Procurement, Cost Reduction, and Vertical Integration[*]

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Abstract

We study a two-stage model of vertical integration that sheds new light on two important questions: Does vertical integration reduce procurement costs? Does it increase economic efficiency? In our model, a buyer who wants to procure an input of a given quality runs a first-price procurement auction. In the first stage, the competing suppliers make simultaneous investment decisions that reduce their expected costs of production. In stage two, each producer observes his cost realization and makes his bid. Without vertical integration, the buyer procures from the supplier who submits the lowest bid. Therefore, absent vertical this is a standard procurement auction augmented by a cost reducing investment stage. With vertical integration, the buyer has access to the production technology of one supplier and procures from a non-integrated supplier if and only if the lowest submitted bid is less than her own production cost. Whether or not the buyer is vertically integrated affects the investment decisions of all suppliers. If the problem of minimizing expected production cost is convex then non-integration is the efficient market structure. With vertical integration, the integrated supplier overinvests and non-integrated suppliers underinvest relative to first-best. If investments shift the mean of the cost distributions, a vertical merger decreases (increases) total investment if the marginal cost of investment is convex (concave). With an exponential cost distribution and quadratic investment costs, non-integration can be efficient but vertical integration is jointly profitable. If the cost distribution is uniform and investment cost is quadratic, vertical integration is efficient if absent integration the number of suppliers is two.

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1 Introduction

That vertical market structure matters for investment incentives is understood. Williamson (1985) argues that asset specificity, bounded rationality, and opportunism conspire to undermine efficient investments. Grossman and Hart (1986) echoes the sentiment by modeling how incomplete contracting causes a holdup problem that diminishes the investment incentive of the party lacking control rights. Bolton and Whinston (1993) add that vertical integration may cause investment distortions motivated by the pursuit of a bargaining advantage.

This paper revisits these issues by examining the consequences of vertical integration for investment in cost reduction in the context of a simple procurement model. The model features incomplete contracts in the sense that any transaction between a customer and an external supplier is determined by a reverse auction in which the supplier must bid the low price to win the supply contract. The model also features asset specificity by assuming that potential suppliers make relationship-specific investments in cost reduction before commencement of the auction. Vertical integration is modeled as a prior acquisition of a potential supplier who then becomes a preferred supplier. The preferred supplier has the option to produce after observing the bids of the external potential suppliers, and therefore elects to do so whenever its own cost is below the low bid. While internal sourcing enables the vertically integrated firm to avoid paying profits to external suppliers, the acquiring firm must compensate the acquisition target for the expected value of foregone profits. Furthermore, as an instance opportunism, vertical integration distorts the sourcing decision, which, in turn, also distorts investments in cost reduction. In particular, vertical integration leads external suppliers to underinvest in cost reduction in anticipation of sourcing distortions. Consequently, it is not clear *a priori* whether vertical integration is on balance an attractive strategy for reducing expected procurement costs.

In this procurement environment, the nonintegrated market structure results in socially efficient investments in cost reduction if diseconomies of investment are sufficiently pronounced and the variance of cost outcomes (given investments) is sufficiently great. In such circumstances, the distortions arising from vertical integration raise expected production costs. Nevertheless, a vertical acquisition may be a profitable strategy because it squeezes the profits of the remaining external suppliers. Indeed, for a special case of an exponential cost distributions and quadratic investment costs, we show that a vertical acquisition reduces expected procurement costs even though it compromises social welfare by increasing expected production cost. On the other hand, if the benefits of multiple potential suppliers are sufficiently small, then vertical integration may confer the social benefit of accomplishing an asymmetric pattern of investment that is more closely aligned with socially optimal cost reduction. We demonstrate this for the special case of a uniform cost distribution and quadratic cost of investment.

Our idea that vertical is motivated by reducing the profits of external suppliers is reminiscent of Bolton and Whinston (1993)'s idea that vertical integration is motivated by the creation of bargaining advantages. Bolton and Whinston (1993) considers how forward integration enables an upstream supplier to extract rents from downstream customers who make relationship-specific investments, whereas our model turns the incentives around to consider how backward integration extracts rents from upstream suppliers who make relationship specific investments in cost reduction. While the direction of vertical integration is mainly a matter of interpretation, there are other important differences between the models. First, the models make different assumptions about information and the market mechansim. The Bolton-Whinston (BW) model assumes complete information and assumes a particular bargaining process to allocate scarce supplies. In contrast, our model features incomplete information about cost reduction and assumes source selection via a first-price auction. Second, the logic of the distortions arising from vertical integration is different. In the BW model, the integrated downstream firm overinvests to create a more powerful outside option when bargaining with an independent custormer, and this investment distortion leads to distortions in the allocation of scarce supplies. In our model, vertical integration leads to sourcing distortions, which in turn lead to investment distortions. Thus the causal relationships between allocation and investment distortions are different.

Vertical integration in our model effectively establishes a preferred supplier, who serves to limit the market power of non-integrated suppliers. The integrated firm avoids giving away rents by allocating production to its upstream division whenever its cost is below the low bid. These allocation distortions from a preferred suppler are similar to those analyzed by Burguet and Perry (2009). Our model goes further by analyzing the consequences for investment in cost reduction. As result of endogenous investments, the preferred supplier has a more favorable cost distribution than the independent suppliers in our model, in contrast to the Burguet and Perry (BP) model which assumes identical cost distributions. Obviously, endogenous investments are an additional dimension along which to consider the consequences of a preferred supplier. We show quite generally that the integrated supplier overinvests in cost reduction and independent suppliers underinvest. For the special case in which investment shifts mean cost, we also provide conditions under which total investment is no larger with vertical integration. Under auch conditions, a vertical merger reduces and shifts investment away from nonintegrated suppliers toward the vertically integrated supplier, and may also reduce total investment if the marginal cost of investment increases too quickly. These investment distortions, in addition to the sourcing distortion from preferred supplier status, account for the social inefficiency of vertical integration when more symmetric investments by potential suppliers are cost minimizing.

2 General model

2.1 Basic setup

A downstream buyer procures a fixed input from an upstream industry consisting of $n \geq 2$ potential suppliers. The value of the final good to the buyer gross of the procurement cost is V. Supplier i makes a costly relationship-specific non-contractible investment x_i, which randomly determines the cost of production c_i according to a cumulative distribution function $F(c_i; x_i)$ with positive density $f(c_i; x_i)$ on its support. We assume the support of $F(c; x)$ is bounded below and let $\mu(x)$ denote the infinum of the support; that is, $F(c; x) = 0$ if $c \leq \mu(x)$.[1] For expositional convenience, we assume for now that that $V = \infty$ and the support is unbounded above for all x, and assume that $\lim_{c\to\infty} cF_x(c; x) = 0$, where $F_x(c; x) \equiv \frac{\partial F(c;x)}{\partial x}$. The cost of the investment is $\Psi(x_i)$. The buyer selects one of the potential suppliers to produce the input.[2]

The product technology for the input is described by $F(c_i; x_i)$ and $\Psi(x_i)$. Higher investment is assumed to shift the cost distribution smoothly according to first-order stochastic dominance,

[1] The support of $F(c; x)$ is extended on the real line in the usual manner, i.e. the support of $F(c)$ is $[c_o, c^o]$ then let $F(c; x) = 0$ for $c < c_o$ and $F(c; x) = 1$ for $c > c^o$.

[2] The implicit assumption justifying a first-price auction is $V = \infty$. Alternatively, if the support of support of $F(c; x)$ is bounded above, then V is above the supremum of the support for the relevant range of x.

and the cost of effort is assumed to be convex increasing and differentiable: $F_x(c;x) > 0$ for all c in the interior of the support of $F(c;x)$ and $\Psi'(x) \equiv \psi(x)$ is strictly positive and strictly increasing for all strictly positive x.

The analysis compares two modes of the procurement. In both modes, cost realizations are the private information of the suppliers. In the non-integrated mode the buyer is independent of the suppliers, and procures the input in a first-price reverse auction in which the suppliers bid a price and the buyer selects the low price supplier. In the vertically-integrated mode, the buyer is integrated with one of the suppliers, who becomes a preferred supplier, and obtains bids from each of the remaining suppliers in a reverse auction with a secret reserve price equal to the realized cost of the preferred supplier.

