Better or More: The demand for product differentiation from generalists and specialists

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Abstract

When goods may be put to several uses to meet a variety of needs, consumers with heterogeneous needs will not only give rise to a demand for horizontal differentiation but also vertical differentiation of specialised goods even for equal incomes. Contrary to the results in the literature on second degree price discrimination it is shown that a monopoly can neither extract all surplus from high nor from low quality specialised varieties. However, rather than distorting product attributes a monopoly has an incentive to offer too little variety and therefore respond to higher incomes by increasing quality rather than variety.

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1 Introduction

Product quality has been analysed in models where consumers with different incomes buy at most one unit of one variety only (see for example Mussa and Rosen (1978), Gabsewicz and Thisse (1979), Donnenfeldt and White (1990) on monopoly, and Johnson and Myatt (2003) on oligopoly). Whilst technological progress or a general increase in income could result in product line extension or changes in quality (see for example Bils and Klenow (2001b) and Sällström (1999)) the underlying assumptions of these models do not allow for that improvements may take the form of a consumer buying *more* rather than *better* goods.[1]

This paper analyses the choice of two attributes (specialised features and quality in general) when goods may be put to several uses and consumers with heterogeneous needs decide *how many* different varieties to buy.[2][3] For example, consider a good such as shoes. A pair of shoes can be used at work, to walk in the street, in the forest or on the mountain under different weather conditions. If a consumer were to buy just one pair to be used for all states welfare would be maximised for high quality and general purpose attributes, whereas if he bought several pairs welfare would be maximised for specialised attributes and shoes of lower quality (since they would be used less frequently). Thus consumers who buy a smaller number of varieties would typically not buy a subset of those who buy a larger number of varieties.[4]

As in Gronau and Hamermesh's (2008) extension of Becker's (1965) model of household production I model goods as inputs for the production of services in different activities. However unlike Gronau and Hamermesh (2008) I assume that consumers may use the same good as an input to generate services in several activities. For example I allow for the possibility that you can use the same shoes when going for

[1]Bils and Klenow (2001a) found, using US Bureau of Labor Statistics item substitution rates for 160 product categories over the period 1980-1996, that 'new varieties do increase spending on a category, as well as drive out or replace incumbent varieties.' When estimating quality growth Bils and Klenow (2001b) used a sample of 66 durable goods. They found that the average growth when weighted by expenditures was 3.7 percent per year. However, only seven product categories had experienced a quality growth in excess of 3 percent. Twice as many goods had had a negative quality growth.

[2]Alger (1999) Analysed multiple purchases in the case of quantitative discrimination.

[3]Suen (1991) showed that consumers will value diversity when goods have many uses. However, he did not model how many different goods a consumer should optimally buy.

[4]This can be contrasted with Murphy, Shleifer and Vishny (1989) where consumers with heterogeneous incomes decide how many different varieties to buy. In their model consumers with lower incomes buy a subset of the goods bought by consumers with higher incomes.

a walk during the weekend and at work. Thus the model corresponds to a situation where goods may be put to several uses and consumers derive utility from the use they put goods to rather than the goods themselves.

The devoted hiker would in this case be prepared to pay more for a pair of high quality boots, than the occasional hiker even if they have the same income. Thus not only will there be a demand for horizontal differentiation of specialised goods to match different activities, but also vertical differentiation to match different intensities in use.

Contrary to previous findings on vertical differentiation the distortion that will arise in a monopoly is to distort variety instead of quality in order to extract more surplus. Thus even though an increase in income may make it socially optimal for consumers to buy more specialised varieties, a monopoly may respond by increasing the quality of its product range rather than increasing the range, that is produce a general purpose variety of higher quality instead of vertically differentiated specialised varieties.

Gronau and Hemermesh (2008) showed that the evidence in Jackson (1984) and Gronau (1986,1997) that consumers with higher incomes demand more variety could be explained as a result of individuals with higher incomes optimally pursuing more activities. In this paper I show that they would demand more varieties even if they do not pursue more activities by using more specialised varieties.

Von Ungern Sternberg (1988) modelled a different mechanism that would generate goods that were too general purpose in equilibrium relative to social optimum in a Salop (1979) style model of horizontal differentiation with entry and endogenous choice of cost of transport (which he gave the interpreation how general purpose a good is).

The Spokes model due to Chen and Riordan (2007) is a generalisation of Hotelling's (1929) model as formalised by d'Aspremont et al. (1979) to n-dimensions. The Spokes can be interpreted as specialised varieties¿ They analysed the effect on prices from entry in this case and found that prices were highest for intermediate incomes.

One feature that these models share is that they rely on the assumption that consumption is single purpose. This assumption implies that it does not matter whether the demand for variety is due to a love of variety (Dixit and Stiglitz (1977), Perloff and Salop (1985), Salop (1979) or heterogeneous tastes in most theoretical and empirical models of the demand for variety as was shown by Anderson et al. (1984) and Deneckere and Rotschild (1992).

The framework for utility in this paper allows for consumption to be single purpose as a special cases. Thus existing models are special cases of a multi-purpose framework for consumption. To allow for multiple uses reveal the trade- off that exists between quality and variety that can explain the evidence that more but not necessarily better products have driven out old varieties. The point being that when we consume more varities in total their optimal attributes will have changed.

The outline of the paper is as follows. Section 2 presents a framework for utility when goods may be put to several uses. Section 3 characterises optimal product attributes and derives a condition for when it is optimal to buy several specialised varieties instead of one general purpose variety. Section 4 caracterizes the set of binding constraints when gneralists and speicalists can choose between vertically differentiated specialised and general purpose varieties. Section 5 analyses product selection in a monopoly. Section 6 concludes the paper with a discussion of directions for future research. All proofs can be found in an appendix.

