IMPERFECT TARGETED ADVERTISING AND PRIVACY REGULATIONS

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Abstract

I investigate how privacy regulations affect welfare. Tougher privacy regulations reduce the accuracy of information collected on consumers. Less accurate information decreases welfare by discouraging targeted advertising. When firms target advertise, privacy regulations have an ambiguous effect on welfare. Less accurate information decreases welfare by inducing a smaller, less-targeted selection of products. Yet less accurate information increases welfare by inducing fewer annoying ads. In extensions, I find that tougher privacy regulations increase the product selection benefit and the ad annoyance cost through reducing ad avoidance and the ad price; and decrease the product selection benefit and the ad annoyance cost through greater marketing costs.

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1 Introduction

Over the past decade, the Federal Trade Commission (FTC) has struggled with how to regulate online "targeted advertising" to protect consumers' personal information. *Targeted advertising* is the personalizing of advertisements to fit consumers' tastes. In 2009, the FTC revised its regulations on online targeted advertising.[1] These regulations include restrictions on what information can

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[1]Despite calling these regulations "self regulating principles," the FTC Commissioner Jon Leibowitz claims that the FTC has the right to and should enforce them.

be collected on consumers and on how this information can be collected. The FTC designed these regulations to balance the benefits of better personalized ads with the costs to consumers' privacy.

When a firm gathers more information on a consumer, it has a more accurate signal on that consumer's taste. I will refer to this as *signal accuracy.* Firms use these signals to determine which consumers to show their ads. Consumers, including myself, appreciate this because they get informed of a greater selection of more personalized products. I appreciate getting ads on products like DVDs and books, instead of ads on feminine hygiene or senior living.

Aside from protecting ourselves from the criminal use of our information, there has been little rational explanation for why consumers value the protection of their personal information from advertisers. Yet even with identity theft protection, regulation, and insurance, people feel uncomfortable with businesses knowing too much of their personal information. In this paper, I propose an explanation for this with advertisement annoyance. When firms get too much information about consumers, too many firms enter the market, pestering consumers with too many annoying ads. Personally, I dislike receiving junk mail and spam email more than I am worried about companies misusing my credit card numbers.[2] Especially, because my credit card provider protects me with account monitoring, password protection, and a guarantee to cover stolen funds.

In this paper, I explore the effect of increasing signal accuracy on social welfare. As each firm gets more accurate information on each consumer's taste, firms switch from advertising to all consumers, or equivalently *mass advertising*, to targeted advertising. Consumers do not benefit from this switch to targeted advertising through the selection of products offered to them. I find that the initial cost of being offered fewer products equals the initial benefit from being offered better matching products, because the signal accuracy is so low. Yet consumers do benefit from this switch to targeted advertising through a reduction in annoying ads. Although targeted advertising induces more firms to enter the market, fewer firms advertise to each consumer, because each firm is advertising to a smaller, targeted segment of the market.

When firms target advertise, signal accuracy has an ambiguous effect on social welfare. A higher signal accuracy increases welfare by inducing firms to offer each consumer more products, which are better matched to that consumer's taste. I refer to this as the *product selection benefit.* Yet a higher signal accuracy decreases welfare by inducing more firms to enter the market and pester consumers with a greater number of annoying ads. I refer to this as the *ad annoyance cost.*

I find that the product selection benefit is increasing and convex in signal accuracy. This means that as firms receive better signals about each consumer's taste, the benefit to consumers from the products they buy increases at an increasing rate. Because of this result, I find that if the ad annoyance is convex enough in the number of ads a consumer sees, then social welfare has an inverted-u shape with respect to signal accuracy. This means that if the

[2] Don't tell the criminals.

marginal annoyance of each additional ad is increasing fast enough, then social welfare could be optimized with an interior signal accuracy. Otherwise social accuracy has a u shape with respect to signal accuracy. Then social welfare is maximized with either firms getting a perfect signal on consumers' tastes or firms getting a signal just accurate enough to induce targeted advertising.

In section 6, I extend my model to consider the case where each firm can pay to get a more accurate signals about consumers' tastes. I call this the *marketing cost*. Firms collect information on consumers through surveys, tracking behavior, locational characteristics, credit scores, etc. As firms collect more information on consumers and receive better signals about each consumer's taste, firms pay for additional marketing. I assume that marketing cost is increasing and convex in signal accuracy. In the targeted advertising model of Iyer et al. (2005), firms pay a fixed marketing cost to have the ability to target advertise, instead of mass advertising. I extend this by making marketing cost an endogenous function of signal accuracy.

I find that firms would pay for no marketing research, when they mass advertise, and for the same amount of marketing research, when they target advertise. This confirms the assumption of Iyer et al. (2005) of a fixed marketing research cost for targeted advertising.

In addition, I investigate how marketing cost affects social welfare. I find that marketing cost adds to the entry cost, discouraging some firms from entering the market. This decreases social welfare by decreasing the product selection benefit. Each consumer gets informed about a smaller, less-targeted selection of products. Yet this increases social welfare by decreasing the ad annoyance cost. Each consumer is pestered by fewer ads. Tougher privacy laws could increase the marketing cost, by making it more expensive for firms to gather information about consumers. This would induce firms to choose a lower signal accuracy. Under the social optimal marketing frictions, I find that the equilibrium signal accuracy could be greater or less than the optimal signal accuracy under no or fixed marketing costs, depending on the convexity of the ad annoyance function. This means that how privacy laws restrict the signal accuracy matters. Sometimes it is more optimal for us to set signal accuracy by picking the information that firms can collect. Other times it is more optimal for us to induce a signal accuracy by changing firms' costs of collecting information.

In section 7, I extend my model to investigate how signal accuracy affects social welfare through an endogenous price of ads. For a low quantity and high price of ads, firms target advertise. For an intermediate quantity and cutoff price of ads, firms *mix advertise*, or equivalently firms mix between targeted advertising and mass advertising. Either all firms advertise to some of the consumers not in their targeted segment, or some firms target advertise and some firms mass advertise. For a high quantity and low price of ads, firms will mass advertise. Increasing the signal accuracy, lowers the cutoff price, inducing more targeting. I find that this unambiguously increases social welfare. When firms target advertise, an increase in signal accuracy increases the demand for advertising, which increases the price of ads. Higher ad prices have an additional effect on social welfare by discouraging firms from entering. This creates an additional decrease in social welfare through the product selection benefit. Consumers get informed about fewer products, because advertising is more expensive. Yet it creates an additional increase in social welfare through the ad annoyance cost. Consumers are pestered by fewer annoying ads, because advertising is more expensive.

In section 8, I will extend my model to allow consumers to choose to avoid or block some ads for a cost. Consumers may choose to block an increasing number of ads by paying the costs and opportunity costs in participate in no-call-or-email lists, spam filters, and driving down roads with fewer billboards. The more that a consumer participates in these programs, the more expensive it becomes and the more ads he blocks. This differs from the ad avoidance tool in the targeted advertising model of Johnson (2010), because he only allows consumers to choose to avoid all ads or not avoid any ads.

I investigate how signal accuracy affects social welfare through an endogenous ad avoidance. I find that increasing signal accuracy, induces consumers to avoid more ads, because more firms enter the market. This may have a positive (or negative) effect on the probability that an ad is avoided. This would cause an additional decrease (or increase) to the product selection benefit and the ad annoyance cost.

In section 9, I will consider the case where firms receive the same common signal about each consumer's type. In my basic model, the signals each firm receives about each consumer are independent. Yet it is reasonable to think that these signals would be interdependent, because the firms could be gathering the same information about each consumer. In section 10, I conclude.

2 Literature Review

This paper is most similar to Bergemann and Bonatti (2011) and Johnson (2010). All three papers explore the effect of signal accuracy on social welfare. Bergemann and Bonatti (2011) and Johnson (2010) tend to interpret improving signal accuracy as a result of changing technology, while I interpret reducing signal accuracy as a result of increasing privacy regulations. In Bergemann and Bonatti (2011) and Johnson (2010), firms do not compete in the product market, while in this paper, firms are monopolistically competitive in the product market. In this paper, each consumer chooses to buy one of the products advertised to him or an outside option.

In all three papers, an improvement in signal accuracy increases social welfare through increasing the chance that a consumer will like a product advertised to him and decreases social welfare through the market for advertisements.

In this paper and Johnson (2010), improved signal accuracy induces more firms to enter the market, increasing the ad annoyance cost faced by consumers. The big difference between our papers is that in Johnson (2010) this effect is intertwined with an ad avoidance or ad blocking effect on social welfare. In Johnson (2010), an improvement in signal accuracy encourages firms to advertise to more consumers, which encourages more consumers to block advertisements.

This additional ad avoidance hurts firms profits and discourages advertising, which reduces the product selection benefit and ad annoyance cost. In section 8, I add ad avoidance to my basic model to show how it changes the effect of signal accuracy on welfare.

In Bergemann and Bonatti (2011), improved signal accuracy effects social welfare through changes in the prices of advertisements. It induces firms to advertise in fewer markets, decreasing the welfare gained from consumers finding products they like. In their model, consumers have no ad annoyance effect, because consumers are delivered a fixed number of ads. Firms buy these ads through perfect competition or from a monopolist publisher. In section 7, I extend my model to allow for endogenous advertisement costs through a general supply of advertisements. I show the additional effect of ad pricing on social welfare through product selection (similar to Bergemann and Bonatti, 2011) and through ad annoyance.

