

Mass and *Elite* in the Market for News: a Model with Heterogeneous Audience

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Abstract

In this paper we model a market for news where two sources issuing reports compete for the audience. Individuals are heterogeneous with respect to their tastes for the news-other contents mix of the reports. We show that there exists: a) an equilibrium where one of the reports is read by all the population (*mass* source), while the second one is read only by one class of readers (*élite* source). b) an equilibrium where the audience splits between the two reports. We allow for the existence of non-readers. The Sources endogenously provide reports with different combination of news-other contents and different levels of news accuracy.

Preliminary version.

1 Introduction

A widespread evidence from media markets shows that demand and supply for media and contents sharply differ across countries. As a matter of facts, the structure of media markets is characterized, on the supply side, by a mix of different sources of information (broadcasting, newspapers, radio, on-line outlets) and, on the demand side, by a population of heterogeneous consumers with different preferences about resorting to mass media. For instance, Table 1 shows the penetration of different media, by countries.

Table 1					
	Newspaper ¹	Free Press ¹	TV viewing ²	Radio ²	Internet ²
Austria	324.9	79.5	190	30	170
Belgium	157.9	28.7			
Canada	146.5	53.3	125	90	105
Denmark	235.0	126.6			
Finland	462.0	25.9			
France	152.0	53.8			
Germany	278.7		225	180	85
Japan	458.3	0.8	200	70	90
Ireland	217.5	40.3			
Italy	91.7	77.1	169*	46*	87*
Norway	538.3				
Netherlands	260.3				
Poland	98.0	13.8			
Portugal	59.5	38.5			
UK	284.7	47.7			
Spain	99.2	66.1			
Sweden	422.2	9.21			
Switzerland	322.0	22.7			
USA	192.0	9.5	300	270	100
¹ Diffusion x1000 citizens, 2009; ² Minutes per day, 2010					
Source: WAN, World Press Center, 2010; WAN, World Press Trends 2011; *ISTAT, Report, 2011					

It is well known that individual characteristics, such as gender, income level, education, geographical location, and so forth, affect both the consumers' propensity to access the different media and their tastes for contents, within a specific type of medium. For instance, focusing on the newspaper readership, Table 2 and 3 describe the most favorite sections according to the household income and the education level.

Table 2				
USA newspaper sections looked at (Mon-Fri) - 2011				
	readers 100%	HI<50,000\$	50\$ K<HI<99\$K	HI>100,000\$
Main news/ Front page section	79.9%	78.1%	81.2%	81.7%
Local news section	74.6%	74%	75.8%	74.1%
Sports section	55.7%	53.4%	57.5%	57.6%
Entertainment/ Life style section	54.5%	53.2%	55.8%	54.9%
International/ National news	53.2%	52%	53.7%	54.8%
Comics	53%	54.7%	52.7%	50%
Advertising/ Inserts/ Flyers	51.4%	51.8%	52.7%	48.8%
Food/ Cooking section	51%	50.9%	51.4%	50.4%
Editorial/ Opinion section	49.8%	48.8%	50.6%	50.8%
Classified advertising	49.7%	53.2%	49%	43.9%
Business/ Finance section	48.9%	45.2%	50.3%	53.9%
Movie listings and reviews	47.2%	47.6%	47.5%	45.7%
Home and garden section	47.1%	46.5%	47.8%	47.3%
TV or radio listings	45.%	46.8%	44.7%	41.8%
Science and technology section	43.9%	43%	44.4%	44.8%
Fashion section	42.3%	42.4%	42.7%	41.6%
HI=Household Income				
Source: NAA, 2011				

Newspaper sections looked at (Mon-Fri) - 2011			
	High School or less	Any College	College Grad or more
Main news/ Front page section	77.6%	81.5%	82.5%
Local news section	74.1%	75%	75.4%
Sports section	55.3%	55.9%	57.2%
Entertainment/ Life style section	52.4%	55.9%	57.2%
International/ National news	51.2%	54.6%	56.8%
Comics	53.4%	52.7%	52.2%
Advertising/ Inserts/ Flyers	52.1%	51%	50.2%
Food/ Cooking section	50.3%	51.4%	51.7%
Editorial/ Opinion section	48%	51.1%	53%
Classified advertising	53%	47.4%	45.1%
Business/ Finance section	45.8%	50.9%	54%
Movie listings and reviews	46.7%	47.4%	47.6%
Home and garden section	46.6%	47.5%	48%
TV or radio listings	46.2%	44.2%	43.9%
Science and technology section	42.6%	44.7%	46.3%
Fashion section	42.5%	42.2%	42.6%
Source: NAA, 2011			

In spite of this remarkable heterogeneity, which advertisers, marketing executives and media operators are certainly well aware of, economic literature still lacks an exhaustive explanation of the interaction between supply and demand sides in shaping the media equilibrium structure.

In the last few years, economic literature has paid great attention to the influence of mass media on information disclosure, capture and ideological bias to affect individuals behavior, mainly voting, as well as politics and policy (for an extensive survey, see Pratt and Stromberg (2011)). A substantial part of the contributions focuses on the media capture, as the influence exerted by political message senders (lobbies, parties or governments) to affect information disclosure, with important implications for voting behavior (Stromberg (2004); Besley and Prat (2006); Corneo (2006); Vaglio (2006) Di Tella and Franceschelli, (2009)). Another important stream of the literature analyzes the different dimensions of media bias - both supply and demand driven - with particular regards to the ideological stand bias (Gentzkow and Shapiro (2006); Mullainathan and Shleifer (2005); Baron (2006), Chan and Suen (2008); Durante and Knight (2009); Elmann and Germano (2009)). However, most of the contributions deals with the information role of mass media and their influence on voting and political behavior, without tackling

the role of the demand side in shaping the structure and the equilibrium in the market for news.

The present paper tries to fill this gap, by modelling a media market equilibrium where two profit-maximizing sources compete for the audience. We assume the advertising market to be competitive, with a uniform advertising fee taken as given by media firms. We also rule out by assumption the existence of influential pressure groups of any kind. Our main interest is the analysis of the implications of the individual choice between acquiring or not acquiring information in the media market and of heterogeneity across individuals from this viewpoint. More precisely, the audience is composed by rational individuals who choose whether or not to access to the outlets provided by the sources and how many of them. Media outlet contain a mix of news and "other services" (e.g. comics, weather forecasts, TV programs). Individuals differ as their favorite news/other services mix. Getting informed is a time-consuming activity; the opportunity cost of time depends on individual characteristics. Therefore, we analyze the market interaction between heterogenous individuals and sources providing contents. From an interpretive point of view, our model could be applied to a wide range of media. Nevertheless, to make the exposition more intuitive, we will refer to the newspaper market.

Notice also that the demand for information has a peculiar feature with respect to other goods. In fact, not only the quality, or the accuracy, of the information is valued by the consumers, but also the possibility to gather multiple information and to compare them might have a positive value for the consumers, namely the benefit of pluralism. In fact, media literature, both at theoretical and empirical level, has recognized that consumers are not only passive receivers of the news, but they actively affect news provisions by means of their ideological position, their education level and their behavior, the so-called "demand-driven bias" (e.g. Mullainathan.-Shleifer (2005), Gentzkow-Shapiro (2006, 2010), Burke, J. (2008), Chan, J.- Suen W., (2008); Corneo (2006); Elmann and Germano (2009); Larcinese (2009); Sobbrío (2011)). However, our work show some remarkable difference with the ongoing literature. A basic point in our analysis is that individuals get informed in order to find the optimal private allocation decision, where voting might be a relevant option, but not the only one. Information acquisition is costly, therefore, our framework allows for equilibria where at least some of the individuals do not read any reports. Furthermore, in our approach the individuals resort to media outlet because they are interested in hard news as well as other contents. While sources are defined by specific ratio between the news accuracy and the entertainment. Therefore, the report is de-

scribed as a bundle of characteristics (news and entertainment) which individuals have different preferences upon. Notice that the interaction of these two components deeply affect the media outlet demand; in other words the entertainment contents matters on the individuals' propensity to resort to media and to get informed.

