Estimating Market Power with a Generalized Supply Relation: Application to an Airline Antitrust Case

Alessandro V. M. Oliveira[*]

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Abstract

The empirical assessment of market power using conduct parameter models has the weaknesses of lacking proper grounding in oligopoly theory and typically producing inconsistent supply-side estimates. This paper develops an alternative framework for evaluating market power with a freely estimated parameter. This model is based on a marginal profits ratio that balances the marginal profit obtained from collusion with the marginal profit from best-response behavior in a repeated game. It builds upon the concept of efficient collusion, and also allows for a set of possible penal codes for deviation punishment and profits persistence. The resulting generalized supply relation has the advantage of not imposing a particular static noncooperative equilibrium - a strong restriction that is commonly found in the literature. The model nests important benchmarks of oligopoly theory as special cases, with convenient expressions for homogeneous or differentiated, single or multi-product, price or quantity competition settings. As the proposal is particularly suitable for antitrust investigations, it is applied to a price-fixing case from the airline industry, with the hypothesis of coordinated market power not being rejected by the data. However, the alleged price parallelism between carriers did not significantly shift market outcome in the investigated period.

Key words: conduct parameter, supply relations, market power, airlines.

JEL Classification: L0, L4, L93, C1.

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[*] Latin American Center for Transportation Economics. Aeronautical Institute of Technology, Brazil. Homepage: www.nectar.ita.br/oliveira. Email: alessandro.oliveira@pq.cnpq.br. The author wishes to thank Fapesp, CAPES and CNPq for financial support, Michael Waterson, Jeremy Toner, Margaret Slade, Peter Mackie, Eliseu Lucena, Flávio Mendes, Paulo Ivo Queiroz, Carlos Müller, Frode Steen, José Maria Silveira, Elizabeth Farina, Rogério Coimbra, Ronei Glanzmann, Cristian dos Reis, Wellington Costa, Humberto Bettini, Moisés Vassallo, Lucia Helena Salgado. Special thanks to Elena Oliveira, Rachel Oliveira and Maria Clara Oliveira.

1 Introduction

Economists have been intrigued by the formation of supra-competitive prices since the early stages of industrial organization. The situation of a firm exercising market power, thus setting a price above its marginal cost, was soon conceived as stemming from either a monopolistic or an oligopolistic market structure and, therefore, was regarded as harmful to economic welfare. Subsequent study of the behavior of firms in situations of few rivals was leveraged by the advent of modern techniques of game theory and, on the empirical side, of econometrics. Even with these notable developments, however, economists have yet to reach agreement upon a way to empirically investigate the competitive conduct of firms. The "competitive conduct" term usually suggests myriad possible situations between the extremes of perfect competition and monopoly, and within which one may find any oligopoly under consideration. The closest we came to a consensus in the empirical study of competitive conduct was with the framework of "conjectural variations", also known as "conduct parameter models", CPMs. In a typical CPM approach, the researcher assumes a set of market outcomes nested within a single behavioral equation: a supply relation derived from a first-order condition for profit maximization. Moreover, the researcher employs instrumental variables methods to econometrically identify a conduct parameter that allows hypotheses tests of the alternative market outcomes. This approach was extremely popular throughout the 1980s due to its simplicity and apparent close connection to oligopoly theory. Additionally, from an antitrust standpoint, the conduct parameter method is straightforwardly applicable.

The popularity of this approach culminated with the review of Bresnahan (1989). In that survey, the author coined the expression "new empirical industrial organization" (NEIO) to designate the whole literature that employed conduct parameter models. This literature was also characterized by the hypothesis of non-observability of the costs side and the suspicion of estimates from cross-section regressions across industries. More recently, Einav and Levin (2010) suggested that the advances in empirical IO have shifted the standards for empirical evidence in merger reviews and antitrust litigation. Paradoxically, the review of Bresnahan also marked the beginning of the decay of NEIO's early approach, more focused on the CPM method. Today, one of CPM's recognized weaknesses is its lack of proper grounding in oligopoly theory, which was initially one of its apparent benefits. Through a conjectural variations framework, the researcher could have only a few micro-founded values assumed by the estimated parameter: the outcomes of Bertrand, Cournot and cartel. However, there is no theoretical justification for the unpleasant, but typical, situation when the estimated parameter lies within those references. In fact, the IO literature has not yet developed an equilibrium concept that supports a conduct parameter estimated "between Cournot and cartel" or "between Bertrand and Cournot".[1]

In addition to the lack of theoretical background, other criticisms have contributed to the decay of the CPM approach. Genesove and Mullin (1998), for example, suggest that the breakdown requires strong assumptions on the functional form of demand (e.g. linearity),

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[1] There are some recent attempts of developing a theoretical basis for conjectural variations, based on the concept of varying competitive toughness. See d'Aspremont and Dos Santos Ferreira (2009).

which ultimately generates even stronger restrictions on the pass-through of costs to prices, thus invalidating inferences on competitive conduct when the particular adopted demand model is not true. This has been addressed by the notable evolution of the literature on discrete-choice models in empirical IO. Huang, Rojas and Bass (2008) investigate the biases of assuming a misspecified demand model with favorable results to the logit model, considering the case when the true model was not logit.

An even stronger criticism of the CPM approach is the so-called Corts critique (Corts, 1999). Corts argues that the typical CPM framework does not resist the insertion of simple elements of repeated interaction among firms. In particular, by modeling the behavior of a grim trigger cartel with space-state variables, Corts finds that, in most cases, the estimated conduct parameter is biased. In addition, because, in general, dynamic and repeated-game models are usually considered more realistic representations of the market behavior of firms, he highlights a serious under-specification problem of conjectural variations with a potentially large bias in the estimation of the conduct parameter. This bias leads the researcher to reject the hypothesis of collusion more frequently, even if a cartel outcome is observed in the data. Recently, Puller (2009) addressed the Corts Critique with a general empirical model robust to efficient collusion and that allowed for consistent estimation of the conduct parameter in a homogeneous-product setting. The resulting estimating equation makes use of time fixed effects to condition out unobservables that are common to all firms and related to the incentive-compatibitility constraint of the cartel.

In short, as a result of the criticism from the IO literature over the past two decades, the practice of estimating the competitive conduct as a free parameter in a supply relation regression was virtually abolished in mainstream literature. Applied researchers still lack a theoretical framework that unites simplicity, direct econometric tract, close connection with modern oligopoly theory and flexibility enough to identify different non-cooperative and cooperative equilibrium concepts from the data.

The modeling approach that currently prevails in the empirical IO literature is that of imposing the restriction of a particular oligopoly model equilibrium solution - typically Bertrand-Nash with price competition and product differentiation - both as a moment condition in a GMM estimation of demand and supply and as a way to identify unobserved marginal costs of firms. Through this approach, which is still far from the popularity of the past achieved by conjectural variations, the current literature makes use of an inversion of the first-order conditions to solve for marginal costs in terms of demand parameters. Berry and Haile (2010) demonstrate that this invertibility condition is satisfied for a set of standard oligopoly models. Another strand of the literature is the so-called menu approach for selecting oligopoly models. With this approach the researcher must run many regression models applying restrictions in the parameters. These restrictions stem from oligopoly theory among a set of plausible models for the case under analysis. For example, if the only two options in a duopoly with homogeneous product are the Cournot or cartel outcomes, then both models must be regressed, each with its own restrictions, and the data will then select the model with the best performance on non-nested hypotheses tests (Gasmi, Laffont and Vuong, 1990).[2] It is important to note

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[2] One recent application of this approach is Salvo (2010).

that neither of the above approaches addresses the Corts critique.

Einav and Levin (2010) note that many of the early NEIO studies attempted to distinguish the unilateral exercise of market power from collusive alternatives. With the recent focus on imposing an assumption of a particular static equilibrium, however, the amount of papers dedicated to collusion has declined considerably. Ciliberto and Williams (2010) and Orcholski (2011), for example, are among the few recent papers addressing market power estimation using conduct parameters aimed at identifying collusion. Ciliberto and Williams (2010) use conduct parameters to assess the impact of multimarket contact and the mutual forbearance hypothesis in the airline industry. They assume a Bertrand-Nash pricing game to develop the first-order condition for profit maximization as a typical CPM study. However, they use a multimarket contact measure plugged into an additional cooperative term in the first-order condition. They aim at using the first-order conditions to estimate both the multimarket-related conduct parameters and the marginal costs functions of firms. Orcholski (2011) uses a reduced-form profit function to identify conjectural parameters of firms and, therefore, to assess the degree of competition in the market. The author also has an application to the US airline industry.

The present paper proposes an alternative approach for studying the competitive conduct of firms in an oligopoly and, consequently, for investigating potential anti-competitive practices. I develop an empirical framework derived from a *marginal profits ratio* to model the relation between of the marginal profits from collusion and the marginal profit from deviation in a context of a repeated game. In this situation, we have efficient collusion in the market, as in Corts (1999) and Puller (2009), and therefore the cartel applies a penal code in such a way that firms have no incentive to defect. Whereas I apply a stick-and-carrot model (Abreu, 1988), a grim-trigger behavior is also accommodated within the framework. The framework builds upon the developments of Bernheim and Whinston (1990), who find that optimal collusion in a situation of multimarket contact produce the equalization of the marginal profits ratios across markets. Here I do not focus on multimarket contact but have the marginal profits ratio concept as a way of nesting different collusive solutions. The model allows for the case of endogenously determined punishments, that is, punishment levels associated with best response profits, and also for the simpler case of exogenous punishments. The marginal profits ratio approach results in a single empirical pricing equation - a generalized supply relation, GSR - that permits nesting alternative oligopoly outcomes in the same spirit of the conduct parameter approach. Consequently, the estimation of several alternative models as with the menu approach, or the imposition of a particular equilibrium concept, as with standard empirical IO papers, is unnecessary.

The proposed model has the following three characteristics. 1) It does not impose restrictions on demand, that is, it can accommodate any demand specification considered and previously estimated by the researcher. 2) It has a direct connection with oligopoly theory by nesting a wide range of possible games under assumptions of homogeneous or heterogeneous product with single product or multiproduct firms and either price- or quantity- competition. 3) It addresses the Corts critique as in Puller (2009), as the marginal profits ratio is a direct result of the application of an efficient collusion concept with a stochastic process in profits and therefore, it does not incur a biased estimation from not controlling for the repeated interaction of firms in the market. The model of Puller (2009) is a special case nested by the proposed framework.

The proposed approach permits a simple decomposition of the market power of firms in an oligopoly into two main components, after controlling for cost shifters: *unilateral market power* - the price premium arising from the independent behavior of prices by firms, such as pricing derived from the lower own-price elasticity caused by a higher quality product; and *coordinated market power* - the price premium stemming from concerted action with other firms by means of explicit or tacit collusion.[3] As the ultimate goal of the proposed model is to allow for inferences about the potential anti-competitive conduct of firms, the present modeling focuses on the analysis and testing of hypotheses about the statistical significance of the coordinated market power component. This is performed after applying the proper unilateral market power restrictions according to the oligopoly setting under consideration.

This paper is divided as follows. In Section 2, I present the theoretical framework for obtaining the marginal profits ratio, with a description of how this model can accommodate a wide range of static games outcomes within a generalized supply relation in Section 3. Additionally, a three-step methodology for empirical research on competitive conduct is presented. Sections 4 and 5 contains an application of the proposed model. The application makes use of data from an antitrust case from the airline industry: the investigation of an alleged price parallelism in the Rio de Janeiro - São Paulo air shuttle market in Brazil in 1999, which was adjudicated by the antitrust authority in 2004. In the demand side, I employ a generalized nested logit (GNL) model with overlapping nests that has the models of Berry (1994) and Ackerberg and Rysman (2005) as special cases. The demand framework allows for single- and multi-outlet strategy by firms and controls for unobserved outlet exclusivity and shelf visibility of products at outlets. It also models shopping frequency of consumers to control for changes in the size of the outside good. In the supply side, I use the proposed GSR for heterogeneous, multi-product settings with price competition and perform a hypothesis test of coordinated market power during the investigated period. A final section concludes.

2 A marginal profits ratio (MPR) framework

Consider a market with N firms playing an infinitely repeated game in which a cartel applies stick-and-carrot two-phase punishment (Abreu, 1988).[4] The ultimate goal of the cartel is to engage in *efficient collusion*, that is, joint profit maximization in which no participating firm has an incentive to deviate by acting non-cooperatively. Depending on the cartel's move, we may observe either a cooperative or a punishment phase at period t. Firms may adhere to, or defect from, the cartel's move. If a given firm adheres to the cartel's choice in t, then there

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[3] In case of a multiproduct firm, it follows that, as in a merger analysis, the market power arising from the internalization of competition between rival products in connection with the merger is incorporated into the unilateral market power component, known as the unilateral effects of the merger.

[4] Grim trigger punishment (Friedman, 1971) is also possible and will be one of the results nested within the marginal profits balance relation below.

will be cooperation in $t+1$. Otherwise, if any firm produces its best-response move, then there will be punishment in the following period.[5] Best response profits by the *k-th* firm in the cooperative phase are denoted by π_{kt}^{br}, and in the punishment phase by $\pi_{kt}^{br,p}$. Adherence profits in the cooperative phase are denoted by π_{kt}, and in the punishment phase by π_{kt}^{p}. We therefore have two incentive compatibility constraints in the optimization problem of the cartel - one for the cooperative phase (IC_1) and the other for the punishment phase (IC_2).

Assume that the strategic variable being played in this game is observed by the researcher. Depending on market characteristics and institutions, the researcher may set either $x_t = q_t$ (vector of quantities) or $x_t = p_t$ (vector of prices). The cartel thus have to coordinate to find $x_t = \{x_{1t}, ..., x_{kt}, ..., x_{Nt}\}$ equal to x_t^*, the vector of efficient collusion for the strategic variable. We then have the following setting:

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\begin{aligned}
& x_t^* = \arg\max_{x_t} \sum_{k=1}^{N} \pi_{kt}(x_t)\text{, subject to} \\
& IC_1\text{: } \pi_{kt}^{br}(x_t) + \rho E_t \pi_{kt+1}^{p} + \sum_{s=1}^{\infty} \rho^{2s}\left[E_t \pi_{kt+2s}^{br} + \rho E_t \pi_{kt+2s+1}^{p}\right] \leq \pi_{kt}(x_t) + \sum_{s=1}^{\infty} \rho^{s} E_t \pi_{kt+s}^{*}, \\
& IC_2\text{: } \pi_{kt}^{br,p}(x_t) + \sum_{s=1}^{\infty} \rho^{s} E_t \pi_{kt+s}^{p} \leq \pi_{kt}^{p}(x_t) + \sum_{s=1}^{\infty} \rho^{s} E_t \pi_{kt+s}^{*}\text{ , } \forall k.
\end{aligned}
\tag{1}
$$

where ρ is the discount factor and E_t is the operator of expectations of future profits conditional on the available information at time t. π_{kt+s}^{*} is firm k's optimal collusive profit in future period $t+s$.[6] IC_1 and IC_2 are the incentive compatibility constraints of the cooperative and the punishment phases, respectively. Note that the cartel solves the constrained optimization problem in (1) by setting the firm-specific values x_{kt} within x_t. By doing this, the cartel directly influence profits obtained from collusion[7] (π_{kt}) and with defection from collusion (π_{kt}^{br}) at time t. For example, if x represents quantities, in IC_1 we have that the lower the quantities supplied by all participants, the higher the profits from collusion and the higher the profits associated with unilateral defection. Adjustments in x_t therefore affect both the incentives of participating and of abandoning the cartel at t.

In order to obtain a solution for (1), we need to model how profits behave over time under different regimes - cooperation, punishment, adherence, defection -, given the adjustments made by the cartel in x_t. This is a crucial step because it dictates the decision making by firms interested in either participating or abandoning the cartel. More specifically, we

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[5] There are four possible paths in this setting: 1. in the cooperation phase: 1a. eternal adherance to the cartel outcome; and 1b. defection from the cartel outcome, punishment and then reversal to the cartel outcome; defection again and reversal again *ad infinitum*. 2. in the punishment phase: 2a. adherance to the cartel behavior and then reversal to the cartel outcome; and 2b. defection from the cartel behavior and then eternal punishment. All cases of defection are treated here as the best-response behavior by defectors.

[6] Note that, in line with Corts (1999) and Puller (2009), I impose stationarity of the equilibrium by setting $\pi_{kt+s} = \pi_{kt+s}^{*}$. This means that firms solve (1) in each period with respect to time-invariant a punishment scheme.

[7] That is, joint profit maximization obtained from an unconstrained optimization problem.

deal with terms $E_t\pi^*_{kt+s}$, $E_t\pi^{br}_{kt+s}$ and $E_t\pi^{p}_{kt+s}$ of (1), $s > 0$. We also need to calculate the expected present discounted value of profits in both IC_1 and IC_2. The most typical framework for modeling the future payoffs of firms is to consider some stochastic process of common knowledge space state variables, for example a demand shifter Y_t. Some applications include observed product or market characteristics as state variables, evolving according to a first-order Markov process with a transition probability function. Typically, a vector of private information shocks is also modeled. This literature follows Ericson and Pakes (1995), which employs a Markov Perfect Equilibrium concept. As these models usually set the evolution of state variables, they explicitly control for the intertemporal impact of adjustments in x_t on firms' stream of future profits - for example, Aguirregabiria and Ho (2011). This approach has its own disadvatanges, however. The problem with dynamic structural models is related to their complexity and lack of tractability - as discussed by Aguirregabiria and Nevo (2010). Additionally, with dynamic models, the researcher typically incurs the "curse of dimensionality", with the dimension of the state space increasing exponentially with the number of competitors.[8]

Here I use a simplified approach to the modeling of intertemporal evolution of profits. I employ a stochastic process in π_{kt} instead of dealing with a vector of state variables determinants of π_{kt}. This approach is less structural but allows straightforward application to the cartels' problem above. By modeling profits as a time series process, I follow the profits persistence literature (Mueller, 1977, Geroski and Jacquemin, 1988 and Glen, Lee and Singh, 2003). The model of future profits is stated in the following proposition.

Proposition 1 *Assume that π_{kt} follow an $ARMA(m,n)$ process of the type $\pi_{kt} = a_k + \sum_{i=1}^{m} b_{ki}\pi_{kt-i} + \sum_{i=1}^{n} g_{ki}z_{kt-i} + z_{kt}$, in which a_k, b_{ki} and g_{ki} are unknown parameters and z_{kt} is the error term. Consider the predictor of this process at future time s, $\widehat{\pi}_{kt+s}$, and the expected present discounted value of profits at t, $E_tV(\pi_{kt})$. It follows that with convenient manipulation and recursive substitution, we have $\widehat{\pi}_{kt+s} = \widetilde{\alpha}_{k,t,s,m} + \widetilde{\beta}_{k,s,m}\pi_t$, where here $\widetilde{\alpha}_{k,t,s,m}$ and $\widetilde{\beta}_{k,s,m}$ are functions of the parameters of π_{kt}, the time horizon s and the autoregressive order m. Additionally, we have $E_tV(\pi_{kt}) = \alpha_{kt} + (1+\beta_k)\pi_{kt}$, where $\alpha_{kt} = \overline{\alpha}_{kt}\rho(1-\rho)^{-1}$, $\beta_k = \overline{\beta}_k\rho(1-\rho)^{-1}$, where $\overline{\alpha}_{kt}$ and $\overline{\beta}_k$ are intertemporal weighted averages of, respectively, $\widetilde{\alpha}_{k,t,s,m}$ and $\widetilde{\beta}_{k,s,m}$, $s = 1,..,\infty$.*

Proof. See the Appendix. ■

With Proposition 1 we have a simple and useful framework for the expected present discounted value of profits in both IC_1 and IC_2 of (1). Moreover, expression $E_tV(\pi_{kt}) = \alpha_{kt} + (1+\beta_k)\pi_{kt}$ accomodates different patterns of persistence in profits. For example, profits are highly persistent and follow a process with *iid* shocks when $\widetilde{\beta}_{k,s,m} = 1$, $\forall s$. In this case, $\overline{\beta}_k = 1$ and $\beta_k = \rho(1-\rho)^{-1}$. Either low persistence in profits or low discount factors produce

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[8] Aguirregabiria and Ho (2011), for example, find that the dimension of the space in their study of the airline industry is $10exp(10,000)$. They had to recourse to Monte Carlo simulation to approximate the transition probability functions of the dynamic model.

low values of β_k. Corts (1999) investigates how varying degres of pesistence of demand shocks impact the estimation of misspecified conduct parameter models (CPM), which do not account for demand dynamics over time. In the present model, any factor that causes profit persistence is explicitly controlled by β_k.

