

Efficient gate revenue sharing in sports leagues

Thomas Peeters

Flanders Research Foundation (FWO) and University of Antwerp¹

November 26, 2011

¹Author correspondence: University of Antwerp, Faculty of Applied Economics, Prinsstraat 13, 2000 Antwerp, Belgium. e-mail: thomas.peeters@ua.ac.be. Acknowledgements: I would like to thank Stefan Késenne, Jan Bouckaert, Mathias Reynaert, Stefan Szymanski and Stephen Layson for comments and suggestions. This paper also benefited from discussions with seminar and conference participants at the University of Antwerp and the Southern Economic Association Conference in Washington DC. I gratefully acknowledge financial support from the Flanders Research Foundation (FWO).

Abstract

Sports leagues constitute one of the few examples of legally operating cartels. In this paper I examine how gate revenue sharing may serve to coordinate talent investments within these cartels. I show that sharing revenues has the potential to raise cartel profits, because it decreases the incentive to invest in playing talent. Leagues consisting of teams with heterogeneous local markets should share less revenues to maximize profits, whereas homogeneous teams should share more.

Keywords: cartel behavior, revenue sharing, sports leagues.

JEL-classification: L41-L83

1 Introduction

Professional team sports is one of the scarce industries in which coordination between competing firms (clubs) is widely accepted. Clubs have consequently organized themselves in legal cartels (leagues). A certain degree of coordination is of course necessary to produce a team sports competition, e.g. for scheduling games. However, leagues have also introduced regulations, which would normally classify as restrictive practices. Today, the most relevant examples of such practices are gate revenue sharing (the home team gives a part of its match-day income to the visiting team), collective sales of media rights (the league monopolizes media rights and distributes the revenues) and salary caps (the league limits the amount teams may spend on player wages). In this paper I focus on the league's rationale to share gate revenues.

<INSERT TABLE 1 ABOUT HERE>

Table 1 shows that gate revenue sharing is quite common in the American major leagues (NFL, MLB and MLS). In contrast, European soccer clubs, along with the AFL and NBA, share (almost) no gate revenues. However, both the AFL and the EPL have had sizeable gate revenue sharing arrangements in the past. Gate revenue sharing is also on the table of the negotiations between NBA players and owners to end the current NBA lockout.¹ These observations raise the question why some cartels share gate revenues while others have chosen not to.

I examine this question using a theoretical model of a sports league, which serves two types of consumers, committed stadium visitors, who prefer to see their team win, and neutral TV viewers, who like to watch a tense and high level competition. I show that leagues whose teams have more homogenous home markets are able to raise profits by engaging in more extensive gate revenue sharing. Likewise, less equal teams

¹See Washington Post, June 29th 2011, "Lengthy NBA lockout looms, with owners and players deeply divided".

should share less gate revenues to be profitable. This leads to the somewhat surprising conclusion that gate revenue sharing is more interesting for leagues who are less likely to suffer from a "competitive balance problem".²

The intuition for this result may be understood as follows. Revenue sharing functions as a taxation on talent investments, because a part of talent related revenues flows away to competing teams. As such, it reduces the incentives to invest in talented players, which in turn reduces costs and increases profits. However, neutral consumers dislike a league in which the talent level is low. If teams raise the amount of revenue sharing too much, the taxation effect shrinks the talent level below its profit-maximizing point. In leagues with heterogeneous markets, small teams reduce their talent investments more drastically, because with revenue sharing they receive a (relatively) important part of their revenues from the large team's market. In this market the small team's investments reduce gate revenues and hence, the incentive not to invest. Likewise, in heterogeneous leagues, the taxation effect hits large teams more severely, because they have to share a (relatively) more important revenue source.

The economics literature has mostly looked at the effect of gate revenue sharing on competitive balance (see for example, El Hodiri and Quirk (1971), Fort and Quirk (1995), Szymanski and Késenne (2004), Feess and Stähler (2009) and Grossman et al. (2010)). Within this literature the works of Szymanski (2001) and Forrest et al. (2006) established the idea that sports leagues serve consumers with differentiated tastes (i.e., committed stadium visitors vs. uncommitted TV viewers). This paper builds on the model in Peeters (forthcoming) to integrate this insight into the analysis of gate revenue sharing. In line with the findings of Feess and Stähler (2009), the competitive balance effect of revenue sharing turns out to be ambiguous. This is a result of the complex nature of team revenues in my model, which depend on the relative

²The main justification for introducing gate revenue sharing has traditionally been the perceived need to protect the uncertainty of outcome in sporting competitions. This line of reasoning is usually referred to as the competitive balance argument in the sports economic literature.

and total talent stock in the league.³ Another related strand of literature explicitly analyzes the functioning of cartels in the sports industry. Examples include Kahn (2007) and Pecorino and Farmer (2010) on labor market restrictions in college sports, Palomino and Sakovics (2004) and Peeters (forthcoming) on sharing rules for media rights revenues and Ferguson et al. (2000) on salary restrictions in MLB. Atkinson et al. (1988) first identified the possibility to use revenue sharing as a coordination device in an empirical study of the NFL. This paper extends their analysis by distinguishing between different types of league revenues. As such, it allows to analyze the factors that influence the profit-maximizing sharing rule for the cartel.⁴

This paper is further organized in four sections. The first part presents the model setup in full detail. Section 3 covers the impact of revenue sharing on talent investments and relates the results to the competitive balance literature. A subsequent part looks at the profitability of gate revenue sharing for the league cartel. In a final section I provide a few concluding remarks.

