Approximating the Price Effects of Mergers: Numerical Evidence and an Empirical Application

Nathan H. Miller   Marc Remer   Conor Ryan   Gloria Sheu*

March 2012

PRELIMINARY DRAFT

Abstract

A merger between competing firms may increase the incentive for the merging partners to raise their prices, which in turn has a negative effect on consumer welfare. Both academic researchers and antitrust authorities have developed and implemented techniques to evaluate this potential harm. We use numerical methods to assess the accuracy of one such technique recently proposed by Jaffe and Weyl (2011), which uses information on the pre-merger equilibrium to calculate approximate price effects. Our numerical experiments measure the accuracy of this method and explore the manner in which it is affected by the curvature of the consumer demand curves and by how much information is available on pre-merger equilibrium conditions. We also discuss issues related to implementation and provide computer code that constructs the approximation.

Keywords: merger approximation; merger simulation; upward pricing pressure
JEL classification: K21; L13; L41

---

*The views expressed herein are entirely those of the authors and should not be purported to reflect those of the U.S. Department of Justice. Address: Economic Analysis Group, Antitrust Division, U.S. Department of Justice, 450 5th St. NW, Washington DC 20530. E-mail: nathan.miller@usdoj.gov, marc.remer@usdoj.gov, conor.ryan@usdoj.gov, gloria.sheu@usdoj.gov. We thank Sonia Jaffe and Glen Weyl for productive conversations.

1 Introduction

Mergers can blunt the incentives of the merging firms to compete, as each merging firm internalizes the impact of aggressive actions on the profits of the other. The literature on antitrust economics often characterizes this effect as arising due to the creation of opportunity costs; each merging firm, when making a sale, forgoes with some probability a sale by the other merging firm (e.g., Farrell and Shapiro (2010a); Jaffe and Weyl (2011)). This interpretation is useful for antitrust policymakers because these opportunity costs can be measured given data on consumer substitution patterns and margins in the pre-merger equilibrium.[1] Building on this logic, Jaffe and Weyl (2011) provide general conditions under which the price effects of mergers can be calculated, to a first-order approximation, simply by multiplying these opportunity costs by an appropriate measure of cost pass-through.[2] This calculation, hereafter referred to as the "JW-approximation," is the subject of our research.

The primary contribution of this paper is an evaluation of the JW-approximation. We use numerical experiments to assess the accuracy of the JW-approximation across a variety of economic environments. The exercise is important because the precision of the JW-approximation is theoretically unclear, except when the opportunity costs approach zero or when the second order properties of the demand and cost functions are unimportant (e.g., as with linear demand and constant marginal costs). Additionally, in practice, information on the pre-merger equilibrium often may be incomplete, and it is unclear a priori how the missing information affects the performance of the JW-approximation.

A forthcoming secondary contribution is an empirical application of the JW-approximation to data, demonstrating how the approximation can be operationalized in practice. This application is still in progress and will appear in a future version of this paper.

We find that when economic conditions are characterized by standard demand and cost

[1] Farrell and Shapiro (2010a) refer to the opportunity costs created by a merger as gross upward pricing pressure (UPP). The Horizontal Merger Guidelines of the U.S. Department of Justice and the Federal Trade Commission, as revised in 2010, endorse upward pricing pressure as informative of the likely competitive effects of mergers. See Horizontal Merger Guidelines ∮6.1:

> "The value of sales diverted to a product is equal to the number of units diverted to that product multiplied by the margin between price and incremental cost on that that product. In some cases, where sufficient information is available, the Agencies assess the value of diverted sales, which can serve as a diagnostic of the upward pricing pressure.... The Agencies rely much more on the value of diverted sales than on the level of the HHI for diagnosing unilateral price effects in markets with differentiated products."

[2] Froeb, Tschantz, and Werden (2005) derives a similar approximation for the specific case of Nash-Bertrand competition and constant marginal costs.

functions the JW-approximation performs well, except for the case of log-linear demand. The approximation is quite precise when the true price effect resulting from a merger is less than 10%, but loses accuracy as the true price effect increases. Similarly, the JW-approximation loses precision with increasing curvature of demand. We also find that the accuracy of the JW-approximation is, in most circumstances, not substantially affected by limited information on cost pass-through.

Our paper proceeds as follows. In Section 2, we derive and motivate the JW-approximation, discuss how the requisite measure of cost pass-through can be obtained with knowledge of either the second derivatives of demand or cost pass-through in the pre-merger equilibrium, and compare the JW-approximation to merger simulation. We detail the research design of the numerical experiments in Section 3 and provide results in Section 4. Section 5 concludes.

2 Overview of Merger Approximation

2.1 Derivation and Graphical Illustration

We focus on models of Bertrand-Nash competition in which firms face well-behaved, twice-differentiable demand functions. Each firm i produces some subset of the products available to consumers and sets prices to maximize short-run profits, taking as given the prices of its competitors. The profits of firm i have the expression:

$$\pi_i = P_i^T Q_i(P) - C_i(Q_i(P)), \tag{1}$$

where P_i is a vector of firm i's prices, Q_i is a vector of firm i's sales, P is a vector containing the prices of every product, and C_i is the cost of firm i. The following first order conditions characterize firm i's profit-maximizing prices:

$$f_i(P) \equiv - \left[\frac{\partial Q_i(P)^T}{\partial P_i} \right]^{-1} Q_i(P) - (P_i - MC_i) = 0, \tag{2}$$

where MC_i is a vector of firm i's marginal costs (i.e., $MC_i = \frac{\partial C_i(Q_i(P))}{\partial Q_i(P)}$).

Mergers change the pricing incentives of the merging firms, causing each firm to internalize the effect that a change in its price has on the sales of its merging partner. This change in incentives is reflected in a new set of first-order conditions. Consider a merger between firms j and k; the post-merger first order conditions are:

$$h_i(P) \equiv f_i(P) + g_i(P) = 0 \quad \forall i, \tag{3}$$

where

$$g_j(P) = \underbrace{-\left(\frac{\partial Q_j(P)^T}{\partial P_j}\right)^{-1}\left(\frac{\partial Q_k(P)^T}{\partial P_j}\right)}_{\text{Matrix of Diversion from } j \text{ to } k}\underbrace{(P_k - MC_k)}_{\text{Markup of } k} + \underbrace{\Delta MC_j}_{\text{Cost Efficiencies}}, \tag{4}$$

$g_k(P)$ is analogous and $g_i(P) = 0$ for $i \neq j, k$. The diversion matrix in equation 4 represents the fractions of sales lost by firm j's products that shift to firm k's products due to an increase in firm j's prices. When multiplied by the vector of firm k's markups this yields the value of diverted sales; the higher are the value of diverted sales, the greater incentive a firm has to raise price following a merger. Hence, Farrell and Shapiro (2010a) refer to $g_j(P^0)$ and $g_k(P^0)$ as the net *upward pricing pressure* created by the merger.

It is natural to conceptualize upward pricing pressure as the opportunity cost of sales; an additional sale of firm j's products cannibalizes the profit that may be earned on the sale of firm k's products (e.g., Weyl and Fabinger (2009); Farrell and Shapiro (2010a); Farrell and Shapiro (2010b); Kominers and Shapiro (2010); Jaffe and Weyl (2011)). The extent of this cannibalization depends upon the value of diverted sales. The interpretation of upward pricing pressure as an opportunity cost is reinforced by the fact that the value of diverted sales and marginal costs enter linearly into the post-merger first order conditions. That is, the effect on pricing incentives of a change in marginal cost (which is traditionally thought of as the opportunity cost of selling one unit) is linearly counterbalanced by the value of diverted sales.

These altered incentives affect post-merger equilibrium prices, as firms pass through to consumers the opportunity costs created by the merger. Hence the insight of Jaffe and Weyl (2011) – approximate price effects can be calculated given measures of cost pass-through and upward pricing pressure:[3]

Theorem 1 (Jaffe and Weyl 2011). *Let P^0 be the pre-merger equilibrium price vector. If the functions f(P), g(P) and h(P) characterize the pre-merger first order conditions, upward pricing pressure and the post-merger first order conditions, respectively, so that $\frac{\partial h(P)}{\partial P} = \frac{\partial f(P)}{\partial P} + \frac{\partial g(P)}{\partial P}$ and $h(P^0) = g(P^0)$, and if $h(P)$ is invertible, then the price changes due to a merger, to a first approximation, are*

[3]Note that Theorem 1 in Jaffe and Weyl 2011 generalize beyond Bertand-Nash competition to all competitive environments where firms choose a single strategic variable.

given by the vector

$$\Delta P = -\left(\frac{\partial h(P)}{\partial P}\right)^{-1}\Bigg|_{P=P^0} h(P^0). \tag{5}$$

Equation (5), in essence, applies a single step of Newton's optimization method to approximate the price vector that satisfies each firm's post-merger first order conditions. The function $h(P^0)$ characterizes the post-merger first order conditions but is evaluated at pre-merger prices. Note that $h(P^0)$ is equivalent to the gross upward pricing pressure vector, equation (4), because $f(P^0) = 0$ by definition. The matrix $-\left(\partial h(P)/\partial P\right)^{-1}|_{P=P_0}$ can be interpreted as a measure of the extent to which upward pricing pressure is transmitted to consumers; Jaffe and Weyl (2011) refer to the matrix as *merger pass-through.*[4]

To build intuition, we represent graphically a simplified version of the approximation.[5] Figure 1 plots a hypothetical function $h_i(P_i; P^0_{-i})$ for the single-product firm i, holding the prices of other products fixed at pre-merger equilibrium levels. The dashed line is tangent to $h_i(P_i; P^0_{-i})$ at the pre-merger price; the post-merger price is approximated by projecting this tangent to its point of intersection with the horizontal axis. In this example, the convexity of $h_i(P_i; P^0_{-i})$ leads the approximation to understate the optimal price of the product given other prices at pre-merger levels (the optimal price is located at the intersection of h_i with the horizontal axis).[6] Since the convexity or concavity of the function depends on the higher-order properties of demand, in general the approximation could understate or overstate this optimal price.

