Competitive Venture Capital Seed Investment: A Signaling Perspective

Jin-Hyuk Kim* Liad Wagman†

Abstract

A model in which an entrepreneur raises multiple financing rounds is presented. A main feature of the model is that participation by past investors in a subsequent financing round sends a signal to the market about the underlying value of the venture. We show that this form of signaling can work against entrepreneurs by exacerbating equity dilution. In turn, entrepreneurs prefer to finance earlier rounds via liquidity-constrained investors who will not participate in subsequent rounds; however, they may be forced to do the opposite when their own investor choice acts as a signal. We further show that if the additional capital raised in a subsequent round is large, entrepreneurs are better off when new investors are as informed about the venture as prior investors.

*Department of Economics, University of Colorado at Boulder. Email: jinhyuk.kim@colorado.edu
†Stuart School of Business, Illinois Institute of Technology. Email: lwagman@stuart.iit.edu.

1 Introduction

The practice of staged financing in venture capital is well documented (Gompers and Lerner 1999), and an extensive theoretical literature investigates the staging of venture capital financing. In each stage, the entrepreneur aims to meet a certain goal, and venture capitalists monitor the firm's progress and retain the option to discontinue funding if they learn negative information about its future prospects. The earlier literature focused on the agency problem that arises when entrepreneurs may be better informed than the investors (e.g., Brennan and Kraus 1987; Neher 1999), and venture capital contracts were interpreted in light of these agency theories (e.g., Kaplan and Stromberg 2003, 2004). A standard view is that staging is used to mitigate the moral hazard and hold-up problems.

A more recent stream of literature extends this agency view by incorporating the investor's learning process. Some of the papers demonstrate the implications of investor learning in a single investor relationship (e.g., Bergemann and Hege 1998; Cornelli and Yosha 2003) while others allow inside investors to possess private information that is not shared by other investors in the market (e.g., Admati and Pfleiderer 1994; Casamatta 2003). In this paper, we analyze a simple two-stage model in which informational asymmetry creates a signaling role for inside (seed) investors. To be precise, seed investors are more informed about the venture, and whether seed investors do or do not follow on in a subsequent financing round can send a positive or a negative signal about the value of the venture.

The basic mechanism for this type of learning is well known in the literature. In labor markets, for instance, it is typically thought that current employers are better informed about the abilities of their workers than potential outside employers, and outside firms can only infer them by using signals the current employer generates (e.g., Waldman 1984; Greenwald 1986). In our model, the market updates its beliefs about the value of the venture based on the seed investor's participation decision. We show that this form of signaling works against the entrepreneur when he does not know the project's success probability. Consequently, entrepreneurs in our base model prefer to receive seed investments from angels or liquidity-constrained investors who can commit not to take part in a subsequent round.

The reason for this finding is rooted in the informational spillover that is caused by a seed investor in a subsequent round. In particular, competitive pressures from outside investors in a subsequent round ratchet up the value threshold above which a seed investor is willing to participate. As a result, a positive signal about the venture is only transmitted when the value of the venture is relatively high. In other words, the seed investor's behavior is more likely to negatively impact market beliefs regarding the entrepreneur's venture, which in turn increases the expected cost of obtaining financing and exacerbates dilution of the entrepreneur's equity share. Thus, entrepreneurs in our framework prefer to work with an angel investor who can credibly commit not to invest in a future round.

We then extend the analysis in three different ways. In our first extension, we assume that the entrepreneur may have some private information — the entrepreneur obtains a signal at the beginning of the game that reveals information about the venture. Our main finding here is that when entrepreneurs are better informed about the potential of their venture, they choose to finance with venture capitalists rather than with angels, which negatively impacts their *ex ante* expected share in the venture. In our second extension, we introduce investor heterogeneity and show that the costs of overall financing decreases when seed investors are more liquidity constrained. In our third extension, we study the effects of full information disclosure among first and second-round investors. Here, we show that entrepreneurs would prefer such disclosure *ex ante* when second-round capital requirements are relatively large.

Thus, our paper supports the "signaling hypothesis" of venture capital seed investment, and adds a new perspective to studies on staged financing. An emerging literature argues that asymmetric information is not necessary to explain staged financing and provides alternative theories. Dorobantu (2006) argues that syndication is a signaling device in that the high-skilled entrepreneur issues undervalued shares in the syndicated financing round in order to signal his expectation of high gains from managing the follow-on fund. Broughman and Fried (2010) and Ray (2010) consider a model in which inside rounds are more likely as firm values decline. That is, when a firm's value declines, the terms offered by an outside VC firm increases the costs of a down-round financing.

One of the major difference between our paper and those above is that our model explicitly focuses on the effects of asymmetric information between inside and outside investors. In Broughman and Fried (2010), all investors observe the state realization before the second round; and Ray (2010) abstracts from explicit interactions with outside investors, although implications of changes in outside option values are presented. However, the informational asymmetry between inside and outside investors is not new to the literature. For instance, Hochberg et al. (2010) consider a model in which investors in a fund learn whether the fund manager has skills, whereas outside investors do not, giving current investors hold-up power in subsequent fundraising.

Other papers on venture financing with asymmetric information include Bergemann and Hege (2005) and Dessí (2005). Bergemann and Hege study a tradeoff between relationship financing and an arm's length relationship. They find that while close monitoring reduces the rents associated with private learning, lack of commitment to not financing a project with positive net payoffs allows the entrepreneur to divert funds for his private benefit. Dessí considers a setting in which an entrepreneur and an investor may collude, and shows that the entrepreneur may undertake a good project under mixed financing if the entrepreneur's own capital is large. In contrast, in our model, entrepreneurs are equally capital constrained, and the analysis instead focuses on the heterogeneity of information, project types, and investors.

The remainder of the paper is organized as follows. The next section presents the base model and characterizes the equilibrium. Section 3 studies a variant of the model with the introduction of an angel investor who does not participate in a subsequent investment round. Section 4 extends the analysis by considering the possibility of entrepreneurs possessing private information about their ventures. Section 5 considers another extension of the base model in which seed investors have varying degrees of liquidity constraints. Section 6 studies the implications of a disclosure requirement that leads to symmetric information among investors. Section 7 summarizes the empirical implications of our findings and Section 8 concludes. All omitted proofs are provided in an appendix.

2 Model and Analysis

In this section, we present the base model in which seed investors are unable to commit regarding their participation in a subsequent round. In the following sections, we extend the analysis both to the case where some investors are liquidity constrained and to the case where some entrepreneurs are privately informed about their project.

2.1 The Model

There is a continuum of entrepreneurs of unit mass. At the beginning of the game, nature draws a venture idea for each entrepreneur. We assume that entrepreneurs are liquidity constrained and unable to self-finance a new business. We model business development as consisting of two stages. In the first (seed) stage, an entrepreneur raises capital K to turn his idea into a product or service. No revenue is generated at the end of this stage. In the second (follow-on or subsequent) stage, the entrepreneur requires an additional capital F to bring the product to market. At the end of the second stage, the business either succeeds with probability p generating a revenue R, or fails with probability $1-p$, giving zero revenue.

An entrepreneur's idea is characterized by the probability of success p, which is uniformly distributed on $[0, 1]$.[1] To raise capital at each stage, an entrepreneur can approach venture capitalists or VC firms. In our analysis, we primarily restrict attention to per-stage equity financing, but we provide in Subsection 3.1 a comparison to convertible debt and non-dilutable equity shares. When approached by an entrepreneur in either stage, a VC firm can offer a contract, (γ_X, X), which gives the VC firm a $\gamma_X < 1$ share of equity in return for investing X in the particular financing stage. For technical simplicity and at no qualitative

[1]The uniform distribution is not necessary for the main results, but it simplifies the analysis and yields closed form solutions.

loss, we assume that firms face a zero rate of interest between the two stages.