2 A model of utility

This section presents a framework for consumer utility when goods may be put to several uses. Vertical, spatial and non-spatial product differentiation are special cases of this model.

Let it be noted that consumers buy goods because they have needs, such as needing shoes to walk, cycle or run, or apparel for hot and cold weather. These needs can be represented by a set of states, say $s \in \{1, 2\}$, (for example walking and running or hot and cold weather). Consumers derive utility from goods when they are being used to meet these needs.

The utility $v_s(\cdot)$ from using a good in state s, depends on two product attributes. General quality attributes $r > 0$ increase utility for all states $\partial v_s/\partial r > 0$ for $s = 1, 2$. General quality attributes thus include all attributes that will increase the value in use regardless of state, such as durability and reliability. Specialised attributes $t \in [0, k]$ increase utility in one state and reduce it in the other; $\partial v_1/\partial t > 0$ and $\partial v_2/\partial t < 0$. Specialised attributes, on the other hand, are attributes that make a good more desirable in one state at the expense of making them less desirable in another. [5] For example an attribute that makes shoes more suitable for cycling will make them less suitable for walking because the needs are different.

Goods are general purpose if $t = k/2$ and specialised to better suit the needs in state 1 if $t > k/2$ and in state 2 if $t < k/2$. For example let t be the thickness of a jumper and let state 1 represent cold and state 2 hot weather. The utility from using the good in cold weather is then higher the thicker the jumper, and when it is hot the utility is higher the thinner the jumper. Thus the effect from increasing t will be the opposite in the two states.

How much a consumer values these attributes depends on the marginal utility of income $\theta > 0$, and the proportion of time that is spent in state 1, $\lambda \in [0, 1]$ and in state 2, $(1 - \lambda)$ respectively. Consumers have the same income but differ in terms of their needs. Thus a consumer has preferences not so much for the goods in themselves but for the use to which he puts them.

This paper contains one discrete model where consumers are either generalists or specialists. Generalists divide their time equally between the two states, $\lambda_G = 1/2$ and specialists spend all their time in one state only. Hence there are two types of

[5] Note that the degree of specialisation will increase utility in some states and reduce it in other states, whereas in Von Ungern Sternberg (1988) a more specialised variety will only reduce utility in other states.

specialists, that is $\lambda_1 = 1$ and $\lambda_2 = 0$.[6]

The paper also contains a benchmark derived using a continuous version whith a uniform distribution of λ.

The consumer's preferences can be represented by the following utility function. The utility from using the good in state 1 is given by

$$v_1(r, t, \lambda) = \theta(r + t)\lambda, \tag{1}$$

and the utility if used in state 2 is

$$v_2(r, t, \lambda) = \theta(r + k - t)(1 - \lambda). \tag{2}$$

The utility from a good that is used in both states is

$$\sum_{s=1}^{2} v_s(t, r, \lambda) = \theta[r + k - t + \lambda(2t - k)]. \tag{3}$$

From (3) one can infer that all consumers derive the same utility from a general purpose variety. This is because a general purpose variety with $t = k/2$ generates the same value in use for different uses. It therefore does not matter how much it is used in different states.

Taking the first derivative of (3) with respect to λ,

$$\frac{\partial \sum_{s=1}^{2} v_s(t, r, \lambda)}{\partial \lambda} = 2t - k \tag{4}$$

shows that the utility varies more with the consumer's needs λ the more specialised the varieties. Thus consumer heterogeneity is endogenously determined by product characteristics.

[6] Generalists and specialists can be derived in a framework with endogenous choice of time allocated to each state when there is diminishing marginal utility from time spent in each state. Generalists would then be consumers who derived the same value in use from using the good in each state and who would therefore optimally allocate the same amount of time for each state. A specialist would be someone who derives higher marginal utility from spending time in one state than the alternative even if he spends all his time in that state.

Remark 1 *Consumer heterogeneity is increasing in the degree of specialisation.*

Goods may be differentiated with respect to t and r. Thus consider the case where they can be general purpose or more or less specialised, and of high or low quality. The number of available varieties N is finite and determined endogenously chosen by the firm.

2.1 Utility maximisation

When goods may be put to several uses the utility maximisation problem has to be solved in two steps. For each subset of available varieties the consumer needs to decide how to put them to their optimal use. The consumer then picks the subset that gives the highest utility when the varieties have been put to their optimal use.

When there are two states this implies that the consumer will choose between the 'best' bundle that contains only one variety to be used in both states, and the best bundle containing two varieties to be used in different states.[7] This leaves the consumer with the following utility maximisation problem.

Let the varieties be indexed by i, and let the price of variety i be denoted p_i. Since there are only two states a consumer will either purchase one unit of one variety i to be used in both states that maximises

$$\max_i U(i) = \sum_{s=1}^{2} v_s(t_i, r_i, \lambda) - p_i \tag{5}$$

or one unit each of the two varieties i, j to be used in different states that maximise

$$\max_{i,j} U(i,j) = v_1(t_i, r_i, \lambda) + v_2(t_j, r_j, \lambda) - p_i - p_j \tag{6}$$

or nothing which would give zero utility. Note that since goods only generate utility if they are being used, the consumer could not get higher utility buy buying a third variety that is never optimal to use. Thus the only relevant bundles to consider are those that contain, none, one or two varieties.

[7] We can trivially exclude the possibility of having three varieties at positive prices, since the consumer should optimally use only one variety for each state.

