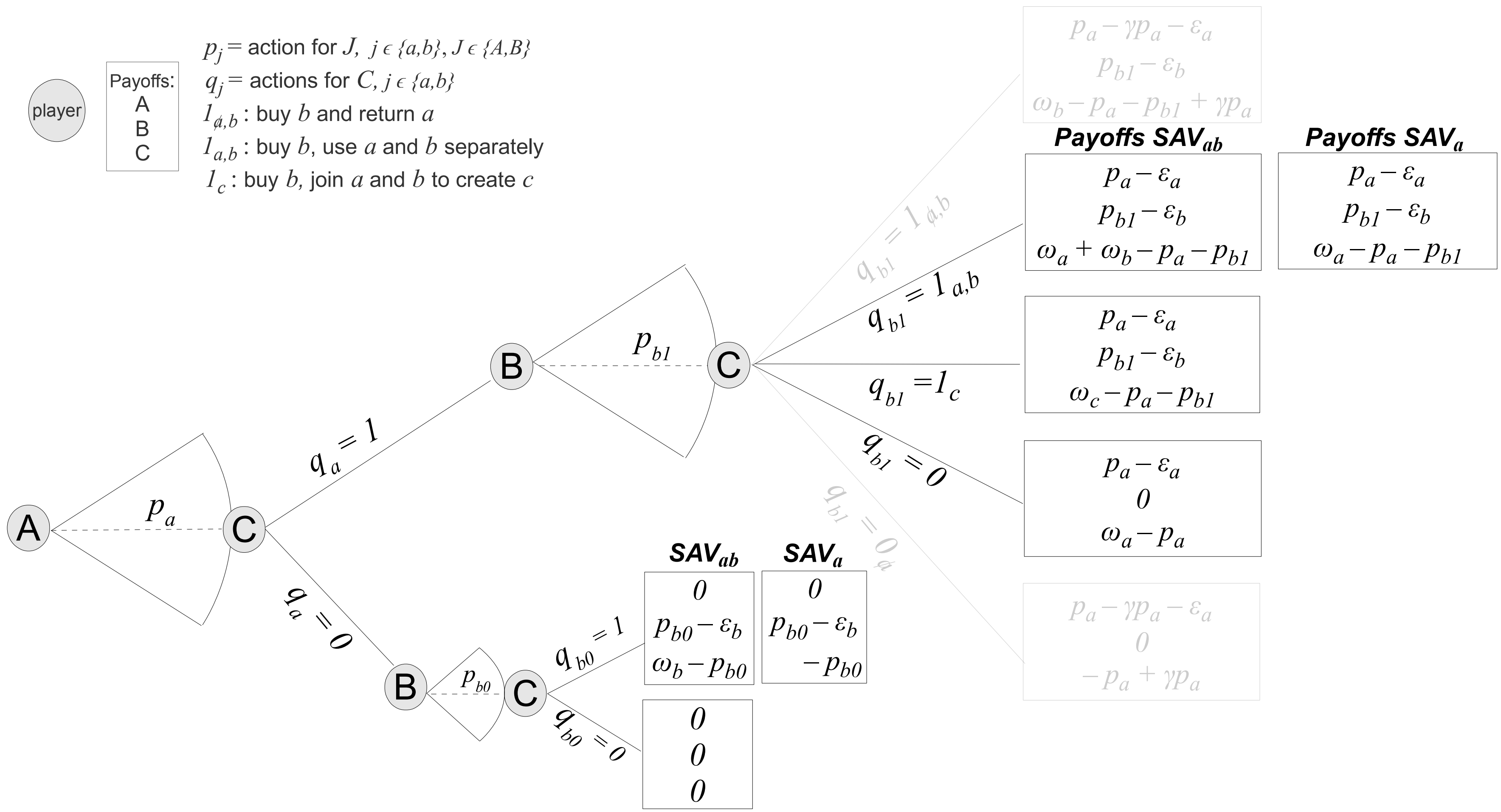

Figure 7: Game of no refund with SAV_{ab} and SAV_a