This paper is also related to Iyer et al. (2005). Both papers analyze firms decisions to either mass advertise or target advertise. Iyer et al. (2005) analyzes the case of a duopoly, and I analyze the case of monopolistic competition. In both papers, the incentive to target advertise is the cost effectiveness of targeted advertising. A firm would advertise to only those consumers more likely to buy its product. In this paper, the incentive to mass advertise comes from the inaccuracy of the signal that a firm would get about a consumer's taste. Sending an ad to any consumer, always has a some positive probability of a sale. In Iyer et al. (2005), the incentive to mass advertise is an added fixed marketing cost that a firm would pay to have the ability to targeted advertise. In section 6, I add marketing costs to my model by allowing firms to pay for the accuracy of the information that they receive on consumers' tastes. This extends Iyer et al. (2005) by making marketing costs endogenous. I find that firms would pay for no marketing research, when they mass advertise, and for the same amount of marketing research, when they target advertise. This confirms the assumption in Iyer et al. (2005) of a fixed marketing research cost for targeted advertising.

Both Iyer et al. (2005) and this paper find that for a low per-consumer advertising cost or a high marketing cost, firms will mass advertise, and for a high per-consumer advertising cost or a low marketing cost, firms will target advertise.[3] I extend this to show that for a low signal accuracy, firms mass advertise, and for a high signal accuracy, firms mass advertise.

This paper is also related to Esteban et al. (2001). In their paper, the specialization of a magazine or other advertising medium is equivalent to signal accuracy. Consumers reading more specialized magazines are more likely to by the product. Esteban et al. (2001) show that if a social planner picks product price and degree of advertisement specialization, then targeted advertising is

[3]Iyer et al. (2005) also show that for an intermediate per-consumer advertising cost or an intermediate marketing cost, one firm will target advertise and the other will mass advertise. There results depend on the fact that consumers who would be willing to buy either product (the comparison shoppers) have the same value r for both products. If both firms were to advertise to the comparison shoppers, then firms would both set a product price of zero in Bertrand style price competition.

socially preferable to mass advertising. I get a similar result. I find that if a social planner picks the signal accuracy, then targeted advertising is socially preferable to mass advertising.

Esteban et al. (2001) also show that a firm might choose to specialize its advertising too much. This might make targeted advertising less socially desirable than mass advertising, because the firm may choose to increase its prices too much.[4] My paper extends this discussion by the addition of ad annoyance costs. I find that if firms are getting too accurate or too inaccurate information on consumers, then targeted advertising might be less socially desirable than mass advertising, because consumers face additional advertisement annoyance costs.

3 Game

There is a sufficiently large number of profit maximizing firms. Each firm may potentially enter the market at an entry cost $F > 0$ and produce a product at a constant marginal cost normalized to zero. There is a unit mass of utility maximizing consumers. Half of the consumers are type 0 and the other half of the consumers are type 1.

Initially, a firm knows the aggregate distribution of types, but does not know the types of individual consumers. If a firm chooses to enter, then it receives a private, i.i.d. signal about each consumer's type that is true with probability $\frac{1+\theta}{2}$ and false with a probability $\frac{1-\theta}{2}$, where $\theta \in [0, 1]$. I interpret θ as the signal accuracy. A θ of one would be a perfect signal about consumers' types and a θ of zero would be a meaningless signal about consumers' types.

Each entrant j simultaneously chooses its product type (type 0 or a type 1), which consumers it will advertise to, and single product price p_j to maximize its own profit Π_j. For each consumer whom it chooses to advertise to, entrant j pays a constant per consumer advertising cost c. These choices are observed by all consumers. If firm j enters, advertises to M_j consumers, and sells Q_j units of its good then it gets a profit of $\Pi_j \equiv p_j Q_j - cM_j - F$.

Each consumer i may buy one unit of his choice from the goods advertised to him, or he may choose to take an outside option.[5] The utility consumer i would get from buying good j is $u_{ij} = R + b\lambda_{ij} - p_j + \epsilon_{ij} - A(N)$, where $R > 0$ is the benefit from consuming any advertised good, $b > 0$ is the benefit from buying a good personalized to a consumer's type, λ_{ij} is one if consumer i's type matches product j's type and zero otherwise, p_j is the price of good j, ϵ_{ij} is an i.i.d. stochastic shock to consumer i's value for good j with a c.d.f. of $F(\epsilon) = e^{-e^{-\epsilon/\mu}}$, and $A(N)$ is the ad annoyance suffered by a consumer from seeing ads from N different firms. I assume that for all $N \geq 0$ I have $A'(N) > 0$. The utility consumer i would get from buying the outside option is $u_{i0} = \epsilon_{i0} - A(N)$, where

[4] He gets this result when mass advertising wastes few ads and when product demand is sufficiently inelastic.

[5] This may simply be the option not to buy any good

ϵ_{i0} is an i.i.d. stochastic shock to consumer i's value for the outside option with a c.d.f. of $F(\epsilon) = e^{-e^{-\epsilon/\mu}}$.

4 Equilibrium

In this section, I find a pure strategy Nash Equilibrium that is symmetric in pricing and advertising with an equal number of firms of each type. I start by finding the equilibrium prices and number of entrants in each market for any given signal accuracy $\theta \in [0, 1]$. Then I examine how social welfare changes with signal accuracy θ.

If entrant j expects a profit from advertising to a consumer with a signal of type $l \in \{0, 1\}$, then entrant j expects a profit from advertising to each and every consumer with a signal of type l. Therefore a firm only chooses one of four possibilities: 1) advertise to all consumers (mass advertise), 2) advertise to all consumers with a signal of its type and only some of the other consumers (mixed advertise), 3) advertise to only those consumers with a signal of its type (target advertise), and 4) not advertise to any consumers (not enter the market).

If entrant j of type $k \in \{0, 1\}$ expects a profit from advertising to a consumer with a signal of type $l \in \{0, 1\}$, then every entrant of type k expects a profit from advertising to each and every consumer with a signal of type l, and by symmetry, every entrant of type $1 - k$ expects a profit from advertising to each and every consumer with a signal of type $1-l$. Therefore I have one of three situations: 1) all entrants advertise to all consumers (the mass advertising equilibrium), 2) all entrants target their advertising (the targeted advertising equilibrium), and 3) entrants are indifferent between mass advertising and targeted advertising (the mixed advertising equilibrium).

4.1 Mass Advertising Equilibrium

If firm j of type $k \in \{0, 1\}$ advertises to all consumers, then half of the consumers it advertises to will be of type k and half of the consumers it advertises to will be of type $1 - k$. In addition, if all firms mass advertise, then half of the firms that advertise to a consumer will be of his type, and half of the firms that advertise to a consumer will not be of his type. It is straight-forward to show that if $2N$ firms enter the market (including firm j) and if all firms set a price of p, firm j will sell to a share $e^{\frac{R+b-p_j}{\mu}}/K_{MA}$ of the type k consumers and to a share $e^{\frac{R-p_j}{\mu}}/K_{MA}$ of the type $1-k$ consumers, where $K_{MA} \equiv 1+N[e^{\frac{R+b-p}{\mu}} + e^{\frac{R-p}{\mu}}]$.[6] Therefore firm j's quantity Q_j sold as a function of its price p_j is given by (1).

$$Q_j = \frac{\frac{1}{2}[e^{\frac{R+b-p_j}{\mu}} + e^{\frac{R-p_j}{\mu}}]}{K_{MA}} \tag{1}$$

[6] These market shares are found in Anderson et al. (1992, p. 39-40) for a more general framework. My addition is the separation into two market types.

In this paper I analyze the case of monopolistic competition. Similar to the basic logit monopolistic competition model presented in Anderson et al. (1992, p. 221-226), this market structure is the limit case where there are so many firms that an individual firm's decisions do not impact the market variable K_{MA}, or an individual firm does not consider its impact on the the market variable K_{MA} (as in Dixit and Stiglitz, 1977).

Optimizing firm j's profit over its price p_j and using symmetry, I find that each advertiser sets a price of $p = \mu$ and sells the same quantity Q of their good. Because firm j would advertise to all of the consumers, it would pay an advertising cost c. By free entry, I have the zero profit condition $F + c = pQ$, which solving for N becomes (ZPC-MA).

$$N = \frac{\mu}{2(F+c)} - \frac{1}{e^{\frac{R+b-\mu}{\mu}} + e^{\frac{R-\mu}{\mu}}} \equiv N_{MA} \qquad \text{(ZPC-MA)}$$

Here $\frac{\mu}{2(F+c)}$ is the number of firms that would enter each sub-market if there were no outside option. It is also the effect of the entry cost F and the advertising cost c on the entry. And $[e^{\frac{R+b-\mu}{\mu}} + e^{\frac{R-\mu}{\mu}}]^{-1}$ is the number of firms discouraged from entering each sub-market due to the outside option.

Because firms make zero profits, total social welfare is total consumer welfare. This is the sum of the aggregate consumer surplus from sales minus the aggregate ad annoyance cost. Using the derivation of consumer surplus in Anderson et al. (1992, p. 60-61), the total welfare TW is given by (TW-MA).

$$TW = R - \mu + \mu\Big[\ln(e^{b/\mu}+1) + \ln(\frac{\mu}{2(F+c)})\Big] - A(2N_{MA}) \equiv TW_{MA} \qquad \text{(TW-MA)}$$

Here $R - \mu + \mu[\ln(e^{b/\mu} + 1) + \ln(\frac{\mu}{2(F+c)})]$ is the aggregate consumer surplus gained from sales, while $A(2N_{MA})$ is the aggregate ad annoyance cost.[7] Higher entry costs F or advertising costs c would hurt consumers though a worse product selection by way of the term $\ln(\frac{\mu}{2(F+c)})$ and would benefit consumers through less ad annoyance $A(2N_{MA})$ by way of less entry, see equation (ZPC-MA).