According to this framework, we generate different patterns of reading in the case where the individual reads at maximum one report and in the case where they read up to two reports. Notice that, given the same structure of supply, there may exist different patterns in the use of the available information. It is also worthwhile to mention that the market size is endogenously determined, as well as the news accuracy and the weight addressed to entertainment.

Finally, let point out that these patterns correspond to different notions of pluralism. In the single reader case, pluralism consists in the possibility to choose among differentiated sources. While, in the double reader case pluralism consists in comparing information coming from different sources.

The paper is organized as follows. Section 2 describes the information value, while Section 3 explains the set up of the model, with respect of the individual demand and the sources supply. Then in Sections 4 and 5 respectively, we solve the model in the single-reader case and in the double-reader one. Some conclusions end the work.

2 Value of information

Suppose that there exist two possible states of the world, called A and B , with π as the prior probability that the state of the world is A . Individuals must choose among two possible actions, one which is appropriate when the state of the world is A and the other one which is appropriate in state B . This is what we call a *private allocation decision*. The actual state of the world is not known to the individual, at the moment of choice. If the state is A (B) and the appropriate action has been chosen, the ex post payoff is ω^A (ω^B). If instead the wrong action has been chosen, the ex post outcome, in the two cases is respectively l^A (l^B). We assume that

$$\pi\omega^A + (1 - \pi)l^B > \pi l^A + (1 - \pi)\omega^B \quad (1)$$

i.e. the action which would be appropriate in state A is optimal ex-ante, on the basis of the prior. If we define: $x^A = \pi(\omega^A - l^A)$ and $x^B = (1 - \pi)(\omega^B - l^B)$, (1) translates into

$$x_A > x_B \quad (2)$$

The expected value of utility from choosing the ex-ante optimal action is therefore (given the prior π):

$$V^0 = \pi\omega^A + (1 - \pi)l^B \quad (3)$$

Before choosing, the individual must make a *reading decision*. There exist two reports. Each report contains a statement concerning the state of the world. Such statements are correct with probabilities, q_1 and q_2 , respectively, which we call "accuracies" or "quality levels" of the two reports¹. In the next section we shall explain how these accuracy levels are determined.

If the individual reads just one report of quality q_i ($i = 1, 2$) his ex ante utility from trusting that report is:

$$V^I(q_i) = q_i(x_A + x_B) + \pi l_A + (1 - \pi)l_B \quad (4)$$

When the reader reads both reports, the information contained is exploited according to the following behavioral pattern: the reader trusts the sources when they agree about the state of the world, while he chooses the action appropriate to state A (the ex-ante best action) when they disagree. Any other pattern of behavior would be equivalent to trusting just one of the sources.² The ex-ante utility, in this case, is:

$$V^{II}(q_1, q_2) = \pi \{ (\omega^A - l^A) [q_1 + q_2 (1 - q_1)] + l^A \} + (1 - \pi) \{ (\omega^B - l^B) q_1 q_2 + l^B \} \quad (5)$$

$$V^{II}(q_1, q_2) = x_A [q_1 + q_2 (1 - q_1)] + \pi l^A + (1 - \pi) \{ (\omega^B - l^B) q_1 q_2 + l^B \} \quad (6)$$

Defining $F(q) = x_A(1 - q) + x_B q$, (5) can be rewritten as:

$$V^{II}(q_1, q_2) = F(q_1)q_2 + x_A q_1 = F(q_2)q_1 + x_A q_2 + \pi l_A + (1 - \pi)l_B \quad (7)$$

It is easy to see that: $F(q_1)q_2 + x_A q_1 = F(q_2)q_1 + x_A q_2$. Given q_i ($i = 1, 2$), the condition $q_j \geq 0.5$ ($j \neq i$) is necessary for the pattern just mentioned to be optimal. Otherwise it would be dominated by an alternative pattern where the reader "interprets" the statement "The state of the world is A (B)" from report j as if it were "The state of the world is B (A)" and then acts according to the first pattern. To avoid the unnecessary complications attached to this case ("you are so

¹See Ellman-Germano (2009) for a thorough discussion of the possible interpretations of q

²In Battaglion-Vaglio (2012) we show that this pattern is consistent with Bayesian belief updating.

unreliable that I believe the contrary of what you say") ,we shall assume from now on that $q_i \quad (i : 1, 2) \geq 0.5$. For the sake of simplicity, in what follows we shall also assume $l^A = l^B = 0$.

3 Publishing costs and the cost of reading

We analyze a situation where there exist two media firms (or sources) which we call Source One and Source Two respectively. Each source receives an independent signal about the actual state of the world. The probability of correctly receiving the signal is chosen by each firm at some cost and it is denoted as q_1 for Source One and q_2 for Source Two. Each source issue a *report* which, as we said in the Introduction, is a bundle containing a truthful statement about the signal received and an additional characteristic which we call *entertainment* (e_1, e_2 for the two firms respectively). A source choosing a value of q for accuracy and e for entertainment incurs a cost $\frac{c}{2}q^2 + re$, with $c, r > 0$. Revenue comes entirely from advertising, and it equals the exogenous per-reader fee w , times the number of readers (for the sake of simplicity $w = 1$).

The value of information, as described in the previous section, is assumed to be the same across the population. Reading a piece of entertainment e gets the reader an additional utility ke , with $k > 0$. Conversely, readers are assumed to be heterogenous with respect to their reading costs. The cost of reading consists entirely in the opportunity cost of time. This has two components. The first one is uniform across readers: they need tq units of time to read a piece of information whose level of accuracy is q ($t > 0$). The second one is the heterogenous component and it is related to the information-entertainment mix. Each individual is characterized by a parameter h_i ; if the individual reads a report whose entertainment-information ratio lies below h_i then he incurs an additional reading cost: $\varphi(qh_i - e)$, with $\varphi > 0$. Then, in fact the total cost of reading is for the h_i individual:

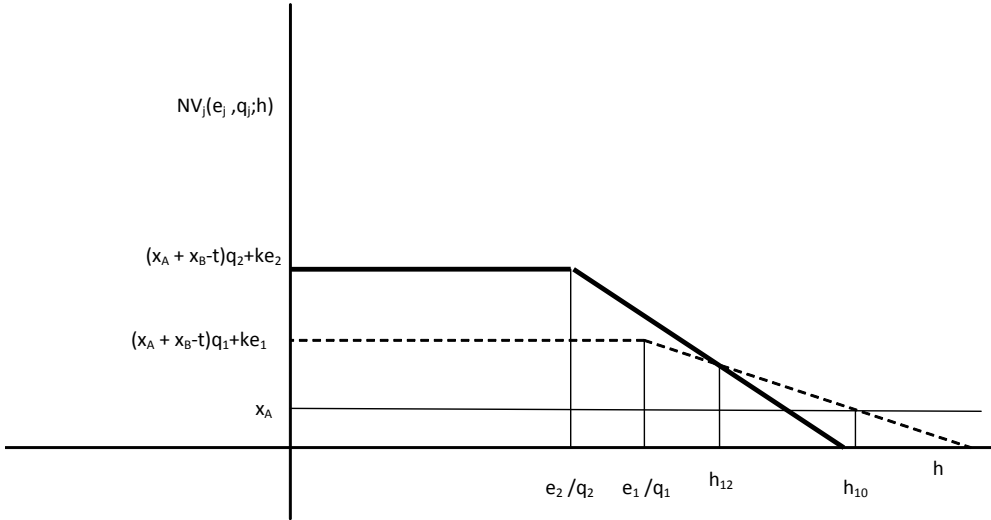
$$tq + \max[\varphi(qh_i - e), 0] \tag{8}$$

We assume that readers' population is uniformly distributed over the interval $[\underline{h}, \bar{h}]$.

