Targeting Informative Messages to a Network of Consumers

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Abstract

This paper considers duopolistic firms targeting informative messages to consumers who share information locally with their neighbors in the network graph. A monopolist targets efficiently by sending messages to a parsimonious set of nodes that leave the whole population informed either directly or by word-of-mouth. A duopolist faces a tradeoff between this efficient targeting and possible preemption by a competitor's message. Under gentle price competition, duopolists saturate the network when sending messages is cheap, and target sets similar to the monopolist's when sending messages is costly. Under fierce price competition, duopolists' messages divide the network in an intermingled patchwork segmentation.

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1   Introduction

Word-of-mouth is an important way in which consumers can learn about a product, and there is evidence that the existence and nature of word-of-mouth have a tangible impact on consumer behavior (Godes and Mayzlin (2004), Godes and Mayzlin (2009)). Communication partners, however, are not selected at random. The structure of such communication - who talks to whom - is therefore of interest to a firm or other entity that would like to spread information among those consumers: when some consumers learn a piece of information, it can be amplified in a particular way by retransmission to others they interact with, and so informing a given set of people need not require *directly* informing them. In particular, the problem for a direct marketer who can send messages to consumers is affected by the extent and structure of word-of-mouth among the population who will receive the messages. In this spirit, Zubcsek and Sarvary (2011) considers the problem for a firm (or firms) that can choose a number of direct marketing messages to send to a population of consumers who are arranged in some network whose links capture channels along which information can flow; Galeotti and Goyal (2009) considers both this case and the case in which the firm can target individuals of specific degree. These contributions uphold the notion that incorporating information on the structure of the network enables the firm to improve the efficiency of its marketing strategy.

However, as technology advances and the structure of communication networks becomes ever more observable, the targeting of such campaigns can become still more specific - and its consequences more tangible. The graph of connections among users of Twitter, for example, is, despite its complexity, public knowledge; the retransmission through 'retweeting' of a message sent by a business to its followers can be readily observed. Rather than simply deciding how many messages to send to potential customers, knowing that an opaque process of word-of-mouth could follow, marketers are gaining an unprecedented ability to send messages to precisely chosen locations in a well-defined communication network. Indeed, early in the life of Twitter, businesses enthusiastically adopted it as a marketing tool;[1] business schools including those at Harvard and Columbia now offer courses in social media marketing.[2] There

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[1] "Marketing Small Businesses With Twitter" by Claire Cain Miller, *New York Times*, July 22, 2009
[2] "B-Schools All A-Twitter Over Social Media" by Sommer Saadi, *Bloomberg Businessweek*, July 26, 2010

are surely a variety of motivations for the use of social media by a marketer: to engage customers as de facto spokespeople for a product or to engender loyalty, for example. One particular motivation that this paper will focus on is linked to the increasing transparency of communication: if a firm seeks to disseminate some new piece of information and can *observe* the graph of communication channels among consumers, to which individuals should it send that information?

This question of targeted dissemination is familiar from an extensive literature on the targeting of 'influentials' or 'opinion leaders' in marketing a product, dating at least to the seminal study of the targeting by pharmaceutical marketers of influential physicians by Coleman, Katz, and Menzel (1966).[3] Following Rogers and Cartano (1962), Iyengar, Van den Bulte, and Valente (2011) taxonomizes the identification of such individuals as possible via self-designation, other-designation or 'sociometric' techniques designed to reverse-engineer the structure of network of influence to identify 'central' individuals. The model below speaks to this third means; consumers are assumed homogeneous except for their position on the network in order to isolate the effect of network structure on the pattern of who should be targeted directly in various competitive environments.

These targeting approaches are naturally constrained by the fact that a network graph is highly complex, and so identifying the 'right' locations to target is correspondingly difficult. Indeed, a rich literature in computer science has considered the closely related topic of the value of capturing a consumer when they can go on to influence others recursively across network links (Domingos and Richardson (2001), Richardson and Domingos (2002), Kempe, Kleinberg, and Éva Tardos (2003)); such problems in graph coverage are known to be NP-hard (Garey and Johnson (1979)). This paper proposes a theoretical model that captures key features of the dissemination problem in situations in which communication patterns in the relevant network are understandable by the decision-maker. Settings that can be literally captured by the model developed in the present paper must therefore operate over 'small' networks in order to be tractable. However, this can mean a small number of individuals (as in the Coleman, Katz, and Menzel (1966) study which constructed a picture of the influence

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[3]A question re-analyzed in Van den Bulte and Lilien (2001), which considers in detail the confounding effects of advertising volume on the conclusion of the original study.

network among physicians local to the target area), or, exploiting that the model can naturally be interpreted in the abstract, interpreting the network with a coarser definition of a node. For example, say a software company is marketing a new piece of software that is designed for video game designers. The *firm-level* network of collaborative relationships among game developers could capture for the software company relevant channels of communication along which news of its product will spread from users to the uninformed. Then the number of individuals in the relevant market may be large, but a practically relevant and tractable communication network can nevertheless be identified.