Initially, the probability of success is unknown both to the entrepreneur and to all VC firms.[2] If a VC firm invests in the first round, however, this "inside" VC firm learns the venture's success probability, p. In the second financing round, outside investors observe whether the inside VC firm offers to invest in the second round, and submit their own financing offers. The timeline of the game is thus given as follows:

1. Entrepreneur raises first financing round of size K, investors share common prior.

2. Seed investor learns venture's success probability, p.

3. Seed investor announces its second-round participation decision.

4. Entrepreneur raises second financing round of size F at market valuation.

We assume free entry of VC firms in both financing rounds. We further assume that the entrepreneur accepts the best financing terms offered and stays with the inside VC firm when all other things are equal.[3] We employ Perfect Bayesian Equilibrium as our solution concept.

2.2 Equilibrium Characterization

Throughout the analysis, we maintain the following assumption on the parameter values:

Assumption 1. $4KR \leq (R-2F)^2$.

Assumption 1 requires that the revenue from a successful venture is sufficiently large. In particular, it ensures that when a seed investor chooses not to participate in a follow-on round,

[2]Although VC firms can initially screen projects, we believe that both entrepreneurs and VC firms face considerable uncertainty about the project returns at this stage. In fact, a number of VC firms do make seed investments without much screening effort for the option value at a later stage. One interpretation is that the entrepreneurs in our model meet a certain minimum standard, so that investing without serious screening is a viable strategy for VC firms.

[3]Hence, we abstract from other reasons for why an entrepreneur might want to consider financing with another VC firm at equal or even worse terms. For instance, the entrepreneur might benefit from a network of more established venture capitalists and their mentorship. However, bringing aboard outside investors at a later stage raises issues of control and cash flow rights, so that it is typically in down rounds when the ownership of the venture changes (Bengtsson and Sensoy 2011).

financing by outside investors still takes place. That is, we focus on cases where investing in the venture is still worthwhile to outside investors in a "down round". The assumption will necessarily be satisfied for a sufficiently large R and, holding other parameters constant, is more likely to be satisfied the smaller the initial capital requirement K.

Consider the following strategy for a seed investor (an inside VC firm) in the second round: finance the venture according to the terms (γ_F, F) if and only if the probability of success is above a threshold value p^*. Given this, outside VC firms revise expectation of the success probability conditional on the inside VC firm's financing decision. Specifically, if the inside VC firm continues to finance the second round, then the expected probability of success is $\frac{1+p^*}{2}$, whereas it is $\frac{p^*}{2}$ if the VC firm does not participate in the second round. We refer to these two events as an "up round" and "down round" financing, respectively.[4]

Because of free entry, the financing terms offered to the entrepreneur are characterized by a zero-profit condition, $\frac{1+p^*}{2}\gamma_F R - F = 0$. Accordingly, the inside VC firm's negotiated share of the equity in the second round will be bid down to $\gamma_F^+ = \frac{2F}{(1+p^*)R}$. In a similar fashion, the financing terms made to the entrepreneur who does not have an offer from the inside VC are bid down to a fraction $\gamma_F^- = \frac{2F}{p^* R}$ of the start-up firm's equity. We note that the first-round investor's share is diluted in both up and down rounds. Specifically, if the first-round investor owns a share γ_K of the venture at the end of the first round, then the value of this share drops to $\gamma_K(1-\gamma_F)$ in the second round.[5] It follows that the inside VC firm will finance the second round if and only if p satisfies

$$p(\gamma_K(1-\gamma_F^+) + \gamma_F^+)R - F \geq p(\gamma_K(1-\gamma_F^-))R. \tag{1}$$

[4] To be more precise, the first-round firm valuation is $\frac{R}{2}$ because given uniform distribution the expected probability of success is $\frac{1}{2}$. It will be shown in the proof of Proposition 1 that Assumption 1 implies $0 < p^* < 1$. Therefore, $\frac{p^*}{2} < \frac{1}{2} < \frac{1+p^*}{2}$, implying that the second-round venture valuation either increases or decreases compared to its first-round valuation.

[5] At this point, we abstract from anti-dilution protections that are applicable in the down round. Such protection, if exercised, magnifies the dilutive effect of the down round for all non-protected equity holders. Section 3.1 provides a detailed comparison.

On the left-hand side of (1) is the expected profit from financing the second round. The inside VC firm's share is the sum of its initial share γ_K, implicitly diluted by a factor $(1-\gamma_F^+)$, and the newly acquired share γ_F^+, in return for making the additional investment F. On the right-hand side is the inside VC firm's expected profit when it does not participate in the follow-on round, letting its share being diluted by outside investment. By Assumption 1, the entrepreneur who does not have an inside offer can still finance the second round through an outside VC firm. Hence, the inside VC firm's share from the first round is not worthless even if it decides not to participate in the subsequent round.

Proposition 1 *If the realization of an entrepreneur's success probability is above p^*, where $p^* = 1 - 2\gamma_K$, then the inside VC firm finances the second round and obtains an additional share of $\gamma_F^+ = \frac{2F}{(1+p^*)R}$. If the realization is below p^*, then an outside VC firm finances the second round and obtains a new share of $\gamma_F^- = \frac{2F}{p^*R}$.*

The equilibrium displays properties that are consistent with the "signaling hypothesis" of venture capital seed investment. That is, if a VC invests in a start-up firm and participates (does not participate) in the future round, then it sends a positive (negative) signal to outside investors who lack inside information about the company. Moreover, when negotiating financing terms, an investor only matches what other investors are willing to offer. Therefore, for a given capital amount F, the negotiated share γ_F depends only on whether the inside VC stands to finance the second round. That is, although the probability of success is known to the inside VC, the financing terms do not further depend on the observed success probability. The inside VC firm does not have an incentive to reveal additional information on p through the financing term γ_F because then the entrepreneurs whose businesses are undervalued would demand a higher share and this adverse selection problem could unravel the inside VC firm's optimal strategy.

Another property of the equilibrium is that the inside VC firm's cutoff value of the success probability is negatively related to the firm's initial share of the start-up firm. The intuitive reason for this is that the share acquired in the first-round gives the inside VC firm an incentive to create an up round because the rate of dilution is less than in a down round. Suppose the VC firm's first-round share were zero. Then, the cut-off probability p^* would degenerate to 1. What happens is that, at the cut-off margin p^*, the VC firm negotiates terms at less than fair odds because the market perceives the start-up firm (in expectation) better than it really is. Having a zero fallback position, the inside firm can invest only in firms with the very highest success probability. Now, by having a non-zero initial share γ_K, the cutoff value will be strictly less than 1, as the differential rate of dilution in up and down rounds drives a wedge between two fallback positions and gives the inside VC firm a higher return from financing in an up round.

From the perspective of the entrepreneur, increasing the seed investor's equity share, γ_K, has two effects. The direct effect is the dilution of the entrepreneur's share, negatively impacting his expected payoff. The indirect effect works to the benefit of the entrepreneur by lowering the threshold probability p^*, resulting in more beneficial financing terms in the second round. However, as the following corollary shows, the direct effect dominates, and the entrepreneur always prefers to keep the seed investor's equity share as small as possible.

Corollary 1 *The entrepreneur's expected equity share at the end of the second financing round is decreasing in the seed investor's first-round equity share, γ_K.*

Proof. The entrepreneur's expected share is given by $(1-\gamma_K)(1-p^*\gamma_F^- - (1-p^*)\gamma_F^+)$. Substituting for p^*, γ_F^-, and γ_F^+, the entrepreneur's expected share is given by $1-\frac{2F}{R}-\gamma_K$, which is decreasing in γ_K. ■

To complete the analysis, we now derive the equilibrium share γ_K^* of the first-round investor. Given Proposition 1, the first-round investor has an option to invest in the second round if the expected profit from doing so is higher than not participating in the second round. Because these fallback positions yield a strictly positive profits as long as γ_K is positive, the equilibrium share of the first-round investor can be characterized by a zero-profit condition. A zero-profit condition does not entail that VC firms end up with zero profit at the end of the game; rather, it implies that due to competition, the expected return on investment is zero (normal) rate of return, but the actual return could be much higher if the company succeeds.