Vertical integration can be interpreted as forward integration by an upstream supplier to acquire the property rights of the downstream buyer. Suppose that the buyer has property rights over the technology to produce the downstream product with a value V gross of procurement costs for a required input. Each of the n upstream suppliers has property over a production technology for the required input. Vertical integration occurs when one of the upstream firms acquires the downstream production rights. Alternatively, and for our purposes equivalently, the buyer can be viewed as integrating backwards to acquire one of the upstream suppliers. On this interpretation, the buyer acquires to control rights to direct the investment of the acquired supplier and to observe its realized cost. There is an obvious question of why the buyer stops at only one acquisition. A possible answer is that competition laws prevent a consolidation of the upstream industry.[3] A systematic investigation of this competition policy issue, however, is beyond the scope of this paper.

2.2 Nonintegration

The timing of the game under nonintegration is as follows:

- Suppliers simultaneously choose investments x_i and observe costs c_i.
- Suppliers simultaneously submit bids b_i.
- The low-bid supplier, say i', produces the input and incurs cost $c_{i'}$.

The payoff of the buyer is $V - b_{i'}$, the payoff of the low-bid supplier is $b_{i'} - c_{i'} - \Psi(x_{i'})$, and the payoff of the others is $-\Psi(x_i)$. This might be thought of as an extensive form game in which suppliers choose investments in the first stage, and submit bids in the second stage. The appropriate equilibrium concept is subgame perfection. Since the investments are unobserved, the normal formal of the game has firms simultaneously choosing an investment and bidding strategy. We focus on symmetric equilibria, by which we mean Nash equilibria in which all firms choose the same investment level x^*, so that all firms draw their costs independently from the same distribution $F(c) \equiv F(c;x^*)$ and accordingly employ the same bidding function $b(c)$.

The structure of equilibrium bidding is well understood from auction theory. Consider the bidding incentives of a representative firm with cost realization c when rival bidders use an invertible bid strategy $b(c)$. A representative bidder chooses β to maximize $(\beta - c)[1 - F(b^{-1}(\beta))]^{n-1}$. Therefore, a symmetric equilibrium bidding strategy $b(c)$ is such that

$$c = \arg\max_z \left\{ [b(z) - c] \, [1 - F(c)]^{n-1} \right\}, \tag{1}$$

[3] Another possible explanation is that first-best may be achieved by vertically integrating with only one supplier, which may, for example, occur in the model with uniform distributions.

or

$$b(c) = c + \frac{\int_c^\infty [1 - F(z)]^{n-1} \, dz}{[1 - F(c)]^{n-1}}. \tag{2}$$

Note that $b(c)$ is an increasing function and is indeed invertible on the support $F(\cdot)$.[4]

Next consider equilibrium investment incentives. Even if a "deviant" firm had a different distribution of costs, the deviant would still follow the equilibrium bidding strategy if it expects its rivals to do so; similarly, rivals also would follow the equilibrium bidding strategy because the deviation is unobserved. Consequently, in considering conditions for a symmetric Nash equilibrium of the normal form game, it is enough to consider an isolated investment deviation.

Suppose that a representative firm were to deviate and choose $x^* + \varepsilon$ instead of x^*. The deviant would have cost distribution $G(c;\varepsilon) \equiv F(c; x^* + \varepsilon)$, but, as noted above, would continue to follow the same bidding strategy given by (2). Let $U(c) = (b(c) - c)(1 - F(c))^{n-1}$ be the expected payoff of a firm with cost c when placing the equilibrium bid $b(c)$. By the envelope theorem, $U'(c) = -[1 - F(c)]^{n-1}$. Therefore, using integration by parts, the deviant's expected profit gross of investment cost is

$$\begin{aligned} \Pi(\varepsilon) &= \int_{\mu(x^*+\varepsilon)}^{\infty} U(c) dG(c;\varepsilon) \\ &= \int_{\mu(x^*+\varepsilon)}^{\infty} [1 - F(c)]^{n-1} G(c;\varepsilon) dc \end{aligned} \tag{3}$$

and the derivative of $\Pi(\varepsilon)$ is

$$\Pi'(\varepsilon) \equiv \int_{\mu(x^*+\varepsilon)}^{\infty} [1 - F(c)]^{n-1} G_\varepsilon(c;\varepsilon) dc - [1 - F(\mu(x^* + \varepsilon))]^{n-1} \mu'(x^* + \varepsilon) g(\mu(x^* + \varepsilon);\varepsilon) \tag{4}$$

where $G_\varepsilon(c;\varepsilon) \equiv \frac{\partial G(c;\varepsilon)}{\partial \varepsilon}$ and $g(c;\varepsilon) \equiv \frac{\partial G(c;\varepsilon)}{\partial c}$. A necessary condition for a symmetric equilibrium is $\psi(x^*) = \Pi'(0)$, or, equivalently,

$$\psi(x^*) = \int_{\mu(x^*)}^{\infty} [1 - F(c; x^*)]^{n-1} F_x(c; x^*) dc - f(\mu(x^*); x^*) \mu'(x^*) \tag{5}$$

since $F(\mu(x^*)) = 0$. Therefore, for $c \geq \mu(x^*)$, the equilibrium bid function is

$$b(c) = c + \frac{\int_c^\infty [1 - F(z; x^*)]^{n-1} \, dz}{[1 - F(c; x^*)]^{n-1}} \tag{6}$$

For the rest of this section, we maintain the assumption that these equilibrium conditions are not only necessary but also sufficient for an equilibrium of the normal form game.

In a symmetric equilibrium, the low-cost firm wins the procurement auction. Thus, if all firms have the same investment x, then the realized production cost is determined by the distribution of the minimum order statistic for n independent draws from $F(c; x)$. The distribution of this minimum order statistic is

$$L(c; x, n) = 1 - [1 - F(c; x)]^n$$

[4]While this definition is strictly correct if the support of $F(\cdot)$ has no upper bound, it readily extends to the case of bounded support. If the supremum of the support of $F(\cdot)$ is ν, then it is convenient to define $b(c) = c$ on the extended support where $c \geq \nu$.

and the expected production cost is

$$C(x) \equiv \int_{\mu(x)}^{\infty} cdL(c; x, n). \tag{7}$$

Consequently, using integration by parts and $\lim_{c\to\infty} cF_x(c;x) = 0$, the cost reduction from a symmetric marginal increase in investment is

$$\begin{aligned} C'(x) &= -\int_{\mu(x)}^{\infty} L_x(c; x, n)dc - \mu'(x)l(\mu(x); x, n) \qquad (8)\\ &= -n\int_{\mu(x)}^{\infty} [1 - F(c; x)]^{n-1} F_x(c; x)dc - nf(\mu(x); x)\mu'(x) \end{aligned}$$

where $L_x(c; x, n) \equiv \frac{\partial L(c;x,n)}{\partial x}$ and $l(c; x, n) \equiv \frac{\partial L(c;x,n)}{\partial c}$. It follows from (5) and (8) that

$$\psi(x^*) = -\frac{1}{n}C'(x^*) \tag{9}$$

at a symmetric equilibrium. In other words, each firm fully internalizes the expected cost reduction from a marginal increase in its investment. The result is summarized as follows.

Proposition 1 *In equilibrium under non-integration, downstream investments minimize expected production plus effort costs, assuming that this minimum is achieved with symmetric investments.*

2.3 Vertical Integration

2.3.1 Model

Under vertical integration, the downstream buyer is vertically integrated with upstream firm 1, and the independent suppliers are labeled $i = 2, ...n$. The timing of the game is the same as for nonintegration, except at the last stage the low-bid independent firm ($i' \neq 1$) is selected to produce only if $b_{i'} < c_1$. Otherwise the vertically-integrated firm produces and incurs c_1. The payoff of the integrated firm is $V - \min\{b_{i'}, c_1\} - \Psi(x_1)$, the payoff of the low-bid independent firm is $b_{i'} - c_{i'} - \Psi(x_{i'})$ if selected and $-\Psi(x_{i'})$ otherwise, and the payoff of the others is $-\Psi(x_i)$. Since the integrated firm and independent firms are positioned asymmetrically, the analysis focuses on equilibria in which the integrated firm invests x_I and the independent firms symmetrically invest x_N.