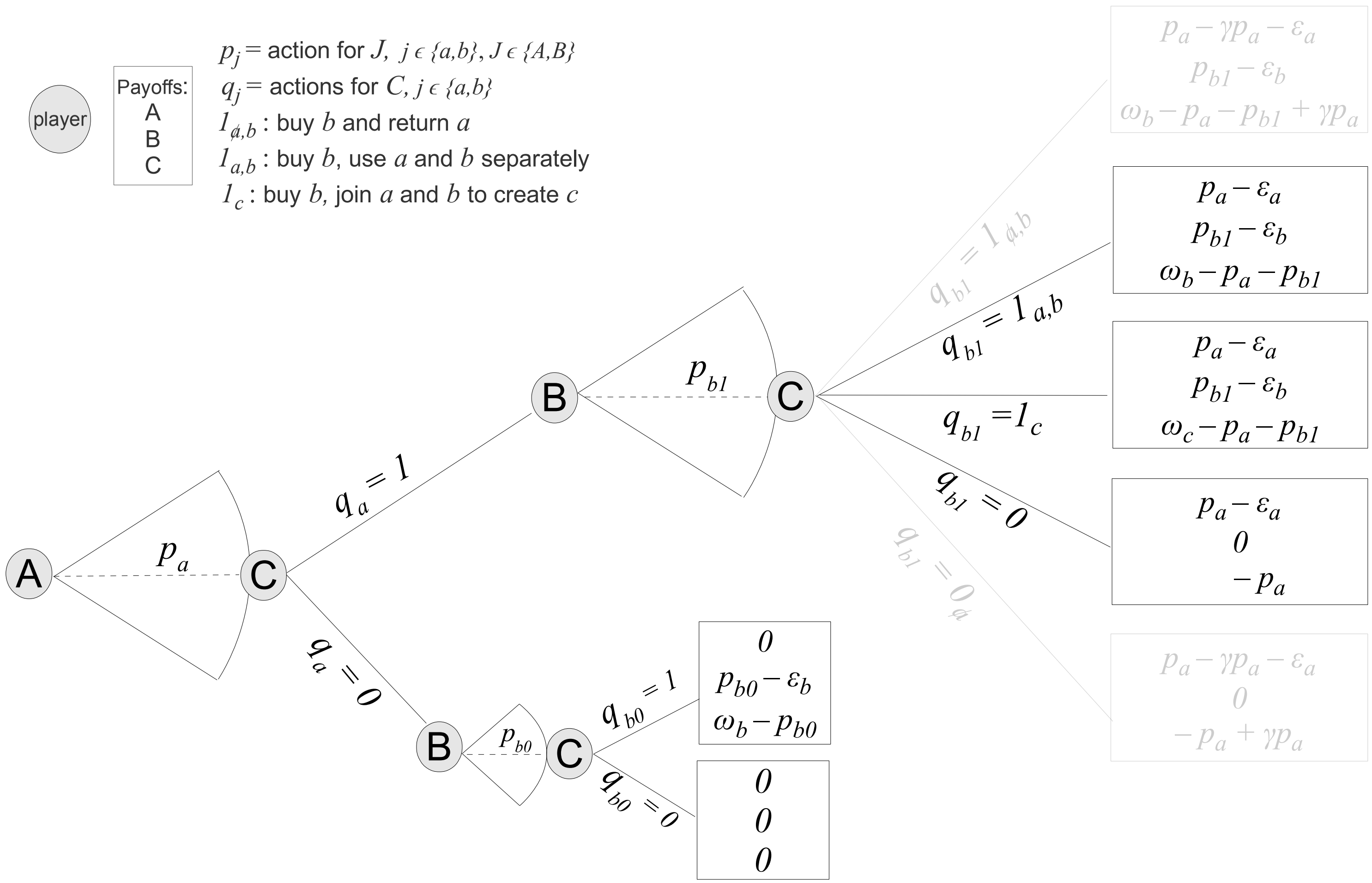

Figure 8: Game of no refund and SAV_b

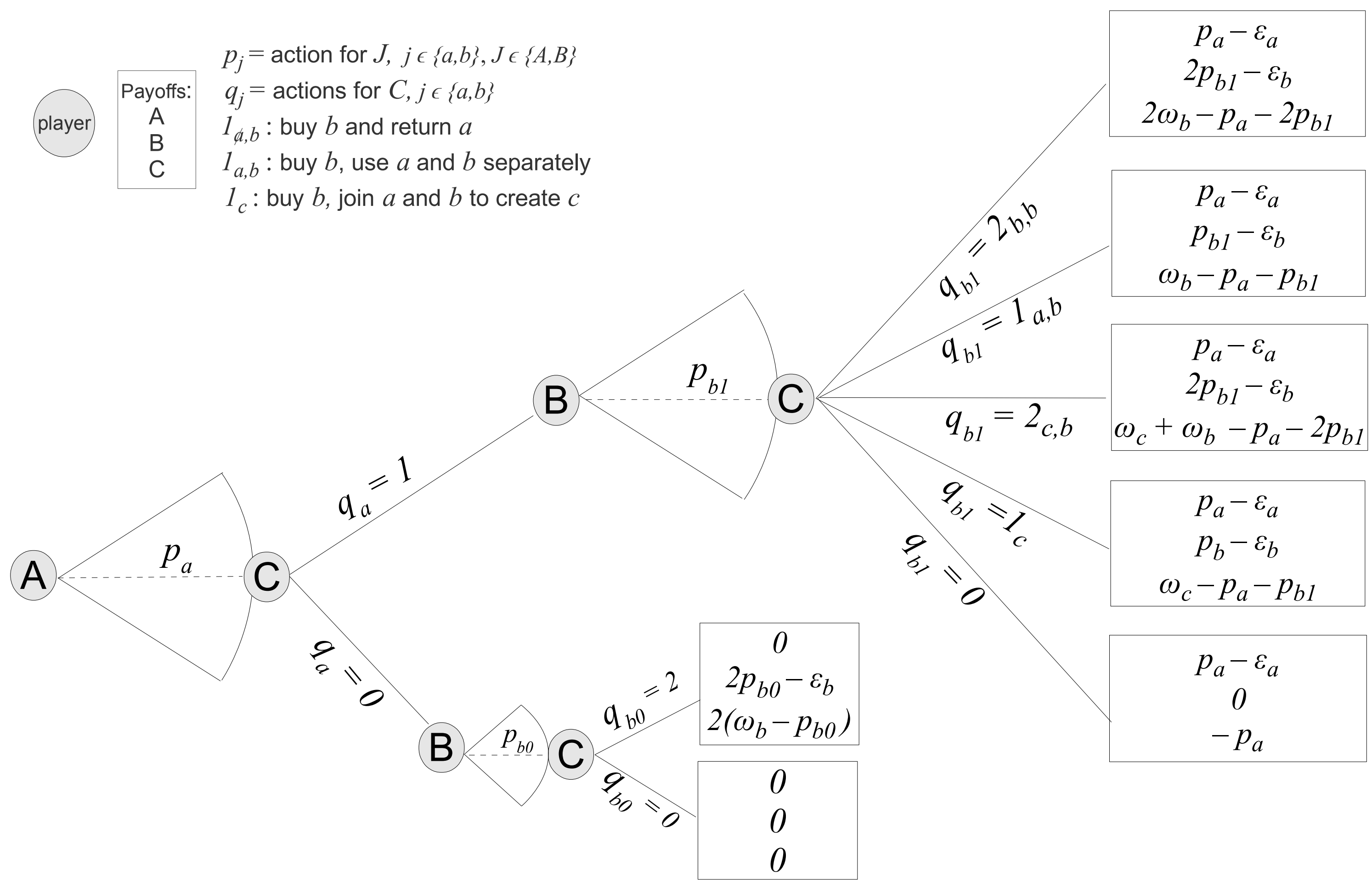

Figure 9: Game of no refund and SAV_b with unconstrained capacity in b and no price discrimination