Proposition 2 *The equilibrium share of the VC firms investing in the first stage of financing is* $\gamma_K^* = \frac{2K}{R-2F}$.

Proof. Given Proposition 1, γ_K can be characterized by using a zero-profit condition. Since the second round terms are $\gamma_F^+ = \frac{2F}{(1+p^*)R}$ and $\gamma_F^- = \frac{2F}{p^*R}$ with probability $1-p^*$ and p^*, respectively, the expected profit of the first round investor is

$$-K + (1-p^*)\left(\frac{1+p^*}{2}(\gamma_K + (1-\gamma_K)\frac{2F}{(1+p^*)R})R - F\right) + p^*\left(\frac{p^*}{2}(\gamma_K(1-\frac{2F}{p^*R}))R\right) \geq 0.$$

Substituting $p^* = 1-2\gamma_K$ and simplifying yield

$$-K + \frac{\gamma_K R}{2} - \gamma_K F \geq 0.$$

Therefore, $\gamma_K^* = \frac{2K}{R-2F}$. ■

Before concluding this section, we note one empirical implication thus far. In equilibrium, the share of the first-round investor, γ_K^*, is positively related to the size of the initial capital requirement, K. Thus, if K decreases, the cutoff probability of success, p^*, would increase. Therefore, the model predicts that, as initial capital requirements become smaller, holding other things constant, down rounds in subsequent financing stages are more likely. Moreover, the devaluation of a venture in a down round would become smaller, whereas its appreciation in an up round would be steeper.

3 Commitment to (Not) Participate

In the previous section, we assumed that all investors are unable to commit in the first stage to (non) participation in the second round of investment. We thus ignored the possibility that some investors specialize in seed and/or angel investments. That is our focus here. Let us consider the case where some first-round investors (henceforth referred to as "angels") can commit to stay out of the follow-on round (e.g., due to an organizational policy or scope). We consider the same framework as in the base model but assume that in the first round, the entrepreneur receives offers from angels as well as from VCs.

There are two possible outcomes in equilibrium. First, the entrepreneur accepts a seed investment from a VC firm, who cannot commit to not participating in the second round. In this case, the rest of the equilibrium would be described by Propositions 1 and 2. That is, the inside VC firm finances the second round if and only if the observed probability of success is above a certain threshold. Further, this necessitates an adjustment of the company's valuation by outside investors. The second possibility is for the entrepreneur to accept a financing offer from an angel investor. Which of these two outcomes prevails in equilibrium depends on the specific offer terms made by angels and VC firms.

Let us suppose that the entrepreneur financed the first round through an angel investor. Since angels do not participate in the second round, outside VC firms in the second stage do not learn from the insider's investment behavior. Hence, second-round investors bid down their required equity term in return for the investment F until the expected profit from investment is driven down to zero, that is, $\frac{1}{2}\gamma_F R - F = 0$. Thus, an outside investor's share is given by $\gamma_F^0 = \frac{2F}{R}$. The angel investor's share, γ_K, is also determined by a zero-profit condition, where its initial share of the venture is diluted by a fraction $(1-\gamma_F^0)$ in the second round.

Let $(\hat{\gamma}_K, K)$ denote the first-round offer made by an angel investor (as opposed to the offer (γ_K^*, K) that is made by a VC firm who may participate in the second round). With two types of investors in the first round, an entrepreneur can affect future financing terms based on his choice of first-round investor. Specifically, if the entrepreneur receives a first-round investment from an angel, his second-round financing term is (γ_F^0, F), whereas if he takes seed money from a VC firm, then the second-round financing term is given by (γ_F^-, F) with probability p^* and by (γ_F^+, F) with probability $1 - p^*$. Note that due to changes in firm valuation, $\gamma_F^+ < \gamma_F^0 < \gamma_F^-$. Because the entrepreneur accepts whichever offer gives him the higher overall share of the company, he accepts an offer from an angel if and only if $1 - [\hat{\gamma}_K(1 - \gamma_F^0) + \gamma_F^0] > 1 - [\gamma_K^*(1 - E\gamma_F) + E\gamma_F]$, where $E\gamma_F = p^*\gamma_F^- + (1 - p^*)\gamma_F^+$. The following proposition shows that this inequality is weighed in favor of the angel investor.

Proposition 3 *Entrepreneurs prefer to finance the seed round with angels rather than with VC firms.*

Proposition 3 shows that entrepreneurs who have no *ex-ante* private information about the success probability of their ideas are better off by taking seed investments from angel investors. The reason for this result is that the informational spillover that is caused by an informed investor in the second round is more likely to work against the entrepreneur. In other words, the second-round investment behavior of an informed VC firm tends to negatively impact the market valuation of the entrepreneur's start-up, which in turn increases the cost of obtaining additional financing and dilutes the entrepreneur's share.

An implication of Proposition 3 is that market information about the entrepreneur's venture can actually work against the entrepreneur. Thus, an entrepreneur in our framework prefers to work with a first-round investor who can credibly commit not to invest in a future round. This is reminiscent of Hirshleifer (1971), who showed that more information in advance of trading could be damaing to the society because it removes risk sharing opportunities in the economy. Similarly, in our model, more information known to outside investors reduces the *ex-ante* private return to the entrepreneurs.

3.1 Convertible Debt and Non-Dilutable Shares

Two common investment instruments that are used by early-stage investors are convertible debt and non-dilutable shares. In our framework, they work similarly.[6] Non-dilutable share holders maintain their equity positions in future rounds, whereas a convertible debt note allows the holder to convert the debt into equity shares at a future investment round. In our framework, a first-round investor has the option to convert debt into equity in the second round at the round's share-per-dollar-invested ratio.

In the case of angel investment, we have shown that a second-stage investor's share is given by $\gamma_F^0 = \frac{2F}{R}$. It follows that the share-per-dollar-invested ratio in the second stage is given by γ_F^0/F. Hence, given a competitive investment market, an angel can convert a first-round convertible debt note of K into second-stage equity in the amount of $\gamma_K^c = \frac{\gamma_F^0}{F}K = \frac{2K}{R}$.[7]

Corollary 2 *Angel investors are indifferent between using traditional equity financing, non-dilutable equity, and convertible debt notes for first-round investments.*

Proof. We have shown in Proposition 3 that an angel investor's first-round share when employing equity financing is given by $\hat{\gamma}_K = \frac{2K}{R-2F}$. Given that $\gamma_F^0 = \frac{2F}{R}$, the angel's share after second-round financing is given by $\frac{2K}{R-2F}(1 - \frac{2F}{R}) = \frac{2K}{R}$, which is equivalent to a convertible debt note of K with share-per-dollar-invested ratio of $\frac{\gamma_F^0}{F} = \frac{2}{R}$ in the second round. In terms of non-dilutable shares, competition in the first round of investment will bid γ_K down such that $\frac{1}{2}\gamma_K R - K = 0$, also resulting in the angel (equivalently) holding $\gamma_K = \frac{2K}{R}$ in non-dilutable equity. ■

[6] Convertible debt has the additional feature of granting the holder interest payments; however, we set these to zero at no qualitative loss. In practice, angel investors tend to prefer convertible debt due to its simplicity and low cost of legal implementation.

[7] Since equity shares must sum to 1, an implicit requirement here is that $\gamma_F^0 + \frac{2K}{R} = \frac{2(F+K)}{R} \leq 1$, which holds for R large relative to F and K.

4 Private Information and Investor Choice

In the base model, we assumed that the entrepreneur had no private information about the success likelihood of his venture in the seed round. Let us suppose now that the entrepreneur obtains a signal about the probability of success, p, at the beginning of the game. The signal technology works as follows. With probability θ, the entrepreneur perfectly learns p, and with probability $1-\theta$, the entrepreneur learns no new information. In the case where $\theta = 0$, i.e., when the entrepreneur has no private information about the venture, Proposition 3 shows that entrepreneurs prefer to finance the seed round with angel investors. In contrast to the results from the previous section, entrepreneurs who learn that their venture has a high success probability may be more inclined to finance with VC firms — by doing so, they are able to signal positive news to outside investors about the success likelihood of their venture. It is thus natural to consider threshold equilibria of the following form.