The intuition behind our set-up is that there exists a sort of imperfect complementarity between what we call entertainment and information. In fact, we do not completely rule out substitution between entertainment and information. We simply assume that when the entertainment-information ratio goes below an individual specific threshold, the rate of substitution jumps. Considering a type- h individual reading *one report* containing q_j, e_j , with $j = 1, 2$, his net utility is:

$$NV_j(q_j, e_j; h) = (x_A + x_B) q_j + k e_j - t q_j - \max[\varphi(q_j h_i - e_j), 0] \quad (9)$$

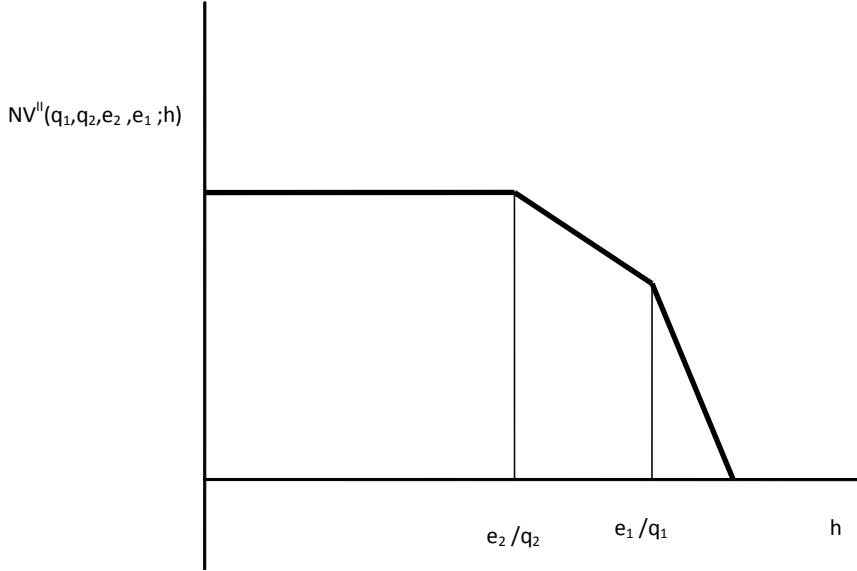
Some intuition can be gained if one keeps e and q constant and lets h change. Then $NV_j(q_j, e_j; h)$ as a function of h is constant for all $h \leq \frac{e_j}{q_j}$, the value of such constant being $(x_A + x_B - t) q_j + k e_j$, while for $h > \frac{e_j}{q_j}$ $NV_j(q_j, e_j; h)$ declines as h increases with slope φq_j , as the following graph shows. Here, the bold line represents $NV_2(q_2, e_2; h)$, while the dotted line represents $NV_1(q_1, e_1; h)$. We have assumed, for the purposes of this representation, that $\frac{e_1}{q_1} > \frac{e_2}{q_2}$, with $q_2 \geq q_1$. h^{12} and h^{10} (to be more precisely defined below) are respectively the individual types indifferent between report One and report Two and between report One and not reading (x_A being the expected utility from choosing based on prior information).



The net utility for an individual reading both reports is given by:

$$NV^{II}(q_1, q_2, e_1, e_2; h) = F(q_2) q_1 + x_A q_2 + k(e_1 + e_2) - \max[\varphi(q_1 h - e_1), 0] - \max[\varphi(q_2 h - e_2), 0] \quad (10)$$

Then, keeping q_1, q_2, e_1, e_2 constant, NV^{II} as a function of h can be plotted as follows. Notice that $NV^{II}(q_1, q_2, e_1, e_2; h)$ declines as h increases with a slope of φq_2 between $\frac{e_2}{q_2}$ and $\frac{e_1}{q_1}$ and with a slope $\varphi(q_1 + q_2)$ for $h \geq \frac{e_1}{q_1}$.



The timing of the model is as follows

- at stage 0 Nature picks up the state of the world
- at stage 1 the sources choose simultaneously their levels of q and e .
- at stage 2 the sources learn the signals, while the readers, knowing q_1, q_2 and e_1, e_2 (but not the signals) make their reading decisions.
- at stage 3 the readers read the report(s) or not and make their private allocation decisions
- at stage 4 the outcomes are observed.

Our analysis focuses on stages 1 and 2.

4 Single reading case

In this section we analyze a set of market configurations which can be simply described as follows: each individual reads at most one report, so that the population is divided into readers of source One, readers of source Two and non-readers. The two reports differ in the entertainment-information ratio, $\frac{e_i}{q_i}$ for $i = 1, 2$, chosen by the sources. The report with the lowest ratio (intuitively, with a large news content with respect to the entertainment one) is preferred by the individuals

whose types h are lowest, while at the other extreme, the individuals who do not read are of the highest types. Readers who prefer the source with the highest entertainment-information ratio are therefore those whose types lie in an intermediate interval. Then, if we define as h^{12} the type of the reader who is indifferent between report One and report Two and as h^{10} the type of the reader who is indifferent between reading report One and not reading at, we expect that:

$$\bar{h} > h^{10} > h^{12} > \underline{h} \quad (11)$$

What we want to show is that, under suitable conditions, such a configuration can be an equilibrium. Before distinguishing among the different cases that will be listed in what follows, there are two general conditions which must be satisfied if (11) must hold, and if we want to rule out double reading. If we expect source Two to sell its report to individuals belonging to the lowest h types, source Two must be preferred to source One at least by individuals with $h \leq \min\left(\frac{e_1}{q_1}, \frac{e_2}{q_2}\right)$. Then, from expression (9), setting $NV_2(q_2, e_2) \geq NV_1(q_1, e_1)$, $h \leq \min\left(\frac{e_1}{q_1}, \frac{e_2}{q_2}\right)$, we get:

$$(x_A + x_B - t)(q_1 - q_2) + k(e_1 - e_2) \leq 0 \quad (12)$$

According to (12), $e_1 > e_2$ implies $q_1 < q_2$. In order for individuals with $h \leq \min\left(\frac{e_1}{q_1}, \frac{e_2}{q_2}\right)$ to prefer reading source Two to reading both we must have:

$$x_A q_1 (1 - q_2) + x_B q_2 (q_1 - 1) + k e_1 - t q_1 \leq 0 \quad (13)$$

(13) was obtained from (9) and (10), setting $NV_2(q_2, e_2; h) \geq NV^{II}(q_1, e_1, q_2, e_2; h)$, for all $h \leq \min\left(\frac{e_1}{q_1}, \frac{e_2}{q_2}\right)$,

Strictly speaking, there are four possible configurations which are consistent with (11) and command our attention.