The approximate price effects of Theorem 1 are precise when upward pricing pressure is arbitrarily small. The approximation is also precise for profit functions that are quadratic in price, e.g., with linear demand and constant marginal costs. Outside of these special cases, however, the precision of the JW-approximation is theoretically ambiguous. While, intuitively, the accuracy of the approximation may be expected to decrease with the magnitude of upward pricing pressure and the degree of curvature in $h(P)$ over the relevant range of prices, it is unclear how these factors interact and at what rate the precision degrades.

[4]In Section 2.2, we precisely state the relationship between merger pass-through and marginal cost pass-through in the pre-merger equilibrium.

[5]For ease of exposition, we impose that $\partial h(P)/\partial P$ is diagonal, thereby ignoring how the price of a product is affected by upward pricing pressure from other products.

[6]This optimum typically does not characterize the post-merger price because the point of intersection shifts as the prices of other goods re-equilibrate. Whether the post-merger price is higher or lower than this optimum depends on whether products are strategic complements or substitutes, as defined by Bulow, Geanakoplos, and Klemperer (1985).

Thus, in Section 4 we conduct numerical experiments in order to evaluate the accuracy of the approximation in such settings.

2.2 Obtaining Merger Pass-Through

The calculation of approximate price effects based on Theorem 1 of Jaffe and Weyl (2011) requires knowledge of the merger pass-through matrix, i.e., knowledge of $-\left(\partial h(P)/\partial P\right)^{-1}|_{P=P_0}$. Merger pass-through depends on both the first and the second derivatives of demand, which can be ascertained from equations (2), (3) and (4).[7] One approach to obtaining these demand derivatives is to estimate them from data.[8] An alternative approach, one that we outline next, is to infer the second derivatives of demand based on pre-merger pass-through rates.[9] Such pass-through rates have been estimated in the academic literature (e.g., Besanko, Dube, and Gupta (2005)) and in conjunction with antitrust litigation (e.g., see Ashenfelter, Ashmore, Baker, and McKernan (1998) on the Staples-Office Depot merger case).

The connection between pass-through rates and the second order properties of demand has been emphasized in the recent theoretical literature (e.g., Weyl and Fabinger (2011)). Consider the imposition of a per-unit tax on each product, which serves to perturb marginal costs, and denote the vector of taxes t. Since marginal costs enter quasi-linearly into the first order conditions of each firm, as expressed in equation 2, the post-tax, pre-merger first order conditions can be written as

$$f(P) + t = 0.$$

Differentiating with respect to t obtains

$$\frac{\partial P}{\partial t}\frac{\partial f(P)}{\partial P} + I = 0,$$

and algebraic manipulations then yield:

$$\frac{\partial P}{\partial t} = -\left(\frac{\partial f(P)}{\partial P}\right)^{-1}. \tag{6}$$

The inverted Jacobian of $f(P)$, which incorporates first and second derivatives of demand, provides the pass-through rates of the pre-merger equilibrium when it is evaluated at pre-

[7] We provide an explicit expression for merger pass-through in Appendix A.

[8] For instance, the translog demand model (Christensen, Jorgenson, and Lau (1975)) and the AIDS model (Deaton and Muellbauer (1980)) each have somewhat flexible second order properties. Given sufficient data, these can be estimated.

[9] Knowledge of the first derivatives of demand is also required.

merger prices.[10] Pre-merger pass-through is closely related to merger pass-through, which is provided by the inverted Jacobian of $h(P)$ evaluated at pre-merger prices.

It is possible to infer merger pass-through from pre-merger pass-through because both depend on the first and second derivatives of demand. In particular, if the first derivatives are known then second derivatives can be selected to rationalize the pre-merger pass-through rates. This entails numerical optimization. Suppose that one observes (or estimates) a pre-merger pass-through rate matrix W. Then one can search for candidate second derivatives that satisfy

$$\frac{1}{N^2}\left\| W + \left(\frac{\partial f(P,X)}{\partial P}\right)^{-1}\Bigg|_{P=P_0}\right\| < \delta, \tag{7}$$

where N is the number of products, X contains the candidate second derivatives, and δ is some user-specified tolerance. These second derivatives can then be used, in conjunction with the first derivatives, to obtain merger pass-through.

Since the matrices that appear in equation (6) are of dimensionality $N \times N$, the relationship between cost pass-through and the Jacobian of the pre-merger first order conditions provides N^2 equations with which to identify unknown second derivatives. An assumption that demand satisfies Slutsky symmetry is sufficient for full identification in the special case of a merger among single product duopolists.[11] In other cases, second derivatives of the form $\frac{\partial^2 Q_i}{\partial P_j \partial P_k}$, for $i \neq j$, $i \neq k$ and $j \neq k$, are not identified from equation (6) even with Slutsky symmetry. These second derivatives are plausibly small, however, and it may be reasonable to normalize them to zero. Alternatively, Jaffe and Weyl (2011) suggest the following "horizontality" assumption on demand:

$$Q_i(P) = \psi\left(P_i + \sum_{j \neq i} \mu_j(P_j)\right), \tag{8}$$

for some $\psi : \mathbb{R} \to \mathbb{R}$ and $\mu : \mathbb{R} \to \mathbb{R}$, which is sufficient for full identification. The needed

[10] See Appendix A for the full expression of $\partial f(P)/\partial P$.

[11] Slutsky symmetry implies $\frac{\partial Q_i}{\partial P_j} = \frac{\partial Q_j}{\partial P_i}$ and it follows that:

$$\frac{\partial^2 Q_i}{\partial^2 P_j} = \frac{\partial}{\partial P_j}\frac{\partial Q_i}{\partial P_j} = \frac{\partial}{\partial P_j}\frac{\partial Q_j}{\partial P_i} = \frac{\partial^2 Q_j}{\partial P_j \partial P_i}.$$

second derivatives then take the form:

$$\frac{\partial^2 Q_i}{\partial P_j \partial P_k} = \frac{\partial^2 Q_i}{\partial^2 P_i} \frac{\frac{\partial Q_i}{\partial P_j}\frac{\partial Q_i}{\partial P_k}}{\left(\frac{\partial Q_i}{\partial P_i}\right)^2}. \tag{9}$$

The numerical experiments that we conduct explore how these identifying assumptions affect the accuracy of the approximation in a variety of economic environments.

2.3 Comparison to Merger Simulation

It is useful to compare approximate price effects as defined in Theorem 1 with the technique of merger simulation, which is employed routinely by researchers and antitrust authorities to predict the price effects of mergers (e.g., Nevo (2000); Werden and Froeb (2008)). Merger simulation begins with the specification of a function for the demand system, which is typically assumed by the practitioner. Then, the structural parameters are estimated to bring the implied first derivatives of demand close to those implied by the data.[12] Alternatively, the structural parameters can be calibrated with evidence on price-cost margins and consumer substitution patterns gleaned from accounting tools, surveys, marketing studies, or other documentary evidence.[13] With the specified demand system and appropriate structural parameters, post-merger prices can be calculated as the P^* that solves

$$h(P^*) \equiv f(P^*) + g(P^*) = 0, \tag{10}$$

where the functions f, g, and h are as defined in Section 2.1. This step often, but not necessarily, entails numerical optimization.[14]

The functional form of demand determines how elasticities in the simulation model

[12]In the literature, demand estimation is often discussed in the context of elasticity estimation. Note that, in practice, structurally estimating demand elasticities is operationally equivalent to estimating first derivatives. We frame our discussion of merger simulation in the context of first derivative estimation, so as to facilitate the comparison to the JW-approximation.

[13]Demand estimation has received considerable attention in the academic literature (e.g., Berry, Levinsohn, and Pakes (1995); Nevo (2001)), in part because price-cost margins are difficult to measure with public data but can be inferred given estimates of the demand elasticities. Demand calibration is less commonly employed by academic researchers because it often requires access to confidential information. However, firms have a strong incentive to understand their costs and the relevant patterns of consumer substitution, and the resulting documentation often becomes available to economists employed by the Antitrust Division and the Federal Trade Commission under the Hart-Scott-Rodino Act.