Itis useful to distinguish the concepts of production cost and procurement cost. Production cost is the cost of actually producing the input, while procuremnt cost is the expense incurred by the buyer which may include a profi margin for the supplier. Under non-integration, the distinction is simple. The distribution of minimum production cost is $L(c; x^*, n)$, and the distribution of the price incurred by the buyer is $L(b^{-1}(b); x^*, n)$. Thus the probability that procurement cost is no more than $b(c)$ is also $L(c; x^*, n)$.

Under vertical integration, the distributions of production and of procurement costs are illustrated in Figure 1. On the vertical axis is the cost of the integrated firm c_I, and on the horizontal axis is the minimum cost draw of a nonintegrated firm c_N. The line above the 45-degree line is $b(c_N)$. The integrated firm procures from the lowest-cost independent supplier if

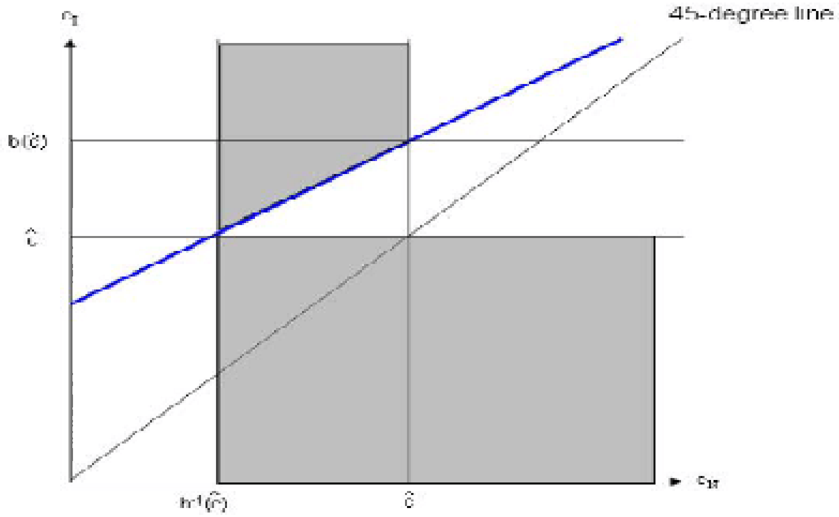

Figure 1: Distribution of production and of procurement costs

and only if $c_I > b(c_N)$. Consequently, the probability that realized procurement cost (excluding its investment cost $\Psi(x_I)$) is at least $b(\hat{c})$ is given by the probability mass in the rectangle to the northeast of the point $(\hat{c}, b(\hat{c}))$. This probability is

$$[1 - F(b(\hat{c}); x_I)][1 - L(\hat{c}; x_N, n-1)]$$

and consequently the probability that this cost is not more than $b(\hat{c})$ is

$$P(b(\hat{c}); x_N, x_I) = 1 - [1 - F(b(\hat{c}), x_I)][1 - L(\hat{c}; x_N, n-1).$$

On the other hand, the probability that actual production cost is less than $\hat{c}$ is given by the probability mass over the shaded area, which is given by $R(c; x_N, x_I)$. The actual production cost generally is higher than the minimum production cost because of the sourcing distortion.

The fist-order approach to equilibrium analysis proceeds similarly to the nonintegration case, except (a) there is one fewer non-integrated firm, (b) the upstream division of the integrated firm is a preferred supplier, and (c) there are different equilibrium investments for the integrated and nonintegrated suppliers.

2.3.2 Nonintegrated suppliers

Let $b(c)$ and x_N denote the symmetric equilibrium bid strategy and investment for nonintegrated suppliers, and x_I the investment of the integrated supplier. In a symmetric equilibrium, the expected profit of a nonintegrated firm bidding $b(c)$ with cost c is

$$U_N(c) = (b(c) - c)(1 - F(c; x_N))^{n-2}(1 - F(b(c); x_I)). \tag{10}$$

This equilibrium profit for a non-integrated firm reflects that the integrated firm will self supply if its cost is below the low bid $b(c)$. Invoking the revelation principle and the envelope theorem,

$$U_N'(c) = -(1 - F(c; x_N))^{n-2}(1 - F(b(c)); x_I). \tag{11}$$

and imposing the boundary condition

$$\lim_{c \to \infty} U_N(c) \to 0, \tag{12}$$

simple integration implies

$$U_N(c) = \int_c^\infty (1 - F(t; x_N))^{n-2}(1 - F(b(t)); x_I)dt. \tag{13}$$

From these relationships, $b(c) = c + \frac{\int_c^\infty (1-F(t;x_N))^{n-2}(1-F(b(t);x_I))dt}{(1-F(c;x_N))^{n-2}(1-F(b(c);x_I))}$.

If a nonintegrated supplier deviates and invests $x_N + \epsilon$, then the expected profit of the deviant is

$$\int_{\mu(x_N+\epsilon)}^\infty U_N(c)dF(c; x_N + \epsilon) \tag{14}$$

A representative non-integrated firm's investment problem is therefore

$$\max_\epsilon \int_{\mu(x_N+\epsilon)}^\infty U_N(c)dF(c; x_N + \epsilon) - \Psi(x_N + \epsilon), \tag{15}$$

yielding the equilibrium first order condition

$$\begin{aligned} \psi(x_N) &= \int_{\mu(x_N)}^\infty U_N(c)dF_{x_N}(c; x_N) - U_N(\mu(x_N))f(\mu(x_N); x_N)\mu'(x_N) \quad (16) \\ &= \int_{\mu(x_N)}^\infty (1 - F(c; x_N))^{n-2}(1 - F(b(c); x_I)F_{x_N}(c; x_N)dc \\ &= \frac{1}{n-1}\int_{\mu(x_N)}^\infty (1 - F(b(c); x_I)L_x(c; x, n-1)dc \end{aligned}$$

where the second equality follows from integration by parts and substitution of (**??**), and the third inequality is definitional.

The investment incentives of non-integrated frims can be understood alternatively with reference to the distribution of procurement cost. Since

$$P_{x_N}(b(c); x_N, x_I) = [1 - F(b(c); x_I)]L_x(c; x_N, n-1)] \tag{17}$$

for $c > \mu(x_N)$, in a symmetric equilibrium,

$$\begin{aligned} \psi(x_N) &= \int_{\mu(x_N)}^\infty P_{x_N}(b(c); x_N, x_I)dc \quad (18) \\ &= -\int_{\mu(x_N)}^\infty c dP_{x_N}(b(c); x_N, x_I). \end{aligned}$$

where the second equality follows from integration by parts.

This alternative characterization leads to the conclusion that noniintegrated firms underinvest in cost reduction. The distribution of actual production costs is

$$R(c; x_N, x_I) \equiv P(b(c); x_N, x_I) - \int_c^{b(c)} [1 - L(b^{-1}(t)); x_N, n-1)]dF(t; x_I). \tag{19}$$

Notice that $R(c; x_N, x_I)$ is the probability that the production cost is not greater than c. For $c \le \mu(x_N)$, the distribution of production cost is simply $R(c; x_N, x_I) = F(c; x_I)$. Therefore the expected cost of production in the integrated case is

$$\begin{aligned} C(x_N, x_I) &\equiv \int_{\mu(x_I)}^\infty c dR(c; x_N, x_I) \quad (20) \\ &= \int_{\mu(x_N)}^\infty c dR(c; x_N, x_I) + \int_{\mu(x_I)}^{\mu(x_N)} c dF(c; x_I) \quad (21) \end{aligned}$$

with

$$dR(c;x_N,x_I) = dP(b(c);x_N,x_I) - [1 - L(c;x_N,n-1)]dF(b(c);x_I) + [1 - L(b^{-1}(c);x_N,n-1)]dF(c;x_I) \tag{22}$$

for $c > \mu(x_N)$ and $dR(c;x_N,x_I) = dF(c;x_I)$ otherwise. This characterization leads to the conclusion that nonintegrated firms underinvest in cost reduction.