2 Model Setup

In this model two profit-maximizing clubs (i and j) join together to form a league. In a first stage the league sets a sharing rule (α) for gate revenues, where α reflects the part of gate revenue a club is allowed to keep. Full sharing then means $\alpha = \frac{1}{2}$, while no sharing implies $\alpha = 1$. The league therefore sets a rule on the interval $\alpha \in [\frac{1}{2}, 1]$.

In a second stage clubs make decisions on talent investments. Talent is available at constant marginal cost, but as in Palomino and Sakovics (2004) and Peeters (forthcoming) has a discrete nature. Clubs may make either a high investment at cost h or a low investment at cost l . These talent investments translate in win probabilities

³See Feess and Stähler (2009) for a survey of the competitive balance effects of sharing under different assumptions about team revenues.

⁴I do not model the internal decision making in the cartel, as is done in Easton and Rockerbie (2005) or Peeters (forthcoming).

given by

$$w_i(t_i, t_j) : \left\{ \begin{array}{l} w_i(h, l) = \beta \\ w_i(l, l) = w_i(h, h) = \frac{1}{2} \\ w_i(l, h) = 1 - \beta \end{array} \right\} \quad (1)$$

where $\beta > 1/2$.

In the final stage clubs set ticket prices (p_i^f) and sell media rights. Clubs are assumed to be monopolists in their home markets of size m_i and m_j , where they face demand for stadium seating from committed fans. One club (from now on club 1) has a strictly larger market than the other (further on club 2), so $m_1 > m_2 > 0$. Since talent investments are sunk in this stage and all other costs are neglected, the price setting problem boils down to a revenue-maximization problem. To meet demand for media appearances the league pools media rights and sells them to the highest-bidding broadcaster in an auction setting. The broadcasting market is assumed to be competitive, so the league is able to extract all relevant profits. She splits the proceeds equally among clubs, as is common in the US major leagues.⁵ The broadcaster who obtains the rights faces demand from neutral fans. The size of the neutral consumer market is given by n , which may be interpreted as the number of TV households. The cost of broadcasting is neglected, so all relevant costs to the broadcaster are sunk. He too should therefore set a pay-per-view price (p^b) that simply maximizes total revenues.

As first proposed by Szymanski (2001), consumers of sports contests are not a homogeneous group. Instead, the model follows Peeters (forthcoming) in distinguishing demand from hard-core club fans and neutral sports fans. Both groups have a different appreciation for the characteristics of sports contests. Neutral fans appreciate tension and a high-level of play in a game. This implies that a higher level of play only increases their perceived quality ($b(t_i, t_j)$), when it does not result in a deterioration

⁵In the US the league usually sells national media rights, while local rights are sold by individual clubs. Under an alternative interpretation of this model local media revenues (stemming from local, committed fans) are shared under the gate revenue sharing arrangement.

of competitive balance, as such

$$b(h, h) > b(h, l) = b(l, h) = b(l, l). \quad (2)$$

Committed club fans prefer to see their team win. Consequently, they rank possible outcomes according to their team's expected winning percentage. I assume that a decline in winning percentage leads to a similar decline in quality, whether it is on the "winning" or the "losing" segment. This means (team-specific) quality ($f_i(t_i, t_j)$) may be expressed by

$$f_i(h, l) > f_i(h, h) = f_i(l, l) > f_i(l, h) \quad (3)$$

where,

$$f_i(h, l) - f_i(l, l) = f_i(l, l) - f_i(l, h).$$

Demand of both groups follows from (3) and (2) by assuming consumers have individual specific preferences x_v^b and x_v^f , which are uniformly distributed along the interval $[0, 1]$. Consumers then face a utility maximization problem of the form,

$$\begin{aligned} & \max \left\{ 0, x_v^f f_i(t_i, t_j) - p_i^f \right\} \\ & \max \left\{ 0, x_v^b b(t_i, t_j) - p^b \right\}. \end{aligned}$$

This implies market demand is linearly decreasing in prices and increasing in quality at a decreasing rate,

$$D_i^f = m_i \frac{f_i(t_i, t_j) - p_i^f}{f_i(t_i, t_j)} \quad (4)$$

$$D^b = n \frac{b(t_i, t_j) - p^b}{b(t_i, t_j)}. \quad (5)$$

Given the demand (4) and (5), revenues from both fan groups are maximized by

setting

$$\begin{aligned} p_i^f &= \frac{f(t_i, t_j)}{2} \\ p^b &= \frac{b(t_i, t_j)}{2}. \end{aligned}$$