Another advantage of using $E_t V(\pi_{kt})$ as in Proposition 1, is that it is linear in π_{kt}, and therefore can be conveniently used for algebraic manipulations and, in particular, for insertion direct in (1). To enhance flexibility, in what follows I perform these manipulations by employing different parameters α_{kt} and β_k for each strategic regime. Again, let us use superscripts br, p, br and p. By considering $E_t V(\pi_{kt})$ of Proposition 1, along with a slight modification of its expression to incorporate the present value of profits in the case eternal oscilation between defection and reversal to cartel outcome,[9] the lagrangian related to (1) is thus the following:

$$\begin{aligned}\mathcal{L}_t = &\sum_{k=1}^{N} \pi_{kt}(x_{kt}) \\ &- \mu_{kt}^{(1)} \left\{ \left[\alpha_{kt}^{br} + \left(1+\beta_k^{br_1}\right)\pi_{kt}^{br}(x_{kt}) + \beta_k^{br_2}\pi_{kt}(x_{kt})\right] - \left[\alpha_{kt} + (1+\beta_k)\pi_{kt}(x_{kt})\right]\right\} \\ &- \mu_{kt}^{(2)} \left\{ \left[\alpha_{kt}^{br,p} + \left(1+\beta_k^{br,p}\right)\pi_{kt}^{br,p}(x_{kt})\right] - \left[\pi_{kt}^{p}(x_{kt}) + \alpha_{kt}^{p} + \beta_k^{p}\pi_{kt}(x_{kt})\right]\right\} \end{aligned} \tag{2}$$

where $\mu_{kt}^{(1)}$ and $\mu_{kt}^{(2)}$ are shadow values of marginal relaxation in the incentive compatibility constraints. For $k = j$, we have the following first-order condition for profit maximization in which $\partial \mathcal{L}_{jt}/\partial x_{jt} = 0$:

$$\begin{aligned}\frac{\partial \pi_{jt}(x_{jt})}{\partial x_{jt}} - \mu_{jt}^{(1)} &\left[\left(1+\beta_j^{br_1}\right)\frac{\partial \pi_{jt}^{br}(x_{jt})}{\partial x_{jt}} + \beta_j^{br_2}\frac{\partial \pi_{jt}(x_{jt})}{\partial x_{jt}} - \left(1+\beta_j\right)\frac{\partial \pi_{jt}(x_{jt})}{\partial x_{jt}}\right] \\ - \mu_{jt}^{(2)} &\left[\left(1+\beta_j^{br,p}\right)\frac{\partial \pi_{jt}^{br,p}(x_{jt})}{\partial x_{jt}} - \frac{\partial \pi_{jt}^{p}(x_{jt})}{\partial x_{jt}} - \beta_j^{p}\frac{\partial \pi_{jt}(x_{jt})}{\partial x_{jt}}\right] = 0. \end{aligned} \tag{3}$$

Note that both in (3) and in IC_2 of (1) we have π_{kt}^{p} as a function of x_{kt}. Thus, differently from Corts (1999) and Puller (2009), here the cartel may set punishment levels - and profits - according to the benefits of defection. With endogenously-determined punishment levels, we therefore have $\pi_{jt}^{p}(x_{jt}) = \pi_{jt}^{p}\left[\pi_{jt}^{br}(x_{jt})\right]$. Another feature of (1) is that the benefits of deviating from punishment may depend on the harshness of punishment and therefore $\pi_{jt}^{br,p}(x_{jt}) = \pi_{jt}^{br,p}\left\{\pi_{jt}^{p}\left[\pi_{jt}^{br}(x_{jt})\right]\right\}$. For simplicity, assume $\partial \pi_{jt}^{p}(x_{jt})/\partial x_{jt} = \kappa_{jt}^{p}\left[\partial \pi_{jt}^{br}(x_{jt})/\partial x_{jt}\right]$ and $\partial \pi_{jt}^{br,p}(x_{jt})/\partial x_{jt} = \kappa_{jt}^{br,p}\kappa_{jt}^{p}\left[\partial \pi_{jt}^{br}(x_{jt})/\partial x_{jt}\right]$, with κ_{jt}^{p} and $\kappa_{jt}^{br,p}$ being unkown parameters.[10]

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[9] See Appendix C for a demonstration.

[10] See Appendix B for details.

We then finally reach the following relation:

$$\begin{aligned}\frac{\partial \pi_{jt}\left(x_{jt}\right)/\partial x_{jt}}{\partial \pi_{jt}^{br}\left(x_{jt}\right)/x_{jt}} &= \lambda_{jt}, \\ \text{where } \lambda_{jt} &= \frac{\mu_{jt}^{(1)}\left(1+\beta_j^{br_1}\right)+\mu_{jt}^{(2)}\kappa_{jt}^{p}\left[\left(1+\beta_j^{br,p}\right)\kappa_{jt}^{br,p}-1\right]}{1+\mu_{jt}^{(1)}\left[1+\left(\beta_j-\beta_j^{br_2}\right)\right]+\mu_{jt}^{(2)}\beta_j^{p}}. \end{aligned} \tag{4}$$

The marginal profits ratio (MPR) of (4) was used for tacit collusion analysis in a multimarket situation by Bernheim and Whinston (1990). It describes the relative magnitude of marginal profits from collusion with respective to the marginal profit from deviation of firm j. The higher the λ_{jt} the higher the incentive to engage in collusion of firm j. The problem of the cartel is to balance this relation of marginal profits in order to produce the efficient collusion outcome, that is, to set the correct value of λ_{jt}. Note that λ_{jt} is an unknown parameter to the researcher. Additionally, it accommodates different possible collusion concepts. With (4) it is possible to nest a set of repeated-interaction models, by considering different theoretical values of λ_{jt}. With (4), we have the efficient collusion model used by Puller (2009) to address the Corts Critique, nested as a special case. Also, a grim-trigger punishment strategy is another case nested by λ_{jt}.[11] Below are some of the possible cases of collusion nested within (4), according to the following restrictions in the parameters of (4):

a) efficient collusion with temporal interdependence of profits, stick-and-carrot cartel and endogenously determined punishment: λ_{jt} as in (4), that is, no restrictions;

b) efficient collusion with temporal interdependence of profits, stick-and-carrot cartel and independent best responses to punishment: $\kappa_{1j}^{br,p}=0$;

c) efficient collusion with temporal interdependence of profits, stick-and-carrot cartel and exogenously determined punishment: $\kappa_{1j}^{p}=0$;

d) efficient collusion with temporal interdependence of profits and grim-trigger cartel: $\kappa_{1j}^{br,p}=0$, $\kappa_{1j}^{p}=0$, $\beta_j^{br_1}=\beta_j^{br_2}=\beta_j^{br}$ and $\mu_{jt}^{(1)}=\mu_{jt}^{(2)}=\mu_{jt}$;

e) Puller's efficient collusion: $\kappa_{jt}^{br,p}=0$, $\kappa_{jt}^{p}=0$, $\beta_j^{br_1}=\beta_j^{br_2}=\beta_j^{p}=\beta_j=0$ and $\mu_{jt}^{(1)}=\mu_{jt}^{(2)}=\mu_t$;

f) static joint profits maximization: $\mu_{jt}^{(1)}=\mu_{jt}^{(2)}=0$.

With respect the model of Puller (2009), the restrictions imposed on λ_{jt} are such that $\lambda_t=\mu_t/\left(1+\mu_t\right)$. By implictly imposing nullity of the $\beta' s$, the author considers a simplified process of profit evolution, with intertemporally independent shocks in profits.[12] In this

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[11] See the Appendix for the demonstration.

[12] The author also assumes a grim-trigger cartel ($\mu_{jt}^{(1)}=\mu_{jt}^{(2)}=\mu_t$), exogenous punishments ($\theta_{jt}^{br,p}=0$) and independent best-responses to punishment ($\theta_{jt}^{p}=0$).

model, a shock in profits does not affect firms' efficient collusion solution. This result is not changed when profits are white noise, or fully persistent as discussed by Corts (1999).[13] In order to address the Corts Critique, Puller makes use of an efficient collusion setup in which firms do not consider profit dynamics over time. In the MPR of (4) this restriction may be one of the possible patterns of behavior by firms. Another may be the cartel considering profit shocks interdependence. Simpler processes of profit evolution, such as random walk or white noise - with or without a drift - are nested by the case of $\beta_j = 1$ and $\beta_j = 0$, respectively. And finally, the simplest case - nested both by Puller and the present framework - is static joint profit maximization, in which firms are myopic and set prices according to $\partial\pi_{jt}/\partial x_{jt} = 0$, given that $\lambda_{jt} = 0$. Important to note that with impatient firms (or high interest rates), and thus with $\rho \to 0$, we have a situation that brings Puller's result back. In the other extreme, with either patient firms or low interest rates ($\rho \to 1$), or when $\mu_{jt}^{(1)}, \mu_{jt}^{(2)} \to 0$, we have that $\lambda_{jt} \to 0$. In this case firms interact as in a static collusive game. Finally, the researcher may also consider the case of symmetric firms with respect to (4), that is, disregarding index j such as $\lambda_{jt} = \lambda_t$.[14]

In all situations discussed above we assumed that λ_{jt} is non-observable. We will keep this assumption but considering that λ_{jt} (or λ_t) may be empirically handled. In the next section we will see how the framework of (4) may produce an estimating equation for empirical evaluation of λ_{jt} (or λ_t) and, by consequence, of the coordinated market power of firms.

3 A generalized supply relation

With the development of the marginal profits ratio (MPR) of (4), it is straightforward to reach an empirical model of market power with a freely estimated coordination parameter in the same fashion as classic conduct parameter models. As seen, the MPR framework nests as special cases some efficient collusion references and also the model of Puller (2009). In this section, I show that it also accommodates important references from classic static games. Ultimately, the MPR results in a first-order condition specification that incorporates a wide range of static and repeated games and that can be used for market power estimation. In order to reach this *generalized supply relation* (GSR), we must first solve the static games according to the market setting under analysis. As discussed before, it is the researcher's deliberation to decide which market setting to use in the particular case of analysis, according to the idiosyncrasies of players, products, markets and institutions. In what follows, I present the algebraic developments for one possible market setting - the differentiated, multiproduct market with competition in prices. This is certainly one the most popular competition models used in the empirical literature.

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[13] In a fully persistent case we have $\pi_{kt+s} = \pi_{kt+s-1}, \forall s$.

[14] I shall come back to this point when discussing the empirical implementation of the MPR in the next section.

3.1 Differentiated multiproduct, price competition

a. Nash equilibrium: The problem of the multiproduct firm is to maximize the total profits of all F products belonging to its product portfolio $\mathcal{F}$. There are N products in the market. Assume price competition, that is, $x = p$.[15] The problem of the owner of product j ($j \in \mathcal{F}$) is therefore

$$\max_{\{p_k : k=1,...j,...,F\}} \textstyle\sum_{k\in\mathcal{F}} p_k q_k - TC_k,$$

where p_k, q_k and TC_k are, respectively, the price, quantity and total cost of product k. Marginal profit of j is then equal to

$$\partial\pi_j/\partial p_j = q_j - S_{jj}\left(p_j - c_j\right) + \textstyle\sum_{k\in\mathcal{F},k\neq j} S_{kj}\left(p_k - c_k\right), \tag{5}$$

where $S_{kj} = \text{abs}(\partial q_k/\partial p_j)$, $k = 1,..,j,..,N$. Denote $S_{-j,\mathcal{F}} = \sum_{k\in\mathcal{F},k\neq j} S_{kj}$. Also, denote $p_{-j,\mathcal{F}} = \sum_{k\in\mathcal{F},k\neq j} S_{kj}p_k/S_{-j,\mathcal{F}}$ and $c_{-j,\mathcal{F}} = \sum_{k\in\mathcal{F},k\neq j} S_{kj}c_k/S_{-j,\mathcal{F}}$. By multiplying and dividing the last term on the right of (5) by $S_{-j,\mathcal{F}}$ and with some manipulation we then have

$$\partial\pi_j/\partial p_j = q_j - S_{jj}\left(p_j - c_j\right) + S_{-j,\mathcal{F}}\left(p_{-j,\mathcal{F}} - c_{-j,\mathcal{F}}\right). \tag{6}$$

Note that (6) represents the marginal profits associated with best response in the efficient collusion setting, that is, $\partial\pi_j^{br}/\partial p_j$.

b. Static joint-profit maximization: The problem of the multiproduct firm is now to engage in joint-profit maximization not only with its F products of portfolio $\mathcal{F}$, but also to coordinate with all the other $N - F$ products in the market:

$$\max_{\{p_k : k=1,...j,...,N\}} \textstyle\sum_{k=1}^{N} p_k q_k - TC_k \tag{7}$$

Marginal profit of j is equal to

$$\partial\pi_j/\partial p_j = q_j - S_{jj}\left(p_j - c_j\right) + \textstyle\sum_{k\in\mathcal{F},k\neq j} S_{kj}p_k + \sum_{k\notin\mathcal{F}} S_{kj}p_k - \sum_{k\neq j} S_{kj}c_k.$$

Denote again $S_{-j,\mathcal{F}} = \sum_{k\in\mathcal{F},k\neq j} S_{kj}$ and $p_{-j,\mathcal{F}} = \sum_{k\in\mathcal{F},k\neq j} S_{kj}p_k/S_{-j,\mathcal{F}}$ Also, denote $S_{-\mathcal{F}} = \sum_{k\notin\mathcal{F}} S_{kj}$, $p_{-\mathcal{F}} = \sum_{k\notin\mathcal{F}} S_{kj}p_k/S_{-j,\mathcal{F}}$, $S_{-j} = \sum_{k\neq j} S_{kj}$ and $c_{-j} = \sum_{k\neq j} S_{kj}c_k/S_{-j}$. By using these terms and with some manipulation we reach

$$\partial\pi_j/\partial p_j = q_j - S_{jj}\left(p_j - c_j\right) + S_{-j,\mathcal{F}}p_{-j,\mathcal{F}} + S_{-\mathcal{F}}p_{-\mathcal{F}} - S_{-j}c_{-j}. \tag{8}$$

c. Efficient collusion: By substituting (8) and (6) into the marginal profits ratio of (4) we then have $\left(\partial\pi_j/\partial p_j\right)/\left(\partial\pi_j^{br}/\partial p_j\right) = \lambda_j$. This is equivalent to $\partial\pi_j/\partial p_j - \lambda_j\partial\pi_j^{br}/\partial p_j = 0$

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[15] Note that I disregard index t in this section.

and thus we have:

$$\begin{aligned}\partial\pi_j/\partial p_j - \lambda_j \partial\pi_j^{br}/\partial p_j &= \left[q_j - S_{jj}\left(p_j - c_j\right) + S_{-j,\mathcal{F}}p_{-j,\mathcal{F}} + S_{-\mathcal{F}}p_{-\mathcal{F}} - S_{-j}c_{-j}\right] \\ &\quad -\lambda\left[q_j - S_{jj}\left(p_j - c_j\right) + S_{-j,\mathcal{F}}\left(p_{-j,\mathcal{F}} - c_{-j,\mathcal{F}}\right)\right] = 0, \qquad (9)\end{aligned}$$

And, with some manipulations:

$$p_j = c_j + \left(\frac{1}{S_{jj}}q_j + \frac{S_{-j,\mathcal{F}}}{S_{jj}}p_{-j,\mathcal{F}}\right) + \Lambda_j\left(\lambda_j\frac{S_{-j,\mathcal{F}}}{S_{jj}}c_{-j,\mathcal{F}} - \frac{S_{-j}}{S_{jj}}c_{-j} + \frac{S_{-\mathcal{F}}}{S_{jj}}p_{-\mathcal{F}}\right) \qquad (10)$$

where $\Lambda_j = 1/\left(1-\lambda_j\right)$. Note that with (10), we have price p_j additively separable and decomposed into $c_j + UMP_j + CMP_j$, in which $UMP_j = \left(1/S_{jj}\right)q_j + \left(S_{-j,\mathcal{F}}/S_{jj}\right)p_{-j,\mathcal{F}}$ and $CMP_j = \Lambda_j\left[\lambda_j\left(S_{-j,\mathcal{F}}/S_{jj}\right)c_{-j,\mathcal{F}} - \left(S_{-j}/S_{jj}\right)c_{-j} + \left(S_{-\mathcal{F}}/S_{jj}\right)p_{-\mathcal{F}}\right]$. Denote UMP_j and CMP_j by, respectively, *unilateral market power* and *coordinated market power* of j. In order to produce an estimating pricing equation, I impose non-nullity of UMP_j, by moving it to the left-hand side of (10). We then have

$$p_j - \frac{1}{S_{jj}}q_j - \frac{S_{-j,\mathcal{F}}}{S_{jj}}p_{-j,\mathcal{F}} = c_j + \Lambda_j\lambda_j\frac{S_{-j,\mathcal{F}}}{S_{jj}}c_{-j,\mathcal{F}} - \Lambda_j\frac{S_{-j}}{S_{jj}}c_{-j} + \Lambda_j\frac{S_{-\mathcal{F}}}{S_{jj}}p_{-\mathcal{F}},$$

and then

$$\widetilde{p}_j = c_j + \lambda_j\Lambda_j\widetilde{c}_{-j,\mathcal{F}} - \Lambda_j\widetilde{c}_{-j} + \Lambda_j\widetilde{p}_{-\mathcal{F}}, \qquad (11)$$

where $\widetilde{p}_j$ is equal to $p_j - UMP_j = p_j - \left(1/S_{jj}\right)q_j + \left(S_{-j,\mathcal{F}}/S_{jj}\right)p_{-j,\mathcal{F}}$, being therefore net of unilateral market power effects stemming from either product-specific perceived quality or the internalization of competition from other products in the multiproduct setting. Additionally, $\widetilde{c}_{-j,\mathcal{F}} = \left(S_{-j,\mathcal{F}}/S_{jj}\right)c_{-j,\mathcal{F}}$, $\widetilde{c}_{-j} = \left(S_{-j}/S_{jj}\right)c_{-j}$ and $\widetilde{p}_{-\mathcal{F}} = \left(S_{-\mathcal{F}}/S_{jj}\right)p_{-\mathcal{F}}$. Finally, consider the following specification of (11):

$$\widetilde{p}_j = C_j + \Lambda_j\widetilde{p}_{-\mathcal{F}}, \qquad (12)$$

where $C_j = c_j + \lambda_j\Lambda_j\widetilde{c}_{-j,\mathcal{F}} - \Lambda_j\widetilde{c}_{-j}$ is an expanded marginal cost term that account for variability of the price of j due to the marginal cost of j and of j's rivals.

3.2 Development of a generalized supply relation

Consider the following specification for the first-order condition for the optimization problem of (1):

$$\widetilde{p}_j = C_j + \theta_j\widetilde{x}_{-\varphi}, \qquad (13)$$

where, $\widetilde{p}_j$ is the price of alternative j net of unilateral effects, C_j is the expanded marginal cost term for j, $\widetilde{x}$ is the (transformed) strategic variable being played, x =price or quantity. $-\varphi$ is an index of the average effective opponent of j, being equal to either $-j$ (all j's rival firms, in case of a single product setting) or $-\mathcal{F}$ (j's rival products pertaining to other firms, in case of a multiproduct setting). We have therefore that $\widetilde{x}_{-\varphi}$ means a transformation of the strategic variable of the average, not internalized, opponent.

The novelty permitted by (13) is related to θ_j, a coordinated market power parameter indicative of the degree of tacit collusion of j in the market. θ_j may assume a broad range of values that are consistent with (1), and therefore is grounded in a efficient collusion concept. In particular, if $\theta_j > 1$ then θ_j is consistent with the assuption of $\theta_j = \Lambda_j$, $\Lambda_j > 1$, where Λ_j is a direct function of the structural marginal profits ratio term λ_j. If $\Lambda_j > 1$, then we have evidence of efficient collusion being played. Note that as $0 \leq \lambda_j < 1$ then $\Lambda_j \geq 1$. In case of the testable hypothesis $\theta_j = \Lambda_j = 1$, we have static joint profit maximization. Cases of $\Lambda_j < 1$ do not have economic meaning. The coordination parameter may be null, however. In case $\theta_j = 0$ we then have non-cooperative behavior of alternative j.[16] Remember that with $\widetilde{p}_j$ we impose the restriction on non-null unilateral market power and therefore marginal cost pricing is not nested within the above framework.

I denote (13) as a *generalized supply relation* (GSR) on account of two properties. First, contrary to classic supply relations of conduct parameter models, the term θ_j above is sufficiently flexible to accommodate a wide range of possible repeated-interaction games by means of $\Lambda_j(\lambda_j)$. Although with (13) the researcher cannot indentify any of the terms embedded in $\Lambda_j(\lambda_j)$ like $\kappa_{jt}^{br,p}$, κ_{jt}^{p}, $\beta_j^{br_1}$, $\beta_j^{br_2}$, $\mu_{jt}^{(1)}$ or $\mu_{jt}^{(2)}$ of (4), it is robust to alternative values assumed by these parameters. We therefore have that, like Puller (2009), the proposed GSR addresses the Corts Critique (Corts, 1999). Different from Puller, however, it accommodates a broad range of possible specifications of $\Lambda_j(\lambda_j)$.

The second property makes (13) a general relation is related to the possibility of accommodation of other market configurations than the heterogeneous, multiproduct setting with price competition typically found in the recent empirical IO literature. Other alternative configurations related to either homogeneous or differentiated product, single or multiproduct firms, and quantity- or price-competition, are also desirable and are actually embedded in (13).