[14]The post-merger prices can be computed as the vector $\widetilde{P}$ that satisfies $\frac{1}{N} \parallel h(\widetilde{P}) \parallel < \delta$. Analytical solutions are available for the case of linear demand and constant marginal costs.

change as prices move away from the pre-merger equilibrium and, as a result, the predicted price effects can be sensitive to functional form assumptions (e.g., Shapiro (1996); Crooke, Froeb, Tschantz, and Werden (1999)). Absent efficiencies, merger simulations based on demand systems with little or no curvature (e.g., linear demand) produce smaller predicted price increases than simulations based on demand systems that are more convex (e.g., log-linear demand).[15] It is worth pointing out that demand estimation is generally employed to recover the first derivatives of demand within the range of the data, but not to recover the second derivatives of demand. Rather, the second derivatives are dictated by the assumed functional form which is rarely selected with demand curvature in mind.[16]

The JW-approximation differs from merger simulation primarily in how the second derivatives of demand are treated, or equivalently, in how demand elasticities are projected to change as we move away from the pre-merger equilibrium. Whereas merger simulation requires an assumption on the second derivatives, imposed via the functional form of demand, the JW-approximation utilizes knowledge of cost pass-through or the second-order properties of demand around the pre-merger equilibrium. Thus, the basic insight of Froeb, Tschantz, and Werden (2005) and Jaffe and Weyl (2011) is that when high quality information on cost pass-through or demand curvature around the pre-merger equilibrium is available, this information can enable researchers and practitioners to approximate the effect of a merger without restricting them to specify a full demand system.

2.4 Alternative first order conditions

It is well understood that first order conditions can be manipulated to yield various expressions, each of which characterizes the same profit-maximizing prices. How the first order conditions are written is unimportant for merger simulation, as predictions are unaffected,

[15]Shapiro (1996) considers a merger between two single-product firms with identical margins (m) and diversion ratios (d) and shows that $\Delta p/p = md/2(1-d)$ if demand is linear and $\Delta p/p = md/(1-m-d)$ if demand is log-linear. For any m and d, the predicted price effects with log-linear demand are more than double those with linear demand. See also Crooke, Froeb, Tschantz, and Werden (1999), which conducts numerical experiments and documents that for a given set of pre-merger elasticities, a log-linear demand specification yields substantially greater price increases than logit or AIDS specifications, which in turn yield greater price increases than a linear specification.

[16]More important is the trade-off between the tractability of estimation and the reasonableness of implied consumer behavior. For instance, the almost ideal demand system (AIDS) model of Deaton and Muellbauer (1980) allows for flexible substitution patterns but suffers from the curse of dimensionality as N^2 price coefficients must be estimated (N being the number of products). By contrast, the logit demand system has only a single price coefficient but restricts substitution patterns. The random coefficients logit model of Berry, Levinsohn, and Pakes (1995) is widely used in the academic literature because it does not suffer from the curse of dimensionality while allowing for flexible substitution patterns.

but this is not the case for approximate price effects. Rather, both the approximated post-merger prices and their accuracy are affected by the form of the first order conditions.

The JW-approximation as originally derived makes use of a transformation of the post-merger first order conditions, which is specified in equation (3) and defines the function $h_i(P)$. All terms in the first order condition are multiplied by the inverse of the matrix of demand first derivatives. This transformation is innocuous when $h_i(P)$ is evaluated at the post-merger equilibrium prices, where the first order conditions equal zero by definition. However, to approximate post-merger prices, the JW-approximation evaluates $h_i(P)$ at the pre-merger equilibrium prices, where the first order conditions do not equal zero. In this case, the multiplication changes the first order condition. Consequently, the transformation used to derive $h_i(P)$ may have real implications on the accuracy of the JW-approximation.

An alternative specification of the post-merger first order conditions is derived in Froeb, Tschantz, and Werden (2005). Therein the first order conditions are constructed in the usual manner by taking the derivative of the profit function with respect to price, which is expressed as follows:

$$f_i^{alt}(P) \equiv Q_i(P) + \left(\frac{\partial Q_i(P)}{\partial P_i}^T\right)(P_i - MC_i) = 0. \tag{11}$$

Considering a merger between firms j and k, the post-merger first order conditions are:

$$h_i^{alt}(P) \equiv f_i^{alt}(P) + g_i^{alt}(P) = 0, \tag{12}$$

where

$$g_j^{alt}(P) = \left(\frac{\partial Q_k(P)^T}{\partial P_j}\right)(P_k - MC_k) - \left(\frac{\partial Q_j(P)}{\partial P_j}^T\right)\Delta MC_j. \tag{13}$$

This modified formula can also be used to approximate merger price effects just as in Theorem 1. There is no reason, a priori, to expect the JW-approximation to perform better or worse than this alternative specification, and in practice both methods require the same set of primitives. In Section 4, we evaluate the performance of both formulations.

It worth noting, however, that marginal costs do not enter these alternative first order conditions linearly, and therefore the interpretation of $g_j^{alt}(P^0)$ and $h_j^{alt}(P^0)$ as opportunity costs is less straight-forward than in the JW-approximation.[17] Further, while the alternative

[17] The emphasis of the antitrust literature on upward pricing pressure, as expressed in equation (4), rather than on $g_j^{alt}(P)$, stems from the fact that upward pricing pressure can be calculated with diversion ratios whereas $g_j^{alt}(P)$ requires knowledge of demand elasticities. This advantage of upward pricing pressure does not extend to the calculation of approximate price effects, which requires knowledge of these elasticities regardless of how the first order conditions are expressed.

merger pass-through matrix $-(\partial h^{alt}(P)/\partial P)^{-1}|_{P=P^0}$ retains its interpretation as a measure of how $g_j^{alt}(P)$ is transmitted to consumers through prices, its connection to pre-merger cost pass-through rates is tenuous because the inverse Jacobian of the alternative first order conditions does not yield pre-merger cost pass through, as it does for the baseline first order conditions in equation (6).

3 Research Methodology

We use numerical experiments to evaluate the accuracy of the JW-approximation across a range of economic environments. In each experiment, we first posit the demand and cost functions that fully characterize the market, and treat these as the "truth." Then, a merger between two firms in the market is simulated, which yields the true price effect of the merger and provides a baseline against which to measure the accuracy of the JW-approximation. To be clear, as discussed in the previous section, the tool of merger simulation is typically used to *estimate* the price effect of a merger. In our experiments, by contrast, merger simulation is conducted with full knowledge of the underlying demand system and thus provides the true price effects.

To perform the JW-approximation, we start by deriving the first derivatives and pass-through rates that arise in pre-merger equilibrium from the true demand and cost functions. In practice, to implement the JW-approximation these values will either be estimated from data or, in the case of antitrust enforcement, may be gleaned from company documents. As our purpose is to evaluate the precision of the JW-approximation, we start with the true pre-merger first derivatives and pass-through rates implied by the underlying demand system. With this information, we then approximate the second derivatives of demand using the relationship defined in equation (7) and the horizontality of demand assumption expressed in equation (8). We choose this method for obtaining second derivatives, as opposed to using the true values, because, in practice, second derivatives are not typically observed from data. In Section 4.4, we evaluate the accuracy of the JW-approximation using various alternative specifications for the second derivatives, including the true values.

This information is then sufficient to perform the JW-approximation and compare its predicted price effects to the true price effects. Thus, each numerical experiment enables us to measure how closely the approximation captures the true price effects under a specific set of demand and supply conditions. We iterate this procedure over a range of demand and cost conditions to generate evidence regarding (1) the overall accuracy of the approximation,

(2) how market conditions affect accuracy, and (3) how the approximation performs when information on the pre-merger equilibrium is incomplete.

In each iteration, we consider an industry with three single-product firms and evaluate a merger between the first two firms. We draw the pre-merger market shares of firms 1 and 2 over the range 5%-42% and let firm 3 capture the remaining sales. We normalize pre-merger prices to one, assume constant marginal costs, and specify firm 1's margins alternately to be 0.2, 0.4, 0.6, and 0.8. These data are sufficient to calibrate a logit demand model of the form:

$$q_i = \frac{e^{(\eta_i - p_i)/\tau}}{\sum_k e^{(\eta_k - p_k)/\tau}} M, \tag{14}$$

where M is the overall size of the market, which we normalize to one, and the parameters to be calibrated are η_i and τ.[18] The implied elasticities evaluated at the pre-merger equilibrium are:

$$\epsilon_{jk} = \begin{cases} -(1 - s_j)/\tau & \text{if } j = k \\ s_k/\tau & \text{if } j \neq k \end{cases}, \tag{15}$$

and the margins of firms 2 and 3 are obtainable from the Lerner index. We use these elasticities and margins to calibrate two additional demand systems: PC-AIDS and log-linear.[19] The PC-AIDS demand system of Epstein and Rubinfeld (1999) takes the form:

$$w_i = \alpha_i + \sum_j \beta_{ij} \log p_j, \tag{16}$$

where w_i is the expenditure share of firm i (i.e., $w_i = p_i q_i / \sum_k (p_k q_k)$), and the log-linear demand system takes the form

$$\ln(q_i) = \kappa_i + \sum_j \epsilon_{ij} \ln p_j. \tag{17}$$

Given this data generating procedure, each demand system will be evaluated at the same set of pre-merger equilibrium prices, shares, elasticities, and margins;[20] however, the curvature of demand at the pre-merger equilibrium will vary across demand systems. This allows us to analyze how well the JW-approximation performs within different underlying

[18] We defer the details of calibration to Appendix B.

