Wage Inequality, Imitation and Social Institutions: A Theoretical Analysis*

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Abstract

The paper develops a four sector closed economy model with two primary factors-skilled labour and unskilled labour. One of the two production sectors produces varieties of innovated products and the other sector imitates those innovated products and produces with unskilled labour as the only input. A R&D sector develops blue-prints of new products using skilled labour as the only input. We here introduce endogenous imitation and assume that a social institution has control over this endogenous imitation. This social institution produces an imitation preventing public good with skilled labour as the only input. It is shown that an increase in skilled (unskilled) labour endowment raises (has no effect on) the rate of growth and raises (lowers) the skilled-unskilled wage ratio. However, an improvement in the imitation preventing efficiency of the public good raises the skilled-unskilled wage ratio though it has no effect on growth rate. A change in skilled labour endowment or a change in unskilled labour endowment has no effect on the imitation rate. However, an improvement in the imitation prevention efficiency of the public good lowers the imitation rate. We also analyse the effects of change in different parameters on the level of social welfare.

JEL classification: F13, J31, O10, O31, O34, O40.

Keywords: Skilled labour, Unskilled labour, Wage Inequality, Innovation, Endogenous Imitation, Social Institution, Product Variety, Monopolistic Competition, Welfare.

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* This is a part of the research work of the author, which is in progress for the prospective Ph.D. degree of Indian Statistical Institute. Helpful comments have been received from Prof. Manash Ranjan Gupta to whom the author is thankful. All remaining errors are of the author alone .

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1. Introduction

Growing wage inequality between skilled labour and unskilled labour is a widely discussed topic in Development economics. Empirical studies point out this feature in U.S.A. during 1960s[1] and in European countries between 1978 and 1988[2]. We find similar observations in many developing countries too. Wage inequality has gone up in many Latin American and South Asian countries in the mid 1980s[3]. However, the experience of East Asian countries between 1960s and 1970s advocates the conventional theory that a greater openness to the rest of the world leads to a decrease in the skilled-unskilled wage gap[4]. Different empirical studies provide different explanations for this growing income inequality. Trade liberalization and technological progress appear to be the main two controversial reasons of this phenomenon[5].However, we have other explanations of this increasing wage inequality; and these include, international outsourcing[6], increase in the relative price of the skill intensive good[7], entry of unskilled labour surplus low income countries into the global market[8] etc.

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[1] See, for example, Hoe et. al. (2005), Bound and Johnson (1992), Juhn et. al. (1993), Autor et. al. (1998) Leamer (2000), Acemoglu (2002a) etc. Accoding to Lee and Wolpin (2010) wage differentials by education increased during the period 1968-2000.

[2] See, for example, Lawrence (1994), Katz et. al. (1992) etc.

[3] See, for example, Mollick (2009), Wood (1997), Dev (2000), Borjas and Ramey (1993), Banga (2005), Beyer et. al. (1999) etc. Accoding to Mollick (2009), wage differentials by skilled labour actually increased in Mexico during the period 1990-2006.

[4] See, for example, Wood (1997).

[5] According to Wood (1998), Beyer et. al. (1999), Green et. al. (2001), Behrman et. al. (2000), Isgut (2001) etc. trade liberalization is to blame for this growing wage inequality. However, Wood (1997, 1998), Dev (2000) and Görg and Strobl (2002) are of the view that technological progress worsens wage inequality through an increase in the relative demand for skilled labour. Esquivel and Lo´pez (2003) shows that technological change aggravates but trade liberalization lowers wage inequality in Mexico.

[6] See Feenstra and Hanson (1997) in this context.

[7] See Harrison and Hanson (1999), Hanson and Harrison (1999) and Beyer et. al. (1999) in this context.

[8] See Wood (1997) in this context.

Many theoretical models deal with the problem of growing wage inequality. Some of them are static competitive general equilibrium models[9] of small open economies with two different types of labour- skilled and unskilled. There are some other static general equilibrium models[10] which deal with this growing wage inequality problem using a product variety structure and assuming monopolistic competition in markets of different varieties. Some authors develop dynamic models[11] and explain skilled-unskilled wage inequality problem in the long run equilibrium of their models. The ratio of the wage rate of the skilled labour to that of the unskilled labour is considered to be a measure of wage inequality in all these models. Neither these static models nor those dynamic models deal with the role of imitation on the skilled-unskilled wage inequality. Grossman and Helpman (1991a) spends a few chapters to explain this growing wage inequality. However, models developed in those chapters do not analyze the role of imitation. North-South models of Grossman and Helpman (1991a) and Helpman (1993) analyse the role of imitation on the long run rate of growth and on the North-South relative wage. However, these Nort-South models do not distinguish between skilled labour and unskilled labour; and hence can not focus on the role of imitation on skilled-unskilled wage inequality.

No model in the existing literature, except Thoeing and Verdier (2003), has analysed the effects of imitation on this skilled-unskilled wage inequality problem. In Thoeing and Verdier (2003), innovating firms use skill intensive technology to get rid of the threat of imitation. This raises the relative demand for skilled labour and thus worsens the problem of skilled-unskilled wage inequality. Empirical works like Kanwar and Evenson (2003, 2009), Park (2008), Ginarte and Park (1997) etc. show that there is significant improvement in the worldwide patent protection during the period 1960-2005. Any improvement in patent protection would reduce the threat of imitation; and, according to the prediction of Thoeing and Verdier (2003) model,

[9] See, for example, Beladi et. al. (2008), Chaudhuri and Yabuuchi (2007, 2008), Chaudhuri (2004, 2008), Marjit and Kar (2005), Yabuuchi and Chaudhuri (2007), Marjit and Acharyya (2003), Marjit (2003), Xu (2003), Marjit et. al. (2004), Marjit and Acharyya (2006), Kar and Beladi (2004), Zhu and Trefler (2005), Gupta and Dutta (2011a, 2010a, 2010b) etc. in this context. Beladi and Oladi (2009) consider an open economy with a non-traded good sector and a sector producing exportables; and shows that the degree of skilled-unskilled wage inequality depends on the elasticity of import demand.

[10] See for example Anwar and Rice (2009), Anwar (2009, 2006), Glazer and Ranjan (2003) etc.