1. $h^{10} > h^{12} > \max\left(\frac{e_1}{q_1}, \frac{e_2}{q_2}\right)$;
2. $h^{10} > \frac{e_1}{q_1} > h^{12} > \frac{e_2}{q_2}$
3. $\frac{e_1}{q_1} > h^{10} > h^{12} > \frac{e_2}{q_2}$
4. $h^{12} < \min\left(\frac{e_1}{q_1}, \frac{e_2}{q_2}\right)$

In general, we obtain from expression (9) the equation defining h^{12} as:

$$(x_A + x_B)q_1 + ke_1 - tq_1 - \max[\varphi(q_1h^{12} - e_1), 0] = (x_A + x_B)q_2 + ke_2 - tq_2 - \max[\varphi(q_2h^{12} - e_2), 0]$$

The expression of h^{12} changes according to the case under examination. Leaving aside, for the moment, cases 3 and 4, we now provide the expression of h^{12} when $h^{12} > \max\left(\frac{e_1}{q_1}, \frac{e_2}{q_2}\right)$ (expression (14)) and when $\frac{e_1}{q_1} > h^{12} > \frac{e_2}{q_2}$ (expression (15)).

$$\bar{h}^{12}(q_1, q_2, e_1, e_2) = \frac{(x_A + x_B - t)}{\varphi} + \frac{(k + \varphi)(e_1 - e_2)}{(q_1 - q_2)} \quad (14)$$

and

$$\tilde{h}^{12}(q_1, q_2, e_1, e_2) = \frac{(x_A + x_B - t)(q_2 - q_1) + (k + \varphi)e_2 - ke_1}{\varphi q_2} \quad (15)$$

Notice that according to (14) readers with $h < h^{12}$ prefer report 2 if and only if $q_2 > q_1$. On the contrary, this is not necessary according to (15).

Analogously, as regards h^{10} , from (9) we get the equation defining h^{10} in general:

$$(x_A + x_B)q_1 + ke_1 - tq_1 - \max[\varphi(q_1h^{10} - e_1), 0] = x_A$$

Therefore, if $h^{10} > h^{12} > \max\left(\frac{e_1}{q_1}, \frac{e_2}{q_2}\right)$ the above expression becomes:

$$h^{10}(q_1, e_1) = \frac{(x_A + x_B - t)}{\varphi} + \frac{(k + \varphi)e_1 - x_A}{\varphi q_1} \quad (16)$$

As we shall see $h^{10}(q_1, e_1)$ represents the upper boundary to the demand for report One: individuals with $h > h^{10}(q_1, e_1)$ do not read. Then understanding $h^{10}(q_1, e_1)$ means understanding the incentives to invest in news accuracy or in entertainment which are specific to source One. As regards news accuracy, such incentives are conflicting. The larger is x^A , the larger is the expected utility of an individual who does not read, and therefore the larger is the accuracy required to convince such an individual to read: this makes an incentive to invest in accuracy. At the same time, increasing q_1 results in a decrease of $\frac{e_1}{q_1}$. and such a decrease is sharper, the higher the value of e_1 . Then, the higher e_1 the stronger is the effect of an increase in q_1 in discouraging individuals with high h types from reading. The incentive to invest in e_1 , instead,

is always positive, and the larger the smaller is q_1 . In what follows, we shall see that the sign of $(k + \varphi)\bar{e} - x_A$ will be crucial.

Starting with **Case 1.**, the profit functions for Source One and Two are, respectively:

$$\Pi_1(q_1, q_2, e_1, e_2) = h^{10}(q_1, e_1) - \bar{h}^{12}(q_1, q_2, e_1, e_2) - \frac{c(q_1)^2}{2} - re_1 \quad (17)$$

$$\Pi_2(q_1, q_2, e_1, e_2) = \bar{h}^{12}(q_1, q_2, e_1, e_2) - \frac{c(q_2)^2}{2} - re_2 \quad (18)$$

It is important to understand a formal feature of our model. Take for instance source One in the present case (but similar remarks apply to all other cases) When choosing q_1 and e_1 , the source assumes that the situation is the one described by the inequality

$$h^{10} > h^{12} > \frac{e_1}{q_1} > \frac{e_2}{q_2} \quad (19)$$

, that no individual is willing to read more than one report, and that readers with lowest h -types prefer source Two (i.e. (12) and (13) hold). The source does not include these constraint implicitly in its decision problem and we assume that, at equilibrium, the values chosen for e_1, q_1, e_2 and q_2 satisfy all of them.

It should be noticed that, in this case as well as in those which follow, e_1 and e_2 may take only corner values. When stating the first-order conditions we shall always express them in way which ensures that source One chooses the upper bound value \bar{e} and source Two the lower end value \underline{e} . This requirement translates into requiring that r , the unit cost of entertainment lies in some specified interval. Since r does not appear in any other necessary condition, it can be assumed that such a requirement is satisfied, without interfering with other necessary or sufficient conditions. The only problem regarding the optimal values of e_1 and e_2 consists in making sure that the critical interval to which r must belong is non-empty, an issue we shall discuss case by case. We can now proceed to the first-order conditions:

$$\frac{\partial \Pi_1(q_1, q_2, e_1, e_2)}{\partial q_1} = -\frac{(k + \varphi)e_1 - x_A}{\varphi(q_1)^2} + \frac{(k + \varphi)(e_1 - e_2)}{(q_1 - q_2)^2} - cq_1 = 0 \quad (20)$$

$$\frac{\partial \Pi_2(q_1, q_2, e_1, e_2)}{\partial q_2} = \frac{(k + \varphi)(e_1 - e_2)}{(q_1 - q_2)^2} - cq_2 = 0 \quad (21)$$

$$\frac{(k + \varphi)}{\varphi q_1} - \frac{(k + \varphi)}{(q_1 - q_2)} - r \geq 0 \quad (22)$$

$$-\frac{(k + \varphi)}{(q_1 - q_2)} - r \leq 0 \quad (23)$$

The next lemma shows that, conditional on $(k + \varphi)\bar{e} - x_A$ being positive, if a solution exists to equations (20) through (23), it is one with $q_1 < q_2$.

Lemma 1 *If there exists a pair q_1^*, q_2^* such that q_1^*, q_2^* and $e_1 = \bar{e}, e_2 = \underline{e}$ satisfy conditions (20) through (23) and if $(k + \varphi)\bar{e} - x_A > 0$, then $q_1^* < q_2^*$.*

Proof. According to (20) and (21)

$$\left(-\frac{(k+\varphi)\bar{e}-x_A}{(q_1)^2\varphi} + \frac{(k+\varphi)(e_1-e_2)}{(q_1-q_2)^2} \right) \frac{1}{c} = q_1^* = q_2^* - \frac{(k+\varphi)\bar{e}-x_A}{(q_1)^2\varphi} \quad (24)$$

Then $q_1^* < q_2^*$ follows from the assumption $(k + \varphi)\bar{e} - x_A > 0$. ■

Notice that if $q_1^* < q_2^*$, the interval $\left[-\frac{(k+\varphi)}{(q_1-q_2)}, \frac{(k+\varphi)}{\varphi q_1} - \frac{(k+\varphi)}{(q_1-q_2)} \right]$ to which r must belong is non-empty. Moreover, $q_1^* < q_2^*$ obviously implies $\frac{\bar{e}}{q_1^*} > \frac{\underline{e}}{q_2^*}$.