Definition 1 *An equilibrium is said to be a* threshold equilibrium *at* $\tilde{p} \in [0,1]$ *if entrepreneurs with* $p < \tilde{p}$ *finance with angels and those with* $p \geq \tilde{p}$ *finance with VC firms.*

Since off-equilibrium beliefs can be arbitrary, we apply the following sensible refinement to the equilibrium concept we employ.[8]

Definition 2 (Off-equilibrium beliefs) *Let* p^* *denote the marginal type above which VC firms follow on in a subsequent round in the base model (i.e., with* $\theta = 0$*). Then when no entrepreneur types finance with either VC firms or with angels, off-equilibrium beliefs are specified by a conditional prior over supports* $[0, p^*)$ *and* $[p^*, 1]$*, respectively.*

In the following proposition, we begin by addressing the case where all entrepreneurs initially learn the success probability of their ventures, i.e., where $\theta = 1$.

Proposition 4 *If all entrepreneurs learn their ventures' success probabilities prior to raising seed rounds, they pursue financing with VC firms rather than with angels.*

[8]Without this specification, equilibria where all entrepreneurs finance with an angel can be supported by less plausible off-equilibrium beliefs, e.g., such that entrepreneurs who finance with VC firms have $p = 0$.

It thus follows that when entrepreneurs have prior knowledge about the success of their ventures, all entrepreneur types pool and finance with VC firms, which, from Proposition 3, results in lower expected shares for the entrepreneurs relative to angel financing. The outcome thus has the flavor of a Prisoner's Dilemma game: If commitment were possible, it would be a Pareto improvement for all entrepreneur types to commit *ex ante* to angel financing. The following corollary highlights this observation.

Corollary 3 *If entrepreneurs could commit to a seed investor choice prior to learning their ventures' success likelihoods, they would commit to angel financing.*

Let us now consider intermediate cases where $\theta \in (0,1)$, that is, where an entrepreneur has a chance of learning his venture's probability of success with probability θ. For a given venture characterized by p, there are then 3 potential types of entrepreneurs: (i) entrepreneurs who learn p and choose to finance with a VC firm; (ii) entrepreneurs who learn p and choose to finance with an angel; and (iii) entrepreneurs who do not learn p. The following proposition characterizes the threshold equilibria in this case.

Proposition 5 *Given $\theta \in [0,1]$ threshold equilibria are characterized as follows. There are values, θ' and θ'', where $0 < \theta' < \theta'' < 1$, such that: (i) For $\theta < \theta'$, all entrepreneurs finance with angels. (ii) For $\theta' \leq \theta \leq \theta''$, entrepreneurs who learn $p \geq \tilde{p}(\theta)$ finance with VC firms, while other entrepreneurs finance with angels; furthermore, within this range, the threshold $\tilde{p}(\theta)$ is decreasing in θ. (iii) For $\theta > \theta''$, all entrepreneurs finance with VC firms.*

Combining Propositions 4 and 5, it follows that when entrepreneurs have private information about the viability of their ventures (e.g., through the development of prototypes), a Prisoner's Dilemma situation may ensue where all entrepreneurs finance with VC firms. However, when the availability of such information is diminished, entrepreneurs may self select a seed investor based on their private information, with high types financing with

VC investors and low types with angels. As such information becomes less and less available (e.g., when prototypes are not developed or when their development does not privately convey new information about viability), entrepreneurs switch to angel financing.

5 Liquidity Constraints and Heterogeneous Investors

Thus far, we have considered two types of investors. First, in Section 2, first-round investors were free to participate in the second round. In Section 3, angel investors only invested in the seed round. In this section, we maintain the base model assumption of a competitive market with the ex ante identical entrepreneurs but assume that a first-round investor i has a probability λ_i of having the necessary liquidity to participate in the second round, where $\lambda_i \in [0, 1]$. For instance, individual investors are likely to have less liquidity than institutional investors. What type of an investor would the entrepreneur prefer, one with limited or high liquidity? To answer this question, we first characterize the equilibrium following steps similar to those in Section 2.

First, if the seed investor has the necessary liquidity to invest in the second round and chooses to participate, then as in Section 2, we obtain $\gamma_{F,i}^{+} = \frac{2F}{(1+p_i^*)R}$, where p_i^* denotes the threshold above which investor i chooses to participate, and $\gamma_{F,i}^{+}$ denotes the equity share granted in exchange for financing in an up round. On the other hand, if investor i does not participate in the second round, then from the perspective of other second-stage investors there are two possible cases: (i) the first-round investor has the necessary liquidity but chooses not to follow on, or (ii) the first-round investor does not have the necessary liquidity. Then the equity term $\gamma_{F,i}^{-}$ (denoting an outside investor's share in exchange for financing F in a down round) would be bid down to satisfy a zero profit condition, $\lambda_i \frac{p_i^*}{2} \gamma_{F,i}^{-} R + (1 - \lambda_i)\frac{1}{2}\gamma_{F,i}^{-} R - F = 0$. Solving for $\gamma_{F,i}^{-}$, we obtain $\gamma_{F,i}^{-} = \frac{2F}{(1-\lambda_i(1-p_i^*))R}$.

We observe that if $\lambda_i = 1$, then $\gamma_{F,i}^{-} = \frac{2F}{p^* R}$ as in the base model, whereas if $\lambda_i = 0$, then $\gamma_{F,i}^{-} = \frac{2F}{R}$, as in the case of an angel investor. Furthermore, for all $\lambda_i \in [0, 1)$, we have

$\gamma^{-}_{F,i} < \frac{2F}{p^*_i R}$, implying that the shares of both the entrepreneur and the first-round investor sustain less dilution in a down round compared to the case where all investors have the necessary liquidity to follow on. This creates a complex equilibrium relationship between λ_i and the first-round investor's equity share; however, the general trend is that because the seed investor is ex ante uncertain of whether he can follow on a profitable opportunity or not, the seed investor will demand higher initial share of the firm in the seed stage.

Proposition 6 *The share of first-round equity that is granted to a seed investor in exchange for an investment K, γ_K, is non-monotonic in the liquidity parameter λ.*

The following figure numerically shows that an intermediate level of liquidity is coupled with less favorable seed financing terms. Here we set $R = 100$ and $F = K = 10$ and plot γ_K as a function of λ.

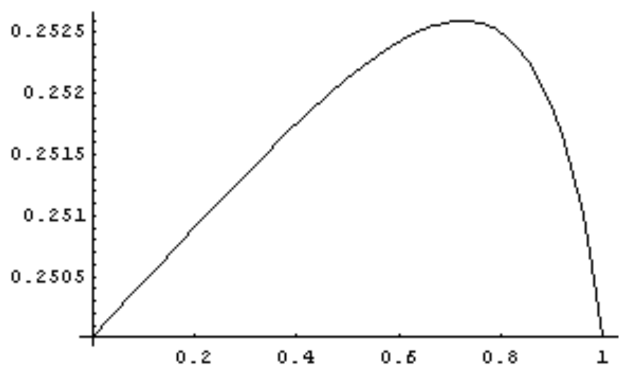

Figure 1: The first-round equity share γ_K as a function of λ when $R = 100$ and $F = K = 10$.

As Figure 1 shows, the first-round equity term γ_K is non-monotonic in λ_i, reaching a peak at an intermediate level of λ. Intuitively, intermediate levels of λ generate the most volatility in the second round. This is because the signal that outside investors perceive from the seed investor's participation decision has the largest amount of "noise," particularly in the case where the latter chooses not to participate. This noise is due to the fact that outside investors are unsure whether the seed investor is staying out of the round because of (i) liquidity constraints or (ii) a low valuation for the venture.