This leads to total per game revenues from both consumer groups of the form

$$\begin{aligned} R_i^f &= \frac{f(t_i, t_j)}{4} \\ R^b &= \frac{b(t_i, t_j)}{4}. \end{aligned}$$

Given that talent investments are the only relevant costs to clubs, it is straightforward to express profits as a function of α and the club's talent investments,

$$\pi_i = \frac{n}{4}b(t_i, t_j) + \alpha \frac{m_i}{4}f(t_i, t_j) + (1 - \alpha) \frac{m_j}{4}f(t_j, t_i) - t_i. \quad (6)$$

The league's objective is to maximize joint profits of both clubs. This means her objective function boils down to

$$\pi_L = \frac{1}{4} [2nb(t_1, t_2) + m_1f(t_1, t_2) + m_2f(t_2, t_1)] - t_1 - t_2. \quad (7)$$

3 Talent Investments

In the second stage clubs decide on their talent investments. Since each club has two options to choose from, four different outcomes may arise, mutually high investments ($t_1 = h, t_2 = h$), mutually low investments ($t_1 = l, t_2 = l$) and domination by the large ($t_1 = h, t_2 = l$) or the small team ($t_1 = l, t_2 = h$).

Proposition 1 *There exists no sharing rule $\alpha \in [\frac{1}{2}, 1]$ for which domination by the small team is a Nash-equilibrium of the talent investment stage.*

Proof. see appendix. ■

To get some insight in the intuition for proposition 1, first compare the revenue potential of both teams when clubs share no gate revenues ($\alpha = 1$). In that case, a small market team dominating the competition has to offset high investment costs using the increased willingness to pay from its (small) market of committed fans. At the same time, the large market team has the option to match high investments. It may offset these investments through higher earnings from neutral fans and from its (large group of) committed fans. Therefore it always pays for the large team to match high investments, when it pays for the small team to make them. Introducing gate revenue sharing decreases the incentives from both local markets. Yet, as long as $\alpha > \frac{1}{2}$, the incentive to invest from local fans is guaranteed to be larger for the large team.

To characterize the remaining talent investment outcomes I define thresholds (θ_h and θ_l) on the values of the model parameters, which define the regions for which each situation constitutes a Nash-equilibrium. In order to ensure the presence of unique Nash-equilibria at all parameter values (except for the thresholds), it is necessary to impose that both clubs have sufficiently different markets, i.e. $m_1 - m_2 \geq n \frac{b(h,h) - b(h,l)}{f(h,l) - f(h,h)}$.

Proposition 2 *The Nash-equilibrium of the talent investment stage is given by*

- *Mutually high investments* $\Leftrightarrow m_2 > \theta_h = \frac{1}{\alpha} \left(\frac{4(h-l) - n[b(h,h) - b(h,l)]}{f(h,l) - f(l,l)} + (1 - \alpha) m_1 \right)$
- *Mutually low investments* $\Leftrightarrow m_1 < \theta_l = \frac{1}{\alpha} \left(4 \frac{h-l}{f(h,l) - f(l,l)} + (1 - \alpha) m_2 \right)$
- *Large market domination* $\Leftrightarrow m_2 < \theta_h$ and $m_1 > \theta_l$

Proof. see appendix. ■

As a direct consequence of proposition 1, the large team never deviates from mutually high investment before the small team. So, the threshold θ_h is on the size of

the small club's market. The reverse is true for mutually low investments, such that θ_l constitutes a threshold on the large club's market size. Naturally, proposition 2 finds that higher investment costs ($h - l$) result in lower incentives to invest. Further, a larger media market (n) increases the probability of an equilibrium in mutually high investments.

More importantly, proposition 2 shows that increasing revenue sharing pushes up θ_h and θ_l , meaning that it reduces the incentives to invest in talent for both teams. Gate revenue sharing has two distinct effects on incentives. First, it effectively functions as a taxation on talent investments. Instead of being able to retain the revenues from winning, clubs see part of the gain flowing to their competitor. In response teams decline their talent investments.⁶ A second effect, which works in the same direction, is caused by clubs internalizing part of the externality they create through their talent investments.⁷ When revenue sharing is introduced, clubs obtain part of their revenue from their competitor's market. So, they have an incentive not to hurt their competitor's revenue by investing heavily in talented players. Increasing revenue sharing strengthens this tendency, but the effect is stronger for the small team, because the revenues it obtains from sharing are relatively more important. In my model this shows in the thresholds through the interaction term consisting of α and the size of the rival team's market size.

