Gender-Based Price Discrimination in Matching Markets*

Thomas Trégouët†

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Abstract

I analyze the impact of a ban on third-degree (or gender-based) price discrimination in matching markets. When third-degree price discrimination is not allowed, the cost of revealing information is higher but a monopoly intermediary may have stronger incentives to implement an efficient allocation. I provide necessary and sufficient conditions under which a ban on third-degree price discrimination has a positive impact on total welfare. The general idea is that a ban is more likely to be harmful the more symmetric the matching environment is.

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†Thema, Université de Cergy-Pontoise. Email: thomas.tregouet@u-cergy.fr

1 Introduction

Discount prices for women in bars or dating services is a well-known strategy aimed at attracting men who are – supposedly – willing to pay more for mingling with women than vice-versa. The legality of this practice has been challenged recently in the US by civil rights organizations and by the promulgation of laws directed at eliminating gender-based price discrimination.[1] These actions resulted in the ban on 'Ladies' nights' in several states in the US and the adoption of equal pricing policies by major dating websites.[2] The impact of these decisions has yet to be assessed and should not be neglected: online dating services are used by millions of people and the market is still growing.[3]

Economists have long noticed that third-degree price discrimination may either reduce or raise social welfare:[4] in a monopoly market, moving from non-discrimination to discrimination raises the firm's profits, harms consumers in markets where prices increase and benefits the consumers who face lower prices. While this logic may translate to matching markets, it should be amended in two important ways. First, men and women demands for intermediation services are interdependent: lower prices for women result in higher demand from men, and conversely. A "two-sided" logic is at stake: in equilibrium, prices should internalize the complementarity between men and women participation and, therefore, a ban on gender-based price discrimination is likely have a negative effect.[5] Second, and perhaps more importantly, matching markets are plagued by adverse selection so that men and women use membership in dating services to signal their types or preferences: a man subscribing to an expensive premium account intends to signal women that he is serious about finding someone,

[1] For instance, the Gender Tax Repeal Act, 1995 (California, USA) stipulates that "no business establishment of any kind whatsoever may discriminate, with respect to the price charged for services of similar or like kind, against a person because of the person's gender." In Europe, concerns about gender-based price discrimination are for now focused on the insurance market: in 2011, the Court of Justice of the European Union ruled that different premiums for men and women constitute sex discrimination.

[2] 'Ladies' nights' are considered unlawful in California, New-Jersey, Maryland, Pennsylvania and Wisconsin.

[3] In 2010, there were 40 millions users per month of online dating services in the US, 38.2 millions in Europe and 140 millions in China; 17% of couples who married in 2010 in the US met online.

[4] The analysis of third-degree price discrimination goes back to the seminal works of Pigou (1920) and Robinson (1933), later taken forward by Schmalensee (1981) and Varian (1985). More recently Aguirre, Cowan, and Vickers (2010) provide general conditions on the curvature of demand functions under which third-degree price discrimination has a positive or a negative impact on total welfare.

[5] See the discussion in Wright (2004) and, more generally, the recent literature on two-sided markets by Armstrong (2006), Caillaud and Jullien (2003), Rochet and Tirole (2003 and 2006).

and women should presumably believe him provided that the price he paid is high enough.[6] In this context, change in prices for intermediation services may dramatically change the matching between men and women.

The purpose of this article is to understand the impact of a ban on gender-based price discrimination on matching efficiency. The analysis builds on a one-to-one two-sided matching model where a profit-maximizing intermediary offers matching services to populations of men and women. The matchmaker offers menus of fees that are subsequently used by men and women to signal their types (high or low). Men and women form beliefs about each other types that depend only on the prices chosen by each participant. Given these beliefs, the matching between men and women who join the matchmaker should be stable.

In the first part of the paper, I analyze the problem of the matchmaker when gender-based price discrimination is allowed. First, I show that the matchmaker implements either a *shutdown* or a *separating* allocation. In the shutdown allocation, the intermediary only serves the high type men and women and, therefore, the low type men and women remain unmatched. In the separating allocation, the intermediary serves all agents, men and women are matched assortatively and the total surplus is maximized. When choosing between these two allocations, the matchmaker faces a rent-extraction/efficiency trade-off: on the one hand, a shutdown allocation allows the matchmaker to capture the entire high type surplus, but no surplus is captured from low-type agents; on the other hand, a separating allocation allows the matchmaker to capture the entire low type surplus, but high-type men and women obtain an information rent.

When gender-based price discrimination is not allowed, the matchmaker has to offer the same set of prices on both sides of the market. The cost of information revelation increases: information rents are higher and some users who were left with zero surplus prior to the ban now receive an information rent. Suppose that, before the ban, high type men and women were offered p^m and p^w respectively to signal their types. Then, following the ban, if the matchmaker offers $\{p^m, p^w\}$ on both sides, it may be the case that both the high

[6] Dating websites have long recognized this and typically provides different services, ranging from basic intermediation services to more exclusive (and expensive) offers. For instance, users of Match.com and Chemistry.com –two websites owned by the same company– are charged \$30 and \$50 respectively (on a monthly basis), and Chemistry.com is supposed to attract people interested in long-term relationships, i.e. people who might be more willing to pay a higher subscription fee.

type men and women choose $\min\{p^m, p^w\}$ if this "signaling" strategy is dominated for low type agents. In other words, either the high type men or women now receive an *additional* information rent: $\max\{p^m, p^w\} - \min\{p^m, p^w\}$. In the second part of the paper I apply this argument to characterize the matchmaker's optimal strategy and profits and the matching that is implemented in equilibrium.

In the last part of the paper, I compare the matching that is implemented when gender-based price discrimination is allowed or not. I obtain necessary and sufficient conditions under which the ban has a positive impact on total welfare. The general idea is that a ban is more likely to be harmful the more symmetric the matching environment is. More precisely, if the allocation or distribution of the matching surplus is symmetric, the ban has a negative impact on total welfare. On the other hand, I show that in a matching market dominated by men, i.e., where there are more profits to be made on the men side of the market, the ban has a positive impact if low type women obtain more than low type men from a match with a high type. Finally, with these conditions in mind, I discuss the empirical evidence on the sharing of the marital surplus that may support the assumption of symmetry or asymmetry of the matching environment.

Related literature. Formally I analyze the problem of a monopolist that can use both second-degree and third-degree price discrimination in a case where demands are interdependent. Layson (1998) extends the classical analysis of third-degree price discrimination to the case of interdependent demands. Problems with both second- and third-degree price discrimination arise naturally in insurance markets. The literature on risk classification (see, e.g, Crocker and Snow (2000)) discusses the implication for efficiency and equity of third-degree price discrimination in insurance markets.

The literature on matching tournaments (see, e.g., Chiappori, Iyigun, and Weiss (2009), Hoppe, Moldovanu, and Sela (2009), Mailath, Postlewaite, and Samuelson (2011), Peters and Siow (2002)) studies how pre-marital investment or investment before trading shape the matching between men and women/buyers and sellers/etc. In my paper, the set of available signals/investments is endogenous: it is chosen by a profit-maximizing matchmaker.[7]

[7] A similar question arises in Rayo (2005), who analyzes the problem of a discriminating monopolist serving a population of consumers who use the goods as a signaling device (conspicuous goods).

Therefore, there may be too few available signals to sustain a separating equilibrium in the matching market and inefficient allocations can be implemented in equilibrium.

Damiano and Li (2007) analyzes second-degree price discrimination in one-to-one two-sided matching markets.[8] With a continuum of (one-dimensional) types and complementarities in production, they provide sufficient conditions under which a monopoly matchmaker implements a separating – a necessary condition for efficiency – allocation. Yet, in their framework, it seems difficult to characterize the privately optimal allocation in full generality. I consider a model with two types on each side of the market in which I am able to fully characterize the optimal allocation.[9]

Last there is a burgeoning empirical literature on online dating and dating services. Hitsch, Hortacsu, and Ariely (2010) and Lee (2009) both estimate marital preferences using data from matchmaking websites. Lee and Niederle (2011) design an experiment where participants in a dating website were given the opportunity to signal their preferences to a limited number of potential partners. They conclude to a significative positive impact of signaling on the chance of successfully meet someone.

The paper is organized as follows. Section 2 describes the model. Then I derive the monopoly outcome when gender-based price discrimination is allowed (Section 3) or not (Section 4). Section 5 investigates the impact of a ban on gender-based price discrimination on total welfare. Section 6 concludes.

2 The model

Men and women. There are two populations of agents: men and women. The two populations have the same size, normalized to one. There are two types of men and women: a proportion $0 < \lambda < 1$ of men (or women) are of type h and the others are of type l. Type is private information of each agent. A match between a type-i man and a type-j woman, $(i, j) \in \{l, h\}^2$, creates surplus u_{ij} (≥ 0) to the man and v_{ij} (≥ 0) to the woman.

[8] Gomes and Pavan (2011) analyze price-discrimination in many-to-many matching markets.

[9] In a companion paper (Trégouët (2011)) I analyze price discrimination in matching markets where one side is exempted from payment. The focus is on the labor market where intermediation services are usually free to workers.

Denote by $s_{ij} = u_{ij} + v_{ij}$ the total surplus of a match between a type-i man and a type-j woman. I assume homogenous preferences, $u_{ih} \geq u_{il}$ and $v_{hi} \geq v_{li}, \forall i \in \{l, h\}$, and increasing differences (or complementarities in matching):

$$u_{hh} - u_{hl} \geq u_{lh} - u_{ll} \text{ and } v_{hh} - v_{lh} \geq v_{hl} - v_{ll}.$$

This last assumption is the analogous of the single-crossing condition in screening problems. It insures that information revelation will be possible. Agents are risk neutral and have quasi-linear preferences. They only care about the difference between the expected match value and the subscription fee they pay. An unmatched agent gets a payoff of 0, regardless of his type. Men and women cannot find a partner by themselves. They can only be matched by a matchmaker.

