Expectations, Network Effects and Platform Pricing

Andrei Hagiu*and Hanna Halaburda†

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Abstract

In markets with network effects, users must form expectations about the total number of users who join a given platform. In this paper, we distinguish two ways in which rational expectations can be formed, which correspond to two different types of users—sophisticated and unsophisticated. Only sophisticated users adjust their expectations in response to platforms' price changes. We study the effect of the fraction of sophisticated users on platform profits. A monopoly platform's profits are always increasing in the fraction of sophisticated users. The profits of competing platforms in a market of fixed size are decreasing in the fraction of sophisticated users. When market expansion is introduced, the fraction of sophisticated users that maximizes competing platforms' profits may be positive and is strictly lower than 1. We also investigate the possibility of platforms investing in "educating" unsophisticated users. In a competitive environment, such education is a public good among platforms and therefore the equilibrium level is lower than the one that would maximize joint industry profits.

1 Introduction

In markets with network effects, the value that users derive from platforms is determined by the number of other users of the same type who join the same platform (direct network effects) or the number of users of a different type that join (cross-group network effects). Examples include social networks like Facebook or Google+, payment systems like PayPal or Visa, videogame systems like PlayStation 3 and Xbox 360, smartphone platforms like

*Harvard University (HBS Strategy Unit), ahagiu@hbs.edu.

†Harvard University (HBS Strategy Unit), hhalaburda@hbs.edu.

Apple's iPhone or Google's Android, etc. Users typically rely on external information (e.g. press announcements, market reports, word of mouth) to form expectations about the total number of other users that join a given platform. In most real-world settings, however, users are unable to calculate the effect of platforms' prices on adoption by other users. In other words, they do not take price into account when forming expectations. This may be for a number of reasons. For example, the real price charged to other users may be obscured by discounts and special offers. Or, in two-sided contexts, users on one side may simply not be aware of the price charged to the other side (e.g. few videogame console users are aware of the royalties that console manufacturers charge to third-party game developers). And even when all prices are known to everyone, users may not have sufficient information about aggregate user demand to compute its responsiveness to price changes.

The majority of the existing literature on platform pricing in the presence of network effects (one-sided or two-sided) typically assumes that users have full information about prices and the ability to perfectly compute their impact on platform adoption. Thus, users are assumed capable to perfectly adjust their expectations in response to changes in platform prices. These stark assumptions are obviously made for analytical tractability and do not drive the key insights obtained to date (e.g. qualitative impact of the magnitude of network effects on platform prices). Still, it is important to ask whether they lead to any loss of economic substance. The goal of this paper is to demonstrate that different assumptions about how *rational* expectations are formed have different and meaningful economic implications regarding market outcomes and firm profits.

An individual user's platform adoption decision depends on whether the expected value of a given platform (a function of the number of other users who adopt) is above or below its price. We compare two types of rational expectations. When users do not take into account the impact of prices on total expected platform adoption, they form *fixed rational* expectations. This means that they do not change their expectations regarding total adoption of a given platform when the latter changes its access prices. They do however change their individual adoption decisions based on price. We call such users "unsophisticated". On the other hand, when users take into account the impact of prices on expected platform adoption, they form *responsive rational* expectations. In this case, they adjust their expectations about platform adoption in response to price changes. Thus, when expectations are responsive rational, price affects user participation decisions both directly and indirectly, through its impact on the expected value of joining a given platform. We call such users "sophisticated".

It is worth emphasizing that both types of expectations we use are rational, i.e., fulfilled in equilibrium. Thus, we do not introduce any behavioral biases. We show however, that the equilibrium outcomes (prices, quantities and profits) are different under the two types of expectations. Specifically, we consider settings in which agents with both types of expectations coexist. We show that a monopoly platform's profits are strictly increasing in the proportion of sophisticated users. The reason is that sophisticated users are more responsive to platform prices. Indeed, a price cut has a double positive effect on sophisticated users: not only does it make platform adoption more affordable (lower price) but it also makes it more valuable (more *expected* users). Thus, when the proportion of sophisticated users increases, the same price cut attracts more users. Results are reversed in the case of two platforms competing for share in a market of fixed size: equilibrium platform profits are now decreasing in the proportion of sophisticated users. The underlying logic is the same as before but the consequence is different: more sophisticated users drive more intense price competition, which results in lower profits if the market cannot expand. We also show that when market expansion is introduced, equilibrium platform profits can be non-monotonic in the proportion of sophisticated users. Finally, we also derive implications for platforms' incentives to "educate" users, i.e., make costly investments that increase the share of sophisticated users. In particular, we show that the total equilibrium level of investment in user education with two competing platforms is lower than the level that would maximize industry profits. This is an instance of the tragedy of the commons, due to the positive externalities between platforms' investments.

2 Related literature

The existing literature on platform pricing in the presence of network effects (one-sided and two-sided) contains two different approaches to modelling user expectations.[1]

The majority of models assume *perfect* adjustments of expectations (and therefore demand) to price variations. In other words, users are assumed capable of perfectly computing the effect of price changes on overall user demand for a given platform. This approach originated with Katz and Shapiro (1986), who study competition between incompatible technologies with direct (one-sided) network effects. Most of the recent literature on two-sided markets (e.g. Armstrong (2006), Armstrong and Wright (2007), Caillaud and Jullien (2003),

[1] In models of platform adoption with network effects where platforms do not make pricing decisions (e.g. Church and Gandal (1992), Farrell and Saloner (1985) and (1986)), the distinction between fixed rational and responsive rational expectations is meaningless.

Economides (1996), Hagiu (2009), Rochet and Tirole (2006), Weyl (2010)) also assumes perfectly adjustable (i.e., responsive rational) user expectations. The logic for a monopoly platform setting is as follows. Start with two-sided demand functions $n_i = D_i(n_j, p_i)$, where $i \neq j \in \{1, 2\}$ are the two sides, n_j is realized demand on side j (which is equal to expected demand for all prices), p_i is the price charged by the platform to side i and $D_i(.,.)$ is increasing in its first argument and decreasing in its second argument. Then all authors adopt one of two methods. They either solve directly for (n_1, n_2) as a function of (p_1, p_2) only, and then maximize the resulting profit expression over (p_1, p_2). Or they invert $n_i = D_i(n_j, p_i)$ to express p_i as a function of (n_i, n_j), then replace in the expression of profits and maximize over (n_1, n_2). Note indeed that (n_1, n_2) determines a unique (p_1, p_2). The problem (which arises with both methods) is that the reverse may not be true: there may be multiple equilibrium solutions (n_1, n_2) for a given (p_1, p_2). This issue is usually side-stepped by assuming platforms have the ability to coordinate users on the allocation (n_1, n_2) they prefer. Parker and Van Alstyne (2005) use a slightly different approach, by directly assuming two-sided demands that depend on both prices: $n_1 = D_1(p_1) + e_{12} D_2(p_2)$ and $n_2 = D_2(p_2) + e_{21} D_1(p_1)$. Argenziano (2007) applies the global games methodology to the study of competition between one-sided networks. While each user receives a noisy signal regarding the intrinsic (standalone) value of a given platform, she is still assumed capable of calculating the optimal adoption strategies for all other users as a function of platform price. Ambrus and Argenziano (2009) show that multiple asymmetric networks can coexist in equilibrium when agents are heterogeneous. In their model too, each individual agent observes all platform prices and calculates the resulting adoption decisions by all other agents. Thus, all of these papers rely on responsive rational expectations.