Assessing the effect of revenue sharing on competitive balance in the present model is not straightforward, as there are only three equilibrium outcomes. Playing strengths are either equal (mutually high or mutually low) or unequal (in large market domination). As such, the chance of ending up in large market domination may be interpreted as a measure of competitive (im)balance. Revenue sharing influences this probability in several ways. On the one hand, sharing enhances the likelihood of domination

⁶Feess and Stähler (2009) show the robustness of this "dulling effect" result in various previous models such as Szymanski and Késenne (2004) and Fort and Quirk (1995).

⁷Szymanski and Késenne (2004) label this the "free riding" effect. In their model, it is responsible for the result that sharing decreases competitive balance.

instead of mutually high investments. On the other hand, it decreases the probability of domination in favor of mutually low investments. In total, the effect of gate revenue sharing on competitive balance therefore depends on parameter values. The present model thus confirms the findings of Feess and Stähler (2009), who show that the balance effect of gate revenue sharing is indeterminate, when competitive balance, total playing quality and relative quality influence club revenues (as is the case in this model).

4 Setting a profitable sharing rule

In this section the focus is on determining the league's optimal talent investment outcome, again defining thresholds ($\lambda^{l,h}$, $\lambda^{u,h}$ and $\lambda^{u,l}$) on the model parameters. The superscripts indicate which situations are being compared, where h stands for mutually high investments, l for mutually low and u for large market domination.

Proposition 3 *Small market domination is never optimal for league profits. The league prefers*

- *mutually high investments when $\lambda^{l,h} \leq n$ and $\lambda^{u,h} \geq m_1 - m_2$*
- *mutually low investments when $\lambda^{l,h} \geq n$ and $\lambda^{u,l} \geq m_1 - m_2$*
- *large market domination when $\lambda^{u,l} \leq m_1 - m_2$ and $\lambda^{l,h} \geq n$ or $\lambda^{u,h} \leq m_1 - m_2$ and $\lambda^{l,h} \leq n$*

Proof. see appendix. ■

Proposition 3 shows that each talent investment outcome, except small market domination, optimizes leaguelike profits for certain ranges of the parameter values. If both home markets are sufficiently different, it is optimal for the large team to dominate the competition. Therefore the thresholds $\lambda^{u,h}$, $\lambda^{u,l}$ are defined on the difference

in home market size $m_1 - m_2$, where a larger difference implies a preference for unbalanced investments. This means that profit-maximizing leagues should not always try to enhance competitive balance. As the willingness-to-pay of hard-core fans is the same for mutually low and high investments, the choice between them depends on the size of the neutral consumer market. So, $\lambda^{l,h}$ describes a limit on the amount of neutral fans, n , where larger n implies mutually high investments are preferred.

In the framework of this model, the league's problem is to devise a sharing rule, which prevents talent investment outcomes from arising in equilibrium when they are not optimal and which stimulates their occurrence when they are preferable. Section 3 has shown that revenue sharing lowers talent investment incentives. As such, a rationale for positive gate revenue sharing exists, when talent investments are too high without sharing.

Proposition 4 *When no gate revenue sharing takes place ($\alpha = 1$)*

- *mutually high investments always arise when they are optimal for joint profits*
- *mutually low investments are always optimal for joint profits when they arise.*

Proof. see appendix. ■

Proposition 4 indicates that engaging in gate revenue sharing is a profitable strategy for the league. It provides a way to decrease talent investments to a more profitable level. The question remains what level of gate revenue sharing is optimal, because a high percentage of sharing might lead to underinvestment in talent. However, as long as the result of proposition 4 holds for a given level of revenue sharing α , the league should not be worried about underinvestment.

Corollary 1 *The results of proposition 4 are guaranteed to hold for all values of $\alpha > \frac{m_1}{m_1+m_2}$.*

Proof. see appendix. ■

Corollary 1 implies that on the interval $[1, \frac{m_1}{m_1+m_2}[$ any league is sure that increasing the amount of sharing raises profitability. So, a clear rationale is present to push back the percentage of revenues a team may keep (α) to $\frac{m_1}{m_1+m_2}$, i.e. the relative size of the large team's home market. In other words, leagues with more heterogeneous clubs have to share less revenues to be profitable. To grasp the intuition for this result, first think about the taxation effect of revenue sharing. When the large club has a relatively big market, a small percentage of revenue sharing imposes a large burden on its revenues. The taxation effect for the large team is therefore *ceteris paribus* more important when teams differ a lot. This in turn increases the risk of underinvestment in talent relative to the profit-maximizing solution. Second, the revenue from sharing is relatively more important for the small team in a heterogeneous league. The small club therefore internalizes the damage, which its investments bring to the large team's gate revenues, more quickly. This leads it to shrink its investments faster than would be the case in a homogeneous league. A somewhat counter-intuitive implication of corollary 1 is that leagues with a high chance of low competitive balance should select a low amount of gate revenue sharing. Consequently, sharing is more likely to be employed where the need to improve competitive balance is less. It is clearly hard to reconcile this finding with the defence of revenue sharing based on a competitive balance argument.