Under these assumptions, the total surplus from matching is maximal if (i) all men and women are matched, (ii) men and women are matched assortatively, i.e. h-types together and l-types together. Condition (i) stems from the fact that $u_{ij} \geq 0$ and $v_{ij} \geq 0$ for all (i, j). Condition (ii) is implied by increasing differences of the matching surplus.

The matchmaker. A monopoly matchmaker, unable to observe types of men and women, offers a pair of fee schedules Σ^m and Σ^w, where Σ^m and Σ^w are closed subsets of $\mathbb{R}^+$.[10] In order to join the matchmaker, a man (a woman) must pick one price in Σ^m (Σ^w). The agents who join the matchmaker observe the prices chosen by each participant and a matching occurs. I postpone the description of the matching to the presentation of my equilibrium concept (see below). As will be apparent, the sole role of the matchmaker is to provide men and women with a signaling device: once users have paid the entrance fees, the matching occurs without the help of the matchmaker. The objective of the matchmaker is to maximize the sum of subscription fees collected from men and women.

Timing and equilibrium. The timing of the game is as follows:

1. The matchmaker announces prices Σ^m and Σ^w.

[10]The 'closed set' assumption insures that $\inf \Sigma^i \in \Sigma^i$.

2. Men and women pick one price in Σ^m and Σ^w respectively, or do not participate.

3. Men and women who join in stage 2 observe each other choices. The matching occurs.

In stage 3, men and women form beliefs about each others types. These beliefs depend only on the prices chosen by participants in stage 2. Intuitively, if a man paid a very high price, then, women should reasonably believe that he is a high type. On the other hand, women should believe that a man who joins the matchmaker for free has a low type.

The matching should depend on these beliefs. For instance, if a man is believed to be of type l while there are plenty of men who are believed to be of type h, then, a woman should reject the l-man and try to match with a h-man. Let us give a more formal statement. Let $\mathcal{M}$ and $\mathcal{W}$ denote the sets of men and women who join the matchmaker in stage 2. For all $i \in \mathcal{M}$ and $j \in \mathcal{W}$, let $m(i)$ and $w(j)$ be men and women (homogenous) posterior beliefs on man i's and woman j's type: participants believe that man i (woman j) has type h with probability $m(i)$ ($w(j)$). We require the matching to be *stable* in the following sense: there do not exists pairs of matched men and women (i, j) and (i', j') such that

$$m(i) > m(i') \text{ and } w(j) < w(j').$$[11]

In words, in equilibrium, there cannot be a man and a woman who *both believe* they would be better off being matched to one another compared to their current assignment. Stability has a straightforward implication in our framework: in equilibrium, men and women are matched assortatively according to beliefs. To be concrete, suppose that two men, i and i', and two women, j and j', join the matchmaker with associated beliefs $m(i) = 1$, $m(i') = 0$, $w(j) = 1$ and $w(j') = 0$. Then, stability requires that man i is matched with woman j, and i' with j'.

A matching should also be *feasible*: if two men and one woman join the matchmaker, then, at least one man will remain single. A precise definition of a feasible matching is rather complex and not very informative. Such a definition can be found in Appendix A.1. For expositional clarity, let us adopt the following "loose" definition: a *feasible* matching is a measure-preserving function from $\mathcal{M}$ to $\mathcal{W}$.

[11] My notion of stability is a convenient adaptation of the notion of pairwise stability defined in complete information two-sided matching model (see Chapter 2 in Roth and Sotomayor (1992)).

In the end, a perfect Bayesian equilibrium of the game specifies: prices Σ^m and Σ^w, men and women participation $\mathcal{M}$ and $\mathcal{W}$, beliefs and a matching. There can be multiple equilibria for two reasons: first, men and women play a coordination game in stage 2; second, multiple equilibria can be sustained in stage 3 by choosing appropriate out-of-equilibrium beliefs. To get rid of these potential issues I focus on the equilibria with maximal participation in stage 2 and I require beliefs to be intuitive in the sense of Cho and Kreps (1987).

First assumptions. The model can be solved without facing any major technical difficulties. However, the analysis may be tedious because of the many different cases to be discussed. Assumption 1 below reduces the number of parameters from eight (u_{ij} and v_{ij}) to five and allow me to obtain more clear-cut results.

Assumption 1. *There exists $\beta \in [0,1]$ such that, for all $(i,j) \in \{l,h\}^2$, $u_{ij} = \beta \cdot s_{ij}$.*

Assumption 1 says that the sharing rule of the matching surplus does not depend on the man's and woman's types. Hereafter, denote by $\beta^m = \beta$ and $\beta^w = 1 - \beta$ the men and women share of the matching surplus respectively.

3 Gender-based price discrimination is allowed

In this section, I derive the matching implemented by the matchmaker when gender-based price discrimination is allowed.

The matchmaker could possibly implement many different allocations. For instance, if $\Sigma^m = \Sigma^w = \{0\}$, all agents participate, no information is revealed and, therefore, men and women are matched randomly to one another. In this case, we will say that a *pooling* allocation is implemented by the matchmaker. On the other hand, if Σ^m and Σ^w contain only high prices, then, no one participate and a *null* allocation is implemented.

We have seen that different allocations can be implemented. Yet some allocations are better candidates than others: in a *shutdown* allocation, the matchmaker only serves the high type men and women; in a *separating* allocation, the matchmaker serves all men and women, and men and women are matched assortatively. In the following I derive the matchmaker's profits if it implements a shutdown or a separating allocation. Then, I argue that the matchmaker cannot obtain more profits by implementing other allocations. Last, I provide conditions under which a shutdown or a separating allocation is implemented.

Implementation of a shutdown allocation. The matchmaker can implement a shutdown allocation by proposing singletons $\Sigma^m = \{p^m\}$ and $\Sigma^w = \{p^w\}$ since, in a shutdown allocation, only the high type men and women participate. Let us find conditions that p^m and p^w must satisfy. First high type men and women must be willing to participate:

$$\beta^m s_{hh} - p^m \geq 0 \text{ and } \beta^w s_{hh} - p^w \geq 0. \tag{1}$$

Second low type men and women must not be willing to participate:

$$\beta^m s_{lh} - p^m < 0 \text{ and } \beta^w s_{hl} - p^w < 0. \tag{2}$$

If the matchmaker offers $\Sigma^m = \{p^m\}$ that satisfies conditions (1) and (2), participants should believe that a man who chooses p^m in stage 2 is a high type since p^m is dominated for low type men. Similarly, p^w should "signal" a high type woman. Clearly, maximum profits are obtained by capturing the entire high type surplus, i.e. $p^m = \beta^m s_{hh}$ and $p^w = \beta^w s_{hh}$. We conclude:

Lemma 1. *The maximum profit in a shutdown allocation is $\Pi_{Sh} = \lambda s_{hh}$. It is achieved for instance with $\Sigma^m = \{\beta^m s_{hh}\}$ and $\Sigma^w = \{\beta^w s_{hh}\}$.*

Implementation of a separating allocation. The matchmaker can implement a separating allocation by offering pairs of prices $\Sigma^m = \{p_l^m, p_h^m\}$ and $\Sigma^w = \{p_l^w, p_h^w\}$. Let us find conditions that Σ^m must satisfy to sustain a separating allocation.[12] Prices p_l^m and p_h^m must

[12]Conditions on Σ^w can be derived similarly.

satisfy a set of participation and incentive constraints:

$$\begin{aligned}
\beta^m s_{hh} - p_h^m &\geq 0, \\
\beta^m s_{hh} - p_h^m &\geq \beta^m s_{hl} - p_l^m, \\
\beta^m s_{ll} - p_l^m &\geq 0, \\
\beta^m s_{ll} - p_l^m &\geq \beta^m s_{lh} - p_h^m.
\end{aligned}$$

Increasing differences of the matching surplus ensures that the above inequalities define a non-empty set of $\mathbb{R}^2$. Notice that the last inequality implies that $p_h^m \geq p_l^m$. Price p_l^m "signals" a low-type man since this is the lowest price in Σ^m and it satisfies the low type men participation constraint. Price p_h^m "signals" a high type man because it is dominated for low type men and compatible with high type men incentive and participation constraints. Standard arguments then show that maximum profits are obtained by capturing the entire low type surplus and the entire high type surplus minus an information rent: $p_l^m = \beta^m s_{ll}$ and $p_h^m = \beta^m s_{hh} - \beta^m(s_{hl} - s_{ll})$. Similarly, on the women side of the market, profits are maximized when $p_l^m = \beta^m s_{ll}$ and $p_h^w = \beta^w s_{hh} - \beta^w(s_{lh} - s_{ll})$. We conclude:

Lemma 2. *The maximum profit in a separating allocation is:*

$$\Pi_{Sep} = \lambda s_{hh} + (1-\lambda)s_{ll} - \lambda(\beta^m(s_{hl} - s_{ll}) + \beta^w(s_{lh} - s_{ll})).$$

It is achieved for instance with $\Sigma^m = \{\beta^m s_{sll}, \beta^m(s_{hh} - (s_{hl} - s_{ll}))\}$ *and* $\Sigma^w = \{\beta^w s_{sll}, \beta^w(s_{hh} - (s_{lh} - s_{ll}))\}$.

Other allocations? In this paragraph, I argue that maximum profits are obtained either with a shutdown or a separating allocation. This can be established formally by writing the general problem of the matchmaker and solving for the optimal prices. The proof is rather long and not very informative. Therefore I only provide the two reasons why the result is obtained. First, there are only two types of agents on both sides and the men and women incentive problems can basically be solved separately. With more than two types, conflict between local and global incentive constraints may lead to the implementation of pooling allocations.[13] Second, the matchmaker's problem is linear so that "corner" allocations are implemented: in order to reduce the high type information rent, the matchmaker stops serving low type men and women.