Proposition 2 *The generalized supply relation of (13) nests non-cooperative and cooperative equilibirum outcomes of the following market settings: a) homogeneous, single product market with competition in quantities; b) homogeneous, multiproduct market, with competition in quantities; c) differentiated, single product market, with competition in quantities; d) differentiated, single product market, with competition in prices; e) differentiated, multiproduct market, with competition in quantities; and f) differentiated, multiproduct market, with competition in prices.*

Proof. See the Appendix. ■

Below is a summary of results accommodated by (13), obtained from Proposition 2. For each market setting, both the non-cooperative outcome (Nash equilibrium) and the cooperative outcome (joint profit maximization) are developed and plugged into the MPR in (4) in order to produce the respective definitions of $\widetilde{p}_j$, C_j and $\widetilde{x}_{-\varphi}$. We therefore have:

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[16] Note that when $\theta_j = 0$ we have evidence that the game being played is static. In this case, it is as if $\Lambda_j = 0$ and therefore all terms of the expanded marginal cost C_j become null.

a) homogeneous, single product market with competition in quantities:
$\{\widetilde{p}_j = P - Rq_j; C_j = c_j; \widetilde{x}_{-\varphi} = Rq_{-j}\}$;

b) homogeneous, multiproduct market, with competition in quantities:
$\{\widetilde{p}_j = P - Rq_j - RQ_{-j,\mathcal{F}}; C_j = c_j; \widetilde{x}_{-\varphi} = Rq_{-\mathcal{F}}\}$;

c) differentiated, single product market, with competition in quantities:
$\{\widetilde{p}_j = p_j - R_{jj}q_j; C_j = c_j; \widetilde{x}_{-\varphi} = R_{-j}q_{-j}\}$;

d) differentiated, single product market, with competition in prices:
$\left\{\widetilde{p}_j = p_j - \frac{1}{S_{jj}}q_j; C_j = c_j - \Lambda\frac{S_{-j}}{S_{jj}}c_{-j}; \widetilde{x}_{-\varphi} = \frac{S_{-j}}{S_{jj}}p_{-j}\right\}$;

e) differentiated, multiproduct market, with competition in quantities:
$\{\widetilde{p}_j = p_j - R_{jj}q_j - R_{-j,\mathcal{F}}q_{-j,\mathcal{F}}; C_j = c_j; \widetilde{x}_{-\varphi} = R_{-\mathcal{F}}q_{-\mathcal{F}}\}$;

f) differentiated, multiproduct market, with competition in prices:
$\left\{\widetilde{p}_j = p_j - \frac{1}{S_{jj}}q_j - \frac{S_{-j,\mathcal{F}}}{S_{jj}}p_{-j,\mathcal{F}}; C_j = c_j + \Lambda\lambda\frac{S_{-j,\mathcal{F}}}{S_{jj}}c_{-j,\mathcal{F}} - \Lambda\frac{S_{-j}}{S_{jj}}c_{-j}; \widetilde{x}_{-\varphi} = \frac{S_{-\mathcal{F}}}{S_{jj}}p_{-\mathcal{F}}\right\}$.

where $P = D(Q)$ is the inverse demand function, with P and Q being, respectively, the market price and quantity. $Q_{-j,\mathcal{F}} = \sum_{k\in\mathcal{F},k\neq j} q_k$. Also, $R =$abs$(\partial P/\partial Q)$ and $R_{kj} = $ abs$(\partial p_k/\partial q_j)$, $\forall k = 1, ..., j, ..., N$. $R_{-j,\mathcal{F}}$, $R_{-\mathcal{F}}$ and R_{-j} are terms derived from R_{kj}. See the Appendix for details. S_{jj}, S_{-j}, $S_{-\mathcal{F}}$ and $S_{-j,\mathcal{F}}$ are the same defined before.

To sum up, it is possible to conclude that, with suitable definitions of $\widetilde{p}_j$, C_j and $\widetilde{x}_{-\varphi}$, the generalized supply relation of (13) accommodates any of the market settings listed by Proposition 2. Expression (13) can therefore be used in different empirical contexts aiming at coordinated market power estimation.

3.3 Empirical implementation issues

A relevant issue concerning the generalized supply relation in (13), and its possible configurations of Proposition 2, is related to empirical implementation with respect to the econometric assessment of competitive conduct. By means of (13), we have a single framework to perform hypotheses tests regarding potential anticompetitive behavior of firms via the estimation of the coordinated market power parameter. We reached a framework that allows for a freely estimated parameter in the same fashion as the conduct parameter models - but without some of its most notable drawbacks. With the focus on empirical implementation, let us perform the following developments. Assume the researcher employs panel data to estimate (13). Consider then the following specification for firm (brand) j, market i and time t:

$$\widetilde{p}_{jit} = C_{jit} + \theta_{jt}\widetilde{x}_{-\varphi it} + \varepsilon_{jit}. \tag{14}$$

where ε_{jit} is the error term, associated with unobserved price and cost factors. Note that, as Bresnahan (1989), I let θ vary both by product (firm) and time and therefore have an overparameterized model. In typical applications, however, the researcher has to put some structure on the variability of θ over individuals and periods. This structure depends on the data set, being therefore application-specific.

In order to accomplish estimation of (14), we need first proper demand estimates for computing the transformations in $\widetilde{p}_{jit}$ and $\widetilde{x}_{-\varphi it}$. The GSR framework does not impose any restriction on the researcher's choice of demand model and therefore popular specifications such as the random coefficients logit, nested logit, distance-metric, AIDS, among others, may be used. Additionally, the researcher needs to model the unobserved marginal costs side. A typical specification of marginal costs makes use of cost shifters as in Bresnahan (1989) and Berry, Levinson and Pakes (1995). We then have

$$C_{jit} = C\left[c_j\left(W_{jit}, Z_{jit}, q_{jit}\right), \widetilde{c}_{-\varphi it}, \varepsilon^c_{jit}\right], \tag{15}$$

where W_{jit}, Z_{jit} and ε^c_{jit} are, respectively, input prices, other cost controls and the econometric error term of marginal costs. ε^c_{jit} accounts for both unobserved cost heterogeneity and latent cost shocks, and therefore has an error-component structure, entering additively in (14). An important note is that, for proper implementation of (13), the researcher must include transformed cost shifters ($\widetilde{c}_{-\varphi it}$), which could be a transformed input price of the average opponent, for example. However, in such framework, the researcher must deal with the trade-off between including several transformed terms in (15) and inevitable multicolinearity incurred by deeper cost specifications. Also, he may include q_{jit}, allowing for marginal cost to vary with output to control for either increasing, constant or decreasing returns. This is an important aspect of the proposed GSR, regarding the identification issue in the sense of Bresnahan (1982) for quantity competition settings. As we have transformed variables in (14), the identification of the coordinated market power parameter is feasible even if the marginal cost is not constant, but is also a function of quantities. The reason for that is because $\widetilde{q}_{-\varphi it}$ (considering $x = q$) and q_{jit} are not perfectly collinear and therfore may appear in the right-hand side of the GSR. We therefore have the advantage of a relaxation of the a typical NEIO assumption of constant marginal costs in quantity-competition settings. With price competition, identification is also guaranteed.

In order to empirically implement the above GSR framework, consider the following three-step approach:

i) In possession of at least product-level data, estimate demand with a suitable demand or choice model to obtain estimates of own and cross price-elasticities. With price-elasticities estimates, make the transformations according to the appropriate market setting (Proposition 1) and get $\widetilde{p}_{jit}$, $\widetilde{x}_{-\varphi it}$ and, if applicable, $\widetilde{c}_{-\varphi it}$.

ii) Specify marginal costs as in (15) and then estimate the GSR of (14). Appropriate instrumentation is required, as $\widetilde{x}_{-\varphi it}$, and, if applicable, $\widetilde{c}_{-\varphi it}$, are endogenous.

iii) Test of hypothesis regarding the coordinated market power parameter ($\theta = 0$, $\theta = 1$).

With the three steps above it is possible to infer the competitive conduct of firms by using a model in which data can "select" a wide range of game outcomes and which, at the same time, is robust to repeated interaction and avoids the problems of bias due to model mispecification of the Corts Critique. However, as we employ transformed variables in (14), it is necessary to correct the standard error of estimates to account for the fact that $\widetilde{p}_{jit}$, $\widetilde{x}_{-\varphi it}$ and $\widetilde{c}_{-\varphi it}$ are calculated using previously estimated demand parameters. Traditional methods for standard error correction are bootstrapping and the procedures suggested by Murphy and Topel (1985) and Slade (2004).

4 Application to an airline antitrust case

To illustrate how the proposed framework may be used in an empirical analysis of the competitive conduct of firms, I use an application with real data coming from a closed antitrust case in Brazil. In this case, antitrust authorities of the country prosecuted major airlines for price parallelism on the Rio de Janeiro - São Paulo route in 1999. This case became known as the "air shuttle cartel" and was adjudicated in 2004. All involved airlines were convicted and fined 1% over total operating revenues for the year.

Since the early 1990s, air transportation in Brazil has been characterized by some degree of economic liberalization. Full deregulation occurred by the end of the decade when major regulatory devices controlling fares and flight frequencies were removed, thus airlines are currently free to operate in domestic markets without interference of the regulator,[17] with the exception of routes involving slot-controlled airports that are subject to specific rules limiting the number of take-offs and landings per hour.

Simultaneously, the country has observed a tremendous improvement in its antitrust enforcement capabilities. Although antitrust policy has long been deficient and lacked strength in Brazil, there has been a notable effort in the opposite direction in more recent times. For example, Brazil's antitrust authorities are now regarded by practitioners and agency officials throughout the world as among the best in combating cartels.[18] Between 2007 and 2010, the enforcers conducted 267 dawn raids that resulted in more than two hundred executives facing criminal action.[19] It is possible to state that, during the past two decades, the country has released a set of relevant markets from strict economic regulation. However, the country has also imposed vigilant antitrust monitoring of potential anticompetitive practices and unfair competition. The airline industry was one of the first that authorities targeted for cartel investigation.

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[17] The airline regulator in Brazil is the National Civil Aviation Agency (ANAC).

[18] Source: "*Brazil emerges as among the world's best cartel busters*", Global Competition Review, October, 7, 2010. For a portrait of the country's problems related to antitrust, see "*Competition policy in Brazil - Too little, too late*". The Economist, July 7, 2011.

[19] Source: Administrative Council for Economic Defense's website (2010) and previous footnote.

4.1 The case

On August 3, 1999, at the Sofitel Hotel in São Paulo, there was a meeting between the CEOs of the four major Brazilian airlines, Varig, Tam, Vasp and Transbrasil (TB). These airlines accounted for more than 90% of the market on that year.[20] This meeting was widely reported in the local media. Since the late 1960s, airline managers in Brazil had been meeting together to discuss overall industry conditions. During the strict regulatory period, fare regulations were based on a cost of service mechanism and therefore meetings between airline managers and the regulator, aimed at discussing costs figures for the annual fare review process, were commonly observed. From time to time, major public hearings on airline regulatory reform were called for by the government. These were known as the Civil Aviation Conferences. These occasions, in practice, were traditionally used as opportunities for managers to coordinate actions and to jointly lobby the government for or against the rules under reform. The 1999 meeting episode was the first - and the last - publicly announced meeting after deregulation.[21]

The factual antecedents of the August 1999 meeting were as follows. In 1998, the airline industry in Brazil experienced intense competition due to a round of liberalization measures. In particular, because of a new, anti-grandfathering, slot allocation scheme at the downtown airports of major cities such as Rio de Janeiro and São Paulo, competition became fiercer, with intense price movements emerging in many markets. In March 1998, an unprecedented fare war episode dropped prices in the air shuttle market by almost 30%.

In January 1999, however, a change in the foreign currency regime by the macroeconomic authorities represented a major source of instability in the economic environment of the airline industry in Brazil. Indeed, many airlines' production inputs are not quoted in local currency (BRL) but actually quoted in US dollars (USD), with notable examples being aviation fuel, aircraft insurance, spare parts and aircraft leasing. As a result, exchange rate variations are quickly passed-through to airline costs in subsequent contracts with suppliers.[22] As the change in the exchange rates regime caused an immediate overshooting of the BRL/USD parity, all airlines' costs were severely impacted. In fact, after being 1.21 in December 1998, the BRL/USD rate went up to 1.97 (a 63% increase) in October 1999 and to 3.81 in October 2002 (a 215% increase). As a result, in 1999, the domestic cost per available seat-kilometer (CASK) increased 31%, causing operational results of the whole industry to plunge from a profit of USD 25 million in 1998, to a loss of USD 105 million in 1999.[23]

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[20] Source: Brazil's Statistical Yearbook of Air Transportation, 2000.

[21] On the occasion, the airline CEOs were so clearly unaware of the government's new antitrust watchdog attitude, that they allowed a journalist to take a picture of them together laughing at the camera and to publish it in one of the country's major magazines. Source: Veja, April, 5, 2000, p. 136.

[22] Fuel hedge is a frequently used instrument for airlines to protect against fuel costs fluctuations. It was not a common practice in the industry in the late 1990s, however.

[23] Source: Brazil's Statistical Yearbook of Air Transportation, 1998-2000. Note that I am reporting figures in USD for convenience. From now on, I employ the BRL 2 = USD 1 exchange rate, which prevailed by the end of the sample period.

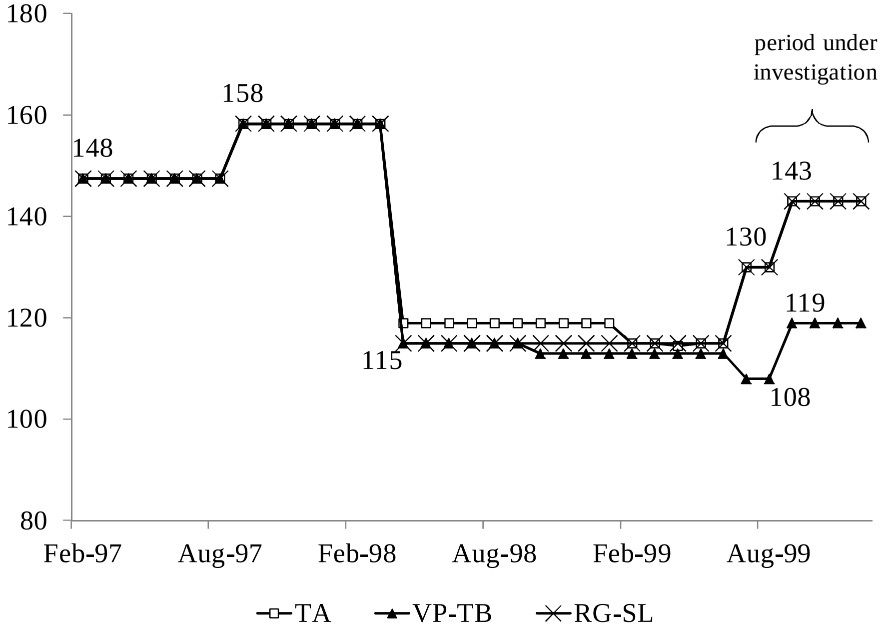

Figure 1: Evolution of full return fares (in USD) in the period. Source: regulator.

The 1999 meeting between the four airline CEOs was apparently motivated by the perspective of a worsening of the crisis in the industry that year. All four airline managers declared, on that occasion, that they only aimed to promote a broad discussion of the industry's problems and to find alternatives to avoid a widespread contagion of the financial instability led by the shock in costs of January 1999. Possible mergers and incorporations of airlines towards a greater consolidation of the industry were under negotiation. In contrast, from an antitrust perspective, the meeting was the evidence that clearly revealed the airlines' intention to implement coordinated behavior in the market. In the following week (August 9, 1999), all four airlines raised their prices by 10% on the same day and on the same route: the air shuttle between Rio de Janeiro and São Paulo, the densest and most profitable market in the country. Indeed, airlines conjointly increased their full fares and abolished the then-available discounted fares. The 10% price increase was, therefore, fully absorbed by air shuttle consumers. Figure 1 presents the evolution of full fares in the market during the period under investigation.

In Figure 1 it is possible to see the 1998 fare war in which full return fares dropped from USD 158 to USD 115. Prices then remained stable until mid-1999. With the cost shock from early 1999, Varig and Tam decided to increase their fares in June. Then in August, the 10% increase by all airlines occurred. Varig and Tam fares increased from USD 130 to USD 143, and Vasp and Transbrasil increased from USD 108 to USD 119. All fares, however, were still below the levels that prevailed in the period immediately prior to the then-recent liberalization measures. These price movements, however, were used to sustain the claims that airlines were coordinating strategies, and they constituted the main evidence for the antitrust case against the airlines.

On March 27, 2000, the Secretariat of Economic Defense[24] decided to prosecute Varig,

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[24] There are three governmental entities responsible for antitrust in Brazil: the Secretariat for Economic

Tam, Vasp and Transbrasil based on the evidence of a price-fixing arrangement that resulted from the uniformity of actions of August, 9, 1999. Conscious price-fixing conduct constitutes a violation of the 1994 Brazilian antitrust code, which explicitly forbids any form of setting prices or conditions of sales in combination with competitors. The additional evidence resulting from the meeting between the airline managers seven days before the price increase episode was regarded as a "plus factor", therefore used to reinforce the allegations. Finally, antitrust authorities investigated the use of an electronic device for the dissemination of price change information among carriers – the ATPCO (Airline Tariff Publishing Company) system.[25] The ATPCO had already been subject to investigation by US antitrust authorities regarding the US airline industry in 1991. Its key tools allow carriers to signal their willingness to coordinate with rivals through a fare structure dataset submitted to global distribution systems such as Amadeus and Galileo. In particular, a footnote device with future ticket date information of a price increase may be submitted and become visible to others, ultimately being used as an invitation to follow suit. Accordingly, the ATPCO tools may act as facilitating practices to sustain collusion, thus serving as a system that permits, to some degree, both information sharing and communication between players in airline markets. As in the US case, the Brazilian antitrust authorities had sufficient evidence on the use of ATPCO tools prior to the price increase of August 9.

It was until 2004, however, that the case was deliberated by the Administrative Council for Economic Defense, the final administrative body for antitrust cases in Brazil.[26] All involved airlines were convicted and each carrier was fined 1% of its annual revenue on the route in 1999. The total fines amounted to USD 300,000 for Tam and USD 1.5 million for Varig. On May 9, 2008, the defendants were also convicted by the Federal Court.[27]

4.2 The market

For decades, the Rio de Janeiro - São Paulo route was marked by an alliance of airlines named "Ponte Aérea". This agreement, formally established in 1959, aimed at restricting flight frequency competition between participating airlines to coordinate towards evenly spread schedules throughout a day of operations. This feature was viewed as highly beneficial to time-sensitive businessmen who formed the majority of travelers. Thus, the agreement soon conquered the sympathy of the regulator, which granted informal antitrust immunity. The

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Monitoring (SEAE), the Secretariat of Economic Defense (SDE), and the Administrative Council for Economic Defense (CADE), an independent administrative tribunal linked to the Ministry of Justice.

[25] The Airline Tariff Publishing Company (ATPCO) is a company that keeps a computerized airline price data system for information dissemination. This system contains past, current and future information on prices, fares, routes, first and last ticket dates and first and last travel dates for airlines and travel agents.

[26] CADE's decisions may only be reviewed by federal courts.

[27] On the subject, the Judge of the Third Federal Court stated: "*It is not arithmetics that has to be considered in order to deliberate on the existence of anticompetitive practices, but the effects of business conduct of agents present in the same geographic and relevant market*". The statement concludes: "(...) *all the evidence gathered for the case leads to the inevitable conclusion that the defendants made horizontal agreement to fix uniform prices. They built a cartel and harmed consumers.*"

concept was the origin of the "air shuttle" scheme that later was established on major routes around the world, such as with Eastern Airlines US West Coast shuttle and Iberia's Madrid-Barcelona "Puente Aereo". Traditionally, air shuttles have been a synonym for frequent air service on short-haul corridors, walk-on flights with even intervals, and no need for reservations.

The Ponte Aérea market consists of the downtown airports of Rio de Janeiro (Santos Dumont, IATA three-letters code SDU) and São Paulo (Congonhas, IATA code CGH), in a nonstop flight of 225 miles and approximately 45 minutes flight time. The route connects the two largest financial centers and services in the country. In fact, the São Paulo and Rio de Janeiro metropolitan regions together account for almost one fifth of the total population of Brazil, with 29 million inhabitants, and one third of the GDP of the country, USD 175 billion, more than double the national GDP per capita.[28]

The typical passenger in this market consists of the last-minute business traveler with a very short stay at the destination. These passengers are very time-sensitive and usually not sensitive to price.[29] Nearly 60% of the passengers are between 21 and 40 years of age, 70% are male and 83% are corporate travelers. Additionally, close to 50% of these travelers earn more than 85,000 USD a year, which is well above the national average income. The typical shuttle traveler is a frequent flier who makes between 2 and 3 air trips per quarter.[30]

The SDU-CGH route is the densest airport-pair of the Americas with 3.4 million passengers annually.[31] This is equivalent to 135 enplanements every quarter of an hour.[32] With almost 15% of the total passengers within the country, the route is notable for its very high service levels. In 1998, the average time between flights in a given direction on the route was 20 minutes, 15 minutes during peak hours.[33] The Ponte Aérea market is highly relevant to the income statement of any major airline in Brazil. With total annual revenues of roughly USD 125 million, this single market accounted for 10% of the total operating revenues and 15% of the total domestic enplanements within the country in 1997.[34]

In the sample period being considered here, the airport-pair SDU-CGH was the only relevant market for travel between Rio de Janeiro and São Paulo. Differently from travel markets between major metropolitan regions across the world, this airport-pair alone accounted for

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[28] Source: Brazilian Institute of Geography and Statistics (IBGE), 2000.

[29] Actually, seven out of ten declared they prefer last-minute flexibility to price in the survey, as indicated in Table 4.

[30] Source: Market research, Transportation Department of São Paulo State, 2005.

[31] Source: Statistical Yearbook of Air Transportation (2000). For comparison, Atlanta (ATL) - Orlando (MCO) was the densest airport-pair in the US market, with 2.8 million passengers in 2008 (source: US DOT). Los Angeles (LAX) - San Francisco (SFO) had 2.8 million and Boston (BOS) - New York (LGA) shuttle had 1.1 million passengers.

[32] Considering that CGH is open only 17 hours each day.

[33] Source: ANAC's airline schedule sheets (1998).

[34] Source: Brazil's Statistical Yearbook of Air Transportation (1997). Over the years, however, the SDU-CGH route has lost considerable market share in the Brazilian market. As air transportation increased in popularity across the country and began to rapidly expand, the Ponte Aérea market's participation dropped to less than 5% of the total revenues and less than 8% of the total enplanements in 2008.