In the model, a firm (variously a monopolist or one half of a duopoly) seeks to disseminate some piece of information, perhaps about a special offer or a new product, to a population of initially uninformed consumers,[4] who are arranged in a publicly observable social network. The firm can choose a set of consumers to whom it will send (costly) direct marketing messages containing the piece of information. There are two ways for individual consumers to learn: a consumer learns the information today if the firm directly informs the consumer, or learns the information tomorrow if they are linked in the network to someone who was directly informed.

Several contributions that analyze similar settings to this model raise doubts that it is best to target the most connected individuals (for example Tucker (2008), Watts and Dodds (2007)), and these doubts are further validated here. For the case of a monopolist, a strategy that targets the most connected individuals overlooks any weakly-interconnected regions of the network; yet, a strategy that sends messages to randomly chosen locations does not acknowledge that well-connected individuals can generate the most word-of-mouth. To maximize the impact of direct marketing messages instead requires a more subtle targeting strategy that acknowledges both the outsized influence of the well-connected *and* the existence of more barren regions. In the focal case of the model in which information travels at most one degree from its recipient, the monopolist does best by targeting messages to the set of nodes corresponding to the graph-theoretic concept of the *minimum dominating set*. Targeting this set maximizes the impact of a given number of messages and generally is very

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[4]Advertising here therefore performs a role as in (Butters 1977), and 'social influence' operates purely as information transfer, as set forth in Katz and Lazarsfeld (1955).

different than targeting the well-connected. The spirit of this result is readily generalizable upward to richer assumptions on the prevalence and distance of information transmission. One implication is that even when individuals differ only in their network position, those who appear ex-post 'influential' - as a result of being targeted first - are located in diverse, almost idiosyncratic positions that can be sensitive to small changes in network structure.

The competitive case brings a new tension for firms. There remains an incentive to rely on word-of-mouth rather than on costly direct marketing, but there may also be an incentive to avoid an individual learning about the competitor's product first. To assess the impact of this competitive pressure, below the competitive case is further subdivided into two according to the extent to the loss for one firm associated with an individual knowing also of the other firm. In both cases the firm does better whenever it reaches a consumer first, but the value obtained from reaching a consumer at the same time as a competitor can be variously half of the first-arriver value or nil.

When competitive pressure is high, the cost of sharing the market outweighs the benefit to choosing the word-of-mouth-maximizing set, and firms are driven to send messages to distinct regions of the network. However, the existence of word-of-mouth means that this segmentation takes a specific form, which in the two-period case corresponds to the firms sending messages to *disjoint dominating sets* of the network graph. This means that the two firms segment the network in an overlapping 'patchwork' rather than, say, dividing the network east-west. This is because an east-west division would give either firm incentive to send fewer direct messages and exploit word-of-mouth, and thus in turn give its competitor incentive to preempt them in that region. In this sense competitive pressure damages each firm's ability to exploit word-of-mouth, since doing so would expose them to preemption.

In the case with lower competitive pressure, the cost of sharing the market is low relative to the benefit of targeting the set of consumers most efficient in generating word-of-mouth, so equilibria can be symmetric, but their nature depends on the cost of sending a message. When the cost of marketing is high, the firms restrict their marketing volume in a similar way as would a monopolist in order to exploit word-of-mouth. In particular, neither informs a *redundant set* of consumers, and each consumer hears of at least one firm either directly

or indirectly. This means that no message is sent purely in order to capture the consumer that receives it; each message must have a word-of-mouth effect that is irredundant to that generated by other messages. If competitive pressure is low and the cost of sending a message is low, sending a message that captures only the consumer that receives it is profitable even if it means 'sharing' that consumer. There is then an 'arms-race' effect: in the unique equilibrium sees both firms engage in a mass-marketing campaign and send messages to everyone.

The case in which potential price competition drives perfect segmentation has parallels in the partly related settings considered in Banerji and Dutta (2009) and Roy (2000). Banerji and Dutta (2009) consider a model of network externalities in a market with two producers. Their framework assumes network benefits to adopting a technology are present only across links in the network graph and not over the whole population, and find that local network externalities permits strong market segmentation and therefore positive profits, even when the firms are a priori identical and are Bertrand competitors. Roy (2000) analyzes a model in which two firms first target information to a set of consumers and then engage in price competition, finding that perfect segmentation and pure local monopoly emerges. The present model confirms a similar outcome for the case with word-of-mouth, with the associated particular segmentation structure. Another related contribution which considers a competitive targeting problem is Iyer, Soberman, and Villas-Boas (2005), where consumers are heterogeneous in their idiosyncratic preference for each firm's product, and targeting is by type.

The remainder of the paper proceeds as follows. Section 2 outlines the framework of the model. Section 3 discusses the monopolistic case, and its Proposition 1 establishes the solution to the monopolist's problem. Section 4 discusses the duopolistic case. Proposition 2, establishes equilibria in the case with high intensity of competition, and Propositions 3 and 4 equilibria in the case with low intensity of competition. Section 5 considers implications of the suggestive results of the model for the structure of industries that function as information intermediaries between firms and consumers. Section 6 concludes.