[11] These include Galor and Moav (2000), Aghion et. al. (1999), Aghion (2002), Beladi and Chakrabarty (2008), Ripol (2005), Kiley (1999), Acemoglu (1998, 2002a, 2002b) etc.

innovating firms would reduce the skill intensity of the production process which in turn should lower the skilled-unskilled relative wage. However, according to Acemoglu and Verdier (1998), property rights are never perfect in terms of implementation. Social infrastructure is very crucial for monitoring of these written laws. Difference in intuitional framework can have huge impact on the effective implementation of these laws. Many empirical studies focus on the relationship between the presence of the appropriate social institution and the strength of the intellectual property right. Magge (1992) estimates significant benefits to strong legal systems. His empirical approach implicitly assumes an endogenous institutions model where a fraction of population is hired to build and maintain those institutions. Khan (2003), in the context of British patent system, argues that patent laws are regarded only when they are monitored. Khan and Sokoloff (2001) provide extensive evidence to justify that early development of broad access to IPR institutions with strict enforcement was crucial for USA to move from a net importer to a net exporter of patents. Hall and Jones (1999) and Grigorian and Martinez (2002) argue that social institutions as measured by government bureaucracy quality, corruption, risk of appropriation and government repudiation of contracts are important factors for cross-country differences in output per worker. North and Thomas (1973) shows social infrastructure or Government institutions help social agents to capture the full returns of their actions by reducing uncertainty and transaction costs. According to Rodrik (1995), social institutions protect property rights. So threat of imitation can not be reduced only by introducing laws.

In the model developed in the present paper, we also plan to analyse the role of imitation and the role of immigration of skilled labour and unskilled labour on skilled-unskilled wage inequality using a Grossman-Helpman (1991a) type of product variety structure in which innovation gives birth of new varieties. We consider a four sector closed economy model with two primary factors- skilled labour and unskilled labour; and this present model is basically an extension of the product variety model developed in chapter 3 of Grossman and Helpman (1991a) in which there exists only one production sector producing varieties of innovated products with labour as the only input. Like Grossman and Helpman (1991a) and Helpman (1993), a R&D sector develops blue-prints of new products in this model using skilled labour as the only input. We here introduce endogenous imitation and assume that a social institution has control over this endogenous imitation rate. This social institution produces an imitation preventing public good with skilled labour as the only input; and the rate of imitation varies inversely with the size of this institution which is also endogenously determined. However the efficiency parameter of this sector is exogenous and its value stand for the efficiency of the institution. There are two production sectors of which one sector produces varieties of innovated products and the other sector imitates those innovated products without bearing any cost of imitation[12]. Both the production sectors use only unskilled labour as input. However, the innovated sector that derives benefits from this social institution must bear the burden of financing the cost of production of this public good. This cost is financed by lump sum tax imposed on all firms producing innovated varieties.

We derive many interesting results from this model. First, there exists a constant rate of growth in this model and it is independent of the attainment of steady-state equilibrium. Secondly, an increase in skilled labour endowment raises the rate of growth (expansion of varieties) but a change in unskilled labour endowment has no effect on it. Thirdly, the change in skilled labour endowment or unskilled labour has no effect on imitation rate. An improvement in the imitation prevention efficiency of the public good lowers the imitation rate. Fourthly, an increase in unskilled labour endowment and/or an improvement in the efficiency of imitation preventing public good raises the skilled-unskilled wage ratio in our model. If the monopoly power of each firm in the innovated sector is very low, then an increase in skilled labour endowment lowers the skilled-unskilled wage ratio. Finally, in the steady state equilibrium, an increase in the level of unskilled labour endowment raises the level of social welfare but an increase in skilled labour endowment and an improvement in the efficiency of imitation prevention of the public good has an ambiguous effect on it.

The paper is organized as follows. Section 2 describes the model and section 3 analyses its results. Rate of growth and rate of imitation are derived in subsection 3.1 and 3.2 respectively; and the stability of the steady-state equilibrium is analysed in subsection 3.3. Effects of parametric changes on the degree of wage inequality in the steady-state growth equilibrium are described in subsection 3.4. The rate of interest is determined in subsection 3.5;

[12] We assume this following Helpman (1993) being fully aware that imitation activity is not at all cost less in the real world.

and comparative static effects on welfare are analysed in sub section 3.6. Concluding remarks are made in section 4.

2. The Model

We consider a closed economy with four sectors and two primary factors- skilled labour and unskilled labour. Sector 1 produces varieties of innovated products and sector 2 produces varieties of imitated products; and unskilled labour is the only input used in both these two sectors[13,14]. Also there is a R&D sector developing blue-prints of new products and it uses skilled labour as the only input. Skilled labour is also used to produce a public good that prevents imitation. It is a social institution or an intellectual property rights (IPR) protecting institution. Innovators can derive its service without paying any price. However, sector 1 who derives benefits from this institution must bear the burden of financing the cost of production this public input. So this cost of producing the public input is financed by the lump sum tax imposed by the government on all firms of sector 1. There are studies which show that cross-country differences in imitation rate are driven not only by differences in government policies but also by institutions[15].

Let the rate of innovation of new products per unit time be denoted by $\dot{n}$. Then the production function in the R&D sector is given by

$$\dot{n} = \frac{lK_S}{a} \quad (1).$$

Here, l is the amount of skilled labour employed in the R&D sector; K_S is the existing stock of knowledge and a is the per unit labour requirement in the R&D sector. Following Grossman and

[13] Generally varieties innovated in a country are imitated in other countries. This model may also represent the world economy with free trade, perfect mobility of factors, identical production technology across countries and with intercountry variations in the degree of implementation of intellectual Property Right (IPR) protection Acts.

[14] None of the imitated products, in reality, is produced without the use of skilled labour. However, unskilled workers acquire some production specific skill through learning by doing and can replace skilled workers in many skill intensive stages of production once products are imitated. Our concept of skilled labour does not include this learning by doing skill of unskilled workers.

[15] See, for example, Hall and Jones (1999), Grigorian and Martinez (2002).

Helpman (1991a), Helpman (1993) etc. we assume that $K_S = n$ where n is the total number of varieties innovated as well as imitated. So we can modify equation (1) as follows.

$$g = \frac{\dot{n}}{n} = \frac{l}{a} \qquad (2).$$

where g is the rate of growth of new products.

The production function of the imitation preventing public good producing sector is given as follows.

$$y_m = n{l_m}^{\beta} \text{ , with } 0 < \beta < 1 \qquad (3).$$

Here y_m stands for the level of output of this public good and l_m is the amount of skilled labour employed in this public good sector. β is the labour elasticity of output. $0 < \beta < 1$ implies that there is diminishing returns to labour in this sector.[16] Productivity of skilled labour in this sector also varies proportionately with the stock of knowledge, n, because expansion of the stock of knowledge enhances the level of skill of the worker.