From the entrepreneur's perspective, his *ex-ante* expected equity stake at the end of the second round is given by

$$(1-\gamma_{K,i})\big[\lambda_i(1-p_i^*\gamma_{F,i}^- - (1-p_i^*)\gamma_{F,i}^+) + (1-\lambda_i)(1-\gamma_{F,i}^-)\big]. \tag{2}$$

Figure 2 plots the entrepreneur's expected share as a function of λ for the case where $R = 100$ and $F = K = 10$. As the figure shows, the entrepreneur's cost of financing the innovation is

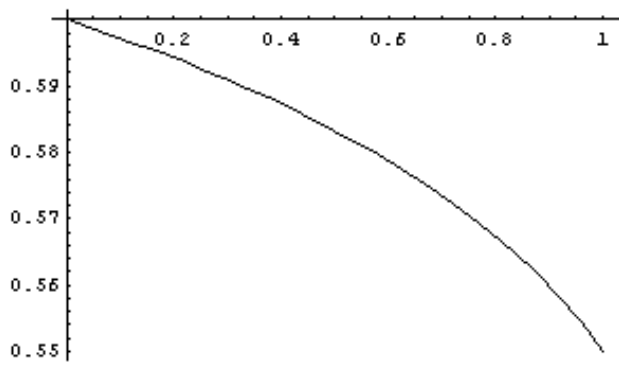

Figure 2: Entrepreneur's expected share as a function of λ when $R = 100$ and $F = K = 10$.

smallest when the first-round investor is liquidity constrained (i.e., when $\lambda_i = 0$). Although this is difficult to prove analytically, we obtain similar findings for a wide range of parameter specification on R, K, and F. An investor's liquidity constraint acts as a commitment device for ensuring non-participation in the second round. Seed investors who are highly liquid are more likely to negatively impact the entrepreneur's share in the second round, as outside investors put more weight on their participation or lack of as a signal about the venture's viability.

6 Disclosure Requirement

In this section, we discuss the model's implications for disclosure policy. As in the base model, we make the assumption that the VC firms investing in the first round cannot commit to non-participation. However, instead of the inside VC firms keeping private information on their first round investments, let us suppose that the government requires VC firms to disclose details of their portfolio companies.[9] Under such a restriction, information asymmetry between inside and outside investors is reduced. For technical simplicity, we assume that outside VC firms perfectly learn the entrepreneur's success probability through the inside investor's disclosure in follow-on rounds.

As in the previous analysis, we start by deriving the second-round financing terms. Under symmetric learning, for each entrepreneur characterized by p, a zero-profit constraint would determine the second-round financing terms $(\tilde{\gamma}_F, F)$ offered by the market. Thus, in equilibrium, the VC firms would demand a $\tilde{\gamma}_F = \frac{F}{pR}$ share of the venture in return for investing F in the second round. Notice that, if the realization of the entrepreneur's success probability is below a certain threshold $\bar{p} = \frac{F}{R}$, then no investor would invest in the second round. This results because the share of equity held by investors cannot exceed one. Given this, the first-round investor's financing terms $(\tilde{\gamma}_K, K)$ can be derived by the use of a zero-profit condition.

The first-round offer term is $\tilde{\gamma}_K = \frac{2K}{(1-\bar{p}^2)R-2F(1-\bar{p})}$. We can understand this result intuitively by comparing it to the first-round offer term made by VC firms in the base model. In Section 2, we showed that a VC firm would invest K in return for an initial share of $\gamma_K^* = \frac{2K}{R-2F}$. In fact, it holds that $\gamma_K^* > \tilde{\gamma}_K$. The reason why the share of the first-round investor decreases under symmetric learning is that the expected second-round profit increases, as investors can now avoid investing in projects whose expected surplus is negative, that is, those with realized probability of success below $\bar{p}$. On the other hand, the effect of symmetric information on the expected second-round offer terms is in general ambiguous. This can be seen by observing that $E\tilde{\gamma}_F = \int_{\bar{p}}^1 \frac{F}{pR} dp = \frac{F}{R}(-\ln \frac{F}{R})$ is an inverted U-shaped function of $\frac{F}{R}$ whereas both γ_F^+ and γ_F^- are linear functions of $\frac{F}{R}$.

[9] One potential example of such disclosure policy is the Dodd-Frank Act. The Act exempts investment advisers who manage only venture capital funds from the registration requirements, but venture capital fund advisers must still comply with federal record-keeping, reporting and disclosure requirements. Furthermore, venture capital fund advisers may still be subject to state registration requirement.

We now compare the entrepreneur's expected share of the company under disclosure to that of the benchmark case.

Proposition 7 *There is a value ϕ, $0 < \phi < \frac{1}{2}$, such that entrepreneurs with the veil of ignorance prefer disclosure policy if and only if $\frac{F}{R} > \phi$.*

Proposition 7 characterizes the condition under which the expected share of the entrepreneur is larger with disclosure than without it. That is, if the additional capital requirement F is relatively large, then it is more likely that the disclosure policy will benefit potential entrepreneurs. The logic behind this result is as follows. With disclosure, a larger follow-on investment means that the possibility of investing in the entrepreneur's second round is smaller. Thus, the investors' expected second-round shares would not be as high as those in the base model. This means that the entrepreneur's expected share would increase, so entrepreneurs would prefer to have a disclosure requirement when F is relatively large.

On the other hand, a VC firm's expected profit is always zero from an ex ante perspective, so one might think that VC firms would be indifferent as to whether a disclosure policy is imposed on them. However, after the first round of investment, if an inside VC firm learns that its portfolio company would be undervalued by the market, then it would not have an incentive to disclose that information prior to the second round of financing. This means that after the first round of financing (that is, for those existing VC firms), such disclosure requirement would deprive them of the opportunity to recoup their investment. Therefore, our model is not inconsistent with the real world observation that VC firms tend to prefer non-disclosure.

As a final point, we consider the social welfare implications of the government policy that requires investors to disclose detailed information about their portfolio companies. Given our parameter restriction in Assumption 1, without disclosure, all entrepreneurs are financed in the second round. Consequently, some entrepreneurs whose success probabilities are less than acceptable for second-round financing are actually funded. This results because the signal generated by a seed investor's participation decision in the second round is imperfect. That is, without a disclosure requirement that leads to symmetric learning between inside and outside investors, there is an inefficiency in the capital market. Therefore, by requiring full disclosure, the government can reduce this inefficiency associated with imperfect learning by market participants.

7 Empirical Implications

The theoretical framework introduced in this paper generates a number of empirical predictions, some of which have been mentioned in the previous sections. This section brings them all together.

Our base model gives rise to the prediction that ventures that are financed by the same investors over multiple financing rounds are valued more and are more likely succeed. Furthermore, when entrepreneurs have some private information about the viability of their ventures prior to seed financing (e.g., from the development of prototypes), our model predicts that VC investors are more likely to follow on their seed investments in subsequent rounds. The reason for this prediction is that, as shown by Proposition 5, entrepreneurs may self select a seed investor based on their private information. In contrast, our analysis in Sections 3 and 5 predicts that entrepreneurs with no *ex ante* private information about the viability of their ventures (e.g., when prototypes had not been developed) are more likely to finance seed rounds with liquidity-constrained and angel investors. Finally, Section 6 gives rise to the prediction that entrepreneurs prefer a policy of transparency in their disclosures when seeking to raise large amount of funds in subsequent rounds.

8 Conclusion

In this paper, we constructed a simple model consistent with the signaling hypothesis of venture capital seed investment, and then analyzed the model under several different assumptions: asymmetric learning with, without, and with partial commitment to follow on, and symmetric learning due to a disclosure requirement. A main feature of the model was that participation by past investors in subsequent financing rounds sent a signal to the market about the value of a venture. We showed that this form of signaling can work against entrepreneurs by exacerbating equity dilution. In turn, entrepreneurs preferred to finance seed rounds via liquidity-constrained investors who could effectively commit not to participate in subsequent rounds. We then showed that when entrepreneurs have private information about the viability of their ventures, a Prisoner's Dilemma situation may ensue where all entrepreneurs finance with VC firms. However, when the availability of such information is diminished, entrepreneurs self select a seed investor based on their private information, with high types financing with VC investors and low types with angels. We also showed that if the additional capital requirement in a subsequent round was relatively large, entrepreneurs were better off when venture-pertinent information was disclosed to outside investors.