4.2 Targeted Advertising Equilibrium

If firm j of type $k \in \{0, 1\}$ advertises to only those consumers with a signal of k, then firm j will only advertise to half of the consumers: $\frac{1+\theta}{2}$ of whom will be of type k, and $\frac{1-\theta}{2}$ of whom will be of type $1-k$. In addition, if all firms target advertise, then $\frac{1+\theta}{2}$ of the firms that advertise to a consumer will be of his type, and $\frac{1-\theta}{2}$ of the firms that advertise to a consumer will be not of his type. It is straight-forward to show that if $2N$ firms enter the market and if all firms sets a market price of p, firm j will sell to a share $e^{\frac{R+b-p_j}{\mu}}/K_{TA}$ of the type k consumers and a share $e^{\frac{R-p_j}{\mu}}/K_{TA}$ of the type $1-k$ consumers, where

[7] Note that consumers see ads from both types of firms, so they receive ads from $2N$ firms.

$K_{TA} \equiv 1 + N[\frac{1+\theta}{2}e^{\frac{R+b-p}{\mu}} + \frac{1-\theta}{2}e^{\frac{R-p}{\mu}}]$.[8] Therefore firm j's quantity Q_j sold as a function of its price p_j is given by (2).

$$Q_j = \frac{\frac{1}{2}[\frac{1+\theta}{2}e^{\frac{R+b-p_j}{\mu}} + \frac{1-\theta}{2}e^{\frac{R-p_j}{\mu}}]}{K_{TA}} \tag{2}$$

Note the difference between equations (1) and (2). Under targeted advertising, firm j would advertise to half the consumers than it would under mass advertising, which explains the differences in the numerators of equations (1) and (2). Yet under targeted advertising, a consumer would only see half of the ads, while under mass advertising, a consumer would see all of the ads. In addition under targeted advertising, a consumer would see $\frac{1+\theta}{2}$ of its ads from products of its type and $\frac{1-\theta}{2}$ of its ads from products not of its type. This explains the difference between K_{MA} and K_{TA}.

Similar to section 4.1, by optimizing firm j's profit over its price p_j and using symmetry, I find that each advertiser sets a price of $p = \mu$ and sells the same quantity Q of their good. Because firm j would advertise to half of the consumers, it would pay an advertising cost $c/2$. By free entry, I have the zero profit condition $F + c/2 = pQ$, which solving for N becomes (ZPC-TA).

$$N = \frac{\mu}{2(F+c/2)} - \frac{1}{\frac{1+\theta}{2}e^{\frac{R+b-\mu}{\mu}} + \frac{1-\theta}{2}e^{\frac{R-\mu}{\mu}}} \equiv N_{TA}(\theta) \tag{ZPC-TA}$$

Here $\frac{\mu}{2(F+c/2)}$ is the number of firms that would enter each sub-market if there were no outside option. It is also the effect of the entry cost F and the advertising cost $c/2$ on the entry. And $[\frac{1+\theta}{2}e^{\frac{R+b-\mu}{\mu}} + \frac{1-\theta}{2}e^{\frac{R-\mu}{\mu}}]^{-1}$ is the number of firms discouraged from entering each sub-market due to the outside option. Note that, unlike (ZPC-MA), this term depends on the signal accuracy θ, because here firms are using their signals to decide which consumers to show their ads. Coincidentally the fact that θ only affects entry through the outside option is a result of the form of the demand function.

Because firms make zero profits, total social welfare is total consumer welfare. This is the sum of the aggregate consumer surplus from sales minus the aggregate ad annoyance cost. Using the derivation of consumer surplus in Anderson et al. (1992, p. 60-61), the total welfare TW is given by (TW-TA).

$$TW = R - \mu + \mu\Big[\ln(\frac{1+\theta}{2}e^{b/\mu} + \frac{1-\theta}{2}) + \ln(\frac{\mu}{2(F+c/2)})\Big] - A(N_{TA}) \equiv TW_{TA}(\theta) \tag{TW-TA}$$

Here $R - \mu + \mu\Big[ln(\frac{1+\theta}{2}e^{b/\mu} + \frac{1-\theta}{2}) + ln(\frac{\mu}{2(F+c/2)})\Big]]$ is the aggregate consumer surplus gained from sales, while $A(N_{TA})$ is the aggregate ad annoyance cost.

[8]These market shares are found in Anderson et al. (1992, p. 39-40) for a more general framework.

In the mass advertising equilibrium, the signal accuracy θ doesn't matter, because firms advertise to all consumers anyway. Here, in the targeted advertising equilibrium, the signal accuracy θ does matter, because firms only advertise to those consumers with signals that match their product characteristic. Higher signal accuracy θ would benefit consumers though better product selection by way of the term $\ln(\frac{1+\theta}{2}e^{b/\mu} + \frac{1-\theta}{2})$ and would hurt consumers through higher ad annoyance $A(N_{TA})$ by way of more entry, see equation (ZPC-TA).

4.3 Equilibrium Mass and Targeting Conditions

For most signal accuracies θ there is only one possible equilibrium. In this section I show that there exists a threshold signal accuracy $\widehat{\theta}$ such that for $\theta < \widehat{\theta}$ I have the mass advertising equilibrium and for $\theta > \widehat{\theta}$ I have a targeted advertising equilibrium.

Under the mass advertising equilibrium, a firm j with a product characteristic k must expect profit from advertising to consumers with signal $1-k$: $\frac{1-\theta}{2}$ of whom will be of type k, and $\frac{1+\theta}{2}$ of whom will be of type $1-k$. The quantity of consumers with signal $1-k$ who buy his product would therefore be q_{MA}, given by (3). Therefore I have $c/2 \leq pq_{MA}$, which reduces to $\theta \leq \widehat{\theta}$ where $\widehat{\theta}$ is given by (4).

$$q_{MA} = \frac{\frac{1}{2}[\frac{1-\theta}{2}e^{\frac{R+b-p}{\mu}} + \frac{1+\theta}{2}e^{\frac{R-p}{\mu}}]}{K_{MA}} \tag{3}$$

$$\hat{\theta} \equiv \frac{F}{F+c}\frac{e^{b/\mu}+1}{e^{b/\mu}-1} \tag{4}$$

Under the targeted advertising equilibrium, a firm j with a product characteristic k must not expect profit from advertising to consumers with signal $1-k$: $\frac{1-\theta}{2}$ of whom will be of type k, and $\frac{1+\theta}{2}$ of whom will be of type $1-k$. Similar to the mass advertising case, the quantity of consumers with signal $1-k$ who buy from firm j with a product characteristic k would be q_{TA}, given by (5). Therefore I have $c/2 \geq pq_{TA}$, which reduces to $\theta \geq \widehat{\theta}$ where $\widehat{\theta}$ is given by (4).

$$q_{TA} = \frac{\frac{1}{2}[\frac{1-\theta}{2}e^{\frac{R+b-p}{\mu}} + \frac{1+\theta}{2}e^{\frac{R-p}{\mu}}]}{K_{TA}} \tag{5}$$

Therefore $\widehat{\theta}$ is the minimum signal accuracy θ needed for targeted advertising to be profitable. When $\theta < \widehat{\theta}$ I have the mass advertising equilibrium and when $\theta > \widehat{\theta}$ I have the targeted advertising equilibrium. When $\theta = \widehat{\theta}$, I can have either the mass advertising equilibrium, the targeted advertising equilibrium, or the mixed advertising equilibrium. The mixed equilibrium is only feasible when $\theta = \widehat{\theta}$, because firm j with a product characteristic k needs to be indifferent between advertising and not advertising to consumers with signal $1-k$.

From (4), I have that the threshold signal accuracy $\widehat{\theta}$ increases with a higher entry cost F, a lower per-person advertising cost c, a lower benefit b from consuming a product that matches your type, and a lower variation μ in consumers' tastes. I interpret this as if entry costs are high enough, advertising is cheap enough, the benefit from buying a good matching a consumer's type is low enough, and consumers' tastes are varied enough, then an entrant might as well sell to the less profitable group of consumers, those consumers with signals that don't match its product type.

Note that choosing whether or not to target advertise does not change the market variable K_{MA} in (3) and the market variable K_{TA} is (5), because I am analyzing the case of monopolistic competition (see page 8). This extends the standard logit monopolistic competition assumption that there are so many firms that an individual firm's pricing decision does not impact the market variable K (as in Anderson et al., 1992, p. 221-226), or an individual firm does not consider the impact of its pricing on the the market variable K (as in Dixit and Stiglitz, 1977), to the firms' advertising decisions.

5 Impact of Signal Accuracy on Welfare

In this section I consider how changing the signal accuracy θ affects the equilibrium found in section 4. In particular I am concerned with finding the effect of θ on total welfare TW, because I interpret stricter privacy laws as creating noisier signals about consumers' tastes. This section would be useful in determining how much we should protect the privacy of personal information to maximize a society's welfare.

5.1 To Target or Not To Target

In 4.3, I showed how signal accuracy θ impacts whether we are in the mass advertising equilibrium or a targeted advertising equilibrium. I showed that for $\theta < \widehat{\theta}$ I have the mass advertising equilibrium and for $\theta > \widehat{\theta}$ I have a targeted advertising equilibrium, where $\widehat{\theta}$ is given by (4). Here I show that when $\widehat{\theta} \leq 1$, there exists a targeted advertising equilibrium socially preferable to the mass advertising equilibrium.[9]

I do this by considering the targeted advertising equilibrium for $\theta = \hat{\theta}$. This is the case where entrants are indifferent between targeting their advertisements and not. Putting $\theta = \hat{\theta}$ into (ZPC-TA), I have the relation between the number $2N_{TA}$ of entrants under targeted advertising and the number $2N_{MA}$ of entrants under mass advertising, given by (6).

$$N_{TA}(\hat{\theta}) = \frac{F + c}{F + c/2} N_{MA} \tag{6}$$

Putting $\theta = \hat{\theta}$ into (TW-TA), I have that the aggregate consumer surplus from sales under targeted advertising is equal to the aggregate consumer surplus

[9] When $\widehat{\theta} > 1$, only the mass advertising equilibrium is possible.

from sales under mass advertising. I find that the initial cost of being offered fewer products equals the initial benefit from being offered better matching products, because the signal accuracy is so low. Therefore I have that the total welfare TW_{TA} from targeted advertising is greater than the total welfare TW_{TA} from mass advertising, as shown in (7).

$$\begin{aligned} TW_{TA}(\hat{\theta}) &= R - \mu + \mu\Big[\ln(e^{b/\mu}+1) + \ln(\frac{\mu}{2(F+c)})\Big] - A(\frac{F+c}{F+c/2}N_{MA}) \\ &> R - \mu + \mu\Big[\ln(e^{b/\mu}+1) + \ln(\frac{\mu}{2(F+c)})\Big] - A(2N_{MA}) = TW_{MA} \end{aligned} \tag{7}$$

Therefore as long as long as $\widehat{\theta} \leq 1$, or equivalently as long as it is possible to have a targeted advertising equilibrium, then there is at least one targeted advertising equilibrium socially preferable to mass advertising. This result is similar to the result in Esteban et al. (2001) that if a social planner picks product price and degree of advertisement specialization, then targeted advertising is socially preferable to mass advertising. While I show that if a social planner picks the signal accuracy, then targeted advertising is socially preferable to mass advertising.

