Sales of perishable goods without consumer heterogeneity or competition: exploring the consumers' option value*

Alexei Alexandrov†
University of Rochester‡

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Abstract

I show that storability, many periods, consumer heterogeneity, and competition are not necessary to generate temporary price discounts as an optimal pricing scheme. The model has full information (except for the price realization), one period, one firm, and one consumer who requires a positive expected utility to patronize the firm. The reason for the potential profitability of sales is that, given standard assumptions, consumer utility is convex in price, and thus the consumer becomes as if risk-loving in price. The consumer adjusts his quantity purchased (and consumed) based on price. However, the firm's profit is concave in price, so the firm is as-if risk-averse. Thus, when the consumer is relatively more risk-loving, sales are profitable.

1 Introduction

Sales (temporary price cuts) are prevalent in the marketplace. There are many explanations for why we see sales in the real world, but most of them have to do either with a mixed outcome of a competitive price setting game, price discrimination along some dimension of consumer heterogeneity, or dynamic effects of consumer and/or firm inventory levels.[1]

I present an explanation which relies on neither of the mechanisms above. In particular, I consider a monopolist selling a non-storable product, with one consumer in the market. The consumer has a standard utility function from purchasing the product, resulting in a downward sloping demand. The only somewhat non-standard assumption is that the consumer requires some positive utility (known to the firm) to come to the store to shop.

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‡Assistant Professor of Economics and Management, email: Alexei.Alexandrov@Simon.Rochester.edu

[1]See, for example, Varian (1980), Narasimhan (1988), and Lal and Villas-Boas (1998) for the first type, Salop (1977) for the second type, and Sobel (1984) and Aguirregabiria (1999) for the third type. See Berck, Brown, Perloff, and Villas-Boas (2008), and references within for empirical tests of many of these hypotheses. See Rao (1991) for a competitive outcome of sales where promotions are explicitly modeled.

An easy way to view this requirement is a consideration set argument. If the consumer does not get a reasonable expected utility from patronizing the store/brand, the consumer switches to an outside option. The utility is expected because if a firm is randomizing prices, the consumer knows the distribution of the randomization (the probability of a given price occurring), but not the realization. For simplicity, I assume that the game lasts one period, and that the firm cannot change the price distribution once the consumer comes to the store. This could be endogenized by building a reputation model.

The key to understanding why sales might be profitable in this model is the observation that the consumer's utility is convex in price. The fact that the price is random gives consumer an option value of adjusting the purchase quantity. If the product is relatively cheaper, then the consumer buys and consumes more. If the product is more expensive, the consumer buys and consumes less. If the consumer is not able to adjust consumption/purchase quantity, then any random price fluctuations do not matter in expectation, since the consumer utility is linear in price. Crucially, due to the option value of quantity adjustment, the consumer becomes as if risk-loving in price, and prefers a price lottery over a certain mean of the lottery.[2] Note, that while the consumer's ability to store increases this option value, it is not necessary, and I do not assume it in my model.

The sales are not always a good idea for the firm. The problem is that while consumer utility is convex in price, we generally assume that the firm's profit is concave in price, and that makes price randomization unattractive. Thus, the key to sales being successful is a profit function which is relatively close to linear (not too concave) in price, and a utility function which is relatively far from being linear (really convex). An issue that complicates the analysis is that demand is derived from the utility function, which then enters the profit function, making the two interrelated.

The closest strand of literature outside of the sales articles is the similar idea that a firm's profit is convex in its netput prices when the firm is in perfect competition and has a production function characterized by diseconomies of scale, see Oi (1961) and Mas-Colell, Whinston, and Green (1995). See Alexandrov (2011) showing similar results for both cost and demand shocks in all other market structures, with any nonlinear cost (including decreasing marginal cost, as long as it satisfies the standard second order conditions).

2 Model and General Results

There is one period. The monopolist has a cost of $C(q)$. The consumer derives utility of

$$U(q) = u(q) - pq, \tag{1}$$

where function u is concave and increasing, $u(0) = 0$, p is the price, and q is the quantity purchased. For a given price, the consumer maximizes utility with respect to quantity, resulting in

$$u'(q^*) = p, \tag{2}$$

[2] I assume that the consumer is risk-neutral. Sufficient risk-aversion ensures no sales in this model.

which implicitly defines consumer demand, $q(p)$.