Conclusion. Lemma 1 and 2 together give:

Proposition 1. *When gender-based price discrimination is allowed:*

- *if* $s_{ll} < \lambda(\beta^m s_{hl} + \beta^w s_{lh})$, *the matchmaker implements a shutdown allocation and makes profits:*

$$\Pi_{Sh} = \lambda s_{hh};$$

- *if* $s_{ll} \geq \lambda(\beta^m s_{hl} + \beta^w s_{lh})$, *the matchmaker implements a separating allocation and makes profits:*

$$\Pi_{Sep} = \lambda s_{hh} + (1-\lambda)s_{ll} - \lambda(\beta^m(s_{hl} - s_{ll}) + \beta^w(s_{lh} - s_{ll})).$$

A standard rent extraction/efficiency trade-off is at stake in Proposition 1 . On the one hand, in a shutdown allocation, the matchmaker captures the entire high type surplus, λs_{hh}, but is unable to capture the low type surplus. On the other hand, in a separating allocation, the matchmaker captures the entire matching surplus, $\lambda s_{hh} + (1-\lambda)s_{ll}$, minus information rents left to high type men and women, $\lambda(\beta^m(s_{hl} - s_{ll}) + \beta^w(s_{lh} - s_{ll}))$.

4 Gender-based price discrimination is not allowed

Assume now that gender-based price discrimination is not allowed so that the matchmaker must offer the same set of prices on both sides of the market: $\Sigma^m = \Sigma^w = \Sigma$. In the following, I characterize the maximum profits in a shutdown and in a separating allocation. Contrary to Section 3, I will not give conditions under which one allocation or the other is implemented since this would imply discussing numerous cases without yielding clearcut results. Section 5 will discuss special cases of interest where the comparison is made easier.

[13] See Chapter 3 in Laffont and Martimort (2001).

4.1 Implementation of a shutdown allocation

Suppose first that the matchmaker offers the same prices as when gender-based price discrimination were allowed: $\Sigma = \{\beta^m s_{hh}, \beta^w s_{hh}\}$ and high type men and women are supposed to choose $\beta^m s_{hh}$ and $\beta^w s_{hh}$ respectively. In the following, assume for instance that $\beta^m \geq \beta^w$. Let us find conditions under which this can be sustained in equilibrium. A high type woman never chooses $\beta^m s_{hh}$ ($\geq \beta^w s_{hh}$) since this would violate her participation constraint. Things are more complicated on the other side of the market. Suppose that a high type man deviates and chooses the lower price $\beta^w s_{hh}$. This deviation is profitable if women believe that only a high type man could have chosen $\beta^w s_{hh}$. This is the case when $\beta^w s_{hh}$ is a dominated strategy for low type men, i.e. if $\beta^m s_{lh} - \beta^w s_{hh} < 0$. Proposition 2 below discusses the different cases.

Proposition 2. *Assume that $\beta^m \geq \beta^w$. When gender-based price discrimination is not allowed, the matchmaker implements the shutdown allocation with:*

- *if $\beta^w s_{hh} \leq \beta^m s_{lh}$,*

$$\Sigma = \{\beta^w s_{hh}, \beta^m s_{hh}\} \text{ and } \Pi_{Sh} = \lambda s_{hh},$$

- *if $\beta^m s_{lh} < \beta^w s_{hh} \leq \beta^m \frac{s_{hh}+s_{lh}}{2}$,*

$$\Sigma = \{\beta^m s_{lh}, \beta^m s_{hh}\} \text{ and } \Pi_{Sh} = \lambda \beta^m (s_{hh} + s_{lh}),$$

- *if $\beta^m \frac{s_{hh}+s_{lh}}{2} < \beta^w s_{hh}$,*

$$\Sigma = \{\beta^w s_{hh}\} \text{ and } \Pi_{Sh} = 2\lambda \beta^w s_{hh}.$$

The case where $\beta^w > \beta^m$ can be stated similarly.

Proof. See Appendix A.2. □

The first case in Proposition 2 states that, if the difference between $\beta^m s_{hh}$ and $\beta^w s_{hh}$ is sufficiently large, then, a high type man will never consider choosing the price $\beta^w s_{hh}$ since this would signal for sure that he is a low type. Therefore the matchmaker is still able to capture the entire high type surplus.

In the second and third case, the difference between $\beta^m s_{hh}$ and $\beta^w s_{hh}$ is not large enough so that a high type man may not consider choosing $\beta^w s_{hh}$. The matchmaker has two options: either it captures the entire high type man surplus and leaves an information rent to high type women (second case) or it captures the entire high type women surplus and leaves an information rent to high type men (third case). The matchmaker chooses the first option when there is more surplus to be extracted from high type men. In this case, the low price, i.e. the price for high type women, must be low enough so that it would signal a low type if it were taken by a man. The highest price that has this property is the highest price a low type man would be willing to pay in order to be matched with a high type woman: $\beta^m s_{lh}$. High type women therefore receive an information rent $\beta^w s_{hh} - \beta^m s_{lh}$. On the other hand, if this information rent is too high (third case), the matchmaker is better off capturing the entire high type women surplus: $\beta^w s_{hh}$ is the lowest available price in Σ. Since this price also signals a high type on the man side of the market, high type men obtain an information rent $\beta^m s_{hh} - \beta^w s_{hh}$.

To summarize, when $\beta^m \geq \beta^w$, the "cost" of a ban on gender-based price discrimination is that high type men or women now receive an information rent if $\beta^m s_{lh} < \beta^w s_{hh}$.

4.2 Implementation of a separating allocation

Let Σ the set of prices offered by the matchmaker to implement a separating allocation. In a separating equilibrium, low type men and women choose prices that "signal" they have low types. Therefore low type men and women choose the lowest price in Σ since in the worst case this signals a low type. This has an important implication: in a separating allocation, low type men or women receive an information rent. Let $i \neq j$, $(i,j) \in \{m, w\}$, such that $\beta^i \geq \beta^j$. Since price $\inf \Sigma$ is chosen by low type $j-$agents, we have $\inf \Sigma \leq \beta^j s_{ll} \leq \beta^i s_{ll}$: low type $i-$agents receive an information rent. Notice also that the low type $i-$agents information rent passes to the high type $i-$agents. In other words, high type $i-$agents should receive higher information rents. These are the first effects of a ban on gender-based price discrimination when a separating allocation is implemented.

Notice that w.l.o.g we can assume Σ contains only three elements: $\Sigma = \{\underline{p}, p^m, p^w\}$, where p^m, p^w and $\underline{p} = \inf \Sigma$ are chosen by $h-$type men, $h-$type women and $l-$type men and women respectively. Prices p^m, p^w and $\underline{p}$ must satisfy participation constraints:

- High type men participation constraint:

$$\beta^m s_{hh} - p^m \geq 0, \tag{3}$$

- High type women participation constraint:

$$\beta^w s_{hh} - p^w \geq 0, \tag{4}$$

- Low type men and women participation constraint:

$$\min\{\beta^m, \beta^w\} s_{ll} - \underline{p} \geq 0. \tag{5}$$

Prices $\underline{p}$, p^m and p^w must also satisfy a total of eight incentive constraints, two for each gender/type combination:

- High type men incentive constraints:

$$\beta^m s_{hh} - p^m \geq \beta^m s_{hl} - \underline{p}, \tag{6}$$

$$\beta^m s_{hh} - p^m \geq \mu \cdot \beta^m s_{hh} + (1-\mu) \cdot \beta^m s_{hl} - p^w, \tag{7}$$

  where $\mu = \Pr\{h{-}\text{man}|p^w\}$ is women' beliefs that a man who chooses p^w has type h.

- High type women incentive constraints:

$$\beta^w s_{hh} - p^w \geq \beta^w s_{lh} - \underline{p}, \tag{8}$$

$$\beta^w s_{hh} - p^w \geq \nu \cdot \beta^w s_{hh} + (1-\nu) \cdot \beta^w s_{lh} - p^m, \tag{9}$$

  where $\nu = \Pr\{h{-}\text{woman}|p^m\}$ is men' beliefs that a woman who chooses p^m has type h.

- Low type men incentive constraints:

$$\beta^m s_{ll} - \underline{p} \geq \beta^m s_{lh} - p^m, \tag{10}$$

$$\beta^m s_{ll} - \underline{p} \geq \mu \cdot \beta^m s_{lh} + (1-\mu) \cdot \beta^m s_{ll} - p^w. \tag{11}$$

- Low type women incentive constraints:

$$\beta^w s_{ll} - \underline{p} \geq \beta^w s_{hl} - p^w, \tag{12}$$

$$\beta^w s_{ll} - \underline{p} \geq \nu \cdot \beta^w s_{hl} + (1-\nu) \cdot \beta^w s_{ll} - p^m. \tag{13}$$

In the end, the matchmaker's problem writes:

$$\begin{aligned} \max_{(p^m, p^w, \underline{p})} \Pi \quad &= \quad \lambda(p^m + p^w) + 2(1-\lambda)\underline{p} \\ \text{s.t.} \quad & (3) \text{ to } (13), \\ & \underline{p} \leq p^m, p^w. \end{aligned}$$

In order to simplify the problem, we would like to identify *a priori* the binding constraints. The standard approach suggests to order agents according to their incentives to misrepresent their true type and, then, to ignore the "upward" incentive constraints. Unfortunately there is no such natural ordering in our context. In the following, I describe the case where high type men are more likely to pretend having a low type than high type women, i.e. where constraint (6) is "above" constraint (8). The opposite case can be treated similarly.

Assumption 2. $\beta^m(s_{hh} - s_{hl}) \geq \beta^w(s_{hh} - s_{lh})$.