For $N = 1$ this trivially implies that the highest price a consumer would accept solves $U(i) = 0$ that is

$$p = \theta[r + k - t + \lambda(2t - k)]. \tag{7}$$

If $2t > k$ a consumer is more likely to buy the higher is λ.[8]

$N = 2$ represents the simplest case when consumers have a choice that involves choosing not only which variety to buy but also how many. Since the consumer has only two distinct needs this becomes a choice between buying one or two varieties. Now suppose that there are two specialised varieties to choose from with attributes $t_1 > \frac{k}{2} > t_2$, and r_1, r_2. A consumer will then buy both if that gives a higher net surplus than buying just one of them, that is (6) is greater than (5). This gives,

$$V_1(1, 2) = \theta(\Delta t + \Delta r)\lambda \geq p_1 \tag{8}$$

and

$$V_2(1, 2) = \theta(\Delta t - \Delta r)(1 - \lambda) \geq p_2, \tag{9}$$

where $\Delta t = t_1 - t_2 > 0$ and $\Delta r = r_1 - r_2$. The left hand side is the Shapley value $V_i(1, 2)$ of each variety $i = 1, 2$. For these prices the consumer gets a positive surplus given by,

$$U(1, 2) = \theta\left[\lambda(r_2 + t_2) + (1 - \lambda)(r_1 + k - t_1)\right] \geq 0. \tag{10}$$

The consumer will get more surplus the higher the quality and the less differentiated the goods. This is because each variety is then more usable for several purposes and the consumer is therefore reluctant to buy several varieties unless offered a very favourable price.

For a generalist with $\lambda = 1/2$ the conditions(8) and (9) are more likely to be satisfied the more specialised the goods, that is when Δt is higher[9], the higher the income

[8] Thus if consumers needs are uniformly distributed the demand for a specialised variety will be a downward sloping linear demand function.

[9] Making a good more specialised has two effects. First it makes the good more valuable for its intended use. Second it makes it less valuable for alternative uses. Thus if a consumer buys one

and the lower the price. Multi-purpose consumption thus provides an explanation for why consumers demand more variety with higher real incomes.[10]

For a specialist, on the other hand, either (8) or (9) can be satisfied for positive prices, since the Shapley value will then be zero for one of the varieties. Specialists will therefore demand only one variety that maximises (5).[11]

The question is whether firms will set prices in equilibrium such that generalists optimally purchase one or two varieties.

2.2 A Specialised duopoly

To answer this question it will be useful to derive a benchmark when λ is uniformly distributed on $[0, 1]$ that shows that if two firms sold one specialised variety each they would never set prices so low that any consumers would buy both be they generalists or specialists.

Proposition 1 (Hotelling) *Suppose that two firms compete in price. They sell one specialised variety each of the same quality and have the same cost per unit c. If λ is uniformly distributed on $[0, 1]$ the equilibrium price will be $p_i = \theta\Delta t + c$ and specialists and generalists alike will buy one variety only.*

This shows that consumers who use goods differently will give rise to Hotelling style competition where the 'cost of transport' parameter is indirectly chosen through choice of product attributes. Thus it shows why the cost of transport also can be

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specialised pair of shoes he would be willing to pay more for another pair of specialised shoes than he would have been had he bought a general purpose pair of shoes instead.

[10]Other explanations that have been modelled are that consumers get satiated with each variety and therefore demand more variety with higher income, Murphy et al. (1989) or that higher income consumers optimally pursue more activities and since each variety can only be used in one activity therefore demand more variety, Gronau and Hamermesh (2008).

[11]Thus the models for demand for variety in Perloff and Salop (1984) and Rotschild and Deneckere (1992) can be derived from the case where consumption is single purpose and all consumers are specialists.

used as a measurement of the degree of product differentiation. It is interesting to note that it increases with θ. Thus consumers treat goods as more differentiated the higher their income.

Whilst firms could induce consumers to increase total demand by lowering their prices they will not have an incentive to do so. This is because they cannot discriminate between specialists and generalists. The point is that it is neither profit maximising nor socially optimal for specialists and generalists to consume specialised goods of the same quality. This is because consumers value goods more the more use they get from them. A specialist (e.g. the devoted hiker) will therefore value quality r and specialised attributes t more on the margin than the generalist (e.g. the occasional hiker) even if they have the same marginal utility of income, that is the same θ. Thus heterogeneous needs do not only give rise to a demand for horizontal differentiation of goods but also vertical differentiation.

Remark 2 *Horizontal heterogeneity generates a demand for vertical differentiation of specialised goods.*

What makes this form of vertical differentiation interesting is that it is analytically distinct from the models of vertical differentiation by Mussa and Rosen (1978) and Gabsewicz and Thisse (1979) thereby showing that differences in willingness to pay for quality as a result of different needs is different from differences in income.

Before deriving the demand for vertically differentiated specialised varieties when consumers are either generalists $\lambda = 1/2$ or specialists $\lambda \in \{0, 1\}$ and its implication for prices and product selection we shall derive the social optimum to identify the relevant product selection from a welfare point of view.

3 Social Optimum

To derive optimal attributes we need to make assumptions about the cost side. For simplicity consider the case where there is a unit cost function $c(r, t)$ that takes the following form,

$$c(r,t) = a + cr^2 + g(\frac{k}{2} - t)^2. \quad (11)$$

This cost function captures that there is a cost a of producing a good regardless of its attributes. For example there is a cost of assembling a product which is independent of its material quality and its specialised attributes. Making a good more specialised to match state 1 or state 2 is costly. Hence the cost function for t reaches a minimum at $k/2$.[12] It is also more costly on the margin to increase quality.

Welfare maximising attributes depends on how many states each variety should optimally be used in. Since there are two states this implies that optimal attributes will either be the solution to

$$\max_{r,t} W(i) = \sum_{s=1}^{2} v_s(r,t,\lambda) - c(r,t). \quad (12)$$

if a variety will be used in both states; or the solution to

$$\max_{r,t} W(i) = v_s(r,t,\lambda) - c(r,t), \quad (13)$$

if it is optimal to have two specialised varieties to be used in one state only.

