Auctions in Markets: Common Outside Options and the Continuation Value Effect*

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Abstract

In this paper, we study auctions with outside options provided by future market interaction focusing on the revenue effects of some information revelation policies. We show that auctions with less information revelation may yield higher revenues. In particular, we show that it is never optimal for the auctioneer to reveal information after the auction. Moreover, it is also not optimal to reveal information before the auction unless bidders already have precise information on their own. Our model provides a novel explanation for the prevalence of opaque trading mechanisms, and it offers insights into information sharing in dynamic models of trade.

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1 Introduction

Few auctions take place in isolation. Instead, losing bidders have outside options. For example, in many of the auctions commonly studied by economists losing bidders can buy substitutes at other auctions or through other channels. Some examples include timber auctions, eBay auctions, as well as auctions for US municipality and corporate bonds.

We study how the presence of outside options influences whether an auctioneer prefers "opaque" or "transparent" auctions, which differ according to the information that bidders receive. In particular, we show that an auctioneer might have incentives to choose opaque auctions in order to reduce the value of the bidders' outside option.

Understanding the choices of individual traders between transparent and opaque trading mechanisms helps to identify conditions under which transparent or opaque mechanisms arise in decentralized markets. An extensively studied example is the over-the-counter asset market, where individual dealers often employ opaque mechanisms; see Duffie (2012).[1] Bessembinder and Maxwell (2008) describe the introduction of a mandatory price reporting system in the US corporate bond market. They report that trading costs of bonds for which trades were made public were cut in half. In broad accordance with our finding that opaque auctions may increase sellers' revenues, Bessembinder and Maxwell (2008) note that their results "are consistent with the reasoning that dealers extracted rents from the customers in the opaque market" (p. 226).

We develop a two-stage model to study the preferences of an auctioneer over different information disclosure policies. In the first stage, buyers participate in an auction for an indivisible good. In the second stage, each losing bidder exercises his outside option. In particular, each losing bidder takes an action in a single-person decision problem under uncertainty. Each player's payoff in the decision problem depends on his own action and a state of nature. The state of nature is unknown. The state of nature may be interpreted as the state of the market where buyers may buy a substitute if they lose in the initial auction.

Before entering the auction, bidders privately observe noisy signals about the state of nature. Bidders may learn additional information through the auction. By varying the information available to the bidders, the auctioneer can influence the bidders' beliefs about the value of the outside option. Since equilibrium bids depend on the expected value of the outside option, the choice of the auction format and the information disclosure policy affect the auctioneer's revenue.

[1] For example, Schultz (p. 678, 2001) states about over-the-counter asset markets, "the potential bond trader cannot observe all quotes in a central location or on a computer screen. Instead, the institution must call several dealers for quotes or broadcast a list of bonds to sell (or buy) to various dealers through Bloomberg."

Our model captures three salient features of auctions with outside options: (i) Bidders have a common outside option; (ii) the value of the outside option is uncertain and individual bidders have private information about the value of the outside option; and (iii) losing bidders take further actions when pursuing outside options. Examples with those features include auctions in which losing bidders may submit bids in subsequent auctions[2], as it is often the case on eBay, or where losing bidders engage in costly search activities in an external market.

We begin our analysis of the seller's information policy by considering the case where the auctioneer knows the state fully. This way we can simplify the analysis and clearly isolate the relevant factors at hand. This assumption is later relaxed.[3] We ask whether the auctioneer prefers to reveal the state of nature to the bidders. Following the mechanism design literature, we posit that the auctioneer can commit before learning the state (ex ante) whether or not he reveals the state. We decompose the revenue impact of revealing an informative signal about the state into two opposing effects. The first effect we identify is akin to the *linkage principle*. Milgrom and Weber (1982) observed that the auctioneer prefers to reveal any signal that is affiliated with the value of the object. The fact that the linkage principle is active in our environment is due to similarities of our setting to a standard common-value auction as explained at the end of Section 3.1. Our analysis also identifies a new effect, the *continuation value effect*. The continuation value effect favors opaque auctions, and works roughly as follows: A transparent auction reveals information about outside market conditions, which improves the payoffs of losing bidders in the decision problem by allowing them to make better choices. Because the outside option to losers becomes more attractive, buyers bid less aggressively, thereby decreasing the auctioneer's revenue.[4]

We use the decomposition to provide comparative static results for the information revelation's revenue impact with respect to the informativeness of the bidders' signals. We show that the auctioneer prefers revealing the state if bidders' signals are informative, and prefers not revealing the state when bidders' signals are uninformative. Thus, the auctioneer hides the state exactly when information would be most valuable to the bidders.

We also ask whether facilitating information exchange amongst the bidders is beneficial for the auctioneer. We compare the English-auction where bidders observe all the losing bids, with the less transparent second-price format, where a bidder only learns whether he

[2] In Section 6, we analyze a dynamic trading game where bidders may participate in a sequence of auctions.

[3] In Section 5.3, we study an extension where the auctioneer has a noisy signal instead, and we show that our analysis does not change qualitatively.

[4] Technically, the continuation value effect is related to the convexity of the continuation payoffs in buyers' posteriors. Revealing information leads to a mean-preserving spread in the distribution of posteriors. Therefore, when continuation payoffs are (strictly) convex, revealing information increases expected continuation payoffs. Convexity of continuation values in beliefs is likely to be a robust feature of many environments; see our discussion in Section 6.

lost or won. We show that the second-price auction may yield higher revenues than the English-auction because it provides lower continuation values for the losing bidders.