Evans and Schmalensee (2010) study platform adoption in the presence of network effects with imperfect, dynamic adjustments of user participation decisions. At a high level, our paper is related to theirs in the effort to formally capture imperfections in the mechanisms through which users form expectations—a prevalent phenomenon in real-world settings. The difference is that in their model, platform prices are fixed and the focus is on determining conditions (critical mass) under which the imperfect dynamic adjustment process converges to positive levels of platform adoption.

To the best of our knowledge, Katz and Shapiro (1985) is the only paper in the traditional network effects literature (including two-sided markets) that explicitly distinguishes fixed rational expectations from responsive rational expectations.[2] In particular, the paper studies

[2] Matutes and Vives (1996) do so in a model of financial intermediation.

Cournot competition between n firms (technologies) with direct network effects, each of which can set price strategically. In the main text of the paper, the authors use the notion of fulfilled expectations Cournot equillibrium: each firm chooses its output taking other firms' decisions and users' expectations regarding firms' outputs as fixed. This case corresponds to our fixed rational expectations. In the appendix, the authors also analyze the case when users' expectations adjust (correctly) based on firms' output decisions, i.e., the case with responsive rational expectations. They confirm that most of their analysis goes through but do not compare firms' equilibrium profits and prices under the two types of expectations. Nor do they treat the mixed case in which some users form responsive rational expectations, while others form fixed rational expectations. The same Cournot model with fixed and fulfilled expectations is also used by Economides (1996) to study the incentives of a network leader to invite entry by competing followers.

Gabszewicz and Wauthy (2004) is the only two-sided model we are aware of that relies on fixed rational expectations. In their paper, users on both sides are differentiated by the intensity of their indirect network effects and all users form fixed rational expectations. The methodology is similar to Katz and Shapiro (1985) and its basic logic for a monopoly platform runs as follows (note the contrast with the logic of responsive rational expectations summarized above). Two-sided demands are $n_i = D_i\left(n_j^e, p_i\right)$ where $i \neq j \in \{1, 2\}$ are the two sides, n_j^e is expected demand on side j, p_i is the price charged by the platform to side i and $D_i(.,.)$ is increasing in its first argument and decreasing in the second argument. The platform maximizes $\Pi = p_1 D_1\left(n_2^e, p_1\right) + p_2 D_2\left(n_1^e, p_2\right)$ over (p_1, p_2) treating (n_1^e, n_2^e) as exogenously given. Then rational expectations require $(n_1, n_2) = (n_1^e, n_2^e)$, which closes the loop by determining equilibrium demands and prices. The same approach is used for competing platforms.

It is worthwhile to note that all of the papers listed above adopt either one of the two approaches for modelling expectations (with the exception of Katz and Shapiro (1985)), but none of them discusses the relative merits of one approach over the other. Our paper renders the comparison explicit and, most importantly, derives several substantive implications of the difference between the two methods regarding platform profits, market outcomes and incentives to "educate users". As suggested by our terminology (framing the difference in terms of user sophistication), we believe that both approaches have economic merit and reflect meaningful differences in the behavior of agents in real markets.

3 Two notions of expectations

In this section we introduce and specify the difference between the two types of expectations that we use throughout the paper: fixed rational vs. responsive rational. Rationality means that expectations are always fulfilled *in equilibrium*. For the sake of clarity, we use the simplest possible setting here: a monopoly platform with direct (one-sided) network effects. Suppose the demand faced by such a platform is $n = D(n^e, p)$, where n^e denotes users' expectation about total user adoption of the platform; p is the price charged by the platform to users; $D(.,.)$ is increasing in its first argument and decreasing in its second argument.

First, if expectations are *responsive* and rational then expectations and realized demand adjust perfectly so that they are equal for *any* given price p. Given such expectations, the full equilibrium is calculated as follows:

(1) Solve $n = D(n, p)$ in order to obtain $n = \widehat{n}(p)$.

(2) Maximize $\Pi = p\widehat{n}(p)$ over p, yielding p_r^* and $n_r^* \equiv \widehat{n}(p_r^*)$.

Second, if expectations are *fixed* and rational, each user expects a *fixed* total number of users to join the platform. This number, denoted by n_f, does not respond to price changes. In this case, the platform takes n_f as exogenously given when setting its price. In equilibrium, expectations must be equal to realized demand. The full equilibrium is then derived as follows:

(1) Maximize $\Pi = pD(n_f, p)$ over p, which yields $\widehat{p}(n_f)$ and $n(n_f) \equiv D(n_f, \widehat{p}(n_f))$.

(2) Impose the rationality condition $n(n_f) = n_f$, which yields $n_f = n_f^*$ and $p_f^* = \widehat{p}\left(n_f^*\right)$.

With both approaches, all agents converge on the correct expectation about platform adoption in equilibrium. Furthermore, both types of expectations can be viewed as the result of dynamic adjustment processes. The key difference is that in the case of responsive rational expectations, each user takes into account the effect of price changes on the behavior of *other* users, whereas that effect is ignored in the case of fixed rational expectations. In other words, users with fixed rational expectations adjust their expectations based solely on external information regarding platform adoption (e.g. market studies, media announcements) and are unable to calculate the effect of price changes on the behavior of other users. Throughout the paper we refer to users holding responsive rational expectations as "sophisticated" and to users holding fixed rational expectations as "unsophisticated".

More formally, the two types of expectations correspond to two different timings of the game played by users and platforms. Fixed rational expectations are equivalent to a game in which users "choose" their expectations first and the platform chooses prices second, taking user expectations as given. In contrast, responsive rational expectations correspond to a game in which the platform chooses prices first and users set their expectations second, based on the platform's price. Thus, fixed rational expectations can also be interpreted as a form of credible commitment by users, which the platform cannot influence with its price. Note that it is not necessary to assume that fixed rational expectations are observed by the platform. Instead, it is sufficient to assume that users commit to their expectations, so that both users and the platform solve $n_f = n(n_f)$ and then converge upon the "correct" expectation n_f^*.

Throughout the paper we allow sophisticated and unophisticated users to simultaneously co-exist in the market: a fraction λ of sophisticates and a fraction $(1-\lambda)$ of unsophisticates. Sophisticated users are aware of λ when computing their expectations. We are primarily interested in how the presence of sophisticated users affects platforms' profits.

For simplicity of exposition, in the remainder of the paper we only analyze one-sided platforms with direct network effects and rely on linear demand functions. All of our results hold however with general demand formulations (assuming second order conditions for all optimization problems are satisfied) and for two-sided platforms, as we briefly show in Appendices A and B.

4 Monopoly

Consider a one-sided platform with direct network effects and linear user demand:

$$n = V + \alpha n^e - p\,,$$

where $V > 0$ is the standalone value of the platform to every user.

Equilibrium outcomes depend on the nature of user expectations. Let us first compare the two polar cases in which: (i) all users are sophisticated ($\lambda = 1$); and (ii) all users are unsophisticated ($\lambda = 0$).