[19] In the case of PC-AIDS, only two margins are required for calibration. We take from the logit calibration the margins for the first two goods and let the third margin vary according to the relationships implied by the model.

[20] In the case of PC-AIDS, only the first two margins are identical to those in the other demand systems. The PC-AIDS model only requires two margins for calibration, and then implies a value for the third.

demand environments.

Then, to further isolate the effect of demand curvature on the accuracy of the JW-approximation we evaluate it when the demand system is characterized by the following functional form,

$$q_i = \mu_i + \sum_j \delta_{ij} p_j^{\gamma}, \tag{18}$$

which we call the "price-exponent" demand system. Here, higher values of γ correspond to greater curvature of demand. While this demand system is not commonly used in practice,[21] evaluating the accuracy of the JW-approximation under different values of γ allows us to more precisely pin down the effect of demand curvature than does looking across different demand systems.

In addition, we more generally investigate the connection between the magnitude of price effects and the accuracy of the JW-approximation. For arbitrarily small true price effects the JW-approximation should be exact, as it depends upon information local to the pre-merger equilibrium. Thus, we expect that as the true post-merger price change moves farther away from the pre-merger equilibrium the first-order approximation will be less accurate. Section 4.1 tests this hypothesis by estimating how the true magnitude of the price change affects the precision of the JW-approximation.

As mentioned in Section 2.4, the specification of first-order conditions affect the predictions of the JW-approximations. Section 4.3 considers the specification in Jaffe and Weyl (2011), equation (2), as well as another commonly used functional form, equation (11), and investigates the impact on the JW-approximation.

4 Results

4.1 Overall Accuracy

We begin assessing the accuracy of the JW methodology by comparing its predicted post-merger prices to those produced by a structural demand model, which is assumed to completely characterize the true economic environment. The demand functional forms we consider are logit, log-linear, and PC-AIDS, and we generate a large set of parameterizations by varying the primitives fed into the calibrations.

The results of this exercise are summarized in Table 1. Across demand systems, the JW-approximation predicts price changes that are, on average, 15% lower than the actual

[21] Note, however, that $\gamma = 1$ corresponds to the case of linear demand.

price change.[22] However, the mean masks substantial variation, as 90% of the approximated price changes fall between 82% below and 50% above the actual price change. While this finding suggests a large amount of dispersion between the predicted and actual price changes, a closer examination reveals that this is largely due to the poor performance of the JW-approximation when demand is log-linear. On average, the approximated price effects are 76% below the true effects when demand is log-linear (90% of the observations are between −91% and 74%). When demand is logit or PC-AIDS, however, the JW-approximation performs much better; the approximated effects are on average 6% below the true effect for logit (90% of the observations are between −13% and 9%), and for PC-AIDS the average difference between the approximation and the truth is 23% (90% of the observations are between 5% and 57%).

Taking a closer look, the approximation errors appear to be related to the size of the price change implied by the true demand model. In particular, the error tends to increase as the true price change becomes larger. This pattern is apparent in Figure 2, which graphs the approximated change in the price of good 1 relative to the true change in the price of good 1. As the true change in the price of good 1 increases, the dots tend to move farther away from the 45 degree line, indicating greater error. This result is intuitive; the JW methodology uses only information local to the pre-merger equilibrium, in lieu of leveraging structural assumptions that would define demand across all scenarios. Consequently, if the post-merger equilibrium is actually far away from the observed starting point, then the approximation is more likely to perform poorly. It is comforting that for true price changes of less than 10%, both the logit and PC-AIDS approximations are reasonably accurate.

4.2 Relationship to Curvature

The previous section makes clear that the accuracy of the JW-approximation varies across demand specifications. Indeed, these differences persist even when distinct demand systems imply similar post-merger price increases. Here we explore how variation in the curvature of demand can drive these differences in accuracy.

Curvature dictates how demand changes as a firm's price moves away from the pre-merger equilibrium. If demand is linear, for example, knowing the slope of demand at the pre-merger equilibrium is sufficient to be able to trace out the entirety of the demand function. If instead the demand function is curved, knowing the slope at the pre-merger

[22] To be clear, if the true price change is 5% and the JW-approximation predicts a 4.25% price change, then the JW-approximation predicts a price increase that is 15% lower than the truth ($\frac{4.25-5}{5} = -.15$).

equilibrium will only provide an approximation to what demand looks like elsewhere. The more curved the function is at the pre-merger equilibrium, the worse this approximation method becomes.

In our analysis, curvature changes as we move across demand specifications. As Crooke, Froeb, Tschantz, and Werden (1999) show, curvature tends to increase between logit, PC-AIDS, and log-linear, with the latter tending to be the most curved. Of course, as we move between these demand specifications, other aspects of demand change as well, particularly with respect to the nature of substitution between products.

In order to isolate the effect of curvature, we focus on the price-exponent demand system defined in equation (18). The advantage of using this functional form is that it has a single parameter, γ, that allows us to vary curvature without entirely switching the underlying functional form. See Figure 3, which graphs out a single-product price-exponent demand for different values of γ. As γ increases from 0 to 1, the demand function moves from a convex shape to a straight line (that is, linear demand is included as a special case when $\gamma = 1$). As γ increases beyond 1, the demand function moves from a straight line to a concave shape.

We calibrate the price-exponent demand system for different values of $\gamma = \{0.25, 0.50, 0.75, 1.25, 1.5, 2\}$ and then follow the same procedure described in Section 3 to generate data. The results of this exercise are summarized graphically in Figure 4. Each panel depicts results for a different value of γ, displaying the approximate price change in product 1 against the actual price change. For all values of γ, the approximation is quite accurate for small true price changes; differences only become apparent as the size of the true price change increases. In particular, for values of γ farther away from 1, the approximation is less accurate, which is evidenced by the increased distance away from the 45 degree line. Thus, for large true price changes (greater than 10%) the accuracy of the JW-approximation decreases as the curvature of demand increases. This accords with our finding that the JW-approximation tends to perform most poorly when the underlying demand is log-linear, which is relatively more curved compared to the logit and PC-AIDS.

Furthermore, for convex cases ($\gamma < 1$), the approximation tends to overestimate the price change, evidenced by the dots generally being above the 45 degree line. For the concave cases ($\gamma > 1$), the approximation tends to underestimate the price change.[23]

[23] Although omitted from the figure due to space constraints, the analogous graph for a linear demand system ($\gamma = 1$) has all its dots lined up perfectly along the 45 degree line.

4.3 FOC Transformations

Here we compare the baseline JW-approximation with the alternative formulation specified in equation 12. To do so, we employ the same procedure utilized in Section (4.1), and depict the results in Figure 5. When the underlying demand conditions are characterized by logit or PC-AIDS the alternative form tends to increase the spread of results, indicating a deterioration in accuracy. For log-linear, the level of accuracy is similarly poor with both methods. In addition, the alternative form tends to under predict the true price change more often than the baseline JW-approximation.

4.4 Different Informational Regimes

In theory, the JW methodology assumes that the full pre-merger pass through matrix is known and uses this matrix to construct a post-merger analog. This is accomplished by exploiting the relationship between pass-through and the second derivatives of demand, which is specified in equation (6). Still, this equation only provides enough restrictions to identify a subset of second derivatives; the remaining second derivatives are identified by invoking the horizontality of demand assumption. Thus, it is important to explore the impact of the horizontality assumption on the accuracy of the JW-approximation. Furthermore, in practice there may be a limited amount of information available on cost pass-through. Therefore, we also evaluate the JW-approximation when it is supplied with incomplete information on firm-specific cost pass-through.

Six informational regimes are considered: (A) the baseline case with full information on pass-through and when horizontality of demand is assumed, (B) pass-through is known, but the horizontality assumption is dropped, leaving non-identifiable second derivatives to be set to zero, (C) the full set of second derivatives is known, so that they do not have to be imputed from pass-through, (D) the pre-merger pass through matrix is used in lieu of the post-merger matrix, (E) only the pass-through for the merging firms are known[24] and (F) only the merging firms' own pass-through is known.[25]

For each case, we compare the predictions of the JW-approximation to the true price changes implied by the calibrated demand systems; the results for logit demand are summarized graphically in Figure 6. Panel A is the baseline JW-approximation, reproducing the

[24] In this case, we use the average of the merging firms' pass through rates to fill in the non-merging firms' rates.