We then introduce a dynamic auction model, in which bidders can participate in a sequence of a large number of simultaneously conducted auctions. This model is similar to Satterthwaite and Shneyerov (2007) and other matching-and-bargaining games.[5] In our model, bidders are uncertain about the *expected* number of competitors. We show a formal equivalence between the second-stage decision problem in our original model and bidding in a sequence of future auctions in the dynamic auction model. We use this equivalence to provide further interpretation of our second-stage decision problem and to discuss consequences for information policy in models of dynamic market interaction.

Our article contributes to the literature on auction design, focusing on information policy.[6] Pure common-value auctions are nested as a special case in our model. In pure common-value auctions, revealing the value of the object (the state) leads to Bertrand competition between bidders, allowing the auctioneer to extract all the rents of trade. Thus, ex ante, the seller prefers to reveal a perfectly informative signal. The linkage principle by Milgrom and Weber (1982) extends this observation: releasing any affiliated signal increases ex ante expected revenues. Subsequent work has qualified the linkage principle.[7] Our work differs from this subsequent analysis on the scope of the linkage principle in three ways: (i) in our model, information disclosure directly affects the value of winning through a novel channel; (ii) in contrast to most models of common-value auctions, even revealing a fully informative signal may not be beneficial; and (iii) we provide a precise decomposition of the revenue impact of information disclosure.

In a recent contribution, Atakan and Ekmekci (2012) also consider an auction setting in which bidders take actions after the auction. However, in Atakan and Ekmekci (2012), it is the winners who take an action, the auction is for multiple units, and they study a different question, namely, information aggregation.

The rest of the paper is organized as follows. Section 2 describes the model, Section 3 characterizes the bidding equilibrium and the revenue decomposition result, and Section 4 offers comparative statics. Section 5 discusses additional aspects of the auctioneer's infor-

[5] Such games are often used to model decentralized markets such as over-the-counter asset markets. Duffie, Garleanu and Peddersen (2005) studies information transmission in this framework.

[6] Other works on disclosure policies in auctions include Bergemann and Pesendorfer (2007), Bergemann and Horner (2010), and Forand (2010). In these models (as here), the auctioneer cannot charge fees to buyers for information. Among others, Gershkov (2009), and Eső and Szentes (2007) consider the case where information is sold.

[7] Perry and Reny (1999) show that the linkage principle does not extend to multi-unit auctions. Board (2009) shows that the linkage principle does not apply if new information can change the *ranking* of bidder's valuations (allocation effect); see Krishna (2008) for further discussions of the scope of the linkage principle.

mation policy choice, and Section 6 connects our analysis with a model of dynamic markets. Most of the proofs are in the Appendix.

2 Model and preliminary analysis

There is a single auctioneer with an indivisible object and N bidders. First, the auctioneer and the N bidders receive signals about the state of the world. Second, the auctioneer runs an auction for an indivisible object. Third, to model outside options we assume that each losing bidder takes an action in a single-person decision problem under uncertainty.

Information There are two states of the world, $w \in \{H, L\}$, and the realization is not observed by the bidders. The ex ante probability of the high state is ρ_0. Bidder i receives a private signal s_i. In state w, the signals are distributed independently and identically according to a distribution function G_w with support $[\underline{s}, \overline{s}]$ with a differentiable density function g_w. Given a signal s and prior ρ_0, the Bayesian posterior probability of the high state is $\widetilde{\rho}(s) = \rho_0 g_H(s) / ((1-\rho_0) g_L(s) + \rho_0 g_H(s))$. We assume that the likelihood ratio of the signal g_H/g_L is strictly increasing, and thus the posterior is strictly increasing in s. To simplify exposition we assume that signal $\overline{s}$ is perfectly informative, that is $\widetilde{\rho}(\overline{s}) = 1$.

Auction Bidders participate in an auction of a single indivisible object. The auction is a second-price auction: the highest bidder wins and he pays the second highest bid.

Preferences and payoffs The winning bidder receives the object and pays a price p. The valuation for the object, v, is the same for all bidders. The payoff of the winner is equal to $v - p$. The losers' payoff is equal to their continuation payoffs defined below.

Outside Option After the auction, each losing bidder proceeds to a decision problem. In this decision problem, the payoff from an action depends on the state. Let $u_w(a)$ be the payoff in state w of a buyer who takes action $a \in A$, where u_w is continuous for $w = H, L$, and A is a compact subset of $\mathbb{R}$. Let $a^*(\rho)$ be an optimal action when the high state has probability ρ. Introducing notation $U_w(\rho) = u_w(a^*(\rho))$, the continuation value can be written as $V(\rho) = \rho U_H(\rho) + (1-\rho) U_L(\rho)$.

By standard arguments, maximized payoffs V are continuous and convex. Continuity is implied by the maximum theorem. Convexity of V follows from a standard revealed preference argument: For any beliefs ρ and ρ', optimality implies that $V(\rho) \geq \rho U_H(x) + (1-\rho) U_L(x)$ and $V(\rho') \geq \rho' U_H(x) + (1-\rho') U_L(x)$ for any $x \in [0,1]$. Let ρ_λ be a convex combination of ρ and ρ', that is, $\rho_\lambda = \lambda\rho + (1-\lambda)\rho'$, $\lambda \in (0,1)$. By construction of ρ_λ, $V(\rho_\lambda) =$

$\rho_\lambda U_H(\rho_\lambda)+(1-\rho_\lambda)U_L(\rho_\lambda) = \lambda[\rho U_H(\rho_\lambda)+(1-\rho)U_L(\rho_\lambda)]+(1-\lambda)[\rho' U_H(\rho_\lambda)+(1-\rho')U_L(\rho_\lambda)]$. Substituting $x = \rho_\lambda$ into the previous two inequalities establishes the desired convexity,

$$V(\rho_\lambda) \leq \lambda V(\rho) + (1-\lambda)V(\rho').$$

In Section 6, we show that the decision problem is formally equivalent to bidding in a sequence of future auctions. Alternative interpretations of the decision problem include a decision on home production, or whether to conduct costly search upon losing the auction.