2 Model

There is a set $A = \{1, ..., n\}$ of consumers.[5] Each consumer is a node in an undirected and connected[6] graph (A, g), where g is a real-valued $n \times n$ matrix in which g_{ij} represents the relationship between consumers i and j.[7] The graph represents the *social network* across which information can flow among consumers. $g_{ij} = 1$ if there is a link between i and j and 0 otherwise; since the graph is undirected $g_{ij} = g_{ji}$ for all i and j, and $g_{ii} = 0$ for all i. The *network size* is n, and since A is assumed fixed, for convenience denote the graph g.

The *open neighborhood* of consumer i is $N(i) = \{j : g_{ij} = 1\}$, the set of consumers that i is linked to. The *closed neighborhood* of i is $N[i] = N(i) \cup \{i\}$. Let $S \subseteq A$ be some set of consumers and let $s = |S|$ be its cardinality. The open neighborhood of S is $N(S) = \cup_{i \in S} N(i)$. Let $\Omega(S) = N(S) - S$; this is the set of consumers in the neighborhood of S who are not themselves in the set S. For notational consistency, let $\Omega(i) \equiv N(i)$. Following this, let $\omega(i) = |\Omega(i)|$ denote the number of neighbors to consumer i (equivalent to i's degree), and similarly let $\omega(S) = |\Omega(S)|$. Some $l \in S$ has a *private neighbor* outside S if there is some vertex m in the open neighborhood of l that is not in the open neighborhood of any other vertex in S (following, for example, Fellows, Fricke, Hedetniemi, and Jacobs (1994)).

There are two firms indexed $k = 1, 2$ which produce a homogeneous product at zero cost (below we also consider a baseline case in which there is only a single firm). Let there be two periods, $t = 1, 2$. Initially consumers do not know that either firm's product exists, but firms have the chance to inform consumers: each firm can observe the social network g and simultaneously chooses at $t = 1$ a set of consumers $S_k \subseteq A$ to whom it will send a *direct marketing message* at a cost c per message.[8] All $i \in S_k$ (those consumers who receive a direct marketing message from firm k) learn at $t = 1$ that firm k's product exists. All consumers $i \in \Omega(S_k)$ (those consumers who are linked to at least one consumer who received a direct

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[5]As mentioned above, the setting in question will define the level at which a network can be defined over the relevant market. Depending on the setting, we may thus interpret these "consumers" as individuals or as larger clusters.

[6]That the graph is connected is without loss of generality, since the analysis of an unconnected graph will be identical to the separate analysis of its connected components.

[7]Notation for the graph follows convention, as in Jackson (2008).

[8]The constant cost per message follows Zubcsek and Sarvary (2011) and is a conservative assumption; in many settings this will be convex in s_k.

marketing message) learn at $t = 2$ that firm k's product exists. This implies non-optional and costless information transmission along links of the social network. Information travels only one degree, mirroring Zubcsek and Sarvary (2011).

The payoff earned by firm k depends on (i) the number of consumers who learn about its product, and (ii) whether those consumers also know about the other firm's product. Assume that the firm k earns some value v from a consumer who learns of k before she learns of l, and zero from a consumer who never learns of k. The analysis below considers separately two cases: one in which firm k earns value $\frac{1}{2}v$ from a consumer who learns of both firms simultaneously, and one in which firm k earns a value 0 from such a consumer. One motivation for the former case is that two firms 'share' consumers who know about both firms; one motivation for the latter case is that two firms compete destructively in some way over consumers who know about both firms. These two cases thus follow the parameter in Zubcsek and Sarvary (2011) representing 'intensity of competition'.

3   The monopolistic case

First consider the case in which there exists only a single firm m. Let the firm earn a payoff according to the number of consumers who learn at any time about the product, so that when the monopolist chooses a set S_m, the firm's payoff is given by

$$\pi(S_m) = (s_m + \omega(S_m))v - s_m c. \tag{3.1}$$

Assume $v > c$. The solution to the monopolist's problem is given in the following result:

Proposition 1. *A payoff-maximizing S_m^* for the monopolist is a* minimum dominating set *of g. That is:*

$$S_m^* \cup \Omega(S_m^*) = A \tag{3.2}$$

$$\nexists S' : S' \cup \Omega(S') = A, s' < s_m^* \tag{3.3}$$

The proof (and others to follow) appears in Appendix A. 3.2 says that S_m^* is a *dominating*

set of the graph g. This is some set so that all consumers are either in the set or its neighborhood. 3.3 says that S_m^* is a *minimum dominating set* of g. This means that there does not exist a dominating set with a lower cardinality than S_m^*. The result therefore states that the best strategy for the monopolist is to choose the smallest (equivalently cheapest) set of consumers such that all consumers will learn about the firm's product. The cardinality of such a set in a given network g is called the *domination number* of the graph and is denoted $\gamma(g)$; the number of messages sent by a monopolist is therefore bounded above by this number. By Ore (1962), the domination number of a connected graph is at most half of the number of vertices, but in general the bound will be tighter. Denote (for later reference) the set of minimum dominating sets in g by $MDS(g)$.