The solution to (12) r^*_{GH}, t^*_{GH} can be found by partial differentiation of $W(i)$ with respect to quality r and specialisation t;

$$\sum_{s=1}^{2} \frac{\partial v_s(r,t,\lambda)}{\partial r} - c_r(r,t) = 0 \quad (14)$$

and

$$\sum_{s=1}^{2} \frac{\partial v_s(r,t,\lambda)}{\partial t} - c_t(r,t) = 0. \quad (15)$$

When a good is used in several states the marginal value of increasing a product attribute depends on the sum of the marginal value of this attribute for each state in which it will be used. For general quality attributes this implies that the marginal benefit will be higher the more states in which the good will be used since the value

[12]For example, a standard fabric is cheapest, whereas a fabric with specialised attributes to match either hot or cold weather is more expensive.

in use from higher general quality is increasing for all states by definition. For the specialised quality attribute, on the other hand, the effect in different states will be the opposite. In particular if $\lambda = 1/2$ the positive effect from making a good more specialised for one state will be off set by the negative effect on utility when it is used in the other state. Thus both utility and unit cost will be minimised for $t = k/2$ if $\lambda = 1/2$. Since this general purpose variety will be of high quality it will be indexed $i = GH$.

If the generalist instead were to use different goods in different states the optimal attributes r^*_{iL}, t^*_{iL} $i = 1, 2$, solve

$$\frac{\partial v_s(r, t, \lambda)}{\partial r} - c_r(r, t) = 0 \tag{16}$$

and

$$\frac{\partial v_s(r, t, \lambda)}{\partial t} - c_t(r, t) = 0, \tag{17}$$

which are the first order conditions to (13). In this case the marginal return to general quality r will be lower since the good will be used in one state only. However, the marginal return from making a good more specialised will be higher even though the good is used less frequently. This is because the use has become more specialised. Since these goods should optimally be of low quality but have specialised attributes they will be indexed $1L, 2L$.

The solution to (13) for specialists $\lambda \in \{0, 1\}$, on the other hand, is a high quality good that is even more specialised, provided that θ is not too high, to be indexed $1H, 2H$. Since specialists will make maximum use of a specialised variety it should optimally be of higher quality and more specialised. If θ is very high it will be optimal to make both high and low quality specialised varieties as specialised as they can be.

Table 3 summarises welfare maximising attributes t^*_{ij} and r^*_{ij},$i = G, 1, 2$ $j = H, L$ if generalists buy a general purpose variety versus if they buy specialised varieties and the resulting unit cost and welfare. In Table 3 the same is done for products that are optimised for specialists. Comparing the welfare from a general purpose variety with a high quality specialised variety when bought by a specialist, one can see that the

Table 1: Optimal product attributes for generalists

	GH	1L	2L
r^*	$\frac{\theta}{2c}$	$\frac{\theta}{4c}$	$\frac{\theta}{4c}$
t^*	$\frac{k}{2}$	$\min\left\{k, \frac{k}{2}+\frac{\theta}{4g}\right\}$	$\max\left\{0, \frac{k}{2}-\frac{\theta}{4g}\right\}$
c	$a+\frac{\theta^2}{4c}$	$a+\frac{\theta^2}{16g}+\frac{\theta^2}{16c}$	$a+\frac{\theta^2}{16g}+\frac{\theta^2}{16c}$
W	$\frac{\theta}{2}\left(\frac{\theta}{2c}+k\right)-a$	$\frac{\theta}{2}\left(\frac{\theta}{8c}+\frac{\theta}{8g}+k\right)-a$	$\frac{\theta}{2}\left(\frac{\theta}{8c}+\frac{\theta}{8g}+k\right)-a$

Table 2: Optimal product attributes for specialists

	1H	2H
r^*	$\frac{\theta}{2c}$	$\frac{\theta}{2c}$
t^*	$\min\left\{k, \frac{k}{2}+\frac{\theta}{2g}\right\}$	$\max\left\{0, \frac{k}{2}-\frac{\theta}{2g}\right\}$
c	$a+\frac{\theta^2}{4g}+\frac{\theta^2}{4c}$	$a+\frac{\theta^2}{4g}+\frac{\theta^2}{4c}$
W	$\frac{\theta}{2}\left(\frac{\theta}{2c}+\frac{\theta}{2g}+k\right)-a$	$\frac{\theta}{2}\left(\frac{\theta}{2c}+\frac{\theta}{2g}+k\right)-a$

latter is higher. Hence for specialists it will always be optimal to use a high quality specialised variety, since there are no fixed costs of each variety in this model.[13] The total surplus for a generalist, on the other hand, will either be maximised using one general purpose variety GH of high quality or two specialised varieties of low quality $1L, 2L$. A generalist should therefore optimally consume either quality or variety depending on income (θ).[14]

Proposition 2 (The Quality-Variety Tradeoff) *Social welfare W for a generalist with $\lambda = 1/2$ is higher for two low quality specialised varieties than a high quality*

[13]If there were a fixed cost of variety, social optimum for low incomes might entail all consumers buying the high quality general purpose variety.

[14]This will be true when there are no complementarities between r and t. This happens when how specialised a good can be is independent of its material quality.

general purpose variety if

$$\frac{\theta^2}{8}\left[\frac{1}{g}-\frac{1}{c}\right] > a. \tag{18}$$

Consumers can only gain from more variety if the marginal cost of making a good more specialised is lower than the marginal cost of increasing the general quality. Hence, a necessary condition is that $g < c$. However, this condition is not sufficient. It is also required that the income is high enough relative to the unit cost a. Comparative statics of θ would thus imply that it would be optimal to increase variety for goods with low a, such as shoes already for intermediate incomes, whereas for goods with a high a such as cars, a much higher income would be required to increase variety.