Proposition 2 *In equilibrium under vertical integration, non-integrated firms symmetrically invest less effort than if they minimized actual expected production plus effort costs.*

Proof. We prove the statement by showing that

$$\psi(x_N) = -\int_{\mu(x_N)}^{\infty} c dP_{x_N}(b(c);x_N,x_I) < -\int_{\mu(x_N)}^{\infty} c dR_{x_N}(c;x_N,x_I) = \frac{\partial C(x_N,x_I)}{\partial x_N} \tag{23}$$

The first equality follows from (18) and the second from (**??**), so we are left to establish the inequality. From (22) we get

$$dR_{x_N}(c;x_N,x_I) = dP_{x_N}(b(c);x_N,x_I) + L_{x_N}(c;x_N,n-1)]dF(b(c);x_I) - L_{x_N}(b^{-1}(c);x_N,n-1)dF(c;x_I). \tag{24}$$

Inserting this into (23) and canceling terms, the inequality in (23) is equivalent to

$$\int_{\mu(x_N)}^{\infty} cL_{x_N}(b^{-1}(c);x_N,n-1)dF(c;x_I) - \int_{\mu(x_N)}^{\infty} cL_{x_N}(c;x_N,n-1)]dF(b(c);x_I) \geq 0. \tag{25}$$

A change of variables reveals that the first integral is equal to

$$\int_{\mu(x_N)}^{\infty} b(c)L_{x_N}(c;x_N,n-1)dF(b(c);x_I). \tag{26}$$

Therefore, the inequality in (23) is equivalent to

$$\int_{\mu(x_N)}^{\infty} [b(c)-c]L_{x_N}(c;x_N,n-1)dF(b(c);x_I) \geq 0. \tag{27}$$

which follows because $b(c) > c$ and $L_{x_N}(c;x_N,n-1) \geq 0$. ■

Propositions 2 seems intuitive on the surface. Non-integrated suppliers are discouraged, at the margin, from exerting effort because they do not enjoy the benefits from investments in some of the instances when they are the low-cost potential supplier. This is because the integrated firm opportunistically sources internally to avoid paying profit margins to the nonintegrated suppliers. Note, however, that the definition of expected procurement cost already accounts for the sourcing decision. Thus non-integrated firms underinvest taking the sourcing rule as given. The reason is that a nonintegrated supplier does not fully internalize the benefit of reducing the sourcing distortion (by shifting the cost distribution downward) because of the monotonicity, i.e even though the independent supplier might beat the cost of the integrated supplier, its success is uncertain and the winning price is lower.

2.3.3 Integrated Supplier

The integrated supplier chooses x_I to minimize its expected procurement cost, which equals payments to independent suppliers plus production costs of self-supply plus the investment cost. Assuming $\mu(x_I) < b(\mu(x_N))$, a sufficient condition for which is $x_I \geq x_N$, expected procurement cost is given by

$$\Theta(x_I, x_N) \equiv \int_{\mu(x_N)}^{\infty} b(c) dP(b(c); x_N, x_I) + \int_{\mu(x_I)}^{b(\mu(x_N))} c dF(c; x_I) + \Psi(x_I) \tag{28}$$

and the first order condition for the integrated firm is given by

$$\begin{aligned} \psi(x_I) &= -\int_{\mu(x_N)}^{\infty} b(c) dP_{x_I}(b(c); x_N, x_I) \\ &\quad - \int_{\mu(x_I)}^{b(\mu(x_N))} c dF_{x_I}(c, x_I) + \mu'(x_I)\mu(x_I) f(\mu(x_I); x_I). \end{aligned} \tag{29}$$

Proposition 3 *In equilibrium under vertical integration, the integrated supplier invests more effort than if it minimized expected production plus effort costs.*

Proof. The partial derivative of expected production cost $C(x_I, x_N)$ as given in (**??**) with respect to x_I is

$$\begin{aligned} \frac{\partial C(x_I, x_N)}{\partial x_I} &= \int_{\mu(x_N)}^{\infty} c dR_{x_I}(c; x_I, x_N) + \int_{\mu(x_I)}^{\mu(x_N)} c dF_{x_I}(c; x_I) \\ &\quad - \mu'(x_I)\mu(x_I) f(\mu(x_I); x_I) \end{aligned} \tag{30}$$

Making use of the expression for $dR(c; x_I, x_N)$ in (22), this derivative can be written as

$$\begin{aligned} \frac{\partial C(x_I, x_N)}{\partial x_I} &= \int_{\mu(x_N)}^{\infty} c dP_{x_I}(b(c); x_I, x_N) - \int_{\mu(x_N)}^{\infty} c(1 - L(c; x_N, n-1)) dF_{x_I}(b(c); x_I) \\ &\quad + \int_{\mu(x_N)}^{\infty} c(1 - L(b^{-1}(c)); x_N, n-1)) dF_{x_I}(c; x_I) \\ &\quad + \int_{\mu(x_I)}^{\mu(x_N)} c dF_{x_I}(c; x_I) - \mu'(x_I)\mu(x_I) f(\mu(x_I); x_I). \end{aligned}$$

Re-write the term on the second line to get

$$\int_{b(\mu(x_N))}^{\infty} c(1 - L(b^{-1}(c), ; x_N, n-1)) dF_{x_I}(c; x_I) + \int_{\mu(x_N)}^{b(\mu(x_N))} c dF_{x_I}(c; x_I). \tag{31}$$

Substituting, $\frac{\partial C(x_I, x_N)}{\partial x_I}$ can now be rewritten as

$$\begin{aligned} \frac{\partial C(x_I, x_N)}{\partial x_I} &= \int_{\mu(x_N)}^{\infty} c dP_{x_I}(b(c); x_I, x_N) - \int_{\mu(x_N)}^{\infty} c(1 - L(c; x_N, n-1)) dF_{x_I}(b(c); x_I) \\ &\quad + \int_{b(\mu(x_N))}^{\infty} c(1 - L(b^{-1}(c); x_N, n-1)) dF_{x_I}(c; x_I) \\ &\quad + \int_{\mu(x_I)}^{b(\mu(x_N))} c dF_{x_I}(c; x_I) - \mu'(x_I)\mu(x_I) f(\mu(x_I); x_I). \end{aligned}$$

Using a change of variables,

$$\begin{aligned} &\int_{b(\mu(x_N))}^{\infty} c(1-L(b^{-1}(c);x_N,n-1))dF_{x_I}(c;x_I) \\ = &\int_{\mu(x_N)}^{\infty} b(c)(1-L(c;x_N,n-1))dF_{x_I}(b(c);x_I). \end{aligned} \tag{32}$$

Consequently,

$$\begin{aligned} \frac{\partial C(x_I,x_N)}{\partial x_I} = &\int_{\mu(x_N)}^{\infty} cdP_{x_I}(b(c);x_I,x_N) \\ &+\int_{\mu(x_N)}^{\infty}(b(c)-c)(1-L(c;x_N,n-1))dF_{x_I}(b(c);x_I) \\ &+\int_{\mu(x_I)}^{b(\mu(x_N))} cdF_{x_I}(c;x_I)-\mu'(x_I)\mu(x_I)f(\mu(x_I);x_I). \end{aligned} \tag{33}$$

Observe next that

$$\begin{aligned} \frac{\partial \Theta(x_I,x_N)}{\partial x_I} = &\int_{\mu(x_N)}^{\infty} b(c)dP_{x_I}(b(c);x_N,x_I) \\ &+\int_{\mu(x_I)}^{b(\mu(x_N))} cdF_{x_I}(c;x_I)-\mu'(x_I)\mu(x_I)f(\mu(x_I);x_I). \end{aligned} \tag{34}$$

Thus, $\frac{\partial\Theta(x_I,x_N)}{\partial x_I} \leq \frac{\partial C(x_I,x_N)}{\partial x_I}$ is equivalent to

$$\int_{\mu_N}^{\infty}(b(c)-c)dP_{x_I}(b(c);x_I,x_N) \leq \int_{\mu_N}^{\infty}(b(c)-c)(1-L(c;x_N,n-1))dF_{x_I}(b(c);x_I). \tag{35}$$

Since

$$dP_{x_I}(b(c);x_I,x_N) = (1-L(c;x_N,n-1))dF_{x_I}(b(c);x_I) - F_{x_I}(b(c);x_I)dL(c;x_N,n-1) \tag{36}$$

this inequality is equivalent to

$$0 \leq \int_{\mu_N}^{\infty}(b(c)-c)F_{x_I}(b(c);x_I)dL(c;x_N,n-1). \tag{37}$$

The right hand side is positive because $b(c)-c \geq 0$ and $F_{x_I}(b(c);x_I) \geq 0$. Thus, we have established $\frac{\partial\Theta(x_I,x_N)}{\partial x_I} \leq \frac{\partial C(x_I,x_N)}{\partial x_I}$, which is equivalent to $-\frac{\partial\Theta(x_I,x_N)}{\partial x_I} \geq -\frac{\partial C(x_I,x_N)}{\partial x_I}$. Since $\psi(.)$ is an increasing function and the equilibrium level of investment satisfies $\psi(x_I) = -\frac{\partial\Theta(x_I,x_N)}{\partial x_I}$, the proof is complete. ■

Proposition 3 is quite intuitive. As the integrated firm obtains the additional benefit of saving procurement costs $b(c)$ in some instances where it is not the lowest cost firm, it has an additional incentive to invest.