Two implications of this result are that the ability to perfectly target is valuable, in the sense that (i) the firm does better than in the case without any targeting, in which it chooses a number of messages to send to random nodes, and (ii) the firm does better than in the case with targeting based on some non-locational statistics like the degree of each node. To see the spirit of (i), consider a network structure as in Figure 1. In this network there are

Figure 1: Line network

four minimum dominating sets: four ways to locate two messages such that all consumers learn about the product. When the monopolist targets such a set, it pays $2c$ and earns $4v$. If, however, it cannot perfectly target messages and sends two messages at random, the firm again pays $2c$ but now earns (in expectation) only $\frac{11}{3}v$, since there is a probability $\frac{1}{3}$ that the messages arrive so that one consumer never learns about the product. The wedge between the payoff to optimal targeting and random targeting represents a premium that the firm would be willing to pay to discover the precise network structure, or for a service that could perfectly target messages over a service that randomly targeted messages.

On (ii), consider the network in Figure 2. Sending messages to the minimum dominating
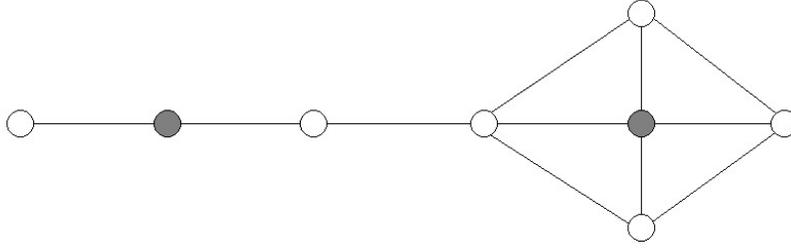
Figure 2: Network with minimum dominating set marked

set (the shaded nodes) results in all eight consumers learning about the firm's product at a cost of only 2 messages. Say instead that the firm sent the same number of messages but used a rule-of-thumb that specified targeting the most well-connected consumers. Then the firm would send messages to two of the consumers with four neighbors; this results in at most six consumers learning about the firm's product. Targeting that maximizes the impact of a given number of messages cannot generally be achieved by such rules. When targeting is possible, knowing the precise network structure is more valuable than knowing some summary statistic like the degree of each node.

4 The duopolistic case

Now return to the duopolistic case. The two firms $k = 1, 2$ observe g and simultaneously choose a set S_k of consumers to whom they will send direct marketing messages. As outlined above, we will consider separately two models that differ in the intensity of competition between the two firms over 'common' consumers - those who learn about both firms simultaneously.

4.1 High intensity of competition over common consumers

First we consider a setting in which the intensity of competition is 'high': firm k earns a value v per consumer who learns about firm k before they learn about firm l, and 0 per consumer who either learns of both firms simultaneously or does not learn about firm k at all. This

captures a situation in which a firm benefits from getting to a given consumer first, but that competition nullifies this benefit when the firms arrive at the same time.

The following result characterizes Nash equilibria in this one-shot, simultaneous move game played between the two firms.

Proposition 2. *In the high intensity of competition case, the pair (S_1, S_2) is a Nash equilibrium in pure strategies if and only if S_1 and S_2 are disjoint dominating sets of the network graph and $S_1 \cup S_2 = A$.*

This outcome is one of perfect segmentation: although by $t = 2$ all consumers have learned of both firms, each firm enjoys one period of sole capture of those consumers to whom it sent messages. This is not surprising: the firms will never each send a message to any common consumer in equilibrium, since then both pay c and receive 0 for that consumer; also, all consumers must receive one message in equilibrium, since if not any one firm could earn v and pay only c to send a message to such a consumer.

However, the segmentation has a striking feature: the firm's chosen targets are intermingled throughout the network. Consider again the simple line network. Figure 3a shows a
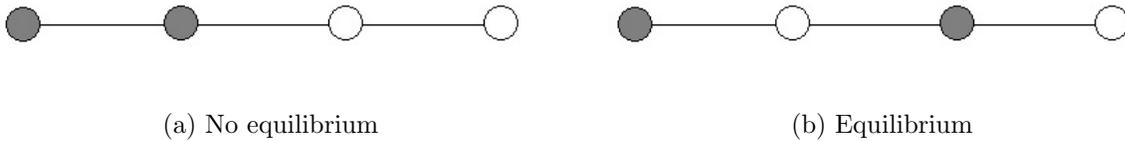

(a) No equilibrium   (b) Equilibrium

Figure 3: Illustrating Proposition 2

'geographic' segmentation pattern that has each firm taking a 'side' of the network: one firm sends messages to the shaded consumers, the other to the unshaded; this is not an equilibrium. Figure 3b shows a different segmentation pattern (disjoint dominating sets) that does constitute an equilibrium.[9] An important feature of this equilibrium pattern is that each consumer learns of one firm today and of the other firm tomorrow by word-of-mouth.

Why should this be? This is an interplay of the two competing incentives for each firm. Neither firm will send a message to a consumer that would otherwise have learned of the firm

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[9]Not that this is certainly not unique, since there are many pairs of disjoint dominating sets in this example.

by word-of-mouth before the other firm reached them: this is the opportunity afforded by word-of-mouth to reduce direct marketing effort. All consumers must receive some message, however, since intense competition means that is is certainly better for a single firm to send a message that results in sole capture of a consumer than to leave that consumer unserved *or* to 'share' that consumer by word-of-mouth. Informally, word-of-mouth drives each firm to be parsimonious with its messages in a given local area of the network, but this parsimony drives the competing firm to send messages that fill in these gaps. The paradoxical consequence is that both firms are unable to exploit word-of-mouth in equilibrium, since in doing so they are exposed to preemption.