Corollary 1 *If consumers have heterogeneous needs it will be optimal to vertically differentiate specialised varieties provided that their income is high enough.*

Hence, in the absence of fixed costs of variety welfare will be maximised for three horizontally differentiated high quality varieties or four vertically differentiated specialised varieties depending on θ.

Having thus derived welfare maximising product attributes for consumers who are either specialists or generalists we shall turn to how much consumers are prepared to pay for these varieties if they can choose any subset of all available varieties.

4 Demand for Vertically Differentiated Specialised Varieties

When consumers can choose any subset of all available varieties their willingness to pay may be constrained by other alternatives. For example a specialist may buy a subset of varieties intended for a generalist, and a generalist may buy just one high or low quality variety instead of two specialised ones. The question is whether the highest price that can be charged for each variety will be determined by a binding incentive constraint or a binding participation constraint.

This section shows that whilst heterogeneous needs give rise to a demand for vertical differentiation of specialised varieties, it does not have the same predictions for prices as a Mussa and Rosen (1978) style model of vertical differentiation where the incentive constraint is binding for a 'high type' and the participation constraint for the 'low type'. Instead I find that:

1. if there are III varieties $(1H, 2H, GH)$ to choose from, prices are determined by binding participation constraints for all varieties;

2. if there are IV varieties $(1L, 2L, 1H, 2H)$ to choose from, prices are determined by binding incentive constraints for all varieties.

This will be shown to have interesting implications for product attributes, number of varieties and responses to an increase in income in a monopoly.

If there are III varieties to choose from a specialist with preference for state 1 $(\lambda = 1)$ will buy the specialised variety $1H$ if

$$\theta(r_{1H} + t_{1H}) - p_{1H} \geq 0, \tag{19}$$

$$\theta(r_{1H} + t_{1H}) - p_{1H} \geq \theta(r_{2H} + t_{2H}) - p_{2H}, \tag{20}$$

$$\theta(r_{1H} + t_{1H}) - p_{1H} \geq \theta(r_{GH} + \frac{k}{2}) - p_{GH}. \tag{21}$$

Similar constraints can be written for a specialist with a preference for state 2. A generalist will buy a general purpose variety if

$$\theta(r_{GH} + \frac{k}{2}) - p_{GH} \geq 0, \tag{22}$$

$$\theta(r_{GH} + \frac{k}{2}) - p_{GH} \geq \theta(r_{2H} + \frac{k}{2}) - p_{2H}, \tag{23}$$

$$\theta(r_{GH} + \frac{k}{2}) - p_{GH} \geq \theta(r_{1H} + \frac{k}{2}) - p_{1H}. \tag{24}$$

Note that specialists do not value a general purpose variety more than a generalist. Generalists in turn value a specialised variety less than the specialists. The incentive constraints for specialists (20;21) and generalists (23;24) will therefore be satisfied if prices are determined by the binding participation constraints for generalists (22) and specialists (19). Since $t_{1H} > \frac{k}{2}$ by definition equilibrium prices for specialised varieties will be higher than the price for a general purpose variety $p_{1H} = \theta(r_{1H}+t_{1H}) > p_{GH} = \theta(r_{GH} + \frac{k}{2})$. Consumers are in this case left with no surplus.

When there are IV varieties to choose from specialists and generalists alike have options such as buying a subset of what was intended for the other that will limit how much they are willing to pay.

Since a specialist has one use only for a good she will compare bundles containing one variety. A specialist with $\lambda = 1$ will prefer variety $1H$ at price p_{1H} to all other available varieties if the following constraints hold. First the participation constraint

$$\theta(r_{1H} + t_{1H}) - p_{1H} \geq 0. \tag{25}$$

Then the vertical constraint, that is a specialised variety of lower quality should not give a higher surplus,

$$\theta(r_{1H} + t_{1H}) - p_{1H} \geq \theta(r_{1L} + t_{1L}) - p_{1L}. \tag{26}$$

There is also a horizontal constraint, that is another specialised variety of the same quality should not give a higher surplus,

$$\theta(r_{1H} + t_{1H}) - p_{1H} \geq \theta(r_{2H} + t_{2H}) - p_{2H}. \tag{27}$$

Finally there is a diagonal constraint, that is a different specialised variety of lower quality should not give a higher surplus,

$$\theta(r_{1H} + t_{1H}) - p_{1H} > \theta(r_{2L} + t_{2L}) - p_{2L}. \tag{28}$$

The situation has now changed. Whilst a specialist would not have an incentive to buy the combination of products intended for the generalist, a specialist would have an incentive to buy a subset of what the generalist is buying (that is one specialised low quality variety), unless left with some surplus. Thus the following constraint will be binding for specialists.

Lemma 1 *The vertical constraint for a specialist will bind.*

There are two reasons for this. First, a specialist has more use for a specialised variety and thus values it more, which is the same as in a Mussa and Rosen style model. Second, because a generalist will not pay his full value of the product either. The net effect being that even less surplus can be extracted from the specialists than in the Mussa and Rosen style model. To see why consider the choice made by a generalist.

