Effect of Market Structure and the Regulatory Franchise in Reputation-Dependent Industries

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Abstract

Practitioners and academics alike assume that the value of reputation is so important to reputation-dependent industries such as credit rating agencies and auditors that they would never put their reputation at risk to collude with a client. However, both the current global financial crisis and the accounting scandals at the beginning of the decade seem to call this assumption into question: even if none of the failures were profit-driven, only one failure of the many led to a firm collapse. This paper explores this assumption by modeling a monopolist certifier, whose certificate of viability is required for capital-constrained entrepreneurs to take their projects to the capital markets for funding. Results suggest that the monopolist will sell fraudulent certificates of viability to at least some fraction of the non-viable projects in at least some periods.

1 Introduction

The first decade of the 21st Century saw massive failures in two critical gatekeepers in the finance industry: the auditors during the accounting scandals in first part of the decade and the credit rating agencies in financial crisis of the latter part of the decade. Auditing and credit rating share some industry characteristics, and the ways in which they failed in their duties are related to those shared characteristics.

It has long been assumed that informational gatekeepers will not risk their reputation for financial inducements. Practitioners often resist criticism and government regulation by making the argument like this one made by KPMG auditor, Brendan Nelson:

> Auditing firms have only one real asset and that is their reputation. Every professional is acutely aware of the need to protect at all costs, without it you have no business, because quite simply you would have no clients. (Nelson, 2010)

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This has been accepted as true in the academic literature surrounding auditors (e.g. DeAngelo, 1981; Datar & Alles, 1999) and credit rating agencies (Mathis et al., 2009). In particular, even in work such as Mathis et al. (2009) which suggest that ratings agencies may indeed risk their reputation for profit, the literature assumes that once it is determined that a firm has been cheating with a client, their other clients will evaporate. The experience of Arthur Andersen's disintegration after its indictment for obstruction of justice in the case of Enron helped confirm this intuition.

But Andersen is more the exception than the rule. There have been many more audit failures by the Big Four where the audit firm appeared willfully incompetent at best, or collusive at worst, and yet the firm lived to see another day. Likewise, despite the critical role played by the Big Three ratings agencies in facilitating the creation of the toxic AAA-rated mortgage-backed securities that led to the financial crisis, not one of them has suffered the evaporation of clients that should be the consequence of such a failure if reputation operates the way we have assumed.

Partnoy (1999, 2006) offers an explanation for why we may see the disjunction between historical experience and theory: in addition to the value added by auditors' or credit rating agencies' reputation, both entities provide a certificate needed for regulatory compliance. This paper explores Partnoy's argument more formally, investigating the possibility that reputation can come uncoupled with quality of monitoring in a fraction of an informational gatekeeper's clients if the conditions are suitable. Both auditors and credit raters provide services that mitigate information asymmetries and carry a signalling value that increases with the reputation for independence and diligence of the service provider. However, large segments of their clients also purchase their services to satisfy regulatory requirements or contractual obligations with third parties. Both providers are organized in relatively tight oligopolies: before the relevant crises, there were five major accounting firms and three credit rating agencies. I demonstrate here that the combination of a regulatory franchise and market power can result in monitoring entities that accept bribes from some clients in exchange for a fraudulent certification.

2 Model

To explore the role market structure plays in the quality decisions of reputation-dependent monitors, I construct a simple model of an entrepreneurial economy that suffers from information asymmetries.

2.1 Setup

Assume an economy with a class of entrepreneurs who have a three period life: t_{it} where $t \in 0, 1, 2$ and $i \in 1...Q$. In every period t, a new class of entrepreneurs enter the market, so they form a market of overlapping generations (but one with no bequests). In t_{i0}, every entrepreneur draws a project whose potential value is determined by: $V_i(v) = v_i\theta$. The element v_i determines the basic viability of the project. With probability π, $v_i = v_g$ and with probability $(1-\pi)$, $v_i = v_b$. For project that are v_g, $P(v_g = 1) = 1$, and for those that are v_b, $P(v_b = 1) = \delta$ and $P(v_b = 0) = (1-\delta)$. The term v_i could be interpreted a measure of the entrepreneur's basic competence that is observable to the entrepreneur but not to the market in t_{i0}, and where incompetent entrepreneurs nevertheless have a (small) chance (δ) of having "dumb luck" and their project succeeding despite their incompetence.