5.2 In the Targeted Advertising Equilibrium

In 5.1, I showed that targeted advertising (for some values of θ) would be socially preferable to mass advertising. Here I consider how changing the signal accuracy θ affects the targeted advertising equilibrium. Because I interpret stricter privacy laws as creating noisier signals about consumers' tastes, this subsection would be useful in determining how much we should protect the privacy of personal information, while still allowing firms to collect enough information to target their advertisements.

Differentiating the total welfare under the targeted advertising equilibrium (given by (TW-TA)) by the signal accuracy θ, I have (8).

$$TW_{TA}{}'(\theta) = \lambda(\theta)\Big[\mu(\frac{1+\theta}{2}e^{\frac{R+b-\mu}{\mu}} + \frac{1-\theta}{2}e^{\frac{R-\mu}{\mu}}) - A'\big(N_{TA}(\theta)\big)\Big] \tag{8}$$

$$\text{where } \lambda(\theta) \equiv \frac{\frac{1}{2}(e^{\frac{R+b-\mu}{\mu}} - e^{\frac{R-\mu}{\mu}})}{\big(\frac{1+\theta}{2}e^{\frac{R+b-\mu}{\mu}} + \frac{1-\theta}{2}e^{\frac{R-\mu}{\mu}}\big)^2} = N_{TA}{}'(\theta) > 0$$

Here $\lambda(\theta)\mu(\frac{1+\theta}{2}e^{\frac{R+b-\mu}{\mu}} + \frac{1-\theta}{2}e^{\frac{R-\mu}{\mu}}) > 0$ is the aggregate consumer surplus gained from increasing θ through more goods being offered to consumers (from a higher number of firms) and through an increase in the chance of products matching consumers' tastes (from more accurate signals), and $\lambda(\theta)A'(N_{TA}(\theta)) > 0$ is the aggregate ad annoyance gained from increasing θ through a higher number of advertisers. These two forces, the product selection effect and the ad annoyance effect, can make increasing signal accuracy θ either increase or decrease total social welfare. Furthermore by taking the derivative of (8) and evaluating it when it is equal to zero, I have Proposition 1.

Proposition 1. *If* θ^{opt} *solves* ${TW_{TA}}'(\theta^{opt}) = 0$ *and:*

a) if $A''\big(N_{TA}(\theta^{opt})\big) > \mu(\frac{1+\theta^{opt}}{2}e^{\frac{R+b-\mu}{\mu}} + \frac{1-\theta^{opt}}{2}e^{\frac{R-\mu}{\mu}})$
then θ^{opt} *is a local maximum of* $TW_{TA}(\theta)$

b) if $A''\big(N_{TA}(\theta^{opt})\big) < \mu(\frac{1+\theta^{opt}}{2}e^{\frac{R+b-\mu}{\mu}} + \frac{1-\theta^{opt}}{2}e^{\frac{R-\mu}{\mu}})$
then θ^{opt} *is a local minimum of* $TW_{TA}(\theta)$

Proposition 1 shows how the shape of the advertising annoyance function influences the shape of the total welfare function TW_{TA}. If the ad annoyance function $A(N)$ satisfies (a) for critical value(s) in $[\widehat{\theta}, 1]$, then the total welfare function is an inverted-u shape. Then society would be better off with firms getting somewhat noisy information about the consumer characteristic (i.e. $\theta \in (\widehat{\theta}, 1)$). Yet if the ad annoyance function $A(N)$ satisfies (b) for critical value(s) in $[\widehat{\theta}, 1]$, then the total welfare function is a u shape. Then society would be better off with firms getting either an accurate signal (i.e. $\theta = 1$) or a noisy signal barely accurate enough to encourage targeted advertising (i.e. $\theta = \widehat{\theta}$).

5.2.1 Example: Linear Ad Annoyance

For example, suppose the ad annoyance function were of the form $A(N) = aN$, where a is the additional annoyance cost to a consumer per ad. By Proposition 1, any θ^{opt} that solves ${TW_{TA}}'(\theta^{opt}) = 0$ would be a minimum. Therefore $TW_{TA}(\theta)$ is a u shape. Therefore society would be better off with firms getting either an accurate signal (i.e. $\theta = 1$) or a noisy signal barely accurate enough to encourage targeted advertising (i.e. $\theta = \widehat{\theta}$). Comparing $TW_{TA}(1)$ to $TW_{TA}(\widehat{\theta})$ using (TW-TA), I have Corollary 3.

Corollary 1. *If ad annoyance is of the form* $A(N) = aN$*, if* $\widehat{\theta} \leq 1$*, and:*

a) if $a > \mu e^{\frac{R+b-\mu}{\mu}}(e^{b/\mu} + 1)\ln[\frac{e^{b/\mu}+1}{e^{b/\mu}}\frac{F+c/2}{F+c}]$
then the social welfare maximizing θ *is one.*

b) if $a < \mu e^{\frac{R+b-\mu}{\mu}}(e^{b/\mu} + 1)\ln[\frac{e^{b/\mu}+1}{e^{b/\mu}}\frac{F+c/2}{F+c}]$
then the social welfare maximizing θ *is* $\widehat{\theta}$*.*

5.2.2 Example: Quadratic Ad Annoyance

For example, suppose the ad annoyance function were of the form $A(N) = aN^2$, where a is some ad annoyance parameter. Then Proposition 1 shows that critical values could be either local minima or local maxima. Evaluating (8) for ${TW_{TA}}'(\theta) = 0$, I have a local maximum at θ_1 and a local minimum at $\theta_2 > \theta_1$, where θ_1 and θ_2 are given by (9) and (10).

$$\theta_1 \equiv \frac{\frac{a}{2(F+c/2)} - \sqrt{(\frac{a}{2(F+c/2)})^2 - 4\frac{a}{\mu}} - (e^{\frac{R+b-\mu}{\mu}} - e^{\frac{R-\mu}{\mu}})}{e^{\frac{R+b-\mu}{\mu}} + e^{\frac{R-\mu}{\mu}}} \tag{9}$$

$$\theta_2 \equiv \frac{\frac{a}{2(F+c/2)} + \sqrt{(\frac{a}{2(F+c/2)})^2 - 4\frac{a}{\mu}} - (e^{\frac{R+b-\mu}{\mu}} - e^{\frac{R-\mu}{\mu}})}{e^{\frac{R+b-\mu}{\mu}} + e^{\frac{R-\mu}{\mu}}} \tag{10}$$

Under sufficient parameter restrictions, $TW_{TA}(\theta)$ has an inverted-u shape. Then society would be better off with firms getting somewhat noisy information about the consumer characteristic (i.e. $\theta = \theta_1 \in [\widehat{\theta}, 1]$). This is given by Corollary 2.

Corollary 2. *For some $a_1, a_2 > 0$, if ad annoyance is of the form $A(N) = aN^2$, if $\widehat{\theta} \leq 1$, and if $a_1 \leq a \leq a_2$, then the social welfare maximizing θ is θ_1.*

Proof. By Parts:
Restriction 1: Because $\frac{\partial \theta_2}{\partial a} > 0$, as long as $a \geq \widetilde{a_1}$ for some $\widetilde{a_1}$, any $\theta > \theta_2$ would be infeasible.
Restriction 2: By (9), $\theta_1 \leq 1$ becomes $a(\frac{2e^{\frac{R+b-\mu}{\mu}}}{F+c/2} - \frac{4}{\mu}) \geq 4e^{2\frac{R+b-\mu}{\mu}}$.
Because $4e^{2\frac{R+b-\mu}{\mu}} > 0$, I have that if $\frac{2e^{\frac{R+b-\mu}{\mu}}}{F+c/2} - \frac{4}{\mu} > 0$ and $a \geq \widehat{a_1} \equiv 4e^{2\frac{R+b-\mu}{\mu}} / (\frac{2e^{\frac{R+b-\mu}{\mu}}}{F+c/2} - \frac{4}{\mu})$, or if $\frac{2e^{\frac{R+b-\mu}{\mu}}}{F+c/2} - \frac{4}{\mu} \leq 0$, then $\theta_1 \leq 1$.
Restriction 3: By (4) and (9), $\theta_1 \geq \widehat{\theta}$ becomes $a \leq a_2 \equiv \frac{\psi^2}{\frac{4}{\mu} + \frac{\psi}{F+c/2}}$ where $\psi \equiv \frac{F}{F+c} \frac{e^{\frac{R+b-\mu}{\mu}} + e^{\frac{R-\mu}{\mu}}}{e^{\frac{R+b-\mu}{\mu}} - e^{\frac{R-\mu}{\mu}}} + (e^{\frac{R+b-\mu}{\mu}} - e^{\frac{R-\mu}{\mu}}) > 0$
Therefore: If $\widehat{\theta} \leq 1$ and if $\max\{\widetilde{a_1}, \widehat{a_1}\} \leq a \leq a_2$, then the social welfare maximizing θ is θ_1. □

6 Marketing Costs

In this section, I will consider the case where firms can pay to get a more accurate signal θ about consumers' tastes. Here, each firm may potentially enter the market at an entry cost $f + M(\theta)$, where $f > 0$ is the fixed entry cost and $M(\theta)$ is the cost of getting a signal accuracy of θ. I interpret $M(\theta)$ as the marketing research cost.