The results of corollary 1 provide a further explanation for the observations of table 1. The dp measure indicates that European soccer teams differ substantially in home market size. Consequently, gate revenue sharing is a rare phenomenon in these leagues.⁸ Rather than being a shortcoming of league authorities or stemming from a desire to comply with competition policy, this may simply constitute an example of profit-maximization. On the other side of the Atlantic, both the MLS, NFL and MLB

⁸In the EPL and Bundesliga, first division teams usually play less than ten cup games a year, meaning that sharing only applies to a small fraction of games.

force their teams to share local and gate revenues. Again, this may constitute a profit-maximizing strategy, especially since the most homogeneous league (NFL) shares more than both others. The NBA currently has no gate sharing, which is somewhat at odds with the results of the model, as it appears to have quite homogeneous markets. Yet, the NBA team owners are reportedly pushing to introduce gate sharing in the new collective bargaining agreement.⁹ Another puzzle is the AFL, which appears to have relatively homogeneous teams, but abolished gate sharing in 1999.

5 Concluding Remarks

This paper has analyzed how a sports league may use gate revenue sharing to coordinate talent investments. Revenue sharing depresses talent investments by all teams, which has an initial profit-enhancing effect. A further finding is that more unequal leagues have less to gain from high amounts of revenue sharing than more homogeneous leagues. This implies that revenue sharing persists more, when there is less chance of a low competitive balance.

6 References

1. Atkinson, S. E., Stanley, L. R. & Tschirhart, 1988. Revenue Sharing as an Incentive in an Agency Problem: an example from the National Football League. *RAND Journal of Economics*, Vol.19, No. 1, Spring 1988.
2. Booth, R., 2006. The economic development of the Australian Football League. In: Andreff, W. and Szymanski, S., 2006. *Handbook on the economics of sport*. Edward Elgar Publishing, Cheltenham, UK.
3. Easton, E. & Rokerbie, D., 2005. Revenue sharing, conjectures and scarce talent

⁹See Washington Post, June 29th 2011, "Lengthy NBA lockout looms, with owners and players deeply divided".

- in a sports league model, *Journal of Sports Economics*, vol.6 No. 4, November 2005, 359-378.
4. El Hodiri, M. & Quirk, J. 1971. An Economic Model of a Professional Sports League. *Journal of Political Economy*, 79, 1302-1319.
 5. Farmer, A. & Pecorino, P., 2010. Is the coach paid too much?: coaching salaries and the NCAA cartel. *Journal of Economics & Management Strategy*, Volume 19, Number 3, Fall 2010, 841-862.
 6. Feess, E. & Stähler, F., 2009. Revenue Sharing in Professional Sports Leagues. *Scottish Journal of Political Economy*, vol. 56, No. 2, May 2009, 255-265.
 7. Ferguson, D. G., Jones, J. C. H. & Stewart, K. G., 2000. Competition within a cartel: league conduct and team conduct in the market for baseball player services. *The Review of Economics and Statistics*, August 2000, 82(3): 422-430.
 8. Forrest, D., Simmons, R. & Buraimo, B., 2005. Outcome uncertainty and the couch potato audience. *Scottish Journal of Political Economy*, vol. 52, No. 4, September 2005, 641-661.
 9. Fort, R. & Quirk, J., 1995. Cross-subsidization, Incentives and Outcomes in Professional Team Sports Leagues, *Journal of Economic Literature*, 33, 1265-1299.
 10. Grossman, M., Dietl, H. & Lang, M., 2010. Revenue Sharing and Competitive Balance in a Dynamic Contest Model, *Review of Industrial Organization* (2010) 36:17–36.
 11. Kahn, L. M., 2007. Cartel Behavior and Amateurism in College Sports. *Journal of Economic Perspectives*. Vol. 21, No.1, Winter 2007, p.209-226.
 12. Palomino, F. & Sakovics, J., 2004. Inter-league competition for talent vs. competitive balance, *International Journal of Industrial Organization*, 22 (2004), p.

783-797.

13. Peeters, T., forthcoming. Media Revenue Sharing as a Coordination Device in Sports Leagues. *International Journal of Industrial Organization*, (2011), doi 10.1016/j.ijindorg.2011.07.004.
14. Szymanski, S., 2001. Income inequality, competitive balance and the attractiveness of team sports: some evidence and a natural experiment from English soccer, *Economic Journal*, 111(February), F69-F84.
15. Szymanski, S. & Késenne, S., 2004. Competitive balance and revenue sharing in team sports. *Journal of Industrial Economics*, LII/1, p. 165-177.

7 Appendix

In this appendix I provide proof of all propositions.