We note for reference in the next section that the integrated firm can be viewed equivalently as maximizing the procurement cost savings from self-supply. The gross procurement cost saving of an integrated supplier who invests x_I when non-integrated suppliers invest x_N is

$$\begin{aligned} &\int_{\mu(x_N)}^{\infty}\int_{\mu(x_I)}^{b(c_N)}[b(c_N)-c]\,dF(c;x_I)dL(c_N;x_N,n-1)dc \\ = &\int_{\mu(x_N)}^{\infty} K(b(c_N);x_I)dL(c_N;x_N,n-1) \end{aligned} \tag{38}$$

where

$$K(b;x)=\int_{\mu(x)}^{b} F(c;x)dc. \tag{39}$$

The equilibrium investment choice of the integrated supplier therefore can be viewed as balancing the marginal cost of investment to the marginal reduction in expected procurement cost:

$$\psi(x_I)=\int_{\mu(x_N)}^{\infty} K_x(b(c_N);x_I)dL(c_N;x_N,n-1) \tag{40}$$

where

$$K_x(b;x)=\int_{\mu(x)}^{b} F_x(z;x)dz. \tag{41}$$

It is straightforward to show that that this alternative characterization of equilibrium investment is equivalent to (29). This representation of investment incentives for the integrated supplier is useful for considering the shifting support model that follows.

3 Shifting Support Model

We now turn to a specialization of the the general model in which increases in investment effort maintain the shape of the cost distribution but shift its support downward; that is, $F(z;x)=F(z+x;0)$ and $\mu(x)=\mu_0-x$. For notational ease, we let $f(z+x)\equiv\frac{\partial F(z;x)}{\partial z}$. Notice that under the shifting support assumption we have $F_x(z;x)=f(z+x)$. It follows that

$$K_x(b;x_I)=\int_{\mu_0-x_I}^{b} F_x(z;x_I)dz=F(b+x_I;0). \tag{42}$$

Keeping the assumption that first-order conditions are necessary and sufficient, we have

$$\begin{aligned}\psi(x_N) &= \int_{-\infty}^{\infty}[1-F(c;x_N)]^{n-2}[1-F(b(c);x_I)]F_x(c;x_N)dc \\ &= \int_{-\infty}^{\infty}[1-F(c+x_N;0)]^{n-2}[1-F(b(c)+x_I;0)]f(c+x_N)dc \\ &= \frac{1}{n-1}\int_{-\infty}^{\infty}[1-F(b(c)+x_I;0)]dL(c;x_N,n-1)\end{aligned} \tag{43}$$

and

$$\psi(x_I)=\int_{-\infty}^{\infty} F(b(c)+x_I;0)dL(c;x_N,n-1). \tag{44}$$

Hence

$$(n-1)\psi(x_N)+\psi(x_I)=1. \tag{45}$$

This implies that the equilibrium aggregate effort depends on the shape of the effort cost function.

Proposition 4 *In the shifting support model, aggregate investment under vertical integration is the same, higher or lower than under non-integration if, for all $x\geq 0$, $\psi''(x)=0$, $\psi''(x)<0$ or $\psi''(x)>0$.*

Proof. Under nonintegration, equilibrium effort is given by $\psi(x^*) = \frac{1}{n}$. On the other hand, rewriting the consolidated equilibrium condition with vertical integration, (45), as $\frac{n-1}{n}\psi(x_N) + \frac{1}{n}\psi(x_I) = \frac{1}{n}$, it follows from Jensen's inequality that $(n-1)x_N + x_I = nx^*$ if $\psi'' = 0$ and $(n-1)x_N + x_I > (<)nx^*$ if $\psi'' < (>)0$. ■

Expected production costs are minimized under non-integratio, assuming a symmetric solution to the cost minimization problem. Outcomes under vertical integration depart from this benchmark in three important ways. First, if $\psi''(x) \neq 0$, then equilibrium aggregate effort is either too high or too low under vertical integration. Second, even assuming $\psi''(x) = 0$ so that aggregate investment is fixed, vertical integration equilibrium inefficiently shifts investment toward the integrated supplier. This misallocation not only increases expected production cost, but also the cost of effort because the marginal cost of effort is increasing. Third, the sourcing decision is distorted in favor of the vertically integrated firm. This sourcing biases increases expected production cost, even though the integrated firm is motivated to reduce procurement cost.

4 Exponential-quadratic model

4.1 Cost minimization

There are n potential suppliers whose costs are independent and identically distributed draws from an exponential distribution that shifts with investment:

$$F(c;x) = 1 - e^{-\lambda(c+x-k)} \tag{46}$$

Assuming symmetric investments, the minimum cost of production is distributed according to the minimum order statistic:

$$L(c,x,n) = 1 - e^{-\lambda n(c+x-k)} \tag{47}$$

The expected minimum production cost is therefore

$$\begin{aligned} C(x,n) &= \lambda n \int_{k-x}^{\infty} c e^{-\lambda n(c+x-k)} dc \\ &= \frac{1}{\lambda n} + k - x \end{aligned} \tag{48}$$

If in addition investment cost is quadratic, i.e.

$$\Psi(x) = \frac{1}{2}x^2 \tag{49}$$

then total expected cost is

$$C(x,n) + n\Psi(x) = \frac{1}{\lambda n} + k - x + \frac{n}{2}x^2 \tag{50}$$

and is minimized at $x = \frac{1}{n}$.

The more general statement of the cost minimization problem allows for asymmetric investments. Assume without loss of generality that $x_1 \geq x_{2.....} \geq x_n$. Then expected minimum production cost is:

$$\bar{C}(x_1,...x_n) = \lambda \left(\sum_{j=1}^{n-1} je^{-\lambda \sum_{h=1}^{j} x_h}\right) \int_{k-x_j}^{k-x_j+1} ce^{-j\lambda(c-k)}dc + n\lambda e^{-\lambda \sum_{h=1}^{n} x_h} \int_{k-x_n}^{\infty} ce^{-n\lambda(c-k)}dc. \quad (51)$$

For $\lambda < 1$, $\bar{C}(x_1,...x_n)$ is minimized at the symmetric solution $x = \frac{1}{n}$. This is easiest to see for $n = 2$, in which case the two first order conditions for a minimum are $\frac{\partial \bar{C}(x_1,x_2)}{\partial x_1} = -1 + \frac{1}{2}e^{-\lambda(x_1-x_2)} + x_1 = 0$ and $\frac{\partial \bar{C}(x_1,x_2)}{\partial x_1} = -\frac{1}{2}e^{-\lambda(x_1-x_2)} + x_2 = 0$. Subtracting the first from the second yields, the difference equation

$$1 - e^{-\lambda\Delta} = \Delta, \quad (52)$$

where $\Delta = x_1 - x_2$. Since $1 - e^{-\lambda\Delta}$ is concave in Δ and its slope is λ at $\Delta = 0$, it follows that $\Delta = 0$ is the unique solution for $\lambda < 1$. Plugging $\Delta = 0$ back into the first order conditions then gives the result. Though the argument is somewhat more complicated for $n > 2$, the basic idea generalizes directly to arbitrary n.