To see this concretely, observe that in the case of Figure 3a, either firm could reduce the number of messages it sends by 1 but still remain the first firm to reach 2 consumers; since this yields a higher payoff than the proposed pattern of segmentation, such a pattern is not an equilibrium. Word-of-mouth thus polices segmentation: in the equilibrium pattern in Figure 3b, neither firm can reduce the number of messages it sends without giving up the first shot at a consumer. This result therefore demonstrates that the existence of the incentive given by word-of-mouth to reduce the number of direct messages sent can lead to patterns of segmentation across communication networks that appear somewhat different from a 'simple' local monopolization pattern.

Section 5 below discusses potential implications of this suggestive result for the type of information intermediaries that competing firms may seek to employ to disseminate their messages when the structure of their market corresponds to this case. For the firms, as we would expect, industry profits are lower here than in the monopolistic case. Segmentation means that revenue per consumer is higher, but competitive pressure drive the firms to a greater combined advertising volume than the parsimonious monopolistic solution.

4.2 Low intensity of competition over common consumers

Second, consider a case in which the nature of competition is different. Again let v be the value to firm k per consumer who learns first about firm k, and again let the value to firm k be 0 per consumer who learns about firm l first. Now, however, assume that firm k earns

$\frac{1}{2}v$ per consumer who learns about both firms at the same time. For example, say that the two firms are promoting essentially identical email services. To a consumer there is no relevant difference between the two, so she adopts whichever he hears about first and chooses at random when she hears about both together. Since the product is free to the consumer, a firm receives the same value whether from that subscriber whether she chose that firm while ignorant of the other or at random.

The following result demonstrates that in this case, when the cost of sending marketing messages is sufficiently low, the unique equilibrium features both firms engaging in mass-marketing and sending a message to every consumer:

Proposition 3. *In the low intensity of competition case, if $c < \frac{1}{2}v$ the unique Nash equilibrium in pure strategies for any network g is $S_1 = A$, $S_2 = A$.*

Intuitively, for low cost of direct marketing, the net benefit of relying on word-of-mouth rather than directly informing a consumer and the net loss of sharing consumers with the competitor are both small relative to the loss associated with being preempted by the competitor, and so a marketing arms-race drives both firms to saturate the network with messages.

For the case in which the cost of sending a message is above the threshold in Proposition 3 we will require some further definitions for the network g. Denote by $ID(g)$ the set of *irredundant sets* in g, defined as those sets of vertices such that no vertex can be removed from the set without reducing the number of vertices in the closed neighborhood of the set (Bollobás and Cockayne (1979)). Denote by $OID(g)$ the set of *open irredundant sets* in g, defined as those sets S such that each $i \in S$ has at least one private neighbor outside the set S (Farley and Schacham (1983), Fellows, Fricke, Hedetniemi, and Jacobs (1994)). Similarly, denote by $IDS(g)$ the set of *open irredundant dominating sets* in g, defined as those open irredundant sets that are also dominating sets. A set that is not irredundant is a *redundant set*. Denote by $IR(g)$ the *upper irredundance number*, defined as the maximum cardinality of an irredundant set in g (Cockayne, Hedetniemi, and Slater (1978)).

Then the following result characterizes equilibria when the cost of sending a message is sufficiently high:

Proposition 4. *In the low intensity of competition case when $c > \frac{1}{2}v$:*

a. *In equilibrium neither firm chooses an open redundant set, and so each firm sends at most $IR(g)$ messages.*

b. *In equilibrium each consumer learns of at least one firm by tomorrow.*

c. *There exists no symmetric equilibrium in which the firms each inform a set of consumers that is not an open irredundant dominating set.*

Leaving aside the technical details of the graph-theoretic definitions, the intuition for these results is the following: when the cost of direct marketing is high, the net benefit of relying on word-of-mouth rather than directly informing a consumer is high, and so both firms are driven to reduce their direct marketing volume to exploit word-of-mouth. In equilibrium, each message sent by a firm must yield either first shot at at least one consumer, or a simultaneous shot at at least two consumers. The most important contrast with the fiercely competitive setting is that word-of-mouth is relevant in equilibrium, in the sense that it can be that some consumers do not receive direct messages from either firm but learn of at least one firm tomorrow by communication with fellow consumers. This is because symmetric equilibria (and more generally equilibria with overlap) are sustainable for two broad reasons. First, the cost of sharing a consumer is lower than before. Second, the competitor does not necessarily have incentive to plug a gap left by one firm in its effort to exploit word-of-mouth; a consumer shared by word-of-mouth is more profitable than sending an extra message to today monopolize that consumer directly.

Part a. of Proposition 4 states that neither firm chooses a set of consumers such that informing one fewer consumer affects only the payoff obtained over that consumer; in this sense each direct marketing message must be associated with some word-of-mouth effect in equilibrium. Part b. says that no consumer is left out of both the messages and word-of-mouth, which follows directly from the primitive assumption that v, the value to a firm of sole capture of a consumer, exceeds c, the cost of sending one message. Part c. is in some sense a combination of a. and b., and says that in any symmetric equilibrium, all consumers hear of both firms either by direct message or word-of-mouth, and that the set chosen by both firms respects the irredundancy condition in a..