In equilibrium, real wage rate of skilled labour is equal to its average physical productivity in the public good producing sector because the objective of the institution providing the public good is to maintain a no profit no loss equilibrium, i.e., its budget must be balanced. So

$$n{l_m}^{(\beta-1)} = W_S \qquad (4).$$

Here W_S represents the wage rate of the skilled labour in the public good sector. Firms in sector 2 do imitations without bearing any direct cost. The rate of imitation is assumed to vary inversely with the size of the imitation preventing public good sector and positively with the existing stock of knowledge, n. So the imitation rate, denoted by m, is defined as follows.

$$m = \frac{\dot{n}^U}{n^S} = \frac{n}{y_m b} \qquad (5).$$

Here n^S and n^U represent total number of varieties produced by sector 1 and sector 2, respectively. b is a parameter measuring the efficiency of imitation prevention done by the public input.

Sector 1 does not produce any variety already imitated by sector 2.

[16] It does not mean that the assumption of constant returns is empirically rejected. We assume constant returns in the R&D sector and so an interior allocation of skilled labour can not be obtained with constant returns in both the R&D sector and in the public good sector.

So we have

$$n^S + n^U = n \qquad (6).$$

The fraction of goods not imitated by sector 2 is denoted by ξ. Hence

$$\xi = \frac{n^S}{n} \qquad (7).$$

Now, from equation (7), we obtain[17]

$$\dot{\xi} = g - (g + m)\xi \qquad (8).$$

Equation (8) shows how the rate of change in the fraction of unimitated (innovated) products varies with the growth rate and the imitation rate.

In the steady-state equilibrium, the fraction of unimitated goods remains unchanged over time. Hence $\dot{\xi} = 0$. So we obtain

$$\xi = \frac{g}{(g+m)} \qquad (9).$$

So equation (9) implies that the fraction of innovated products in the steady-state equilibrium varies positively with the growth rate and inversely with the imitation rate.

All individuals in this model have identical preferences. The representative household maximizes the discounted present value of instantaneous utility over the infinite time horizon; and it is given by

$$U(t) = \int_t^{\infty} e^{-\rho(\tau - t)} \log u(\tau) d\tau \qquad (10).$$

The intertemporal budget equation of that representative household[18] is given by

$$\int_t^{\infty} e^{-r(\tau-t)} E(\tau) d\tau = \int_t^{\infty} e^{-r(\tau-t)} I(\tau) d\tau + A(t) \quad \forall t \qquad (11).$$

Here, $u(\tau)$, $E(\tau)$, $I(\tau)$ and $A(\tau)$ stand for levels of instantaneous utility, instantaneous expenditure, instantaneous income and current assets respectively at the time point τ. ρ and r stand for the subjective discount rate and the nominal interest rate respectively; and, for the sake of simplicity, ρ and r are assumed to be time independent.

The instantaneous utility function of the representative consumer is given by

[17] Detailed derivation of equation (8) is given in the Appendix.

[18] We assume that the representative household owns both skilled labour endowment and unskilled labour endowment. Even if we consider two representative households- one with skilled labour endowment and the other with unskilled labour endowment, aggregate demand functions for varieties would remain unchanged provided that their preferences are identical.

$$u(\tau) = \left[\int_0^n x(j)^\alpha dj\right]^{\frac{1}{\alpha}} \qquad \text{with } 0 < \alpha < 1 \qquad (12).$$

Here, $x(j)$ is the level of consumption of jth variety. This instantaneous utility function is of CES type. It satisfies all standard properties and is symmetric in its arguments. Maximizing the discounted present value of instantaneous utility given by equation (10) subject to the intertemporal budget constraint given by equation (11), we obtain the following optimality condition[19].

$$\frac{\dot{E}}{E} = r - \rho \qquad (13).$$

The aggregate demand function for jth variety can be derived as follows.

$$x(j) = p(j)^{-\varepsilon} \frac{E}{P^{1-\varepsilon}} \qquad (14).$$

Here $\varepsilon = \frac{1}{1-\alpha} > 1$ is the price elasticity of demand for the representative variety. Here, $p(j)$ is the price of the jth variety, E is the aggregate expenditure on all these varieties, and P is a price index defined as

$$P = \left[\int_0^n p(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}} \qquad (15).$$

Sector 1 produces each of these innovated products with unskilled labour as input; and labour-output coefficient in this sector is assumed to be unity. So the wage rate of unskilled labour is the marginal cost of production of each of these innovated varieties. The producer of each of these innovated varieties is a monopolist. So it charges a monopoly price of its product which is given by

$$P^S = \frac{1}{\alpha} W_U \qquad (16).$$

Here, P^S represents the price of the representative innovated variety and W_U stands for the wage rate of unskilled labour.

Sector 2, that produces varieties of imitated products with unskilled labour as the only input, faces a competitive market for each of those varieties; and hence charges a price equal to the marginal cost of production. Here also marginal cost is equal to the wage rate of unskilled labour because labour-output coefficient is assumed to be unity. So we have

$$P^U = W_U \qquad (17),$$

[19] Detailed derivations of equations (13) and (14) are given in the Appendix.

where, P^U is the price of the representative imitated variety.

Out of total n products, each of n^S products are sold at the price, P^S, and each of n^U products are sold at the price, P^U. Hence, using equations (4) and (5), equation (13) can be expressed as follows.

$$P = n^{\frac{1}{1-\varepsilon}}\left[\xi(P^S)^{(1-\varepsilon)} + (1-\xi)(P^U)^{(1-\varepsilon)}\right]^{\frac{1}{1-\varepsilon}} \quad (18).$$

Let x^S and x^U be levels of output of the representative varieties to be produced in sector 1 and sector 2, respectively. L^S and L^U represent skilled labour endowment and unskilled labour endowment respectively. Markets for each of these two types of labour are assumed to be competitive. So market clearing conditions of these two types of labour, who are perfectly mobile among their using sectors, are given by following two equations.

$$ag + l_m = L^S \quad (19);$$

and,

$$n^S x^S + n^U x^U = L^U \quad (20).$$

We assume free entry of firms of sector 1 into the R&D sector. The return from this R&D activity, denoted by v^S, is basically the value of the blue print; and this is equal to the discounted present value of profit of the producer of the representative innovated variety defined over the infinite time horizon. Under competitive equilibrium, return from this R&D activity must be equal to its cost; and, if W_S represents the wage rate of skilled labour in the R&D sector, then $\frac{W_S a}{n}$ is the cost of developing a blueprint because only skilled labour is used in the R&D sector. So in equilibrium

$$v^S = \frac{W_S a}{n} \quad (21).$$

Skilled labour is perfectly mobile between the R&D sector and the imitation preventing public good sector; and the level of outputs of both these sectors are expressed in same unit. So the skilled wage rates in these two sectors are also expressed in same units, and are equal in migration equilibrium. So W_S is the wage rate in the skilled labour market. We assume that $W_S > W_U$ in the initial equilibrium and also assume that comparative static effects are too small to reverse this inequality. We do so because our objective is to focus on the wage inequality problem. However, there is no guarantee that $W_S > W_U$ is always satisfied following comparative static effects.