I normalize $M(0) = 0$. In addition, I assume that (A1) M is continuous, (A2) $M'(0) = 0$, (A3) $M'(\theta) > 0$ for all $\theta \in (0, 1]$, and (A4) $M''(\theta) > 0$ for all $\theta \in [0, 1]$. I interpret assumptions (A2) to (A4) as a diminishing marginal effect of marketing research. As a firm increases the accuracy of the signal θ, additional accuracy gets more expensive.

Note that I have intentionally used different notation for the fixed entry cost f (in this section) and the entry cost F (in sections 3 through 5), because there is no reason for f to equal F. In 6.4, I will use this distinction when I consider whether it is socially preferable to have a marketing cost or to restrict θ.

6.1 Equilibrium

The mass advertising equilibrium with marketing costs is the same as the mass advertising equilibrium found in section 4.1 with an entry cost of $F = f$ and a signal accuracy $\theta = 0$, because firms would not pay for information about consumers' tastes if they are going to advertise to all consumers.

The targeted advertising equilibrium with marketing costs is the same as the targeted advertising equilibrium found in section 4.2 with an entry cost of $F = f + M(\theta^*)$ and a signal accuracy $\theta = \theta^*$, where θ^* solves a firm's first order condition for θ, which reduces to (11).[10]

$$v(\theta^*) = \theta^* + \frac{e^{b/\mu} + 1}{e^{b/\mu} - 1} \tag{11}$$
$$\text{where } v(\theta) \equiv \frac{f + M(\theta) + c/2}{M'(\theta)}$$

Here $v(\theta)$ is the inverse hazard of the targeted advertising total cost function $f + M(\theta) + c/2$. My assumptions on the function M, guarantee a unique value of θ^*.

From (11), I have θ^* increases with a higher fixed entry cost f and a higher per-person advertising cost c. I interpret this as when fewer firms enter the market because of a higher fixed entry cost f or a higher advertising cost c, each firm will benefit more from better information about consumers' tastes. In addition, I have θ^* increases with a higher benefit b from consuming a product that matches your type and a lower variation μ in consumers' tastes. I interpret this as when each consumer is more likely to prefer a good designed for his type, then firms will benefit more from better information about consumers' tastes.

6.2 Equilibrium Mass and Targeting Conditions

For most targeted advertising equilibrium signal accuracies θ^* there is only one possible equilibrium. Similar to the $\widehat{\theta}$ found section 4.3, in this section I show that there exists a threshold $\tilde{\theta}$ such that for $\theta < \tilde{\theta}$ I have the mass advertising equilibrium and for $\theta > \tilde{\theta}$ I have the targeted advertising equilibrium.

Under the mass advertising equilibrium, a firm j with a product characteristic k must expect a profit from advertising to consumers with signal $1 - k$. Advertising to these consumers (in addition to those consumers with signal k)

[10] When the θ^* that solves (11) is greater than one, then firms would choose the profit maximizing $\theta = 1$.

means that the firm has to pay to advertise to an additional half of the consumers and it no longer values signal accuracy θ. Instead of paying for a signal accuracy of $\theta = \theta^*$, the firm chooses to pay for a signal accuracy of $\theta = 0$. Therefore I have $c/2 - M(\theta^*) \leq pq_{MA}$, where q_{MA} is given by (3). This reduces to $\theta^* \leq \tilde{\theta}$, where $\tilde{\theta}$ is given by (12).

$$\tilde{\theta} \equiv \frac{f + 2M(\theta^*)}{f + c} \frac{e^{b/\mu} + 1}{e^{b/\mu} - 1} \tag{12}$$

Under the targeted advertising equilibrium, a firm j with a product characteristic k must expect a loss from advertising to consumers with signal $1 - k$. Similar to the mass advertising equilibrium, I must have $c/2 - M(\theta^*) \geq pq_{TA}$, where q_{TA} is given by (5). This reduces to $\theta^* \geq \tilde{\theta}$, where $\tilde{\theta}$ is given by (12).

Note the difference between $\tilde{\theta}$ and $\widehat{\theta}$, given by (4) and (12). The fixed entry cost f, affect $\tilde{\theta}$ both directly and through the equilibrium θ^*.

Similar to the effect of the entry cost F on $\widehat{\theta}$, increasing the fixed entry cost f increases $\tilde{\theta}$. I interpret this as if fixed entry costs are high enough, then enough firms will be deterred from entering so that an entrant might as well sell to the less profitable group of consumers, those consumers with signals that don't match its product characteristic. Unlike their effects on $\widehat{\theta}$, the effects of the advertising cost c, the benefit b from consuming a product that matches your type, and the variation μ in consumers' tastes on $\tilde{\theta}$ is ambiguous. Decreasing the advertising cost c, increasing the benefit b from consuming a product that matches your type, and decreasing the variation μ in consumers' tastes both directly increases $\tilde{\theta}$ as shown in (12) and indirectly decreases $\tilde{\theta}$ through decreasing θ^* as shown in (11). See Appendix A for details.

6.3 Impact of Marketing Costs on Welfare

Under the mass advertising equilibrium, the impact of changing the function M on the social total welfare TW_{MA} is zero, because firm would not choose the minimal signal accuracy $\theta = 0$ and I have restricted $M(0) = 0$.

Under the targeted advertising equilibrium, increasing the function M would effect total social welfare TW_{TA} in three ways (see (TW-TA)): 1) a negative product selection effect (through increasing the fixed cost $F = f + M(\theta)$), 2) a positive ad anoyance effect (through decreasing the number of firms N_{TA} advertising to a consumer), and 3) a possibly negative or positive indirect effect by altering the firms' choice of θ (see (8), its following discussion, and (11)).

6.3.1 Example: Power-σ Marketing Cost Function

For example, suppose that the marketing cost function is of the form $M(\theta) = m\theta^\sigma$, where $\sigma > 1$ to satisfy the condition $M''(\theta) > 0$, and where $m > 0$. I interpret m as the frictional cost associated with gathering a more accurate signal. Then by the equilibrium condition given by (11), I have (13).

$$\frac{\partial \theta^*}{\partial m} = -\frac{f + c/2}{f + m\theta^{*\sigma} + c/2}\frac{\theta^*}{m(\sigma - 1)} < 0 \tag{13}$$

I interpret (13) as increasing the frictional cost m to gathering a more accurate signal about consumers' tastes induces firms to invest less in signal accuracy. At first glance, it might appear that increasing the fixed entry cost f or the advertising cost c would decrease the size of this effect, or in other words $\frac{\partial_2 \theta^*}{\partial m \partial f} > 0$ and $\frac{\partial_2 \theta^*}{\partial m \partial c} > 0$. This is usually the case. Yet when M is only weakly convex and equilibrium signal accuracy θ^* is low enough, it is possible that $\frac{\partial_2 \theta^*}{\partial m \partial f} < 0$ and $\frac{\partial_2 \theta^*}{\partial m \partial c} < 0$, because increasing f and c decreases the choice in the equilibrium θ^*. This is shown by Proposition 2 shown in Appendix B.

Differentiating my total social welfare TW_{TA} under the targeted adverting equilibrium by the frictional cost m of a better signal, I have (14).

$$\frac{\partial TW_{TA}}{\partial m} = \frac{\partial \theta^*}{\partial m} {TW_{TA}}'(\theta^*) - \phi(\theta^*)(2(f + m\theta^{*\sigma} + c/2) - A'(N_{TA})) \tag{14}$$

$$\text{where } \phi(\theta) \equiv \frac{\mu\theta^\sigma}{2(f + m\theta^\sigma + c/2)^2} > 0$$

Here $-\phi(\theta^*)2(f + m\theta^{*\sigma} + c/2)$ is the negative product selection effect on social welfare through increasing the entry cost $F = f + M(\theta)$. $\phi A'(N_{TA})$ is the positive ad annoyance effect through decreasing the number of firms N_{TA} advertising to a consumer. And $\frac{\partial \theta^*}{\partial m} {TW_{TA}}'(\theta^*)$ is the possibly negative or positive indirect effect by altering the firms' choice of θ; This will depend on whether ${TW_{TA}}'(\theta^*)$ is positive or negative, see (8).

6.4 Should Marketing Cost or Data Restrictions Protect Privacy?

Here I consider whether it is better to set signal accuracy θ (as in section 5) or change some friction associated with the marketing cost function M (as in 6.3). I am considering whether it is better to restrict the information firms gather on consumers or to increase the cost of gathering information. By doing so, I am considering how we should restrict privacy.

If we could choose the fixed component of entry cost, and we could choose θ or m, then the targeted advertising equlibria in the fixed signal accuracy model (the base model presented in section 3) and the endogenous signal accuracy model (the extension presented in this section) would be the same. We could choose θ^* by choosing m and adjust entry costs to be equivalent. Yet $\tilde{\theta} > \widehat{\theta}$, therefore there are some targeted advertising equilibria that are feasible under fixed signal accuracy that are not feasible under endogenous signal accuracy. Under endogenous signal accuracy, the incentive to mass advertise would be greater, because if a firm mass advertises, then it doesn't pay for marketing research. Iyer et al. (2005) assumed this result to give firms an incentive not to target advertise in his model.