In **Case 2**, the profit function for source One is

$$\Pi_1(q_1, q_2, e_1, e_2) = h^{10}(q_1, e_1) - \tilde{h}^{12}(q_1, q_2, e_1, e_2) - \frac{c}{2}q_1^2 - re_1$$

while the profit function for source Two is

$$\Pi_2(q_1, q_2, e_1, e_2) = \tilde{h}^{12}(q_1, q_2, e_1, e_2) - \frac{c}{2}q_2^2 - re_2$$

The first-order conditions are

$$\frac{\partial \Pi_1(q_1, q_2, e_1, e_2)}{\partial q_1} = -\frac{(k + \varphi)e_1 - x_A}{(q_1)^2\varphi} + \frac{(x_A + x_B - t)}{\varphi q_2} - cq_1 = 0 \quad (25)$$

$$\frac{\partial \Pi_2(q_1, q_2, e_1, e_2)}{\partial q_2} = \frac{(x_A + x_B - t)q_1}{\varphi(q_2)^2} - \frac{k(e_2 - e_1) + \varphi e_2}{\varphi(q_2)^2} - cq_2 = 0 \quad (26)$$

$$\frac{(k + \varphi)}{q_1\varphi} + \frac{k}{\varphi q_2} - r \geq 0 \quad (27)$$

$$\frac{k + \varphi}{\varphi q_2} - r \leq 0 \quad (28)$$

The following Proposition sets out a sufficient condition for the existence of an equilibrium :

Proposition 2 *If: a) $\frac{(x^A + x^B - t)}{c\varphi + (k + \varphi)\bar{e} - x_A} < \sqrt[3]{\frac{(x^A + x^B - t) - [k(\underline{e} - \bar{e}) + \varphi\underline{e}]}{\varphi c}}$ and b) $\sqrt[3]{-\frac{k(\underline{e} - \bar{e}) + \varphi\underline{e}}{c\varphi}} < 1$ then an equilibrium exists with $q_1^*, q_2^* \in (0, 1)$ and $e_1 = \bar{e}, e_2 = \underline{e}$.*

Proof. See Appendix 7.1 ■

The conditions ensuring that $q_1 < q_2$, are not as sharp as those ensuring existence. However, their general meaning is very similar to the meaning of the condition $(k + \varphi)\bar{e} - x_A > 0$ mentioned in Lemma (1) above. By subtracting (25) from (26) and setting $e_1 = \bar{e}, e_2 = \underline{e}$ we get :

$$\begin{aligned} & \frac{(x_A + x_B - t)}{q_2} \left[\frac{q_1}{q_2} - 1 \right] + \\ & + \frac{[(k + \varphi)\bar{e} - x_A](q_2)^2 - [k(\underline{e} - \bar{e}) + \varphi\underline{e}](q_1)^2}{(q_2)^2 (q_1)^2} = \varphi c (q_2 - q_1) \end{aligned} \quad (29)$$

It is easy to see that the sign of the LHS of (29) is ambiguous; also, the condition $(k + \varphi)\bar{e} > x_A$ is not necessary nor sufficient to ensure that $q_2 - q_1 > 0$, although it certainly works in that direction. Other things being equal, the larger the value of \bar{e} the more it is likely that $q_2 > q_1$.

Finally, in **Case 3**, $\frac{e_1}{q_1} > h^{10} > h^{12} > \frac{e_2}{q_2}$, and in **Case 4**, $h^{12} < \min\left(\frac{e_1}{q_1}, \frac{e_2}{q_2}\right)$ we show that the only possible equilibrium is a monopoly structure.

Proposition 3 *If a) $\frac{e_1}{q_1} > h^{10} > h^{12} > \frac{e_2}{q_2}$ or b) $h^{12} < \min\left(\frac{e_1}{q_1}, \frac{e_2}{q_2}\right)$ the equilibrium is a Monopoly.*

Proof. See Appendix 7.2 ■

Up to now, we have characterized the equilibrium in the single-reading case, in the following section we deal with the double reading one.

5 Double reading case

We now concentrate on a set of market configurations where some readers - those with the lowest h types - prefer to read both reports rather than any of them alone. This means that

$$NV^{II}(q_1q_2, e_1, e_2, h) \geq NV_1(q_1, e_1, h), NV_2(q_2, e_2, h) \quad (30)$$

at least on some interval of h values starting at $h = \underline{h}$. More explicitly $NV^{II}(q_1q_2, e_1, e_2, h) \geq NV_1(q_1, e_1, h)$ if

$$F(q_1)q_2 + ke_2 - tq_2 - x^Bq_1 - \max[\varphi(q_2h - e_2), 0] \geq 0 \quad (31)$$

Now, either (31) holds as $h \leq \frac{e_2}{q_2}$ or it never holds. Then a necessary condition is:

$$(F(q_1) - t)q_2 + ke_2 - x^Bq_1 \geq 0 \quad (32)$$

In a similar fashion, $NV^{II}(q_1q_2, e_1, e_2, h) \geq NV_2(q_2, e_2, h)$ if

$$F(q_2)q_1 + ke_1 - tq_1 - \max[\varphi(q_1h - e_1), 0] - x^Bq_2 \geq 0 \quad (33)$$

(33) must hold at least for $h \leq \frac{e_1}{q_1}$, which implies

$$(F(q_2) - t)q_1 + ke_1 - x^Bq_2 \geq 0 \quad (34)$$

Notice that (34) is the reverse of (13) above.

Let us define $h_{II,1}$ as the type of the reader who is indifferent between reading both reports and reading report One only. Using (31), we can determine $h_{II,1}$ a:

$$h_{II,1} = \frac{F(q_1)q_2 + (k + \varphi)e_2 - tq_2 - x^Bq_1}{\varphi q_2} \quad (35)$$

or, more explicitly:

$$h_{II,1} = \frac{-q_1x_B + ke_2 + \varphi e_2}{\varphi q_2} + \frac{x_A - q_1(x_A - x_B) - t}{\varphi} \quad (36)$$

Obviously, if (32) holds, $h_{II,1} \geq \frac{e_2}{q_2}$. Defining instead $h_{II,2}$ as the type of the reader who is indifferent between reading both reports and reading report Two only, and using (33), we find that the following expression for $h_{II,2}$

$$h_{II,2} = \frac{F(q_2)q_1 + (k + \varphi)e_1 - tq_1 - x^Bq_2}{\varphi q_2} \quad (37)$$

and, if (34) holds, $h_{II,2} \geq \frac{e_1}{q_1}$. Then we can prove the following lemma.

Lemma 4 *If $\frac{e_1}{q_1} > h_{12} > \frac{e_2}{q_2}$ and (34) holds, then all readers of report 2 are double readers.*

Proof. The assumption (34) implies that $h_{II,2} \geq \frac{e_1}{q_1}$. Then since by assumption $\frac{e_1}{q_1} > h_{12} > \frac{e_2}{q_2}$, we can conclude that $h_{II,2} > h_{12}$, i.e. all those who prefer report Two to report One also prefer reading both reports to report Two alone. ■

When instead $h_{12} > \frac{e_1}{q_1}$, the above lemma does no longer hold and there possibly exist also individuals who read report Two alone.

Let us now examine the equilibria in the two cases.