Firms of sector 1 issue equities to finance their R&D investments. $\frac{\Pi^S}{v^S}$ represents the rate of dividend and $\frac{\dot{v}^S}{v^S}$ is the rate of growth of the value of the firm. Since m stands for the rate of imitation, $\left(\frac{\Pi^S}{v^S} + \frac{\dot{v}^S}{v^S} - m\right)$ is the net rate of return from investment in the stock market. This net rate of return should be equal to the interest rate obtained from the loan market. Hence we have

$$\frac{\Pi^S}{v^S} + \frac{\dot{v}^S}{v^S} = r + m \qquad (22).$$

Π^S is the profit of the representative firm in sector 1. All firms in sector 1, who produce innovated varieties, have to bear the cost of producing the public good as it protects imitation. This cost takes the form of lump sum tax imposed by the government. So Π^S is defined as follows.

$$\Pi^S = (1-\alpha)P^S x_S - \frac{W_S l_m}{n^S} \qquad (23).$$

Here, $W_S l_m$ is the cost of producing the imitation preventing public good because skilled labour is the only input in that sector; and this amount is taken by the government in the form of lump sum taxes.

3. Working of the model

The value of the firm, v^S, is normalized to unity following Lai (1998), Mondal and Gupta (2008), Gupta and Dutta (2011) etc. Hence,

$$v^S = 1 \qquad (24).$$

3.1. Rate of growth

Using equations (4), (19) and (24), we obtain[20]

[20] Detailed derivation is given in the Appendix.

$$g = \frac{L^S - (a)^{\frac{1}{(1-\beta)}}}{a} \qquad (25).$$

Equation (25) shows the constant rate of product development (growth) in this model; and this rate is independent of whether the economy is in the steady-state growth equilibrium or not. While deriving equation (25), we have never used equation (9) i.e., the steady-state equilibrium condition of this model. In Helpman (1993) or in Grossman and Helpman (1991a), the rate of product development is constant only in the steady-state growth equilibrium. Here the rate of expansion of varieties (rate of growth) is determined by values of some parameters like skilled labour endowment, the productivity parameter in the R&D sector and the labour elasticity parameter in the imitation preventing public good sector. We need appropriate restrictions on the values of those parameters to ensure that $g > 0$[21].However, equation (25) shows that g varies positively with L^S and inversely with a and β. Also, g is independent of change in L^U and b. Here b stands for the efficiency parameter of the imitation prevention of the public good. So we can establish the following proposition.

<u>PROPOSITION-1</u>: *An increase in skilled labour endowment raises the rate of growth (expansion of varieties) but a change in unskilled labour endowment or an improvement in the imitation prevention efficiency of the public good has no effect on it.*

3.2. <u>Rate of imitation</u>

Using equations (3), (5), (19) and (25), we obtain[22]

$$m = \frac{1}{(a)^{\frac{\beta}{(1-\beta)}} b} \qquad (26).$$

Equation (26) shows that imitation rate is independent of L^S. However, it changes with respect to change in other parameters, a, b and β.

So we can establish the following proposition.

[21] The rate of growth is positive if $L^S > (a)^{\frac{1}{(1-\beta)}}$.

[22] Detailed derivation is given in the Appendix.

PROPOSITION-2: *The long run rate of imitation is independent of changes in skilled labour endowment and unskilled labour endowment. An improvement in the imitation prevention efficiency of the public good lowers the imitation rate.*

3.3. The stability of steady-state equilibrium

Using equations (8), (25) and (26), we obtain

$$\dot{\xi} = \frac{L^S - (a)^{\frac{1}{(1-\beta)}}}{a} - \left(\frac{L^S - (a)^{\frac{1}{(1-\beta)}}}{a} + \frac{1}{(L^S - ag)^{\beta} b} \right) \xi \qquad (27).$$

In the steady-state growth equilibrium, $\dot{\xi} = 0$. Hence, the steady-state growth equilibrium value of ξ is given by

$$\xi^* = \frac{\frac{L^S - (a)^{\frac{1}{(1-\beta)}}}{a}}{\frac{L^S - (a)^{\frac{1}{(1-\beta)}}}{a} + \frac{1}{(L^S - ag)^{\beta} b}}.$$

Since ξ^* is a constant, then $\frac{n^S}{n^U} = \frac{\xi}{1-\xi}$ is also so. Hence, in the steady-state growth equilibrium, we have $\frac{\dot{n}^S}{n^S} = \frac{\dot{n}^U}{n^U} = \frac{\dot{n}}{n} = g$. This equation (27) shows that $\dot{\xi}$ is a negative function of ξ. So the steady state growth equilibrium is stable. If the economy initially starts with a higher (lower) fraction of goods not imitated, then that fraction falls (rises) over time and converges to its steady-state growth equilibrium value. We can establish the following proposition.

PROPOSITION-3: *The steady-state growth equilibrium is stable.*

In models of Helpman (1993), Grossman and Helpman (1991a) etc., the steady-state equilibrium is a saddle point because g is a constant in none of those models. In each of these models, we find another differential equation like

$$\dot{g} = g(g, \xi);$$

and the stability property of the dynamic equilibrium in that model is to be investigated by solving the time path of ξ and g simultaneously. In our model, equation (25) shows that $\dot{g} \equiv 0$; and so the stability property is analyzed using only the time path of ξ.

3.4. Interest rate:

Using equations (16), (17), (18) and (20), we obtain[23]

$$E = W_U L^U \left[\frac{\xi\alpha^{(\varepsilon-1)}+(1-\xi)}{\alpha^{\varepsilon}\xi+(1-\xi)}\right] \quad (28).$$

In the steady-state growth equilibrium, ξ takes a constant value and g is always a constant. Hence, from equations (21), (24) and (28), we have

$$\frac{\dot{v}^S}{v^S} + g = \frac{\dot{W}^S}{W^S} = \frac{\dot{W}^U}{W^U} = \frac{\dot{E}}{E}.$$

As, v^S, is normalized to unity; hence, $\dot{v}^S = 0$; and so we have

$$\frac{\dot{E}}{E} = g \quad (29).$$

Using equations (13) and (28) we have $r = \rho + g$; and then using equation (25), we can solve for r in the steady-state growth equilibrium. Obviously r and g behave in similar ways with respect to changes in parameters.