When comparing the fixed signal accuracy model to my endogenous signal accuracy extension, we need to be careful how we compare the entry costs. Marketing costs could add to the existing fixed entry cost (i.e. $F = f$). Or marketing costs could already be considered in the fixed signal accuracy model as part of the already present entry cost F (i.e. $F = f + M(\theta^*)$).

If marketing cost add to the existing fixed entry cost (i.e. $F = f$), then the additional marketing costs could either help or hurt social welfare. Marketing costs discourage more firms from entering, so social welfare may improve through a reduction of ad annoyance and possibly better signal accuracy (through an endogenous θ). Yet social welfare would also be hurt by a worse product selection (through fewer entrants), possibly a worse signal accuracy (through an endogenous θ), and a bigger incentive to mass advertise. These results vary depending on the ad annoyance function A and marketing cost function M.

Consider the example presented in section 6.3.1, where the marketing cost function is of the form $M(\theta) = m\theta^\sigma$. Suppose marketing costs are already in the fixed signal accuracy model as part of the already present entry cost F (i.e. $F = f + M(\theta^*)$). Suppose we could choose to implement the socially optimal $\theta = \theta^{opt}$ (in the fixed signal model), and we could choose to implement the socially optimal $m = m^{opt}$ (in the endogenous signal model). Then the fixed signal accuracy model would be preferable, because we could always choose θ to be the equilibrium θ^* in the endogenous signal accuracy model. Furthermore, signal accuracy θ^{opt} in the fixed signal accuracy model could be greater than or less than the endogenous signal accuracy $\theta^*(m^{opt})$, as shown in Corollary 3.

Corollary 3. *If the marketing cost function is of the form* $M(\theta) = m\theta^\sigma$*, if* $\frac{\partial_2 TW_{TA}}{\partial m^2}(\theta^*(m^{opt})) < 0$*, if* $\frac{\partial_2 TW_{TA}}{\partial \theta}(\theta^{opt}) < 0$*, if* $F = f + M(\theta^*(m^{opt}))$*, and:*

a) if $\frac{\mu}{2(F+c/2)} > \frac{1}{\frac{1+\theta^*}{2}e^{\frac{R+b+\mu}{\mu}} + \frac{1-\theta^*}{2}e^{\frac{R+\mu}{\mu}}}$ *then* $\theta^{opt} > \theta^*(m^{opt})$

b) if $\frac{\mu}{2(F+c/2)} < \frac{1}{\frac{1+\theta^*}{2}e^{\frac{R+b+\mu}{\mu}} + \frac{1-\theta^*}{2}e^{\frac{R+\mu}{\mu}}}$ *then* $\theta^{opt} < \theta^*(m^{opt})$

Proof. This follows directly from evaluating (14) at the m such that $\theta^* = \theta^{opt}$. By (8), $\mu\big(\frac{1+\theta^*}{2}e^{\frac{R+b-\mu}{\mu}} + \frac{1-\theta^*}{2}e^{\frac{R-\mu}{\mu}}\big) = A'\big(N_{TA}(\theta^*)\big)$. □

Recall that $\frac{\mu}{2(F+c/2)}$ is the number of firms that would enter each sub-market if there were no outside option. And that $[\frac{1+\theta}{2}e^{\frac{R+b-\mu}{\mu}} + \frac{1-\theta}{2}e^{\frac{R-\mu}{\mu}}]^{-1}$ is the number of firms discouraged from entering each sub-market due to the outside option. Therefore I interpret Corollary 3 as the optimal signal accuracy θ^{opt} will be lower than the equilibrium signal accuracy $\theta^*(m^{opt})$ when the outside option and signal accuracy plays a big role in discouraging firms from entering the market.

7 Endogenous Advertising Prices

In the previous sections, I considered a perfectly elastic supply of advertisements. I considered a constant per consumer advertising cost c. In this section, I consider a more general supply of ads. I assume that it costs the same to send an ad to every consumer.[11] Yet I allow the supply of ads to vary with price c. I have a supply function $S(c)$ of advertisements, where I assume $S'(c) \geq 0$ for all c. After firms enter the market, they buy ads in a perfectly competitive advertising market.

7.1 Equilibrium

The mass advertising equilibrium and targeted advertising equilibrium would be the same as found in sections 4.1 and 4.2, with the addition of an endogenous advertising price c.

In equilibrium, the advertising price c would be such that firms make zero profit. If c were too low, firms would make a profit, which would induce firms to enter the market until they make zero profit. If c were too high, firms would make a loss, which would induce firms to leave the market until they make zero profit.

Under mass advertising, $2N_{MA}$ firms would advertise to all of the consumers. Therefore by (ZPC-MA) the demand $D_{MA}(c)$ for ads would be given by (15). Under targeted advertising N_{TA} firms would advertise to half of the consumers, and another N_{TA} firms would advertise to the other half of the consumers. Therefore by (ZPC-TA) the demand $D_{TA}(c)$ for ads would be given by (16).

$$D_{MA}(c) = 2\left[\frac{\mu}{2(F+c)} - \frac{1}{e^{\frac{R+b-\mu}{\mu}} + e^{\frac{R-\mu}{\mu}}}\right] \tag{15}$$

$$D_{TA}(c) = \frac{\mu}{2(F+c/2)} - \frac{1}{\frac{1+\theta}{2}e^{\frac{R+b-\mu}{\mu}} + \frac{1-\theta}{2}e^{\frac{R-\mu}{\mu}}} \tag{16}$$

Note that $D_{MA}(c)$ and $D_{TA}(c)$ are both decreasing in the advertising price c and in entry cost F. I interpret this as when advertising or entry gets more expensive fewer firms enter the market, which creates a lower quantity demanded of ads.

7.2 Equilibrium Mass, Mixed, and Targeting Conditions

Similar to the $\widehat{\theta}$ found section 4.3, in this section I show that there exists a threshold ad price $\widehat{c}$ such that: for $c < \widehat{c}$, I have the mass advertising equilibrium; for $c = \widehat{c}$, I have the mixed advertising equilibrium; and for $c > \widehat{c}$, I have the targeted advertising equilibrium.

[11] This is not unreasonable because in equilibrium all consumers should receive the same number of ads.

Under the mass advertising equilibrium, a firm j with a product characteristic k must expect a profit from advertising to consumers with signal $1-k$. By the same reasoning in section 4.3, I have the condition $c/2 \le pq_{MA}$, which reduces to $c \le \widehat{c}$ where $\widehat{c}$ is given by (17). Similarly under the targeted advertising equilibrium, a firm j with a product characteristic k must expect a loss from advertising to consumers with signal $1-k$. Therefore I have the condition $c/2 \ge pq_{TA}$, which reduces to $c \ge \widehat{c}$ where $\widehat{c}$ is given by (17).

$$\widehat{c} \equiv F(\frac{1}{\theta}\frac{e^{b/\mu}+1}{e^{b/\mu}-1} - 1) \tag{17}$$

I interpret $\widehat{c}$ as the lowest advertising price such that firms would not make more money by mass advertising.

Note that while $D_{MA}(c)$ is not necessarily always greater that $D_{TA}(c)$ for all values of c, I have that $D_{MA}(\widehat{c}) > D_{TA}(\widehat{c})$ by (6). Therefore I have that the inverse demand D^{-1} for advertisements as a function of the number n of ads is given by (18).

$$D^{-1}(n) = \begin{cases} D_{TA}^{-1}(n) & \text{if } n \le D_{TA}(\widehat{c}) & \text{(targeted advertising)} \\ \widehat{c} & \text{if } D_{TA}(\widehat{c}) \le n \le D_{MA}(\widehat{c}) & \text{(mixed advertising)} \\ D_{MA}^{-1}(n) & \text{if } n \ge D_{MA}(\widehat{c}) & \text{(mass advertising)} \end{cases} \tag{18}$$

Therefore for a low supply of ads, I have targeted advertising. For an intermediate supply of ads, I have mixed advertising. And for a high supply of ads, I have mass advertising.

7.3 Impact of Signal Accuracy on Welfare

In this section I consider how changing the signal accuracy θ affects the equilibrium found in section 7.1. In particular I am concerned with finding the effect of θ on total welfare TW, because I interpret stricter privacy laws as creating noisier signals about each consumer's taste. This section differs from the discussion in section 5 by the addition of a general supply $S(p)$ of ads.

Similar to section 5, under mass advertising (for a high supply of ads), firms ignore the signal accuracy θ so small changes in θ would not impact social welfare. Therefore in this section I will focus on the effect of changing θ on the mixed and the targeted advertising equilibria.

Under the mixed advertising equilibrium (for an intermediate supply of ads), I have that $c = \widehat{c}$ or $\theta = \hat{\theta}$. Recall putting $\theta = \hat{\theta}$ into (TW-TA), I have that the aggregate consumer surplus from sales under targeted and mass advertising are equivalent. Similarly setting $c = \widehat{c}$, I have that the aggregate consumer surplus from sales under targeted, mixed, and mass advertising are equivalent and equal to $R - \mu + \mu[\ln(e^{b/\mu}+1) + \ln(\frac{\mu}{2(F+\widehat{c})})]$. By (17), I have that $\widehat{c}$ is decreasing in θ. Therefore under the mixed advertising equilibrium, I have that total social welfare is strictly increasing in θ.