Basically, Assumption 2 implies that p^m will be higher that p^w, therefore allowing us to ignore women' upward incentive constraints (9) and (13). Then, following the standard analysis of price discrimination, we ignore the high type men and women participation constraints (3) and (4). In the end, we consider the relaxed problem:

$$\max_{(p^m, p^w, \underline{p})} \Pi = \lambda(p^m + p^w) + 2(1-\lambda)\underline{p}$$
$$\text{s.t.} \quad (5),\ (6),\ (7),\ (8),\ (10),\ (12).$$

Clearly the low type men and women participation constraint (5) is binding so that $\underline{p} = \min\{\beta^m, \beta^w\} s_{ll}$. We are still left with five constraints. Notice that if constraint (7), that describes the incentives of high type men to choose p^w, were absent, we could treat the men and women incentive problems separately, as in Section 3. Constraint (7) says that the price for high type men should not be too high compared with the price for high type women. In other words, there is now a link between the men and women incentive problems. Constraint (7) rewrites:

$$p^m \leq p^w + (1-\mu)\beta^m(s_{hh} - s_{hl}),$$

where $\mu = 1$ if women believe that a man who chooses p^w has type h. In words, if p^w "signals" a high type ($\mu = 1$), the matchmaker cannot charge a higher price p^m for high type men. This happens if p^w is sufficiently high so that choosing p^w is (equilibrium) dominated for low type workers (i.e. if $\beta^m s_{lh} - p^w < \beta^m s_{ll} - \underline{p}$):

$$\mu = \begin{cases} 1 & \text{if } p^w > \underline{p} + \beta^m(s_{lh} - s_{ll}), \\ 0 & \text{otherwise.} \end{cases}$$

There are three cases to consider depending on whether μ is constant or not for all p^w compatible with women incentive constraints (8) and (12):

- when $\beta^m(s_{lh} - s_{ll}) > \beta^w(s_{hh} - s_{lh})$, the highest price compatible with women incentive constraints is lower than the lowest price compatible with men incentive constraints. Hence, p^w can only signal a low type ($\mu = 0$) so that the men and women incentive problems can be treated separately. Incentive constraints (6) and (8) are binding: $p^m = \underline{p} + \beta^m(s_{hh} - s_{hl})$ and $p^w = \underline{p} + \beta^w(s_{hh} - s_{lh})$.

- when $\beta^w(s_{hl} - s_{ll}) > \beta^m(s_{lh} - s_{ll})$, the lowest price compatible with women incentive constraints is higher than the lowest price compatible with men incentive constraints.

Hence, p^w can only signal a high type ($\mu = 1$) so that $p^m = p^w$. Incentive constraints (7) and (8) are binding: $p^m = p^w = \underline{p} + \beta^w(s_{hh} - s_{lh})$.

- when $\beta^w(s_{hh} - s_{lh}) > \beta^m(s_{lh} - s_{ll}) > \beta^w(s_{hl} - s_{ll})$, the highest (lowest) price compatible with women incentive constraints is higher (lower) than the lowest price compatible with men incentive constraints. Hence, a low p^w signals a low type, while a high p^w signals a high type. There are two candidate prices in this case (see Figure 1 below): either the matchmaker offers the same price for high type men and women or it sets different prices.

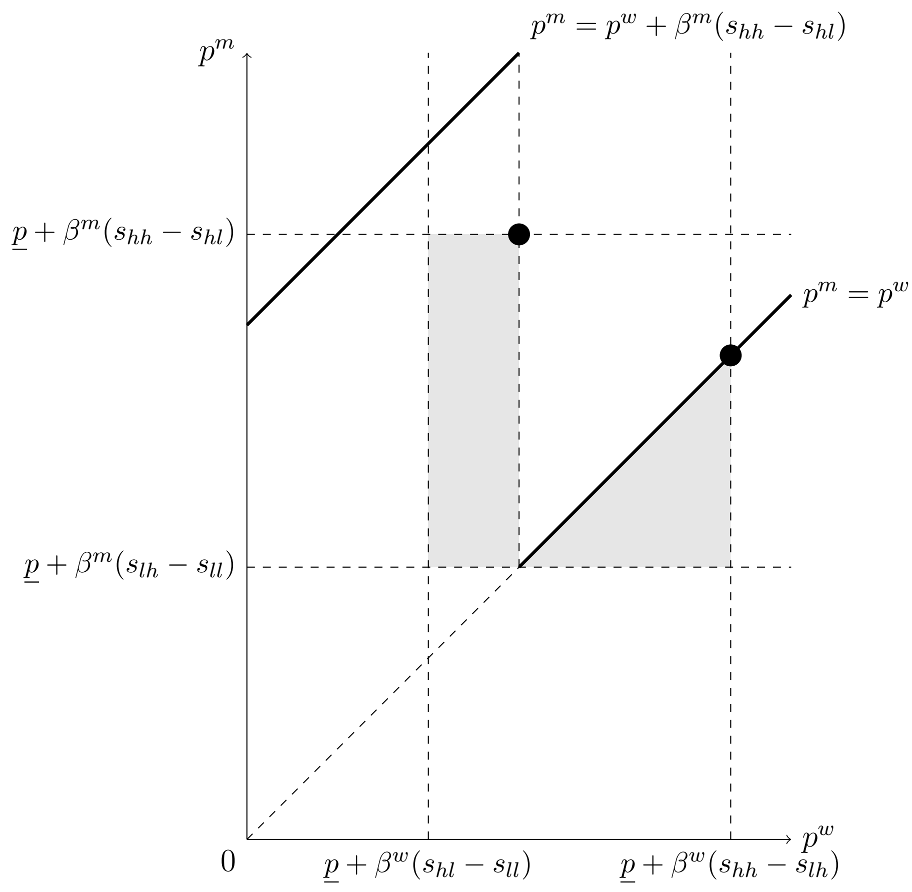

Figure 1: The set of prices p^m and p^w compatible with incentive and participation constraints (grey area) when $\beta^m(s_{hh}-s_{hl}) > \beta^w(s_{hh}-s_{lh}) > \beta^m(s_{lh} - s_{ll}) > \beta^w(s_{hl} - s_{ll})$. *Thick line:* constraint $p^m \leq p^w + (1 - \mu)\beta^m(s_{hh} - s_{hl})$; *Black circles:* candidate prices.

We conclude:

Proposition 3. *Assume that* $\beta^m(s_{hh} - s_{hl}) \geq \beta^w(s_{hh} - s_{lh})$. *Let* $\underline{\beta} = \min\{\beta^m, \beta^w\}$. *When gender-based price discrimination is not allowed, the matchmaker implements the shutdown allocation with:*

1. *if* $\beta^m(s_{lh} - s_{ll}) \geq \beta^w(s_{hh} - s_{lh})$, $\Sigma = \{\underline{p}, p^m, p^w\} = \{\underline{\beta}s_{ll}, \underline{\beta}s_{ll} + \beta^m(s_{hh} - s_{hl}), \underline{\beta}s_{ll} + \beta^w(s_{hh} - s_{lh})\}$ *and* $\Pi_{Sep} = 2\underline{\beta}s_{ll} + \lambda(s_{hh} - \beta^m s_{hl} - \beta^w s_{lh})$;

2. *if* $\beta^w(s_{hl} - s_{ll}) \geq \beta^m(s_{lh} - s_{ll})$, $\Sigma = \{\underline{p}, p^m(= p^w)\} = \{\underline{\beta}s_{ll}, \underline{\beta}s_{ll} + \beta^w(s_{hh} - s_{lh})\}$ *and* $\Pi_{Sep} = 2\underline{\beta}(s_{ll} + \lambda(s_{hh} - s_{lh}))$;

3. *if* $\beta^w(s_{hh} - s_{lh}) \geq \beta^m(s_{lh} - s_{ll}) \geq \beta^w(s_{hl} - s_{ll})$,

   (a) *if* $\beta^m(s_{hh} - s_{hl} + s_{lh} - s_{ll}) \geq 2\beta^w(s_{hh} - s_{lh})$, $\Sigma = \{\underline{p}, p^m, p^w\} = \{\underline{\beta}s_{ll}, \underline{\beta}s_{ll} + \beta^m(s_{hh} - s_{hl}), \underline{\beta}s_{ll} + \beta^m(s_{lh} - s_{ll})\}$ *and* $\Pi_{Sep} = 2\underline{\beta}s_{ll} + \lambda\beta^m(s_{hh} - s_{hl} + s_{lh} - s_{ll})$;

   (b) *if* $\beta^m(s_{hh} - s_{hl} + s_{lh} - s_{ll}) < 2\beta^w(s_{hh} - s_{lh})$, $\Sigma = \{\underline{p}, p^m(= p^m)\} = \{\underline{\beta}s_{ll}, \underline{\beta}s_{ll} + \beta^w(s_{hh} - s_{lh})\}$ *and* $\Pi_{Sep} = 2\underline{\beta}(s_{ll} + \lambda(s_{hh} - s_{lh}))$.

The case where $\beta^m(s_{hh} - s_{hl}) < \beta^w(s_{hh} - s_{lh})$ *can be stated similarly.*

Proof. See Appendix A.3. □

Before we discuss the different cases in Proposition 3, recall first that, in any case, the lowest price is $\min\{\beta^m, \beta^w\}s_{ll}$ so that either the low type men or women receive an information rent $|\beta^m - \beta^w|s_{ll} \geq 0$. Recall also the high type men and women information rents, $\hat{U}_h^m$ and $\hat{U}_h^w$, in a separating allocation when gender-based discrimination were allowed:[14]

$$\hat{U}_h^m = \beta^m(s_{hl} - s_{ll}) \text{ and } \hat{U}_h^w = \beta^w(s_{lh} - s_{ll}).$$

Hereafter denote by U_h^m and U_h^w the information rents that accrue to high type men and women respectively.

In case 1 in Proposition 3, the high type men and women incentive problems can be treated separately. High type men or women may receive a higher information rent compared with

[14]See Lemma 2.

the situation where gender-based price discrimination were allowed only because the price for low type men or women is lower:

$$\begin{aligned} U_h^m - \hat{U}_h^m &= (\beta^m - \min\{\beta^m, \beta^w\}) s_{ll}, \\ U_h^w - \hat{U}_h^w &= (\beta^w - \min\{\beta^m, \beta^w\}) s_{ll}. \end{aligned}$$

In case 2, the two incentive problems completely overlap so that an incentive compatible p^w always signals a high type. In other words, the matchmaker cannot charge a higher price for high type men ($p^m = p^w$). High type men' information rent thus increases for two reasons: first, again, if low type men receive an information rent, it passes to high type men and, second, the matchmaker cannot prevent high type men from choosing the price for high type women:

$$\begin{aligned} U_h^m - \hat{U}_h^m &= \beta^m s_{hh} - (\min\{\beta^m, \beta^w\} s_{ll} + \beta^w (s_{hh} - s_{lh}) - (\beta^m s_{hl} - \beta^m s_{ll}), \\ &= \underbrace{(\beta^m - \min\{\beta^m, \beta^w\}) s_{ll}}_{\geq 0} + \underbrace{\beta^m (s_{hh} - s_{hl}) - \beta^w (s_{hh} - s_{lh})}_{\geq 0}. \end{aligned}$$

The high type women information rent is: $U_h^w = \hat{U}_h^w + (\beta^w - \min\{\beta^m, \beta^w\}) s_{ll}$.