99% of the local traffic between the endpoint cities.[35] Other possible airport-pairs within the same city-pair, for example, São Paulo's Guarulhos Airport (GRU) and Rio de Janeiro's Galeão (GIG), were traditionally used for transfer traffic, that is, passengers on the route for domestic and international flight connections at GRU. Both GRU and GIG are international gateways located relatively distant from downtown.[36] Other transportation means are not relevant for market delimitation as there is no fast train service available between the two cities and travel by coach takes more than six hours.

Many empirical studies have already found evidence of economies of spoke density, for example, Brueckner, Dyer and Spiller (1992) and Berry, Carnall and Spiller (2006).[37] In this situation, the higher the aircraft seat density of airlines, the higher their capability of enhancing traffic by allocating more connecting passengers on the same spoke flights. Such economies may be particularly strong for air shuttle markets where traffic density is far above average and, by consequence, the number of daily flights is typically very high. Additionally, the literature usually associates economies of density with scope effects that may emerge in hub-and-spoke networks. Scope economies arise as more itineraries include the same hub, creating a network effect. For hub-and-spoke monopolists, Brueckner, Dyer and Spiller (1992) found that the traffic enhancement caused by combinations of passengers with different origin-destinations in the same aircraft allows a more effective exploitation of increasing returns. This fact, together with the already dense traffic density of a shuttle, provides evidence for the existence of strong economies of density in that market. Moreover, the higher the number of cities served by the network of a carrier out of an endpoint airport, the higher the resulting traffic permitted by more connecting passengers and therefore, the stronger the economies of density.[38]

Of the two endpoint airports, only CGH is usually regarded as suitable for viable flight connections.[39] In 2000, for example, approximately 1 million enplanements at the airport were related to connecting passengers.[40] This represented 20% of the total airport traffic, making it the most important domestic hub in the country. The airport was particularly relevant to passengers with destinations in the southern part of the country. For example, in December 1998, 85% of the available flights between SDU and the airport of Curitiba, in southern Brazil, had a connection in CGH.[41] As a result, the flight frequencies of the Ponte

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[35] Source: ANAC, 2000. Note that, based on this evidence, the antitrust authorities have already declared SDU-CGH to be the only relevant travel market between Rio de Janeiro and São Paulo for antitrust investigation purposes.

[36] Note that, as discussed before, SDU-CGH's share of passengers has decreased along the 1990s and 2000s. Currently, other adjacent airport-pairs within the city-pairs must be considered for antitrust analysis.

[37] In contrast, Armantier and Richard (2003) found no significant relationship between marginal costs and quantities in their sample markets. For a recent study of economies of density in other network industries, see Holmes (2011).

[38] On average, there were 56 destinations directly connected to CGH and 21 destinations directly connected to SDU during the sample period.

[39] Flight connections or connecting traffic consists of passengers whose trips requires at least one change of plane.

[40] Source: airport enplanements and deplanements statistics report, 2000. Infraero Airports Operator.

[41] Source: Panrotas Flights Guide, 1998. Other major southern cities of Brazil also had the majority of

Aérea market are naturally available to passengers with connections at CGH. Roughly 10% of the passengers aboard Ponte Aérea flights are actually connecting passengers[42]. At the other endpoint, the SDU airport was clearly not used as a hub by carriers, as only 1% of its total domestic traffic resulted from connections.[43]

4.3 The players

During the sample period (1997-2001), there were five airlines in the Ponte Aérea market: Varig (RG), Rio Sul (SL), Tam (TA), Vasp (VP) and Transbrasil (TB). RG was Brazil's flagship carrier and for decades was the dominant airline on most domestic and international markets. It had the highest market share not only on the route but also at the airport and at city endpoints. With an outstanding in-flight service, a dedicated departure lounge with amenities at CGH, and a broader portfolio of destinations, RG was known for its appeal to the mainstream shuttle passenger, that is, those frequent-flier travelers willing to accumulate mileage points. SL was a subsidiary of RG and, since 1998, operated under a codeshare agreement with RG. TA was a former regional carrier, a rising star that ultimately conquered the top of the domestic rank in the mid-2000s. VP and TB were the low-fare, but not low-cost, alternatives with a reputation for low quality. They went bankrupt and exited the market in 2001 and 2004, respectively. Table 2 presents the evolution of market shares on the route.

Table 2 - Market share evolution[44]

Year	SL	RG	TA	TB	VP
1997	6.9	41.4	10.2	16.4	25.1
1998	14.5	33.6	20.2	13.4	18.4
1999	15.7	40.8	22.1	14.3	7.2
2000	7.8	42.7	27.9	5.5	16.1
2001	6.7	40.4	31.8	1.1	20.0

In Table 2, it is possible to observe that there was profuse variability of market share positions caused by the intense competitive movements in the market. RG sustained itself as the dominant firm of the route throughout the sample period, even after losing a considerable share of the market to TA and SL in the immediate post-deregulation of 1998. TA had an ever-increasing path, more than tripling its market share from 10.2% (1997) to 31.8% (2001). VP's market share declined from 25.1% in 1997 to less than 10% in 1999, but later reinforced its low-fare characteristics in the market and, thus, increased its market share to 20% in 2001. TB's share remained between 10% and 20% until mid-2000 when it left the market for the first time as a result of the exchange rate crisis of 1999. After trying to reposition itself in the

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available passenger routings with connections at CGH, for example, Porto Alegre (POA) and Florianópolis (FLN).

[42] Source: Sabre, 2008. Ponte Aérea travelers account for, respectively, 16% and 35% of total enplanements of CGH and SDU (Source: calculations based on available data set and Infraero data, 2000).

[43] Source: airport enplanements and deplanements statistics report, 2000. Infraero Airports Operator.

[44] Percent of total revenue passengers. Source: calculations using the available database.

low-fare niche in early 2001, TB exited for the second time by the end of June of that year. Ultimately, the considerable market share variability within the sample was beneficial to the present estimation of demand and supply models.

The brand positioning of airlines on the Ponte Aérea route was inspected with a combination of secondary and primary data. The secondary data comprise the available dataset and some statistics gathered from the regulator, to develop some metrics on the market attributes of existing airlines. The primary dataset consists of a survey especially designed for the purpose of assessing consumer perceptions and attitudes in the market. This survey was conducted by the author in 2000. With respect to the secondary data, Table 3 presents the main attributes of the carriers in the shuttle market.

Table 3 - Carrier characteristics - 2000[45]

Characteristics	RG-SL	TA	TB	VP
Average round trip fare on the route (USD)	169.5	152.6	115.7	104.2
Route share of passengers (%)	51.3	26.7	7.5	14.5
Route share of flight frequencies (%)	40.9	31.5	11.9	15.8
Route share of flight frequencies during peak hours (%)	42.0	32.2	11.2	14.6
Airport share of flight frequencies (%)	50.0	36.9	4.8	8.3
Average time interval between flights (minutes)	36.2	46.2	56.6	84.9
Network size (number of domestic destinations)	81	60	32	34
Availability of seat assignement	*yes*	*yes*	*no*	*no*
International alliance membership	*yes*	*yes*	*no*	*no*
Frequent flier program with elite status	*yes*	*yes*	*no*	*no*
Dedicated departure lounge with ammenities	*yes*	*yes*	*no*	*no*

We observe that both TB and VP were the low-fare alternatives in the market, with fares that were, on average, 40% lower than their rival's fares. The share of the low-fare market segment was small, however, as TB and VP carried less than a fifth of the passengers on the route. Firms with a higher market share had several attributes that supported their higher prices, however. In particular, RG-SL and TA had a greater share of the total number of flights and, therefore, had a broader portfolio of flights, which considerably increased their flight-time convenience for passengers. Indeed, RG-SL owned 40.9% of the frequencies on the route (42% at peak times), and TA had 31.5% (32.2% at peak times). The average time intervals between flights of these carriers was thus significantly lower than that of the low-fare carriers with 36 and 46 minutes for RG-SL and TA, respectively, compared with 57 and 85 minutes for TB and VP, respectively. With only half the time between flights, RG-SL and TA naturally received higher scores for preference and quality from time-sensitive passengers. In addition, these companies also dominated the endpoint airports, with more flights to a greater number of domestic cities being served - actually, more than double the number of served cities. This dominance allowed RG-SL and TA to enhance their presence in the airports' zone of influence

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[45] RG and SL operated under a codeshare agreement in 2000. Source: author's calculations using the available database and airlines' advertisements.

and to exert increased influence on travel agents.[46] Additionally, these carriers had important benefits for frequent-flier passengers, such as availability of seat assignments, international membership alliances, elite status and a departure lounge with dedicated amenities at CGH. The distinction between the two groups of airlines (RG-SL and TA versus TB and VP) in terms of attributes was quite clear.

Table 4 - Passenger characteristics and preferences - 2000[47]

Characteristics	RG-SL	TA	TB	VP
Business travelers (%)	75.6	66.3	43.4	52.5
Frequent fliers (%)	80.2	69.9	32.9	36.1
Last-minute purchasers (%)	86.8	81.9	79.0	78.7
Stated "airline service" as "important/very important" (%)	82.4	84.3	59.2	55.7
Stated "price" as "not/slightly/moderately" important (%)	61.5	53.0	17.1	24.6
Prefer "punctuality" than "discount" (%)	77.2	72.0	47.4	65.6
Prefer "in-flight service" than "payment installments" (%)	55.6	60.5	47.3	39.3
Prefer "flight time options" than "price" (%)	73.7	69.5	51.3	52.5
Prefer "last-minute flexibility" than "price" (%)	75.1	72.0	60.5	65.6

To check whether the above metrics are in accordance with the standpoint of passengers on the route, I use the results of the 2000 survey. This survey consisted of a set of questions aimed at prompting consumers to reveal their purchasing behavior and attitudes towards attributes of alternative airlines on the route. The market research consisted of questionnaires presented to 402 passengers at the departure lounge of CGH in January 2000. Likert scale-type questions were used in the survey.[48] I also employed dichotomous confirmation questions to check the validity of the responses. Table 4 presents the most important questions and a summary of the results for each carrier.

The results from the survey clearly indicated not only that there is heterogeneity among the consumers, but also that some specific brands are more attractive to the typical passenger, that is, the more time-sensitive and less price-sensitive consumer. For example, whereas more than 80% of the RG-SL and TA passengers stated that the airline's service is an "important" or a "very important" attribute, this amounts to less than 60% of the VP and TB's surveyed passengers. More than 50% of the RG-SL and TA's travelers stated they consider price as "not important", "slightly important" or "moderately important", whereas more than a 25% of passengers of VP and TB did so. Combining the above information from brand attributes and the survey information on passenger heterogeneity, it was possible to deduce the existence of two different market segments. The first is the "mainstream shuttle", which is formed by

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[46] Note that in the covered period, purchasing tickets by consumers over the Internet was still not a common practice.

[47] RG and SL operated under a codeshare agreement in 2,000. Source: authors' survey with shuttle passengers at CGH airport, January 2000.

[48] Likert points were with points being "unimportant", "slightly important", "moderately important", "important" and "very important".

RG, SL and TA. The second is composed of the "low-fare" alternatives of VP and TB, which may not be regarded as low-cost carriers, but due to market positioning, they represent a clear low price/low quality option to consumers.[49]

A final feature of the players was the codeshare agreements that were commonly observed during the sample period in this market. Codesharing is a common agreement in the airline industry in which a carrier allows one or more carriers to market and sell seats on some of its flights.[50] The first of such agreements was the Ponte Aérea "Pool", an alliance of airlines, formed by RG, VP and TB, which had a dominant position for almost forty years. The "Pool" was significantly impacted by the liberalization measures of the late 1990s and ceased operations in 1998. After Ponte Aérea's dissolution, new code share agreements were formed: VP-TB (September 1998 to May 1999) and TA-TB (May 2000 to January 2001).

5 The empirical study of market power

5.1 The demand side

In what follows, I present the demand side framework for the empirical model of competition in the air shuttle market. To accomplish that, I employ a nested-logit model (NL) structure with an outside option at the top level of the tree and two intermediate levels.[51] The main characteristic of the proposed demand framework is the unobservability by the researcher of both the bottom and the top levels. For the bottom level, I employ a Generalized Nested Logit (GNL) with overlapping nests in which each alternative is related to different nests in different degrees.[52] The motivation for that is the case of *shelf visibility* of a product at outlets - supermarkets, for example. Shelf visibility is a measure of the degree of interrelation between products and outlets. High shelf visibility share means higher outlet dominance, with the extreme situation being outlet exclusivity by single products. With this approach, the present framework has the models of Berry (1994) and Ackerberg and Rysman (2005) as special cases. For the top level, I use a *shopping frequency term* to model the size of the outside good and therefore to control for between-group substitutability.

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[49] The first low-cost carrier that entered the air shuttle market was Gol Airlines in 2002.

[50] See, for example, Armantier and Richard (2008) and Gayle (2008).

[51] Aguirregabiria and Ho (2011) also use a nested-logit model specification for airline demand.

[52] The GNL model was introduced by Wen and Koppelman (2001). It is a member of the Generalized Extreme Value (GEV) family, in which alternatives are grouped into nests but each alternative can be a member of more than one nest. It includes the paired combinatorial logit (PCL) and the cross-nested logit (CNL) as special cases. For a recent application, see Thomassen (2010).

The bottom level Consider the indirect utility u_j^{ir} of consumer i in retail outlet r regarding product j. Let $u_j^{ir} = \delta_{rj} + v_j^{ir}$, where δ_{rj} is a product–outlet specific portion of utility[53] and v_j^{ir} is a disturbance term related to unobserved consumer idiosyncrasies. Assume $\delta_{rj} = \delta_r + \delta_j$. Consider a nested logit framework in which the bottom level is formed by the grouping of retail outlets of each product. The immediate superior level is formed by the available products. For simplicity, assume there are no other intermediate levels.[54] The top level consists of the choice between the inside and the outside alternatives. Assume the researcher can only observe data aggregated at the product level but not at the outlet level.[55] In other words, the first nesting level is non-observable. Also, assume a generalized nested logit (GNL) with overlapping nests. In this NL setting, we have the more general case in which each outlet is allocated not necessarily to exactly one product (nest) in the tree, but may be related to different products in different degrees. The share of retail outlet r conditional on product j is therefore equal to

$$s_{rj} = \frac{\left(v_{rj} e^{\delta_{rj}}\right)^{1/(1-\widetilde{\sigma})}}{\sum_{m \in j} \left(v_{mj} e^{\delta_{mj}}\right)^{1/(1-\widetilde{\sigma})}}, \tag{16}$$

where $\widetilde{\sigma}$ is the correlation within the non-observable level, that is, among the outlets in which product j is available. v_{rj} is an allocation parameter that reflects the extent to which outlet r is a "member" of product j ($v_r \geq 0$, $\forall r$). If we set $\sum_{m \in j} v_{mj} = 1$ we have a GNL with the interpretation of v_{rj} being a measure of shelf visibility, or shelf space share, for product j to consumers at retail outlet r. In the product level, we have that the share of product j conditional on group g obeys the following relation:

$$s_{jg} = \frac{e^{I_{jg}/(1-\sigma)}}{\sum_k e^{I_{kg}/(1-\sigma)}}, \tag{17}$$

where σ is the correlation among products and I_{kg} is an inclusive value of product k with respect to group g. For product j, we have that $I_{jg} = (1-\widetilde{\sigma}) \ln \sum_{m \in j} \left(v_{mj} e^{\delta_{mj}}\right)^{1/(1-\widetilde{\sigma})}$. As $\delta_{mj} = \delta_m + \delta_j$, we then have $I_{jg} = \delta_j + (1-\widetilde{\sigma}) \ln \sum_{m \in j} \left(v_{mj} e^{\delta_m}\right)^{1/(1-\widetilde{\sigma})}$. In the Appendix I show that the inclusive value I_{jg} may be conveniently approximated by expression $I_{jg} = \delta_j + \ln \phi_{0j} + (1-\widetilde{\sigma}) \phi_j \ln R_j$, where R_j is the number of retail outlets in which product j has any visibility and ϕ_{0j} is a function of a proxy for mean quality ($\widetilde{\delta}_{rj}$) and mean visibility ($\overline{v}_j$) of product j's outlets.[56] ϕ_j is an adjustment parameter for mean shelf visibility, $0 \leq \phi_j \leq 1$. The higher the ϕ_j the higher the visibility, with the extreme case being full product visibility ($\phi = 1$), when product j has exclusivity of all its outlets.[57] If $\phi = 0$, then product j has null

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[53] Note that I assume $\delta_j = \delta_j^{ir}$ for all $r \in j$.

[54] See next section for a model with one additional intermediate level.

[55] This assumption is also part of the model of Ackerberg and Rysman (2005).

[56] Note that $\widetilde{\delta}_{rj} \geq 1$ and $0 \leq \overline{v}_j \leq 1$.

[57] This case nests the case of contract clauses that confine retailers exclusively to only one producer. Antitrust authorities may consider these practices as ways of restricting access to the market and an infringement

visibility at all outlets but one, meaning the complete absence of a multi-outlets strategy by producers.[58]

Considering that $s_j = s_{jg} s_g$ and using (16) and (17), the corresponding estimating equation using Berry (1994) inversion procedure is $\ln(s_j/s_0) = I_{jg} + \sigma \ln s_{jg}$, where s_j and s_0 are the market share of j and of the outside good, respectively. We then have the following empirical specification:[59]

$$\ln(s_j/s_0) = \delta_j + \ln\phi_{0j} + (1-\widetilde{\sigma})\,\phi_j \ln R_j + \sigma \ln s_{jg}. \tag{18}$$

Note that R_j appears in (18) in a similar way as with the proposal of Ackerberg and Rysman (2005) to control for congestion in the unobserved product characteristic space.[60] The present framework provides a more structural approach for congestion, as it is based on the bottom level of the decision-making tree. Besides that, it incorporates the authors' model and also Berry (1994) as special cases. We have Berry's model when $\phi_{0j} = 1$ and $\phi = 0$, which is the case when outlets have zero mean utility[61] and products have null visibility at outlets.[62] Ackerberg and Rysman (2005) also implicitly assume zero mean outlet utility ($\phi_{0j} = 1$) but do not assume null product visibility. Instead, they assume a restricted form of visibility in which $\phi_j = \phi = (1-\sigma)/(1-\widetilde{\sigma})$, that is, visibility is such to make the correlation between outlets ($\widetilde{\sigma}$) proportional to the correlation between products (σ). The more flexible approach here allows data to select any of these extreme parameters but also other intermediate values. In sum, the present model allows for any mean outlet quality and product visibility at outlets.

In empirical terms, it is possible to estimate (18) by using product fixed effects to control for $\ln\phi_{0j}$, assuming that outlet quality and visibility are constant over time in the sample period. Also, it is possible to use a proxy for the number of outlets to account for $\ln R_j$. Although it is not possible to have separate identification of ϕ_j and $\widetilde{\sigma}$, this is irrelevant for empirical purposes.[63] Another important issue is that, as with the coordinated market power parameter, discussed in 3.3, here we have overparameterization of ϕ_j, that is, product-specific ϕ terms. In line with Ackerberg and Rysman (2005), we may put some structure on ϕ_j by, for example, setting $\phi_j = \phi$. Other restrictions are possible and, again, are application-specific. Finally, with both $\ln\phi_{0j}$ and $\ln R_j$ controlled, we have not only congestion, but also unobserved outlet quality, controlled in a single demand framework. As these factors are typically correlated with price, thus controlling for them in (18) prevents omitted variables bias.[64]

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of competition law.

[58] See the Appendix for details.

[59] Contrary to Ackerberg and Rysman (2005), I employ the same σ of Berry (1994). Hence, $\sigma = 1 - \sigma_{AR}$, where σ_{AR} is the correlation term employed by the authors.

[60] Their proposal is for the logit family. As with the framework of the authors, this correction may be applied to random-coefficients models, by simply adding term $\ln\phi_{0j} R_j^{\phi(1-\widetilde{\sigma})}$.

[61] That is, $\widetilde{\delta}_{rj} = 1$. Note that Berry (1994) also assumes zero mean utility of the outside good. This is equivalent to impose the restriction that outlets and outside good have the same utility.

[62] That is, $\overline{v}_j = 0$. See the Appendix for details.

[63] Ackerberg and Rysman (2005) also emphasize this issue in their specification.

[64] It is still necessary to instrument R_j, however. See 5.5 for detais on the instrumentation strategy.

The top level The top level of the NL tree represents the consumer choice between the outside good and the inside good. As with the bottom level, I assume lack of unobservability by the researcher, by assuming a non-observed outside good size q_0. Total population is POP. *Market penetration*, that is, the probability of an individual of population effectively choosing the inside good is Ψ.[65] *Potential consumer shopping frequency* is equal to τ, the average number of purchases that a consumer potentially makes within a period.[66] Total market size in number of purchases is $M = q + q_0$, where q is the total inside good ($q = \sum_{k,k\neq 0} q_k$) and $q_0 = (POP \times \tau)(1 - \Psi)$. We therefore have $M = POP \times \tau$.[67] Both Ψ and τ are unobserved by the researcher. The left-hand side of (18) becomes

$$\ln(s_j/s_0) = \ln q_j - \ln(POP \times \widetilde{\tau}) = \ln(q_j/POP) - \ln \widetilde{\tau}, \tag{19}$$

where $\widetilde{\tau} = \tau(1 - \Psi)$ is an adjusted potential purchase term. $\widetilde{\tau}$ acts as a scaling factor making the unobserved ratio s_j/s_0 exactly proportional to the observed ratio q_j/POP. The higher the $\widetilde{\tau}$ the higher the outside good. Although the researcher does not observe $\widetilde{\tau}$, he can control for it. This is straightforward, as we may have $\ln \widetilde{\tau}$ displaced in the right-hand side of the Berry's estimating equation. In this case it is possible to control it with both market and time fixed effects and some aggregate demand shifters.[68] This proposal does not make either τ or Ψ identifiable from data, but is relatively easy to implement and avoid arbitrary assumptions regarding the size of the outside good and, consequently the price elasticities of demand.[69]

Yet with respect to the issue of identification, a key aspect in the above framework is related to the separability of $\ln \widetilde{\tau}$ from both the constant term of the utility function δ_j - say δ_{0j} - and the term $\ln \phi_{0j}$ in (18). In principle, these terms are not separable and therefore identification of the size of the outside good may be impacted. It is possible to address this problem with some basic procedures, however. First, we may use outside good (shopping frequency) shifters in the empirical specification to account for a substantial portion of $\ln \widetilde{\tau}$. Second, with respect to the constant of the empirical specification of $\ln \widetilde{\tau}$, it is possible to take advantage of the fact that the absolute level of utility is irrelevant from the decision maker's

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[65] Factors such as price, income, employment, tastes or outlet proximity are key to the decision of individuals regarding either the inside or the outside option.