Projects can either stay private or the entrepreneurs can bring them to the capital markets. The main benefit to bringing a project to market is that it allows the full potential value of the project to be realized—private enterprises are capital constrained and cannot therefore be expanded to their full value. The value of θ is the discounted expected payoff of the project in t_{i2} if it is funded by the capital markets. We can assume that $\theta > 0$. If the entrepreneur does not go to the capital markets with his viable project, he can only realize a fraction, $a_i \sim U(\underline{a}, \bar{a})$, of the full potential value of the project, where $0 < \underline{a} < \bar{a} < 1$. If a project is capitalized privately, but fails (i.e. $v_b = 0$), the entrepreneur will lose $c\theta$. The loss, c, is such that:

$$\delta\bar{a}\theta - (1-\delta)c\theta \leq 0.$$

This ensures that those entrepreneurs with unviable projects who do not sell those projects to the capital markets abandon them and earn their reservation wage, which is normalized to zero.

In t_{i1}, every entrepreneur has the option of holding full rights to his project or selling those rights to the capital markets. As a consequence, we can construct a market supply curve for all entrepreneurs, indexed by the price they are willing to sell:

$$\nu_s = \begin{cases} 0 & \text{if} \quad q \leq (1-\pi)Q \\ \underline{a}\theta + \frac{(\bar{a}-\underline{a})\theta}{\pi Q}\left(q - (1-\pi)Q\right) & \text{if} \quad (1-\pi)Q < q \leq Q \end{cases} \tag{1}$$

The capital markets are fully liquid, competitive and frictionless, and the zero profit condition holds for investors. Therefore entrepreneurs that go to the capital markets to raise funds will be offered a price, ν, that is equal to the expected value of the project, conditional on the fact that the entrepreneur has decided to take the project to the markets.

2.2 Information structure

At t_{i0}, every entrepreneur learns v_i and a_i costlessly and perfectly but they are not observed by the market. The value of θ is common knowledge, as is the distribution of a_i and the probability δ.

2.3 Market without signaling

To motivate the role of a signaling service, we can first consider the entrepreneurs' interactions with the capital markets in the absence of such a service, which is essentially a basic adverse selection story. Clearly, entrepreneurs with non-viable projects have nothing to lose by attempting to raise capital at any price. Entrepreneurs with viable projects have the option of being patient and assuming the risk associated with staying private. Therefore, they will only go to the capital markets if they can get a price, ν, for their project that improves their utility. If all entrepreneurs bring their projects to market, the market price for projects will be

$$\nu = (\pi + \delta(1-\pi))\,\theta \equiv \gamma\theta.$$

The full market will participate if the entrepreneurs with the best ability to self-fund who have viable projects decide to participate:

$$\gamma\theta \geq \bar{a}\theta \tag{2}$$

If this condition is not met, some or all of the viable projects will choose to self-fund, with the market valuation of certified projects dropping as viable projects drop out, raising the possibility that the market for projects will dissolve into a lemons market (Akerlof, 1970).

Clearly, those entrepreneurs with viable projects would be willing to pay to differentiate themselves from the non-viable projects if the terms are right. In tension with the demand is the willingness of those entrepreneurs with non-viable projects to go to almost any expense to have the ability to pass themselves off as viable. This sets the stage for an entity that has the capacity to signal project viability to the market, as well as the temptations the signaler will have to dilute the quality of her signal.