The role of irredundancy in Proposition 4 is similar in spirit to the selection of a minimum dominating set by a monopolist, but slightly weaker. In this competitive case, it is again true that neither firm can send one fewer message without some consumer not learning either directly or indirectly the firm's information; a minimum dominating set is certainly open irredundant dominating, but the converse is not true. However, the rationale for this strategy is identical: in both this and the monopolistic case the incentive to exploit word-of-mouth is sufficiently strong to drive the firm to send messages that each generate some irredundant word-of-mouth effect.

To see concretely the role of open irredundance in relation to minimum dominance, consider again the four-consumer line network and the minimum dominating set marked in Figure 4. This minimum dominating set is not open irredundant, in the sense that one of

Figure 4: An open redundant minimum dominating set

its constituent nodes can be dropped without reducing the number of nodes who remain neighbors to at least one node in the set. Such a minimum dominating set cannot form a strategy for a firm in symmetric equilibrium when the cost of informing a consumer is high. Consider firm 2's best response if firm 1 was to inform this set. If firm 2 also informs this set, it will earn $\frac{1}{2}v$ for all four consumers, since all will learn of the two firms simultaneously (either today or tomorrow), at a cost of $2c$. If firm 2 instead informs only consumer 3, it earns $\frac{1}{2}v$ for only three consumers, since consumer 1 never learns of firm 2, at a cost of c. When $c > \frac{1}{2}v$, this minimum dominating set is not a best response to itself, and so cannot be played in a symmetric equilibrium. The underlying issue is that a open redundant minimum dominating set admits the possibility of reducing the number of costly direct messages sent without the indirect consequence of reducing word-of-mouth.

Once again, industry profits are lower than in the monopolistic case, and again this is due to an increased volume of advertising. However, in this case with low intensity of competition

the symmetry of the competitors' strategies means that the increased volume is more starkly 'wasteful' in the sense of being a duplication of effort. Consider a setting that shares some features with this case: the milk industry in the United States. The pervasive existence of price supports (Manchester and Blayney (2001)) means that the intensity of competition among individual producers is somewhat constrained, despite the product being, to a first approximation, homogeneous. The existence of industry-wide advertising cooperatives ('Got Milk?') in this industry reflects the room for a potential 'collusive' marketing agreement in a such a setting, to overcome wasteful replication of marketing effort.

5 Information intermediaries

Interpreting real settings in the context of the model requires us to understand the possibilities and constraints on the ability of the informer to reach subsets of consumers. As discussed in the beginning of the paper, the model literally describes a situation in which the informer can precisely construct or observe a relevant communication network operating at some level over its market and send direct marketing messages to a subset of those consumers. In many applications, disseminating information is fuzzier, and best achieved through information intermediaries - media, advertising, or public relations firms, for example. This section therefore briefly considers how the ideas of the model can translate to settings with looser assumptions on the availability of precise targeting by considering the problem for information intermediaries.

An intermediary selling direct marketing services can in principle negotiate with a client a contract that defines a price to inform a given subset of the market. The extent to which this reachable audience is also a strategic variable may itself vary. For some intermediaries, the subset of the market that they can offer to a client is fixed; a newspaper's audience, for example, is fixed in the short run. For others, perhaps an online advertising platform like Google's AdWords, it may be possible to offer a menu of various subsets of the target population.

The market structure in the informer's industry and the shape of the communication network among the target population can plausibly associate to various approaches by an

information intermediary. On price, Propositions 3 and 4 admit the possibility that the volume of direct marketing desired by an informer can be *negatively* associated with its price: as the price of direct marketing falls, there is an incentive for competing firms to switch from a strategy that targets the word-of-mouth-maximizing network locations to one that saturates the network with messages. A profit-maximizing intermediary with discretion over the set of consumers it can reach may thus prefer to set a low price for its services in order to induce the 'arms race' that drives clients in weakly competitive industries to mass-marketing. Similarly, all else equal, this effect could lead an intermediary tied to a mass audience to rationally set a lower price per impression for advertising services than an intermediary tied to a niche audience. The direct competitors to mass media outlets for an advertising contract with a firm that has significant market power are therefore not only other mass media outlets, but include higher price-per-impression niche media outlets that might superficially seem better tailored to niche products.

On the audience served by the intermediary, for an industry characterized by a high intensity of competition, Proposition 2's segmentation result drives firms to market to distinct but intermingled sets of consumers. This could correspond to competitors advertising in different newspapers or websites, for example; competing newspapers serve approximately distinct readerships, and while communication channels may be richer among than between these readerships, the between-channels are not trivial. This suggests that natural partners for an information intermediary that serves an audience that has little overlap with that served by other intermediaries are firms in highly competitive industries; the fiercely competitive model predicts that such a firm will seek to deliver its messages to an audience distinct from its competitor.

Propositions 3 and 4, by contrast, suggest that low intensity of competition can beget symmetric direct marketing strategies. The burden on an information intermediary or media market to offer a variety of outlets that capture distinct audiences is correspondingly less. Instead, mass media can more valuable to the informing firm. In this fashion the problem for an information intermediary seeking to fill advertising space is informed by the audience it reaches: the characteristics of firms that will find a given media outlet most valuable depend

on the nature of that firm's industry.