(i) **100% sophisticated users (responsive rational expectations).** Given price p, user demand $\widehat{n}(p)$ for the platform is defined by

$$\widehat{n}(p) = V + \alpha\widehat{n}(p) - p\,,$$

leading to $\widehat{n}(p) = \frac{V-p}{1-\alpha}$. The platform chooses p to maximize $p\frac{V-p}{1-\alpha}$, which yields

$$p_r^* = \frac{V}{2} \text{ and } n_r^* = \frac{V}{2(1-\alpha)}.$$

The resulting platform profits are

$$\Pi_r^* = \frac{V^2}{4(1-\alpha)}.$$

(ii) **100% unsophisticated users (fixed rational expectations).** Given the users' (fixed) expectation n_f about total platform adoption, the platform's optimal price is

$$\widehat{p}(n_f) = \arg\max_p \{p(V + \alpha n_f - p)\} = \frac{V + \alpha n_f}{2},$$

leading to $n(n_f) = \frac{V+\alpha n_f}{2}$. In equilibrium, it must be that $n_f = n(n_f)$; therefore,

$$n_f^* = \frac{V}{2-\alpha} \text{ and } p_f^* = \frac{V}{2-\alpha}.$$

The resulting profits are

$$\Pi_f^* = \frac{V^2}{(2-\alpha)^2}.$$

By comparing prices, quantities and profits in the two cases, we obtain:

Lemma 1 *The equilibrium price is lower while the resulting number of platform users is larger when all users are sophisticated relative to the case when all users are unsophisticated* ($p_r^* < p_f^*$ *and* $n_r^* > n_f^*$)*. The platform's equilibrium profits are higher when all users are sophisticated* ($\Pi_r^* > \Pi_f^*$)*.*

The results contained in Lemma 1 do not depend on linear demand[3] — the proof in the appendix (page 18) is provided for the general case. The main driving force behind these results is the fact that sophisticated users are more responsive to price changes by the platform than unsophisticated users. Sophisticates see a two-fold benefit from a price decrease. First, they have to pay less for joining the platform. Second, they understand that a lower price affects other users' purchasing decisions and adjust their expectations about market size accordingly, i.e., upwards. In contrast, unsophisticated users ignore the effect

[3]The result that total equilibrium demand (output) is higher with responsive rational expectations confirms a similar finding by Katz and Shapiro (1985) in a Cournot model. It holds generally for any market structure (monopoly or competing platforms) and independently of the specific modelling approach adopted (Cournot vs. Bertrand).

of the price cut on other users and maintain the same fixed expectations. This explains why the optimal price when the platform faces sophisticated users is lower than when it faces unsophisticated users. The reason profits are higher with sophisticated users is closely related. Since expectations are fulfilled in both equilibria, the platform can replicate the equilibrium outcome of the case with unsophisticated users even when it faces sophisticated users: charging p_f^* leads to realized demand n_f^* in both cases. Starting from this allocation, if users are sophisticated then the platform can strictly increase its profits by slightly lowering its price. Indeed, the resulting increase in demand (accompanied by a perfect adjustment of expectations) is larger than the one that would occur if users were unsophisticated, which by definition of $\left(p_f^*, n_f^*\right)$ would leave the platform indifferent.

Another way to interpret the difference in profits is to recall that fixed rational expectations are a form of credible user commitment to ignore the platform's price changes. It seems then intuitive that the platform's profits are lower relative to the case in which users are unable to credibly commit *not* to adjust their expectations in response to the platform's price choice.

Having considered the extreme cases, let us now turn to the mixed case, in which a fraction $\lambda \in (0,1)$ of the user population is sophisticated, i.e., has responsive expectations, while the remaining fraction $1-\lambda$ is unsophisticated, i.e., holds fixed rational expectations. For any platform price p, realized demand n is determined by

$$n = \lambda \left(V + \alpha n - p\right) + \left(1-\lambda\right)\left(V + \alpha n_f - p\right) \ ,$$

where $(V + \alpha n - p)$ is the part of demand that comes from sophisticated users and $(V + \alpha n_f - p)$ is the part of demand coming from unsophisticated users. This follows from our assumption that sophisticated users' expectation about network size is equal to realized demand for all p, while unsophisticated users maintain their fixed expectation n_f regardless of price.

We can therefore express demand n as a function of price and the fixed expectation of unsophisticated users:

$$n\left(p, n_f\right) = \frac{V - p + \left(1-\lambda\right)\alpha n_f}{1 - \lambda\alpha} \ .$$

Given this function, the platform chooses $\widehat{p}\left(n_f\right)$ which maximizes profits $p\, n(p, n_f)$. And in equilibrium it must be that fixed expectations are fulfilled, hence $n\left(\widehat{p}\left(n_f\right), n_f\right) = n_f$. This leads to the following result.[4]

[4] We defer the remaining details to the appendix (page 19).

Proposition 1 *The platform's optimal price and profits as a function of the fraction* $\lambda \in [0,1]$ *of sophisticated users in the market are* $p^*(\lambda) = \frac{V(1-\lambda\alpha)}{2-\alpha-\lambda\alpha}$ *and* $\Pi^*(\lambda) = \frac{V^2(1-\lambda\alpha)}{(2-\alpha-\lambda\alpha)^2} \in \left[\Pi_f^*, \Pi_r^*\right]$. *We have:* $\frac{\partial p^*(\lambda)}{\partial\lambda} < 0$ *and* $\frac{\partial \Pi^*(\lambda)}{\partial\lambda} > 0$.

Thus, the platform's optimal price is decreasing in the proportion λ of sophisticated users while its profits are increasing in λ. The reason is that, as noted earlier, sophisticated users are more responsive to price decreases. Increasing λ is then equivalent to increasing demand elasticity (this is easily seen in the expression of $n(p, n_f)$ above), which leads to a lower profit-maximizing price. We can also use a similar heuristic reasoning to the one developed after Lemma 1 to explain why profits are increasing in λ. Start with the optimal allocation $(p^*(\lambda), n^*(\lambda))$ for a fraction λ of sophisticated users. Suppose that fraction increases to $\lambda + \Delta\lambda$. If the platform charges $p^*(\lambda)$ it still attracts $n^*(\lambda)$ users.[5] But now it can do strictly better by slightly lowering its price: the same price decrease that kept profits constant with λ sophisticates results in a larger increase in demand with $\lambda + \Delta\lambda$ sophisticates.

Finally, note that profits and prices are increasing in the network effect parameter α for all λ. Thus, if one is solely interested in the qualitative impact of the strength of network effects on market outcomes, the two approaches to modelling expectations are equivalent in that they lead to the same insights. As we have seen however, market outcomes are different.

5 Competition

In this section, we study competition between two symmetric platforms, located at the extreme ends of a $[0, 1]$ Hotelling segment. In the first subsection, the Hotelling segment represents the entire market, i.e., market size is fixed (in equilibrium both firms always serve exactly half of the consumers). In the second subsection, we allow for market expansion beyond both ends of the Hotelling segment, so that platform prices affect not only market shares but also market size. This feature has important consequences for the way in which the nature of expectations affects platform profits.

In both subsections, we assume a fraction λ of customers is sophisticated and the remainder $(1-\lambda)$ is unsophisticated. The distribution of sophisticated and unsophisticated users is independent of their location on the Hotelling line.

[5] Whenever demand is $n = \lambda D(n, p) + (1-\lambda) D(n_f, p)$ and expectations are fulfilled, we have $n = n_f = D(n, p)$, which yields $n = n_f = \widehat{n}(p)$, a function that is independent of λ.