[66] I still assume a random-utility model in which an individual consumes one unit of the product that yields the highest utility. However, it is commonly observed that the analyst does not have data frequency that is exactly consistent with the timing of the consumer choice. For example, with monthly data a given consumer may have thirty serving of breakfast cereals. In this example, potential market is one serving per capita per day (Nevo, 2000). For emerging countries it often a strong assumption to consider the same market potential of highly developed countries. For example, potential market for cereal breakfast may well be 15 serves a month, ie, 0.5 serving per day. Therefore in some markets is not reasonable to set τ equal to 1. Albuquerque and Bronnenberg (2009) also have a framework for estimating unobserved market penetration rates.

[67] Berry (1994), p. 247, was the first to suggest that when total market size is not directly observed, it would have to be estimated or parameterized as depending on shifters that affect product aggregate demand, such as population.

[68] For other attempts to deal with the problem of unobservability of the outside good, see Huang and Rojas (2010).

[69] See Slade (2009) for a discussion on how price elasticities of demand are sensitive to the size of the outside good.

perspective in a discrete-choice model. As it is impossible to estimate specific constants of the utility function for *all* alternatives, the researcher must normalize the absolute levels of these constants. The standard procedure in this case is the normalization of the constant term of one of the alternatives to zero (Train, 2009, p. 25). That is, for a base-case alternative, we impose $\delta_{0j} = 0$. By using the same reasoning, we impose $\phi_{0j} = 1$ for the same base-case alternative. With the constant of the utility function of the base-case alternative normalized to zero, it is then possible to identify the constant of the outside good by considering the original (unnormalized) constant of the alternative. The resulting constant of the outside good holds for all alternatives. The only caveat of this procedure is that, as the size of the outside good may be sensitive to the choice of the base-case alternative chosen by the researcher, it is crucial to employ outside good shifters and to experiment with different base cases.

Application to the air shuttle market I use the above GNL framework to model passenger demand in the air shuttle market. Assume that passengers choose flights according to their most preferred departure times. Also, consider the above products-outlets nesting scheme to represent the flight portfolio of airlines.[70] When flights are marketed by a single carrier for only local passengers, then airlines have full product visibility, that is, exclusivity of flights for use on the route. When carriers engage in code share agreements or have transit passengers on board, however, then flights are shared and a subset of seats of a subset of flights is designated to either allying carrier(s) or to other origin-destination product(s) of the same airline. In this case, airlines do not have exclusivity of flights and therefore have only partial product visibility in the market. As the number of seats allocated to codesharing partners is private information, then the codesharing scheme is unobserved by the researcher. Also, the dataset does not contain information on transit/transfer passenger traffic.[71] Consistent with the outlets model, we then have to control for the effects of airline-flights visibility and flight-specific quality.

We then have a nested logit tree with four levels. Levels 1 and 2 are formed by, respectively, available flights and airlines, with level 1 being unobserved.[72] I also consider airline segmentation (level 3), with carriers classified into "mainstream air shuttles" (carriers RG, TA and SL) and "low fare air shuttles" (carriers TB and VP).[73] This segmentation was discussed in 4.2 and is based on the 2000 market research on carrier and passenger characteristics. Fi-

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[70] Modeling consumer choice of outlets is consistent with an address approach where geographically dispersed consumers choose locations. Thill (1997) extends the location theory to assume multi-outlet firms serving multiple locations. Borestein and Netz (1999) and Salvanes, Steen and Sogard (2005), apply location theory to model airline competition. In these models, airline schedules may be interpreted as location on a line representing a daily time schedule, with passengers choosing their most preferred departure times.

[71] Transfer traffic is formed by connecting passengers. Transit traffic is composed by passengers remaining on board the aircraft.

[72] The available dataset does not contain information on the distribution of demand across flights. This situation of data aggregated at the market-carrier level is typical in airline studies.

[73] Remember from the discussion of Section 4 that there was no low cost airline on the route in the sample period, but, due to market positioning, VP and TB were the low fare alternatives to consumers.

nally, level 4 comprises passenger decision regarding the outside good. Figure 2 presents the nesting structure for the market.

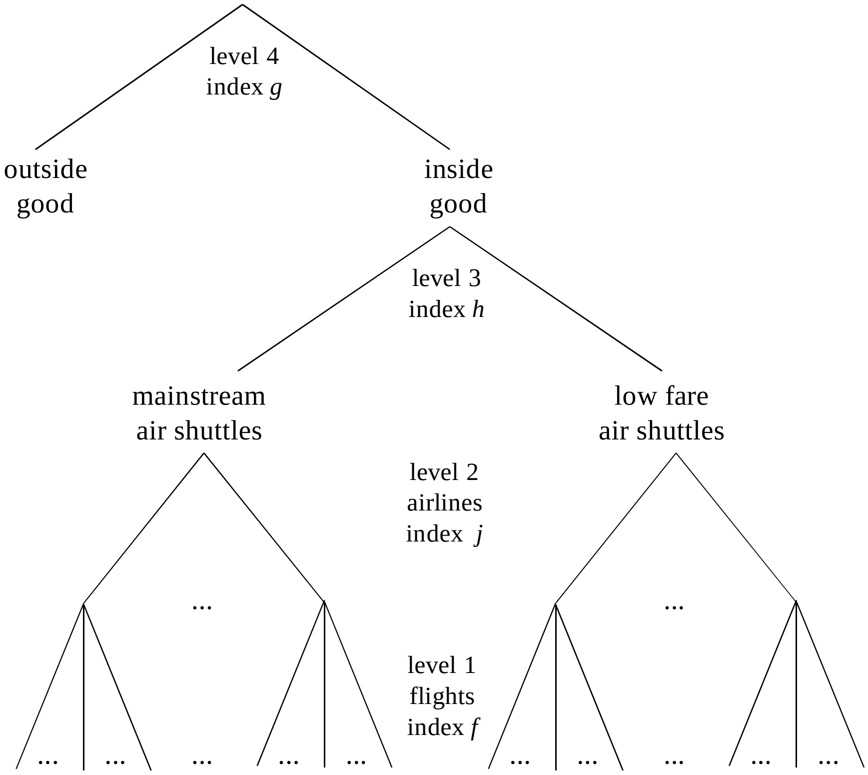

Figure 2: Nesting structure of the air shuttle market

Assume passenger utility function equal to $u_j^{if} = \delta_j + \upsilon_j^{if}$. With four levels, υ_j^{if} therefore has the following error component structure: $\upsilon_j^{if} = \epsilon^i + \epsilon_g^i + (1 - \sigma_2)\,\epsilon_{hg}^i + (1 - \sigma_1)\,\epsilon_{jh}^i + (1 - \widetilde{\sigma})\,\epsilon_j^{if}$, where f, j, h, and g represent nesting levels one to four. ϵ^i, ϵ_g^i, ϵ_{hg}^i, ϵ_{jh}^i, and ϵ_j^{if} are, respectively, passenger i's taste, group-specific taste, segment-specific taste, airline-specific taste, and flight-specific taste. And $\widetilde{\sigma}$, σ_1, σ_2 are correlation measures. Assuming that υ_j^{if} follow a type-I extreme value distribution, we then have the following brand-level market shares:

$$s_r = s_{rj} s_{jh} s_{hg} s_g = \frac{e^{\delta_j/(1-\widetilde{\sigma})}}{e^{I_{jh}/(1-\widetilde{\sigma})}} \frac{e^{I_{jh}/(1-\sigma_1)}}{e^{I_{hg}/(1-\sigma_1)}} \frac{e^{I_{hg}/(1-\sigma_2)}}{e^{I_g/(1-\sigma_2)}} s_g, \tag{20}$$

where s_r is the market share of outlet r, s_{rj} is the market share of outlet r conditional on product j, s_{jh} is the market share of product j conditional on subgroup h, s_{hg} is the market share of subgroup h conditional on group g, and s_g is the market share of group g. I_{jh}, I_{hg} and I_g are inclusive values. Aggregating in s_j, we have that the resulting estimating equation consistent with (18) is therefore

$$\ln(s_j/s_0) = \delta_j + \ln\phi_{0j} + \phi_j\,(1 - \widetilde{\sigma})\ln F_j + \sigma_1 \ln s_{jh} + \sigma_2 \ln s_{hg}, \tag{21}$$

where F_j is the total number of flights of airline j. We expect partial visibility, that is, $0 < \phi_j \leq 1$, as all airlines have transit/transfer traffic in their flights, and as the airlines engaged in codesharing within the sample period. ϕ_{0j} controls for unobserved flight-specific quality and visibility. With respect to the outside good, using (19) we have $\ln(s_j/s_0) = \ln(q_j/POP) - \ln\widetilde{\tau}$, with $\widetilde{\tau} = \tau(1-\Psi)$ being our term for potential purchase frequency. We then finally have:

$$\ln(q_j/POP) = \ln\widetilde{\tau} + \delta_j + \ln\phi_{0j} + \phi_j(1-\widetilde{\sigma})\ln F_j + \sigma_1 \ln s_{jh} + \sigma_2 \ln s_{hg}, \qquad (22)$$

which is our empirical framework for estimation.

5.2 The data

The available sample for this study consists of market level data on output, prices, costs and capacity specific for the air shuttle route.[74] Data were obtained from the Brazilian airline regulator – the National Agency for Civil Aviation. Differently from studies for the US airline market, the core data source here is not a sample of tickets like the US DOT's Origin-Destination Survey Databank. It contains information submitted by regulated firms to the authority according to specific legislation on economic and operating data disclosure. Data are published in Brazil's Air Transportation Yearbooks. The yearbooks contain detailed economic data at the airline-aircraft level for the aggregate national, international and regional markets. The only disaggregated data is related to the shuttle market, being available in a yearly basis. Monthly data was sent upon request. With these data, it is possible to construct airline-market specific monthly figures on the number of passengers carried in each direction (CGH-SDU and SDU-CGH), proxies for average fares and average costs for the most important airline production inputs such as fuel, maintenance, flight crew, etc. All monetary data are deflated by using a consumer price index.[75] Traffic data consists of all local revenue passengers, and therefore do not include either transfer or transit passengers on the route.[76]

On airline schedules, airline-route-specific data on flight operations were monthly collected at HOTRAN database. HOTRAN is similar to the more widely known Official Airline Guide (OAG) and is used by the regulator to register all scheduled flights within the country. It contains information on time and itinerary for all nonstop and direct flights of Brazilian commercial airlines, being therefore disaggregated by airline/flight code.

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[74] Berry and Haile (2010) note that dealing with market level data is a typical situation in empirical IO, where the researcher observes market characteristics, product quantities, prices and characteristics, and product/market level cost shifters - but does not observe individual choices and firms' costs.

[75] Source: consumer price index, Brazilian Institute of Geography and Statistics (IBGE), 1997-2001.

[76] Here I do not observe either transfer/transit traffic on shuttle flights or traffic in alternative routings in the same origin-destination market, eg. flights with intermediate stops or connections. These data were not collected by the regulator. However, other traffic than from nonstop flights on the route was virtually null. As Neven and de la Mano (2010) discuss, on short-haul routes one-stop services do not generally represent a significant competitive constraint to non-stop services. For the episodes of codeshare agreements, traffic is always accounted to the marketing carrier and not to the operating carrier.

Each observation in the final data set is an airline-directional airport-pair-month combination. The data set consist of a panel of airlines for the sample period from January 1997 to December 2001, for carriers RG, SL, TA, VP and TB. The database thus comprises 60 points for every firm in each directional airport-pair. The only exception is the TB, which exited the market in June 2000 and reentered in February 2001, and therefore has 46 sampling points for each directional airport-pair. Thus, the whole set is a sample with 572 observations for model estimation.

5.3 The empirical specifications

As I use panel data, for the empirical specification of (22) and of the proposed supply relation of (14), we have subscripts j (airline), r (direction) and t (month). For simplicity, I disregard indexes r[77] and t. Below I present the variables used in the demand-side and supply-side specifications.

For the demand side we have, on the left-hand side of (22), q_j as the average number of scheduled daily revenue local passengers in each directional market. Also, POP is the sum of the population of the endpoint cities.[78] The regressor of the demand equation is therefore $\ln(q_j/POP)$. On the right-hand side of (22), I model the potential purchase frequency of passengers with expression $\ln\widetilde{\tau} = k_0 + k_1 \ln activity + k_2 nholidays$, where $activity$ is the mean proportion of economically active population of endpoint cities,[79] and $nholidays$ is the number of holidays.[80] These shifters account for changes in potential purchase frequency behavior by typical shuttle passengers. Higher *activity* means economic growth and need of business-related trips. By contrast, higher *nholidays* means more spare time availability for leisure-related trips on other, more tourism-related, routes. I therefore expect potential purchase frequency to increase in *activity* and decrease in *nholidays*. I also control for seasonality (month dummies and airline-specific seasonal dummies of Summer and July vacation periods) and trend (quarter-specific dummies and airline-specific trend).

As we lack observability of the flights level, and consistent with our previous discussion on flight unobserved quality and visibility during codeshare agreement periods, we have ϕ_{0j} of (22) therefore controlled with airline/codeshare periods dummies.[81] Also, on account of panel

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[77] I did not observe any systematic difference between directions. Direction-specific controls were not statistically significant.

[78] Considering total population within the greater metropolitan area of each city. Geometric interpolation of anual figures was performed to produce monthly population figures. Sources: regulator and IBGE, respectively.

[79] Tat is, the geometric mean of proportions of economically active populations. Source: ipeadata.gov.br. Other metrics were also attempted, for example, the sum and geometric mean of economically active population of both cities, the sum and geometric mean of populations, etc. Results were robust to changes in this variable.

[80] k_0, k_1, and k_2 are unknown parameters. I identified k_0 by employing the normalization of the constant of a base-case alternative discussed before. To check for robustness, I experimented with all possible base cases within the sample. In all cases, estimated elasticities did not change significantly: on average, variations were less than 0.1% for own-price elasticities and less than 0.5% for cross-price elasticities.

[81] I also experimented with an interaction between lnf_j and $codeshare_j$, where $codeshare_j$ is a dummy of code share period of airline j. This interaction was not statistically significant in the regression model of

data limitations, I impose $\phi_j = \phi$.[82] With respect to the congestion metric (F_j), I perform the following robustness checks. I first use F_j, the number of flight frequencies of airline j. I also use the specification with K_j, the total number of seats of airline j, in replacement of F_j. Both congestion terms (F_j and K_j) were extracted from HOTRAN reports. I also experiment with a specification without any control of congestion.

The mean utility term δ_j is modeled as $\delta_j = \delta\left(p_j, \xi_j\right)$ in a linear fashion, with p_j being the average price of airline j, and ξ_j being a random term of Berry (1994), which here controls for unobserved airline characteristics. The price variable, p_{jt}, is a proxy for the average fare of airline j on the route at t. More specifically, it is the average revenue per passenger on the route, being airline-specific, but not direction-specific.[83] I employ interactions of p_j with the airline group dummies $mainstream$ (airlines RG, TA and SL) and $lowfare$ (airlines TB and VP), respectivelly, $p_{jt} \times mainstream$ and $p_j \times lowfare$. Therefore, I allow for group-specific price sensitivities in δ_j. Additionally, I employ an error-component structure for ξ_j, that is, $\xi_j = \zeta_j + \varepsilon_j$, where ζ_j is controlled with airline fixed-effects and ε_j are the residuals. Consistent with the fixed-effects procedure, I assume $corr\left(\zeta_j, \varepsilon_j\right) \neq 0$ and $corr\left(\phi_{0j}, \varepsilon_j\right) \neq 0$, that is, airline flight visibility is correlated with airline unobserved characteristics.[84]

For the supply relation of (14), I assume a differentiated, multiproduct, price-competition setting, and therfore employ the following empirical specification:

$$\begin{aligned} \widetilde{p}_j &= C\left(fuel_j, crew_j, q_j, \widetilde{\omega}_{-j}, \widetilde{\omega}_{-j,\mathcal{F}}\right) + \\ &\quad + \theta_1 \widetilde{p}_{-\mathcal{F}} + \theta_2\left(\widetilde{p}_{-\mathcal{F}} \times invper\right) + \epsilon_j, \end{aligned} \tag{23}$$

where $\widetilde{p}_j$ and $\widetilde{p}_{-\mathcal{F}}$ are the own and rival price variable transformations, as indicated for the differentiated, multiproduct setting with price competition - see 3.1 and Proposition 2. $fuel_j$ is the unit cost of jet fuel for airline j.[85] $crew_j$ is the unit costs of direct labor (crew members) for airline j.[86] q_j variable is the same number of passengers, described above.[87] $\widetilde{\omega}_{-j}$ and $\widetilde{\omega}_{-j,\mathcal{F}}$ are proxies for the rivals' cost variables transformations $\widetilde{c}_{-j}$ and $\widetilde{c}_{-j,\mathcal{F}}$ of (11). For these figures, I used the average cost per passenger of labor plus jet fuel. And finally, $invper$ is a dummy variable that control for the period of alleged price parallelism of airlines on the route - that is, from August 1999 to December 1999. Note that, to avoid overparameterization, I consider

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demand.

[82] I experimented with some specifications that attempted to account for heterogeneity in ϕ. For example, by interacting $\ln F_j$ with dummies of airline groups, and by running regressions for each firm separately. In contrast with the simpler specification with $\phi_j = \phi$, none of these attempts resulted in statistically significant coefficients for the ϕ terms.

[83] Source: regulator.

[84] Note that ζ_j and ϕ_{0jt} are non-separable in the empirical specification.

[85] Calculated by dividing the total monthly expenditure on fuel by the total gallons consumed. Cost data were obtained from economic and financial monthly reports of ANAC (regulator). Values were deflated by using the consumer price index.

[86] Calculated by dividing the costs of wages by total flown hours. Cost data were obtained from reports of ANAC (regulator). Values were deflated by using the consumer price index.

[87] Berry and Haile (2010) assume marginal costs as a function of the output quantity of the product, and some observed and unobserved cost shocks within their nonparametric marginal cost function.

non-firm-specific θ terms.[88] Below are descriptive statistics of the original variables of the demand and the supply relation models.

Table 5 - Descriptive statistics[89]

Variables	Mean	Std. Dev.	Min.	Max.
q_j	800.7520	524.9432	42.5000	2311.0000
POP	13.5627	0.3007	13.0676	14.1058
$activity$	57.0265	0.8457	55.2831	58.2207
$nholidays$	1.5839	1.3924	0	5
p_j	151.8260	38.6124	68.4364	253.1509
K_j	1634.5890	946.0239	296.2661	3704.0320
F_j	13.0025	7.4115	2.7432	28.0601
s_{jh}	0.41958	0.2407	0.0512	1
s_{hg}	0.5628	0.2310	0.0876	0.9124
$fuel_j$	437.3966	196.2931	232.4240	979.6028
$crew_j$	695.3778	203.7854	78.31482	1498.0710

5.4 Estimation and empirical tests

The estimation method employed is the two-step feasible efficient generalized method of moments (GMM) estimator for single equations, equal to $\widehat{\vartheta}_{2SEGMM} = \arg\min_{\vartheta} \left[N\overline{g}\left(\vartheta\right)' W\overline{g}\left(\vartheta\right)\right]$, where ϑ are the unknown parameters, N is the sample size, $\overline{g}\left(\vartheta\right)$ is the set of sample moment conditions and W is an optimally chosen weighting matrix. Efficiency is obtained by setting $W = Z^{-1}$ in the second step, where Z is the covariance matrix of orthogonality conditions, estimated in the first step. Estimates are efficient for arbitrary heteroskedasticity and autocorrelation. The bandwidth used in the estimation with a Newey-West kernel is set equal to four - that is, a three-months autocorrelation structure.

Airline fixed effects and quarter-specific effects are employed in both demand and supply relation specifications. Additionally, controls of seasonality (dummies of month) and airline-specific trends are used. And finally, in line with Bai (2009), to control for heterogeneous responses over cross sections, the following interactive effects are introduced: airline-season and airline-codeshare agreement periods effects.

For the estimation of the supply relation, as discussed in 3.3,the use of transformed variables requires correction of the standard errors of estimates in order to account for the fact that they are are calculated using previously estimated demand parameters. I employ bootstrap in order to perform this correction. The BCa procedure is performed for generating

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[88] I experimented with airline group-specific by introduccing variable $\widetilde{p}_{-\mathcal{F}t} \times invper \times lowfare$. This variable was statistically not significant at 1%, but was significant at 5%. This result slightly suggests heterogenous incentives to engage in efficient collusion but deserves further investigation with an increased sample size. Overall, results were robust to the introduction of this variable.