2.4 Demand for signaling

Assume now that there exists a certification industry that has a technology to generate a signal, σ, that predicts the viability of projects. The mechanics of the signal are such that the entity will receive a signal that a project is not viable with the following probabilities:

$$P(\sigma = 1 | v = v_b) = \rho$$
$$P(\sigma = 1 | v = v_g) = 0,$$

If they don't receive any signal (i.e. $\sigma = 0$), they certify the project as viable. Assume as well that the signaling technology generates σ at a constant marginal cost normalized to zero. Since the entrepreneurs are capital-constrained, they pay for the certificate out of the money they are able to raise on the market. So if a non-viable project applies for a certification, but the certification firm receives a signal that the project is not viable and therefore refuse to certify, the entrepreneur is protected from the cost of the fee via bankruptcy protection. This assumption follows Mathis et al. (2009) and simplifies the analysis as well as tracking the institutional arrangements of particularly the credit rating agencies. Auditors are more likely to get some or all of their fee even when they issue a "going concern" opinion.[1]

Clearly, entrepreneurs with viable projects would be willing to pay for a certification that their project is indeed viable, if it improves on the market's assessment of expected viability without a signal certification, since this would fetch them the higher price. Therefore, if p is the price of a certificate, entrepreneurs with viable projects are willing to purchase a certificate if $\nu_c - p$ exceeds their next best option.

I assume that the market does not know the details of the signaling technology, meaning that the value of ρ is private information. Instead, it updates its assessment of the value of the signal based on the failure rate of projects that had received the signal certification in the prior period.

At this point, some additional notation is in order. The quantity of certificates sold in time period t to entrepreneurs with viable projects is q_1^t at a price, p_1^t. Since ex ante the certifier does not know who is viable, all entrepreneurs without viable projects will first attempt to "slip through the cracks" and qualify for a certificate without colluding with the certifier, and, in

[1]Going concern opinions assert that the company is not, in the auditor's opinion, a going concern.

expectation, $L \equiv (1-\rho)(1-\pi)Q$ will succeed. If the certification industry chooses to sell false certificates to entrepreneurs they have discovered to have an unviable project, they can choose to sell q_2^t fraudulent certificates at a price p_2^t.

In addition, the market valuation of a project with a certificate, ν_c, can be written as the expected value of a project that carries a certificate where the expectation (m) is formed by the success rate in the previous period:

$$\nu_c = m^t\theta = \frac{q_1^{t-1} + \delta(q_2^{t-1} + L)}{q_1^{t-1} + q_2^{t-1} + L}\theta \tag{3}$$

Financial statement audits are explicitly mandated by law for all public traded companies, and while the purchase of a rating is not mandated explicitly, a large fraction of purchasers in the bond markets are required by some combination of regulation and charter to restrict their purchases to bonds with certain rating levels (or, at minimum, to privilege such purchases), leading to a de facto regulatory requirement for a credit rating. Therefore, viable projects do not have the option of skipping acquiring a signal but still going to the markets for capital. They still have the option of remaining privately funded, however. Entrepreneurs without a viable project also value the certification: they would be happy to dump their non-viable project on the market for any non-negative price. Furthermore, since they have bankruptcy protection, they are willing to accept a price for the certification that exceeds its ex ante expected value, since the price will only be paid if they get the certification. In these scenarios, the inverse demand function for the honest certificate of viability is:

$$p_1^t = \begin{cases} \nu_c^t - \left(\underline{a}\theta + \frac{(\bar{a}-\underline{a})\theta}{\pi Q}\, q_1^t\right) & \text{if} \quad v_i = v_g, \\ \nu_c^t & \text{if} \quad v_i = v_b. \end{cases} \tag{4}$$

If an entrepreneur with a non-viable project is identified by the certifier as non-viable, the entrepreneur would be willing to purchase a fraudulent certificate instead, again for up to the full value they would receive from the markets for a certified project:

$$p_2^t = \nu_c^t. \tag{5}$$

The single-period profit function of a certifier is therefore:

$$\Pi_t = p_1^t(q_1^t + L) + p_2^t q_2^t. \tag{6}$$

However, the certifier must consider how the sale of fraudulent certificates will affect future revenues as well as its benefit for today's revenues. We can set up an infinite horizon model (with no discounting) that characterizes the problem the certifier needs to solve. The certifier will wish to maximize profits over the infinite horizon, subject to the constraint of ensuring that the market for honest certificates does not collapse and, potentially, to the maximum demand for honest and fraudulent certificates. The program is therefore:

$$\begin{aligned} \max_{q_1^t, q_2^t} \Pi^T = {} & m^0\theta(q_1^0 + q_2^0 + L) - \left(\underline{a}\theta + \frac{(\bar{a}-\underline{a})\theta}{\pi Q} q_1^0\right)(q_1^0 + L) \\ & + \sum_{t=1}^{T} m^t\theta(q_1^t + q_2^t + L) - \left(\underline{a}\theta + \frac{(\bar{a}-\underline{a})\theta}{\pi Q} q_1^t\right)(q_1^t + L) \end{aligned} \tag{7}$$

$$s.t. \quad q_1^t \leq \pi Q \tag{8}$$
$$q_2^t \leq \rho(1-\pi)Q \tag{9}$$
$$\forall t \in (0, T)$$

where m^t is as it is defined in (3), and m^0 is the market's Bayesian assessment of the success rate of a certified project. Solving for this program leads directly to the central observation of the paper:

Theorem 1. *In equilibrium, the certifier will wish to sell:*

$$q_1^* = \text{Min}\left\{\frac{1}{2}\left(\frac{(1-\underline{a})}{(\bar{a}-\underline{a})}\pi Q - L\right), \pi Q\right\}$$

and

$$q_2^* = \rho(1-\pi)Q.$$

The interior solution results in a market assessment of the value of a certified project as:

$$\nu_c^* = \frac{(1-\underline{a})\pi - (1-\rho)(1-\pi)(\bar{a}-\underline{a}) + 2\delta(1-\pi)(\bar{a}-\underline{a})}{(1-\underline{a})\pi - (1-\rho)(1-\pi)(\bar{a}-\underline{a}) + 2(1-\pi)(\bar{a}-\underline{a})}\theta$$

In the case of a corner solution, the value is:

$$\nu_c^* = (\pi + \delta(1-\pi))\theta.$$

Proof. See Appendix. □

This result returns the central insight of this paper: even though the certifier's business is entirely built on their reputation, they will nonetheless be willing to sell as many fraudulent certificates (q_2) as possible.

2.5 Changes in certification technology

We can use the results found above to look at the effects of an exogenous shock on the value of ρ, the detection probability of a non-viable project. A comparative statics analysis of the effect of a marginal increase in ρ on the equilibrium values shows that the certifier will sell more of both q_1 and q_2 (except in the cases where q_1 is already at a corner solution):

$$\frac{\partial q_1^*}{\partial \rho} = \frac{(1-\pi)Q}{2} > 0 \tag{10}$$
$$\frac{\partial q_2^*}{\partial \rho} = (1-\pi)Q > 0 \tag{11}$$

The increase in q_2 is a direct substitution of certificates to those unviable firms who had previously "slipped through the cracks" and therefore the effect of improved detection technology does not change the total number of unviable projects that the get certified, only the price the certifier is able to extract from the entrepreneur. The increasing number of honest certificates sold (barring a corner solution) has the effect of improving the success rate of certified projects. This has the interesting effect of an increase in detection technology increasing the success rate while simultaneously increasing the amount of fraud in the system.

3 Discussion

It is clear from the analysis above that just because an information intermediary's value rests entirely in its reputation that it will never debase that reputation. Instead, it is entirely possible that reputation-dependent firms will profit from debasing their reputation by colluding with some clients.

Cartels The formal analysis assumes a monopoly firm, and the examples used to motivate the analysis are oligopolies. While future research into how the dynamics would change given different equilibrium assumptions in an oligopolistic competition model would be informative, there is suggestive evidence that both the audit and the credit rating firms act as a cartel when it comes to setting quality levels. If it is true that they are behaving like a cartel, the monopoly solution is informative.