Our model can be extended in several different ways. First, we could relax the assumption that outside VC firms do not learn anything about the entrepreneurs who need additional follow-on capital. For instance, outside investors may learn some aspects of the management team through screening. However, as long as there are uncertainties that cannot be evaluated based on the observed characteristics, one interpretation is that our model applies to each subgroup of entrepreneurs who cannot be distinguished by outside investors.[10] Second, we could allow for more than two stages of financing. As the number of rounds increases, the market has a longer history of information concerning the inside investor's decision on whether to follow on in all previous rounds. Therefore, the range of uncertainty regarding the true valuation would become finer, and the signaling effect of insider participation could be attenuated in the later stages of financing.

[10]We also conjecture that the impact of positive (negative) signal by the inside VC firm would be greater for those entrepreneurs who have poor (good) observable characteristics/credentials.

References

[1] Admati, Anat, and Paul Pfleiderer (1994), "Robust Financial Contracting and the Role of Venture Capitalists," *Journal of Finance* 49: 371–402.

[2] Bengtsson, Ola, and Berk Sensoy (2011), "Changing the Nexus: The Evolution and Renegotiation of Venture Capital Contracts," mimeo, Charles A. Dice Center Working Paper No. 2009-19. Available at SSRN: http://ssrn.com/abstract=1475347.

[3] Bergemann, Dirk, and Ulrich Hege (1998), "Venture Capital Financing, Moral Hazard, and Learning," *Journal of Banking & Finance* 22: 703–735.

[4] Bergemann, Dirk, and Ulrich Hege (2005), "The Financing of Innovation: Learning and Stopping," *RAND Journal of Economics* 36: 719–752.

[5] Brennan, Michael, and Alan Kraus (1987), "Efficient Financing Under Asymmetric Information," *Journal of Finance* 42: 1225–1243.

[6] Broughman, Brian, and Jesse Fried (2010), "VC Inside Rounds as Rescue Financing: Theory and Evidence," mimeo, Indiana University.

[7] Casamatta, Catherine (2003), "Financing and Advising: Optimal Financial Contracts with Venture Capitalists," *Journal of Finance* 58: 2059–2085.

[8] Cornelli, Francesca, and Oved Yosha (2003), "Stage Financing and the Role of Convertible Securities," *Review of Economic Studies* 70: 1–32.

[9] Dessí, Roberta (2005), "Start-up Finance, Monitoring, and Collusion," *RAND Journal of Economics* 36: 255–274.

[10] Gompers, Paul, and Josh Lerner (1999), *The Venture Capital Cycle*, The MIT Press: Cambridge, MA.

[11] Greenwald, Bruce (1986), "Adverse Selection in the Labour Market," *Review of Economic Studie*, 53: 325–347.

[12] Hirshleifer, Jack (1971), "The Private and Social Value of Information and the Reward to Inventive Activity," *American Economic Review* 61: 561–574.

[13] Hochberg, Yael, Alexander Ljungqvist, and Annette Vissing-Jorgensen (2010), "Informational Hold-Up and Performance Persistence in Venture Capital," EFA 2009 Bergen Meetings Paper. Available at SSRN: http://ssrn.com/abstract=1260496.

[14] Kaplan, Steven, and Per Stromberg (2003), "Financial Contracting Theory Meets the Real World: An Empirical Analysis of Venture Capital Contracts," *Review of Economic Studies* 70: 281–315.

[15] Kaplan, Steven, and Per Stromberg (2004), "Characteristics, Contracts, and Actions: Evidence from Venture Capitalist Analyses," *Journal of Finance* 59: 2177–2210.

[16] Neher, Darwin (1999), "Staged Financing: An Agency Perspective," *Review of Economic Studies* 66: 255–274.

[17] Ray, Korok (2010), "Staged Investments in Entrepreneurial Financing," mimeo, Georgetown University.

[18] Waldman, Michael (1984), "Job Assignments, Signalling, and Efficiency," *RAND Journal of Economics* 15: 255–267.

Appendix

This appendix contains all of the proofs omitted from the text.

Proof of Proposition 1

Proof. Suppose the inside VC firm's strategy is to follow on in the second round if and only if $p > p^*$, and the offer term is given by $\gamma_F^+ = \frac{2F}{(1+p^*)R}$. Let the outside VC firms' strategy be to offer to finance the second round in return for acquiring a $\gamma_F^+ = \frac{2F}{(1+p^*)R}$ share in an up round and a $\gamma_F^- = \frac{2F}{p^*R}$ in a down round, respectively. We can easily see that this strategy profile satisfies the zero-profit condition. First, in an up round, the expected probability of success is $\frac{1+p^*}{2}$, so that an outside firm's expected profit is $\frac{1+p^*}{2}\gamma_F^+ R - F = 0$. Since the offer terms are the same, the entrepreneur stays with the inside investor by the tie-breaking assumption. Second, in a down round, the expected probability of success is $\frac{p^*}{2}$, so that an outside investor's expected profit is $\frac{p^*}{2}\gamma_F^- R - F = 0$. Since only outside investors offer to finance, the entrepreneur takes any one of such offers.

Now we need to verify that the inside VC firm's cut-off strategy is optimal given outside firms' strategy. When the inside investor decides to follow on in the second round, outside firms believe that the success probability is greater than p^*; and, when it does not follow on, the belief is $p < p^*$. Notice that it is never optimal for the inside VC firms to offer better terms than the market, and offering worse terms does no better because outside firms will offer better terms, which the inside firm has to match. Given this, it is optimal for the inside VC firm to finance an entrepreneur in the second round if and only if the investor's expected profit conditional on $p > p^*$ is greater than its expected profit conditional on $p < p^*$, that is, if and only if the observed success probability p satisfies the following inequality:

$$p(\gamma_K + (1-\gamma_K)\frac{2F}{(1+p^*)R})R - F \geq p(\gamma_K(1-\frac{2F}{p^*R}))R.$$

Substituting p for p^* and simplifying yield $p^* = 1 - 2\gamma_K$. ■

Proof of Proposition 3

Proof. Notice that if an angel invests in the first round, then there is no signal sent to the market at the beginning of the second round. Given our Assumption 1, outside VC firms will invest in the second round with no signal from an inside investor because the expected probability of success is higher than in a down round. The second-round investor's share, $\gamma_F^0 = \frac{2F}{R}$, is pinned down by the zero-profit condition. Given this, an angel investor's expected profit from the first-round investment is $-K + \frac{1}{2}(\gamma_K(1-\frac{2F}{R})R\int_0^1 p(\gamma_K(1-\frac{2F}{R})R)dp$, where the second term is the expected share value. The angel investor's share is derived by the use of a zero-profit constraint, $-K + \frac{1}{2}(\gamma_K(1-\frac{2F}{R})R = 0$, which yields $\hat{\gamma}_K = \frac{2K}{R-2F}$.