Generalists compare bundles with none, one or two varieties. For a generalist to prefer to buy two low quality specialised varieties $(1L, 2L)$ at individual prices p_{1L}, p_{2L} the following constraints have to hold. First, the participation constraint

$$\theta\frac{1}{2}\left[r_{1L} + t_{1L} + r_{2L} + k - t_{2L}\right] - p_{1L} - p_{2L} \geq 0. \tag{29}$$

Furthermore buying these would have to be preferred to buying just one specialised variety of low quality, i.e. the horizontal constraints,

$$\theta\frac{1}{2}\left[r_{1L} + t_{1L} + r_{2L} + k - t_{2L}\right] - p_{1L} - p_{2L} \geq \theta(r_{iL} + \frac{k}{2}) - p_{iL} \tag{30}$$

or just one specialised variety of high quality or two specialised varieties of high quality, i.e., the vertical constraints

$$\theta\frac{1}{2}\left[r_{1L} + t_{1L} + r_{2L} + k - t_{2L}\right] - p_{1L} - p_{2L} \geq \theta(r_{iH} + \frac{k}{2}) - p_{iH} \tag{31}$$

and

$$\theta\frac{1}{2}\left[r_{1L} + t_{1L} + r_{2L} + k - t_{2L}\right] - p_{1L} - p_{2L} \geq \theta\frac{1}{2}\left[r_{1H} + t_{1H} + r_{2H} + k - t_{2H}\right] - p_{1H} - p_{2H}; \tag{32}$$

or any other combination of two varieties, that is composite constraints

$$\theta\frac{1}{2}\left[r_{1L} + t_{1L} + r_{2L} + k - t_{2L}\right] - p_{1L} - p_{2L} \geq \theta\frac{1}{2}\left[r_{1L} + t_{1L} + r_{2H} + k - t_{2H}\right] - p_{1L} - p_{2H} \tag{33}$$

$$\theta\frac{1}{2}\left[r_{1L} + t_{1L} + r_{2L} + k - t_{2L}\right] - p_{1L} - p_{2L} \geq \theta\frac{1}{2}\left[r_{1H} + t_{1H} + r_{2L} + k - t_{2L}\right] - p_{1H} - p_{2L} \tag{34}$$

The binding constraints in this case will depend on parameter values.

Lemma 2 *The horizontal constraint (30) for generalists will be binding if* $\Delta t_H > \Delta t_L$*, whereas the vertical constraint (31) for generalists will be binding if* $\Delta t_H < \Delta t_L$.

Thus there are two possible set of equilibrium prices which depend on how specialised high quality varieties are relative to low quality varieties.

If low quality varieties are more specialised than high quality varieties $\Delta t_H < \Delta t_L$ equilibrium prices can be found by substituting for p_{1L} from (26) in (31) and solve for p_{1H} to get

$$p_{1H} = \theta \left(r_{1H} + t_{1H} - t_{2H} - \frac{1}{2}(r_{1L} + r_{2L} + t_{1L} - t_{2L}) \right) \tag{35}$$

Similarly one can show that

$$p_{2H} = \theta \left(r_{2H} + t_{1H} - t_{2H} - \frac{1}{2}(r_{1L} + r_{2L} + t_{1L} - t_{2L}). \right) \tag{36}$$

Then substitute for p_{1H} in (26) and solve for p_{1L} to get

$$p_{1L} = \theta \left(\frac{1}{2} (t_{1L} + t_{2L} + r_{1L} - r_{2L}) - t_{2H} \right) \tag{37}$$

Similarly

$$p_{2L} = \theta \left(t_{1H} - \frac{1}{2} (t_{1L} - t_{2L} - r_{2L} + r_{1L}) \right). \tag{38}$$

First note that these prices could not prevail for socially optimal attributes since $t^*_{iL} \leq t^*_{iH}$. The question is therefore whether these prices could nonetheless prevail in a monopoly.

Lemma 3 *If a monopoly expected the prices in (35;36;37;38) to prevail it would have an incentive to make the good too specialised for specialists* $t_{1H} > t^*_{1H}$ *and too general purpose for generalists* $t_{1L} < t^*_{1L}$.

However since $t^*_{iH} > t^*_{1L}$ Lemma 3 implies that (31) would not be the binding constraint.

Instead the binding constraint would be (30) giving equilibrium prices

$$p_{1L} = \theta \frac{1}{2} (t_{1L} - t_{2L} + r_{1L} - r_{2L}) \tag{39}$$

$$p_{2L} = \theta \frac{1}{2} (t_{1L} - t_{2L} + r_{2L} - r_{1L}). \tag{40}$$

Substitution of p_{1L} in (26) gives us the highest price that can be charged from a specialist,

$$p_{1H} = \theta \left(r_{1H} + t_{1H} - \frac{1}{2}(r_{1L} + r_{2L} + t_{1L} + t_{2L}) \right) \tag{41}$$

Similarly

$$p_{2H} = \theta \left(r_{2H} - t_{2H} - \frac{1}{2}(r_{1L} + r_{2L} - t_{1L} - t_{2L}) \right). \tag{42}$$

These prices have the following properties. First if the number of specialists are the same for each state the joint revenue from high quality specialised varieties will be increasing in the degree of vertical differentiation and the degree of specialisation of high quality varieties. However, it is independent of the degree of specialisation of low quality specialised varieties, Δt_L:

$$p_{1H} + p_{2H} = \theta(\Delta r_1 + \Delta r_2 + \Delta t_H), \tag{43}$$

where $\Delta r_i = r_{iH} - r_{iL}$, $i = 1, 2$ and $\Delta t_j = t_{1j} - t_{2j}$, $j = L, H$. This is because there are two effects from making the low quality varieties more specialised. The first effect is that the low quality variety becomes a more attractive substitute, thus implying less surplus can be extracted from the specialist. The second effect is that if the low quality varieties are more specialised, the monopoly can charge a higher price for them from the generalists, thus implying more surplus can be extracted. These two effects exactly cancel in this case.

For low quality specialised varieties the joint revenue is a function of their degree of specialisation only:

$$p_{1L} + p_{2L} = \theta \Delta t_L. \tag{44}$$

This is because there are two effects from an increase in quality. On the one hand, it increases the Shapley value for its intended use. On the other hand, it decreases the Shapley value of the other variety. These two effects exactly cancel in this model, thus leaving us with a revenue that depends on the degree of differentiation only.