Let us start from the case where $\frac{e_1}{q_1} > h_{12} > \frac{e_2}{q_2}$. In this situation, all individuals such that $h \leq h^{10}$ read report One, either as the only report or jointly with report Two. Therefore the profit function for source One is given by:

$$\Pi_{II}^1(q_1, e_1) = h^{10}(q_1, e_1) - \frac{c}{2}q_1^2 - re_1 \quad (38)$$

yielding the following first-order conditions are

$$\frac{\partial \Pi_{II}^1(q_1, e_1)}{\partial q_1} = -\frac{1}{\varphi q_1^2} (c\varphi q_1^3 - x_A + ke_1 + \varphi e_1) = 0 \quad (39)$$

$$\frac{\partial \Pi_{II}^1(q_1, e_1)}{\partial e_1} = \frac{k + \varphi}{\varphi q_1} - r \geq 0 \quad (40)$$

(condition (40), as usual, ensures that $e_1 = \bar{e}$). As regards source Two, its readers are those individuals with $h \leq h_{II,1}$. Therefore its profit function is:

$$\Pi_{II}^2(q_1, q_2, e_2) = h_{II,1}(q_1, q_2, e_2) - \frac{c}{2}q_2^2 - re_2 \quad (41)$$

yielding the first-order conditions:

$$\frac{\partial \Pi_{II}^2(q_1, q_2, e_1, e_2)}{\partial q_2} = -\frac{1}{\varphi q_2^2} (c\varphi q_2^3 + ke_2 + \varphi e_2 - q_1 x_B) = 0 \quad (42)$$

$$\frac{\partial \Pi_{II}^2(q_1, q_2, e_1, e_2)}{\partial q_2} = \frac{k + \varphi}{\varphi q_2} - r < 0 \quad (43)$$

Equations (39) and (42) can be explicitly solved (after substituting $e_1 = \bar{e}$ and $e_2 = \underline{e}$) to yield

$$q_1^* = c \frac{\varphi}{x_B} G \quad (44)$$

$$q_2^* = \sqrt[3]{G - \frac{(k + \varphi)}{c\varphi} \underline{e}} \quad (45)$$

where:

$$G = \sqrt[3]{\frac{1}{c^4 \varphi^4} x_B^3 (x_A - (k + \varphi) \bar{e})} \quad (46)$$

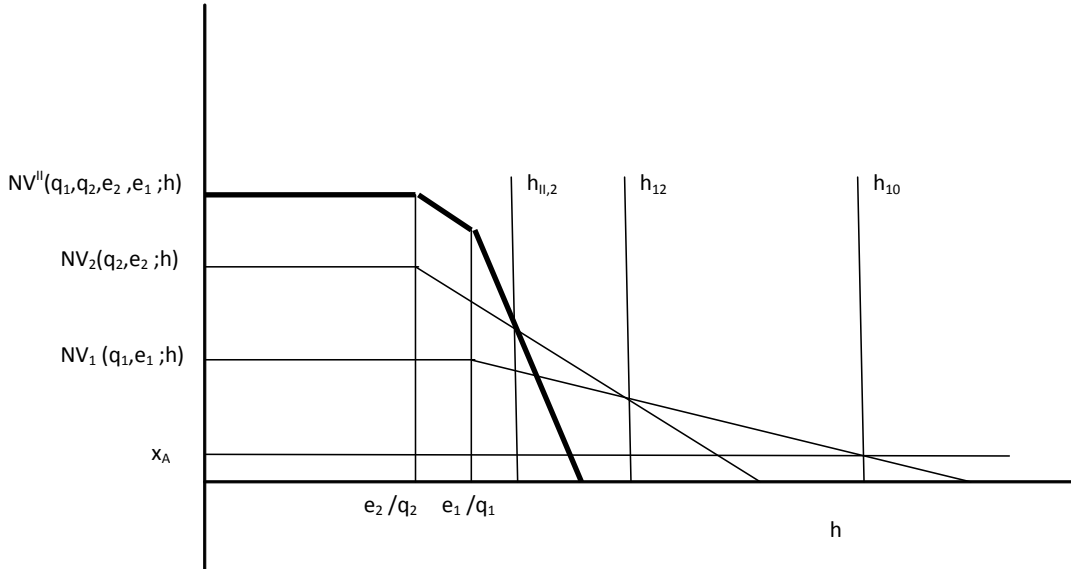
We can now prove the following proposition.

Proposition 5 *If $x_A > (k + \varphi) \bar{e}$ and $\min \left[1 + \frac{(k + \varphi)}{c\varphi} e_2, \frac{x_B}{c\varphi} \right] \geq G > \frac{(k + \varphi)}{c\varphi} \underline{e}$, q_1^*, q_2^* as defined by (44) and (45) and $e_1 = \bar{e}, e_2 = \underline{e}$ represent the unique equilibrium, with $q_2^* > q_1^*$.*

Proof. See the appendix ■

It should be noticed that in this case, if it were $x_A < (k + \varphi) \bar{e}$ as in previous cases, source One would lack any incentive to invest in accuracy.

Now consider the case where $h_{12} > \frac{e_1}{q_1}$. The configuration we are about to examine is represented in the following graph



Here, the market for Source One is represented by individuals with $h \in [h_{12}, h_{10}]$ (single readers) and by individuals with $h \leq h_{II,2}$. Therefore, its profit function is:

$$h^{10} (q_1, e_1) - \bar{h}^{-12} (q_1, q_2, e_1, e_2) + h_{II,2} (q_2, q_1, e_1) - \frac{c}{2} q_1^2 - r e_1$$

or,explicitly:

$$\begin{aligned} & \frac{(x_A+x_B-t)}{\varphi} + \frac{(k+\varphi)e_1-x_A}{\varphi q_1} - \frac{(x_A+x_B-t)}{\varphi} - \frac{(k+\varphi)(e_1-e_2)}{(q_1-q_2)} \\ & + \left(\frac{F(q_2)-t}{\varphi} \right) + \frac{(k+\varphi)e_1-x^B q_2}{\varphi q_1} \\ & - \frac{c}{2} q_1^2 - r e_1 \end{aligned}$$

The market for source Two is represented by those individuals with $h \leq \bar{h}^{12}$, so that profit is given by:

$$\bar{h}^{-12} (q_1, q_2, e_1, e_2) - \frac{c}{2} q_2^2 - r e_2$$

i.e.:

$$\frac{(x_A + x_B - t)}{\varphi} + \frac{(k + \varphi) (e_1 - e_2)}{(q_1 - q_2)} - \frac{c}{2} q_2^2 - r e_2$$

The first-order conditions are

$$-\frac{2(k+\varphi)e_1-x_A-x^B q_2}{\varphi(q_1)^2} + \frac{(k+\varphi)(e_1-e_2)}{(q_1-q_2)^2} - c q_1 = 0 \quad (47)$$

$$\frac{(k+\varphi)(e_1-e_2)}{(q_1-q_2)^2} - c q_2 = 0 \quad (48)$$

$$\frac{2(k+\varphi)}{\varphi q_1} - \frac{(k+\varphi)}{(q_1-q_2)} \geq r \quad (49)$$

$$-\frac{(k+\varphi)}{(q_1-q_2)} \leq r \quad (50)$$

In this case, *mutatis mutandis*, Lemma (1) applies again:

Lemma 6 *If there exists a pair q_1^*, q_2^* such that q_1^*, q_2^* and $e_1 = \bar{e}, e_2 = \underline{e}$ satisfy conditions (47) through (50) and if $(k+\varphi)\bar{e} - x_A > 0$, then $q_1^* < q_2^*$.*

Proof. According to (47) and (48)

$$-\frac{2(k+\varphi)\bar{e}-x_A-x^B q_2}{c\varphi(q_1)^2} + \frac{(k+\varphi)(\bar{e}-\underline{e})}{c(q_1-q_2)^2} = q_1^* = q_2^* - \frac{2(k+\varphi)\bar{e}-x_A-x^B q_2}{c\varphi(q_1)^2} \quad (51)$$

Since $x_A > x^B q_2$, $q_1^* < q_2^*$ follows from the assumption $(k+\varphi)\bar{e} - x_A > 0$.