5.1 Hotelling with fixed market size

Users are distributed along the Hotelling segment $[0,1]$ wih density 1 and transportation costs t. The utility of an unsophisticated user $x \in [0,1]$ from adopting platform 1 is $V + \alpha n_{f1} - p_1 - tx$ and that from adopting platform 2 is $V + \alpha n_{f2} - p_2 - t(1-x)$, where n_{f1} and n_{f2} are the fixed rational expectations of unsophisticated users. For a sophisticated user, the respective utilities are $V + \alpha n_1 - p_1 - tx$ from platform 1 and $V + \alpha n_2 - p_2 - t(1-x)$ from platform 2, where n_1 and n_2 are the *realized* demands.

Total realized demand for platform 1 is therefore determined by

$$n_1 = \lambda \left[\frac{1}{2} + \frac{\alpha(n_1 - n_2) + p_2 - p_1}{2t} \right] + (1-\lambda) \left[\frac{1}{2} + \frac{\alpha(n_{f1} - n_{f2}) + p_2 - p_1}{2t} \right].$$

Demand n_2 for platform 2 is determined in the same way. It is then straightforward to solve for (n_1, n_2) as functions of prices (p_1, p_2) and fixed expectations (n_{f1}, n_{f2}). Platforms simultaneously choose prices to maximize profits. We obtain that the symmetric equilibrium is characterized by $p_1^*(\lambda) = p_2^*(\lambda) = t - \alpha\lambda$ and $n_1^* = n_2^* = \frac{1}{2}$, leading to

$$\Pi_1^*(\lambda) = \Pi_2^*(\lambda) = \frac{1}{2}(t - \lambda\alpha). \quad (1)$$

Proposition 2 *Platform profits in the symmetric competitive equilibrium with fixed market size are strictly decreasing in the fraction of sophisticated users λ.*

In the appendix, we show that this result also holds when platforms are asymmetric, i.e., one platform has a quality advantage over the other. Note that this is the opposite result relative to the monopoly case, in which platform profits were increasing in λ. The reason is straightforward: If platforms compete for share in a market of a fixed size, the individual incentives to lower price are stronger when there are more sophisticated users, because these users are more responsive to price decreases. This can be confirmed in the following way. Start with equal prices and suppose that platform 1 decreases its price by a small amount ε. The number of unsophisticated users that switch to platform 1 is $\frac{\varepsilon}{2t}$. Their purchasing decision is only affected by the change in price, as their expectations are unaffected by this change. In contrast, there are two effects on sophisticated users: the direct effect of platform 1 being more affordable, and the indirect effect operating through the upward adjustment of their expectations regarding platform 1's market share. As a result, the number of sophisticated users who switch is $\frac{\varepsilon}{2t}\left(1 + \frac{\alpha}{t - \alpha\lambda}\right)$, which is larger than $\frac{\varepsilon}{2t}$.

As we show in the next subsection, things are more complicated when market expansion is added: the effect of λ on platform profits may become non-monotonic.

Finally, note that equilibrium profits and price are decreasing in the network effect parameter α for all λ. As in the monopoly case, this implies that the qualitative impact of the strength of network effects on market outcomes does not depend on λ.

5.2 Hotelling with market expansion

Suppose now that the Hotelling segment has density $y \geq 0$ and in addition there are two hinterlands beyond each extremity (from 0 to $-\infty$ and from 1 to $+\infty$) with density $x \geq 0$. The two hinterlands represent market expansion.

Consumers in each hinterland have transportation costs $u > 0$, which may be different from transportation costs along the Hotelling segment (t). We also assume that each platform must set a unique price for all its users, i.e., platforms cannot price-discriminate between users in the Hotelling segment (for which they compete) and users in their respective hinterland (for which each platform is effectively a monopoly). Finally, we assume that V, u and t are large enough relative to x and y so that all stability conditions are met and the Hotelling segment is entirely covered.

We follow the same steps as in the previous subsection. Total realized demand for platform 1 is

$$\begin{aligned} n_1 &= \lambda \left\{ x \left(\frac{V + \alpha n_1 - p_1}{u} \right) + y \left[\frac{1}{2} + \frac{\alpha (n_1 - n_2) + p_2 - p_1}{2t} \right] \right\} \\ &\quad + (1 - \lambda) \left\{ x \left(\frac{V + \alpha n_{f1} - p_1}{u} \right) + y \left[\frac{1}{2} + \frac{\alpha (n_{f1} - n_{f2}) + p_2 - p_1}{2t} \right] \right\} . \end{aligned}$$

As before, it has two components, one coming from sophisticated users (fraction λ) and one from unsophisticated users (fraction $(1 - \lambda)$). In addition, we now have a convex combination of the market share from the Hotelling segment (coefficient y) and the monopoly demand coming from users in the hinterland of platform 1, i.e., who only consider platform 1 (coefficient x).

The derivation of platform demands as functions of prices and fixed expectations is very similar to the previous section. We include it, along with the full derivation of the symmetric equilibrium, in the proof of Proposition 3 in the appendix (page 22). Platform profits in the symmetric equilibrium are

$$\Pi^* (\lambda) = \frac{\left(\frac{y}{2} + \frac{Vx}{u} \right)^2 A (\lambda)}{\left[\frac{x}{u} + A (\lambda) \left(1 - \frac{\alpha x}{u} \right) \right]^2} ,$$

where

$$A(\lambda) \equiv \frac{\frac{y}{2t\left[1-\lambda\alpha\left(\frac{x}{u}+\frac{y}{t}\right)\right]} + \frac{x}{u}}{1 - \frac{\lambda\alpha x}{u}}.$$

Note that $A(\lambda)$ is increasing.

For $x = 0$ the hinterlands are eliminated and therefore the setting is equivalent to the Hotelling case with fixed market size. We can then set $y = 1$ without loss of generality, obtaining $\Pi^*(\lambda) = \frac{t-\lambda\alpha}{2}$, the same as in (1). Note that $\Pi^*(\lambda)$ is decreasing when $x = 0$.

For $y = 0$ the competitive Hotelling segment is eliminated and therefore the setting is equivalent to the pure monopoly case. In our context, there are two independent monopoly platforms. Setting $x = 1$ without loss of generality, we obtain $\Pi^*(\lambda) = \frac{V^2(u-\lambda\alpha)}{[2u-\alpha(\lambda+1)]^2}$, which is strictly increasing in λ, just like the monopoly profit in Proposition 1.[6]

When both x and y are positive, however, it is possible that $\Pi^*(\lambda)$ might be non-monotonic in λ. To see this, note that

$$\begin{aligned} sign\left(\frac{d\Pi^*}{d\lambda}\right) &= sign\left(\frac{d\Pi^*}{dA}\right) \cdot sign\left(\frac{dA}{d\lambda}\right) = sign\left(\frac{d\Pi^*}{dA}\right) \\ &= sign\left[\frac{x}{u} - A(\lambda)\left(1 - \frac{\alpha x}{u}\right)\right]. \end{aligned}$$

This implies that $\Pi^*(\lambda)$ is single-peaked in λ. Indeed, since $A(\lambda)$ is increasing, the term in-between the last square brackets is decreasing in λ. For $\lambda = 1$, the term is negative. There are thus two possible cases:

- If $\frac{x}{u} \leq A(0)\left(1 - \frac{\alpha x}{u}\right)$, then Π^* is decreasing in λ over the entire interval $[0,1]$, so that platform profits are maximized for $\lambda = 0$.