3.5. Wage inequality

Here, we define the ratio of skilled wage to unskilled, i.e., $\frac{W_S}{W_U}$ as Δ. Using equations (9), (13)-(18), (21)-(26) and (29), we derive[24] the ratio of skilled-unskilled wage as follows

$$\Delta = \frac{(1-\alpha)L^U}{a\alpha^{(1-\varepsilon)}\left(\rho + \frac{\frac{L^S}{a}\left\{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}+\frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}\right\}}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}}\right)\left(1-\frac{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}(1-\alpha^{\varepsilon})}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}+\frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}}\right)} \quad (30).$$

Here, $\frac{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}(1-\alpha^{\varepsilon})}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}+\frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}} < 1$ because $\alpha^{\varepsilon} < 1$ and $L^S > (a)^{\frac{1}{(1-\beta)}}$. So equation (30) ensures that $\Delta > 0$. This equation (30) shows how the skilled-unskilled wage ratio in the long run equilibrium varies with changes in different parameters. Here, Δ varies positively with L^U and b**.** The effect

[23] Derivation of equation (28) is given in the Appendix.
[24] Derivation of equation (30) is given in the Appendix.

of change in L^S on Δ is ambiguous. If the value of α is very large, then Δ varies inversely with respect to change in L^S[25]. This leads to the following proposition.

__PROPOSITION-4__: *(i) An increase in the level of unskilled labour endowment and/or an improvement in the imitation prevention efficiency of the public good raises the skilled-unskilled wage ratio*. (ii) *If the monopoly power of the representative firm in the innovated sector is very low, then an increase in skilled labour endowment lowers the skilled-unskilled wage ratio.*

We now provide intuitive explanations for this result. As unskilled labour endowment is increased, there is no effect on growth rate, imitation rate and on the demand for unskilled labour in sector 1 and sector 2. So unskilled wage rate falls and the skilled-unskilled wage ratio rises. Similarly, as the imitation prevention efficiency of the public good is improved, rate of imitation falls. This lowers the demand for unskilled labour because imitated goods are produced with unskilled labour. So the unskilled wage rate is also reduced; and thus the skilled-unskilled wage ratio is increased.

An increase in the skilled labour endowment has two effects. The direct effect implies a fall in the skilled wage rate. However, the innovation rate is also increased implying that more blue prints are produced in the R&D sector. So the proportion of innovated goods is increased and the proportion of imitated goods is reduced in the new steady state equilibrium. Unskilled labour moves from the imitated sector to the innovated sector. However, excess demand for unskilled labour in the innovated sector is less than its excess supply in the imitated sector. So the unskilled wage rate is also reduced. This is the indirect effect. So we have a net ambiguous effect on the skilled-unskilled wage ratio. If the monopoly power of each producer in the innovated sector is very low, then excess demand for unskilled labour in the innovated sector is almost same as its excess supply in the imitated sector. So the decrease in the skilled wage rate is more than the decrease in the unskilled wage rate in this special case.

Our results related to effects of the threat of imitation on skilled-unskilled wage ratio is interesting compared to the corresponding result obtained in Thoeing and Verdier (2003). In our model, an improvement in the efficiency of imitation preventing public good implies a reduction in the threat of imitation. This efficiency improvement lowers the relative demand

[25] Detailed analysis is given in the Appendix.

for unskilled labour and raises the skilled-unskilled wage ratio because production of imitated goods requires only unskilled labour and the change in the threat of imitation has no effect on the technology of producing innovated goods. In Thoeing and Verdier (2003), firms producing innovated products use skill intensive technology to meet the increased threat of imitation; and thus the relative demand for skilled labour is increased leading to an increase in the skilled-unskilled wage ratio when there is an increased threat of imitation.

3.6. <u>Effect on Welfare</u>

The instantaneous utility function of the representative household given by equation (12) is an index of social welfare because all households are identical here. Using equations (9), (12), (14) - (18), (25) and (26) we obtain following modified form of this utility function[26].

$$u = \frac{L^U}{2} n^{\left(\frac{1}{\varepsilon-1}\right)} \left\{ \frac{a}{\left(L^S - (a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}} b} \right\}^{\left(\frac{1}{\varepsilon-1}\right)} \frac{\left[\alpha^{(\varepsilon-1)} \left\{ \frac{\left(L^S - (a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}} b}{a} \right\} + 1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left[\alpha^{\varepsilon} \left\{ \frac{\left(L^S - (a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}} b}{a} \right\} + 1\right]} \quad (31).$$

Here, we normalize the utility function with respect to love for variety effect; and the normalized utility function is given by the following.

$$u^* = \frac{u}{n^{\left(\frac{1}{\varepsilon-1}\right)}} = \frac{L^U}{2} \left\{ \frac{1}{\left\{ \frac{\left(L^S - (a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}} b}{a} \right\} + 1} \right\}^{\left(\frac{1}{\varepsilon-1}\right)} \frac{\left[\alpha^{(\varepsilon-1)} \left\{ \frac{\left(L^S - (a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}} b}{a} \right\} + 1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left[\alpha^{\varepsilon} \left\{ \frac{\left(L^S - (a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}} b}{a} \right\} + 1\right]} \quad (32).$$

In Appendix, it is shown that if $\alpha = \frac{1}{2}$, then equation (32) implies that the nature of relationship between u^* and L^S or b depends on the value of $\left(L^S - (a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}} b$. If

[26] Detailed derivation of equation (31) is given in the Appendix.

$\left(L^S - (a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b > (<)2$ then u^* varies directly (inversely) with both L^S and/or b. Also, equation (32) implies a direct relationship between the unskilled labour endowment and the level of utility of the household. So we can establish the following proposition.

<u>PROPOSITION-5</u>: *In the steady state growth equilibrium, an increase in the level of unskilled labour endowment raises the level of social welfare; and with $\alpha = \frac{1}{2}$, an increase in skilled labour endowment and/or an improvement in the efficiency of imitation prevention of the public good raises (lowers) the welfare level if $\left(L^S - (a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b > (<)2$.*

Here the quantity of skilled labour that social institution employs is given by $l_m = (a)^{\frac{1}{(1-\beta)}}$. So l_m varies directly with β. Thus the welfare effect of an improvement in the efficiency of imitation prevention of the social institution is qualitatively similar to an increase in the level of skilled labour employment in that sector.