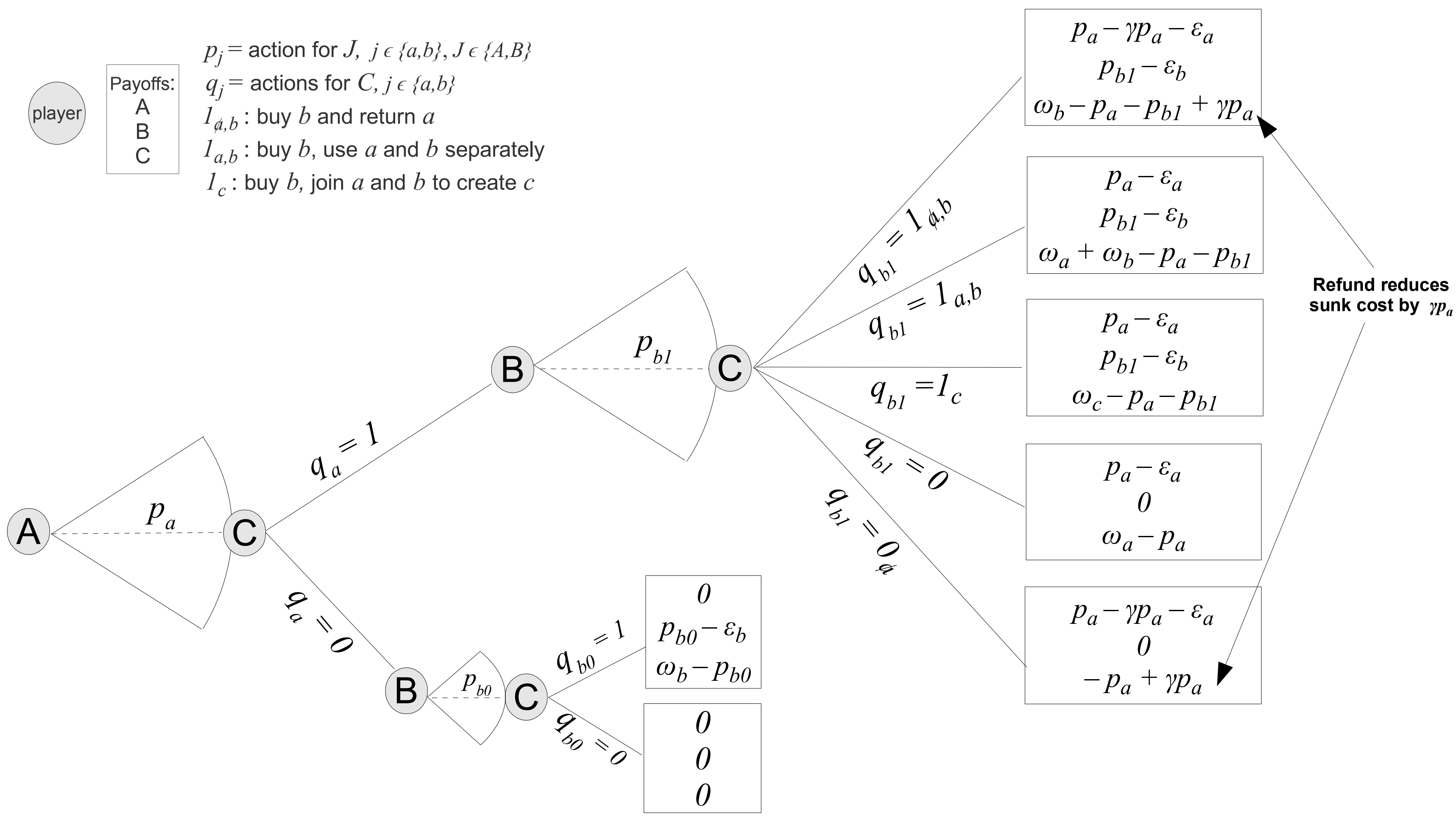

Figure 10: Game of refund $\gamma \in [0, 1]$ with nonnegative SAV_a and SAV_b

Refund level	**Case** PD = Price Discrimination	**Tragedy**	π_A^*	π_B^*	π_C^*
No Refund	Perfect Complements	yes	0	0	0
	SAV_a	no	$\omega_a - \epsilon_a$	$\omega_c - \omega_a - \epsilon_b$	0
	SAV_{ab}	no	$\omega_a - \epsilon_a$	$\omega_c - \omega_a - \epsilon_b$	0
	one unit SAV_b	yes	0	$\omega_b - \epsilon_b$	0
	two units SAV_b/PD	yes	0	$2\omega_b - \epsilon_b$	0
	two units Low SAV_b/No PD	yes	0	$2\omega_b^{\text{low}} - \epsilon_b$	0
	two units High SAV_b/No PD	no	$\omega_c - \omega_b - \epsilon_a$	$2\omega_b^{\text{high}} - \epsilon_b$	0
Partial Refund	Perfect Complements	yes	0	0	0
	SAV_a	no	$\omega_a - \epsilon_a$	$\omega_c - \omega_a - \epsilon_b$	0
	SAV_{ab}	no	$\omega_a - \epsilon_a$	$\omega_c - \omega_a - \epsilon_b$	0
	SAV_b	yes	0	$\omega_b - \epsilon_b$	0
Full Refund	Perfect Complements	no	$\omega_c - \delta - \epsilon_a$	$\delta - \epsilon_b$	0
	SAV_a	no	$\omega_c - \delta - \epsilon_a$	$\delta - \epsilon_b$	0
	SAV_{ab}	no	$\omega_c - \omega_b - \delta - \epsilon_a$	$\omega_b + \delta - \epsilon_b$	0
	SAV_b	no	$\omega_c - \omega_b - \delta - \epsilon_a$	$\omega_b + \delta - \epsilon_b$	0

Table 1: Summary of Results

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