The marginal revenue from making a good more specialised in a monopoly is the same for high and low quality goods, and the monopoly would therefore have an incentive to make these goods equally specialised if it were to produce both. The questions that will be addressed in the next section is whether a monopoly would distort product attributes and or the total number of varieties relative to social optimum, and how it would respond to an increase in θ.

5 Monopoly

In deciding how many varieties to sell, a monopoly will take into account the profit that will accrue once attributes have been chosen optimally.

Lemma 4 *If the monopoly produces one general purpose variety and two specialised varieties for specialists (GH,1H,2H) it will choose socially efficient attributes.*

This is because prices are then determined by binding participation constraints. Maximising monopoly profit therefore coincides with maximising social welfare when the monopoly produces a general purpose variety for generalists and specialised varieties for specialists only.

The question is what the monopoly will do when social welfare is maximised for IV varieties, that is when θ is sufficiently high to motivate specialised varieties for generalists?

In this case prices are determined by binding incentive constraints for specialists and generalists alike. Thus the monopoly can no longer extract all surplus from consumers. The first implication from these binding incentive constraints is that it gives the monopoly an incentive to distort product attributes.

Lemma 5 *If a monopoly were to offer IV varieties it would choose* $r_{iH}^M = r_{iH}^*$, *and* $r_{iL}^M = 0$, *and* $t_{iH}^M = t_{iL}^M = t_{iH}^*$.

The monopoly would have an incentive to make the low quality specialised varieties too specialised and of too low quality relative to social optimum. This is because this allows the monopolist to charge more for the low quality specialised varieties. The monopoly can charge more for each variety the less valuable they are if used as general purpose. This can be achieved by making them more specialised and reducing the quality. Note that the mechanism for distorting the quality of the low quality variety is different from standard models of vertical differentiation, and as a result of that results in quality being even more distorted.

There are two effects which reduces the monopolists incentive to vertically differentiate specialised varieties relative to social optimum. First, the fact that attributes will be distorted implies that welfare is not maximised. Second, the fact that incentive constraints are binding for both generalists and specialists implies that a monopoly can extract less surplus when it vertically differentiates specialised varieties.

Proposition 3 *A monopoly may offer too few varieties.*

Thus an increase in income (θ) may not be sufficient to increase variety in a monopoly. However, the monopoly would have an incentive to increase quality of all varieties if θ increases. A monopoly would therefore increase quality instead of variety in response to an increase in income.

Corollary 2 *Improvements in response to an increase in income will be biased towards quality improvements in a monopoly.*

6 Conclusions

This paper has modelled a mechanism that can explain why quality may optimally be reduced when consumers buy more varieties. The intuition runs as follows. For example if a child has only one toy, it should optimally be very durable and general-purpose so that it can play multiple roles in the child's imagination and last for many years. If a child, on the other hand, has many different toys, their optimal characteristics should be more specialised to better match specific needs, and since they will be used less frequently they should also optimally be of lower quality. The point is that one single plastic toy is not preferable to a high quality toy, but that several specialised plastic toys might be preferable to one high quality general purpose toy,[15] provided that a consumer values function more than quality on the margin.

This paper analysed a highly stylized model where specialists and generalists who can choose to buy any combination of available varieties. This case is relevant for generalisations of the model where consumers also have heterogeneous incomes, since varieties of intermediate quality would then be sold to generalists and specialists with different incomes. If consumers have heterogeneous incomes, the incentive to bundle would only occur for the lowest quality specialised varieties.[16]

When a monopoly can bundle low quality specialised goods we would again get a situation where the participation constraint of all consumers would be binding. This is because a generalist values the combination of two specialised varieties more than the specialist. Thus case IV would then be similar to case III, and product choice would be socially efficient and leave consumers with no surplus. To address issues in bundling as in McAfee, McMillan and Whinston (1989) in the case of specialised varieties is left for future research in a a model where consumers also have heterogneous incomes.

A Proofs

Proof of Proposition 1 Let $p_i > V_i(1,2)$ for all λ, then there exists a consumer λ_M who is indifferent between buying one unit from 1 and one unit from 2. We can find this consumer by solving

$$\theta[r + k(1-\lambda_M) - t_1(1-2\lambda_M)] - p_1 = \theta[r + k(1-\lambda_M) - t_2(1-2\lambda_M)] - p_2 \quad (45)$$

[15]The toy industry is a good example for these phenomena. It is an industry with a few dominant firms and high turnover of small firms (see Pesendorfer (2005)). There is no stability in product varieties and the industry has experienced periods of dramatic variety growth and doubling of sales (see Welsh (1992) for the case of soft toys).

[16]For example high quality socks are usually sold in a single pack, whereas low quality socks of different colours or designs are sold in packs with three or more.

for λ_M. This gives

$$\lambda_M = \frac{\theta\Delta t + p_1 - p_2}{\theta 2\Delta t}. \tag{46}$$

The demand for variety 1 are the consumers with $\lambda \geq \lambda_M$, and the demand for variety 2 are the consumers with $\lambda < \lambda_M$. Thus the profit maximisation problem for the firm selling variety i can be written

$$\max_{p_i}(p_i - c)\left(\frac{\theta\Delta t - p_i + p_j}{\theta 2\Delta t}\right) \tag{47}$$

From first order conditions we get the equilibrium prices

$$p_i = \theta\Delta t + c > V_i(1,2) \tag{48}$$

which are strictly higher than the Shapley value. For these prices the equilibrium profit is

$$\pi_i = \frac{\theta\Delta t}{2}. \tag{49}$$

Could the firm earn more by charging the Shapley value instead? Note that the profit from charging the Shapley value from some marginal consumers must be strictly less than the profit that they would get if all consumers were generalists with $\lambda = 1/2$. In this case all consumers would buy both varieties if prices were $p_i = \theta\Delta t/2$. Thus the profit would be $\frac{\theta\Delta t}{2} - c$. This profit is strictly less than the equilibrium profit. QED.