4. <u>Conclusion</u>

The present paper develops a dynamic product variety model to explain the skilled-unskilled wage inequality in the steady state growth equilibrium. We here introduce endogenous imitation and mainly focus on the role of social institution to control this endogenous imitation rate. This social institution produces a public good using skilled labour as the only input; and the rate of imitation varies inversely with the size of this public good sector. The cost of producing this public good is financed by the lump sum tax imposed on all firms producing innovated products. The R&D sector that develops blue-prints of new products also uses skilled labour. However, firms producing innovated and imitated goods use unskilled labour as the only input.

We derive many interesting results from this model. First, an increase in skilled labour endowment raises the rate of growth (expansion of varieties) but a change in unskilled labour endowment has no effect on it. Secondly, a change in skilled labour endowment or a change in unskilled labour endowment has no effect on the imitation rate in the steady-state equilibrium. An improvement in the imitation prevention efficiency of the public good lowers the imitation rate. Thirdly, an increase in the level of skilled (unskilled) labour endowment lowers (raises) the skilled-unskilled wage ratio. This result is similar to the corresponding result obtained in chapter 5 of Grossman and Helpman (1991a) but is different from the corresponding result obtained by Kiley (1999). In the context of an exogenous change in the skilled (unskilled) labour endowment, our result is also different from (similar to) the corresponding result obtained from chapter 6 of Grossman and Helpman (1991a)[27]. Fourthly, an improvement in the efficiency of imitation preventing social institutions raises the skilled-unskilled wage ratio in our model. Lastly, an increase in the level of unskilled labour endowment raises the level of social welfare but an increase in skilled labour endowment or an improvement in the efficiency of imitation prevention of the public good produces ambiguous effects. Different models available in the existing literature, while analyzing effects on wage inequality, do not analyse effects on welfare; and hence whether wage inequality and welfare move in same or opposite directions is not clear from their analysis.

However, our model fails to consider many important aspects of reality. We assume a closed economy and hence can not analyse the role of international trade on the skilled-unskilled wage inequality. The possibility of unemployment in any of the two labour markets is also ruled out; and both the labour markets are assumed to be competitive. Symmetry assumption in the utility function and the linearity assumption in all production functions are also simplifying ones. It may be a weak excuse to say that all models built on the Grossman and Helpman (1991a) product variety structure suffer from these common limitations.

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[27] Grossman and Helpman (1991a) have not derived these results. We derive these results from their model to compare those to our results.

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Appendix

Derivation of equation (8):

Differentiating both sides of equation (7), with respect to t, we obtain

$$\frac{\dot{\xi}}{\xi} = \frac{\dot{n}^S}{n^S} - \frac{\dot{n}}{n}$$

$$\Rightarrow \dot{\xi} = \left(\frac{\dot{n}}{n^S} - \frac{\dot{n}^U}{n^S} - \frac{\dot{n}}{n}\right)\xi$$

$$\Rightarrow \dot{\xi} = \frac{\dot{n}}{n^S}\frac{n^S}{n} - (g+m)\,\xi$$

$$\Rightarrow \dot{\xi} = g - (g+m)\,\xi \tag{A.1}$$

Equation (A.1) is same as equation (8) in the body of the paper.

<u>Derivation of equation (14):</u>

The consumer maximizes instantaneous utility function given by equation (12) subject to the instantaneous budget constraint which is given by

$$E = \int_0^n p(j)\,x(j)dj \tag{A.2}$$

So, the Lagrange function is given by

$$\mathcal{L} = \left[\int_0^n x(j)^\alpha dj\right]^{\frac{1}{\alpha}} + \lambda\left[E - \int_0^n p(j)\,x(j)dj\right] \tag{A.3}$$

where, λ is the Lagrangian multiplier.

The f.o.c.'s of utility maximization is given by

$$\left[\int_0^n x(i)^\alpha dj\right]^{\frac{1}{\alpha}-1} x(i)^{\alpha-1} = \lambda p(i) \tag{A.4}$$

and,

$$\left[\int_0^n x(j)^\alpha dj\right]^{\frac{1}{\alpha}-1} x(j)^{\alpha-1} = \lambda p(j) \tag{A.5}$$

Using equations (A.4) and (A.5), we obtain

$$\left[\frac{x(i)}{x(j)}\right]^{1-\alpha} = \frac{p(j)}{p(i)} \tag{A.6}$$

and from equation (A.6), we obtain

$$\frac{\widehat{\left[\frac{x(i)}{x(j)}\right]}}{\widehat{\left[\frac{p(j)}{p(i)}\right]}} = \frac{1}{1-\alpha}$$

$$\Rightarrow \varepsilon = \frac{1}{1-\alpha}. \tag{A.7}$$

This ε is the price elasticity of demand for the representative variety.

Multiplying both sides of equation (A.5) by $x(j)$ and summing over all j, we obtain

$$\left[\int_0^n x(j)^\alpha dj\right]^{\frac{1}{\alpha}} = \lambda \int_0^n p(j)\,x(j)dj$$

$$\Rightarrow \lambda = \frac{\left[\int_0^n x(j)^\alpha dj\right]^{\frac{1}{\alpha}}}{\int_0^n p(j)x(j)dj} = \frac{1}{P} \quad \text{(A.8).}$$

Finally, using equations (A.2), (A.5) and (A.8), we obtain

$$x(j) = p(j)^{-\varepsilon} \frac{E}{P^{1-\varepsilon}} \quad \text{(A.9).}$$

Equation (A.9) is same as equation (14) in the body of the paper.

Derivation of equation (13):

Substituting the demand functions given by (14) into equation (12) and then using equation (15), we obtain the indirect utility function

$$log(u) = log(E) - log(P) \quad \text{(A.10).}$$

Differentiating both sides of equation (11), we obtain

$$\dot{A} = I - E + rA \quad \text{(A.11).}$$

The current value Hamiltonian corresponding to this dynamic optimization problem is given by

$$H = log(u) + h(I - E + rA)$$

$$\Rightarrow H = [log(E) - log(P)] + h(I - E + rA)$$

Here h is the co-state variable. The first order optimality condition with respect to E is given by

$$\frac{\partial H}{\partial E} = \frac{\partial u}{\partial E} - h = 0$$

$$\Rightarrow \frac{1}{E} = h$$

$$\Rightarrow \frac{\dot{h}}{h} = -\frac{\dot{E}}{E} \quad \text{(A.12).}$$

The equation motion of the co-state variable, h, should satisfy the following differential equation along the optimal path.

$$\frac{\dot{h}}{h} = r - \rho \quad \text{(A.13).}$$

Using equations (A.12) and (A.13), we obtain

$$\frac{\dot{E}}{E} = r - \rho \quad \text{(A.14).}$$

Equation (A.14) is same as equation (13) in the body of the paper.