7.1 Proposition 1

Proof. From the profit function (6), it is clear that small market domination occurs in Nash equilibrium when both $\frac{n}{4}b(l, h) + \alpha\frac{m_1}{4}f(l, h) + (1 - \alpha)\frac{m_2}{4}f(h, l) - l \geq \frac{n}{4}b(h, h) + \alpha\frac{m_1}{4}f(h, h) + (1 - \alpha)\frac{m_2}{4}f(h, h) - h$ and $\frac{n}{4}b(l, h) + \alpha\frac{m_2}{4}f(h, l) + (1 - \alpha)\frac{m_1}{4}f(l, h) - h \geq \frac{n}{4}b(l, l) + \alpha\frac{m_2}{4}f(l, l) + (1 - \alpha)\frac{m_1}{4}f(l, l) - l$ are simultaneously satisfied. Rearranging terms yields $4(h - l) \geq n[b(h, h) - b(l, h)] + \alpha m_1 [f(h, h) - f(l, h)] + (1 - \alpha)m_2 [f(h, h) - f(h, l)]$ and $\alpha m_2 [f(h, l) - f(l, l)] + (1 - \alpha)m_1 [f(l, h) - f(l, l)] \geq 4(h - l)$. Equating both expressions and using (3) to simplify both sides gives $m_2 [f(h, l) - f(h, h)] \geq n[b(h, h) - b(l, h)] + m_1 [f(h, l) - f(h, h)]$ or $m_2 \geq n\frac{b(h, h) - b(l, h)}{f(h, l) - f(h, h)} + m_1$. As $m_2 < m_1$ and $n\frac{b(h, h) - b(l, h)}{f(h, l) - f(h, h)} > 0$, this is a contradiction. Therefore small market domination never prevails as a Nash equilibrium. ■

7.2 Proposition 2

Proof. Proposition 1 implies that deviating from mutually high investments is more interesting for the small club than for the large club. Club 2 deviates when $\frac{n}{4}b(h, h) + \alpha \frac{m_2}{4}f(h, h) + (1 - \alpha) \frac{m_1}{4}f(h, h) - h \geq \frac{n}{4}b(h, l) + \alpha \frac{m_2}{4}f(l, h) + (1 - \alpha) \frac{m_1}{4}f(h, l) - l$. Rearranging this expression and applying (3) shows immediately that $\theta_h = \frac{1}{\alpha} \left(\frac{4(h-l) - n[b(h, h) - b(h, l)]}{f(h, l) - f(l, l)} + (1 - \alpha) m_1 \right) \leq m_2$. Likewise, mutually low investments are a Nash-equilibrium when the large team has nothing to gain from deviating. This implies $\frac{n}{4}b(h, l) + \alpha \frac{m_1}{4}f(h, l) + (1 - \alpha) \frac{m_2}{4}f(l, h) - h \leq \frac{n}{4}b(l, l) + \alpha \frac{m_1}{4}f(l, l) + (1 - \alpha) \frac{m_2}{4}f(l, l) - l$. Using (2) and (3), allows to derive $m_1 \leq \frac{1}{\alpha} \left(4 \frac{h-l}{f(h, l) - f(l, l)} + (1 - \alpha) m_2 \right) = \theta_l$.

At both thresholds, clubs are indifferent between high and low investments. Therefore multiple equilibria are possible at these values. In order to have unique Nash-equilibria at all other values for the model parameters, it is necessary that both thresholds are never met simultaneously. Observe this is the case when $m_1 < \theta_l = \frac{1-\alpha}{\alpha} m_2 + 4 \frac{h-l}{\alpha(f(h, l) - f(l, l))}$ and $\frac{4(h-l) - n[b(h, h) - b(h, l)]}{\alpha(f(h, l) - f(l, l))} + \frac{1-\alpha}{\alpha} m_1 = \theta_h < m_2$ are not simultaneously true. This is guaranteed by $m_1 - m_2 \geq \frac{n(b(h, h) - b(l, l))}{f(h, l) - f(l, l)}$. To see this plug this into the second condition to get $\frac{4(h-l)}{\alpha(f(h, l) - f(l, l))} - \frac{m_1 - m_2}{\alpha} + \frac{1-\alpha}{\alpha} m_1 < m_2$. From which straightforward derivation shows $\frac{4(h-l)}{\alpha(f(h, l) - f(l, l))} + \frac{1-\alpha}{\alpha} m_2 < m_1$, which violates the first condition. ■