Proof of Proposition 2 The optimal attributes for a variety that will be used in both states by a consumer with needs λ solve,

$$\max_{r,t} \theta\left[\lambda(r+t) + (1-\lambda)(r + \bar{t} - t)\right] - cr^2 - g(\frac{\bar{t}}{2} - t)^2 - a \tag{50}$$

From first order conditions this implies that $r^* = \frac{\theta}{2c}$, $t^* = \frac{\bar{t}}{2} - \theta\frac{1-2\lambda}{2g}$

For $\lambda = 0$ $t^* = \max\{0, \frac{\bar{t}}{2} - \frac{\theta}{2g}\}$

For $\lambda = 1/2$ $t^* = \frac{\bar{t}}{2}$, thus independent of θ

For $\lambda = 1$ $t^* = \min\{\bar{t}, \frac{\bar{t}}{2} + \frac{\theta}{2g}\}$

These generate a total surplus

$$W(1) = \frac{\theta^2}{4}\left[\frac{1}{c} + \frac{(1-2\lambda)^2}{g}\right] - a \tag{51}$$

if specialised varieties are used

$$\max_{r_1,t_1,r_2,t_2} \theta(r_1+t_1)\lambda+(1-\lambda)(r_2+\bar{t}-t_2)-c(r_1^2+r_2^2)-g\left[\left(\frac{\bar{t}}{2}-t_1\right)^2+\left(\frac{\bar{t}}{2}-t_2\right)^2\right]-2a \tag{52}$$

welfare maximising product attributes for specialised varieties are

$$r_1 = \frac{\lambda\theta}{2c} \tag{53}$$

$$t_1 = \frac{\bar{t}}{2} + \frac{\lambda\theta}{2g} \tag{54}$$

$$r_2 = \frac{(1-\lambda)\theta}{2c} \tag{55}$$

$$t_2 = \frac{\bar{t}}{2} - \frac{(1-\lambda)\theta}{2g} \tag{56}$$

The degree of product differentiation is

$$t_1 - t_2 = \frac{\theta}{2g} \tag{57}$$

these generate a total surplus

$$W(2) = (1-2\lambda(1-\lambda)\frac{\theta^2}{4}\left[\frac{1}{c}+\frac{1}{g}\right]-2f \tag{58}$$

$W(GH) < W(1L) + W(2L)$, that is

Total welfare is maximised for two specialised varieties if

$$2\lambda(1-\lambda)\frac{\theta^2}{4}\left[\frac{1}{g}-\frac{1}{c}\right] > a \tag{59}$$

Set $\lambda = 1/2$ QED.

Proof of Lemma 1 For specialists the binding constraint is that it should not give a higher surplus to buy the low quality version of the specialised variety, that is To see that all other constraints will be satisfied when these once are binding note that in a symmetric equilibrium $p_{1j} = p_{2j}$ and $r_{1j} = r_{2j}$, $j = L, H$. Hence $\theta(r_{1H}+t_{1H}) - p_{1H} > \theta(r_{2H}+t_{2H}) - p_{2H}$ and $\theta(r_{1L}+t_{1L}) - p_{1L} > \theta(r_{2L}+t_{2L}) - p_{2L}$

will all be satisfied. Thus a specialist will get strictly less surplus from buying a specialised variety for the other state in which he spends no time.

For a generalist there are two constraints that may be binding depending on parameter values. First rather than buying two specialised low quality varieties, the generalist could buy just one specialised high quality variety to be used as a general purpose variety. For this constraint to be satisfied it is required that,

$$\theta\frac{1}{2}\left[r_{1L}+t_{1L}+r_{2L}+k-t_{2L}\right]-p_{1L}-p_{2L}\geq\theta(r_{iH}+\frac{k}{2})-p_{iH}\quad i=1,2. \tag{60}$$

The other alternative would be to buy just one low quality specialised variety and use it in all states instead of buying two,

$$\theta\frac{1}{2}\left[r_{1L}+t_{1L}+r_{2L}+k-t_{2L}\right]-p_{1L}-p_{2L}\geq\theta(r_{iL}+\frac{k}{2})-p_{iL}\quad i=1,2. \tag{61}$$

The latter will be binding if

$$\theta(r_{1L}+\frac{k}{2})-p_{1L}>\theta(r_{1H}+\frac{k}{2})-p_{1H} \tag{62}$$

rearranging and using (26)

$$\theta(r_{1H}-r_{1L})<p_{1H}-p_{1L}=\theta(r_{1H}-r_{1L}+t_{1H}-t_{1L}). \tag{63}$$

Thus $t_{1H}>t_{1L}$. Similarly one can show that $t_{2H}<t_{2L}$. Hence 30) will be binding if the degree of specialisation is higher for the high quality varieties $\Delta t_H>\Delta t_L$. The reason for this is that the price difference between high and low quality varieties due to the binding constraints for specialists (26) and (**??**).

Proof of Lemma 2: The option of buying one low quality specialised variety (30) will be binding if the incremental utility from using a high quality specialised variety is less than the difference in price, that is if

$$\theta(r_{iH}-r_{iL})\leq p_{iH}-p_{iL}. \tag{64}$$

Now since the difference in price has to satisfy (26) the condition can be written $t_{iH}\geq t_{iL}$, that is if high quality specialised varieties are more specialised than low quality specialised varieties, then the high quality variety will not be a binding option. This is because the prices that can be charged for a high quality specialised variety will be higher the less specialised the low quality variety. Q.E.D.

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