[25] Here, the non-merging firms' own pass through is assumed to be the average of the merging firms', and the cross pass through rates are set to zero.

results seen in Figure 2. In Panel B, the non-identifiable second derivatives are set to zero, instead of being derived using the horizontality assumption. The results are highly similar to the baseline, suggesting that the horizontality assumption is, at worst, innocuous. In Panel C, the full second derivative matrix is used, which only slightly improves the accuracy. In Panel D, the pre-merger pass-through matrix is used, which again produces results similar to the baseline.

The last two panels examine situations where incomplete information is available about pre-merger pass-through. In Panel E, only the merging firms' pass-through rates are known, which, again, does not meaningfully affect the results. In Panel F, only the merging firm's own pass-through is known, leading to a decrease in accuracy relative to the baseline. In this case, the approximation tends to under-predict the true price change. This is likely a result of zeroing out the cross-pass-through rates, and thereby ignoring many of the second-order price effects. While the JW-approximation is least precise in this case, it is important to note that the approximation (i) still performs quite well when the true price change is less than 10% and (ii) tends to underestimate the true effect, which can lead to conservative enforcement policy.

Table 2 summarizes the results depicted in Figure 6 and also includes information for PC-AIDS and log-linear. The results for the other demand systems broadly follow the same pattern as with logit. In the case of PC-AIDS, swapping the zeros assumption (case B) for the horizontality assumption (case A) can improve the approximation. This is because the form of the PC-AIDS demand function tends to imply small values for the cross-price derivatives.[26] Also for PC-AIDS, using only the merging firms' pass-through causes a larger deterioration in accuracy than in the logit specification. Nearly all of the log-linear cases perform similarly poorly.

5 Conclusion

We demonstrate how to implement merger approximation tools and evaluate the conditions under which they provide accurate predictions. When the underlying economic environment is fully characterized by often-used demand and cost functions, the JW-approximation performs well, except for the case of log-linear demand. The approximation is quite precise when the true price effect resulting from a merger is less than 10%, but begins to lose accuracy as the true price effect increases. Related, the JW-approximation loses precisions with

[26]That is, the second derivative of demand for good 1 with respect prices 2 and 3 tends to be small so long as the elasticity of total expenditure is small. If that elasticity is zero, these cross terms are zero as well.

increasing curvature of demand. Finally, and perhaps most importantly for practical considerations, the accuracy of the JW-approximation is robust to being supplied with limited information on cost pass through.

References

Ashenfelter, O., D. Ashmore, J. B. Baker, and S.-M. McKernan (1998). Identifying the firm-specific cost pass-through rate. *FTC Working Paper*.

Berry, S., J. Levinsohn, and A. Pakes (1995, July). Automobile prices in market equilibrium. *Econometrica 63*(4), 847–890.

Besanko, D., J.-P. Dube, and S. Gupta (2005, Winter). Own-brand and cross-brand retail pass-through. *Marketing Science 1*(1), 123–137.

Bulow, J. I., J. D. Geanakoplos, and P. D. Klemperer (1985). Multimarket oligopoly: Strategic substitutes and complements. *Journal of Political Economy 93*(3), pp. 488–511.

Christensen, L., D. Jorgenson, and L. Lau (1975, June). Transcendental logarithmic utility functions. *American Economic Review 65*, 367–383.

Crooke, P., L. Froeb, S. Tschantz, and G. J. Werden (1999). The effects of assumed demand form on simulated post-merger equilibria. *Review of Industrial Organization 15*, 205–217.

Deaton, A. and J. Muellbauer (1980). An almost ideal demand system. *The American Economic Review 70*(3), pp. 312–326.

Epstein, R. J. and D. L. Rubinfeld (1999). Merger simulation: A simplified approach with new applications. *Antitrust Law Journal 69*, 883–919.

Farrell, J. and C. Shapiro (2010a). Antitrust evaluation of horizontal mergers: An economic alternative to market definition. *B.E. Journal of Theoretical Economics: Policies and Perspectives 10*(1).

Farrell, J. and C. Shapiro (2010b). Recapture, pass-through, and market definition. *Antitrust Law Journal 76*(3), 585 – 604.

Froeb, L., S. Tschantz, and G. J. Werden (2005). Pass through rates and the price effects of mergers. *International Journal of Industrial Organization 23*, 703–715.

Jaffe, S. and E. G. Weyl (2011). The first order approach to merger analysis.

Kominers, S. and C. Shapiro (2010). Second-order critical loss analysis.

Nevo, A. (2000). Mergers with differentiated products: The case of the ready-to-eat cereal industry. *The RAND Journal of Economics 31*(3), pp. 395–421.

Nevo, A. (2001). Measuring market power in the ready-to-eat cereal industry. *Econometrica 69*(2), pp. 307–342.

Shapiro, C. (1996, Spring). Mergers with differentiated products. *Antitrust 10*(2), 23–30.

Werden, G. J. and L. M. Froeb (2008). *Handbook of Antitrust Economics*, Chapter Unilateral Competitive Effects of Horizontal Mergers, pp. 43–104. MIT Press.

Weyl, G. E. and M. Fabinger (2009, October). Pass-through as an economic tool.

Weyl, G. E. and M. Fabinger (2011). A restatement of the theory of monopoly.

Appendix

A Merger Pass-Through Defined

In this appendix, we provide an expression for the Jacobian of h(P), which can be used to construct merger pass-through as defined by Jaffe and Weyl (2011). Using the definition $h(P) \equiv f(P) + g(P)$, we have

$$\frac{\partial h(P)}{\partial P} = \frac{\partial f(P)}{\partial P} + \frac{\partial g(P)}{\partial P}. \tag{19}$$

The Jacobian of $f(P)$, can be written as:

$$\frac{\partial f(P)}{\partial P} = \begin{bmatrix} \frac{\partial f_1(P)}{\partial p_1} & \cdots & \frac{\partial f_1(P)}{\partial p_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_J(P)}{\partial p_1} & \cdots & \frac{\partial f_J(P)}{\partial p_N} \end{bmatrix}, \tag{20}$$

where N is the total number of products and J is the number of firms. The vector P includes all prices; we use lower case to refer to the prices of individual products, so that p_n represents the price of product n.

In the case that product n is sold by firm i,

$$\frac{\partial f_i(P)}{\partial p_n} = -\begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{bmatrix} + \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} \left[\frac{\partial^2 Q_i}{\partial P_i \partial p_n}^T\right] \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} Q_i - \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} \left[\frac{\partial Q_i}{\partial p_n}\right], \tag{21}$$

where Q_i and P_i are vectors representing the quantities and prices respectively of the products owned by firm i, and the initial vector of constants has a 1 in the firm-specific index of the product n. For example, if product 5 is the third product of firm 2, then the 1 will be in the 3^{rd} index position when calculating $\partial f_2(P)/\partial p_5$.

If product n is not sold by firm i, the vector of constants is $\vec{0}$, and thus

$$\frac{\partial f_i(P)}{\partial p_n} = \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} \left[\frac{\partial^2 Q_i}{\partial P_i \partial p_n}^T\right] \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} Q_i - \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1} \left[\frac{\partial Q_i}{\partial p_n}\right]. \tag{22}$$

The matrix $\partial g(P)/\partial P$ can be decomposed in a similar manner:

$$\frac{\partial g(P)}{\partial P} = \begin{bmatrix} \frac{\partial g_1(P)}{\partial p_1} & \cdots & \frac{\partial g_1(P)}{\partial p_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_K(P)}{\partial p_1} & \cdots & \frac{\partial g_K(P)}{\partial p_N} \\ \\ 0 & \cdots & 0 \\ \downarrow & & \downarrow \end{bmatrix}, \tag{23}$$

where N is the number of products and K is the number of merging firms. Notice that $\partial g(P)/\partial P$ includes zeros for non-merging firms, because the merger does not create opportunity costs for these firms.

In the case that product n is sold by a firm merging with firm i (this does not include firm i itself),

$$\begin{aligned} \frac{\partial g_i(P)}{\partial p_n} &= -\left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial Q_j}{\partial P_i}^T\right]\begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{bmatrix} \\ &+ \left(\left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial^2 Q_i}{\partial P_i \partial p_n}^T\right]\left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial Q_j}{\partial P_i}^T\right] - \left[\frac{\partial Q_i}{\partial P_i}^T\right]^{-1}\left[\frac{\partial^2 Q_j}{\partial P_i \partial p_n}^T\right]\right)(P_j - C_j), \end{aligned} \tag{24}$$

where Q_j, P_j, and C_j are vectors of the quantities, prices, and marginal costs respectively of products sold by firms merging with firm i, and the vector of 1s and 0s has a 1 in the merging firm's firm-specific index of the product n. For example, if product 5 is the third product of firm 2, and firm 2 is merging with firm 1, then the 1 will be in the 3^{rd} index position when calculating $\partial g_1(P)/\partial p_5$. It is an important distinction that – supposing there are more than two merging parties – the index j refers to all of the merging parties' products, excluding firm i's products.