■

6 Conclusions

The model presented in this paper generates, as possible equilibrium outcomes, three fundamental market structures. It should be noticed that these structures differ as to what concerns the characteristics of the demand side, rather than supply, if by supply side one means the number and type of existing firms. In all cases we discussed, the market is a duopoly³, where firms provide reports with different entertainment/news ratio. If instead one refers to production cost parameters, indeed these contribute, together with demand-side parameters, to determine the existence conditions for the equilibria.

The differences among equilibria refer to the use individuals make of the available information. Individuals may choose one source, the one which they think best suits their tastes and disregard the other one: this is what we called single-reading and it is completely similar to what happens in most conventional good markets. But in the case of information, there is another possibility, namely, using both sources to produce higher-level information by jointly processing the news provided by the reports. The latter option requires higher reading costs (as we called them) than the former, so that two general principles apply:

1. If at least one of the sources provides sufficiently accurate news, the incremental information value from reading both sources might not compensate for the additional effort required, and no one would choose the double reading option

- 2 Given the informational advantage from reading two reports relative to reading just one, the individual with the highest reading costs are more likely to read just one source (if at all).

In our paper, the differences in reading costs are summarized by a single parameter, h , which represents the value of the entertainment/news ratio which is best for the individual. "Best" here means that, if a given report provides an entertainment/news ratio which is smaller than h , the individual incurs an additional reading cost which is proportional to the deviation of e/q with respect to h .

Summarizing we have three basic equilibria

1. Single reading: the population splits into three groups: readers of source Two (lowest h , high accuracy); readers of source One (high h , low accuracy); non-readers (highest h).

2. Double reading (I): the population splits into three groups: readers of both sources (lowest h); readers of source One alone (high h , low accuracy); non-readers (highest h).

³As shown in Proposition (3) also monopoly cases may in principle emerge, but we did not analyze these in depth in this paper.

3. Double reading (II): the population splits into four groups: readers of both sources (lowest h); readers of source Two alone (higher h , higher accuracy); readers of source One alone (even higher h , low accuracy); non-readers (highest h).

It is important to see how entertainment and information interact. All readers value both news and entertainment, but each individual does this differently. This explains why the two sources behave differently at the equilibrium. One of them (source One in our paper) takes care of those individuals who value relatively more entertainment as compared with other individuals, who instead are taken care of by the other source. In the presence of double reading, a source might face the task of balancing the needs of two different categories of readers: the high- h -level individuals who read the report as the only source information and the low- h -level individuals who read both reports. In particular in what we called double reading(II), the two segments related to source One are not even contiguous.

Concluding, we think that our model mimics some important features of media markets: first of all, the variety of individual behaviour as regards the access to media. Secondly the joint use of multiple media or of multiple sources within the same media type. Third, the emergence of mass-elite phenomena, with some sources being accessed by the majority of the population, while others are chosen by a minority. Among the three patterns outlined above, two have a particularly appealing interpretation. The single reading equilibrium has its most convincing interpretation when referred to competition between two sources belonging to the same medium type: competition, say, between two newspapers, with separate readerships or two broadcasting stations with non-overlapping audiences. Double reading (I) seems more akin to competition/coexistence among different media: for example, almost the entire population watches TV (source One in our paper) while a minority also reads newspapers. Double reading (II) summarizes the characteristics of the other equilibria. Here we have both competition (highest h types choose between the two sources) and coexistence (low- h types read both reports). From the descriptive viewpoint, this is a situation where an elite of "sophisticated" double readers confronts a fragmented "mass" of single-report readers.

References

- [1] Anderson, S.P. - Gabszewicz, J. (2005), "The media and advertising: a tale of two-sided markets", CORE Discussion Paper 2005/88: 1-79
- [2] Baron, D.P. (2006) "Persistent Media Bias" Journal of Public Economics, 90:1-36.

- [3] Battaglion, M.R-Vaglio,A. (2012) "The Market for News: a Demand-Oriented Analysis", *Economia Politica*, XXIX, n.1 aprile: 81-110 (*forthcoming*)
- [4] Besley, T.-Prat, A. (2006) "Handcuffs for the Grabbing Hand ? Media Capture and Government Accountability", *American Economic Review*, n.3 June: 720-736.
- [5] Burke, J. (2008), "Primetime Spin:...", *Journal of Economics & Management Strategy*, 17, 3: 633-665.
- [6] Brown, K. - Cavazos, R., (2005), "Why is This Show so Dumb? Advertising Revenue and Program Content of Network Television", *Review of Industrial Organization*, vol.27, 1: 17-34
- [7] Chan, J.- Suen W., (2008), " A Spatial Theory of News Consumption ...", *Review of Economic Studies*, 75: 699-728.
- [8] Corneo, G., (2006), "Media capture in a democracy: The role of wealth concentration", *Journal of Public Economics*, 90: 37-58.
- [9] Di Tella, R. - Franceschelli, I., (2009), "Government Advertising and Media Coverage of Corruption Scandals", *NBER Working paper*, no.15402.
- [10] Durante., R. - Knight, B. G., (2009), "Partisan Control, Media Bias and Viewer Responses: Evidence from Berlusconi's Italy", *NBER Working paper*, no.w14762.
- [11] Ellman, M. -Germano, F., (2009), "What Do the Papers Sell? A Model of Advertising and Media Bias", *The Economic Journal*, 119: 680-704.
- [12] Gentzkow M., Shapiro J., (2006), "Media Bias and Reputation", *Journal of Political Economy*, 114, 2: 280-316.
- [13] Gentzkow M., Shapiro J., (2010), "What Drives Media Slant? Evidence from U.S. Daily Newspapers", *Econometrica*, 78, 1: 35-71.
- [14] Larcinese, V. (2009) "Information Acquisition, Ideology and Turnout: Theory and Evidence from Britain" *Journal of Theoretical Politics*, 21(2): 237-276
- [15] Mullainathan, S.-Shleifer, A. (2005) "The Market for News", *American Economic Review*, n.4 September: 1031-1053.
- [16] Prat A., Strömberg, D., "The Political Economy of Mass Media", *mimeo LSE*
- [17] Sobbrío F., (2011), "A Citizen-Editor Model of News Media", *mimeo*.
- [18] Strömberg, D., (2004), "Mass Media Competition, Political Competition, and Public Policy", *Review of Economic Studies*, 71: 265-84
- [19] Vaglio, A. (2006) "Shaping the Beliefs of the Majority. Pluralism and Competition in Mass Information ", *International Review of Economics*, Vol. 53, n. 3 Sept.:1865-1704