[89] Indexes r and t ommited.

standard deviations. The BCa procedure is a method of setting approximate confidence intervals for the parameters of interest from the percentiles of the bootstrap histogram (DiCiccio and Efron, 1996). Bootstrap samples are taken independently within each airline stratum. A thousand bootstrap samples are drawn. Demand parameters are estimated before supply relation estimation in each bottstrapped sample.

5.5 The identification strategy

In order to estimate the above empirical framework, one needs instrumental variables for prices (p_j), quantities (q_j) and shares (s_{jh} and s_{hg}).[90] In the nested logit equation, one also needs to instrument the congestion terms K_j and F_j. Also, in the supply relation, not only the variables related to $\widetilde{p}_{-\mathcal{F}}$ have to be instrumented, but also the average cost variables $\widetilde{\omega}_{-j}$ and $\widetilde{\omega}_{-j,\mathcal{F}}$.

I assume that the unobservable factors of demand and prices are mean-independent of observable product characteristics and cost shifters as in Berry, Levisohn and Pakes (1995). Not all cost shifters are used as instruments, however. Due to potential effects of unionization on labor costs, instead of $crew_t$, I employed proxies for maintenance unit costs (levels and logs). In terms of product characteristics, I used the number of flight frequencies during peak times within a week of operations, during only weekdays, and during codeshare agreements.[91] I also used the economically active population series, disaggregated by city.

Besides demand and cost shifters, I also consider a set of instruments related to the number of destinations of each carrier out of the endpoint cities. This set of instruments has good variability across airlines and across time within the sample. It includes the number of destinations of airline j and its subgroup h, for both domestic and international markets, and in both levels and logs. Carriers' portfolio of destinations is a strategic variable usually considered fixed in the short-run by the literature. Therefore this variable is typically conceived as not being correlated with unobserved, short-run, shocks of either demand or supply.

On the demand side, the number of destinations available to passengers is certainly a factor that enhances not only demand but also the loyalty of consumers. It is clearly related to the quality of frequent flier programs. For example, the higher the number of destinations out of a given city, the higher the number of possibilities for bonus trips obtained from mileage accumulation. Also, to keep more destinations out of a city require more intensive marketing efforts by airlines. These marketing efforts typically enhance overall carrier visibility to consumers and therefore have spillover effects over the routes from the city. As a result, the broader the portfolio of destinations of an airline out of a city, the higher the levels of consumer brand awareness through reminder advertising, periodic promotions and dominance of channels of distribution in the city.

Another advantage of using the number of destinations as instruments is related to its

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[90] For example, in the above demand setting it is clear that I assume $corr\left(p_{jt}, \zeta_{jt}\right) \neq 0$, and therefore p_{jt} needs to be instrumented.

[91] In this case, the sum of allied carries flight frequencies during peak times were employed. Peak time was considered from 6am to 10am and from 5.30pm to 10pm.

lack of costs side effects. According to classic airline literature, the number of destinations is a proxy for the scale of operations, therefore being used to test the presence of economies of scale. Typically, empirical studies do not find evidence of economies of scale in the airline industry (for example, Caves, Christensen and Tretheway, 1984). In this case, this variable may be regarded as not correlated with the unobservables at the costs side, but correlated with the output and therefore used to instrument the endogenous variables of the right-hand side of the supply relation. I use a C-test of a subset of the overidentifying restrictions (Eichenbaum, Hansen and Singleton, 1988) to check the validity of the number of destinations for the instrumentation. This test requires the computation of the difference between two Hansen J statistics, calculated form the restricted and the unrestricted models. In the unrestricted, the subset of investigated instruments is removed from the list of instruments. The statistic is χ^2 distributed and has the null hypothesis that the specified instruments are orthogonal. Instruments passed the orthogonality test, that is, did not reject the null.[92]

5.6 Estimation results

Table 6 presents the estimation results of the nested logit with alternative specifications. Column (1), labelled "Crowding F_j" has the lowest GMM objective function value and is the preferred model. It is clear from Table 6 that both models of Columns (1) and (2) have estimated coefficients compatible with the interpretation arising from economic theory, regarding signs and statistical significance. The estimated marginal effect of price is negative and significant at 5% level for both the mainstream and the low fare segments. Consistent with the results of the 2000 quality survey with shuttle passengers, we find that mainstream airlines have roughly 30% lower marginal price effect in absolute values, respectively, -0.00076 against -0.00110 of budget carriers. This difference is statisticaly significant at 5%. Regarding the estimated coefficients of $\ln s_{jh}$, $\ln s_{hg}$ and $\ln F_j$, we have that, as expected, these estimates are positive and statistically significant. The outside good shifters $\ln activity$ and $nholidays$ work properly, allowing the control of fluctuations in potential market with statistical significance. As expected for a route with strong participation of business-related trips, the estimated effects of economic activity on the purchase frequency is positive, whereas the estimated effect of number of holidays is negative.

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[92] C-statistic $= 8.584$, $p - value = 0.5720$ ("Crowding F_j" model).

Table 6 - Estimation Results - Nested Logit[93]

	(1) *Crowding* F_j	(2) *Crowding* K_j	(3) *No Crowding*
$p_j \times mainstream$	-0.00076**	-0.00075**	-0.00075**
	[0.00034]	[0.00034]	[0.00034]
$p_j \times lowfare$	-0.00110**	-0.00106**	-0.00094**
	[0.00044]	[0.00043]	[0.00041]
$\ln s_{jh}$	0.97080***	0.97178***	0.99167***
	[0.01354]	[0.01338]	[0.00940]
$\ln s_{hg}$	0.95716***	0.95862***	0.96769***
	[0.02081]	[0.02061]	[0.01863]
$\ln F_j$	0.03346**		
	[0.01427]		
$\ln K_j$		0.03361**	
		[0.01475]	
$\ln activity$	2.74793***	2.71651***	2.82963***
	[0.53623]	[0.53739]	[0.52728]
nholidays	-0.01577***	-0.01581***	-0.01634***
	[0.00153]	[0.00153]	[0.00155]
GMM Obj.	28.57095	28.66873	32.94910

Overall, the estimation results are robust to the substitution of the congestion controls from $\ln F_j$ to $\ln K_j$ in Column (2). Also, in order to assess the impacts of not performing any control of congestion, we have the results of Column (3) - the "No Crowding" specification. Note that the lack of congestion control generates major changes in the estimates of key parameters of the model. In particular, we have roughly a 15% decrease in the absolute value of the marginal effect of the price of low fare carriers ($p_j \times lowfare$). Also, the difference in price effects between low fare carriers and mainstream carriers drops from 30% down to 20%, and thus making nests more similar in terms of price sensitivity than they are when congestion is controlled. Additionally, we have an important increase in the effects of the intra-subgroup and intra-group shares (s_{jh} and s_{hg}), being the first mostly impacted. These estimates ultimately make the estimated price elasticities to become excessively high and often

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[93] Heteroskedasticity and autocorrelation-robust standard errors in brackets. Superscripts *, **, and *** denote, respectively, significance at the 10%, 5%, and 1% levels. Estimated effects of airline-period, airline-codesharing periods, month, quarter and airline-specific trends not reported.

not statistically significant. We believe that in this case the researcher, by not controlling for either $\ln F_j$ or $\ln K_j$, is incurring in bias due to the correlation between prices, shares and the terms of congestion.[94] With respect to price elasticities, we have that both the "Crowding F_j" and "Crowding K_j" specifications had estimates that are consistent with the market positioning of firms described in Section 4. Airlines in the mainstream air shuttle nest (RG, TA and SL) have lower estimated own-price elasticities than airlines of the low fare air shuttle nest (VP and TB).[95]

Table 7 presents the estimation results of the supply relation according to each demand specification. For the "Crowding F_j", I also present results of a simpler supply relation specification, without accounting for the conduct shifters. In all full specifications, it is not possible to reject the nullity of the coordinated market power parameter. Indeed, the coefficients of $\widetilde{p}_{-\mathcal{F}}$ are always statistically significant, meaning that firms could actually exert some coordinated market power. This is consistent with the fact that there was a forty-year old alliance between carriers (Ponte Aérea) that dominated the market. Also, it is clear from the estimation results of Table 7 that there is no evidence of structural break in the coordinated market power during the period of alleged price coordination. In fact, the estimated coefficient of $\widetilde{p}_{-\mathcal{F}} \times invper$, is not statistically significant in any of the specifications. This means that apparently, the episodes of exchange rate overshooting of the early 1999 did not allow firms to pass through the subsequent cost increase to prices and neither to bring a situation of collusion with higher markups. We then reach the conclusion that the period under investigation by the antitrust authorities was probably not characterized by changes in the competitive conduct of firms, given the lack of a statistically significant shift in the coordinated market power parameter. The alleged price parallelism was not sufficient to shift carriers' market power in terms of efficient collusion incentives.

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[94] The bias tends to be positive for the marginal effect of prices, given the positive correlation between the terms of congestion and prices, and between the terms of congestion and airline market share. With respect to the marginal effect of intra-subgroup shares and share of the subgroup, the bias also tends to be positive, given the positive correlation of these variables with the terms of congestion. This is explained by the fact that the possession of outlets increases both variables. The ex-ante correlation between the intra-subgroup share and the terms of congestion is more pronounced, given the greater similarity between firms within the nest. Consistent with this interpretation, in Column (3) we have the apparent bias in the estimation of the parameter s_{jh} being greater than the parameter estimation of s_{hg}.

[95] See the Appendix for details.

Table 7 - Estimation Results - Supply Relation[96]

	(1) *Crowding* F_j	(2) *Crowding* F_j	(3) *Crowding* K_j	(4) *No Crowding*
$fuel_j$	0.14759***	0.04107*	0.13143***	0.02745
	[0.05005]	[0.02107]	[0.04448]	[0.06760]
$crew_j$	0.01403	0.01755***	0.01414	0.01348
	[0.00967]	[0.00506]	[0.00900]	[0.01017]
q_j	-0.10507***	-0.06571***	-0.09744***	-0.06026***
	[0.02259]	[0.01017]	[0.02104]	[0.02335]
$\widetilde{\omega}_{-j}$	-0.07350***		-0.06546***	-0.00932
	[0.02169]		[0.01897]	[0.02428]
$\widetilde{\omega}_{-j,\mathcal{F}}$	0.17353***		0.14935***	0.07208
	[0.04814]		[0.04355]	[0.06102]
$\widetilde{p}_{-\mathcal{F}}$	1.61921***		1.52645***	0.72292**
	[0.32811]		[0.31171]	[0.34663]
$\widetilde{p}_{-\mathcal{F}} \times invper$	-0.04598		-0.03134	0.16285
	[0.15892]		[0.14954]	[0.19025]
GMM Obj.	8.39217	48.29500	7.66077	39.64330

With respect to the costs side results, most own costs shifters had statistically significant coefficients. Also, taken as a group, the F-statistic for the joint test of nullity of all own cost shifters indicated significance at the 0.01% level. The joint nullity of the transformed rival cost variables $\widetilde{\omega}_{-j}$ and $\widetilde{\omega}_{-j,\mathcal{F}}$ is also rejected, at the 0.01% level. Moreover, in accordance with the classic literature of air transportation (eg Caves, Christensen and Tretheway, 1984 and Brueckner, Dyer and Spiller, 1992), we had clear evidence of economies of density, with higher traffic density reducing both fares and the marginal cost of carrying an extra passenger. In fact, in all supply relation specifications, it is possible to estimate a coefficient of q_j that is negative and statistically significant at 1% level. This is an important result, since the traditional approach for conduct analysis - the conjectural variations framework -

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[96] Bootstrapped standard errors in brackets. Superscripts *, **, and *** denote, respectively, significance at the 10%, 5%, and 1% levels. Estimated effects of airline-period, airline-codesharing periods, month, quarter and airline-specific trends not reported. All specifications are based on estimated elasticities of the fully specified models of demand of Table 4.

since Bresnahan (1982) suffers criticism for not allowing proper econometric identification of conduct from costs when marginal costs are not constant.

The second column of Table 7 presents a restricted version of the GSR. In this column, only the effect of own cost shifters are estimated, thus disregarding the terms related to the coordinated behavior model, $\widetilde{\omega}_{-j}$, $\widetilde{\omega}_{-j,\mathcal{F}}$ and $\widetilde{p}_{-\mathcal{F}}$.[97] This is equivalent to the procedure of estimating unknown cost parameters under the assumption of a particular model of firm behavior, as in Berry, Levinsohn, and Pakes (1995), and Genesove and Mullin (1998).[98] This simpler specification is equivalent to imposing static multiproduct Bertrand-Nash outcome as a restriction to estimating costs, and therefore permits assessing the impacts of using the more usual approach to evaluating market power in empirical IO. In case of a non-null collusive component, the researcher incurs a problem similar to the classic identification problem of Bresnahan (1982). When not controlling for collusion, one can not distinguish a Nash-Bertrand oligopoly with high cost from a situation of a low-cost cartel. Marginal costs will be overestimated whenever there is non-null market power from coordinated behavior not controlled by the researcher. Also, from an econometric standpoint, when imposing Nash-Bertrand conduct in order to estimate marginal costs, the researcher will ultimately place the term of markups of rival firms participating in the cartel in the residual term. In this case, using the traditional omitted variable bias analysis, the researcher will incur inconsistent estimation of the marginal effects of cost shifters, particularly when they are correlated with coordinated market power. When positively (negatively) correlated with markup, this bias tends to be positive (negative). We therefore have that both the global estimates of the marginal cost and the parameter estimates of the own cost shifters will be inherently biased whenever the game being played is similar to the model presented here.

A comparative analysis of the unrestricted model - the first column of Table 7 - and the restricted model where the static product differentiated Betrand game is assumed - the second column of Table 7 - shows that the results are considerably impacted in the second case. The estimated coefficient of fuel costs drops by 72%. Thus, on account of a positive correlation between prices and repeated interaction, we believe that a potential negative correlation between repeated interaction and fuel prices would provoke underestimation of fuel effects. Another relevant cost variable that is impacted by not accounting for repeated interaction is traffic density q_j. In this case, the estimated coefficient drops by more than a third. It is possible to attribute this effect to the positive correlation between prices and repeated interaction, together with a positive correlation between repeated interaction and q_j.[99] We therefore have an indication of a positive omitted variables bias when estimating the effects of q_j. These results clearly point to the risk of underestimation of economies of density when the researcher imposes a particular static model of firm behavior, such as in Berry, Carnall

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[97] These terms are components of the markup of rival firms, which were split to form the terms of the GSR. The markups of rivals are positive and positively correlated with the price of firm j.

[98] Berry, Levinsohn and Pakes (1995) assume Bertrand-Nash Equilibrium in order to estimate cost parameters. Genosove and Mullin (1998) find that costs are overestimated when the researcher assumes perfect competition and underestimated when he assumes monopoly.

[99] The reason would be that if prices of rivals increase, then demand increases, *ceteris paribus*. And thus q_j also increases.

and Spiller (2006).

With respect to the specification "Crowding K_j" - Column (3) -, we have that results are robust to changes in the measure of congestion in the space of characteristics. However, by comparing the results of Columns (1) and (3) with Column (4) - the "No Crowding" specification -, it is possible to conclude that many relevant variables are impacted. The most problematic cases are $fuel_j$, $\widetilde{\omega}_{-j}$ and $\widetilde{\omega}_{-j,\mathcal{F}}$, which become not statistically significant. The estimates of economies of density are also impacted, dropping by more than 40%. We have that congestion controls are critical to the adequacy of some of the estimates in this study. From an antitrust standpoint, however, results on $\widetilde{p}_{-\mathcal{F}} \times invper$ are not affected, even in this case of lack of congestion controls. We then have evidence of non-null coordinated market power estimated for the whole period, but no statistically significant shift in market power in the investigated period.

6 Conclusions

This paper presented a proposal for investigating the conduct of firms in an oligopoly. A marginal profits ratio (MPR) was developed from a situation of efficient collusion with a stick-and-carrot cartel. The final model also accommodates grim-trigger punishment behavior, as in Corts (1999) and Puller (2009). With the MPR framework, I developed a generalized supply relation (GSR) that nests a broad portfolio of possible market settings. Through the econometric specification the GSR, some transformed price variables are used to perform a consistent estimation of coordinated market power with a freely-estimated parameter. It is then possible to directly test the existence of some form of market power originating from repeated interaction among firms in the market within a single-equation framework.

To develop an application of the proposed methodology, data from a real antitrust case in the airline industry were used - the Brazilian air shuttle cartel. In the demand side I used a generalized nested logit (GNL) with overlapping nests and outside good size as unobservables. This framework has the estimating equations of Berry (1994) and Ackerberg and Rysman (2005) as special cases. In the supply side, I employed the proposed GSR for heterogeneous, multi-product settings with price competition. Enough evidence was found that firms exerted statistically significant coordinated market power in the period since the full deregulation of the industry, which was probably a result of the route and airport dominance kept by a forty-year-old alliance in the market. Final results indicated the absence of evidence of a change in the coordinated market power of firms during the period under antitrust investigation.

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Appendix

A Proof of Proposition 1

Assume that π_{kt} follow an $ARMA\,(m,n)$ process of the type $\pi_{kt} = a_k + \sum_{i=1}^{m} b_{ki}\pi_{kt-i} + \sum_{i=1}^{n} g_{ki}z_{kt-i} + z_{kt}$, in which π_{kt} is firm k's profit at time t, a_k, b_{ki} and g_{ki} are unknowns and z_{kt} are unobserved profit shocks. For simplicity, disregard index k. Also, denote $\widehat{\pi}_t = E_t\pi_t$ and $\widehat{z}_t = E_t z_t$. Assume, for current time t, $\widehat{\pi}_t = \pi_t$, $\widehat{z}_t = z_t$. Also, $\widehat{z}_t = 0$ in future time $t+s$, $s>0$. Denote the predictor of this process at future time s by $\widehat{\pi}_{t+s}$.

For any forecast horizon s the optimal s-period-ahead forecast $\widehat{\pi}_{t+s}$ is a linear function of π_t, π_{t-1}, $\pi_{t-2,...}$. It is possible to calculate forecasts through a recursive procedure, based on the law of iterated projections (Hamilton, 1994). First, consider an $ARMA(1,1)$ process. We then have profit at time $t+s$ equal to $\pi_{t+s} = a + b_1\pi_{t+s-1} + z_{t+s} + g_1 z_{t+s-1}$. By extracting expectations and using recursion:

$$\widehat{\pi}_{t+1} = E_t\pi_{t+1} = E_t\left[a + b_1\pi_t + z_{t+1} + g_1 z_t\right] = a + b_1\pi_t$$

$$\widehat{\pi}_{t+2} = E_t\pi_{t+2} = E_t\left[a + b_1\pi_{t+1} + z_{t+2} + g_1 z_{t+1}\right] = a\left(1+b_1\right) + b_1^2\pi_t$$

$$\widehat{\pi}_{t+3} = E_t\pi_{t+3} = E_t\left[a + b_1\pi_{t+2} + z_{t+3} + g_1 z_{t+2}\right] = a\left(1+b_1+b_1^2\right) + b_1^3\pi_t$$

$$...$$

$$\widehat{\pi}_{t+s} = E_t\pi_{t+s} = E_t\left[a + b_1\pi_{T+v-1} + z_{T+v} + g_1 z_{T+v-1}\right] = \widetilde{\alpha}_{1,1}\left(a,b_1,s\right) + \widetilde{\beta}_{1,1}\left(b_1,s\right)\pi_t.$$

where $\alpha_{1,1}$ and $\beta_{1,1}$ are parameters derived from the original $ARMA(1,1)$ parameters, a, b_1, and forecast horizon s. As this result is identical for $ARMA(1,n)$, $n = 1,2,..$, I then rename $\alpha_{1,1}$ and $\beta_{1,1}$ as, respectively, $\widetilde{\alpha}_1$ and $\widetilde{\beta}_1$. By applying the same procedure to an $ARMA(2,1)$ process of the type $\pi_{t+s} = a + b_1\pi_{t+s-1} + b_2\pi_{t+s-2} + z_{t+s} + g_1 z_{t+s-1}$, we have

$$\widehat{\pi}_{t+1} = E_t\pi_{t+1} = E_t\left[a + b_1\pi_t + b_2\pi_{t-1} + z_{t+1} + g_1 z_t\right] = a + b_2\pi_{t-1} + b_1\pi_t$$

$$\widehat{\pi}_{t+2} = E_t\pi_{t+2} = E_t\left[a + b_1\pi_{t+1} + b_2\pi_t + z_{t+2} + g_1 z_{t+1}\right] = a\left(1+b_1\right) + b_1 b_2\pi_{t-1} + \left(b_1^2 + b_2\right)\pi_t$$

$$\begin{aligned}\widehat{\pi}_{t+3} = E_t\pi_{t+3} &= E_t\left[a + b_1\pi_{t+2} + b_2\pi_{t+1} + z_{t+3} + g_1 z_{t+2}\right] \\ &= a\left(1+b_1+b_1^2+b_2\right) + \left(b_1^2 b_2 + b_2^2\right)\pi_{t-1} + \left(b_1^3 + 2b_1 b_2\right)\pi_t\end{aligned}$$

$$...$$

$$\widehat{\pi}_{t+s} = E_t\pi_{t+s} = E_t\left[a + b_1\pi_{t+s-1} + b_2\pi_{t+s-2} + z_{t+s} + g_1 z_{t+s-1}\right] = \widetilde{\alpha}_{2,1}\left(a,b_1,b_2,\pi_{t-1},s\right) + \widetilde{\beta}_{2,1}\left(b_1,b_2,s\right)\pi_t.$$

where $\widetilde{\alpha}_{2,1}$ and $\widetilde{\beta}_{2,1}$ are functions of the $ARMA(2,1)$ parameters, a, b_1, b_2, and s. $\widetilde{\alpha}_{2,1}$ is also a function of historic profit π_{t-1} and therefore is updated every period t. Again, this result is identical for $ARMA(2,n)$, $n = 1,2,..$, and therefore rename $\widetilde{\alpha}_{2,1}$ and $\widetilde{\beta}_{2,1}$ as, respectively, $\widetilde{\alpha}_2$ and $\widetilde{\beta}_2$. Then consider an $ARMA(3,1)$:

$$\widehat{\pi}_{t+1} = a + b_1\pi_t + b_2\pi_{t-1} + b_3\pi_{t-2} + \widehat{z}_{t+1} + g_1 z_t = a + b_2\pi_{t-1} + b_3\pi_{t-2} + b_1\pi_t$$

$$\widehat{\pi}_{t+2} = a + b_1\widehat{\pi}_{t+1} + b_2\pi_t + b_3\pi_{t-1} + \widehat{z}_{t+2} + g_1\widehat{z}_{t+1} = a\left(1+b_1\right) + \left(b_1 b_2 + b_3\right)\pi_{t-1} + b_1 b_3\pi_{t-2} + \left(b_1^2 + b_2\right)\pi_t$$

$$\begin{aligned}\widehat{\pi}_{t+3} &= a + b_1\widehat{\pi}_{t+2} + b_2\widehat{\pi}_{t+1} + b_3\pi_t + \widehat{z}_{t+3} + g_1\widehat{z}_{t+2} = \\ &= a\left(1+b_1+b_1^2+b_2\right) + \left(b_1^2 b_2 + b_1 b_3 + b_2^2\right)\pi_{t-1} + \left(b_1^2 b_3 + b_2 b_3\right)\pi_{t-2} + \left(b_1^3 + 2b_1 b_2 + b_3\right)\pi_t\end{aligned}$$

$$\begin{aligned}
&...\\
\widehat{\pi}_{t+s} &= a + b_1\widehat{\pi}_{t+s-1} + b_2\widehat{\pi}_{t+s-2} + b_3\widehat{\pi}_{t+s-3} + \widehat{z}_{t+s} + g_1\widehat{z}_{t+s-1}\\
&= \widetilde{\alpha}_{3,1}\left(a, b_1, b_2, b_{3,}\pi_{t-1}, \pi_{t-2}, s\right) + \widetilde{\beta}_{3,1}\left(b_1, b_2, b_3, s\right)\pi_t.
\end{aligned}$$