3 Analysis

To start our analysis of the effects of the information policy on revenues, we characterize equilibrium bidding behavior and compare revenues for three information policies. In Section 3.1, we consider the case where no information (other than who has won) is released, and in Section 3.2, we consider the case where the auctioneer reveals his (fully informative) information about the state.

3.1 Equilibrium without information revelation

The following proposition characterizes equilibria in which the bidding strategies of the buyers are symmetric and strictly increasing. It also provides conditions for a symmetric and monotone equilibrium to exist. Let y denote the highest signal among the $n-1$ competitors of any given bidder, and let $\rho(s,x) = \Pr(H \mid s, y = x)$ represent the posterior of a bidder with signal s who knows that the highest other type is x. Let

$$\rho_{lose}(s,x) = \Pr(H|s, y \geq x);$$

denote the probability of the high state conditional on losing with a signal s, and the highest other signal being at least x.

Proposition 1 Characterizing Bidding Strategies *If V is strictly decreasing in ρ, then there is an essentially unique monotone and symmetric equilibrium. For almost all s,*

$$b(s) = v - [\rho(s,s)U_H(\rho_{lose}(s,s)) + (1-\rho(s,s))U_L(\rho_{lose}(s,s))]. \tag{1}$$

If V is not strictly monotone, then monotone, symmetric equilibria do not exist.

For most of the analysis, we assume that the decision problem is such that V is decreasing (see Example 1 for a discussion of the non-monotone case), and we concentrate on the monotone equilibrium in the game with no information revelation. In Lemma 1 in the Appendix, we show that V is decreasing if the action that is optimal in the high state yields a higher utility in the low state than in the high state, that is $U_L(a^*(1)) > U_H(a^*(1))$. This condition requires that payoffs are much higher in the low state in the sense that being in the low state is beneficial, even if the action taken was tailored to the high state.

Regarding the bidding strategy in formula (1), note that in a second-price auction with outside options the players bid their valuation v *minus* the value of the outside option, which is given in the bracket on the right-hand side of (1). The relevant outside option (continuation value) must be assessed conditional on tieing ($\rho(s,s)$), since a bidder is indifferent between winning and losing at the optimal bid if he is given the information that he is tieing at the top.[8] The continuation value also depends on the action taken in the continuation problem, which is determined by the posterior conditional on losing, $\rho_{lose}(s,s)$.

We point out a relationship with the standard interdependent-value auctions setup of Milgrom and Weber (1982).[9] In our model, the valuation of a bidder for the object net of the continuation payoff in state w is given by

$$v - U_w(\rho),$$

where ρ is the belief of the bidder that determines his optimal action in the decision problem. In the *special case* where the decision problem is degenerate—optimal actions are independent of beliefs—there exist real numbers U_L^*, U_H^* such that the net valuation of a bidder is $v - U_w^*$ in state w, for arbitrary $\hat{\rho}$. In this case, the equilibrium bids in our dynamic setting are the same as the equilibrium bids in a static auction with pure common values $v - U_L^*$ and $v - U_H^*$. On the one hand, a pure common-value auction in the framework of Milgrom and Weber (1982) with valuations $v - U_L^*$ and $v - U_H^*$ in the two states gives rise to an equilibrium bid function of $\rho(s,s)(v - U_H^*) + (1 - \rho(s,s))(v - U_L^*) = v - [\rho(s,s)U_H^* + (1 - \rho(s,s))U_L^*]$. On the other hand, if the decision problem is degenerate, then the equilibrium bid function of our auction model (1) simplifies to

$$b(s) = v - [\rho(s,s)U_H^* + (1 - \rho(s,s))U_L^*]. \tag{2}$$

[8] In the classic interdependent value setup of Milgrom and Weber (1982), a similar logic applies: the value of winning is assessed assuming the pivotal event of tieing at the top. This similarity to classic common-value auctions is not a coincidence, as we discuss below.

[9] Indeed, the relationship established here is with the pure common-value mineral rights model of the early auction literature, which was generalized by Milgrom and Weber (1982). See Milgrom and Weber (1982) for a list of early contributions to that literature.

Thus, when the decision problem is degenerate, our dynamic auction has the same equilibrium bids as a common-value auction in the framework of Milgrom and Weber (1982) with appropriately defined value functions that take the continuation payoffs into account.

3.2 Revenue decomposition

Take the problem of an auctioneer who knows the state precisely[10] and can commit ex ante to reveal the state at the end of the auction. We compare the ex ante expected revenue if he reveals the state before the auction (R^{Before}), with the ex ante expected revenue if the state is not revealed (R^{None}). In order to obtain insights into the trade-offs involved, we calculate the difference in revenues ($R^{Before} - R^{None}$) as the difference of two opposing effects.

To formally introduce those two effects, let R^{After} denote the ex ante expected revenue in the second-price auction when the state is revealed *after* the auction. We decompose the change in revenue from revealing the state before the auction:

$$R^{Before} - R^{None} = \underbrace{\left(R^{Before} - R^{After}\right)}_{\text{Linkage Effect}} - \underbrace{\left(R^{None} - R^{After}\right)}_{\text{Continuation Value Effect}} .$$

The ex ante expected revenues of the auctioneer when the state is revealed before or after the auction are derived next. When the state is revealed *before* the auction takes place, all bidders bid $v - V(1)$ in the high state, and all bidders bid $v - V(0)$ in the low state in the unique symmetric equilibrium. Therefore,

$$R^{Before} = v - (1 - \rho_0) V(0) - \rho_0 V(1) = v - V(0) + (V(0) - V(1)) \rho_0. \tag{3}$$

When the state is revealed *after* the auction, the full-information optimal action is taken in both states, yielding utilities $V(1)$ and $V(0)$. In this case, our model can be reduced to the standard pure common-value setup as discussed before, and the equilibrium bid function is $v-(\rho(s,s)V(1) + (1-\rho(s,s))V(0))$. Let $g_{(2)}(s)$ denote the unconditional density function of the second largest signal of the N signals from an ex ante perspective.[11] The bidder with the second highest signal determines the revenue in the second-price auction, and the expected revenue is

$$R^{After} = v - V(0) + (V(0) - V(1)) \int_{\underline{s}}^{\overline{s}} \rho(s,s) g_{(2)}(s) ds. \tag{4}$$

[10] In Section 5.3, we discuss the case where the auctioneer's information about the state is not fully precise, and we show that our results still hold qualitatively.