- If $\frac{x}{u} > A(0)\left(1 - \frac{\alpha x}{u}\right)$, then Π^* is maximized by some λ^* strictly between 0 and 1, such that

$$A(\lambda^*) = \frac{\frac{x}{u}}{1 - \alpha\frac{x}{u}}. \tag{2}$$

The condition for Π^* to be maximized by an interior λ^* is thus:

$$\frac{y}{t} < \frac{2\alpha\left(\frac{x}{u}\right)^2}{1 - \frac{\alpha x}{u}}. \tag{3}$$

We summarize the preceding analysis in the following proposition:

Proposition 3 *Consider two competing platforms on a Hotelling segment of positive measure and with positive market expansion ($x, y > 0$). In the symmetric equilibrium:*

[6] When $u = 1$ we obtain the exact same expression as in the monopoly case.

(i) Platform profits are single-peaked in λ and are maximized by $\lambda^* \in [0, 1)$.

(ii) The profit-maximizing λ^* is strictly lower than 1; it is strictly positive if and only if condition (3) holds.

The results in the proposition imply that for any $\lambda > \lambda^*$ platforms prefer a lower number of sophisticated users. In contrast, if $\lambda < \lambda^*$ then platforms prefer to see the number of sophisticated users increase. In the next section, we briefly discuss the possibility that platforms may make costly investments to "educate" users, i.e., to increase the fraction of sophisticates.

Note that condition (3) is less likely to hold for high y, low x and low α. That is, when the competitive segment is sufficiently large in density relative to the monopolistic hinterlands, the platforms prefer to face only unsophisticated consumers; just as they prefer in the Hotelling case with fixed market size. Conversely, if the hinterlands are sufficiently large relative to the Hotelling segment, the platforms perceive a tradeoff when facing a higher proportion λ of sophisticates: Higher λ increases profits coming from the hinterlands—on which each platform is a monopoly—but decreases profits coming from the competitive Hotelling segment. This is why the optimal λ from platforms' perspective is interior.

The strength of the network effect, α, exacerbates the effect of the presence of sophisticated users on platform profits, which means that there are two potentially countervailing effects. On the one hand, stronger network effects increase profits from hinterlands. On the other hand, they decrease profits from the Hotelling segment.

6 Educating the market

In this section we briefly explore the possibility that platforms may make costly investments (e.g. advertising campaigns) in order to increase the fraction of sophisticated users λ, which can be interpreted as educating users. The timing is as follows:

(1) The platform(s) make (simultaneous) investment(s) in educating users.

(2) Unsophisticated users form their fixed expectations.

(3) Platforms choose prices simultaneously.

(4) All users observe prices, sophisticated users form their expectations.

(5) All users make purchase decisions.

We start with the monopoly platform. Suppose that when the platform invests $e > 0$, a fraction $\lambda(e)$ of the user population becomes sophisticated, where $\lambda(e)$ is assumed to be strictly increasing, sufficiently concave and $\lambda(e) \leq 1$ for all e. The platform's profits net of investments in educating users are then (recall the profit expression in Proposition 1)

$$\Pi^*(\lambda(e)) - e = \frac{V^2(1 - \lambda(e)\alpha)}{[2 - \alpha - \lambda(e)\alpha]^2} - e\,.$$

Without any cost of educating users, a monopolist always prefers all users to be sophisticated. If the investment in education is sufficiently effective (i.e., $\lambda(e)$ is increasing at a sufficiently high rate), then the monopolist still finds it optimal to educate all users. Otherwise, it is too costly to educate all users, so the solution is interior.

Suppose that two platforms, competing as in Section 5.2, invest in educating users. We assume that when platform $i = 1, 2$ invests e_i in educating users, the resulting fraction of sophisticated users is $\lambda(e_1 + e_2)$. In other words, the platforms' education efforts are complementary. Platform i's profits are

$$\Pi^*(\lambda(e_1 + e_2)) - e_i = \frac{\left(\frac{y}{2} + \frac{Vx}{u}\right)^2 A(\lambda(e_1 + e_2))}{\left[\frac{x}{u} + A(\lambda(e_1 + e_2))\left(1 - \frac{\alpha x}{u}\right)\right]^2} - e_i\ .$$

Any non-cooperative equilibrium $(\widehat{e}_1, \widehat{e}_2)$ is therefore defined by:

$$\begin{aligned} \widehat{e}_1 &= \arg\max_e \{\Pi^*(\lambda(e + \widehat{e}_2)) - e\} \\ \widehat{e}_2 &= \arg\max_e \{\Pi^*(\lambda(\widehat{e}_1 + \widehat{e})) - e\} \end{aligned} \tag{4}$$

Consider now total industry profits. If the platforms could choose (e_1, e_2) in order to maximize joint profits in stage 1, then the resulting total level of education $e^* = e_1 + e_2$ would be the solution to

$$\max_e \{2\Pi^*(\lambda(e)) - e\}\ . \tag{5}$$

Assuming that $\lambda(.)$ is such that both optimization problems above are well-defined, i.e., e^* and $\widehat{e}_1 + \widehat{e}_2$ are unique, we obtain:

Proposition 4 *With platform competition, the total equilibrium level of investment in user education* $(\widehat{e}_1 + \widehat{e}_2)$ *is always lower than the level that maximizes industry profits* (e^*).

This result is not surprising: It is simply an instance of the tragedy of the commons, given the assumed complementarity between the platforms' investments.

When $y \to 0$ and $\frac{x}{u} = 1$, the total equilibrium investment $2\widehat{e}$ converges to the monopoly optimal investment e^*. Conversely, when $x \to 0$ and $y = 1$, the equilibrium investment $\widehat{e}$ converges to 0, which is the equilibrium investment under Hotelling competition with fixed market size. This is because without market expansion both platforms prefer to keep λ as low as possible, therefore they never want to invest in educating users.

7 Conclusions

We have analyzed and compared two different mechanisms of expectation formation by agents in markets with network effects. The two mechanisms can be interpreted as corresponding to differences in the behavior of agents in real markets. Responsive rational expectations correspond to sophisticated users, who are able to calculate the impact of platform prices on other agents' decisions to join. Fixed rational expectations correspond to unsophisticated users, who only rely on external information about platform adoption and do not take into account the effect of prices on the expected behavior of other users.

Needless to say, there exist other ways to model different degrees of user sophistication when it comes to forming expectations in markets with network effects. Our paper provides a first pass at showing that these differences have substantive implications for firm profits and market outcomes (although the qualitative effects of the magnitude of network effects are unchanged). Specifically, we have shown that firms have different preferences regarding the composition of their user base depending on market structure. Firms with market power always prefer to face more sophisticated users, whereas firms competing for share in a market of fixed size always prefer to face more unsophisticated users. These preferences create different incentives to educate users about the effects of prices under different market structures. Furthermore, our analysis has implications for empirical research on network effects: specifying the mechanism through which expectations are formed is key to accurately estimating the strength of network effects and the impact of prices on demand.