6 Concluding comments

This paper develops and analyzes a theoretical model of the problem for a firm seeking to disseminate information when it is able to locate direct marketing at locations in a communication network among its population to exploit word-of-mouth. A pervasive prediction of the analysis, across the various competitive structures considered above, is of dispersed rather than concentrated targeting of direct marketing. In weakly competitive settings, the role of word-of-mouth in amplifying direct messages precludes sending many messages to any one network locality. In more competitive settings this incentive interacts with the incentive for fierce competitors to segment the market rather than overlap to predict dispersed but *intermingled* targeting of direct messages. Translating this analysis from direct marketing to advertising via intermediaries implies that for a firm trying to disseminate information to some population, media with broad and shallow reach across that population can be more valuable advertising outlets than media with deep capture of a particular sub-population.

The boundaries of the model, imposed primarily by tractability, suggest areas in which it could be extended. First, the model considers purely informative marketing, and abstracts from persuasive advertising and persuasive word-of-mouth. Second, the model is non-specific on how the information is used by those who receive it, assuming simply that a firm does better the more consumers hear of it before hearing of its competitors. Third, the exact nature of information transmission is simplified for tractability to be (i.) strictly local, (ii.) non-optional, and (iii.) non-strategic. Fourth, although Section 5 begins to explore the relationship between the literal precise targeting of the model and more general targeting mechanisms, the model assumes very fine targeting. While the model is suggestive of the relationship between the competitive structure of an industry, the structure of consumers' communication network, and the menu of available marketing channels, these limitations imply that to translate these insights to a given setting requires work to understand the exact nature of information processing and transmission in that setting.

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A Proofs

Proposition 1

Proof. Assume not, so that $S_m \neq S_m^*$. If $s_m < s_m^*$, some consumers will never learn about the firm, since they are more than one link away from a consumer who is directly informed. Informing those consumers directly, by selecting some $\hat{S} \supset S_m$, is profitable since $p > c$. If instead $s_m > s_m^*$, then all consumers will indeed learn about the firm and buy, but this could be achieved at lower cost by reducing s_m by selecting some minimum dominating set of g. Thus $S_m \neq S_m^*$ was not profit-maximizing. $\qquad\square$

Proposition 2

Proof. Take three steps:

a) First we will prove that S_k and S_l must be disjoint in equilibrium. Suppose not; then, there must be at least one consumer a common to both sets. A deviation by firm k to the set $S_k - a$ saves c and costs 0. S_k is not a best response to S_l, so (S_k, S_l) was not an equilibrium.

b) Second, we will prove that $S_k \cup S_l = A$ in equilibrium. Suppose not, so that $\exists b$ such that $b \notin S_k, S_l$. W.l.o.g, it must be the case that either $b \notin \Omega(S_k), \Omega(S_l)$, $b \in \Omega(S_k), \Omega(S_l)$ or $b \notin \Omega(S_k)$, $b \in \Omega(S_l)$; in any of these cases, a deviation by firm k to $S_k + b$ yields v and costs c, which is profitable by assumption. S_k was therefore not a best response to S_l, so the pair $S_k \cup S_l \neq A$ is not an equilibrium.

c) Finally, we will prove that S_k and S_l must be dominating sets in equilibrium. Assume not, so that S_k is not a dominating set. This means that there exists a consumer $d \in S_l$ such that $d \notin S_k, \Omega(S_k)$. Since the graph is connected and since by b) $S_k \cup S_l = A$ it must be that $d \in S_l, \Omega(S_l)$ and $\nexists e \in \Omega(d), e \notin S_k, S_l$. Therefore a deviation by firm l to $S_l - d$ saves c, but loses nothing since consumer d will be served by firm l at price p in round 2. S_l was not a best response to S_k, so the pair cannot constitute an equilibrium.

$\qquad\square$

Proposition 3

First we will show that such an equilibrium exists:

Lemma 1. $S_1 = A$, $S_2 = A$ *is a Nash equilibrium for any network g when $c < \frac{1}{2}v$.*

Proof. Fix $S_j = A$. If firm k selects $S_k = A$, all consumers will be directly informed by both firms in stage 1, so each firm receives value $\frac{1}{2}v$ for each individual. The payoff to firm k is:

$$\pi_k = n(\frac{1}{2}v - c) \tag{A.1}$$

If firm k instead selects $S_k = M \subset A$, those $n - m$ consumers will learn of firm 1 but not firm 2 in stage 1, and yield value zero to firm k. Firm k's payoff is:

$$\pi_k = m(\frac{1}{2}v - c) \tag{A.2}$$

If $c < \frac{1}{2}v$, this is always less than the payoff to $S_k = A$. $S_k = A$ is thus a best response to $S_j = A$, and so the pair $S_1 = A$, $S_2 = A$ constitutes a Nash equilibrium. $\qquad\square$

Second, we establish uniqueness:

Lemma 2. *When $c < \frac{1}{2}v$, $S_1 = A$, $S_2 = A$ is the unique Nash equilibrium for any network g.*