In case 3, the matchmaker trades off between giving higher information rent to high type women (case 3.a) or to high type men (case 3.b). In case 3.a, the matchmaker sets p^w low enough so that it signals a low type if it is chosen by a man, therefore making it unattractive for high type men. In other words, high type women now receive a higher information rent:

$$\begin{aligned} U_h^w - \hat{U}_h^w &= \beta^w s_{hh} - (\min\{\beta^m, \beta^w\} s_{ll} + \beta^m (s_{lh} - s_{ll})) - (\beta^w s_{lh} - \beta^w s_{ll}), \\ &= \underbrace{(\beta^w - \min\{\beta^m, \beta^w\}) s_{ll}}_{\geq 0} + \underbrace{\beta^w (s_{hh} - s_{lh}) - \beta^m (s_{lh} - s_{ll})}_{\geq 0}. \end{aligned}$$

The high type men receive the same information rent as in case 1. Last, information rents in case 3.b are the same as in case 2.

To summarize the "cost" of a ban on gender-based price discrimination is that low type men or women now receive an information rent, high type men or women receive a higher information rent and, possibly, both high type men and women receive a higher information rent.

4.3 Optimal allocation

The previous section unveils that when gender-based price discrimination is not allowed the men and women incentive problems may be linked. Loosely speaking we move from two 2-types screening problems to a single 4-types screening problem. As already mentioned, it is well known that with more than two types a pooling or partially pooling allocation may be optimal. The same phenomenon arises in my model: for some parameter values a pooling allocation is implemented by the matchmaker.[15] While interesting, this makes the comparison with Section 3 difficult and unclear. Therefore, in the following, I do not characterize these situations and I focus instead on situations where either a shutdown or a separating allocation is implemented.

5 Welfare analysis

We are now in position to derive necessary and sufficient conditions under which a ban on gender-based price discrimination has a positive impact on total welfare, define as the sum of men and women utilities and of the matchmaker's profits. More precisely we are aiming at finding conditions on β^i, s_{ij} and/or λ such that if a separating allocation is implemented without the ban, then, it is also implemented under the ban. Indeed, under such conditions the total welfare would be (weakly) higher under the ban since the total welfare is maximized in a separating allocation.

5.1 Necessary conditions

In this section, I provide conditions under which a ban on gender-based price discrimination has a negative impact on total welfare.

To begin with, we notice that if the sharing rule of the matching surplus is symmetric, then, the ban has a negative impact.

Proposition 4. *If $\beta^w = \beta^m$, then, the total welfare is (weakly) higher when gender-based price discrimination is allowed.*

[15] Proof is available upon request.

Proof. See Appendix A.4. □

The intuition is the following. If $\beta^m = \beta^w$, then, the matchmaker can capture the entire high type surplus at no cost in a shutdown allocation when gender-based price discrimination is not allowed (see Proposition 2). Therefore, if a shutdown allocation were optimal without the ban, it is also optimal under the ban. In other words, a separating allocation is implemented "less often" under the ban if $\beta^m = \beta^w$.

The next proposition shows that if the distribution of the matching surplus is symmetric and log-supermodular, then, the ban has a negative impact.

Proposition 5. *Assume* $s_{hl} = s_{lh}$. *If* $s_{ll}/s_{lh} \geq s_{lh}/s_{hh}$, *then, the total welfare is (weakly) higher when gender-based price discrimination is allowed.*

Proof. See Appendix A.5. □

Proposition 5, illustrated by Example 1 below, describes situations where the "cost" of a ban on gender-price discrimination is higher if the matchmaker implements a separating allocation than a shutdown allocation. It should be emphasized that most papers in the matching literature assume a symmetric allocation and/or distribution of the matching surplus when utility is non-transferable, and would therefore conclude that a ban has a negative impact.[16]

Example 1. Negative impact of a ban on gender-based price discrimination.

$$S = (s_{ij}) = \begin{pmatrix} 3 & 5 \\ 5 & 10 \end{pmatrix}$$

5.2 Sufficient conditions

In this section, I show that asymmetry in both the allocation and the distribution of the matching surplus is required to find a positive impact of a ban on gender-based price discrimination.

[16]For instance, Damiano and Li (2007) assume that the a match between a type x man and a type y woman creates surplus xy to both the man and the woman.

Definition 1. *The matching market is said to be dominated by men (women) if*

$$\beta^m > (<)\ \beta^w \text{ and } \beta^m(s_{hh} - s_{hl}) > (<)\ \beta^w(s_{hh} - s_{lh}).$$

Proposition 6. *In a matching market dominated by men (women), the total welfare is higher when gender-based price discrimination is not allowed if* $\beta^w s_{hl} > (<)\ \beta^m s_{lh}$.

Proof. See Appendix A.6. □

In words, Proposition 6 (illustrated by Example 2 below) states that in a matching market dominated by men, i.e. where there are more profits to be made on the men side of the market, the ban has a positive impact if a low type woman benefits more from being matched with a high type man than a low type man benefits from being matched with a high type woman. This can arise when the population of women is more homogenous than the population of men. Intuitively, in this case, the women marginal benefits from being matched with a better man is higher than the man marginal benefits from being matched with a better woman.

Example 2. Positive impact of a ban on gender-based price discrimination.

$$\beta^w = 0.4 \text{ and } S = (s_{ij}) = \begin{pmatrix} 1 & 3 \\ 5 & 10 \end{pmatrix}$$

5.3 Discussion

The general idea of Propositions 4 to 6 is that a ban on gender-based price discrimination is more likely to be harmful the more symmetric the matching environment is. More precisely, Proposition 6 suggests favorable conditions for finding a positive impact are: first, asymmetry in the sharing of the matching surplus; second, asymmetry in men and women information rents or, saying it differently, asymmetry in men and women deviation payoffs in an assortative matching. In the following I briefly discuss the empirical evidence that may support the assumption of symmetry or asymmetry of the matching environment.

The intra-household allocation of resources is a favorite topic of family economics.[17] A

[17] See Chiappori and Donni (2011) for a comprehensive survey of theoretical models of household behavior.

number of studies have estimated women' share of the marital surplus on the basis of household consumption data.[18] Conclusions are quite similar from one study to another: on average women receive half of the marital surplus but there is a large dispersion in the population and deviation from average can be explained by intra-household asymmetry (i.e. differences in age/income/etc. between the man and the woman). In other words this literature provides evidence that support both the symmetry and the asymmetry assumptions.

Choo and Siow (2006) and Chiappori, Salanié, and Weiss (2011) propose an alternative strategy for estimating the women' share of the marital surplus. They estimate the gains from marriage in a frictionless marriage market *à la* Becker-Shapley-Shubik in which the sharing of the matching surplus is endogenously determined in equilibrium. Chiappori, Salanié, and Weiss (2011) find for instance that the wife's share of the total surplus in a marriage with a man with similar education is significantly higher than 50%. On the other hand high-school and college-educated women married with college-educated men obtain respectively 40.4% and 62.5% of the marital surplus. They also find that the (deterministic part) of the marriage surplus is 0.233 when the man has college education and the woman has high school education, while it is only 0.098 when the man has high school education and the woman has college education.[19] This suggests at least a strong asymmetry in the distribution of the matching surplus ($s_{lh} \neq s_{hl}$ in the sense of my model).

Asymmetric gains from matching could also be explain by different degrees of heterogeneity among men and women. Intuitively, competition for attracting the more desirable partner should be more intense among the more homogenous, i.e. less differentiated, group and should therefore result in asymmetric gains from matching. This argument was for instance proposed by Anderson (2007) to explain the transition from bride prices to dowries (i.e. "groom" prices) in pre-industrial societies.[20] Revenue is a key determinant in part-

[18]See, e.g, Browning, Bourguignon, Chiappori, and Lechene (1994), Browning and Bonke (2009), Couprie, Peluso, and Trannoy (2010), Haddad and Kanbur (1990).

[19]See Table 3 in Chiappori, Salanié, and Weiss (2011).

[20]Anderson (2007) writes: "dowry payments emerge due to quality differentiation amongst grooms as found in socially stratified societies and are consistent with a development process where women do not directly reap the benefits of modernization and men are the primary recipients of the new economic opportunities." Fafchamps and Quisumbing (2005) make similar observation in their study of marriage in rural Ethiopia: "If the difference between grooms is large relative to the difference between brides, brides must bring more to fend off competition from lower-ranked brides who wish to improve their ranking".

ner selection.[21] It is well documented that the average wage is usually higher in the male population. However, depending on the country, wages can be either more or less dispersed among men than women.[22] Men and women also evaluate potential partners on multiple anthropometric traits: age, height, weight, etc. The body mass index (BMI) is a popular indicator of body fatness.[23] The BMI is for instance one of the statistics collected by the National Center for Health Statistics in the US. It is also widely used on dating websites to indicate the overall appearance of an individual. Interestingly the men and women BMI distributions are asymmetric: while they have approximately the same means, the variance is higher in the women population.[24]

Finally anecdotal evidence may suggest existence of asymmetries between men and women in the marriage market. Men are usually willing to pay more than women for dating services or "similar" services like nightclubs. This may be seen as proof that men benefits more matching. Also it seems that the range of ages at which someone is considered "attractive" is much larger for men than for women.[25] To be concrete a 40 years old man has better chance to find a woman willing to meet him than the opposite, while it is the opposite for a 25 years old man. This therefore suggests that "deviation payoffs" from an (age-) assortative matching are highly asymmetric.[26]

6 Conclusion

This paper has built a a theory of third-degree price discrimination in matching markets. It has also provided necessary and sufficient conditions on the allocation and distribution of the matching surplus under which a ban on gender-based price discrimination (i.e. on "third-degree" price discrimination) has a positive impact on total welfare.