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A Appendix: Proofs

A.1 Monopoly Profits with General Demand

Proof of Lemma 1 (page 8) with general (non-linear) demand

The result that $\Pi_f^* < \Pi_r^*$ is independent of any assumptions about the shape of the demand function. Indeed, when expectations are fixed rational we have $D\left(n_f^*, p_f^*\right) = n_f^*$ in equilibrium, so that $n_f^* = \widehat{n}\left(p_f^*\right)$, which means $\Pi_f^* = p_f^* \widehat{n}\left(p_f^*\right)$. But we know that p_r^* maximizes $p\widehat{n}(p)$ while p_f^* does not. Therefore $\Pi_f^* < \max_p p\widehat{n}(p) = \Pi_r^*$.

Let us now prove that $p_r^* < p_f^*$. Suppose all second-order conditions are well behaved and solutions are interior. Consider first the case of responsive rational expectations. The first-order condition defining p_r^* is

$$p\widehat{n}'(p) + \widehat{n}(p) = 0\,,$$

and the left-hand side term is decreasing in p so that p_r^* is the unique solution.

Since $\widehat{n}(p) = D(\widehat{n}(p), p)$, we can use the implicit function theorem to obtain

$$\widehat{n}'(p) = \frac{D_2(\widehat{n}(p), p)}{1 - D_1(\widehat{n}(p), p)}\,,$$

where $D_i(.,.)$ is the derivative in the i'th argument of $D(.,.)$, $i \in \{1, 2\}$. Note that $D_2(.,.) < 0$ and we must have $0 < D_1(.,.) < 1$.

Thus, we can re-write the first-order condition that defines p_r^* as

$$p\frac{D_2\left(\widehat{n}\left(p\right),p\right)}{1-D_1\left(\widehat{n}\left(p\right),p\right)}+\widehat{n}\left(p\right)=0\,. \tag{6}$$

Consider now the first-order condition that defines p_f^*:

$$D\left(n_f^*,p_f^*\right)+p_f^*D_2\left(n_f^*,p_f^*\right)=0\,,$$

where n_f^* verifies $D\left(n_f^*,p_f^*\right)=n_f^*=\widehat{n}\left(p_f^*\right)$. We can therefore re-write the equation determining p_f^* as

$$p_f^*D_2\left(\widehat{n}\left(p_f^*\right),p_f^*\right)+\widehat{n}\left(p_f^*\right)=0\,. \tag{7}$$

Evaluating the left hand side of (6) at $p=p_f^*$ and using (7), we clearly obtain a negative number since $D_2\left(.,.\right)<0$ and $0<1-D_1\left(.,.\right)<1$. Since the left hand side of (6) is decreasing in p, we can conclude that $p_r^*<p_f^*$, which also implies $n_r^*>n_f^*$.

A.2 Mixed Monopoly

Proof of Proposition 1

Consider the one-sided model and suppose now that a fraction λ of users is "sophisticated", i.e., has rational expectations which can be influenced by the platform's prices; whereas a fraction $(1-\lambda)$ has fixed rational expectations. In this case, for any platform price p, realized demand is

$$n=\lambda\left(V+\alpha n-p\right)+\left(1-\lambda\right)\left(V+\alpha n_f-p\right)\,,$$

which yields

$$n\left(p,n_f\right)=\frac{V-p+\left(1-\lambda\right)\alpha n_f}{1-\lambda\alpha}\,.$$

Thus, realized demand is a function of price and also of the fixed demand expectation formed by the“unsophisticated" users.

The platform solves

$$\max_p\left\{p\cdot\frac{V-p+\left(1-\lambda\right)\alpha n_f}{1-\lambda\alpha}\right\}\,,$$

which yields

$$\widehat{p}\left(n_f\right)=\frac{V+\left(1-\lambda\right)\alpha n_f}{2}\,,$$

$$n\left(n_f\right)=\frac{V+\left(1-\lambda\right)\alpha n_f}{2\left(1-\lambda\alpha\right)}\,.$$

In equilibrium it must be that $n(n_f) = n_f$, therefore,

$$n^*(\lambda) = \frac{V}{2-\alpha-\lambda\alpha},$$

$$p^*(\lambda) = \frac{V(1-\lambda\alpha)}{2-\alpha-\lambda\alpha}.$$

Finally, platform profits are

$$\Pi^*(\lambda) = \frac{V^2(1-\lambda\alpha)}{(2-\alpha-\lambda\alpha)^2}.$$

A.3 Hotelling with fixed market size

Here we directly tackle the case when the two platforms can be vertically differentiated. Specifically, the standalone utility derived by any user from platform 1 is V_1, while that derived from platform 2 is V_2, where we assume $\Delta V \equiv V_1 - V_2 \geq 0$. In the main text preceding Proposition 2, we had assumed $\Delta V = 0$.

Following the same reasoning as in the main text, total realized demands for the two platforms are determined by

$$\begin{aligned} n_1 &= \lambda\left[\frac{1}{2}+\frac{\Delta V+\alpha(n_1-n_2)+p_2-p_1}{2t}\right]+(1-\lambda)\left[\frac{1}{2}+\frac{\Delta V+\alpha(n_{f1}-n_{f2})+p_2-p_1}{2t}\right], \\ n_1 &= \lambda\left[\frac{1}{2}+\frac{-\Delta V+\alpha(n_2-n_1)+p_1-p_2}{2t}\right]+(1-\lambda)\left[\frac{1}{2}+\frac{-\Delta V+\alpha(n_{f2}-n_{f1})+p_1-p_2}{2t}\right]. \end{aligned}$$

Recall:

$$\begin{aligned} n_1 &= \lambda\left[\frac{1}{2}+\frac{\Delta V+\alpha(n_1-n_2)+p_2-p_1}{2t}\right]+(1-\lambda)\left[\frac{1}{2}+\frac{\Delta V+\alpha(n_{f1}-n_{f2})+p_2-p_1}{2t}\right], \\ n_1 &= \lambda\left[\frac{1}{2}+\frac{-\Delta V+\alpha(n_2-n_1)+p_1-p_2}{2t}\right]+(1-\lambda)\left[\frac{1}{2}+\frac{-\Delta V+\alpha(n_{f2}-n_{f1})+p_1-p_2}{2t}\right]. \end{aligned}$$

Taking the difference, solving for $(n_1 - n_2)$ and then replacing in the expression of n_1 and n_2, we obtain:

$$n_1 = \frac{1}{2}+\frac{\Delta V+p_2-p_1+\alpha(1-\lambda)(n_{f1}-n_{f2})}{2(t-\alpha\lambda)}, \quad (8)$$

$$n_2 = \frac{1}{2}+\frac{-\Delta V+p_1-p_2+\alpha(1-\lambda)(n_{f2}-n_{f1})}{2(t-\alpha\lambda)}. \quad (9)$$

Platform i's profits are equal to $p_i n_i$. Taking the first order condition in p_i and evaluating it at the equilibrium prices p_i^* and demands n_i^*, we have

$$n_i^* + p_i^* \frac{dn_i}{dp_i} = 0.$$

But $\frac{dn_i}{dp_i} = \frac{-1}{2(t-\alpha\lambda)}$ from the expressions above. Thus,

$$n_i^* = \frac{p_i^*}{2(t-\alpha\lambda)},$$

so that

$$n_1^* - n_2^* = \frac{p_1^* - p_2^*}{2(t-\alpha\lambda)}.$$

We can also evaluate expressions (8) and (9) in equilibrium ($n_{fi} = n_i = n_i^*$), which leads to

$$\begin{aligned} n_1^* &= \frac{1}{2} + \frac{\Delta V + p_2^* - p_1^* + \alpha(1-\lambda)(n_1^* - n_2^*)}{2(t-\alpha\lambda)}, \\ n_2^* &= \frac{1}{2} + \frac{-\Delta V + p_1^* - p_2^* + \alpha(1-\lambda)(n_2^* - n_1^*)}{2(t-\alpha\lambda)}. \end{aligned}$$