[11] Formally, $g_{(2)}(s) = \rho_0 N g_H(s)(1 - G_H(s)) G_H^{N-2}(s) + (1 - \rho_0) N g_L(s)(1 - G_L(s)) G_L^{N-2}(s)$.

Proposition 2 Decomposing the effect of information revelation *The linkage effect and the continuation value effect are given by:*

$$R^{Before} - R^{After} = (V(0) - V(1)) \int_{\underline{s}}^{\overline{s}} g_{(2)}(s) \left(\rho_0 - \rho(s,s)\right) ds, \tag{5}$$

$$R^{None} - R^{After} = \int_{\underline{s}}^{\overline{s}} g_{(2)}[\rho(s,s)(V(1) - U_H(\rho_{lose})) + (1 - \rho(s,s))(V(0) - U_L(\rho_{lose}))]ds. \tag{6}$$

The revenue of the seller is lower if the state is revealed before the auction rather than not revealed at all if and only if the continuation value effect is larger than the linkage effect.

Proof By adding and subtracting terms involving $V(0)$ and $V(1)$ to (1), the ex ante expected revenue from not revealing the state can be rewritten as

$$\begin{aligned} R^{None} &= \int_{\underline{s}}^{\overline{s}} g_{(2)}(s)b(s)ds = v - V(0) + (V(0) - V(1)) \int_{\underline{s}}^{\overline{s}} g_{(2)}\rho(s,s)ds + \\ &+ \int_{\underline{s}}^{\overline{s}} g_{(2)}[\rho(s,s)(V(1) - U_H(\rho_{lose})) + (1 - \rho(s,s))(V(0) - U_L(\rho_{lose}))]ds. \end{aligned}$$

Then the result follows directly from formulas (3) and (4). ∎

Let us interpret the two effects. The linkage effect measures the difference of equilibrium rents the bidders make under the two information policies. This difference is captured by the auctioneer as in Milgrom and Weber (1982). As (5) shows, the larger the difference in the intrinsic payoffs between the states $(V(0) - V(1))$ is, the more pronounced the linkage effect is. The continuation value effect measures how much the relevant bidder's (the one with the second highest signal) continuation value increases when the auctioneer reveals the state. To see this, note that $V(1) - U_H(\rho_{lose}(s))$ and $V(0) - U_L(\rho_{lose}(s))$ measure the increase in the continuation value of a bidder with type s when the state is revealed.

To illustrate the logic of our decomposition, suppose that information has no value, that is, the utility in the continuation problem depends only on the state, not on the action. Inspection of the continuation value effect shows that it is zero. Since only the linkage effect remains active, revealing information is profitable. This is not surprising, because in this case, our model reduces to the Milgrom and Weber (1982) setup (see the discussion after Proposition 1).

We provide an example with non-monotonic value function that is a polar opposite of the above case where the value of the outside option did not depend on any action taken.

Example 1 The two states are symmetric. In particular, $U_H(\rho) = 1 - a(1-\rho)^2$, and $U_L(\rho) = 1 - a\rho^2$. The (unique) optimal action at belief ρ is $a^*(\rho) = \rho$. The value function is $V(\rho) = V(1-\rho)$ for all $\rho \in [0,1]$, and the value function is U-shaped. Consequently, the intrinsic payoff differences between the two states are completely absent, but it is very important to *know* the state when making the decision. Since the two states are symmetric, we concentrate on a *state-symmetric* equilibrium where $b(s) = b(1-s)$ for all s. The posterior upon tieing is $\widetilde{\rho}_{tie}(s) = \Pr(H \mid s_1 = s, s_2 = s \text{ or } s_2 = 1-s) = s$, and the posterior upon losing is $\widetilde{\rho}_{lose}(s) = \Pr(H \mid s_1 = s, s_2 \in (s, 1-s)) = s$. When the state is not revealed, the equilibrium bid function is $b = v - [\widetilde{\rho}_{tie} U_H(\widetilde{\rho}_{lose}) + (1-\widetilde{\rho}_{tie}) U_L(\widetilde{\rho}_{lose})]$, which can be shown to simplify to $b = v - 1 + as(1-s)$. The bid with state revelation is $v - V(0) = v - 1$ in the low state, and $v - V(1) = v - 1$ in the high state. Comparing the bids under the two policies implies that the revenue comparison favors not revealing the state.

Another way to look at Example 1 is through the CV and LP effects. A glance at formulas (5) and (6) reveals that the linkage effect is null (since $V(0) = V(1)$), while the continuation value effect is positive.[12] Therefore, revealing information decreases revenues in a situation like Example 1 where market conditions per se (given the action taken) do not strongly influence payoffs. Therefore, the framework of Milgrom and Weber (1982), and Example 1 can be taken as opposite extremes where revenue comparisons are easily made. In Section 4, we compare the strength of the two effects to obtain insight into the optimal disclosure policy in intermediate situations where both effects are present.