Both the firm and the consumer are risk-neutral. The firm decides on two prices (sale and regular, or low and high), p_L and p_H, and the probability with which each price occurs, α and $1-\alpha$. The consumer must derive a utility (in expectation) of at least $\overline{U}$ to patronize the firm. Let $\overline{p}$ be the price such that $U(\overline{p}) = \overline{U}$ – the price that makes sure that the consumer's constraint is just satisfied. To rule out a trivial case, I assume that $\overline{p} < p_m$, where p_m is the price that an unconstrained firm charges $\left(\frac{\partial \Pi}{\partial p}\Big| p_m = 0\right)$. If consumer expects that this condition is met, then he buys according to the demand function derived above. If the consumer expects the condition not to be met, then he does not patronize the firm, and the firm does not sell anything.

Everything is common knowledge, except for the *realization* of the price. The timing is as follows: the firm decides on the distribution of prices, the consumer decides on whether he will patronize the firm given the price distribution, then the price is drawn from the distribution, and consumer buys according to his demand function (if he decided to patronize the store).

Note that differentiating consumer utility with respect to p after plugging in the optimal consumption quantity results in

$$\frac{\partial U}{\partial p} = u' \frac{\partial q}{\partial p} - q - p\frac{\partial q}{\partial p} = -q, \tag{3}$$

which is simply the Shephard's Lemma. Differentiating with respect to price again results, while implicitly differentiating equation (2), in:

$$\frac{\partial^2 U}{\partial p^2} = -\frac{\partial q}{\partial p} = -\frac{1}{u''(q)} > 0, \tag{4}$$

showing that consumer utility is indeed convex in price.

Thus, the monopolist's problem becomes maximizing the expected profit function with respect to the price distribution (p_L, p_H, and α):

$$max\ \Pi(p_L, p_H, \alpha) = \alpha\pi(p_L) + (1-\alpha)\pi(p_H), \tag{5}$$

subject to the expected utility of the consumer being at least the required value of $\overline{U}$.

$$\alpha U(q(p_L)) + (1-\alpha)U(q(p_H)) \geq \overline{U}, \tag{6}$$

where $\pi(p) = pq(p) - C(q(P))$.

The figure illustrates how sales can be more profitable than charging $\overline{p}$. Note that the regular price (p_H) is likely NOT the price that the firm would charge without the constraint (p_m). At that price, lowering the regular price by a bit is a second order effect on profit, but a first order effect on utility, and thus the firm's regular price is lower than that.

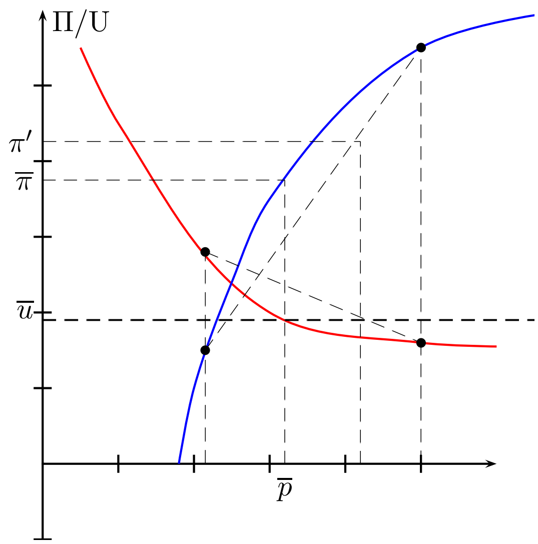

Figure 1: Profit is concave in price, and the utility function is convex. The sales are profitable if there exists a convex combination of two prices, such that the consumer's constraint is satisfied, and the profit is larger than $\overline{\pi}$.

Proposition 1 *The optimal price distribution satisfies*

$$\frac{\pi(p_H) - \pi(p_L)}{U(p_L) - U(p_H)} = \frac{\frac{\partial \pi(p_i)}{\partial p_i}}{q_i}, \tag{7}$$

for $i = L, H$*, and*

$$\alpha = \frac{\overline{U} - U(p_H)}{U(p_L) - U(p_H)}. \tag{8}$$

Proof. It is clear that the constraint must bind, and thus we can solve for α. The FOCs for the prices result in

$$\frac{\partial \alpha}{\partial p_L}(\pi_H - \pi_L) = \frac{\partial \pi_L}{\partial p_L}\alpha, \tag{9a}$$

$$\frac{\partial \alpha}{\partial p_H}(\pi_H - \pi_L) = \frac{\partial \pi_H}{\partial p_H}(1-\alpha). \tag{9b}$$