7.3 Proposition 3

Proof. League profits in each talent investment situation are given by:

$$\pi_L^{h, h} = \frac{1}{4} ((m_1 + m_2)f(h, h) + 2nb(h, h)) - 2h \quad (8)$$

$$\pi_L^{h, l} = \frac{1}{4} (m_1 f(h, l) + m_2 f(l, h) + 2nb(h, l)) - (h + l) \quad (9)$$

$$\pi_L^{l, h} = \frac{1}{4} (m_1 f(l, h) + m_2 f(h, l) + 2nb(l, h)) - (h + l) \quad (10)$$

$$\pi_L^{l, l} = \frac{1}{4} ((m_1 + m_2)f(l, l) + 2nb(l, l)) - 2l \quad (11)$$

Small market domination being optimal for league profits requires $\pi_L^{l,h} \geq \pi_L^{h,l}$. From (9) and (10) this may be filled out to get $\frac{1}{4}(m_1 f(l, h) + m_2 f(h, l) + 2nb(l, h)) - (h + l) \geq \frac{1}{4}(m_1 f(h, l) + m_2 f(l, h) + 2nb(h, l)) - (h + l)$. Simplifying both sides leads to $m_2 \geq m_1$, which is a contradiction. Therefore small market domination is never optimal for league profits. The derivation of the thresholds for the other outcomes proceeds along these lines:

- $t_1 = t_2 = h$ is optimal when $\pi_L^{h,h} \geq \pi_L^{h,l}$ and $\pi_L^{h,h} \geq \pi_L^{l,l}$. Filling this out using (8), (9) and (11) and rearranging terms gives $2n(b(h, h) - b(h, l)) - 4(h - l) \geq m_1(f(h, l) - f(h, h)) - m_2(f(h, h) - f(l, h))$ and $2nb(h, h) - 8(h - l) \geq 2nb(l, l)$. Imputing (3) and (2) leads to $\lambda^{u,h} = 2 \frac{n(b(h,h)-b(l,l))-2(h-l)}{f(h,l)-f(l,l)} \geq m_1 - m_2$ and $n \geq \frac{4(h-l)}{b(h,h)-b(l,l)} = \lambda^{l,h}$.
- $t_1 = t_2 = l$ is optimal when $\pi_L^{l,l} \geq \pi_L^{h,h}$ and $\pi_L^{l,l} \geq \pi_L^{h,l}$. From the reasoning above we know that the first condition implies $n \leq \frac{4(h-l)}{b(h,h)-b(l,l)} = \lambda^{l,h}$. The second condition may be rewritten using (9) and (11) to get $\frac{1}{4}((m_1 + m_2)f(l, l) + 2nb(l, l)) - 2l \geq \frac{1}{4}(m_1 f(h, l) + m_2 f(l, h) + 2nb(h, l)) - (h + l)$. Again from (3) I find that this is equivalent to $\lambda^{u,l} = \frac{4(h-l)}{f(h,l)-f(l,l)} \geq m_1 - m_2$.
- $t_1 = h, t_2 = l$ is optimal in all other cases, i.e. when $n \geq \lambda^{l,h}$ and $\lambda^{u,h} = \frac{2n(b(h,h)-b(h,l))-4(h-l)}{f(h,l)-f(l,l)} \leq m_1 - m_2$ or $n \leq \lambda^{l,h}$ and $\lambda^{u,l} = \frac{4(h-l)}{f(h,l)-f(l,l)} \leq m_1 - m_2$.

Observe also that $n = \frac{4(h-l)}{b(h,h)-b(l,l)} \Leftrightarrow \frac{4(h-l)}{f(h,l)+f(h,h)} = \frac{2n(b(h,h)-b(h,l))-4(h-l)}{f(h,l)-f(h,h)}$. So, all conditions converge at the point where the league is indifferent between mutually high and mutually low investments. ■

7.4 Proposition 4

Proof. The first statement of proposition 4 implies that for $\alpha = 0$

$$\left\{ \begin{array}{l} \lambda^{u,h} = 2^{\frac{n(b(h,h)-b(l,l))-2(h-l)}{f(h,l)-f(h,h)}} \geq m_1 - m_2 \\ \text{and} \\ n \geq \frac{4(h-l)}{b(h,h)-b(l,l)} = \lambda^{l,h} \end{array} \right\} \Rightarrow \theta_h = \frac{4(h-l) - n[b(h,h) - b(h,l)]}{f(h,l) - f(l,l)} < m_2 \quad (12)$$

Now, observe that $n \geq \frac{4(h-l)}{b(h,h)-b(l,l)} \Rightarrow 0 \geq 4(h-l) - n[b(h,h) - b(l,l)]$. From (3), $f(h,l) - f(l,l) > 0$, so I get that $\frac{4(h-l) - n[b(h,h) - b(l,l)]}{f(h,l) - f(l,l)} \leq 0 < m_2$, which provides proof of (12).