4 Comparative statics of information revelation

Let us discuss the major factors that determine whether revealing an informative signal about the state is profitable for the auctioneer. We have seen in Section 3.2 that a large intrinsic payoff difference between the states makes it more profitable for the auctioneer to reveal his information about the state. Following Proposition 2, we also discussed that the CV effect is stronger when information has higher value in the decision problem after losing the auction, and thus the auctioneer has less incentive to reveal the state in this case.

[12] When the state is revealed after the auction, the revenue has to be calculated directly, since bidders may not use monotone bids in equilibrium. This revenue is $R^{After} = v - V(0)$, which is also equal to R^{Before} as $V(0) = V(1)$ in (3). Similarly, the formula for R^{None} changes slightly, but this does not change the fact that $CV = R^{None} - R^{After} > 0$.

In this Section, we discuss how the precision of the bidders' signals affects whether revealing information is profitable. To do so, let us parametrize a family of signal distribution functions G_H^α, G_L^α by a precision parameter α and let $\widetilde{\rho}^\alpha(s) = \frac{\rho_0 g_H^\alpha(s)}{\rho_0 g_H^\alpha(s)+(1-\rho_0)g_L^\alpha(s)}$ denote the posterior upon receiving signal s. We normalize signal precision such that signals are uninformative when $\alpha = 0$, and are perfectly informative when $\alpha = 1$. In particular, we assume that for all s it holds that $G_H^\alpha(s) = G_L^\alpha(s)$ when $\alpha = 0$, so each signal realization s has the same meaning. To formalize that signals become perfectly informative when α is close to 1 we assume that for any $r > 0$ it holds that

$$\lim_{\alpha\to 1} \Pr(\widetilde{\rho}^\alpha(s) \geq r \mid \omega = L) = \lim_{\alpha\to 1} \Pr(\widetilde{\rho}^\alpha(s) \leq 1 - r \mid \omega = H) = 0.$$

Our next result shows that hiding the state is revenue enhancing when bidders have uninformative signals:

Proposition 3 Incentives for information revelation when signals are not too informative *The continuation value effect is strictly greater than the linkage effect if signals are not too informative ($\alpha \leq \overline{\alpha}$ for some $\overline{\alpha} > 0$), and information has value, that is, V is not linear.*

Proof When $\alpha = 0$ for all s it holds that $g_H^\alpha(s) = g_L^\alpha(s)$, and thus $\rho^\alpha(s,s) = \rho_0$ holds[13]. Therefore, $\int_{\underline{s}}^{\overline{s}} g_{(2)}(s)\left(\rho_0 - \rho^\alpha(s,s)\right) ds = 0$ when $\alpha = 0$. This implies that $\alpha = 0 \Rightarrow LP = 0$. On the other hand, as long as V is not linear it holds for a set of beliefs $\rho \in (\underline{\rho}, \overline{\rho})$ that $V(1) - U_H(\rho)$, $V(0) - U_L(\rho) > 0$, and the continuation value effect is strictly positive - see formula (6). The existence of an appropriate $\overline{\alpha}$ then follows from the continuity of the CV and LP effects in α. ∎

This result can also be explained through inspection of the bid functions. When bidders do not have any informative signals, then the bid function without information revelation becomes $b(s) = v - [\rho_0 U_H(\rho_0) + (1-\rho_0)U_L(\rho_0)]$, while with state revelation the expected value of the bid is $Eb = v - [\rho_0 U_H(1) + (1-\rho_0)U_L(0)]$, which is smaller by construction.

It is instructive to consider a numerical example to assess the strength of the two effects when bidders have signals that are not completely uninformative.

Example 2 There are two bidders and the two states are equally likely ex ante. The signal distribution function is $G_L(x) = 1 - (1-x)^{1/(1-\alpha)}$ in the low state, and $G_H(x) = x^{1/(1-\alpha)}$ in the high state for some $\alpha \in [0,1]$. The family of signals (G_L, G_H) satisfies our earlier

[13] To see this, note that in this case $\widetilde{\rho}^\alpha(s) = \frac{\rho_0 g_H^\alpha(s)}{\rho_0 g_H^\alpha(s)+(1-\rho_0)g_L^\alpha(s)} = \rho_0$, and $\rho^\alpha(s,s) = \frac{\rho^\alpha(s) g_H^\alpha(s)(1-G_H^\alpha)^{N-2}}{\rho^\alpha(s) g_H^\alpha(s)(1-G_H^\alpha)^{N-2}+(1-\rho^\alpha(s))g_L^\alpha(s)(1-G_L^\alpha)^{N-2}} = \rho_0$ for all s, since $G_H^\alpha = G_L^\alpha$ also holds.

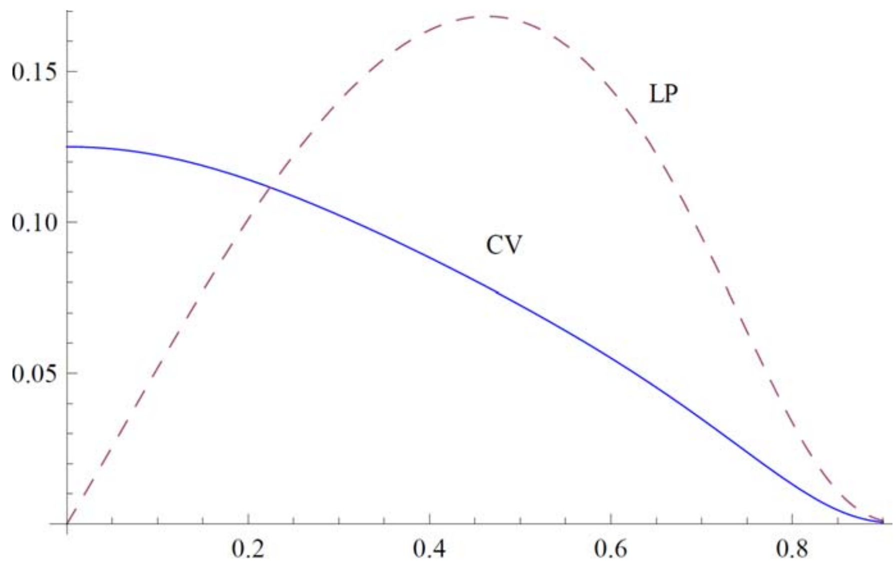

Figure 1: CV and LP as functions of signal precision α in Example 2. If bidders' signals are imprecise, the CV effect dominates and revealing information reduces revenue. If signals are precise, the LP effect dominates and revealing information increases revenue.