From the binding constraint, it is easy to show that (by implicit differentiation)

$$\frac{\partial \alpha}{\partial p_L} = \frac{q_L}{U_L - U_H}\alpha, \tag{10a}$$

$$\frac{\partial \alpha}{\partial p_H} = \frac{q_H}{U_L - U_H}(1-\alpha). \tag{10b}$$

Plugging these into the FOCs, we get the result in the proposition. ■

Corollary 1 *If the firm's marginal cost is zero, and the elasticity of demand is monotone in price,* *sales are not profitable.*

Proof. Marginal cost of zero results in the RHS of the equations defining the price to become $1+\eta_i$, where η_i is the elasticity at price p_i. Since LHS is the same, monotone elasticity implies that the two equations are satisfied only if $p_H = p_L$. ■

Suppose that the prices p_H and p_L are given. Then it is easy to check whether sales are profitable.

Corollary 2 *Suppose for a given price $\overline{p}$, p_H and p_L are considered as the potential regular and sale price. Sales are profitable if and only if the ratio of the differences in profits between the regular price and the certain price to the difference in profits between the regular price and the low price is higher than the ratio of the corresponding utility differences.*

$$\frac{\overline{U} - U_H}{U_L - U_H} < \frac{\pi_H - \overline{\pi}}{\pi_H - \pi_L}, \tag{11}$$

moreover, if the sales are profitable than the left hand side of the inequality is the probability of a sale.

Proof. The constraint binds, and thus one can solve for the optimal α. Plugging that into the profit function, it's easy to see when that is larger than $\overline{\pi}$. ■

It is impossible (at least for now), to provide a general characterization of when the sales are profitable. Thus, I simplify the demand function in the next section to arrive at a more manageable conclusion.

3 Demand on a grid

Continuing with the setup from last section, suppose that the firm's cost is zero. Suppose that $v_3 > v_2 > v_1$, and that the derived demand is as follows:

Table 1: Consumer's Derived Demand

Quantity	**Price**
q_1	$[0, v_1]$
q_2	$(v_1, v_2]$
q_3	$(v_2, v_3]$
0	$(v_3, +\infty)$

Also, assume that $v_3q_3 > v_2q_2 > v_1q_1$, in other words out of the three potential optimal prices, a higher price results in a higher profit.

Suppose that the consumer must receive at least $\overline{U} = U(p = v_2) = (v_3 - v_2)q_3$ in expectation to patronize the firm. In other words, if the distribution is degenerate, then the firm cannot charge more than $p = v_2$. I consider sales where $p_H = v_3$ and $p_L = v_1$.

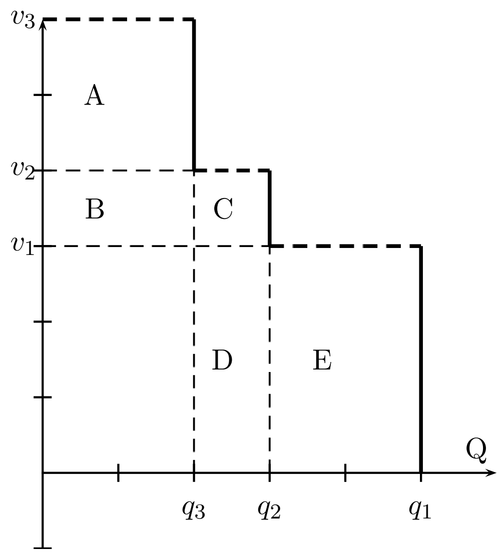

Figure 2: Step demand function with consumer surplus and profit differences breakdown.

Examining the figure, from Corollary 2, sales are profitable iff

$$\frac{A}{A+B+C} < \frac{A-D-C}{A+B-D-E}, \tag{12}$$

where my assumptions imply that $E < C$ and $C + D < A$. It is easy to see that E almost the same size as C and a low B guarantees the profitability of sales. In particular, (v, q) pairs of $(10, 9); (7, 10); (6, 11)$ guarantee that sales are more profitable than a single price.[3]

4 Conclusion

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[3] $\pi_H = 90$, $\overline{\pi} = 70$, $\pi_L = 66$, $U_H = 0$, $\overline{U} = 27$, $U_L = 37$. Thus $\alpha = \frac{27}{37}$, and $90 \times \frac{10}{37} + 66 \times \frac{27}{37} > 70$.

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