4.2 Nonintegration

The equilibrium bid for the exponential model with n symmetric bidders is a fixed markup on cost:

$$\begin{aligned} b(c) &= c + \frac{\int_c^{\infty} e^{-\lambda(n-1)(t+x-k)}dt}{e^{-\lambda(n-1)(c+x-k)}} \\ &= c + \frac{1}{\lambda(n-1)} \end{aligned} \quad (53)$$

If x is a candidate symmetric equilibrium investment and $\mu = k - x$, then a deviant bidder who increases investment by choosing $x + \varepsilon$ with $\varepsilon > 0$ earns an expected profit:[5]

$$\begin{aligned} \Pi(\varepsilon, x) &= \frac{1}{\lambda(n-1)} \int_{k-x}^{\infty} e^{-\lambda n(c+x-k)} \lambda e^{-\lambda(c+x+\varepsilon-k)}dc \\ &\quad + \int_{k-x-\varepsilon}^{k-x} \left[\frac{1}{\lambda(n-1)} + k - x - c\right] \lambda e^{-\lambda(c+x+\varepsilon-k)}dc \\ &\quad - \frac{1}{2}(x+\varepsilon)^2 \\ &= \frac{e^{-\lambda\varepsilon}}{\lambda}\frac{n-1}{n} - \frac{1}{\lambda}\frac{n-2}{n-1} + \varepsilon - \frac{1}{2}(x+\varepsilon)^2. \end{aligned} \quad (54)$$

[5]For following formulas are useful for this derivation:

$$\begin{aligned} \int_{\mu}^{\infty} ce^{-\lambda n(c-\mu)}dc &= \frac{1}{\lambda n}; \\ \int_{\mu-\varepsilon}^{\mu} e^{-\lambda(c-\mu)}dc &= -\frac{1}{\lambda} + \frac{1}{\lambda}e^{\lambda\varepsilon}; \\ \int_{\mu-\varepsilon}^{\mu} ce^{-\lambda(c-\mu)}dc &= -\frac{\mu}{\lambda} + \frac{\mu-\varepsilon}{\lambda}e^{\lambda\varepsilon} - \frac{1}{\lambda^2} + \frac{1}{\lambda^2}e^{\lambda\varepsilon}. \end{aligned}$$

The first partial derivative is

$$\frac{\partial \Pi(\varepsilon, x)}{\partial \varepsilon} = -e^{-\lambda\varepsilon}\frac{n-1}{n} + 1 - (x+\varepsilon), \tag{55}$$

which at $\varepsilon = 0$ if $x = \frac{1}{n}$. Therefore, in a symmetric equilibrium with n suppliers, total investment in the exponential-quadratic model is equal to unity.

It remains to consider second-order conditions for profit maximization to show that a symmetric equilibrium exists. The second partial derivative of the deviant's profit function (for any value of x) is

$$\frac{\partial^2 \Pi(\varepsilon, x)}{\partial \varepsilon^2} = \lambda e^{-\lambda\varepsilon}\frac{n-1}{n} - 1. \tag{56}$$

Since $e^{-\lambda\varepsilon}$ is a decreasing function of ε, $\frac{\partial^2 \Pi(\varepsilon,x)}{\partial \varepsilon^2} \leq \lambda\frac{n-1}{n} - 1$. This is nonpositive if and only if

$$\lambda \leq \frac{n}{n-1}. \tag{57}$$

Thus, condition (57) is necessary and sufficient for the profit function to be concave in an increase in effort ε starting from any candidate, symmetric equilibrium effort level. Alternatively, consider a decrease in investment of by $\varepsilon > 0$ from the conjectured symmetric equilibrium level $x = 1/n$; the profit function is

$$\begin{aligned} \Pi(-\varepsilon, \frac{1}{n}) &= \frac{1}{n-1}\int_{k-x+\varepsilon}^{\infty} e^{-\lambda n[(c+x-\varepsilon-k)]}dc - (x-\varepsilon)^2/2 \\ &= \frac{1}{\lambda}\frac{1}{n(n-1)}e^{-\lambda(n-1)\varepsilon} - \frac{1}{2}\left(\frac{1}{n} - \varepsilon\right)^2, \end{aligned} \tag{58}$$

which follows using similar arguments as above. The first partial derivative is

$$\frac{\partial \Pi(-\varepsilon, \frac{1}{n})}{\partial \varepsilon} = -\frac{1}{n}e^{-\lambda(n-1)\varepsilon} + \left(\frac{1}{n} - \varepsilon\right), \tag{59}$$

which is indeed 0 at $\varepsilon = 0$, and the second partial derivative is

$$\frac{\partial^2 \Pi(-\varepsilon, \frac{1}{n})}{\partial \varepsilon^2} = \frac{\lambda(n-1)}{n}e^{-\lambda(n-1)\varepsilon} - 1, \tag{60}$$

which is non-positive for all $\varepsilon \geq 0$ if and only if (57) is satisfied. Thus, (57) is necessary and sufficient for the existence of a unique symmetric equilibrium. In this equilibrium, $x = 1/n$.[6]

Proposition 5 *In the exponential-quadratic model, the socially efficient and unique symmetric equilibrium outcome under nonintegration is for each supplier to invest* $\frac{1}{n}$. *This equilibrium exists if and only if* $\lambda \leq \frac{n}{n-1}$.

The equilibrium expected procurement cost to the buyer under nonintegration equals the expected low bid:

$$\begin{aligned} \bar{P} &= \int_{k-x}^{\infty} b(c)dL(c,x,n) = \lambda n\int_{k-x}^{\infty} ce^{-\lambda n(c+x-k)}dc + \frac{1}{\lambda(n-1)} \\ &= k - \frac{1}{n} + \frac{1}{\lambda n} + \frac{1}{\lambda(n-1)} \\ &= k - \frac{1}{n} + \frac{1}{\lambda}\frac{2n-1}{n(n-1)} \end{aligned} \tag{61}$$

[6] Notice that x does not appear in any of the second partial derivatives; thus the profit function is globally concave in any symmetric equilibrium. Consequently, the symmetric equilibrium is unique, provided it exists.

Expected production cost, on the other hand, is

$$C(\frac{1}{n}, n) = \frac{1-\lambda}{\lambda}\frac{1}{n} + k \tag{62}$$

as for the planning problem. The expected profit of a representative supplier is

$$\bar{\Pi} = \frac{1}{\lambda n(n-1)} - \frac{1}{2}\frac{1}{n^2}. \tag{63}$$

4.3 Vertical Integration

4.3.1 Nonintegrated suppliers

Suppose now there is one integrated bidder I who invests x_I, and and $n-1$ nonintegrated suppliers who invest x_N. Conjecture that the nonintegrated suppliers use a fixed markup bid function $b(c) = c + \alpha$, where $\alpha > 0$ is the markup. Substituting the conjecture into the right-hand side of the equilibrium bid function confirms that $\alpha = \frac{1}{\lambda(n-1)}$:

$$\begin{aligned} b(c) &= c + \frac{\int_c^\infty (1-F(t;x_N))^{n-2}(1-F(b(t);x_I))dt}{(1-F(c;x_N))^{n-2}(1-F(b(c);x_I))} \\ &= c + \frac{\int_c^\infty e^{-\lambda(n-1)z}dx}{e^{-\lambda(n-1)c}} \\ &= c + \frac{1}{\lambda(n-1)}. \end{aligned} \tag{64}$$

Thus, $n-1$ independent suppliers under vertical integration use the same bid function as with n symmetric suppliers under nonintegration.