Derivation of equation (25):

Using equations (4) and (24), we obtain

$$l_m = (a)^{\frac{1}{(1-\beta)}} \tag{A.15}$$

Using equations (19) and (A.15), we have

$$L_S - ag = (a)^{\frac{1}{(1-\beta)}}$$

$$\Rightarrow g = \frac{L^S - (a)^{\frac{1}{(1-\beta)}}}{a} \tag{A.16}$$

Equation (A.16) is same as equation (25) in the body of the paper.

<u>Derivation of equation (26):</u>

Using equations (3) and (5), we obtain

$$m = \frac{1}{{l_m}^{\beta} b} \tag{A.17}$$

Using equations (A.15) and (A.17), we have

$$m = \frac{1}{(a)^{\frac{\beta}{(1-\beta)}} b} \tag{A.18}$$

Equation (A.18) is same as equation (26) in the body of the paper.

<u>Derivation of equation (28):</u>

From equations (14) and (20), we obtain

$$\frac{n^S (P^S)^{-\varepsilon} E}{P^{1-\varepsilon}} + \frac{n^U (P^U)^{-\varepsilon} E}{P^{1-\varepsilon}} = L^U \tag{A.19}$$

Using equations (7) and (A.19), we obtain

$$\frac{En}{P^{1-\varepsilon}} [\xi (P^S)^{-\varepsilon} + (1-\xi)(P^U)^{-\varepsilon}] = L^U \tag{A.20}$$

Using equations (18) and (A.20), we obtain

$$\frac{E\left[\xi (P^S)^{-\varepsilon} + (1-\xi)(P^U)^{-\varepsilon}\right]}{\left[\xi (P^S)^{(1-\varepsilon)} + (1-\xi)(P^U)^{(1-\varepsilon)}\right]} = L^U$$

$$\Rightarrow \frac{E}{P^U}\left[\frac{\left[\xi\left(\frac{P^S}{P^U}\right)^{-\varepsilon}+(1-\xi)\right]}{\left[\left[\xi\left(\frac{P^S}{P^U}\right)^{(1-\varepsilon)}+(1-\xi)\right]\right]}\right] = L^U \qquad \text{(A.21).}$$

Using equations (16), (17) and (A.21), we obtain

$$\frac{E}{W_U}\left[\frac{\left[\xi\left(\frac{1}{\alpha}\right)^{-\varepsilon}+(1-\xi)\right]}{\left[\left[\xi\left(\frac{1}{\alpha}\right)^{(1-\varepsilon)}+(1-\xi)\right]\right]}\right] = L^U$$

$$\Rightarrow E = W_U L^U \left[\frac{\xi\alpha^{(\varepsilon-1)}+(1-\xi)}{\alpha^{\varepsilon}\xi+(1-\xi)}\right] \qquad \text{(A.22)}$$

Equation (A.22) is same as equation (28) in the body of the paper.

<u>Derivation of equation (30):</u>

Using equations (22), (23) and (24), we obtain

$$(1-\alpha)P^S x_S = r + m + \frac{l_m}{a\xi} \qquad \text{(A.23).}$$

Using equations (13), (14), (28) and (A.23), we get

$$\frac{(1-\alpha){P^S}^{(1-\varepsilon)}E}{P} = g + \rho + m + \frac{(L_S - ag)}{a\xi}$$

$$\Rightarrow \frac{(1-\alpha){P^S}^{(1-\varepsilon)}E}{P} = \frac{L^S}{a\xi} + \rho \qquad \text{(A.24).}$$

Using equations (18) and (A.24), we get

$$\frac{(1-\alpha){P^S}^{(1-\varepsilon)}E}{n\left[\xi\left(P^S\right)^{(1-\varepsilon)}+(1-\xi)(P^U)^{(1-\varepsilon)}\right]} = \frac{L^S}{a\xi} + \rho$$

$$\Rightarrow \frac{(1-\alpha)E}{n\left[\xi+(1-\xi)\alpha^{(1-\varepsilon)}\right]} = \frac{L^S}{a\xi} + \rho$$

$$\Rightarrow \frac{(1-\alpha)E}{n\alpha^{(1-\varepsilon)}\left[\xi\alpha^{(\varepsilon-1)}+(1-\xi)\right]} = \frac{L^S}{a\xi} + \rho \qquad \text{(A.25).}$$

Using equations (28) and (A.25), we obtain

$$\frac{(1-\alpha)W_U L^U}{n\alpha^{(1-\varepsilon)}[\alpha^{\varepsilon}\xi+(1-\xi)]} = \frac{L^S}{a\xi} + \rho$$

$$\Rightarrow \frac{(1-\alpha)W_S L^U}{n\alpha^{(1-\varepsilon)}\Delta[\alpha^{\varepsilon}\xi+(1-\xi)]} = \frac{L^S}{a\xi} + \rho \qquad \text{(A.26);}$$

where, $\Delta = \frac{W_S}{W_U}$

Using equations (24) and (A.26), we get

$$\frac{(1-\alpha)L^U}{a\Delta\alpha^{(1-\varepsilon)}[\alpha^\varepsilon\xi+(1-\xi)]}=\frac{L^S}{a\xi}+\rho$$

$$\Rightarrow \Delta=\frac{(1-\alpha)L^U}{a\alpha^{(1-\varepsilon)}\left(\frac{L_S}{a}+\rho+m\right)[1-\xi(1-\alpha^\varepsilon)]} \qquad \text{(A.27).}$$

Using equations (9), (26) and (A.27), we obtain

$$\Delta=\frac{(1-\alpha)L^U}{a\alpha^{(\varepsilon-1)}\left(\frac{L^S\left(g+\frac{1}{(L^S-ag)^\beta b}\right)}{ag}+\rho\right)\left(1-\frac{g(1-\alpha^\varepsilon)}{g+\frac{1}{(L^S-ag)^\beta b}}\right)} \qquad \text{(A.28).}$$

Using equations (25) and (A.28), we obtain

$$\Delta=\frac{(1-\alpha)L^U}{a\alpha^{(1-\varepsilon)}\left(\rho+\frac{\frac{L^S}{a}\left\{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}+\frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}\right\}}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}}\right)\left(1-\frac{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}(1-\alpha^\varepsilon)}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}+\frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}}\right)} \qquad \text{(A.29).}$$

Equation (A.29) is same as equation (30) in the body of the paper.