In my model I made the implicit assumption that any information volunteered by a

[21] See, e.g., the estimated marital preferences in Hitsch, Hortacsu, and Ariely (2010).

[22] For instance, the male wages dispersion is higher in the US, while it is lower in France and Germany.

[23] The body mass index is calculated as follows: BMI = weight(kilograms)/height(meters)^2.

[24] See the National Health Statistic Reports for anthropometric data in the US.

[25] See http://blog.okcupid.com/index.php/the-case-for-an-older-woman/ for evidence from the dating website okcupid.com.

[26] Similarly there are many dating websites for (rich) old men willing to meet young women, while the converse is much less frequent. This suggests that older men are comparatively more attractive to young women than older women are for young men.

participant beyond what is signaled by his or her choice of price is not credible and therefore cannot be used by the matchmaker or the other participants. However, in the real life, dating websites users spend a huge amount of time polishing their online profile, therefore suggesting that it conveys information. The same users also often complain that profiles may not reflect reality. The conditions under which costless signaling (or cheap talk) may improve matching efficiency is an interesting question that I will explore in future work.

A Appendix

A.1 Feasible and stable matching

Let $\mathcal{M}$ and $\mathcal{W}$ denote the sets of men and women who join the matchmaker. Let $\rho : \mathcal{M} \to \Sigma^m$ and $\alpha : \mathcal{W} \to \Sigma^w$ the Lebesgue-measurable functions describing the prices chosen by men an women. Let λ^m and λ^w be the measures induced on $\rho(\mathcal{M})$ and $\alpha(\mathcal{W})$ by the agents' price choices: for Borel sets $\sigma^m \subset \rho(\mathcal{M})$ and $\sigma^w \subset \alpha(\mathcal{W})$,

$$\lambda^m(\sigma^m) = \lambda\{i \in \mathcal{M} \ : \ \rho(i) \in \sigma^m\} \text{ and } \lambda^w(\sigma^w) = \lambda\{i \in \mathcal{W} \ : \ \alpha(i) \in \sigma^w\},$$

where λ is Lebesgue measure.

Definition 2. *Let $\mathcal{M}_1 \subset \mathcal{M}$ and $\mathcal{W}_1 \subset \mathcal{W}$ such that $\lambda^m(\rho(\mathcal{M}_1)) = \lambda^w(\alpha(\mathcal{W}_1))$. A feasible matching between $\mathcal{M}_1$ and $\mathcal{W}_1$ is a pair of measure-preserving functions $\mu^m : (\rho(\mathcal{M}), \lambda^m) \to (\alpha(\mathcal{W}), \lambda^w)$ and $\mu^w : (\alpha(\mathcal{W}), \lambda^w) \to (\rho(\mathcal{M}), \lambda^m)$satisfying:*

$$\begin{aligned} &\mu^m(\mu^w(p^w)) = p^w \textit{ for all } p^w \in \alpha(\mathcal{W}_1), \\ \textit{and} \quad &\mu^w(\mu^m(p^m)) = p^m \textit{ for all } p^m \in \rho(\mathcal{M}_1). \end{aligned}$$

For all $p^m \in \rho(\mathcal{M})$ and $p^w \in \alpha(\mathcal{W})$ let $b^m(p^m) = \Pr\{h|p^m\}$ and $b^w(p^w) = \Pr\{h|p^w\}$ denote agents' posterior beliefs on each other types. For all $(i, j) \in \{l, h\}^2$ let $u(i, j) = u_{i,j}$ and $v(i, j) = v_{i,j}$.

Definition 3. *An equilibrium is a pair of subsets $\mathcal{M}_1 \subset \mathcal{M}$ and $\mathcal{W}_1 \subset \mathcal{W}$ such that $\lambda^m(\rho(\mathcal{M}_1)) = \lambda^w(\alpha(\mathcal{W}_1))$, and a feasible matching between $\mathcal{M}_1$ and $\mathcal{W}_1$, (μ^m, μ^w), that* *satisfies the following condition: there do not exists* $(i, j) \in \mathcal{M} \times \mathcal{W}$ *such that*

$$b^m(\rho(i)) > b^m(\mu^w(\alpha(j))) \text{ and } b^w(\alpha(j)) > b^w(\mu^m(\rho(j))),$$

with the convention that $b^w(\mu^m(\rho(i))) = -\infty$ *if* $i \in \mathcal{M} \setminus \mathcal{M}_1$ *and* $b^m(\mu^w(\alpha(i))) = -\infty$ *if* $j \in \mathcal{W} \setminus \mathcal{W}_1$.

In an equilibrium , men in $\mathcal{M}_1$ are matched with women in $\mathcal{W}_1$, while men in $\mathcal{M} \setminus \mathcal{M}_1$ and women in men in $\mathcal{W} \setminus \mathcal{W}_1$ remain unmatched.

A.2 Proof of Proposition 2

The problem of the matchmaker writes:

$$\begin{aligned}
\max_{(p^m,p^w)} \Pi &= \lambda(p^m + p^w) \\
\text{s.t} \quad & \beta^m s_{hh} - p^m \geq 0, && \text{(14a)} \\
& \beta^m s_{hh} - p^m \geq \mu \cdot \beta^m s_{hh} + (1-\mu) \cdot 0 - p^w, && \text{(14b)} \\
& \beta^w s_{hh} - p^w \geq 0, && \text{(14c)} \\
& \beta^w s_{hh} - p^w \geq \nu \cdot \beta^w s_{hh} + (1-\nu) \cdot 0 - p^m, && \text{(14d)} \\
& \beta^m s_{lh} - p^m < 0, && \text{(14e)} \\
& \mu \cdot s_{lh} + (1-\mu) \cdot 0 - p^w < 0, && \text{(14f)} \\
& \beta^w s_{hl} - p^w < 0, && \text{(14g)} \\
& \nu \cdot s_{hl} + (1-\nu) \cdot 0 - p^m < 0. && \text{(14h)}
\end{aligned}$$

where $\mu = 1$ ($\nu = 1$) if women (men) believe that a man (woman) who chooses p^m (p^w) has type h. If a man deviates when women believe that p^w signals a low type then our definition of an equilibrium matching implies that he can be punished by being left unmatched: he gets a payoff of 0 (see equation (14b)). Since we have continuum populations on both sides of the market, this does not leave any participating women to remain single, and the resulting allocation is feasible, stable and measure preserving. We have:

$$\mu = \begin{cases} 1 & \text{if } p^w > \beta^m s_{lh}, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and } \nu = \begin{cases} 1 & \text{if } p^m > \beta^w s_{hl}, \\ 0 & \text{otherwise.} \end{cases}$$

Assume in the following that $\beta^m \geq \beta^w$. We solve the relaxed problem:

$$\begin{aligned} \max_{(p^m,p^w)} \Pi &= \lambda(p^m + p^w) \\ \text{s.t} & \ \ (14a),\ (14b),\ (14c), (14e),\ (14g). \end{aligned}$$

Constraint (14b) rewrites: $p^m \leq p^w + (1-\mu) \cdot \beta^m s_{hh}$. There are three cases to consider.

Case 1: $\beta^w s_{hh} \leq \beta^m s_{lh}$**.** Then all prices p^w compatible with inequalities (14c) and (14g) are below $\beta^m s_{lh}$ so that $\mu = 0$. In particular, inequality (14b) is satisfied if inequality (14a) and (14g) are satisfied. Therefore, the solution to the matchmaker problem are the maximum prices p^m and p^w that satisfy (14a) and (14c) respectively: $p^m = \beta^m s_{hh}$ and $p^w = \beta^w s_{hh}$.

Case 2: $\beta^m s_{lh} \leq \beta^w s_{hl}$**.** Then all prices p^w compatible with inequalities (14c) and (14g) are above $\beta^m s_{lh}$ so that $\mu = 1$. Inequality (14b) is therefore binding: $p^m = p^w$. The solution to the matchmaker problem is the maximum price p^w that satisfies (14c): $p^w (= p^m) = \beta^w s_{hh}$.

Case 3: $\beta^w s_{hl} < \beta^m s_{lh} \leq \beta^w s_{hh}$**.** On the one hand, if $\beta^w s_{hl} < p^w \leq \beta^m s_{lh}$, $\mu = 0$ so that the maximum price that satisfies (14a) and (14e) is $p^m = \beta^m s_{hh}$. On the other hand, if $\beta^m s_{lh} \leq p^w \leq \beta^w s_{hh}$, $\mu = 1$ so that the maximum price that satisfies (14a) and (14e) is $p^m = p^w$. The matchmaker therefore chooses between the two following options: either $\Sigma = \{p^m, p^w\} = \{\beta^m s_{hh}, \beta^m s_{lh}\}$, in which case $\Pi = \lambda\beta^m(s_{hh} + s_{lh})$; or $\Sigma = \{p^w(= p^m)\} = \{\beta^m s_{hh}\}$, in which case $\Pi = 2\lambda\beta^w s_{hh}$. To conclude, notice that, if the latter option is chosen, Σ is the same as in Case 2 above.

A.3 Proof of Proposition 3

Cases 1 and 2 in Proposition 3 were established in the main text. Case 3 is similar to case 3 in the proof of Proposition 2 (see Section A.2 above). I therefore rely on a graphical proof (see

Figure 1 in the main text). When $\beta^w(s_{hh}-s_{lh}) \geq \beta^m(s_{lh}-s_{ll}) \geq \beta^w(s_{hl}-s_{ll})$ the optimization constraints define a non-convex polyhedron the extreme points of which are: $(p^w, p^m) = (\underline{p} + \beta^w(s_{hh} - s_{lh}), \underline{p} + \beta^w(s_{hh} - s_{lh}))$ and $(p^w, p^m) = (\underline{p} + \beta^m(s_{lh} - s_{ll}), \underline{p} + \beta^m(s_{hh} - s_{hl}))$. Profits are $\Pi = 2\beta^w(s_{ll}+\lambda(s_{hh}-s_{lh}))$ at the former, and $\Pi = 2\beta^w s_{ll}+\lambda\beta^m(s_{hh}-s_{hl}+s_{lh}-s_{ll})$ at the latter. Comparison of this two profits yields the announced result.