Under the targeted advertising equilibrium (for a low supply of ads), I have that the advertising price c increases with θ through the market clearing condition that the supply $S(c)$ of ads equals the demand $D_{TA}(c)$ of ads. Therefore by (TW-TA) and (16), I have (19).

$$TW_{TA}{}'(\theta) = \lambda(\theta)\Big[\mu\big(\frac{1+\theta}{2}e^{\frac{R+b-\mu}{\mu}} + \frac{1-\theta}{2}e^{\frac{R-\mu}{\mu}}\big) - 2(F+c/2)\tau(c) - \big(1-\tau(c)\big)A'\big(N_{TA}(\theta)\big)\Big] \tag{19}$$

where $\lambda(\theta) \equiv \dfrac{\frac{1}{2}(e^{\frac{R+b-\mu}{\mu}} - e^{\frac{R-\mu}{\mu}})}{\big(\frac{1+\theta}{2}e^{\frac{R+b-\mu}{\mu}} + \frac{1-\theta}{2}e^{\frac{R-\mu}{\mu}}\big)^2} > 0$

$$\tau(c) \equiv \frac{\mu}{4(F+c/2)^2}\Big[S'(c) + \frac{\mu}{4(F+c/2)^2}\Big]^{-1} > 0$$

Similar to (8), $\lambda(\theta)\mu(\frac{1+\theta}{2}e^{\frac{R+b-\mu}{\mu}} + \frac{1-\theta}{2}e^{\frac{R-\mu}{\mu}}) > 0$ is the aggregate consumer surplus gained from increasing θ through more goods being offered to consumers (from a higher number of firms) and through an increase in the chance of products matching consumers' tastes (from more accurate signals), and $\lambda(1-\tau(c))(\theta)A'(N_{TA}(\theta)) > 0$ is the aggregate ad annoyance gained from increasing θ through a higher number of advertisers. Yet (19) also includes $2(F+c/2)\lambda(\theta)\tau(c) > 0$ which is an ad pricing effect on social welfare. A higher signal accuracy θ, increases the value and price c for an ad, which induces firms to leave the market. This decreases social welfare through reducing the number of products offered a consumer, and it increases social welfare by reducing the number of annoying ads shown to a consumer.

Note that if the supply S of ads is perfectly elastic, then I get the same results that I found in (8). And if S is perfectly inelastic, then there is no change in ad annoyance and I get a similar results to the effect found in Bergemann and Bonatti (2011), who showed that in addition to a positive product selection benefit, an increasing signal accuracy would create a negative ad price effect on social welfare.

8 Ad Avoidance

In this section I consider the case were consumers choose to ignore n ads for an ad blocking cost $B(n)$, where $B(0) = 0$ and $B'(n) \geq 0$ for all n. These ads are selected at random from the ads shown to a consumer, and consumers do not know their values for the products or the outside option when deciding how many ads to avoid. Consumers benefit from not being annoyed by the avoided ads; ad annoyance A is now a function of the ads a consumer does not avoid. Consumers suffer from not being able to buy the products of avoided ads.

Because there is a chance that a consumer might not see a firm's ad, a consumer might not see a firm's ad, so a firm has an incentive to send multiple messages m to a consumer to improve the probability α that a consumer does not avoid its ad. Firms may do this at a constant rate of c per message.

This section is similar to Johnson (2010). The big differences between our ad avoidance technology is: 1) he only allows consumers to choose to avoid all ads or not avoid any ads and 2) he allows consumers to know their value for the outside option when deciding whether to avoide advertisements. More to follow.

9 Common Signals

It is reasonable to think that many firms receive the same information about a consumer. This would not make their signal's on consumers' tastes independent across firms (as I have assumed in previous sections). I explore the other extreme, where firms have the same information or signal about a consumer, to show how this information sharing would affect my equilibrium and to show how it would influence how signal accuracy affects social welfare. This section would be particularly useful for considering the affects of regulating information sharing between firms.

In the previous sections, for each consumer, each firms receives a separate, independent (across firms and consumers) signal about that consumer's type. In this section, for each consumer, all firms receive the same, independent (across consumers) signal about that consumer's type, which is true with a probability of $\frac{1+\theta}{2}$ and false with a probability of $\frac{1-\theta}{2}$, where $\theta \in [0,1]$. Here I interpret θ as the accuracy of the common signal about each consumer.

For a signal accuracy (when $\theta < \widehat{\theta}$), the mass advertising equilibrium found in section 4.1 still holds. Firms are ignoring the signals, so it doesn't matter if they share the same signal or get independent signals about a consumer.

For an intermediate signal accuracy, the mixed advertising equilibrium I will present in 9.2 holds. If all other firms mass advertised, then the signal is accurate enough so that a firm could make a profit from targeted advertising. Therefore enough firms target advertise, so that firms make zero profit by targeted advertising. If all other firms targeted advertised, then a firm could make a profit mass advertising. By mass advertising, a firm would make a profit selling to those consumers who gave firms a false signal. Therefore enough firms mass advertise, so that firms make zero profit by mass advertising.

I will show in 9.2 that as the signal accuracy decreases, more firms mass advertise and fewer firms target advertise. Eventually (when $\theta = \widehat{\theta}$), no firms target advertise and we will be in the mass advertising equilibrium. I will also show that as the signal accuracy increases, fewer firms mass advertise. Eventually, no firms will mass advertise and we will be in the targeted advertising equilibrium I will present in section 9.1. Therefore for a high signal accuracy, the targeted advertising equilibrium I will present in section 9.1 holds.

9.1 Targeted Advertising Equilibrium

If N_{TA} firms enter each sub-market and all firms target advertise, then each consumer with a true signal will see ads from N_{TA} firms of his type, and each consumer with a false signal will see ads from N_{TA} firms not of his type. If firm j of type $k \in \{0,1\}$ target advertises, then firm j will only advertise to half of the consumers: $\frac{1+\theta}{2}$ of whom will be of type k, and $\frac{1-\theta}{2}$ of whom will be of type $1-k$. Therefore if N_{TA} firms enter each sub-market and if all firms sets a market price of p, firm j will sell to a share $e^{\frac{R+b-p_j}{\mu}}/K^t_{TA}$ of the type k consumers and a share $e^{\frac{R-p_j}{\mu}}/K^f_{TA}$ of the type $1-k$ consumers, where $K^t_{TA} \equiv 1 + N_{TA}e^{\frac{R+b-p}{\mu}}$ and $K^f_{TA} \equiv 1 + N_{TA}e^{\frac{R-p}{\mu}}$.[12] Therefore firm j's quantity Q_j sold as a function of its price p_j is given by (20).

$$Q_j = \frac{1}{2}\Big[\frac{1+\theta}{2}\frac{e^{\frac{R+b-p_j}{\mu}}}{K^t_{TA}} + \frac{1-\theta}{2}\frac{e^{\frac{R-p_j}{\mu}}}{K^f_{TA}}\Big] \tag{20}$$

Note the difference between (20) and (2). In (2), every firm receives an independent signal about a consumer, so ever consumer receives the same number of false-signals and true-signals. Therefore every consumer sees the same number of ads from firms of his type and the same number of ads from firms not of his type. In (20), firms receive the same signal about a consumer. Therefore all the products available to a consumer with a true signal are of his type and all the products available to a consumer with a false signal are not of his type.

By the firms' first order pricing condition under monopolistic competition, I have that all firms set a price of $p = \mu$ and sell the same quantity Q of goods. Firms would enter the market until there is no profit from entering the market. Therefore I have $F + c/2 = pQ$. This determines the number N_{TA} of entrants. The big difference between this and (ZPC-TA), is that this equation is not solvable for N_{TA}. Differentiating this equality with respect to signal accuracy θ, I find (21).

$$N_{TA}'(\theta) = \frac{\Lambda - \Omega}{\frac{1+\theta}{2}\Lambda^2 + \frac{1-\theta}{2}\Omega^2} > 0 \tag{21}$$

$$\text{where } \Lambda \equiv \frac{e^{\frac{R+b-\mu}{\mu}}}{K^t_{TA}} > \Omega \equiv \frac{e^{\frac{R-\mu}{\mu}}}{K^f_{TA}} > 0$$

(21) shows that the number N_{TA} of ads a consumer receives and the number of firms that enter each sub-market, unambiguously increases in signal accuracy. This means that as firms get a better, shared information about consumers, then more firms enter the market and target advertise.

Because firms make zero profits, total social welfare is total consumer welfare. This is the sum of the aggregate consumer surplus from sales minus the aggregate ad annoyance cost. Using the derivation of consumer surplus in Anderson et al. (1992, p. 60-61), the total welfare TW_{TA} is given by (22).

$$TW_{TA}(\theta) = \frac{1+\theta}{2}\ln K^t_{TA} + \frac{1-\theta}{2}\ln K^f_{TA} - A(N_{TA}) \tag{22}$$

[12] These market shares are found in Anderson et al. (1992, p. 39-40) for a more general framework.

Here $\ln K^t_{TA}$ is the product selection benefit to a consumer with a true signal, and $\ln K^f_{TA}$ is the product selection benefit to a consumer with a false signal. Because $\frac{1+\theta}{2}$ of consumers have a true signal, the aggregate social product selection benefit is $\frac{1+\theta}{2} \ln K^t_{TA}$ from consumers with a true signal. Likewise the aggregate social product selection benefit is $\frac{1-\theta}{2} \ln K^t_{TA}$ from consumers with a false signal. In addition, all consumers face an ad annoyance from N_{TA} firms.

To test how signal accuracy affects total social welfare under targeted advertising, I differentiate (22) by signal accuracy θ, given by (23).

$$TW'_{TA}(\theta) = \frac{1}{2}[\ln K^t_{TA} - \ln K^f_{TA}] + N_{TA}{}'(\theta)\Big[\big(\frac{1+\theta}{2}\Lambda + \frac{1-\theta}{2}\Omega\big) - A'(N_{TA})\Big] \quad (23)$$

Here $\frac{1}{2}[\ln K^t_{TA} - \ln K^f_{TA}] > 0$ is the change in the product selection benefit of consumers as they switch from having false signals to having true signals. I refer to this as the *signal switching benefit.* $N_{TA}{}'(\theta)\Lambda > 0$ is the change in product selection benefit of consumers with true signals and $N_{TA}{}'(\theta)\Omega > 0$ is the change in product selection benefit of consumers with false signals as more firms enter the market. I refer to these as the *infra-marginal product selection benefits* of consumers with true and false signals. The aggregate $N_{TA}{}'(\theta)(\frac{1+\theta}{2}\Lambda + \frac{1-\theta}{2}\Omega)$ would be the total social infra-marginal product selection benefit. And $N_{TA}{}'(\theta)A'(N_{TA})$ is the additional ad annoyance cost faced by each consumer from more firms entering the market. Note that like (8), signal accuracy increases the product selection benefit and the ad annoyance cost. Therefore signal accuracy has an ambiguous affect on social welfare.