Taking the difference and solving for $(n_1^* - n_2^*)$ we obtain

$$n_1^* - n_2^* = \frac{\Delta V + p_2^* - p_1^*}{t-\alpha}.$$

Using the two different expressions of $(n_1^* - n_2^*)$ as functions of $(p_1^* - p_2^*)$ we obtain

$$\begin{aligned} p_1^* - p_2^* &= \frac{2(t-\lambda\alpha)\Delta V}{3t-2\lambda\alpha-\alpha}, \\ n_1^* - n_2^* &= \frac{\Delta V}{3t-2\lambda\alpha-\alpha}. \end{aligned}$$

Finally, this allows us to compute n_i^*, p_i^*,m and equilibrium platform profits:

$$\begin{aligned} n_1^* &= \frac{1}{2} + \frac{\Delta V}{2(3t-2\lambda\alpha-\alpha)} \quad \text{and} \quad n_2^* = \frac{1}{2} - \frac{\Delta V}{2(3t-2\lambda\alpha-\alpha)}, \\ p_1^* &= t - \lambda\alpha + \frac{(t-\lambda\alpha)\Delta V}{3t-2\lambda\alpha-\alpha} \quad \text{and} \quad p_1^* = t - \lambda\alpha - \frac{(t-\lambda\alpha)\Delta V}{3t-2\lambda\alpha-\alpha}, \\ \Pi_1^*(\lambda) &= \frac{t-\lambda\alpha}{2}\left[1 + \frac{\Delta V}{3t-2\lambda\alpha-\alpha}\right]^2 \quad \text{and} \quad \Pi_2^*(\lambda) = \frac{t-\lambda\alpha}{2}\left[1 - \frac{\Delta V}{3t-2\lambda\alpha-\alpha}\right]^2. \end{aligned}$$

For the equilibrium to be interior (i.e., $n_2^* > 0$), we must have $\Delta V < 3t - 2\lambda\alpha - \alpha$ for all $\lambda \in [0,1]$, i.e.,

$$\Delta V < t - \alpha.$$

Clearly, $\Pi_2^*(\lambda)$ is decreasing in λ. Meanwhile, the sign of the first-order derivative of $\Pi_1^*(\lambda)$ in λ is equal to the sign of

$$\Delta V - \frac{(3t-2\lambda\alpha-\alpha)^2}{t-2\lambda\alpha-\alpha},$$

which is negative given the inequality above. Thus, $\Pi_1^*(\lambda)$ is also decreasing in λ.

Finally, note that when $\Delta V = 0$, we obtain the profit expressions in the main text:

$$\Pi_1^*(\lambda) = \Pi_2^*(\lambda) = \frac{t - \lambda\alpha}{2}\,.$$

A.4 Platform Competition with Market Expansion

Proof of extended Hotelling competition (Proposition 3 on page 13)

Recall the expression of total realized demand for platform 1:

$$\begin{aligned} n_1 \;&=\; \lambda\left\{x\left(\frac{V + \alpha n_1 - p_1}{u}\right) + y\left[\frac{1}{2} + \frac{\alpha(n_1 - n_2) + p_2 - p_1}{2t}\right]\right\} \\ &\quad + (1-\lambda)\left\{x\left(\frac{V + \alpha n_{f1} - p_1}{u}\right) + y\left[\frac{1}{2} + \frac{\alpha(n_{f1} - n_{f2}) + p_2 - p_1}{2t}\right]\right\}, \end{aligned}$$

which is equivalent to

$$n_1\left(1 - \frac{\lambda\alpha x}{u}\right) = \frac{y}{2} + \frac{x(V - p_1)}{u} + \frac{y(p_2 - p_1)}{2t} + \frac{\alpha y\left[\lambda(n_1 - n_2) + (1-\lambda)(n_{f1} - n_{f2})\right]}{2t} + \frac{(1-\lambda)\alpha x n_{f1}}{u}\,.$$

Similarly,

$$n_2\left(1 - \frac{\lambda\alpha x}{u}\right) = \frac{y}{2} + \frac{x(V - p_2)}{u} + \frac{y(p_1 - p_2)}{2t} + \frac{\alpha y\left[\lambda(n_2 - n_1) + (1-\lambda)(n_{f2} - n_{f1})\right]}{2t} + \frac{(1-\lambda)\alpha x n_{f2}}{u}\,.$$

Taking the difference $n_1 - n_2$ and solving for $(n_1 - n_2)$, then replacing in the two expressions above we obtain n_1 and n_2 as functions of (p_1, p_2, n_1^e, n_2^e) only:

$$n_1\left(1 - \frac{\lambda\alpha x}{u}\right) = \frac{y}{2} + \frac{x(V - p_1)}{u} + \frac{y(p_2 - p_1)}{2t(1 - \lambda\alpha k)} + \frac{(1-\lambda)y\alpha(n_{f1} - n_{f2})}{2t(1 - \lambda\alpha k)} + \frac{(1-\lambda)\alpha x n_{f1}}{u}, \tag{10}$$

$$n_2\left(1 - \frac{\lambda\alpha x}{u}\right) = \frac{y}{2} + \frac{x(V - p_2)}{u} + \frac{y(p_1 - p_2)}{2t(1 - \lambda\alpha k)} + \frac{(1-\lambda)y\alpha(n_{f2} - n_{f1})}{2t(1 - \lambda\alpha k)} + \frac{(1-\lambda)\alpha x n_{f2}}{u}, \tag{11}$$

where

$$k \equiv \frac{x}{u} + \frac{y}{t}\,.$$

In equilibrium, both platforms simultaneously maximize profits $p_i n_i$ over p_i, where $i = 1, 2$. The first order condition for platform i is

$$n_i + p_i\frac{\partial n_i}{\partial p_i} = 0\,,$$

where $\partial n_i / \partial p_i$ can be directly derived from (10).

In equilibrium, $p_1 = p_2 = p^*(\lambda)$, $n_1 = n_{f1} = n_2 = n_{f2} = n^*(\lambda)$, so that the two first-order conditions are both equivalent to

$$n^*(\lambda) \cdot \left(1 - \frac{\lambda\alpha x}{u}\right) = p^*(\lambda) \cdot \left[\frac{y}{2t(1-\lambda\alpha k)} + \frac{x}{u}\right] .$$

The other equation linking $n^*(\lambda)$ and $p^*(\lambda)$ is obtained simply by plugging the equilibrium values into (10):

$$n^*(\lambda) \cdot \left(1 - \frac{\alpha x}{u}\right) = \frac{y}{2} + \frac{x[V - p^*(\lambda)]}{u} .$$

Solving the last two equations for $[n^*(\lambda), p^*(\lambda)]$ we finally obtain the expression of equilibrium platform profits:

$$\Pi^*(\lambda) = p^*(\lambda) n^*(\lambda) = \frac{\left(\frac{y}{2} + \frac{Vx}{u}\right)^2 A(\lambda)}{\left[\frac{x}{u} + A(\lambda)\left(1 - \frac{\alpha x}{u}\right)\right]^2} ,$$

where

$$A(\lambda) \equiv \frac{\frac{y}{2t(1-\lambda\alpha k)} + \frac{x}{u}}{1 - \frac{\lambda\alpha x}{u}}$$

is increasing in λ.