assumption, that is, if $\alpha = 0$, the signal is uninformative and as $\alpha \to 1$ the signal reveals the state fully in the limit. Assume that the utility from taking the action that is optimal with belief ρ is $U_H(\rho) = \rho - 0.5\rho^2 - 0.5 + t$ if the state is high, and $U_L(\rho) = 1 - 0.5\rho^2 + t$ if the state is low. Therefore, the value function becomes $V = t + \rho(\rho - 0.5\rho^2 - 0.5) + (1-\rho)(1-0.5\rho^2)$. Using that $V(0) = U_L(0) = 1 + t$ and $V(1) = U_H(1) = t$, straightforward algebra shows that

$$CV = \int_{\underline{s}}^{\overline{s}} g_{(2)}(s)(0.5\rho_{lose}^2(s,s) + 0.5\rho(s,s) - \rho(s,s)\rho_{lose}(s,s))ds. \tag{7}$$

The fact that $V(0) - V(1) = 1$ implies that the linkage effect can be written as

$$LP = 0.5 - \int_{\underline{s}}^{\overline{s}} g_{(2)}(s)\rho(s,s)ds. \tag{8}$$

Figure 1 depicts the two effects as functions of the information precision parameter α. As the previous Proposition shows, and Figure 1 confirms, the continuation value effect dominates the linkage effect when signal precision α is low. The Figure also shows that the linkage effect dominates the continuation value effect when signals are sufficiently precise. We show that under a mild assumption this is a general feature of the model. Recall that $\widetilde{\rho}^{\alpha}(s)$ is the posterior of a bidder upon receiving signal s. (We superscript this posterior to emphasize its dependence on the signal precision α.) The assumption is as follows:

Assumption CR: *Given a family of signal distributions indexed by* α*, there exists some* $T > 0$ *and* $\widehat{\varepsilon} > 0$ *such that for all* $0 < \varepsilon \leq \widehat{\varepsilon}$

$$\lim_{\alpha \to 1} \frac{\int_{s: \widetilde{\rho}^{\alpha}(s) \geq 1-\varepsilon} g_{(2)}(s)(1 - \widetilde{\rho}^{\alpha}(s))^2 ds}{\int_{s: \widetilde{\rho}^{\alpha}(s) \leq \varepsilon} g_{(2)}(s) \widetilde{\rho}^{\alpha}(s) ds} \leq T.$$

We obtain the following result:

Proposition 4 Incentives for information revelation when signals are very informative *Assume that there are two bidders. Given a family of signal distributions indexed by* α *that satisfies Assumption CR, the continuation value effect is weaker than the linkage effect if the signals are informative, that is, when* $\alpha \in (\widehat{\alpha}, 1)$ *for some* $\widehat{\alpha} < 1$.

Proof See the online Appendix.

To interpret Assumption CR, imagine that for any fixed α the signals are distributed symmetrically in the sense that $\rho_0 = 1/2$, and $\Pr(\widetilde{\rho}^{\alpha}(s) \leq t \mid L) = \Pr(\widetilde{\rho}^{\alpha}(s) \geq 1 - t \mid H)$ for any $t \in [0, 1]$. In this case, the relevant ratio in Assumption CR can be bounded appropriately. The key idea is that the ratio $\lim_{\alpha \to 1} \frac{\int_{s: \widetilde{\rho}^{\alpha}(s) \geq 1-\varepsilon} g_{(2)}(s)(1 - \widetilde{\rho}^{\alpha}(s)) ds}{\int_{s: \widetilde{\rho}^{\alpha}(s) \leq \varepsilon} g_{(2)}(s) \widetilde{\rho}^{\alpha}(s) ds}$ (that is, omitting the square sign from the numerator) is less than 1 for any α, as it is less likely from an ex ante point of view that the lower of the two signals induces a high posterior (above $1 - \varepsilon$) than that it induces a low one (below ε). Additionally, $\lim_{\alpha \to 1} \frac{(1 - \widetilde{\rho}^{\alpha}(s))^2}{1 - \widetilde{\rho}^{\alpha}(s)} \leq \varepsilon$ if $\widetilde{\rho}^{\alpha}(s) \geq 1 - \varepsilon$. So, for all $\widehat{\varepsilon} > 0$ and $0 < \varepsilon \leq \widehat{\varepsilon}$ it holds that $\lim_{\alpha \to 1} \frac{\int_{s: \widetilde{\rho}^{\alpha}(s) \geq 1-\varepsilon} g_{(2)}(s)(1 - \widetilde{\rho}^{\alpha}(s))^2 ds}{\int_{s: \widetilde{\rho}^{\alpha}(s) \leq \varepsilon} g_{(2)}(s) \widetilde{\rho}^{\alpha}(s) ds} \leq \widehat{\varepsilon}$, and thus Assumption CR is satisfied. Therefore, the only way to violate Assumption CR is to assume that the signal precision is very different in the two states in the limit. In particular, such a violation requires that signal precision converges much slower in the high state than in the low state. At the end of the online Appendix, we consider such an example, and we show that the conclusion of our Proposition above fails.