9.2 Mixed Advertising Equilibrium

If N firms enter each sub-market and N_{MA} of those firms mass advertise, then each consumer with a true signal will see ads from N firms of his type and N_{MA} firms not of his type, and each consumer with a false signal will see ads from N_{MA} firms of his type and N firms not of his type. If firm j of type $k \in \{0,1\}$ of target advertises, then firm j will only advertise to half of the consumers: $\frac{1+\theta}{2}$ of whom will be of type k, and $\frac{1-\theta}{2}$ of whom will be of type $1-k$. Similar to section 9.1, firm j's quantity Q_j sold as a function of its price p_j would given by (24), where $K^t_{Mixed} \equiv 1 + Ne^{\frac{R+b-p}{\mu}} + N_{MA}e^{\frac{R-p}{\mu}}$ and $K^f_{Mixed} \equiv 1 + N_{MA}e^{\frac{R+b-p}{\mu}} + Ne^{\frac{R-p}{\mu}}$. If firm j of mass advertises, then firm j will advertise to all of the consumers: half of whom will be of each type. $\frac{1+\theta}{2}$ of each type will have a true signal, and $\frac{1-\theta}{2}$ of each type will have a false signal. Therefore firm j's quantity Q_j sold as a function of its price p_j would given by (25).

$$Q^j_{TA} = \frac{1}{2}\Big[\frac{1+\theta}{2}\frac{e^{\frac{R+b-p_j}{\mu}}}{K^t_{Mix}} + \frac{1-\theta}{2}\frac{e^{\frac{R-p_j}{\mu}}}{K^f_{Mix}}\Big] \quad (24)$$

$$Q^j_{MA} = \frac{1}{2}\Big[\frac{1+\theta}{2}\frac{1}{K^t_{Mix}} + \frac{1-\theta}{2}\frac{1}{K^f_{Mix}}\Big](e^{\frac{R+b-p_j}{\mu}} + e^{\frac{R-p_j}{\mu}}) \quad (25)$$

By the firms' first order condition under monopolistic competition, I have that all firms set a price of $p = \mu$. By symmetry, I have that all firms targeted advertising sell the same quantity Q_{TA} of goods and all firms mass advertising sell the same quantity Q_{MA} of goods. From the zero profit conditions $F + c/2 = pQ_{TA}$ and $F + c = pQ_{MA}$, I can solve for the number N of firms that enter each sub-market and the number N_{MA} of those firms that mass advertise. Yet that is algebraically messy and complicated, so instead I differentiate these zero profit conditions to find $\frac{\partial K^t_{Mixed}}{\partial \theta} = \frac{K^t_{Mixed}}{1+\theta} > 0$ and $\frac{\partial K^f_{Mixed}}{\partial \theta} = -\frac{K^f_{Mixed}}{1-\theta} < 0$. This gives me that the number N of firms that enter each sub-market is increasing in θ, and the number N_{MA} of those firms that mass advertise is decreasing in θ. In addition, it gives me that the number $N + N_{MA}$ of firms advertising to each consumer is decreasing at a constant rate in θ, given be (26).[13] Therefore I conclude that under mixed advertising, increasing signal accuracy unambiguously decreases the ad annoyance faced by each consumer.

$$N'(\theta) + {N_{MA}}'(\theta) = \frac{e^{b/\mu} - 1}{e^{2b/\mu} - 1} \frac{\frac{K^t_{Mixed}}{1+\theta} - \frac{K^f_{Mixed}}{1-\theta}}{e^{\frac{R-\mu}{\mu}}} < 0 \tag{26}$$

Because firms make zero profits, total social welfare is total consumer welfare. This is the sum of the aggregate consumer surplus from sales minus the aggregate ad annoyance cost. Using the derivation of consumer surplus in Anderson et al. (1992, p. 60-61), the total welfare TW_{Mixed} is given by (27).

$$TW_{mixed}(\theta) = \frac{1+\theta}{2} \ln K^t_{Mixed} + \frac{1-\theta}{2} \ln K^f_{Mixed} - A(N + N_{MA}) \tag{27}$$

Here $\ln K^t_{TA}$ is the product selection benefit to a consumer with a true signal, and $\ln K^f_{TA}$ is the product selection benefit to a consumer with a false signal. Therefore the aggregate social product selection benefit is $\frac{1+\theta}{2} \ln K^t_{Mixed} + \frac{1-\theta}{2} \ln K^f_{Mixed}$. In addition, all consumers face an ad annoyance from $N + N_{MA}$ firms.

To test how signal accuracy affects total social welfare under mixed advertising, I differentiate (27) by signal accuracy θ, given by (28).

$$TW_{mixed}'(\theta) = \frac{1}{2}[\ln K^t_{Mixed} - \ln K^f_{Mixed}] - (N'(\theta) + {N_{MA}}'(\theta))A'(N + N_{MA}) > 0 \tag{28}$$

Here $\frac{1}{2}[\ln K^t_{TA} - \ln K^f_{TA}] > 0$ is the signal switching benefit, or equivalently the change in the product selection benefit of consumers as they switch from having false signals to having true signals. Note that this the same as in (23). Also I find that the total social infra-marginal production selection benefit from consumers with true signals is $1/2$, and the total social infra-marginal production selection benefit from consumers with false signals is $-1/2$. These two social

[13]Note that (26) is negative and constant in θ because: when $\theta = \widehat{\theta}$ I have $N = N_{MA}$, and $N''(\theta) + {N_{MA}}''(\theta) = 0$.

affects cancel each other. And $(N'(\theta) + N_{MA}'(\theta))A'(N + N_{MA}) < 0$ is the loss in ad annoyance cost as fewer firms advertise to each consumer.

Because consumers benefit from both an increased product selection and a decreased ad annoyance, under the mixed equilibrium, social welfare unambiguously increases in signal accuracy. This result is similar to the mixed equilibrium in 7.2, where I found, under endogenous prices and a mixed advertising equilibrium, social welfare increases in signal accuracy because as firms switch from mass advertising to targeted advertising, consumers receive fewer ads.

10 Conclusion

To follow.

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A Comparative Statics on $\tilde{\theta}$

By (11) and (12), when $\theta^* \leq 1$:

$$\frac{\partial\widetilde{\theta}}{\partial f} = 2\rho\big[\frac{1}{1-v'(\theta)} + (c/2 - M(\theta^*))\big] > 0 \tag{29}$$

$$\frac{\partial\widetilde{\theta}}{\partial c} = \rho\big[\frac{1}{1-v'(\theta)} - \frac{f + 2M(\theta^*)}{f+c}\big] \tag{30}$$

$$\frac{\partial\widetilde{\theta}}{\partial b} = \rho\omega(\theta^*)\big[\frac{1}{1-v'(\theta)} - \frac{f/2 + M(\theta^*)}{M'(\theta^*)}\big] \tag{31}$$

$$\frac{\partial\widetilde{\theta}}{\partial \mu} = -\frac{b}{\mu}\frac{\partial\widetilde{\theta}}{\partial b} \tag{32}$$

$$\text{where } \rho \equiv \frac{1}{f+c}\frac{e^{b/\mu}+1}{e^{b/\mu}-1} > 0$$

$$\omega(\theta) \equiv \frac{2M'(\theta)e^{b/\mu}}{\mu(e^{b/\mu}-1)^2} > 0$$

B A Couple Second Order Comparative Statics from the Example in Section 6.3.1

Proposition 2. *If the marketing cost function is of the form $M(\theta) = m\theta^\sigma$:*

a) if either:

i) $\sigma > \frac{1+\sqrt{5}}{2}$*, or*

ii) $\theta^* > \left(\frac{[1/\sigma+1-\sigma][f+c/2]}{m[\sigma-1]^2}\right)^{1/\sigma}$

then $\frac{\partial_2\theta^*}{\partial m\partial f} > 0$ *and* $\frac{\partial_2\theta^*}{\partial m\partial c} > 0$

b) if $\sigma \leq \frac{1+\sqrt{5}}{2}$ *and* $\theta^* < \left(\frac{[1/\sigma+1-\sigma][f+c/2]}{m[\sigma-1]^2}\right)^{1/\sigma}$
then $\frac{\partial_2\theta^*}{\partial m\partial f} < 0$ *and* $\frac{\partial_2\theta^*}{\partial m\partial c} < 0$

Proof. From (11), I have:

$$\begin{aligned}
\frac{\partial_2\theta^*}{\partial m\partial f} &= -\frac{M''(\theta^*)[1-v'(\theta^*)] - M'(\theta^*)v''(\theta^*)}{[M'(\theta^*)(1-v'(\theta^*))]^2}\frac{\partial\theta^*}{\partial m} \\
&= -\frac{(\sigma-1)^2 m{\theta^*}^\sigma - (1/\sigma+1-\sigma)(f+c/2)}{{\theta^*}^2[M'(\theta^*)(1-v'(\theta^*))]^2}\frac{\partial\theta^*}{\partial m} \\
\frac{\partial_2\theta^*}{\partial m\partial c} &= \frac{1}{2}\frac{\partial_2\theta^*}{\partial m\partial f}
\end{aligned}$$

□