The second statement of proposition 4 means that for $\alpha = 0$

$$m_1 \leq 4 \frac{h-l}{f(h,l) - f(l,l)} = \theta_l \Rightarrow \left\{ \begin{array}{l} \lambda^{u,l} = \frac{4(h-l)}{f(h,l)+f(h,h)} \geq m_1 - m_2 \\ \text{and} \\ n \leq \frac{4(h-l)}{b(h,h)-b(l,l)} = \lambda^{l,h} \end{array} \right\} \quad (13)$$

First, for line 1 of (13) simply see that since $m_2 > 0$, $m_1 \leq 4 \frac{h-l}{f(h,l)-f(l,l)} \Rightarrow m_1 - m_2 \leq 4 \frac{h-l}{f(h,l)-f(l,l)}$. Then, for the second line of (13) remember that part 1 of this proof shows $n \geq \frac{4(h-l)}{b(h,h)-b(l,l)} \Rightarrow t_1 = h, t_2 = h$. Since mutually high and low investments mutually exclude each other's occurrence, this means that $n \geq \frac{4(h-l)}{b(h,h)-b(l,l)} \Rightarrow \sim(t_1 = l, t_2 = l)$. This in turn implies that $t_1 = l, t_2 = l \Rightarrow \sim\left(n \geq \frac{4(h-l)}{b(h,h)-b(l,l)}\right)$, which proofs line 2 of (13). ■

7.5 Corollary 1

Proof. To allow for any α (12) may be rewritten as

$$\left\{ \begin{array}{l} \lambda^{u,h} \geq m_1 - m_2 \\ \text{and} \\ n \geq \lambda^{l,h} \end{array} \right\} \Rightarrow \theta_h = \frac{1}{\alpha} \left(\frac{4(h-l) - n[b(h,h) - b(h,l)]}{f(h,l) - f(l,l)} + (1 - \alpha) m_1 \right) < m_2$$

Now observe again that $n \geq \frac{4(h-l)}{b(h,h)-b(h,l)} = \lambda^{l,h} \Rightarrow 4(h-l) - n[b(h,h) - b(h,l)] \leq 0$
 $\Rightarrow \frac{4(h-l)-n[b(h,h)-b(h,l)]}{f(h,l)-f(l,l)} + (1-\alpha)m_1 \leq (1-\alpha)m_1$. Since $\alpha \in [\frac{1}{2}, 1]$ this implies $\frac{1}{\alpha}$
 $\left(\frac{4(h-l)-n[b(h,h)-b(h,l)]}{f(h,l)-f(l,l)} + (1-\alpha)m_1 \right) \leq \frac{1-\alpha}{\alpha}m_1$, which leads to $\frac{1}{\alpha} \left(\frac{4(h-l)-n[b(h,h)-b(h,l)]}{f(h,l)-f(l,l)} + (1-\alpha)m_1 \right)$
 $< m_2 \Leftrightarrow \frac{1-\alpha}{\alpha}m_1 < m_2$. So, (12) may be guaranteed for positive values of α as long as
 $\frac{1-\alpha}{\alpha}m_1 < m_2$, which implies $\frac{m_1}{m_1+m_2} < \alpha$.

To show why the same condition on α also guarantees (13), first observe that the reasoning to show the second line of (13) is essentially unchanged. So the logic of the proof for proposition 4 still applies. To show the first line of (13) fill out θ_l for any α

$$\begin{aligned} \text{to get } m_1 &\leq \frac{1}{\alpha} \left((1-\alpha)m_2 + 4\frac{h-l}{f(h,l)-f(l,l)} \right) \\ &\Rightarrow m_1 - m_2 \leq \frac{1}{\alpha}4\frac{h-l}{f(h,l)-f(l,l)} + \frac{1-\alpha}{\alpha}m_2 - m_2 \\ &\Rightarrow (m_1 - m_2)\alpha \leq 4\frac{h-l}{f(h,l)-f(l,l)} + (1-\alpha)m_2 - \alpha m_2 \\ &\Rightarrow m_1 - m_2 \leq 4\frac{h-l}{f(h,l)-f(l,l)} + (1-2\alpha)m_2 + (1-\alpha)(m_1 - m_2) \\ &\Rightarrow m_1 - m_2 \leq 4\frac{h-l}{f(h,l)-f(l,l)} - \alpha(m_2 + m_1) + m_1 \end{aligned}$$

So, the first line of (13) holds as long as $m_1 - \alpha(m_2 + m_1) < 0$, which implies $\alpha > \frac{m_1}{m_1+m_2}$. ■

Table 1: Gate revenue sharing in sports leagues

Country	Sport	League	Gate Revenue Sharing	market size distribution (dp) ¹⁰
USA	American football	NFL	60-40 ¹¹	0.103
USA	baseball	MLB	69-31 ¹²	0.248
USA	basketball	NBA	no	0.101
USA	soccer	MLS	70-30	0.238
Germany	soccer	Bundesliga	cup games	0.403
UK	soccer	EPL	cup games	0.367
Spain	soccer	La Liga	no	0.615
Australia	Australian football	AFL	no	0.208

¹⁰As a proxy for the market size distribution I calculate the "dp" measure on the 2001-2010 seasons. This measure gives the average of the standard deviation of ten-year average attendances divided by the league average attendance on a season-by-season basis.

¹¹Since 2001 sharing is organized through a central pool.

¹²Since 1996 gate revenue sharing proceeds through a central pool. Percentages have varied over time, since 2007 31% of revenues are to be shared.