Again, rename $\widetilde{\alpha}_{3,1}$ and $\widetilde{\beta}_{3,1}$ as, respectively, $\widetilde{\alpha}_3$ and $\widetilde{\beta}_3$. Note that the above results are easily generalized to an $ARMA(m,n)$ process, as we have $\sum_{i=1}^{n} g_{T+v-i}\widehat{z}_{T+v-i} + \widehat{z}_{T+v} = 0$. We thefore have:

$$\begin{aligned}
&ARMA(1,1) : \widehat{\pi}_{t+s} = \widetilde{\alpha}_1\left(a, b_1, m, s\right) + \widetilde{\beta}_1\left(b_1, m, s\right)\pi_t.\\
&ARMA(2,1) : \widehat{\pi}_{t+s} = \widetilde{\alpha}_2\left(a, b_1, b_2, \pi_{t-1}, m, s\right) + \widetilde{\beta}_2\left(b_1, b_2, m, s\right)\pi_t.\\
&ARMA(3,1) : \widehat{\pi}_{t+s} = \widetilde{\alpha}_3\left(a, b_1, b_2, b_{3,}\pi_{t-1}, \pi_{t-2}, m, s\right) + \widetilde{\beta}_3\left(b_1, b_2, b_3, m, s\right)\pi_t.\\
&\quad ...\\
&ARMA(m,n) : \widehat{\pi}_{t+s} = \widetilde{\alpha}_m\left(a, b_1, b_2, ..., b_m, \pi_{t-1}, \pi_{t-2}, ..., \pi_{t-m+1}, m, s\right) +\\
&\qquad\qquad\qquad\qquad \widetilde{\beta}_m\left(b_1, b_2, ..., b_m, m, s\right)\pi_t.
\end{aligned}$$

For convenience, I use the following expression for expected future profits $\widehat{\pi}_{t+s}$:

$$\widehat{\pi}_{t+s} = \widetilde{\alpha}_{t,s,m} + \widetilde{\beta}_{s,m}\pi_t. \tag{24}$$

Expression (24) means that expected future profits may be expressed in terms of current profits if adjusted by an intercept $\widetilde{\alpha}_{t,s,m}$ and a slope $\widetilde{\beta}_{s,m}$. Note that, $\widetilde{\alpha}_{t,s,m}$ depends on the information available at the current time T, namely the history of profits $\pi_{T-1}, \pi_{T-2}, ...$. In opposition, the slope in (24), $\widetilde{\beta}_{s,m}$, does not depend on the information available at t, but is a function only of the $ARMA$ structural parameters and the order of the process, that is, $\widetilde{\beta}_{s,m} = \widetilde{\beta}_{s,m}\left(b_1, b_2, ..., b_m, m, s\right)$.

Now, let us develop an expression for the present value of the $ARMA(m,n)$ process. Denote the expected present discounted value of profits at t, E_tV. Consider a discount factor ρ. We then have

$$E_tV = \pi_t + \rho E_t\pi_{t+1} + \rho^2 E_t\pi_{t+2} + \rho^3 E_t\pi_{t+3}... = \pi_t + \sum_{s=1}^{\infty}\rho^s\widehat{\pi}_{t+s}. \tag{25}$$

Consider the following algebraic manipulation, using $\sum_{i=1}^{\infty}\rho^i$.We then have

$$\begin{aligned}
E_tV &= \pi_t + \sum_{s=1}^{\infty}\rho^s\widehat{\pi}_{t+s}\frac{\sum_{s=1}^{\infty}\rho^s}{\sum_{s=1}^{\infty}\rho^s} = \pi_t + \left(\sum_{s=1}^{\infty}\rho^s\right)\left(\frac{\sum_{s=1}^{\infty}\rho^s\widehat{\pi}_{t+s}}{\sum_{s=1}^{\infty}\rho^s}\right)\\
&= \pi_t + \frac{\rho}{1-\rho}\sum_{s=1}^{\infty}\omega_s\widehat{\pi}_{t+s}, \qquad (26)
\end{aligned}$$

where $\omega_s = \rho^s / \sum_{i=1}^{\infty}\rho^i$ are geometrically decreasing weights over time, $\rho < 1$. By substituting

$\widehat{\pi}_{t+s}$ of (24) in (26) we have:

$$\begin{aligned} E_t V &= \pi_t + \frac{\rho}{1-\rho} \sum_{s=1}^{\infty} \omega_s \left(\widetilde{\alpha}_{t,s,m} + \widetilde{\beta}_{s,m} \pi_t \right) \\ &= \pi_t + \frac{\rho}{1-\rho} \left(\overline{\alpha}_t + \overline{\beta} \pi_t \right), \end{aligned} \tag{27}$$

where $\overline{\alpha}_t = \sum_{s=1}^{\infty} \omega_s \widetilde{\alpha}_{t,s,m}$ and $\overline{\beta} = \sum_{s=1}^{\infty} \omega_s \widetilde{\beta}_{s,m}$ are intertemporal weighted averages of, respectively, $\widetilde{\alpha}_{t,s,m}$ and $\widetilde{\beta}_{s,m}$, $s = 1, .., \infty$. With (27), we finally we reach the following expression:

$$E_t V = \alpha_t + (1 + \beta) \pi_t, \tag{28}$$

in which $\alpha_t = \overline{\alpha}_t \rho (1-\rho)^{-1}$ and $\beta = \overline{\beta} \rho (1-\rho)^{-1}$. By using index k, we have $E_t V_k = \alpha_{kt} + (1 + \beta_k) \pi_{kt}$. In (28) we have an expression that considerably simplifies the calculation of the expected present discounted value of profits $E_t V_k$. Note again that, contrary to α_{kt}, β_k is a fixed slope parameter not dependent on time t. More specifically, $\alpha_{kt} = \alpha(\rho, \pi_{t-1}, \pi_{t-2}, \ldots, \pi_{t-m+1}, a, b_1, b_2, \ldots, b_m, m)$ and $\beta_k = \beta(\rho, b_1, b_2, ..., b_m)$. This is an important result, implying the stability of β_k across decision-making periods in empirical applications that use time series or panel data samples. ■

B MPR for a stick-and-carrot cartel

For the stick-and-carrot cartel, we have the following optimization problem:

$$\begin{aligned} x_t^* = \arg\max_{x_t} \; & \sum_{k=1}^{N} \pi_{kt}(x_{kt}), \text{ subject to} \\ & IC_1 : E_t V_k^{br}(x_{kt}) \leq E_t V_k(x_{kt}), \\ & IC_2 : E_t V_k^{br,p}(x_{kt}) \leq E_t V_k^{p}(x_{kt}), \forall k, \end{aligned} \tag{29}$$

where $E_t V_k(x_{kt})$, $E_t V_k^{br,p}(x_{kt})$ and $E_t V_k^{p}(x_{kt})$ are directly derived from Proposition 1:

$$E_t V_k(x_{kt}) = \alpha_{kt} + (1 + \beta_k) \pi_{kt}(x_{kt}), \tag{30}$$

$$E_t V_k^{br,p}(x_{kt}) = \alpha_{kt}^{br,p} + \left(1 + \beta_k^{br,p}\right) \pi_{kt}^{br,p}(x_{kt}), \tag{31}$$

$$E_t V_k^{p}(x_{kt}) = \pi_{kt}^{p}(x_{kt}) + \alpha_{kt}^{p} + \beta_k^{p} \pi_{kt}(x_{kt}). \tag{32}$$

With respect to $E_t V_k^{br}(x_{kt})$, we have, from Appendix C, that

$$E_t V_k^{br}(x_{kt}) = \alpha_{kt}^{br} + \left(1 + \beta_k^{br_1}\right) \pi_{kt}^{br}(x_{kt}) + \beta_k^{br_2} \pi_{kt}(x_{kt}), \tag{33}$$

where α_{kt}^{br} , $\beta_k^{br_1}$ and $\beta_k^{br_2}$ are parameters. By substituting $E_t V_k^p\left(x_{kt}\right)$, $E_t V_k\left(x_{kt}\right)$, $E_t V_k^{br,p}\left(x_{kt}\right)$ and $E_t V_k^{br}\left(x_{kt}\right)$ in (29) we then have the following lagrangian:

$$\begin{aligned}
\mathcal{L}_t &= \sum_{k=1}^{N} \pi_{kt}\left(x_{kt}\right) \\
&- \mu_{kt}^{(1)} \left\{ \left[\alpha_{kt}^{br} + \left(1+\beta_k^{br_1}\right) \pi_{kt}^{br}\left(x_{kt}\right) + \beta_k^{br_2} \pi_{kt}\left(x_{kt}\right) \right] - \left[\alpha_{kt} + \left(1+\beta_k\right) \pi_{kt}\left(x_{kt}\right) \right] \right\} \\
&- \mu_{kt}^{(2)} \left\{ \left[\alpha_{kt}^{br,p} + \left(1+\beta_k^{br,p}\right) \pi_{kt}^{br,p}\left(x_{kt}\right) \right] - \left[\pi_{kt}^{p}\left(x_{kt}\right) + \alpha_{kt}^{p} + \beta_k^{p} \pi_{kt}\left(x_{kt}\right) \right] \right\} \qquad (34)
\end{aligned}$$

For $k = j$, we have the following first-order condition for profit maximization in which $\partial \mathcal{L}_{jt} / \partial x_{jt} = 0$:[100]

$$\begin{aligned}
\frac{\partial \pi_{jt}\left(x_{jt}\right)}{\partial x_{jt}} - \mu_{jt}^{(1)} & \left[\left(1+\beta_j^{br_1}\right) \frac{\partial \pi_{jt}^{br}\left(x_{jt}\right)}{\partial x_{jt}} + \beta_j^{br_2} \frac{\partial \pi_{jt}\left(x_{jt}\right)}{\partial x_{jt}} - \left(1+\beta_j\right) \frac{\partial \pi_{jt}\left(x_{jt}\right)}{\partial x_{jt}} \right] \\
- \mu_{jt}^{(2)} & \left[\left(1+\beta_j^{br,p}\right) \frac{\partial \pi_{jt}^{br,p}\left(x_{jt}\right)}{\partial x_{jt}} - \frac{\partial \pi_{jt}^{p}\left(x_{jt}\right)}{\partial x_{jt}} - \beta_j^{p} \frac{\partial \pi_{jt}\left(x_{jt}\right)}{\partial x_{jt}} \right] = 0. \qquad (35)
\end{aligned}$$

By assuming $\pi_{jt}^{p}\left(x_{jt}\right) = \pi_{jt}^{p}\left[\pi_{jt}^{br}\left(x_{jt}\right)\right]$ and $\pi_{jt}^{br,p}\left(x_{jt}\right) = \pi_{jt}^{br,p}\left\{\pi_{jt}^{p}\left[\pi_{jt}^{p}\left(x_{jt}\right)\right]\right\}$ we have

$$\begin{aligned}
\frac{\partial \pi_{jt}^{p}\left(x_{jt}\right)}{\partial x_{jt}} &= \frac{\partial \pi_{jt}^{p}}{\partial \pi_{jt}^{br}\left(x_{jt}\right)} \frac{\partial \pi_{jt}^{br}\left(x_{jt}\right)}{\partial x_{jt}} = \kappa_{jt}^{p} \frac{\partial \pi_{jt}^{br}\left(x_{jt}\right)}{\partial x_{jt}}, \text{ and} \qquad (36) \\
\frac{\partial \pi_{jt}^{br,p}\left(x_{jt}\right)}{\partial x_{jt}} &= \frac{\partial \pi_{jt}^{br,p}}{\partial \pi_{jt}^{p}} \frac{\partial \pi_{jt}^{p}}{\partial \pi_{jt}^{br}\left(x_{jt}\right)} \frac{\partial \pi_{jt}^{br}\left(x_{jt}\right)}{\partial x_{jt}} = \kappa_{jt}^{br,p} \kappa_{jt}^{p} \frac{\partial \pi_{jt}^{br}\left(x_{jt}\right)}{\partial x_{jt}}, \qquad (37)
\end{aligned}$$

where $\kappa_{jt}^{br,p}$ and κ_{jt}^{p} are unkown parameters. By substituting (36) and (37) into (35), we then have

$$\begin{aligned}
\frac{\partial \pi_{jt}\left(x_{jt}\right)}{\partial x_{jt}} - \quad & \mu_{jt}^{(1)} \left[\left(1+\beta_j^{br_1}\right) \frac{\partial \pi_{jt}^{br}\left(x_{jt}\right)}{\partial x_{jt}} + \beta_j^{br_2} \frac{\partial \pi_{jt}\left(x_{jt}\right)}{\partial x_{jt}} - \left(1+\beta_j\right) \frac{\partial \pi_{jt}\left(x_{jt}\right)}{\partial x_{jt}} \right] \\
- \quad & \mu_{jt}^{(2)} \left[\left(1+\beta_j^{br}\right) \kappa_{jt}^{br,p} \kappa_{jt}^{p} \frac{\partial \pi_{jt}^{br}\left(x_{jt}\right)}{\partial x_{jt}} - \kappa_{jt}^{p} \frac{\partial \pi_{jt}^{br}\left(x_{jt}\right)}{\partial x_{jt}} - \beta_j^{p} \frac{\partial \pi_{jt}\left(x_{jt}\right)}{\partial x_{jt}} \right] = 0. \; (38)
\end{aligned}$$

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[100] Considering that $\partial \pi_{jt}^{br,p} / \partial x_{jt} = 0$ and $\partial \pi_{jt}^{p} / \partial x_{jt} = 0$.

We then reach the final expression for the MPR:

$$\begin{aligned} \frac{\partial \pi_{jt}\left(x_{jt}\right)}{\partial x_{jt}} &= \lambda_{jt}\frac{\partial \pi^{br}_{jt}\left(x_{jt}\right)}{\partial x_{jt}}, \\ \text{where } \lambda_{jt} &= \frac{\mu^{(1)}_{jt}\left(1+\beta^{br_1}_{j}\right)+\mu^{(2)}_{jt}\kappa^{p}_{jt}\left[\left(1+\beta^{br,p}_{j}\right)\kappa^{br,p}_{jt}-1\right]}{1+\mu^{(1)}_{jt}\left[1+\left(\beta_j-\beta^{br_2}_{j}\right)\right]+\mu^{(2)}_{jt}\beta^{p}_{j}}. \end{aligned} \tag{39}$$

C Expected present value - best response case

Here I show that $E_t V^{br}_{k}\left(x_{kt}\right)=\alpha^{br}_{kt}+\left(1+\beta^{br_1}_{k}\right)\pi^{br}_{kt}+\beta^{br_2}_{k}\pi_{kt}$. For simplicity, let us disregard index k for a moment. We have

$$\begin{aligned} E_t V^{br}\left(x_t\right) &= \pi^{br}_{t}+\rho E_t\pi_{t+1}+\rho^2 E_t\pi^{br}_{t+2}+\rho^3 E_t\pi_{t+3}\ldots \\ &= \left(\pi^{br}_{t}+0+\rho^2 E_t\pi^{br}_{t+2}+0+\rho^4 E_t\pi^{br}_{t+4}\ldots\right)+\left(0+\rho\pi_{t+1}+0+\rho^3 E_t\pi_{t+3}\ldots\right) \\ &= E_t V^{br_1}\left(x_t\right)+E_t V^{br_2}\left(x_t\right), \end{aligned} \tag{40}$$

where $E_t V^{br_1}$ and $E_t V^{br_2}$ are expected present discounted values of profits during best-response (br_1) and cartel-adherance (br_2) periods, respectivelly. First, by using Proposition 1, let us develop a convenient expression for $E_t V^{br_1}\left(x_{kt}\right)$:

$$\begin{aligned} E_t V^{br}\left(x_t\right) &= \pi^{br}_{t}+0+\rho^2\left[\alpha^{br_1}_{t+2}+\beta^{br_1}_{t+2}\pi^{br}_{t}\right]+0+\rho^4\left[\alpha^{br_1}_{t+4}+\beta^{br_1}_{t+4}\pi^{br}_{t}\right]\ldots \\ &= \pi^{br}_{t}+\left(\sum_{s=2,4,\cdots}^{\infty}\rho^s\alpha^{br_1}_{t+s}+\pi^{br}_{t}\sum_{s=2,4,\cdots}^{\infty}\rho^s\beta^{br_1}_{t+s}\right)\frac{\sum_{s=1}^{\infty}\rho^s}{\sum_{s=1}^{\infty}\rho^s} \\ &= \pi^{br}_{t}+\frac{\rho}{1-\rho}\left(\overline{\alpha}^{br_1}_{t}+\overline{\beta}^{br_1}\pi^{br}_{t}\right) \\ &= \alpha^{br_1}_{t}+\left(1+\beta^{br_1}\right)\pi^{br}_{t}. \end{aligned} \tag{41}$$

where $\alpha^{br_1}_{t}=\rho\left(1-\rho\right)^{-1}\overline{\alpha}^{br_1}_{t}$ and $\beta^{br_1}=\rho\left(1-\rho\right)^{-1}\overline{\beta}^{br_1}$. Second, and also employing Proposition 1, we have that a convenient expression for $E_t V^{br_2}\left(x_t\right)$ is

$$\begin{aligned} E_t V\left(x\right) &= 0+\rho\left[\alpha^{br_2}_{t+1}+\beta^{br_2}_{t+1}\pi_t\right]+0+\rho^3\left[\alpha^{br_2}_{t+3}+\beta^{br_2}_{t+3}\pi_t\right]\ldots \\ &= \left(\sum_{s=1,3,\cdots}^{\infty}\rho^s\alpha^{br_2}_{t+s}+\pi_t\sum_{s=1,3,\cdots}^{\infty}\rho^s\beta^{br_2}_{t+s}\right)\frac{\sum_{s=1}^{\infty}\rho^s}{\sum_{s=1}^{\infty}\rho^s} \\ &= \frac{\rho}{1-\rho}\left(\overline{\alpha}^{br_2}_{t}+\overline{\beta}^{br_2}\pi_t\right) \\ &= \alpha^{br_2}_{t}+\beta^{br_2}\pi_t. \end{aligned} \tag{42}$$

where $\alpha_t^{br_2} = \rho(1-\rho)^{-1}\overline{\alpha}_t^{br_2}$ and $\beta^{br_2} = \rho(1-\rho)^{-1}\overline{\beta}^{br_2}$. Finally, by substituting (41) and (42) into (41), we have that the full expected discounted present value $E_tV^{br}(x_t)$ is:

$$\begin{aligned} E_tV^{br}(x_t) &= E_tV_t^{br_1}(x_t) + E_tV_t^{br_2}(x_t) \\ &= \alpha_t^{br_1} + \left(1+\beta^{br_1}\right)\pi_t^{br} + \alpha_t^{br_2} + \beta^{br_2}\pi_t \\ &= \alpha_t^{br} + \left(1+\beta^{br_1}\right)\pi_t^{br} + \beta^{br_2}\pi_t. \end{aligned} \tag{43}$$

By reintroducing index k, we have $E_tV_k^{br}(x_{kt}) = \alpha_{kt}^{br} + \left(1+\beta_k^{br_1}\right)\pi_{kt}^{br} + \beta_k^{br_2}\pi_{kt}$, where $\alpha_{kt}^{br} = \alpha_t^{br_1} + \alpha_t^{br_2}$.