In a relatively clear example of cartel-like coordination on quality, when Coopers & Lybrand broke ranks with its peers and introduced the service of providing reviews of quarterly statements in addition to the full annual audit, the other firms saw this as an attempt by the firm to differentiate itself with respect to quality. (Whether it would indeed have actually been interpreted by the markets as evidence of higher quality audits is unclear, but that was the fear of the rest of the industry.) In response to the move by Coopers, the American Institution of Certified Public Accountants (AICPA), the professional body of accountants, began to facilitate regular meetings with the largest firms to ensure that any new quality advancements were rolled out in a coordinated manner (Olson, 1982).

In a more personal form of cartel enforcement on quality, there were at least two incidents of heads of firms calling out the profession for the level of quality provided. Leonard Spacek, the mid-century head of Arthur Andersen, and then Joe Connor, head of Price Waterhouse in the 1980s, both spoke out about the need for auditors to assume more responsibility for detecting fraud. For example, Spacek said in a 1957 speech, "Let me state the problem simply and frankly. There is only one reason for misleading financial statements. It is the failure or unwillingness of the public accounting profession to square its so-called principles of accounting with its professional responsibilities to the public" (Spacek, 1957). In both cases, the other heads of firms attacked the men and attempted to discredit them or shun them publicly. For example, the successor to Connor reported in an interview that Connor was shunned socially: "'He really wanted to be open about all of this and come clean. Connor made his proposal, and practically no one in the profession would even speak to him"'(Brewster, 2003, 172). Connor left his position shortly after his attempt to reform the profession.

Credit ratings agencies have posted their risk models publicly. While this is not definitive evidence of collusion on quality, it certainly facilitates such collusion, much as publishing price lists does with price coordination.

Threat of Entry The model here helps illuminate the difficulty a new entrant would have in establishing a reputation, even if the firm were committed to not colluding with clients. The market incumbents are able to attract the viable firms with their established reputation, and it is not clear how an entrant would lure enough viable clients to compete with the established reputation of an incumbent. This is a result of the firm's reputation being as much about the fraction of viable clients (i.e. a selection bias) as about either their ability to discern unviable projects or their refusal to collude.

4 Conclusion

There are two main policy conclusions to draw from the analysis presented here. First, reputation is not a cure-all in markets with the characteristics explored here. There need to be other forms of incentives to prevent collusion with a fraction of the unviable market. Second, a key component to ensuring the loyalty of the viable clients is the gatekeeping component of the service. If viable projects could access the markets freely without certification, some might find that to be a more profitable route to take than purchasing a debased signal. If some leave, that reduces the signaling value further, inducing others to leave as well. This would have the possibility of dissolving into a "confidence" run on the certifier—similar to what happened with Arthur Andersen in 2002, when it abruptly appeared to be non-negligibly worse than its competitors.

This suggests that the movement in the Dodd-Frank Financial Reform Act (2010) to reduce regulatory requirements for credit ratings has the potential to add constructive competitive pressure on the ratings agencies to maintain their credibility at a higher level. Similar thought could be given to ending the requirement for audits (though not the requirement for fraud-free financial statements).

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A Mathematical Appendix

Theorem 2. *In equilibrium, the certifier will wish to sell:*

$$q_1^* = \text{Min}\left\{\frac{1}{2}\left(\frac{(1-\underline{a})}{(\bar{a}-\underline{a})}\pi Q - L\right), \pi Q\right\}$$

and

$$q_2^* = \rho(1-\pi)Q.$$

The interior solution results in a market assessment of the value of a certified project as:

$$\nu_c^* = \frac{(1-\underline{a})\pi - (1-\rho)(1-\pi)(\bar{a}-\underline{a}) + 2\delta(1-\pi)(\bar{a}-\underline{a})}{(1-\underline{a})\pi - (1-\rho)(1-\pi)(\bar{a}-\underline{a}) + 2(1-\pi)(\bar{a}-\underline{a})}\theta$$