5 Further aspects of information policy choices

In many settings, sellers might not possess complete information about the state or cannot verifiably disclose such information. We discuss such settings in this section. First, suppose the seller has no private information himself. We ask whether the seller has an incentive to facilitate information exchange between the privately informed bidders. Specifically, in Section 5.1 we ask whether the seller prefers losing bidders to learn the winning bid, and in

Section 5.2 we study whether the seller prefers an English-auction (a "transparent" mechanism) to a second-price auction. Second, in Section 5.3 we ask whether the precision of the seller's signal affects whether he would like to reveal the signal.

5.1 Revenue when the winning bid is revealed

In the previous Section, we showed that revealing the state after the auction decreases revenues. The argument there relied on the fact that the linkage effect is absent when information is revealed after the auction, while the continuation value effect is present. This suggests that revealing any information after the auction lowers the revenues of the auctioneer. We show that this is indeed the case when the winning bid is revealed.

Proposition 5 Hiding the winning bid *If V is strictly convex and strictly decreasing, then revealing the winning bid after a second-price auction decreases revenues compared to not revealing any information.*

Proof Let b_b denote the (unique) symmetric and monotone equilibrium bid function when the winning bid is revealed. The bidders still condition their bids on tieing at the top. The novelty is that a losing bidder learns that he tied at the top, if he did, so his relevant belief upon tieing at the top is $\rho(s,s)$. This belief governs the action a bidder takes who is tied at the top (and loses). Therefore, one can show that

$$b_b(s) = v - [\rho(s,s)U_H(\rho(s,s)) + (1-\rho(s,s))U_L(\rho(s,s))].$$

Comparing the equilibrium bidding strategy with and without bid revelation we obtain that for almost all $s \in [\underline{s}, \overline{s}]$,

$$b_b(s) = v - [\rho(s,s)U_H(\rho(s,s)) + (1-\rho(s,s))U_L(\rho(s,s))] <$$

$$< v - [\rho(s,s)U_H(\rho_{lose}(s,s)) + (1-\rho(s,s))U_L(\rho_{lose}(s,s))] = b(s),$$

where the inequality follows from the fact that $\rho = \arg\max_{q\in[0,1]} \rho U_H(q) + (1-\rho)U_L(q)$. The equilibrium bid function is pointwise lower if the winning bid is revealed, which implies that the expected revenue is lower under bid revelation as well. ∎

5.2 Revenue comparison between auction formats

In this section, we compare the revenues of the auctioneer from the three standard auction formats, fixing the information policy. In particular, we assume that the auctioneer does not reveal any bid information and runs a first-price, second-price or English-auction. Note that an English-auction automatically reveals all losing bids, as bidders can observe when their rivals dropped out of the auction. No such revelation occurs in the other two formats. Using the linkage principle, Milgrom and Weber (1982) shows that in their setup the English-auction yields higher revenue than the second-price auction, which in turn yields higher revenue than the first-price auction. Their argument relies on the effect of information revelation on equilibrium rents, as the equilibrium allocation in their setup is not affected by the information policy. This is not the case in Board (2009), where the information policy's effect on equilibrium allocation modifies the insights of the linkage principle.

First, we observe that in our model the second-price auction revenue dominates the first-price auction just like in Milgrom and Weber (1982). The key is that in both auction formats the losers learn the same information; they only learn that there was a bidder with a higher signal than theirs. This implies that they take the same actions in the continuation decision problems, and therefore the presence of endogenous outside options does not change the comparison between the two formats.[14]

The important novelty when comparing second-price and English auctions is that the English auction reveals more information, so it allows the losers to make better decisions in the aftermarket, and thus the continuation value effect favors the second-price auction. Since the continuation value and the linkage effects work in opposite directions, one needs to assess whether the second-price or the English auction raises higher revenues. To present our result, we concentrate on a three-bidder example, where a monotone equilibrium exists and the auctioneers' revenue is higher in the second-price auction than in the ascending auction.

Example 3 *Let* $\rho_0 = 1/2$, $N = 3$, $g_L(s) = 2(1-s)$ *and* $g_H(s) = 2s$, *and*

$$U_H(\rho) = \rho^{1/10} - (1/11)\,\rho^{11/10},\ U_L(\rho) = d - (1/11)\,\rho^{11/10}.$$

The following observation is established in the online Appendix:

Observation *In Example 3, there exists some* $\overline{d} > 1$ *such that the expected revenue of the English-auction is less than the expected revenue of the second-price auction whenever* $1 \le d < \overline{d}$.

Note that the parameter d is a measure of the payoff differences between the two states. Therefore, the above observation means that the more transparent English auction yields a

[14] In the online Appendix, we show that our first-price auction has the same equilibrium as a standard common-value first-price auction with a particular value function, and that the same holds for our second-price auction with the same constructed value function. Since it is known that in such common-value auctions the second-price format yields higher revenue, it must be the case for our auctions too.