If product n is not sold by any firm merging with firm i (including a product sold by firm i),

$$\frac{\partial g_i(P)}{\partial p_n} = \left(\left[\frac{\partial Q_i}{\partial P_i}^T \right]^{-1} \left[\frac{\partial^2 Q_i}{\partial P_i \partial p_n}^T \right] \left[\frac{\partial Q_i}{\partial P_i}^T \right]^{-1} \left[\frac{\partial Q_j}{\partial P_i}^T \right] - \left[\frac{\partial Q_i}{\partial P_i}^T \right]^{-1} \left[\frac{\partial^2 Q_j}{\partial P_i \partial p_n}^T \right] \right) (P_j - C_j). \tag{25}$$

B Calibration of demand parameters

Coming soon.

Figure 1: Simplified Version of Approximate Price Effects

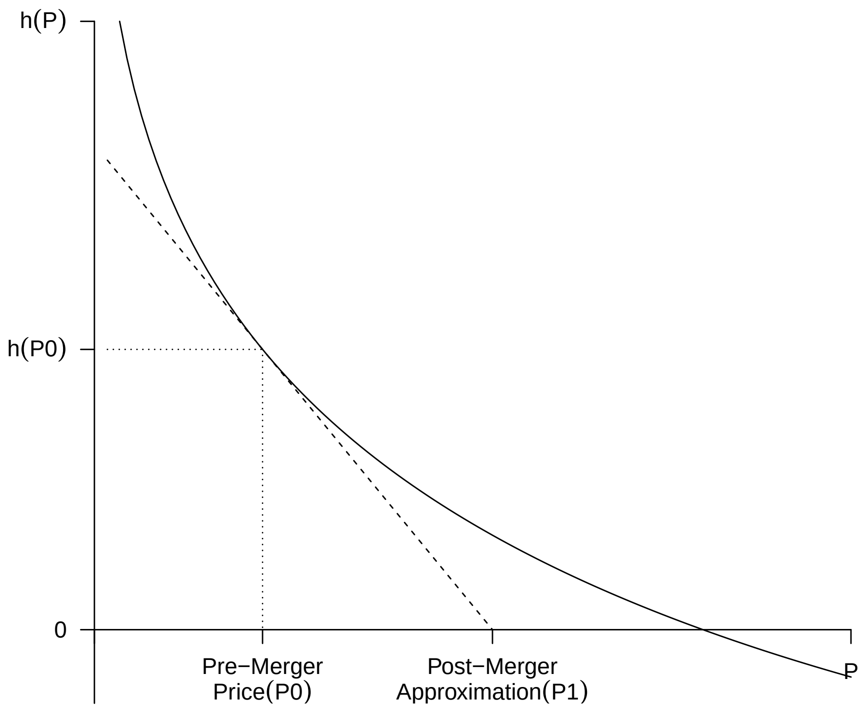
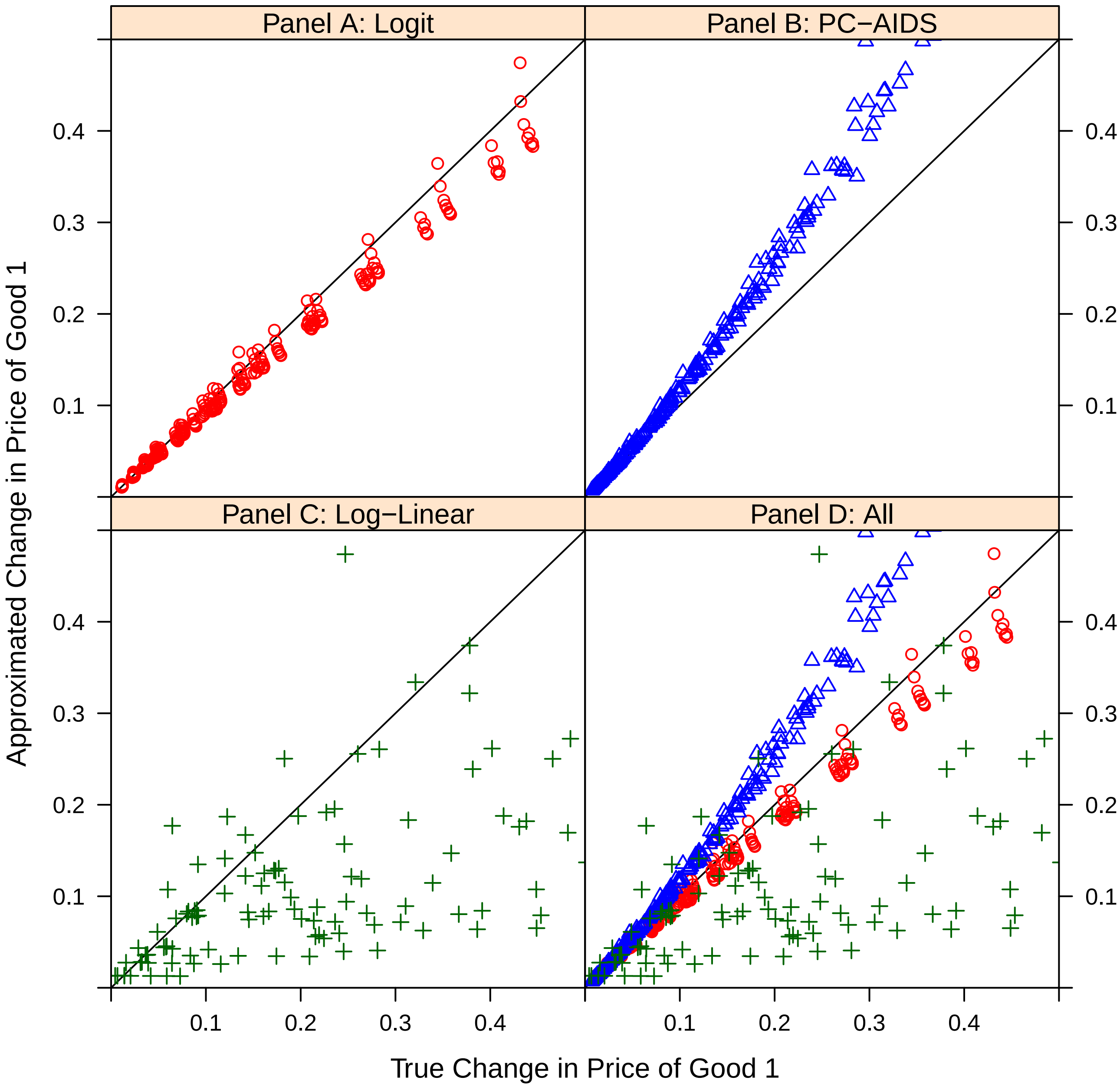
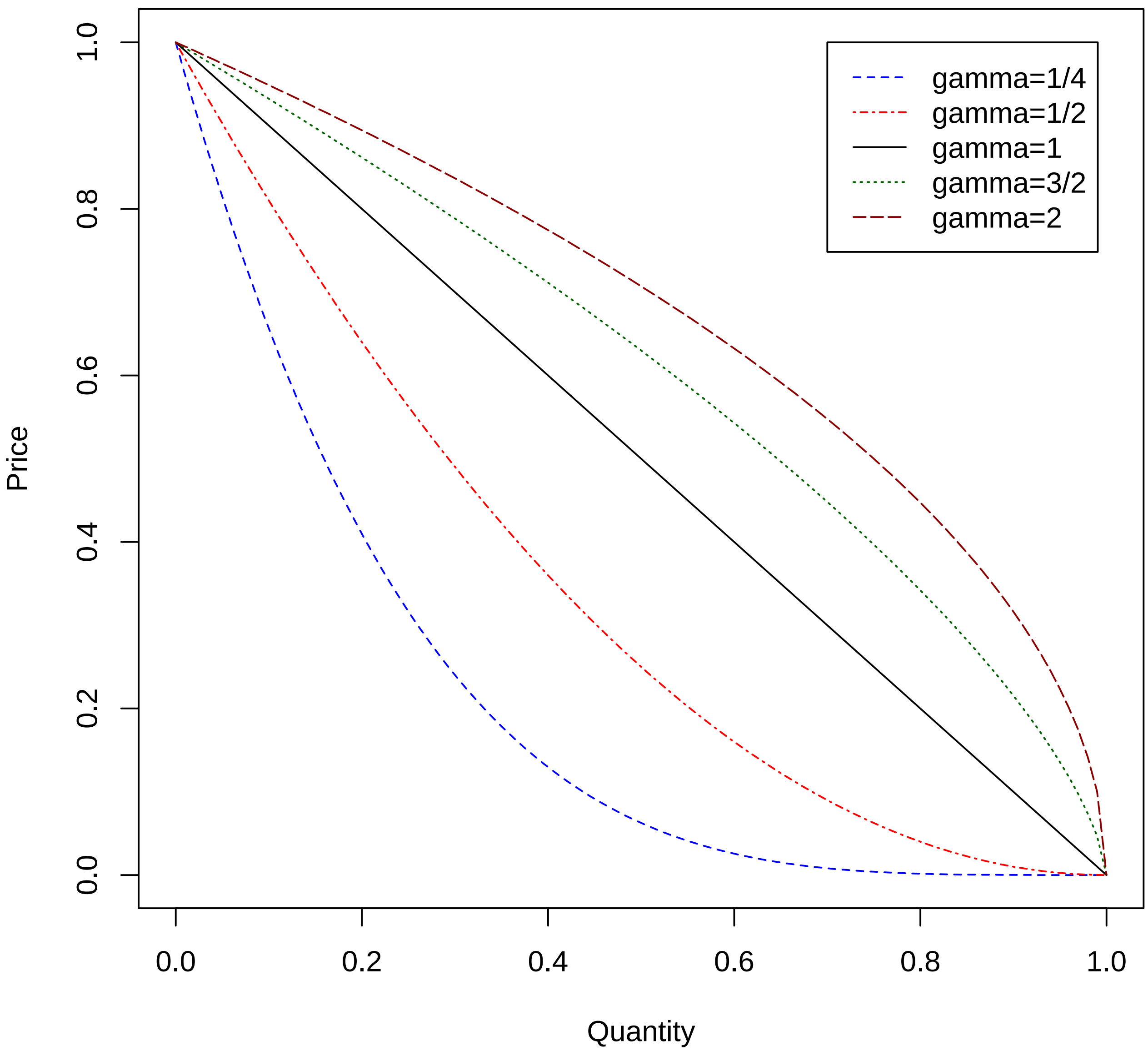