Consider first an independent supplier's investment problem given this fixed markup bid strategy. Let $\mu_N = k - x_N$ and $\mu_I = k-x_I$. Following a decrease of effort to $x_N - \varepsilon$, the representative independent supplier's profit is

$$\begin{aligned} \Pi_N(\varepsilon, \mu_N, \mu_I) &= \frac{1}{\lambda(n-1)}\int_{\mu_N+\varepsilon}^\infty e^{-\lambda(c-\mu_N)(n-2)}e^{-\lambda[c+\frac{1}{\lambda(n-1)}-\mu_I]}\lambda e^{-\lambda(c-\mu_N-\varepsilon)}dc - (k-\mu_I-\varepsilon)^2/2 \\ &= \frac{1}{n-1}\int_{\mu_N+\varepsilon}^\infty e^{-\lambda[(c-\mu_N)(n-1)-\varepsilon]-\lambda[c+\frac{1}{\lambda(n-1)}-\mu_I]}dc - (k-\mu_N-\varepsilon)^2/2. \end{aligned} \tag{65}$$

Letting $\Delta_\mu \equiv \mu_N - \mu_I$ (which is presumably positive), the partial derivative with respect to μ_N is

$$\frac{\partial \Pi_N}{\partial \varepsilon} = -\frac{1}{n-1}e^{-\lambda[\varepsilon(n-1)+\Delta_\mu]-\frac{1}{n-1}} + \frac{\lambda}{n-1}\int_{\mu_N+\varepsilon}^\infty e^{-\lambda[(c-\mu_N)n-\varepsilon+\Delta_\mu]-\frac{1}{n-1}}dc + (k-\mu_N-\varepsilon)$$

Letting $\varepsilon \to 0$, the first order condition becomes

$$\begin{aligned} x_N \equiv k - \mu_N &= \frac{1}{n-1}e^{-\lambda\Delta_\mu - \frac{1}{n-1}} - \frac{\lambda}{n-1}\int_{\mu_N}^\infty e^{-\lambda[(c-\mu_N)n+\Delta_\mu]-\frac{1}{n-1}}dc \\ &= \frac{1}{n-1}e^{-\lambda\Delta_\mu-\frac{1}{n-1}} - \frac{1}{(n-1)n}e^{-\lambda\Delta_\mu-\frac{1}{n-1}} \\ &= \frac{1}{n}e^{-\lambda\Delta_\mu-\frac{1}{n-1}} \end{aligned} \tag{66}$$

Thus, the investment of a nonintegrated supplier is a function of Δ_μ, which remains to be determined in equilibrium.

4.3.2 Integrated supplier

Consider now the integrated firm's investment problem. The integrated firm's benefit $B_I(\varepsilon)$ from increasing effort to $x_I + \varepsilon$ with $\varepsilon \geq 0$ is

$$B_I(\varepsilon,) = \int_{\mu_N}^{\infty} \int_{\mu_I - \varepsilon}^{b(c_N)} [b(c_N) - c_I] dF(c_I; x_I - \varepsilon) dL(c_N; x_N, n-1) - (k - \mu_I + \varepsilon)^2/2. \quad (67)$$

Neglecting the cost of effort for the moment, the expected profit of the buyer from increasing effort to $x_I + \varepsilon$ with $\varepsilon \geq 0$ is

$$\begin{aligned}
& \int_{\mu_N}^{\infty} \int_{\mu_I - \varepsilon}^{b(c_N)} [b(c_N) - c_I] dF(c_I; x_I - \varepsilon) dL(c_N; x_N, n-1) \quad (68) \\
= \; & \lambda^2(n-1) \int_{\mu_N}^{\infty} \int_{\mu_I - \varepsilon}^{c_N + \frac{1}{\lambda(n-1)}} \left[c_N + \frac{1}{\lambda(n-1)} - c_I \right] e^{-\lambda[c_I - \mu_I + \varepsilon]} e^{-\lambda(c_N - \mu_N)(n-1)} dc_I dc_N \\
= \; & \lambda(n-1) \int_{\mu_N}^{\infty} \left[c_N + \frac{1}{\lambda(n-1)} \right] e^{-\lambda(n-1)(c_N - \mu_N)} dc_N \\
& -\lambda(n-1) \int_{\mu_N}^{\infty} \left[\mu_I - \varepsilon + \frac{1}{\lambda} - \frac{1}{\lambda} e^{-\lambda[c_N + \frac{1}{\lambda(n-1)} - \mu_I + \varepsilon]} \right] e^{-\lambda(c_N - \mu_N)(n-1)} dc_N
\end{aligned}$$

Notice that the term in the second to last line is independent of ε. Thus, the integrated firm can be viewed as maximizing

$$\hat{B}_I(\varepsilon, \mu_N, \mu_I) = -\mu_N + \Delta_\mu + \varepsilon - \frac{1}{\lambda} + (n-1) \int_{\mu_N}^{\infty} e^{-\lambda[(c_N - \mu_N)n + \Delta_\mu + \frac{1}{\lambda(n-1)} + \varepsilon]} dc_N - (k - \mu_I + \varepsilon)^2/2. \quad (69)$$

The partial derivative with respect ot ε is

$$\begin{aligned}
\frac{\partial \hat{B}_I(\varepsilon, \mu_N, \mu_I)}{\partial \varepsilon} &= 1 - \lambda(n-1) \int_{\mu_N}^{\infty} e^{-\lambda[(c_N - \mu_N)n + \Delta_\mu + \frac{1}{\lambda(n-1)} + \varepsilon]} dc_N - (k - \mu_I + \varepsilon) \quad (70) \\
&= 1 - \frac{n-1}{n} e^{-\lambda \Delta_\mu - \frac{1}{n-1}} - (k - \mu_I + \varepsilon)
\end{aligned}$$

The first-order condition $\frac{\partial \hat{B}_I(0, \mu_N, \mu_I)}{\partial \varepsilon} = 0$ therefore implies

$$x_I = 1 - \frac{n-1}{n} e^{-\lambda \Delta_\mu - \frac{1}{n-1}}, \quad (71)$$

i.e. the integrated firm's investment also depends on equilibrium Δ_μ.

4.3.3 Equilibrium

Combining (66) and (71), we get

$$\Delta_\mu = 1 - e^{-\lambda \Delta_\mu - \frac{1}{n-1}} \quad (72)$$

as the equilibrium difference in effort levels by the integrated and non-integrated firms. The left hand side is trivially linear while the right hand side is increasing and concave in Δ_μ. At $\Delta_\mu = 0$ the left hand side is smaller than the right hand side while the converse is true at $\Delta_\mu = 1$. Thus, there is a unique $\Delta_\mu(\lambda, n)$ solving equation (72). Moreover, $0 < \Delta_\mu(\lambda, n) < 1$ will hold for any finite $n \geq 2$.

Summarizing, we have:

Proposition 6 *For $\lambda < \frac{n}{n-1}$ in the exponential-quadratic model, there is a unique equilibrium that is symmetric in the decisions of the non-integrated firms. The effort levels in this equilibrium are given by (66) and (71) with Δ_μ determined by (72).*

Uniqueness of the symmetric equilibrium follows from the uniqueness of $\Delta_\mu(\lambda, n)$.

Notice also that the aggregate effort level $x_I + (n-1)x_N = 1$ is constant, consistent with the more general the shifting support model with quadratic effort cost. Furthermore, a higher value of Δ_μ shifts investment toward the vertically integrated. This occurs for higher values of λ and lower values of n:

$$\frac{\partial \Delta_\mu}{\partial \lambda} = \frac{1-\Delta_\mu}{1-\lambda(1-\Delta_\mu)} > 0 \tag{73}$$

and

$$\frac{\partial \Delta_\mu}{\partial n} = -\frac{1}{(n-1)^2}\frac{\partial \Delta_\mu}{\partial \lambda} < 0. \tag{74}$$

These partial derivatives are readily established. Their sign depends on the fact that $1-\lambda(1-\Delta_\mu) > 0$ which is equivalent to $\lambda < \frac{1}{1-\Delta_\mu}$. Since we assume $\lambda < \frac{n}{n-1}$, the left hand side is not bigger than $\frac{n}{n-1}$ while the right hand side is no less than $e^{1/(n-1)}$ since $\Delta_\mu \geq 0$. To see that $\frac{n}{n-1} < e^{1/(n-1)}$ for any finite $n \geq 2$ notice first that this holds at $n = 2$. Since in the limit as $n \to \infty$ the two expressions are both 1 while the derivative of $e^{1/(n-1)}$ is always less than the derivative of $\frac{n}{n-1}$ the result follows.