The entrepreneur's share, if he finances through an angel in the first round, is $1 - [\hat{\gamma}_K(1-\gamma_F^0) + \gamma_F^0]$. On the other hand, if the entrepreneur finances through a VC firm who can follow on in the second round, his expected share is $1 - [\gamma_K^*(1 - E\gamma_F) + E\gamma_F]$, where $E\gamma_F = p^*\gamma_F^- + (1-p^*)\gamma_F^+$, from the previous section. The entrepreneur chooses to finance the first round through an angel rather than a VC firm if and only if $\hat{\gamma}_K(1-\gamma_F^0)+\gamma_F^0 < \gamma_K^*(1-E\gamma_F)+E\gamma_F$. By substitution, $\hat{\gamma}_K(1-\gamma_F^0)+\gamma_F^0 = \frac{2K+2F}{R}$. Since $E\gamma_F = p^*\gamma_F^- + (1-p^*)\gamma_F^+ = p^*\frac{2F}{p^*R} + (1-p^*)\frac{2F}{(1+p^*)R} = \frac{2F}{(1-\gamma_K^*)R}$, it follows that $\gamma_K^*(1-E\gamma_F)+E\gamma_F = \gamma_K^* + \frac{2F}{R}$. Thus, the inequality holds if and only if $\frac{2K+2F}{R} < \frac{2K}{R-2F} + \frac{2F}{R}$, or $\frac{2K}{R} < \frac{2K}{R-2F}$, which is indeed satisfied. ■

Proof of Proposition 4

Proof. Suppose there exists a threshold $\tilde{p} \in (0,1)$ such that entrepreneurs with ideas $p \geq \tilde{p}$ pursue financing with a VC firm (who may follow on) and entrepreneurs with ideas $p < \tilde{p}$ pursue financing with an angel. Let p^*, $p^* \geq \tilde{p}$, denote the threshold level above which a VC firm follows on. Then $\gamma_F^+ = \frac{2F}{(1+p^*)R}$ as in the base model, whereas γ_F^- is determined by $\frac{\tilde{p}+p^*}{2}\gamma_F^- R - F = 0$, giving $\gamma_F^- = \frac{2F}{(\tilde{p}+p^*)R}$. Let γ_K^+ denote the VC firm's share from the first round of financing given that $p \geq \tilde{p}$. Then γ_K^+ is obtained from

$$-K + \frac{1-p^*}{1-\tilde{p}}\left(\frac{1+p^*}{2}(\gamma_K^+ + (1-\gamma_K^+)\frac{2F}{(1+p^*)R})R - F\right) \tag{A1}$$
$$+\frac{p^*-\tilde{p}}{1-\tilde{p}}\left(\frac{\tilde{p}+p^*}{2}\gamma_K^+(1-\frac{2F}{(\tilde{p}+p^*)R})R\right) = 0.$$

Rearranging and simplifying yield

$$\gamma_K^+ = \frac{2K}{(1+\tilde{p})R - 2F}.$$

Notice that, given the VC firm's strategy, an entrepreneur with $p \leq p^*$ who finances with a VC firm has an expected share of

$$(1-\gamma_K^+)(1-\gamma_F^-). \tag{A2}$$

On the other hand, consider entrepreneurs with $p < \tilde{p}$ who pursue angel financing. In the second round, γ_F^0 is bid down such that $\frac{\tilde{p}}{2}\gamma_F^0 R - F = 0$, giving $\gamma_F^0 = \frac{2F}{\tilde{p}R}$. Let γ_K^0 denote the angel's first-round equity stake in return for K. Then γ_K^0 is obtained from $-K + \frac{\tilde{p}}{2}\gamma_K^0(1-\frac{2F}{\tilde{p}R})R = 0$, giving $\gamma_K^0 = \frac{2K}{\tilde{p}R-2F}$.[11] It follows that $\gamma_K^0 > \gamma_K^+$. Notice that an entrepreneur with $p < \tilde{p}$ who pursues angel financing has an expected share of

$$(1-\gamma_K^0)(1-\gamma_F^0). \tag{A3}$$

Suppose an entrepreneur with $p < \tilde{p}$ deviates to VC financing. Comparing (A2) and (A3), since $\gamma_K^+ < \gamma_K^0$ and $\gamma_F^- < \gamma_F^0$, an entrepreneur with any $p < \tilde{p}$ possesses a profitable deviation by instead financing with a VC firm.

Suppose now all entrepreneurs finance with a VC firm, that is, $\tilde{p} = 0$. From Propositions 1 and 2, the VC firm's shares in such a case are $\gamma_F^+ = \frac{2F}{(1+p^*)R}$, $\gamma_F^- = \frac{2F}{p^*R}$, and $\gamma_K^* = \frac{2K}{R-2F}$. A

[11] We assume here that $\tilde{p}$ is sufficiently large. If not, then the angel investor cannot finance the venture for those $p < \tilde{p}$, and these entrepreneurs will pursue financing with a VC firm.

deviation to angel financing leads to $\gamma_F^0 = \frac{2F}{\tilde{p}R}$ and $\gamma_K^0 = \frac{2K}{\tilde{p}R-2F}$, where the angel investor's off-the-equilibrium beliefs are that the entrepreneur's type p is below $p^*(0)$. Since then $\gamma_F^0 \geq \gamma_F^-$ and $\gamma_K^0 \geq \gamma_K^*$, no entrepreneur has an incentive to deviate.

Next, consider the case in which entrepreneurs with ideas $p \geq \tilde{p}$ pursue financing with an angel and those with ideas $p < \tilde{p}$ pursue financing with a VC firm. In such a case, the VC firm's share is $\gamma_F^+ = \frac{2F}{(\tilde{p}+p^*)R}$ in an up round and $\gamma_F^- = \frac{2F}{p^*R}$ in a down round. Using a zero-profit condition, the VC firm's first round share can be derived as above, so that $\gamma_K^+ = \frac{2K}{\tilde{p}R-2F}$. On the other hand, an angel investor's first- and second-round shares are respectively $\gamma_K^0 = \frac{2K}{(1+\tilde{p})R-2F}$ and $\gamma_F^0 = \frac{2F}{(1+\tilde{p})R}$. Following the same logic as above, since $\gamma_K^+ > \gamma_K^0$ and $\gamma_F^- > \gamma_F^+ > \gamma_F^0$, an entrepreneur with any $p \leq \tilde{p}$ possesses a profitable deviation by instead financing with an angel.

Finally, suppose all entrepreneurs finance with an angel. From Proposition 3, the angel investor's shares in such a case are $\gamma_K^0 = \frac{2K}{R-2F}$ and $\gamma_F^0 = \frac{2F}{R}$. A deviation to VC financing leads to $\gamma_K^+ = \frac{2K}{(1+\tilde{p})R-2F}$ and $\gamma_F^+ = \frac{2F}{(1+p^*)R}$, where the VC firm's off-the-equilibrium beliefs are that the entrepreneur's type p satisfies $p \in [p^*, 1]$. Since then $\gamma_F^0 > \gamma_F^+$ and $\gamma_K^0 > \gamma_K^+$, an entrepreneur with $p \geq p^*$ indeed has an incentive to deviate to VC financing. ■

Proof of Proposition 5

Proof. Suppose entrepreneurs with no private information finance with an angel. For a given $\theta \in (0, 1)$, suppose there exists a threshold $\tilde{p}$ such that entrepreneurs with $p < \tilde{p}$ finance with an angel and those with $p \geq \tilde{p}$ finance with a VC firm. Then the expected type of an entrepreneur who finances with an angel is given by

$$p_0 = \frac{\theta\tilde{p}}{1-\theta+\theta\tilde{p}}\frac{\tilde{p}}{2} + \frac{1-\theta}{1-\theta+\theta\tilde{p}}\frac{1}{2} = \frac{1-\theta+\theta\tilde{p}^2}{2((1-\theta)+\theta\tilde{p})}.$$

Using analogous notation to the previous proof, it follows that $\gamma_F^0 = \frac{F}{p_0 R}$ and $\gamma_K^0 = \frac{K}{p_0 R - F}$, where an entrepreneur's share when financing with an angel is given by $(1-\gamma_K^0)(1-\gamma_F^0)$.