D MPR for a grim trigger cartel

We then have the following problem for the grim-trigger cartel:

$$\begin{aligned} x_t^* = \arg\max_{x_t} & \sum_{k=1}^{N}\pi_{kt}(x_{kt}), \text{ subject to} \\ & \pi_{kt}^{br}(x_{kt}) + \sum_{s=1}^{\infty}\rho^s E_t\pi_{kt+s}^{br,p} \leq \pi_{kt}(x_{kt}) + \sum_{s=1}^{\infty}\rho^s E_t\pi_{kt+s}^*, \forall k. \end{aligned} \tag{44}$$

The lagrangian related to (44) is therefore

$$\mathcal{L}_t = \sum_{k=1}^{N}\pi_{kt}(x_{kt}) - \mu_{kt}\left[\pi_{kt}^{br}(x_{kt}) + \sum_{s=1}^{\infty}\rho^s E_t\pi_{kt+s}^{br,p} - \pi_{kt}(x_{kt}) - \sum_{s=1}^{\infty}\rho^s E_t\pi_{kt+s}^*\right] \tag{45}$$

By applying the results of Proposition 1 we then have:

$$\mathcal{L}_t = \sum_{k=1}^{N}\pi_{kt}(x_{kt}) - \mu_{kt}\left[\pi_{kt}^{br}(x_{kt}) + \alpha_{kt}^{br,p} + \beta_k^{br,p}\pi_{kt}^{br,p} - \pi_{kt}(x_{kt}) - \alpha_{kt} - \beta_k\pi_{kt}(x_{kt})\right] \tag{46}$$

For $k = j$, we have the following first-order condition for profit maximization in which $\partial\mathcal{L}_{jt}/\partial x_{jt} = 0$:[101]

$$\frac{\partial\pi_{jt}(x_{jt})}{\partial x_{jt}} - \mu_{jt}\left[\frac{\partial\pi_{jt}^{br}(x_{jt})}{\partial x_{jt}} + \beta_j^{br,p}\frac{\partial\pi_{jt}^{br,p}(x_{jt})}{\partial x_{jt}} - \frac{\partial\pi_{jt}(x_{jt})}{\partial x_{jt}} - \beta_j\frac{\partial\pi_{jt}(x_{jt})}{\partial x_{jt}}\right] = 0. \tag{47}$$

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[101] Considering that $\partial\pi_{jt}^{br,p}/\partial x_{jt} = 0$ and $\partial\pi_{jt}^{p}/\partial x_{jt} = 0$.

By assuming $\pi_{jt}^{br,p}(x_{jt}) = \pi_{jt}^{br,p}\left\{\pi_{jt}^{p}\left[\pi_{jt}^{p}(x_{jt})\right]\right\}$ we have

$$\frac{\partial \pi_{jt}^{br,p}(x_{jt})}{\partial x_{jt}} = \frac{\partial \pi_{jt}^{br,p}}{\partial \pi_{jt}^{p}} \frac{\partial \pi_{jt}^{p}}{\partial \pi_{jt}^{br}(x_{jt})} \frac{\partial \pi_{jt}^{br}(x_{jt})}{\partial x_{jt}} = \kappa_{jt}^{br,p}\kappa_{jt}^{p}\frac{\partial \pi_{jt}^{br}(x_{jt})}{\partial x_{jt}}, \tag{48}$$

where $\kappa_{jt}^{br,p}$ and κ_{jt}^{p} are unkown parameters. By substituting (36) and (37) into (35), we then have

$$\frac{\partial \pi_{jt}(x_{jt})}{\partial x_{jt}} - \left[\mu_{jt}\left(1+\beta_j^{br,p}\theta_{jt}^{br,p}\theta_{jt}^{p}\right)\frac{\partial \pi_{jt}^{br}(x_{jt})}{\partial x_{jt}} - \mu_{jt}\left(1+\beta_j\right)\frac{\partial \pi_{jt}(x_{jt})}{\partial x_{jt}}\right] = 0. \tag{49}$$

$$\left[1+\mu_{jt}\left(1+\beta_j\right)\right]\frac{\partial \pi_{jt}(x_{jt})}{\partial x_{jt}} - \mu_{jt}\left(1+\beta_{jt}^{br,p}\kappa_{jt}^{br,p}\kappa_{jt}^{p}\right)\frac{\partial \pi_{jt}^{br}(x_{jt})}{\partial x_{jt}} = 0. \tag{50}$$

We then reach the final expression for the MPR:

$$\begin{aligned} \frac{\partial \pi_{jt}(x_{jt})}{\partial x_{jt}} &= \lambda_{jt}\frac{\partial \pi_{jt}^{br}(x_{jt})}{\partial x_{jt}}, \\ \text{where } \lambda_{jt} &= \frac{\mu_{jt}\left(1+\beta_{jt}^{br,p}\kappa_{jt}^{br,p}\kappa_{jt}^{p}\right)}{1+\mu_{jt}\left(1+\beta_j\right)}. \end{aligned} \tag{51}$$

E Proof of Proposition 2

E.1 Homogeneous product

Assume a homogenous-product market with N firms in which $P = D(Q)$ is the inverse demand function. P and Q are, respectively, the market price and quantity. Also, say q_j and TC_j are, respectively, the quantity and the total cost of firm j. In case of a multiproduct firm, j represents the $j-th$ product, without loss of generality. We therefore have $P = D(q_j + Q_{-j})$, where Q_{-j} is the output of the remaining firms (or products) in the market, that is, $Q_{-j} = \sum_{k=1,k\neq j}^{N} q_k$. Assume the existence of a unique pure-strategy Nash equilibrium and that market price and individual quantities that support Nash equilibrium are strictly positive.

Homogeneous single product, quantity competition With quantity competition, the problem of the single-product firm j is to solve $\max_{q_j} Pq_j - TC_j$. Marginal profit is therefore $\widetilde{\pi}'_j = P - c_j - Rq_j$, where $R = abs[\partial P/\partial Q] = abs[\partial P/\partial q_j]$ and $c_j = \partial TC_j/\partial q_j$.With joint-profit maximization, the problem of firm j is now to coordinate with the $N-1$ other firms in order to reach joint-profit maximization, ie $\max_{q_k} \sum_k Pq_k - TC_k$, $k = 1, ...j, ..., N$. In this situation, we have $\pi'_j = P - c_j - Rq_j - (N-1)Rq_{-j}$, where $q_{-j} = Q_{-j}/(N-1)$. For efficient collusion, the next step is to substitute the expressions for $\widetilde{\pi}'_j$ and π'_j into the marginal profits ratio

$\pi_j'/\widetilde{\pi}_j' = \lambda$. As we have $\pi_j' - \lambda\widetilde{\pi}_j' = 0$, and by denoting $\Lambda = 1/(1-\lambda)$, we finally reach:

$$P = c_j + Rq_j + \Lambda(N-1)Rq_{-j}. \tag{52}$$

Homogeneous multiproduct, quantity competition The problem of the multiproduct firm is to maximize the total profits of all F products belonging to its product portfolio $\mathcal{F}$. The problem of the owner of product j, $j \in \mathcal{F}$, is to solve $\max_{q_k} \sum_{k\in\mathcal{F}} p_k q_k - TC_k$, $k = 1, ...j, ..., F$. Marginal profit for product j is then equal to $\widetilde{\pi}_j' = P - c_j - Rq_j - RQ_{-j,\mathcal{F}}$, where, again, $R =$ abs$(\partial P/\partial Q) =$ abs$(\partial P/\partial q_j)$, $Q_{-j,\mathcal{F}} = \sum_{k\in\mathcal{F},k\neq j} q_k$. By denoting $\widetilde{P} = P - RQ_{-j,\mathcal{F}}$, we then have $\widetilde{\pi}_j' = \widetilde{P} - c_j - Rq_j$. With joint-profit maximization, the problem of the multiproduct firm is now to engage in coordination not only with its F products of portfolio $\mathcal{F}$, but also to coordinate with all the other $N-F$ products in the market: $\max_{q_k} \sum_k Pq_k - TC_k$, $k = 1, ...j, ..., N$. Marginal profit for product j is $\pi_j' = P - c_j - Rq_j - RQ_{-j,\mathcal{F}} - RQ_{-\mathcal{F}}$, where $Q_{-j,\mathcal{F}} = \sum_{k\in\mathcal{F},k\neq j} q_k$ and $Q_{-\mathcal{F}} = \sum_{k\notin\mathcal{F}} q_k$. Let $q_{-\mathcal{F}} = Q_{-\mathcal{F}}/(N-F)$. And, again, by denoting $\widetilde{P} = P - RQ_{-j,\mathcal{F}}$, we have $\pi_j' = \widetilde{P} - c_j - Rq_j - (N-F)Rq_{-\mathcal{F}}$. For efficient collusion, we have to substitute the expressions for $\widetilde{\pi}_j'$ and π_j' into the marginal profits ratio $\pi_j'/\widetilde{\pi}_j' = \lambda$. As we have $\pi_j' - \lambda\widetilde{\pi}_j' = 0$, and by denoting $\Lambda = 1/(1-\lambda)$, we finally reach:

$$\widetilde{P} = c_j + Rq_j + \Lambda(N-F)Rq_{-\mathcal{F}}. \tag{53}$$

E.2 Differentiated product

Assume a differentiated-product market with N firms in which q_j, p_j and TC_j are, respectively, the quantity, price and total cost of firm j. Again, in case of a multiproduct firm, index j represents the $j-th$ product, without loss of generality. Assume the existence of a unique pure-strategy Nash equilibrium and that prices and quantities that support it are strictly positive.

Differentiated single product, quantity competition The problem of single firm j is to set its quantity to solve $\max_{q_j} p_j q_j - TC_j$. Marginal profit is equal to $\widetilde{\pi}_j' = p_j - c_j - R_{jj}q_j$, where $R_{kj} =$ abs$(\partial p_k/\partial q_j)$, $k = 1, ..., k, ..., N$. The problem of firm j is now to engage in joint-profit maximization with all the other $N-1$ firms in the market when setting its own quantity, ie $\max_{q_k} \sum_k^N p_k q_k - TC_k$, $k = 1, ...j, ..., N$. Marginal profit for firm j is $\pi_j' = p_j - c_j - R_{jj}q_j - \sum_{k\neq j} R_{kj}q_k$. Denote $R_{-j} = \sum_{k\neq j} R_{kj}$ and $q_{-j} = \sum_{k\neq j} R_{kj}q_k/R_{-j}$. By multiplying and dividing the last term on the right by R_{-j} and with some manipulation we then have $\pi_j' = p_j - c_j - R_{jj}q_j - R_{-j}q_{-j}$. We then proceed with substitution into the marginal profits ratio $\pi_j'/\widetilde{\pi}_j' = \lambda$. As we have $\pi_j' - \lambda\widetilde{\pi}_j' = 0$, and by denoting $\Lambda = 1/(1-\lambda)$:

$$p_j = c_j + R_{jj}q_j + \Lambda R_{-j}q_{-j}. \tag{54}$$

Differentiated single product, price competition The problem of the single firm j is to set its price to solve $\max_{p_j} p_j q_j - TC_j$. Marginal profit is equal to $\widetilde{\pi}'_j = q_j - S_{jj}(p_j - c_j)$, where $S_{kj} = \text{abs}(\partial q_k/\partial p_j)$, $k = 1,..,j,..,N$. The problem of firm j is now to engage in joint-profit maximization with all the other $N-1$ firms in the market when setting its own price, ie $\max_{p_k} \sum_k^N p_k q_k - TC_k$, $k = 1,...j,...,N$. Marginal profit for firm j is $\pi'_j = q_j - S_{jj}(p_j - c_j) + \sum_{k \neq j} S_{kj}(p_k - c_k)$. Denote $S_{-j} = \sum_{k \neq j} S_{kj}$, $p_{-j} = \sum_{k \neq j} S_{kj} p_k / S_{-j}$ and $c_{-j} = \sum_{k \neq j} S_{kj} c_k / S_{-j}$ By multiplying and dividing the last term on the right by S_{-j} and with some manipulation we then have $\pi'_j = q_j - S_{jj}(p_j - c_j) + S_{-j}(p_{-j} - c_{-j})$. We then proceed with substitution into the marginal profits ratio $\pi'_j / \widetilde{\pi}'_j = \lambda$. As we have $\pi'_j - \lambda \widetilde{\pi}'_j = 0$, and by denoting $\Lambda = 1/(1-\lambda)$ and $\sigma_{-j} = S_{-j}/S_{jj}$:

$$p_j = c_j - \Lambda \sigma_{-j} c_{-j} + \frac{1}{S_{jj}} q_j + \Lambda \sigma_{-j} p_{-j}. \tag{55}$$

Differentiated multiproduct, quantity competition The problem of the multiproduct firm is to maximize the total profits of all F products belonging to its product portfolio $\mathcal{F}$. The problem of the owner of product j ($j \in \mathcal{F}$) is therefore $\max_{q_k} \sum_{k \in \mathcal{F}} p_k q_k - TC_k$, $k = 1,...j,...,F$. Marginal profit for product j is then equal to $\widetilde{\pi}'_j = p_j - c_j - R_{jj} q_j - \sum_{k \in \mathcal{F}, k \neq j} R_{kj} q_k$, where $R_{kj} = \text{abs}(\partial p_k / \partial q_j)$, $k = 1,...,k,...,N$. Consider $R_{-j,\mathcal{F}} = \sum_{k \in \mathcal{F}, k \neq j} R_{kj}$. By multiplying and dividing the last term on the right by $R_{-j,\mathcal{F}}$, and with some manipulation, we have $\widetilde{\pi}'_j = p_j - c_j - R_{jj} q_j - R_{-j,\mathcal{F}} \sum_{k \in \mathcal{F}, k \neq j} \frac{R_{kj} q_k}{R_{-j,\mathcal{F}}}$. Note that $\sum_{k \in \mathcal{F}, k \neq j} R_{kj} q_k / \sum_{k \in \mathcal{F}, k \neq j} R_{kj}$ is the average quantity of j's oponents within group f, weighted by R_{kj}. Therefore denote this average as $q_{-j,\mathcal{F}} = \sum_{k \in \mathcal{F}, k \neq j} R_{kj} q_k / R_{-j,\mathcal{F}}$. We then have $\widetilde{\pi}'_j = p_j - c_j - R_{jj} q_j - R_{-j,\mathcal{F}} q_{-j,\mathcal{F}}$. By denoting $\widetilde{p}_j = p_j - R_{-j,\mathcal{F}} q_{-j,\mathcal{F}}$, we have $\widetilde{\pi}'_j = \widetilde{p}_j - c_j - R_{jj} q_j$. The problem of the multiproduct firm is to engage in joint-profit maximization not only of its F products of portfolio $\mathcal{F}$, but also to coordinate with all the other $N - F$ products in the market, that is, $\max_{q_k} \sum_k^N p_k q_k - TC_k$, $k = 1,...j,...,N$. Marginal profit for product j is $\pi'_j = p_j - c_j - R_{jj} q_j - \sum_{k \in \mathcal{F}, k \neq j} R_{kj} q_k - \sum_{k \notin \mathcal{F}} R_{kj} q_k$. Denote as before $R_{-j,\mathcal{F}} = \sum_{k \in \mathcal{F}, k \neq j} R_{kj}$ and $q_{-j,\mathcal{F}} = \sum_{k \in \mathcal{F}, k \neq j} R_{kj} q_k / R_{-j,\mathcal{F}}$. Additionally, consider $R_{-\mathcal{F}} = \sum_{k \notin \mathcal{F}} R_{kj}$ and $q_{-\mathcal{F}} = \sum_{k \notin \mathcal{F}} R_{kj} q_k / R_{-\mathcal{F}}$. By multiplying and dividing the last terms on the right of π'_j by, respectively, $R_{-j,\mathcal{F}}$ and $R_{-\mathcal{F}}$, and with some manipulation, we then have $\pi'_j = p_j - c_j - R_{jj} q_j - R_{-j,\mathcal{F}} q_{-j,\mathcal{F}} - R_{-\mathcal{F}} q_{-\mathcal{F}}$. Again, denoting $\widetilde{p}_j = p_j - R_{-j,\mathcal{F}} q_{-j,\mathcal{F}}$:

$$\widetilde{p}_j = c_j + R_{jj} q_j + R_{-\mathcal{F}} q_{-\mathcal{F}}. \tag{56}$$

We then have that (4) nests all the above cases, that is, 1) homogeneous single product, quantity competition, 2) homogeneous multiproduct, quantity competition, 3) differentiated single product, quantity competition, 4) differentiated single product, price competition, 5) differentiated multiproduct, quantity competition. The case of 6) differentiated multiproduct, price competition was presented in 3.1. ■

F Expression for the inclusive value

Consider the inclusive value

$$I_{jg} = \delta_j + (1-\widetilde{\sigma}) \ln \sum_{m=1}^{R_j} \left(v_{mj} e^{\delta_m} \right)^{1/(1-\widetilde{\sigma})}, \tag{57}$$

By conveniently using term $\sum_{m=1}^{R_j} v_{mj}^{1/(1-\widetilde{\sigma})}$ we have

$$\begin{aligned} I_{jg} &= \delta_j + (1-\widetilde{\sigma}) \ln \left[\sum_{m=1}^{R_j} \left(v_{mj} e^{\delta_m} \right)^{1/(1-\widetilde{\sigma})} \frac{\sum_{m=1}^{R_j} v_{mj}^{1/(1-\widetilde{\sigma})}}{\sum_{m=1}^{R_j} v_{mj}^{1/(1-\widetilde{\sigma})}} \right] \\ &= \delta_j + (1-\widetilde{\sigma}) \ln \widetilde{\delta}_{rj} + (1-\widetilde{\sigma}) \ln \sum_{m=1}^{R_j} v_{mj}^{1/(1-\widetilde{\sigma})}, \end{aligned} \tag{58}$$

where $\widetilde{\delta}_{rj} = \sum_{m=1}^{R_j} \left(v_{mj} e^{\delta_m} \right)^{1/(1-\widetilde{\sigma})} / \sum_{m=1}^{R_j} v_{mj}^{1/(1-\widetilde{\sigma})}$, $\widetilde{\delta}_{rj} \geq 1$, is a proxy for weighted mean outlet quality, with outlet visibility terms v_{mj} being weights.

A convenient expression for the summation of the last term of (58) is $\sum_{m=1}^{R_j} v_{mj}^{1/(1-\widetilde{\sigma})} = \left(\overline{v}_j^{1/(1-\widetilde{\sigma})} R_j \right)^{\phi_j}$, where $\overline{v}_j$ is the mean visibility, ϕ_j is an adjustment visibility parameter of product j. We then have

$$I_{jg} = \delta_j + \ln \phi_{0j} + (1-\widetilde{\sigma}) \phi_j \ln R_j, \tag{59}$$

where $\ln \phi_{0j} = (1-\widetilde{\sigma}) \ln \widetilde{\delta}_{rj} + \phi_j \ln \overline{v}_j$. In this framework, ϕ_{0j} is a term that accounts for mean outlet quality and visibility. With (59) we then have the following embedded cases, depending on the values assumed by ϕ_{0j} and ϕ_j:

i) $\left\{ \phi_{0j} = 1, \phi_j = 0 \right\}$: zero mean outlet quality and single outlet strategy: In this case, we assume that only one outlet has full visibility, whereas all other outlets have null visibility of product j. We then have $\sum_{m=1}^{R_j} v_{mj}^{1/(1-\widetilde{\sigma})} = 1$, and consequently $\phi_j = 0$. This setting is equivalent to Berry (1994).

ii) $\left\{ \phi_{0j} = 1, \phi_j = 1 \right\}$: zero mean outlet quality and multi-outlets strategy with outlet exclusivity: In this case, we assume that outlets have no exclusivity of products, but products have exclusivity of outlets. We then have $\sum_{m=1}^{R_j} v_{mj}^{1/(1-\widetilde{\sigma})} = R_j$, and consequently $\overline{v}_j = 1$ and $\phi_j = 1$.

iii) $\left\{ \phi_{0j} = 1, \phi_j = (1-\sigma) / (1-\widetilde{\sigma}) \right\}$: zero mean outlet quality, multi-outlets strategy and a restricted form of visibility: . This setting is equivalent to the congestion framework of Ackerberg and Rysman (2005), when applied to more-general discrete-choice models such as nested logit and random-coefficient models.

G Estimated Price-Elasticities - Crowding F_j

Table 6 - Estimation Results[102]

	Price-Elasticities - Crowding F_j			
Airline	**own**	**cross (same group)**	**cross (rival group)**	**outside good**
SL	-2.5232^{**}	0.6053^{*}	0.3761^{*}	0.0096^{**}
	(1.1808)	(0.3285)	(0.1969)	(0.0043)
TA	-2.0945^{**}	0.8731^{*}	0.5426^{*}	0.0151^{**}
	(0.9632)	(0.4738)	(0.2837)	(0.0068)
TB	-3.800^{***}	0.9579	0.3171^{**}	0.0093^{**}
	(1.3189)	(0.8644)	(0.1443)	(0.0093)
RG	-2.1308^{**}	1.1673^{*}	0.7257^{*}	0.0206^{**}
	(0.9681)	(0.6332)	(0.3793)	(0.0093)
VP	-3.4510^{***}	1.3065	0.4321^{**}	0.0125^{**}
	(1.1697)	(1.1797)	(0.1967)	(0.0050)

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[102] Bootstrapped standard errors in brackets. Superscript * denote significance at the 5% level. Estimated effects of airline-period, airline-codesharing, month, quarter and airline-specific trends not reported.