A.4 Proof of Proposition 4

The proof proceeds in two steps: first, I derive the matchmaker's profits in pooling allocations and I show that the matchmaker can achieve higher profits with a shutdown or a separating allocation if $\beta^m = \beta^w$; second, I show that, if $\beta^m = \beta^w$, if a shutdown allocation is implemented without the ban, then, it is also implemented under the ban.

Step 1: Profits in pooling allocations. There are three different pooling allocations to consider depending on whether all men and/or women join the matchmaker.

(a) All men and women participate. The matchmaker offers $\Sigma = \{p^m, p^w\}$ such that all men and women participate, and men and women are randomly matched. Prices p^m and p^w must satisfy the low type men and women participation constraints:

$$\beta^m(\lambda s_{lh} + (1-\lambda)s_{ll}) - p^m \geq 0 \text{ and } \beta^w(\lambda s_{hl} + (1-\lambda)s_{ll}) - p^w \geq 0.$$

If, for instance, $p^m > p^w$, all men (and women) should choose p^w. Indeed, since p^w is acceptable for both the high type and the low type men, a woman must believe a man who chooses p^w have type l with probability $1-\lambda$ and type h with probability λ. The matchmaker's profits in a pooling allocation are therefore given by:

$$\Pi_{Pool} = 2\min\{\beta^w(s_{ll} + \lambda(s_{hl} - s_{ll})), \beta^m(s_{ll} + \lambda(s_{lh} - s_{ll}))\}.$$

Assume now that $\beta^w = \beta^m$ and, for instance, $s_{hl} \geq s_{lh}$ so that $\Pi_{Pool} = s_{ll} + \lambda(s_{lh} - s_{ll})$. There are two cases to consider.

- If $s_{hl} - s_{ll} \leq s_{hh} - s_{hl}$, then, we have:

$$\Pi_{Sep} \geq s_{ll} + \lambda(s_{hh} - s_{hl}) \geq s_{ll} + \lambda(s_{lh} - s_{ll}) = \Pi_{Pool},$$

  where the first inequality comes from Proposition 3 and the second from increasing differences of the matching surplus.

- If $s_{hl} - s_{ll} > s_{hh} - s_{hl}$, then, by Proposition 3, $\Pi_{Sep} = s_{ll} + \lambda/2(2s_{hh} - s_{hl} - s_{lh})$. Hence

$$\begin{aligned}
\Pi_{Sep} - \Pi_{Pool} &= s_{ll} + \tfrac{\lambda}{2}(2s_{hh} - s_{hl} - s_{lh} - (s_{ll} + (s_{lh} - s_{ll}))), \\
&= \tfrac{\lambda}{2}(2(s_{hh} + s_{ll}) - (3s_{lh} + s_{hl})), \\
&\geq \tfrac{\lambda}{2}(2(s_{lh} + s_{hl}) - (3s_{lh} + s_{hl})) = \lambda(s_{hl} - s_{lh}) \geq 0,
\end{aligned}$$

  where the first inequality comes from increasing differences of the matching surplus.

(b) All men participate, only the high type women participate. We call this allocation a *pooling M - shutdown W allocation.* The lemma below gives an upper bound of the matchmakers' profits if it implements a pooling M - shutdown W allocation:

Lemma 3. *When gender-based price discrimination is not allowed, the matchmaker's profits in a pooling M - shutdown W allocation satisfies:*

$$\Pi \leq \lambda \cdot (\beta^m s_{lh} + \beta^w(\lambda s_{hh} + (1-\lambda)s_{lh})). \tag{15}$$

Proof. Notice first that the matchmaker's profits in a pooling M - shutdown W allocation is lower when gender-based price discrimination is not allowed compared with the case where it is allowed. Then notice that the matchmaker's profits in a pooling M - shutdown W allocation when gender-based price discrimination is allowed is given by:

$$\left.\begin{aligned}
\max_{(p^m,p^w)} \quad & p^m + \lambda p^w \\
\text{s.t.} \quad & \lambda\beta^m s_{lh} - p^m \geq 0 \\
& \beta^w(\lambda s_{hh} + (1-\lambda)s_{lh}) - p^w \geq 0 \\
& \beta^w(\lambda s_{hl} + (1-\lambda)s_{ll}) - p^w < 0
\end{aligned}\right\} = \lambda \cdot (\beta^m s_{lh} + \beta^w(\lambda s_{hh} + (1-\lambda)s_{lh})).$$

This concludes the proof. □

Assume now that $\beta^m = \beta^w$. By Proposition 2, the matchmaker's profits in a shutdown allocation when gender-based price discrimination is not allowed is

$$\Pi_{Sh} = \lambda s_{hh} = \lambda(\beta^m s_{hh} + \beta^m s_{hh}) \geq \lambda(\beta^m s_{lh} + \beta^w(\lambda s_{hh} + (1-\lambda)s_{lh})).$$

(c) Only the high type men participate, all women participate. Similar to case (b).

To summarize, we have shown that, when $\beta^m = \beta^w$, either a shutdown or a separating allocation is implemented when gender-based price discrimination is not allowed.

Step 2: Welfare comparison. Let Π^g_{Sep} and Π^g_{Sh} (Π_{Sep} and Π_{Sh}) denote the matchmakers' profits in a shutdown and separating allocations when gender-based price discrimination is (not) allowed. We already noted that $\Pi^g_{Sh} = \Pi_{Sh}$. Now notice that $\Pi_{Sep} \leq \Pi^g_{Sep}$ so that:

$$\Pi^g_{Sh} - \Pi^g_{Sep} \geq 0 \Rightarrow \Pi_{Sh} - \Pi_{Sep} \geq 0.$$

In words, if a shutdown allocation is implemented without the ban then it is also implemented under the ban. This concludes the proof.

A.5 Proof of Proposition 5

The proof follows the same steps as the proof of Proposition 4. First, I show that the matchmaker can achieve higher profits with a shutdown or a separating allocation if $s_{hl} = s_{lh}$; second, I show that, when $s_{hl} = s_{lh}$ and $s_{ll}/s_{lh} \geq s_{lh}/s_{hh}$, if a shutdown allocation is implemented without the ban, then, it is also implemented under the ban.

Step 1. Profits in pooling allocations.

(a) All men and women participate. When $s_{hl} = s_{lh}$, the matchmaker's profits in a pooling allocation is given by (see the proof of Proposition 4 above):

$$\Pi_{Pool} = 2\min\{\beta^m, \beta^w\}(s_{ll} + \lambda(s_{hl} - s_{ll})).$$

In the following, assume for instance that $\beta^w \leq \beta^m$. There are two cases to consider.

- if $\beta^w(s_{hh} - s_{lh}) \geq \beta^m(s_{hl} - s_{ll})$, then, by Proposition 3:

$$\begin{aligned} \Pi_{Sep} &= \max\{2\beta^w s_{ll} + \lambda\beta^m(s_{hh} - s_{ll}), 2\beta^w s_{ll} + 2\lambda\beta^w(s_{hh} - s_{lh})\}, \\ &\geq 2\beta^w s_{ll} + 2\lambda\beta^w(s_{hh} - s_{lh}), \\ &\geq 2\beta^w s_{ll} + 2\lambda\beta^w(s_{hl} - s_{ll}) = \Pi_{Pool}. \end{aligned}$$

  where the latter inequality comes from increasing differences of the matching surplus.

- if $\beta^w(s_{hh} - s_{lh}) < \beta^m(s_{hl} - s_{ll})$, then, by Proposition 3, $\Pi_{Sep} = 2\beta^w s_{ll} + \lambda(s_{hh} - s_{hl})$. Therefore

$$\begin{aligned} \Pi_{Sep} - \Pi_{Pool} &= 2\beta^w s_{ll} + \lambda(s_{hh} - s_{hl}) - 2\beta^w(s_{ll} + \lambda(s_{hl} - s_{ll})), \\ &= \lambda((s_{hh} - s_{hl}) - 2\beta^w(s_{hl} - s_{ll})) \geq 0, \end{aligned}$$

  where the inequality is obtained by noticing that $s_{hh} - s_{hl} \geq s_{hl} - s_{ll}$ (increasing differences) and $1 \geq 2\beta^w$.

(b) All men participate, only the high type women participate. Let us show that the matchmakers' profits in a shutdown allocation is higher that the upper bound for profits in a pooling M - shutdown W allocation obtained in Lemma 3. There are two cases to consider.

- **Case 1:** $\beta^w \leq \beta^m$**.** By Proposition 2, if $\beta^w s_{hh} \leq \beta^m s_{hl}$, then, $\Pi_{Sh} = \lambda s_{hh} \geq \lambda(\beta^m s_{lh} + \beta^w(\lambda s_{hh} + (1-\lambda)s_{lh}))$. Assume then that $\beta^w s_{hh} > \beta^m s_{hl}$. By Proposition 2, we have

$\Pi_{Sh} = \lambda \max\{2\beta^w s_{hh}, \beta^m(s_{hh} + s_{hl})\}$. Notice then that

$$\begin{aligned}
\beta^m(s_{hh} + s_{hl}) - (\beta^m s_{lh} + \beta^w(\lambda s_{hh} + (1-\lambda)s_{lh})) &\geq \beta^m s_{hh} - \beta^w(\lambda s_{hh} + (1-\lambda)s_{lh}),\\
&\geq \beta^m(s_{hh} - (\lambda s_{hh} + (1-\lambda)s_{lh})),\\
&\geq 0,
\end{aligned}$$

which shows that the matchmaker achieves higher profits in a shutdown allocation than in a pooling M - shutdown W allocation.