Figure 2: JW-Approximation vs. True Price Change



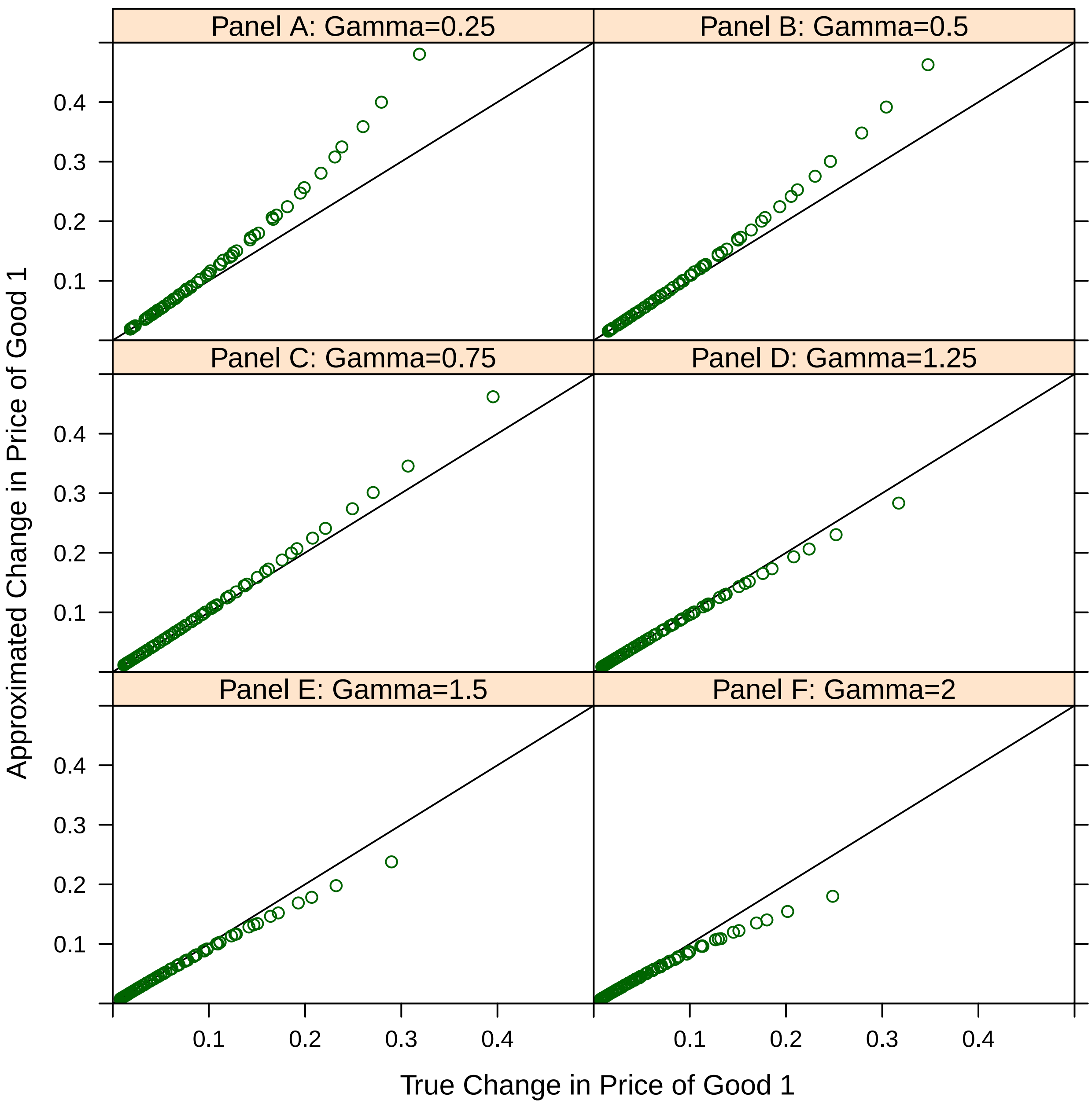
Notes: the vertical axis graphs the JW-approximation post-merger price of good 1 minus the pre-merger price, while the horizontal axis graphs the true post-merger price of good 1 minus the pre-merger price. Each dot represents a separate draw of calibration inputs. The diagonal line is the 45 degree line.

Figure 3: Price-Exponent Demand



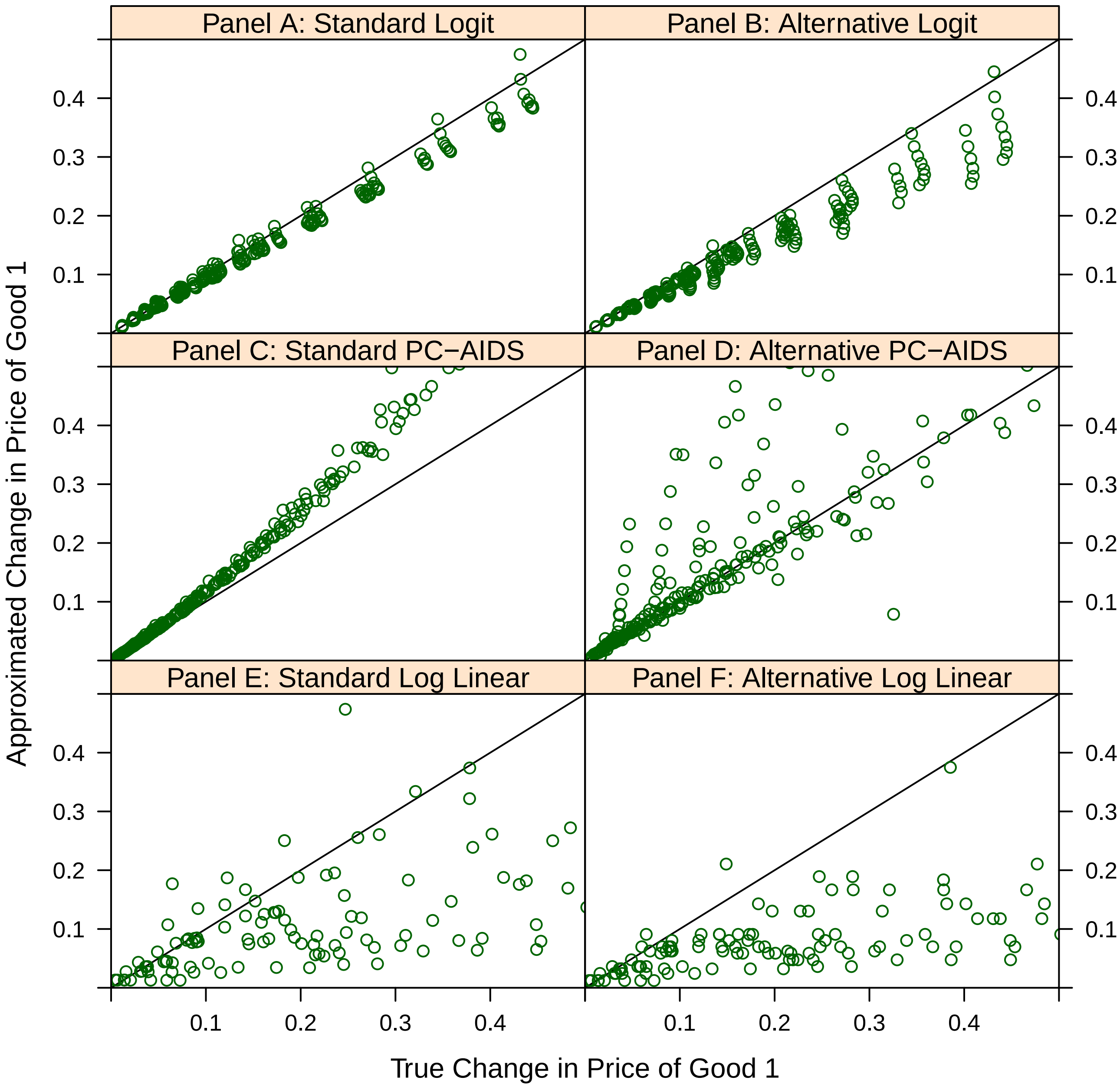
Notes: graphs $q = 1 - p^{\gamma}$ for different values of γ.