A.5 Educating the market

Proof of Proposition 4

We know from section 5 that $\Pi^*(\lambda)$ is maximized by $\lambda < 1$, so that both e^* and $(\widehat{e}_1 + \widehat{e}_2)$ are strictly less than 1. There are two cases. If $\widehat{e}_1 + \widehat{e}_2 = 0$ (which is equivalent to $\widehat{e}_1 = \widehat{e}_2 = 0$) then the result clearly holds: $e^* \geq \widehat{e}_1 + \widehat{e}_2$. Suppose $\widehat{e}_1 + \widehat{e}_2 > 0$. Then we must necessarily have $e^* > 0$. Indeed, (4) and the fact that $\lambda(e)$ is increasing imply:

$$\begin{aligned} \Pi^*(\lambda(\widehat{e}_1 + \widehat{e}_2)) - \widehat{e}_1 &> \Pi^*(\lambda(\widehat{e}_2)) \geq \Pi^*(\lambda(0)) , \\ \Pi^*(\lambda(\widehat{e}_1 + \widehat{e}_2)) - \widehat{e}_2 &> \Pi^*(\lambda(\widehat{e}_1)) \geq \Pi^*(\lambda(0)) . \end{aligned}$$

Summing the two inequalities, we obtain

$$2\Pi^*(\lambda(\widehat{e}_1 + \widehat{e}_2)) - (\widehat{e}_1 + \widehat{e}_2) > 2\Pi^*(\lambda(0)) ,$$

which means that $e = \widehat{e}_1 + \widehat{e}_2$ dominates $e = 0$ from the joint perspective of the two firms.

Thus, both e^* and $(\widehat{e}_1 + \widehat{e}_2)$ must be interior solutions (i.e., strictly between 0 and 1), which means they are given by the first-order conditions:

$$\begin{aligned} 2\Pi^{*\prime}\left(\lambda\left(e^*\right)\right)\lambda'\left(e^*\right) &= 1\,, \\ \Pi^{*\prime}\left(\lambda\left(\widehat{e}_1 + \widehat{e}_2\right)\right)\lambda'\left(\widehat{e}_1 + \widehat{e}_2\right) &= 1\,. \end{aligned}$$

Together with the assumption that second order conditions for unique solutions are satisfied, these last two equations imply $e^* > \widehat{e}_1 + \widehat{e}_2$.

B Appendix: Two-sided Platforms

In this appendix, we briefly show that our analysis also applies to platforms with two-sided network effects. In particular, we derive the profit expressions for two-sided platforms when a variable fraction λ of users on one side (U for users) holds responsive rational expectations, while the remaining fraction holds fixed rational expectations. All users on the other side (D for developers of content or applications) hold responsive rational expectations. The interpretation is straightforward: corporate developers are much more likely to be sophisticated, i.e., fully informed of prices on both sides and able to calculate their impact on user demand.

B.1 Monopoly Two-sided Platform

We assume demands for a monopoly two-sided platform are:

$$\begin{aligned} n^U &= \lambda\left(1 + \alpha^U n^D - p^U\right) + (1-\lambda)\left(1 + \alpha^U n_f^D - p^U\right)\,, \\ n^D &= \alpha^D n^U - p^D\,, \end{aligned}$$

where we must assume

$$\alpha^D + \alpha^U < 2\,.$$

Applying the same methodology described in the text for one-sided platforms, we obtain:

$$n^{U*} = \frac{2}{4 - \left(\alpha^D + \lambda\alpha^U\right)\left(\alpha^D + \alpha^U\right)} \quad \text{and} \quad n^{D*} = \frac{\alpha^D + \lambda\alpha^U}{4 - \left(\alpha^D + \lambda\alpha^U\right)\left(\alpha^D + \alpha^U\right)}\,,$$

$$p^{U*} = \frac{2 - \alpha^D\left(\alpha^D + \lambda\alpha^U\right)}{4 - \left(\alpha^D + \lambda\alpha^U\right)\left(\alpha^D + \alpha^U\right)} \quad \text{and} \quad p^{D*} = \frac{\alpha^D - \lambda\alpha^U}{4 - \left(\alpha^D + \lambda\alpha^U\right)\left(\alpha^D + \alpha^U\right)}\,,$$

$$\Pi^*\left(\lambda\right) = \frac{4 - \left(\alpha^D + \lambda\alpha^U\right)^2}{\left[4 - \left(\alpha^D + \lambda\alpha^U\right)\left(\alpha^D + \alpha^U\right)\right]^2}\,.$$

As in the one-sided case, platform profits are increasing in λ. The intuition is the same: A monopoly platform always prefers to face more sophisticated agents because non-sophisticated agents are akin to agents having credibly committed to under-estimate the value of network effects and therefore to ignore the multiplier effects of small price decreases.

The only difference is the effect of λ on prices:

$$\begin{aligned} sign\left(\frac{dp^{U*}}{d\lambda}\right) &= sign\left(\alpha^U - \alpha^D\right) , \\ sign\left(\frac{dp^{D*}}{d\lambda}\right) &= sign\left[\alpha^D\left(\alpha^U + \alpha^D\right) - 2\right] , \end{aligned}$$

both of which can be either positive or negative. Note however that $\alpha^U - \alpha^D > 0$ implies $\alpha^D\left(\alpha^U + \alpha^D\right) - 2 < 0$ and conversely, $\alpha^D\left(\alpha^U + \alpha^D\right) - 2 > 0$ implies $\alpha^U - \alpha^D < 0$ (under the assumption $\alpha^D + \alpha^U < 2$). This means that at least one of the two optimal prices must be decreasing in λ. Recall that in the case of a monopoly one-sided platform, the optimal price is always decreasing in λ.

B.2 Competing Two-sided Platforms

Consider now two symmetric platforms competing a la Hotelling on both sides (i.e., both sides single-home). Demands for platform $i \in \{1, 2\}$ are:

$$\begin{aligned} n_i^U &= \frac{1}{2} + \frac{\lambda\alpha^U\left(n_i^D - n_j^D\right) + (1-\lambda)\left(n_{fi}^D - n_{fj}^D\right) + p_j^U - p_i^U}{2t^U} , \\ n_i^D &= \frac{1}{2} + \frac{\alpha^D\left(n_i^U - n_j^U\right) + p_j^D - p_i^D}{2t^D} , \end{aligned}$$

where t^U and t^D are the transportation costs on each side, and

$$t^U > \alpha^U \text{ and } t^D > \alpha^D .$$

In the symmetric equilibrium, the platforms split the two sides equally. Equilibrium prices and profits are:

$$p^{U*} = t^U - \alpha^D \quad \text{and} \quad p^{D*} = t^D - \lambda\alpha^U ,$$

$$\Pi^*(\lambda) = \frac{t^U + t^D - \alpha^D - \lambda\alpha^U}{2} ,$$

decreasing in λ as expected.

Results are similar and maintain the same comparative statics in λ in the case of two-sided competition with multi-homing on one side.