We have previously shown that an entrepreneur who finances with a VC firm with the private information that $p \geq \tilde{p}$ has an expected equity share of

$$(1-\gamma_K^+)(1 - \frac{\tilde{p}-p^*}{1-\tilde{p}}\gamma_F^- - (1 - \frac{\tilde{p}-p^*}{1-\tilde{p}})\gamma_F^+).$$

where $\gamma_F^+ = \frac{2F}{(1+p^*)R}$, $\gamma_F^- = \frac{2F}{(\tilde{p}+p^*)R}$, $\gamma_K^+ = \frac{2K}{(1+\tilde{p})R-2F}$, and p^*, $p^* \geq \tilde{p}$, is the threshold for the VC firm's follow-on strategy.

We show that $\tilde{p}$ is a decreasing function of θ. First, notice that p_0 is decreasing in θ as long as $\tilde{p}$ is decreasing in θ. This is clearly seen by rearranging p_0 as follows:

$$p_0 = \frac{1}{2}\left(1 + \tilde{p} + \frac{\tilde{p}}{\theta(1-\tilde{p})-1}\right).$$

Second, therefore, if θ increases, then p_0 decreases, so both γ_F^0 and γ_K^0 increase. For $\tilde{p}$ to be a threshold in equilibrium, the entrepreneur's expected share from angel financing and

VC financing must be the same given the threshold strategy. Thus, γ_F^+, γ_F^-, and γ_K^+ must increase when θ decreases, to keep the payoffs from either type of financing the same. This means that $\tilde{p}$ (as well as p^*) has to decrease, establishing the negative relationship between θ and $\tilde{p}$ assumed above.

From the previous Propositions, we know that the entrepreneurs finance with an angel when $\theta = 0$ and finance with an VC firm when $\theta = 1$. By continuity of the function, $\tilde{p}(\theta)$, and the negative relationship between the two, there exists a value θ' such that all entrepreneurs finance with an angel if $\theta < \theta'$. On the other hand, when $\tilde{p}(\theta)$ is sufficiently low, then an angel financing for those $p < \tilde{p}$ cannot be sustained due to adverse selection, and thus, there exists a value θ'' above which all entrepreneurs finance with a VC firm. When $\theta' < \theta < \theta''$, by construction, the expected shares from VC financing and angel financing are the same for uninformed entrepreneurs. It thus follows that informed entrepreneurs with $p < \tilde{p}$ prefer to finance with angel investors in this case and no entrepreneur type has an incentive to deviate. ■

Proof of Proposition 6

Proof. The threshold probability p_i^* of a first-round investor i who has sufficient liquidity in the second stage is determined from the following indifference condition:

$$p_i^*(\gamma_{K,i} + (1-\gamma_{K,i})\frac{2F}{(1+p_i^*)R})R - F = p_i^*\gamma_{K,i}(1 - \frac{2F}{(1-\lambda_i(1-p_i^*))R})R, \tag{A4}$$

from which we obtain

$$p_i^* = \frac{2\lambda_i(1-\gamma_{K,i}) - 1 + \sqrt{1 + 4\gamma_{K,i}(2 - \lambda_i(3 - \lambda_i\gamma_{K,i}))}}{2(2\gamma_{K,i}(1-\lambda_i) + \lambda_i)}. \tag{A5}$$

Investor i's first-round minimum equity share, $\gamma_{K,i}$, in return for investing K, is implicitly defined by the following indifference condition:

$$-K + \lambda_i\big[(1-p_i^*)(\frac{1+p_i^*}{2}(\gamma_{K,i} + (1-\gamma_{K,i})\gamma_{F,i}^+)R - F) + p_i^*\frac{p_i^*}{2}\gamma_{K,i}(1-\gamma_{F,i}^-)R\big] \tag{A6}$$

$$+(1-\lambda_i)\big[\frac{1}{2}\gamma_{K,i}(1-\gamma_{F,i}^-)R\big] = 0$$

where $\gamma_{F,i}^+ = \frac{2F}{(1+p_i^*)R}$, $\gamma_{F,i}^- = \frac{2F}{(1-\lambda_i(1-p_i^*))R}$, and p_i^* is given by (A5). From (A6), it follows that for $\lambda_i = 0$ or 1, $\gamma_K = \frac{2K}{R-2F}$, as in the base model and the case of an angel investor. Continuity then entails that γ_K is non-monotonic in λ. ■

Proof of Proposition 7

Proof. When the probability of success is perfectly observed by all investors in the second period, investors will offer to finance only if the expected profit is non-negative, that is, when $p\gamma_F R - F \geq 0$. Since the share of the company cannot exceed 1, investors will invest only if $p \geq \bar{p} = \frac{F}{R}$. Given that the offer terms, $\tilde{\gamma}_F = \frac{F}{pR}$, are the same, by the tie-breaking assumption the entrepreneur will stay with the inside VC firm. Knowing its second period share and the cutoff level of probability for which second-round investment yields non-negative return, an inside VC firm's first period share is determined by the zero-profit condition: $-K + \int_{\bar{p}}^{1} \left[p \left(\gamma_K (1 - \frac{F}{pR}) + \frac{F}{pR} \right) R - F \right] dp = 0$, which reduces to $-K + \gamma_K R \frac{1-\bar{p}^2}{2} - \gamma_K F(1-\bar{p}) = 0$. Substituting out $\bar{p} = \frac{F}{R}$ and simplifying yield $\tilde{\gamma}_K = \frac{2KR}{(R-F)^2}$. Since $\gamma_K^* = \frac{2K}{R-2F} = \frac{2KR}{R^2-2FR}$, it follows that $\tilde{\gamma}_K < \gamma_K^*$.

Similar to the proof of Proposition 3, entrepreneurs prefer the disclosure policy if and only if their expected share is higher under disclosure than under no disclosure, that is, if and only if $\tilde{\gamma}_K(1-E\tilde{\gamma}_F)+E\tilde{\gamma}_F < \gamma_K^*(1-E\gamma_F)+E\gamma_F$. We showed that $\gamma_K^*(1-E\gamma_F)+E\gamma_F = \gamma_K^* + \frac{2F}{R}$. Notice that $E\tilde{\gamma}_F = \int_{\bar{p}}^{1} \frac{F}{pR} dp = \frac{F}{R}(-\ln \frac{F}{R})$, which is a concave function of $\frac{F}{R}$. Furthermore, $E\tilde{\gamma}_F$ attains the maximum value of $\frac{1}{e}$ at $\frac{F}{R} = \frac{1}{e}$, $E\tilde{\gamma}_F = 0$ at $\frac{F}{R} = 0$, and $\lim\limits_{F/R \to 0} \frac{dE\tilde{\gamma}_F}{d(F/R)} = \infty$. Since $\frac{2F}{R} = \frac{2}{e}$ at $\frac{F}{R} = \frac{1}{e}$ and $\tilde{\gamma}_K > 0$, $\frac{2F}{R}$ and $(1-\tilde{\gamma}_K)E\tilde{\gamma}_F$ cross once at some value ξ, $0 < \xi < \frac{1}{2}$. The above inequality can be expressed as $\tilde{\gamma}_K + (1-\tilde{\gamma}_K)\frac{F}{R}(-\ln \frac{F}{R}) < \gamma_K^* + \frac{2F}{R}$. Since $\tilde{\gamma}_K < \gamma_K^*$, this inequality holds for $\frac{F}{R} \geq \xi$. If $\frac{F}{R} < \xi$, then $(1-\tilde{\gamma}_K)\frac{F}{R}(-\ln \frac{F}{R}) > \frac{2F}{R}$. Take a small enough value for $\frac{F}{R}$ such that the difference $\left|(1-\tilde{\gamma}_K)\frac{F}{R}(-\ln \frac{F}{R}) - \frac{2F}{R}\right|$ is larger than $|\gamma_K^* - \tilde{\gamma}_K|$ as $\frac{F}{R}$ converges to zero. By the intermediate value theorem, there is a value ϕ, $0 < \phi < \xi < \frac{1}{2}$, such that $\tilde{\gamma}_K + (1-\tilde{\gamma}_K)\frac{F}{R}(-\ln \frac{F}{R}) < \gamma_K^* + \frac{2F}{R}$ if and only if $\frac{F}{R} > \phi$. ■