7 Appendix

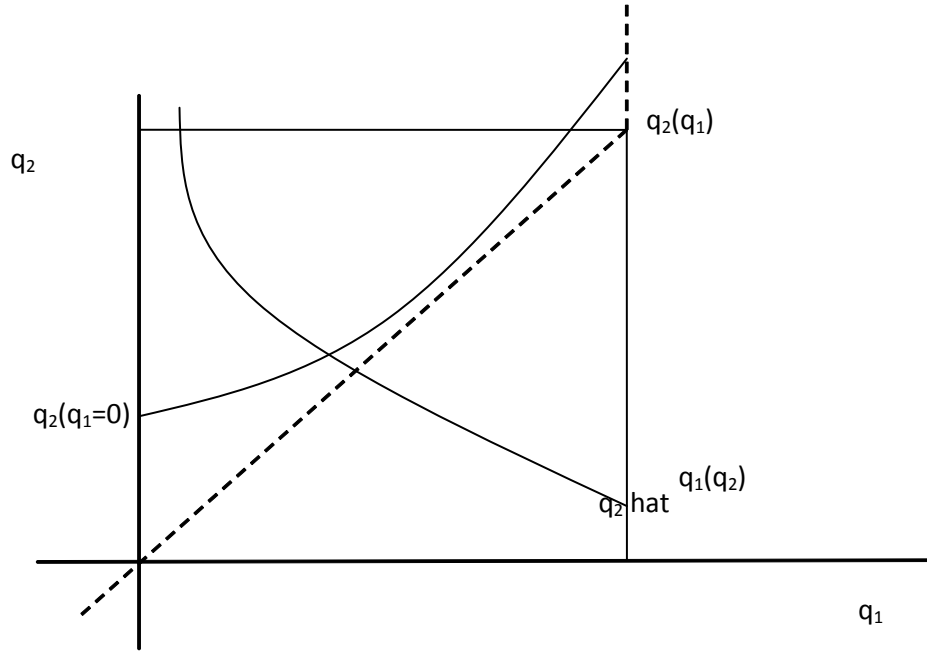
7.1 Proof of Proposition 2

Proof. By differentiating (25) and (26) we find, given second-order conditions, that

$$\frac{dq_1^*(q_2)}{dq_2} < 0 \quad (52)$$

$$\frac{dq_2^*(q_1)}{dq_1} > 0 \quad (53)$$

i.e. the best reply functions for source One and source Two are, respectively, negatively and positively sloped.



As it is clear from the figure, in order to make sure that the best reply functions intersect for values $q_1^*, q_2^* \in (0, 1)$ we need to assume first that

$$\hat{q}_2 < q_2^*(q_1 = 1)$$

where \hat{q}_2 is defined as

$$\hat{q}_2 \text{ s.t. } q_1^*(\hat{q}_2) = 1 \quad (54)$$

Since the values of \hat{q}_2 and $q_2^*(q_1 = 1)$ are respectively:

$$\hat{q}_2 = \frac{(x^A + x^B - t)}{c\varphi + (k + \varphi)e_1 - x_A} \quad (55)$$

and

$$q_2^*(q_1 = 1) = \sqrt[3]{\frac{(x^A + x^B - t) - [k(e_2 - e_1) + \varphi e_2]}{\varphi c}} \quad (56)$$

then inequality a) in the proposition follows. Moreover, according to (25), as q_2 goes to infinity q_1 tends to a positive value, which is defined by:

$$\widehat{q}_1 = \sqrt[3]{-\frac{(k + \varphi) e_1 - x_A}{c\varphi}}$$

Then, given $q_2^*(q_1 = 0)$,

$$q_2^*(q_1 = 0) = \sqrt[3]{-\frac{k(e_2 - e_1) + \varphi e_2}{c\varphi}} \quad (57)$$

it is always true that when q_1 tends to 0, the inverse of reaction function of source 1 lies above $q_2^*(q_1 = 0)$. Assuming that (57) is smaller than 1 (inequality b) in the proposition) is then sufficient to conclude that the best reply functions intersect between 0 and 1. ■

7.2 Proof of Proposition 3

Proof. The Proof goes in 2 parts: a) and b).

a) h^{12} is defined by (15). If any of the individuals with types $h \leq \frac{e_1}{q_1}$ is

indifferent between reading report One and not reading, then the same is true for all individuals in the same set and the following condition holds

$$(x_A + x_B) q_1 + k e_1 - t q_1 - x_A = 0 \quad (58)$$

Therefore, the individuals with $h > \frac{e_1}{q_1}$ certainly give up reading, since in this case the utility from reading is:

$$(x^A + x^B) q_1 + k e_1 - t q_1 - \varphi q_1 \left(h - \frac{e_1}{q_1} \right) < (x^A + x^B) q_1 + k e_1 - t q_1 = x^A \quad (59)$$

Moreover, since $\frac{e_1}{q_1} > h^{12} > \frac{e_2}{q_2}$, some individuals strictly prefer read report Two. Therefore individuals either do not read or read report Two.

b) If any of the individuals with types $h \leq \frac{e_2}{q_2}$ is indifferent between reading report One and report Two, the same is true for all individuals in the same set, and the following condition holds:

$$\begin{aligned} (x_A + x_B) q_1 + k e_1 - t q_1 = \\ (x_A + x_B) q_2 + k e_2 - t q_2 \end{aligned} \quad (60)$$

At the same time, all individuals with $\frac{e_1}{q_1} > h > \frac{e_2}{q_2}$ strictly prefer report One, since:

$$\begin{aligned} (x_A + x_B) q_1 + k e_1 - t q_1 > \\ (x_A + x_B) q_2 + k e_2 - t q_2 - \varphi q_2 \left(h - \frac{e_2}{q_2} \right) \end{aligned} \quad (61)$$

Finally, if $q_1 \leq q_2$

$$\begin{aligned} (x_A + x_B) q_1 + k e_1 - t q_1 - \varphi q_1 \left(h - \frac{e_1}{q_1} \right) > \\ (x_A + x_B) q_2 + k e_2 - t q_2 - \varphi q_2 \left(h - \frac{e_2}{q_2} \right) \end{aligned} \quad (62)$$

i.e., all individuals with $h \geq \frac{e_1}{q_1}$ prefer report One. Again, we have a monopoly, since individuals either do not read or read report One ■

7.3 Proof of Proposition 5

Proof. In order to prove that q_1^*, q_2^* together with $e_1 = \bar{e}, e_2 = \underline{e}$ are an equilibrium we have to prove that $q_1^*, q_2^* \in (0, 1)$. Positiveness is ensured by

$$x_A > (k + \varphi) \bar{e} \quad (63)$$

and

$$G > \frac{(k + \varphi) \underline{e}}{c\varphi} \quad (64)$$

Moreover, q_1^* and $q_2^* \leq 1$ if:

$$G \leq \min \left[1 + \frac{(k + \varphi) \underline{e}}{c\varphi}, \frac{x_B}{c\varphi} \right]$$

$q_2^* \geq q_1^*$ if:

$$\sqrt[3]{G - \frac{(k + \varphi) \underline{e}}{c\varphi}} \geq c \frac{\varphi}{x_B} G \quad (65)$$

which is equivalent to:

$$G - \frac{(k + \varphi)}{c\varphi} \underline{e} \geq \left(c \frac{\varphi}{x_B} \right)^3 \frac{1}{c^4 \varphi^4} x_B^3 (x_A - (k + \varphi) \bar{e}) = \left(\frac{x_A - (k + \varphi) \bar{e}}{c\varphi} \right) \quad (66)$$

and finally

$$Gc\varphi - x_A \geq (k + \varphi) (\underline{e} - \bar{e}) \quad (67)$$

Condition (67) is satisfied whenever the inequalities (63) and (64) hold.

■