The expected procurement cost of the vertically integrated firm equals the expected price paid to the independent suppliers, plus the expected production cost of self supply, plus the investment cost of the integrated supplier. The expected price paid to independent suppliers is

$$\begin{aligned} & \int_{\mu_N}^{\infty}\int_{b(c_N)}^{\infty} b(c_N)dF(c_I;x_I)dL(c_N;x_N,n-1) \qquad (75)\\ = & \int_{\mu_N}^{\infty} b(c_N)[1-F(b(c_N);x_I)]dL(c_N;x_N,n-1)\\ = & \int_{\mu_N}^{\infty}\left\{\left[c_N + \frac{1}{\lambda(n-1)}\right]e^{-\lambda(c_N+1/(\lambda(n-1))-\mu_I)}\right\}dL(c_N;x_N,n-1) \end{aligned}$$

and the expected production cost of self supply is

$$\begin{aligned} & \int_{\mu_N}^{\infty}\int_{\mu_I}^{b(c_N)} c_I dF(c_I;x_I)dL(c_N;x_N,n-1) \qquad (76)\\ = & \int_{\mu_N}^{\infty}\left\{\mu_I + \frac{1}{\lambda} - \left[c_N + \frac{1}{\lambda(n-1)} + \frac{1}{\lambda}\right]e^{-\lambda(c_N+1/(\lambda(n-1))-\mu_I)}\right\}dL(c_N;x_N,n-1) \end{aligned}$$

Adding these two expressions and cancellation of terms yields a simplified expression:

$$\begin{aligned} & \int_{\mu_N}^{\infty}\left[\int_{b(c_N)}^{\infty} b(c_N)dF(c_I;x_I) + \int_{\mu_I}^{b(c_N)} c_I dF(c_I;x_I)\right]dL(c_N;x_N,n-1)\\ = & \int_{\mu_N}^{\infty}\left\{\mu_I + \frac{1}{\lambda} - \frac{1}{\lambda}e^{-\lambda(c_N+1/(\lambda(n-1))-\mu_I)}\right\}\lambda(n-1)e^{-\lambda(c_N-\mu_N)(n-1)}dc_N\\ = & \mu_I + \frac{1}{\lambda} - \frac{1}{\lambda}\frac{n-1}{n}e^{-\lambda\Delta_\mu - 1/(n-1)}\\ = & k + \frac{1-\lambda}{\lambda}x_I. \end{aligned}$$

Finally, adding in the investment cost, the total expected procurement cost under vertical integration is

$$\bar{P}_{INT} = k + \frac{1-\lambda}{\lambda}x_I + \frac{1}{2}x_I^2 \tag{77}$$

where x_I is determined according to Proposition 6.

4.3.4 Incentive for vertical integration

Finally, we turn to analyzing the incentive for vertical integration in the exponential-quadratic model. Toward that end, we interpret the expected costs and profits under alternative market structures as determining the reduced form payoff of an acquisition game. In the the acquisition game, the downstream firm (buyer) sequentially makes take-it-or-leave-it lump-sum offers to the n downstream firms (suppliers). After receiving an offer, a supplier accepts or rejects. If any supplier accepts an offer, then the acquisition game ends and the resulting market structure is vertical integration. If all suppliers reject, then the resulting market structure is nonintegration. Each supplier's objective is to maximize expected profits. The buyer's objective is to maximize procurement cost savings (relative to the nonintegration equilibrium) minus the acquisition price. It is straightforward that in equilibrium the buyer offers a supplier an acquisition price equal to the expected profit under nonintegration and the supplier accepts if and only if this outcome is profitable for the buyer; otherwise, the buyer makes nonserious offers that all suppliers reject. Therefore, the equilibrium outcome is vertical integration if and only if procurement cost savings from vertical integration exceed supplier profit under nonintegration.

Formally, vertical integration is an equilibrium outcome of the acquisition game if $\bar{\Pi} \leq \bar{P} - \bar{P}_{INT}$. Substituting the outcomes for the exponential quadratic model, this condition is equivalent to and vertical integration is the equilibrium outcome of the acquisition game if

$$\Phi(\lambda, n) \equiv [\frac{2n-1}{\lambda n(n-1)} - \frac{1}{n}] - [\frac{1-\lambda}{\lambda}x_I + \frac{1}{2}x_I^2] - [\frac{1}{\lambda n(n-1)} - \frac{1}{2}\frac{1}{n^2}] \geq 0 \tag{78}$$

where x_I is determined by (71) and (72) as a function of λ and n. Note that the parameter k cancels on the left-hand side of this inequality. Since there is no closed form solution for x_I, the condition is evaluated numerically imposing the parameter restriction $\lambda < \frac{n}{n-1}$.

Figure 2 evaluates $\Phi(\lambda, n)$ for different values of λ and n. It shows that vertical integration generally is profitable.

The complete intuition for the result that vertical integration is always profitable in the exponential case remains to be developed. The following factors seem important. In the shifting support model with quadratic effort cost aggregate investment is the same with and without vertical integration (Proposition 4). Since the exponential distribution has a constant hazard rate, this implies that the expected cost of production is the same on the common support.[7]

[7] Denote by $L_\varepsilon(c) = 1 - (1 - F(c + e_N - \varepsilon/(n-1)))^{n-1}(1 - F(c + e_I + \varepsilon))$ the distribution of the minimum order statistic. So

$$\begin{aligned} dL_\varepsilon(c)/d\varepsilon\,|_{\varepsilon=0} &= f(c+e_I)[1-F(c+e_N)]^{n-1} - f(c+e_N)(1-F(c+e_N))^{n-2}(1-F(c+e_I)) > 0 \\ &\Leftrightarrow \frac{f(c+e_N)}{1-F(c+e_N)} < \frac{f(c+e_I)}{1-F(c+e_I)}, \end{aligned}$$

which is a monotone hazard rate condition, i.e. if the hazard rate is increasing, then $L_\varepsilon(c) > L_0(c)$ for $\varepsilon > 0$. So this redistribution will always reduce the expected lowest cost. By the same token, the expected lowest cost will not be affected if the distribution is exponential because it has a constant hazard rate of $1/\lambda$.

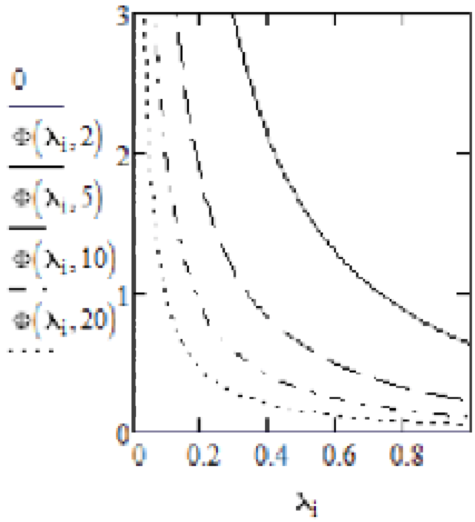

Figure 2: $\Phi(\lambda, n)$ for different values of λ and n.

Because the additional investment of the integrated firm shifts the support downwards, expected production cost falls. On top of that, the integrated firm self-sources (inefficiently) in some instances, thereby reducing its procurement cost compared to the case without vertical integration. The downside to vertical integration for the vertically integrated firm is that its effort cost increases. Notice that revealed preferences arguments cannot be applied directly here: Though it is true that it could keep its investment at the pre-integration level but chooses not to do so, the other firms reduce their investments, and so all we can conclude is that, given that the other firms reduce their investments, the integrated buyer prefers slightly more to less investment, but this does not allow us to conclude that it is better off with integration.

5 Uniform-quadratic model (in preparation)

TBD: Uniform 'counter'example: $n = 2$, quadratic cost with $a = 1$: The socially efficient investment is $x_1 = 1$ and $x_2 = 0$. Without integration, there is no equilibrium in which first-best is achieved. However, with vertical integration such an equilibrium exists. Thus, vertical integration can be beneficial and an equilibrium outcome of the acquisition game.

6 Conclusion

In a procurement environment in which cost minimization requires equal investments by symmetric suppliers, a vertical acquisition raises expected costs by distorting sourcing and investment in favor of the integrated supplier. Additionally, in an environment in which investment shifts expected costs and the marginal cost of investment rises quickly enough, such vertical integration also reduces total investment in cost reduction. Nevertheless, despite the cost inefficiency, there are strong private incentives for vertical integration to reduce expected procurement cost. In contrast, when cost minimization requires asymmetric investment levels, vertical integration can be more efficient than non-integration and may arise endogenously as the outcome of an acquisition game.

7 References

Patrick Bolton and Michael Whinston (1993), "Incomplete Contracts, Vertical Integration, and Supply Assurance," *Review of Economic Studies*, 60, 121-148.

Roberto Burguet and Martin Perry (2009), "Preferred Suppliers in Auction Markets,: *RAND Journal of Economics*, 40, 283-95.

Sanford Grossman and Oliver Hart (1986), "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy*, 94:691-719.

Oliver Williamson (1985), *The Economic Institutions of Capitalism*, The Free Press.