- **Case 2:** $\beta^w > \beta^m$**.** By Proposition 2, if $\beta^m s_{hh} \leq \beta^w s_{hl}$, then, $\Pi_{Sh} = \lambda s_{hh} \geq \lambda(\beta^m s_{lh} + \beta^w(\lambda s_{hh} + (1-\lambda)s_{lh}))$. Assume then that $\beta^m s_{hh} > \beta^w s_{hl}$. By Proposition 2, we have $\Pi_{Sh} = \max\{2\beta^m s_{hh}, \beta^w(s_{hh} + s_{hl})\}$. Notice then that

$$\beta^w(s_{hh}+s_{hl})-(\beta^m s_{lh}+\beta^w(\lambda s_{hh}+(1-\lambda)s_{lh})) = \beta^w(s_{hh}-s_{hl})(1-\lambda)+(\beta^w-\beta^m)s_{hl} \geq 0,$$

which shows that the matchmaker achieves higher profits in a shutdown allocation than in a pooling M - shutdown W allocation.

(c) Only the high type men participate, all women participate. Similar to case (b).

To summarize, we have shown that, when $s_{hl} = s_{lh}$, either a shutdown or a separating allocation is implemented when gender-based price discrimination is not allowed.

Step 2. Welfare comparison. Let Π^g_{Sep} and Π^g_{Sh} (Π_{Sep} and Π_{Sh}) denote the matchmakers' profits in a shutdown and separating allocations when gender-based price discrimination is (not) allowed. We show that:

$$\Pi^g_{Sh} - \Pi^g_{Sep} \geq 0 \Rightarrow \Pi_{Sh} - \Pi_{Sep} \geq 0.$$

Hereafter assume for instance that $\beta^w \leq \beta^m$. By Proposition 2 and 3, we have:

$$\Pi_{Sh} = \lambda \cdot \begin{cases} s_{hh} & \text{if } \frac{\beta^w}{\beta^m} \leq \frac{s_{hl}}{s_{hh}}, \\ \beta^m(s_{hh} + s_{hl}) & \text{if } \frac{s_{hl}}{s_{hh}} < \frac{\beta^w}{\beta^m} \leq \frac{s_{hh}+s_{hl}}{2s_{hh}}, \\ 2\beta^w s_{hh} & \text{if } \frac{s_{hh}+s_{hl}}{2s_{hh}} < \frac{\beta^w}{\beta^m}. \end{cases}$$

$$\Pi_{Sep} = 2\beta^w s_{ll} + \lambda \cdot \begin{cases} s_{hh} - s_{hl} & \text{if } \frac{\beta^w}{\beta^m} \leq \frac{s_{hl}-s_{ll}}{s_{hh}-s_{hl}}, \\ \beta^m(s_{hh} - s_{ll}) & \text{if } \frac{s_{hl}-s_{ll}}{s_{hh}-s_{hl}} < \frac{\beta^w}{\beta^m} \leq \frac{s_{hh}-s_{ll}}{2(s_{hh}-s_{hl}}, \\ 2\beta^w(s_{hh} - s_{hl}) & \text{if } \frac{s_{hh}-s_{ll}}{2(s_{hh}-s_{hl})} < \frac{\beta^w}{\beta^m}. \end{cases}$$

Now notice that since $\frac{s_{ll}}{s_{hl}} \geq \frac{s_{hl}}{s_{hh}}$, we have:

$$\frac{s_{hl} - s_{ll}}{s_{hh} - s_{hl}} \leq \frac{s_{hl}}{s_{hh}} \leq \frac{s_{hh} - s_{ll}}{2(s_{hh} - s_{hl})} \leq \frac{s_{hh} + s_{hl}}{2s_{hh}}.$$

Therefore, there are four cases to consider.

Case 1: $\frac{\beta^w}{\beta^m} \leq \frac{s_{hl}}{s_{hh}}$. Notice that $\Pi_{Sh} = \Pi^g_{Sh}$. Then, since $\Pi_{Sep} \leq \Pi^g_{Sep}$, we have: $\Pi^g_{Sh} - \Pi^g_{Sep} \geq 0 \Rightarrow \Pi_{Sh} - \Pi_{Sep} \geq 0$.

Case 2: $\frac{s_{hh}}{s_{hh}} < \frac{\beta^w}{\beta^m} \leq \frac{s_{hh}-s_{ll}}{2(s_{hh}-s_{hl}}$. We have $\Pi_{Sep} - \Pi_{Sh} = 2\beta^w s_{ll} - \lambda\beta^m(s_{hl} - s_{ll})$. Assume that $\Pi^g_{Sep} - \Pi^g_{Sh} \leq 0$, i.e. $\lambda \geq s_{ll}/s_{hl}$ by Proposition 1. Hence,

$$\begin{aligned} \Pi_{Sep} - \Pi_{Sh} &\leq 2\beta^w s_{ll} - \tfrac{s_{ll}}{s_{hl}}\beta^m(s_{hl} - s_{ll}), \\ &\leq 2s_{ll}\beta^m \left(\tfrac{\beta^w}{\beta^m} - \tfrac{s_{hh}-s_{ll}}{2(s_{hh}-s_{hl}}\right), \\ &\leq 0. \end{aligned}$$

In other words, we have shown $\Pi^g_{Sh} - \Pi^g_{Sep} \geq 0 \Rightarrow \Pi_{Sh} - \Pi_{Sep} \geq 0$.

Case 3: $\frac{s_{hh}-s_{ll}}{2(s_{hh}-s_{hl}} < \frac{\beta^w}{\beta^m} \leq \frac{s_{hh}+s_{hl}}{2s_{hh}}$. Again, assume that $\Pi^g_{Sep} - \Pi^g_{Sh} \leq 0$, i.e. $\lambda \geq s_{ll}/s_{hl}$. Hence,

$$\begin{aligned}
\Pi_{Sep} - \Pi_{Sh} &= 2\beta^w s_{ll} - \lambda(\beta^m(s_{ll}+s_{hl}) - \beta^w(s_{hh}-s_{hl})), \\
&\leq 2\beta^w s_{ll} - \frac{s_{ll}}{s_{hl}}(\beta^m(s_{ll}+s_{hl}) - \beta^w(s_{hh}-s_{hl})), \\
&\leq 2s_{ll}\beta^m \frac{s_{hh}}{s_{hl}}\left(\frac{\beta^w}{\beta^m} - \frac{s_{hh}+s_{hl}}{2s_{hh}}\right). \\
&\leq 0.
\end{aligned}$$

In other words, we have shown $\Pi^g_{Sh} - \Pi^g_{Sep} \geq 0 \Rightarrow \Pi_{Sh} - \Pi_{Sep} \geq 0$.

Case 4: $\frac{s_{hh}+s_{hl}}{2s_{hh}} < \frac{\beta^w}{\beta^m}$. Notice that $\Pi_{Sep} - \Pi_{Sh} = 2\beta^w(s_{ll} - \lambda s_{hl})$. Therefore $\Pi^g_{Sh} - \Pi^g_{Sep} \geq 0 \Leftrightarrow \Pi_{Sh} - \Pi_{Sep} \geq 0$. This concludes the proof.

A.6 Proof of Proposition 6

Assume in the following that the matching market is dominated by men.

Step 1: Profits in pooling allocation.

(a) All men and women participate. Recall that the matchmaker's profits in a pooling allocation is given by (see the proof of Proposition 4):

$$\Pi_{Pool} = 2\min\{\beta^w(s_{ll} + \lambda(s_{hl} - s_{ll})), \beta^m(s_{ll} + \lambda(s_{lh} - s_{ll}))\}.$$

Then notice that, when the market is dominated by men,

$$\begin{aligned}
\Pi_{Sep} &= 2\beta^w(s_{ll} + \lambda(s_{hh} - s_{lh})), \\
&\geq 2\beta^w(s_{ll} + \lambda(s_{hl} - s_{lh})), \\
&\geq 2\min\{\beta^w(s_{ll} + \lambda(s_{hl} - s_{ll})), \beta^m(s_{ll} + \lambda(s_{lh} - s_{ll}))\} = \Pi_{Pool},
\end{aligned}$$

where the first inequality comes from increasing differences of the matching surplus.

(b) All men participate, only the high type women participate. Recall that an upper bound for the matchmaker's profits in a pooling M - shutdown W allocation is

$\lambda(\beta^m s_{lh} + \beta^w(\lambda s_{hh} + (1-\lambda)s_{lh}))$ (see Lemma 3 in the proof of Proposition 4). Then notice that, when the market is dominated by men $\Pi_{Sh} = 2\lambda\beta^w s_{hh}$ and

$$2\beta^w s_{hh} - (\beta^m s_{lh} + \beta^w(\lambda s_{hh} + (1-\lambda)s_{lh})) = (1-\lambda)\beta^w(s_{hh} - s_{lh}) + (\beta^w s_{hh} - \beta^m s_{lh}).$$

To conclude, notice that the second term in the rhm of the above equation is positive since $\beta^w s_{hh} \geq \beta^w s_{hl} > \beta^m s_{lh}$.

(c) Only the high type men participate, all women participate. Similar to case (b).

Step 2: Welfare comparison. Under the assumptions of Proposition 6, Propositions 2 and 3 together give $\Pi_{Sh} - \Pi_{Sep} = 2\beta^w(\lambda s_{lh} - s_{ll})$. Assume that $\Pi_{Sh} - \Pi_{Sep} \geq 0$, i.e. that $\lambda \geq s_{ll}/s_{lh}$. Then, notice that, by Proposition 1,

$$\begin{aligned}\Pi^g_{Sh} - \Pi^g_{Sep} &= \beta^w(\lambda s_{lh} - s_{ll}) + \beta^m(\lambda s_{hl} - s_{ll}), \\ &\geq \beta^m \tfrac{s_{ll}}{s_{lh}}(s_{hl} - s_{lh}).\end{aligned}$$

To conclude, notice that $s_{hl} \geq s_{lh}$ since $\beta^w < \beta^m$ and $\beta^w s_{hl} > \beta^m s_{lh}$.

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