Relationship between skilled-unskilled wage inequality and the skilled labour endowment:

In the denominator of the expression of Δ; $\left(\rho+\frac{\frac{L^S}{a}\left\{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}+\frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}\right\}}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}}\right)$ varies positively with L^S and $\left(1-\frac{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}(1-\alpha^\varepsilon)}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}+\frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}}\right)$ varies negatively with L^S. If, If the value of α is very large, then $\left(1-\frac{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}(1-\alpha^\varepsilon)}{\frac{L^S-(a)^{\frac{1}{(1-\beta)}}}{a}+\frac{1}{(\alpha)^{\frac{\beta}{(1-\beta)}}b}}\right)$ is very small and Δ varies inversely with respect to change in L^S.

Derivation of equation (31):

From equation (A.10), we obtain

$$u = \frac{E}{P} \tag{A.30}$$

From equations (17) and (A.22), we obtain

$$E = P^U L^U \left[\frac{\xi \alpha^{(\varepsilon-1)} + (1-\xi)}{\alpha^{\varepsilon}\xi + (1-\xi)}\right] \tag{A.31}$$

Using equations (16), (17) and (18), we obtain

$$P = n^{\frac{1}{1-\varepsilon}} P^U \left[\xi \alpha^{(\varepsilon-1)} + (1-\xi)\right]^{\frac{1}{1-\varepsilon}} \tag{A.32}$$

Using equations (A.30), (A.31) and (A.32), we obtain

$$u = \frac{L^U}{n^{\frac{1}{1-\varepsilon}}} \frac{\left[\xi \alpha^{(\varepsilon-1)} + (1-\xi)\right]}{\left[\alpha^{\varepsilon}\xi + (1-\xi)\right]\left[\xi \alpha^{(\varepsilon-1)} + (1-\xi)\right]^{\frac{1}{1-\varepsilon}}}$$

$$\Rightarrow u = L^U n^{\left(\frac{1}{\varepsilon-1}\right)} (1-\xi)^{\left(\frac{1}{\varepsilon-1}\right)} \frac{\left[\left\{\frac{\xi}{(1-\xi)}\right\}\alpha^{(\varepsilon-1)} + 1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left[\left\{\frac{\xi}{(1-\xi)}\right\}\alpha^{\varepsilon} + 1\right]} \tag{A.33}$$

Using equations (9) and (A.33), we have

$$u = \frac{L^U}{2} n^{\left(\frac{1}{\varepsilon-1}\right)} \left(\frac{m}{g+m}\right)^{\left(\frac{1}{\varepsilon-1}\right)} \frac{\left[\alpha^{(\varepsilon-1)}\left\{\frac{g}{m}\right\} + 1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left(\alpha^{\varepsilon}\left\{\frac{g}{m}\right\} + 1\right)} \tag{A.34}$$

Using equations (25), (26) and (A.34), we have

$$u = \frac{L^U}{2} n^{\left(\frac{1}{\varepsilon-1}\right)} \left\{\frac{1}{\left\{\frac{\left(L^S - (a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}} b}{a}\right\} + 1}\right\}^{\left(\frac{1}{\varepsilon-1}\right)} \frac{\left[\alpha^{(\varepsilon-1)}\left\{\frac{\left(L^S - (a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}} b}{a}\right\} + 1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left[\alpha^{\varepsilon}\left\{\frac{\left(L^S - (a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}} b}{a}\right\} + 1\right]} \tag{A.35}$$

Equation (A.35) is same as equation (31) in the body of the paper.

<u>Derivation of the relationship between consumers' utility and the parameters:</u>

$$\frac{u}{n^{\left(\frac{1}{\varepsilon-1}\right)}}=\frac{L^U}{2}\left\{\frac{1}{\left\{\frac{\left(L^S-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}\right\}+1}\right\}^{\left(\frac{1}{\varepsilon-1}\right)}\frac{\left[\alpha^{(\varepsilon-1)}\left\{\frac{\left(L^S-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}\right\}+1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left[\alpha^{\varepsilon}\left\{\frac{\left(L^S-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}\right\}+1\right]}$$

Suppose that

$$\frac{\left(L^S-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}b}{a}=C$$

Then we have

$$\frac{u}{n^{\left(\frac{1}{\varepsilon-1}\right)}}=\frac{L^U}{2}(C+1)^{(1-\varepsilon)}\frac{\left[\alpha^{(\varepsilon-1)}C+1\right]^{\left(\frac{\varepsilon}{\varepsilon-1}\right)}}{\left[\alpha^{\varepsilon}C+1\right]}$$

$$\frac{d\left(\frac{u}{n^{\left(\frac{1}{\varepsilon-1}\right)}}\right)}{dC}=\frac{L^U(C+1)^{(-\varepsilon)}\left(\alpha^{(\varepsilon-1)}C+1\right)^{\left(\frac{1}{\varepsilon-1}\right)}}{2(\alpha^{\varepsilon}C+1)^2}\left[(1-\varepsilon)\left(\alpha^{(\varepsilon-1)}C+1\right)(\alpha^{\varepsilon}C+1)+\alpha^{\varepsilon}(C+1)\left\{\alpha^{(\varepsilon-2)}C+\frac{1}{\alpha^2}-\alpha^{(\varepsilon-2)}C-1\right\}\right]$$

Let $\alpha=\frac{1}{2}$. Then $\varepsilon=2$;

and,

$$\left[(1-\varepsilon)\left(\alpha^{(\varepsilon-1)}C+1\right)(\alpha^{\varepsilon}C+1)+\alpha^{\varepsilon}(C+1)\left\{\alpha^{(\varepsilon-2)}C+\frac{1}{\alpha^2}-\alpha^{(\varepsilon-2)}C-1\right\}\right]=\frac{C-2}{8}.$$

We find that

$$\frac{dC}{dL^S}=\frac{(a)^{\frac{\beta}{(1-\beta)}}b}{a}>0;\text{ and }\frac{dC}{db}=\frac{\left(L^S-(a)^{\frac{1}{(1-\beta)}}\right)(a)^{\frac{\beta}{(1-\beta)}}}{a}>0.$$

So, if $C>(<)2$, then

$$\frac{d\left(\frac{u}{n^{\left(\frac{1}{\varepsilon-1}\right)}}\right)}{dL^S}=>(<)0;\text{ and }\frac{d\left(\frac{u}{n^{\left(\frac{1}{\varepsilon-1}\right)}}\right)}{db}=>(<)0.$$