In the case of a corner solution, the value is:

$$\nu_c^* = (\pi + \delta(1-\pi))\theta.$$

Proof. The certifier's maximization problem:

$$\begin{aligned} \max_{q_1^t, q_2^t} \Pi^T = {} & m^0\theta(q_1^0 + q_2^0 + L) - \left(\underline{a}\theta + \frac{(\bar{a}-\underline{a})\theta}{\pi Q}q_1^0\right)(q_1^0 + L) \\ & + \sum_{t=1}^{T} m^t\theta(q_1^t + q_2^t + L) - \left(\underline{a}\theta + \frac{(\bar{a}-\underline{a})\theta}{\pi Q}q_1^t\right)(q_1^t + L) \end{aligned} \tag{12}$$

$$s.t. \quad q_1^t \le \pi Q \tag{13}$$

$$q_2^t \le \rho(1-\pi)Q \tag{14}$$

$$\forall t \in (0, T)$$

This can be solved using Karush-Kuhn-Tucker constrained optimization procedure. The Lagrangian (note that time indices are superscripts):

$$\begin{aligned} \mathcal{L} = {} & m^0\theta(q_1^0 + q_2^0 + L) - (\underline{a}\theta + \frac{(\bar{a}-\underline{a})\theta}{\pi Q}q_1^0)(q_1^0 + L) - \lambda_1^0(q_1^0 - \pi Q) - \lambda_2^0(q_2^0 - \rho(1-\pi)Q) \\ & + \sum_{t=1}^{T} m^t\theta(q_1^t + q_2^t + L) - \left(\underline{a}\theta + \frac{(\bar{a}-\underline{a})\theta}{\pi Q}q_1^t\right)(q_1^t + L) - \lambda_1^t(q_1^t - \pi Q) - \lambda_2^t(q_2^t - \rho(1-\pi)Q) \end{aligned} \tag{15}$$

From this expression, we can derive the following necessary conditions for a selection of (q_1^t, q_2^t) to be an optimal selection:

$$\frac{\partial \mathcal{L}}{\partial q_1^t} = (m^t - \underline{a}) - \frac{(\bar{a}-\underline{a})\theta}{\pi Q}(2q_1^t + L) - \lambda_1^t + \frac{(1-\delta)(q_2^t + L)}{(q_1^t + q_2^t + L)^2}\theta(q_1^{t+1} + q_2^{t+1} + L) = 0 \tag{16}$$

$$\frac{\partial \mathcal{L}}{\partial q_2^t} = m^t\theta - \lambda_2^t - \frac{(1-\delta)q_1^t}{(q_1^t + q_2^t + L)^2}\theta(q_1^{t+1} + q_2^{t+1} + L) = 0 \tag{17}$$

$$0 = \lambda_1^t(q_1^t - \pi Q) \tag{18}$$

$$0 = \lambda_2^t(q_2^t - \rho(1-\pi)Q) \tag{19}$$

$$0 \le \lambda_{1,2}^t \tag{20}$$

If we assume, given the assumption of fixed exogenous parameters, that the optimal choices of $q_1^{t-1} = q_1^t = q_1^{t+1}$ and likewise for q_2, we can substitute in q_1 and q_2 for all quantities with time indices (including those implied by m^t) in (16) and (17). With reductions, these become:

$$0 = (1 - \underline{a})\theta - \frac{(\bar{a} - \underline{a})\theta}{\pi Q}(2q_1 + L) - \lambda_1$$
$$0 = \delta\theta - \lambda_2$$

The reduction of (17) implies that $\lambda_2 > 0 \ \forall t$, and therefore that the upper bound constraint on the fraudulent constraints always binds. Therefore,

$$q_2^* = \rho(1 - \pi)Q \tag{21}$$

In the case of the honest certificates, $\lambda_1 \geq 0$, and therefore if $\lambda_1 = 0$:

$$q_1^* = \frac{1}{2}\left(\frac{(1 - \underline{a})}{(\bar{a} - \underline{a})}\pi Q - L\right) \tag{22}$$

else, which occurs when the above is greater than the number of available projects to certify:

$$q_1^* = \pi Q \tag{23}$$

These values for q_1^* and q_2^* can then be substituted into m for the equilibrium valuation by the market of a certificate. □