Figure 4: JW-Approximation vs. True Price Change Using Price-Exponent Demand



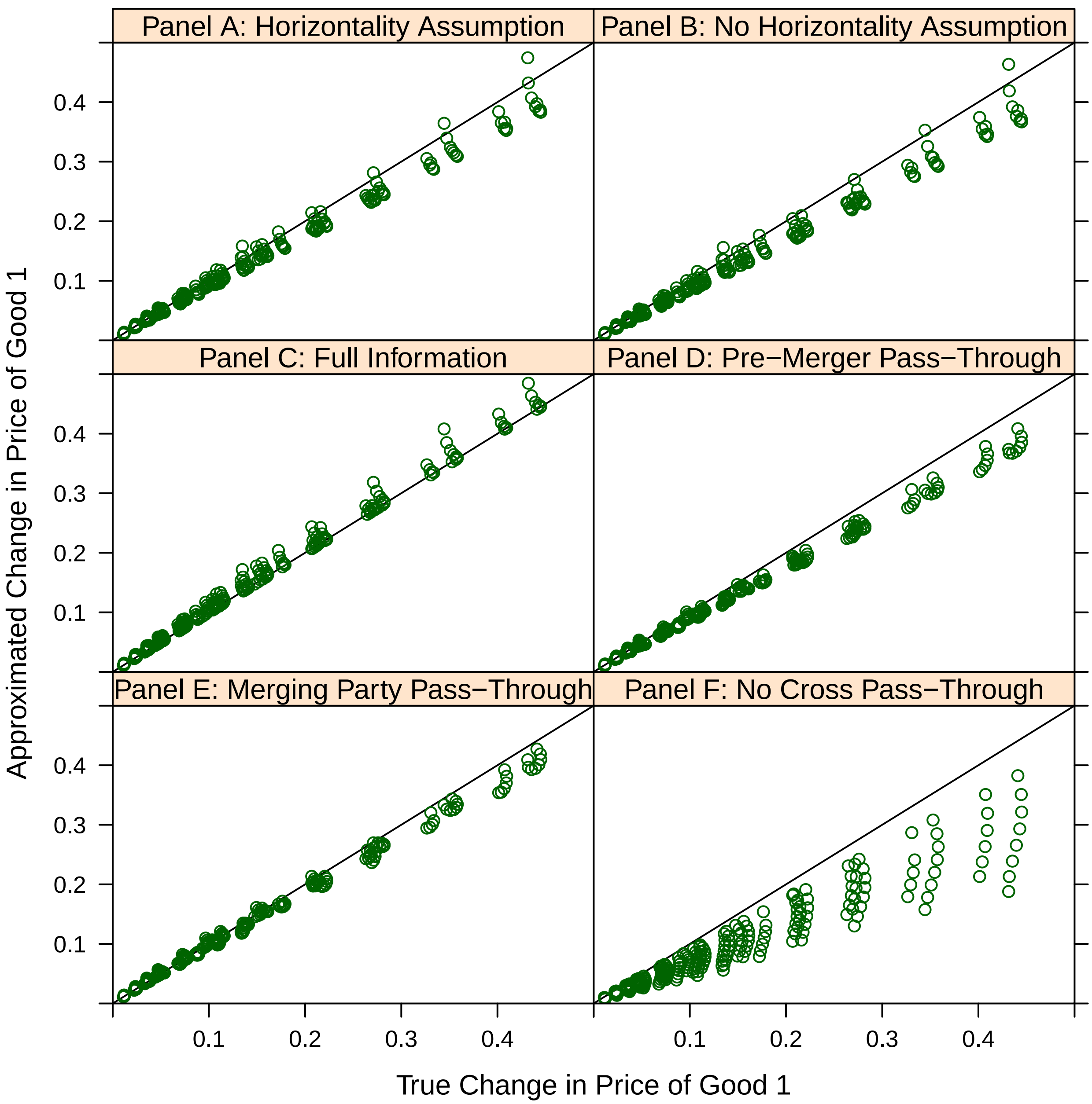
Notes: the vertical axis graphs the JW-approximation post-merger price of good 1 minus the pre-merger price, while the horizontal axis graphs the true post-merger price of good 1 minus the pre-merger price. Each dot represents a separate draw of calibration inputs. The diagonal line is the 45 degree line.

Figure 5: JW FOC vs. Alternative FOC



Notes: the vertical axis graphs the approximated post-merger price of good 1 minus the pre-merger price, while the horizontal axis graphs the true post-merger price of good 1 minus the pre-merger price. Each dot represents a separate draw of calibration inputs. The diagonal line is the 45 degree line.

Figure 6: Different Informational Assumptions



Notes: the vertical axis graphs the approximated post-merger price of good 1 minus the pre-merger price, while the horizontal axis graphs the true post-merger price of good 1 minus the pre-merger price. Each dot represents a separate draw of calibration inputs. The diagonal line is the 45 degree line.

Table 1: JW-Approximation vs. True Price Change

	$\Delta P^{JW} - \Delta P^{*}$	$(\Delta P^{JW} - \Delta P^{*})/\Delta P^{*}$	N Observations
All	-0.14	-0.15	666
	[-0.94, 0.17]	[-0.82, 0.50]	
Logit	-0.01	-0.06	240
	[-0.05, 0.01]	[-0.13, 0.09]	
PC-AIDS	0.06	0.23	240
	[0.00, 0.26]	[0.05, 0.57]	
Log-Linear	-0.57	-0.77	186
	[-2.90, 0.15]	[-0.90, 0.74]	

Notes: The first entry in brackets is the 5^{th} percentile across all simulations, while the second entry is the 95^{th} percentile.

Table 2: Different Informational Assumptions

	$\Delta P^{JW} - \Delta P^{*}$	$(\Delta P^{JW} - \Delta P^{*})/\Delta P^{*}$
	Logit (N Obs.=240)	
Horizontality Assumption	-0.01	-0.06
	[-0.05, 0.01]	[-0.13, 0.09]
No Horizontality Assumption	-0.02	-0.11
	[-0.06, 0.00]	[-0.19, 0.04]
Full Information	0.01	0.06
	[0.00, 0.03]	[-0.00, 0.19]
Pre-Merger Pass-Through	-0.019	-0.09
	[-0.06, 0.00]	[-0.16, 0.02]
Merging Party Pass-Through	-0.01	-0.02
	[-0.04, 0.01]	[-0.10, 0.09]
No Cross Pass-Through	-0.05	-0.31
	[-0.17, -0.00]	[-0.51, -0.12]
	PC-AIDS (N Obs.=240)	
Horizontality Assumption	0.06	0.23
	[0.00, 0.26]	[0.05, 0.57]
No Horizontality Assumption	0.05	0.18
	[0.00, 0.21]	[0.04, 0.47]
Full Information	0.05	0.18
	[0.00, 0.21]	[0.04, 0.47]
Pre-Merger Pass-Through	-0.04	-0.17
	[-0.14, -0.00]	[-0.39, -0.02]
Merging Party Pass-Through	-0.11	-0.57
	[-0.39, -0.01]	[-0.84, -0.22]
No Cross Pass-Through	-0.12	-0.67
	[-0.40, -0.01]	[-0.84, -0.41]
	Log-Linear (N Obs.=186)	
Horizontality Assumption	-0.57	-0.77
	[-2.90, 0.15]	[-0.91, 0.74]
No Horizontality Assumption	-0.57	-0.77
	[-2.90, 0.15]	[-0.91, 0.74]
Full Information	-0.28	-0.07
	[-2.16, 1.26]	[-2.10, 5.41]
Pre-Merger Pass-Through	-0.68	-0.51
	[-3.00, 0.04]	[-0.90, 0.43]
Merging Party Pass-Through	-0.82	-0.78
	[-3.54, -0.01]	[-0.98, -0.40]
No Cross Pass-Through	-0.82	-0.78
	[-3.54, -0.01]	[-0.98, -0.40]

Notes: The first entry in brackets is the 5^{th} percentile across all simulations, while the second entry is the 95^{th} percentile.