Proof. If at least one firm does not send messages to A, there are three possibilities:

i. $S_j = S_k \emptyset$: $\pi_j = 0$; if firm j instead played $S_j = A$, $\pi_j = nv - nc$, which is greater than zero by primitive assumption.

ii. $|S_j| < |S_k|$, $S_k \subseteq A$: firm j earns zero from $i \in S_k, \notin S_j$ since that consumer learns of k before j. If firm j instead played $S_j + i$, it pays c more and earns $\frac{1}{2}v$ from i since then i learns of the two firms at the same time. Since $\frac{1}{2}v > c$, this yields a higher payoff than S_j.

iii. $S_j = S_k \subset A$: firm j earns $\frac{1}{2}v$ from $l \in \Omega(S_j)$ since that consumer leans of both firms at the same time tomorrow. If firm j instead played $S_j + l$, it pays c more and earns

$\frac{1}{2}v$ more from l since l will then learn of j before k. Since $\frac{1}{2}v > c$, this yields a higher payoff than S_j.

In each case the proposed pair is not an equilibrium; there is no Nash equilibrium that does not have $S_1 = A$, $S_2 = A$. $\qquad\square$

This completes Proposition 3.

Proposition 4

Part a.

In equilibrium neither firm chooses a redundant set, and so each firm sends at most $IR(g)$ messages.

Proof. Assume not, so that there exists an equilibrium in which firm j chooses to inform a redundant set R of consumers. Since R is a redundant set, there exists at least one i such that $\{R + \Omega(R)\} = \{R - i + \Omega(R - i)\}$. Consider a deviation by firm j to the set $R - i$.

For all vertices in R, firm j captures the same value as before, since those consumers continue to learn about firm j today when they receive a message. For all vertices in $\Omega(R)$, firm j again captures the same value as before, since those consumers continue to learn about firm j tomorrow by word-of-mouth. For all vertices not in R or $\Omega(R)$, firm j continues to capture zero.

For i, consider separately three exhaustive cases.

i. $i \in S_k$. When $S_j = R$, i learns of both firms today when receiving a message from each, and j receives $\frac{1}{2}v$ from i. When $S_j = R - i$, i learns of firm k before j and so j receives zero from i.

ii. $i \notin S_k, \in \Omega(S_k)$. When $S_j = R$, i learns of j today and k tomorrow, and j receives v from i. When $S_j = R - i$, i learns of j and k tomorrow by word-of-mouth and so j receives $\frac{1}{2}v$ from i.

iii. $i \notin S_k, \notin \Omega(S_k)$. When $S_j = R$, i learns of j today and never learns of k, and j receives v from i. When $S_j = R - i$, i learns of j tomorrow and never learns of k and so j receives v from i.

In each case firm j loses at most $\frac{1}{2}v$ from i. Since informing $R - i$ saves c, the value obtained from all other nodes is identical, and $c > \frac{1}{2}v$, firm j earns a greater payoff from $R - i$ than from R regardless of the choice of firm k. Thus R was not a best response by j and so cannot be part of an equilibrium. $\qquad\square$

Part b.

In equilibrium each consumer learns of at least one firm by tomorrow.

Proof. Assume not, so that there exists an equilibrium at S_j, S_k such that there is some consumer $i \notin S_j, S_k, \Omega(S_j), \Omega(S_k)$. A deviation by firm j to $S_j + i$ costs an extra c and yields an extra v, since i will learn of firm j today and not learn of firm k. Since $v > c$ by assumption, the payoff to firm j from $S_j + i$ is greater than that from S_j. S_j was therefore not a best response to S_k and so the original pair was not an equilibrium. $\qquad\square$

Part c.

There exists no symmetric equilibrium in which the firms each inform a set of consumers that is not an open irredundant dominating set.

Proof. Consider two exhaustive cases.

i. Assume an equilibrium exists at $S_1 = S_2 = Z \notin DS(g)$, where $DS(g)$ is the set of dominating sets of g. Since Z is not a dominating set, there exists some consumer $i \notin Z, \Omega(Z)$ who never learns of either firm. A deviation by firm j to $Z + i$ costs an extra c and yields value v, since i will learn of firm j today and not learn of firm k. $S_j = Z$ is therefore not a best response to $S_k = Z$ and the original pair is not an equilibrium.

ii. Assume an equilibrium exists at $S_1 = S_2 = Y \in DS(g), \notin OIDS(g)$. In such an equilibrium, since $Y \in DS(g)$, all consumers in Y learn of both firms today and all remaining

consumers learn of both firms tomorrow, and so $\pi_j(Y) = \frac{1}{2}nv - yc$. Since Y is not an open irredundant dominating set, there exists some consumer $l \in Y$ such that $\Omega(Y - l) = \Omega(Y)$. A deviation by firm j to $Y - l$ means that $y - 1$ consumers learn of both firms j today, 1 consumer learns only of firm k today, and $n - y$ consumers learn of both firms tomorrow. This yields $\pi_j(Y - l) = \frac{1}{2}(n-1)v - (y-1)c$, which is greater than $\pi_j(Y)$ since $\frac{1}{2}v < c$. $S_j = Y$ is therefore not a best response to $S_k = Y$ and the original pair is not an equilibrium.

$\square$