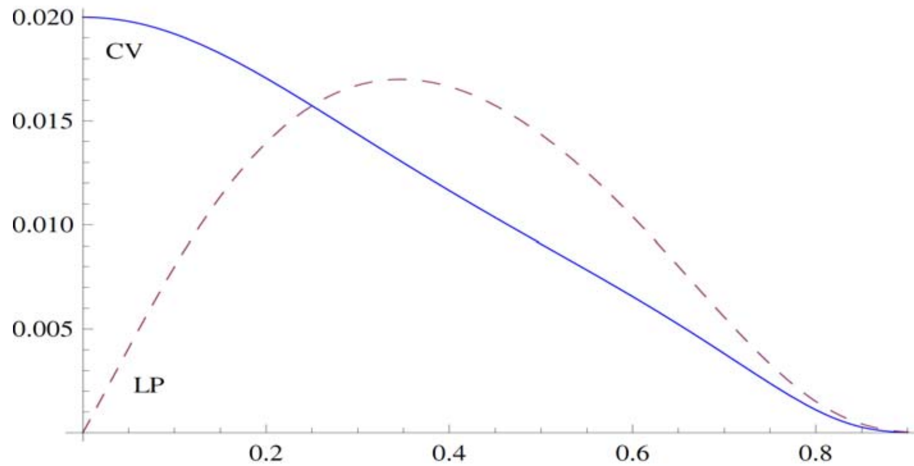

Figure 2: The CV and LP effects as functions of the precision of the buyers' signal α when the precision of the seller's signal is fixed at $z = 0.7$. Similarly to the case of a perfectly informative seller's signal, the auctioneer would like to reveal his signal $(LP > CV)$ if and only if the bidders' signals are already precise.

higher revenue than the second-price if the payoff difference between the two states is large enough. This is similar to what we found when comparing whether the auctioneer should reveal an informative signal in Section 3.

5.3 Imperfect signal for the auctioneer

To show that our results do not depend on the auctioneer's signal being fully informative, we study an example where the auctioneer's signal is noisy. This also allows studying the case where the auctioneer can choose the precision of the signal he wishes to reveal. In the online Appendix, we provide the general framework and analysis, and we conduct some numerical calculations. While few general results are available, two observations still hold. First, when the bidders are fully informed, both effects are null by construction. Second, and more interestingly, the auctioneer still has an incentive to hide his information when the bidders have imprecise signals[15]:

Proposition 6 Sellers with imperfect information *For any fixed $z > 0$, there exists a cutoff $\overline{\alpha} > 0$ such that for all $\alpha \leq \overline{\alpha}$ the seller's revenues are higher when he does not reveal his signal.*

To obtain additional insight, we study Example 2 in the online Appendix, assuming that there are two realizations for the auctioneer's signal, s_L and s_H. Moreover, let $\Pr[s_H \mid H] =$

[15] The proof is omitted, as it is very similar to the proof of Proposition 3.

$\Pr[s_L \mid L] = z \in [0.5, 1]$, that is, z measures the precision of the auctioneer's signal. First, we depict (see Figure 2) the CV and LP effects for a fixed level of precision of the auctioneer's signal ($z = 0.7$), varying the signal precision of the bidders α.[16] The picture is qualitatively similar to the full precision case: the auctioneer reveals information if and only if the bidders have precise private signals. This shows that the trade-offs of an appropriate information policy are robust to the introduction of noisy signals for the seller. Second, we ask whether the auctioneer has any incentive to garble his signal. In particular, suppose that he observes the state fully ($z = 1$), but can commit to announce a signal with an arbitrary precision ζ. Numerical calculations show that in the example studied the auctioneer would always choose to reveal his fully informative signal (choose $\zeta = 1$) or no signal at all ($\zeta = 0.5$), but it is never revenue maximizing to choose a signal with intermediate precision.[17]

6 A dynamic model of market interaction

We introduce a dynamic auction model similar to existing matching-and-bargaining games. In this model, losing bidders can continue bidding in other auctions for a substitute. This is the case in many real-world auctions mentioned at the beginning of our introduction to this paper. We show that the dynamic auction model can be analyzed using a two-stage model equivalent to the model from Section 2, validating our reduced form two-stage model.

The model is similar to the steady-state dynamic auction model by Satterthwaite and Shneyerov (2007). There is a continuum of buyers and a continuum of sellers in every period.[18] The traders exchange an indivisible, homogeneous good. Buyers and sellers are matched randomly each period and each trader who does not transact has a chance to be rematched with other partners in the next period. The assumptions of a large market and random matching imply that no two traders are rematched again with positive probability. Departing from Satterthwaite and Shneyerov (2007), we assume that there is an unknown state of nature that determines the aggregate market conditions. Specifically, as in Wolinsky (1990) and in Golosov, Lorenzoni, and Tsyvinski (2009), there are two states of nature, a high state and a low state $w \in \{H, L\}$. Both states are equally likely. The realized state of nature is fixed throughout and unknown to the traders. The state of nature determines the constant and exogenous number of new traders born each period, d_H and d_L. We consider steady-state equilibria in stationary strategies. The endogenous ratio of the mass of buyers

[16] Fixing z at a different level does not change the main features of the Figure.

[17] We do not consider asymmetric policies that reveal signals with different precision for different bidders. The usefulness of such signals was shown by Mares and Harstad (2003).

[18] The model from this section is also analyzed in Lauermann, Merzyn and Virag (2011). In that paper, we concentrate on comparative statics analysis of the market allocation with respect to market frictions.

to the mass of sellers in the market in state w is denoted by μ_w. The distribution of beliefs of buyers is denoted by Φ_w, that is, in state w, a share $\Phi_w(\rho)$ of buyers believes that the probability of the high state is smaller than ρ.

Each period consists of several steps:

1. Entry occurs (the *inflow*): A mass one of sellers and a mass d_w of buyers is born. The mass of sellers who are born each period is one, independent of the state. Each buyer enters with a signal $s \in [\underline{s}, \overline{s}]$. The structure of the signals of the entering buyers is the same as in the Section 2, and signals are distributed according to the cumulative distribution function G_w.

2. Each buyer from the market is randomly matched with one seller. The probability that a seller is matched with $k = 0, 1, 2, ...$ buyers is Poisson distributed and equal to $e^{-\mu_w}\mu_w^k/k!$. An implication of the Poisson distribution is that this is also the probability that a buyer has $k = 0, 1, 2, ..$ competitors. The larger μ_w, the more competitors a buyer expects.

3. Each seller runs a sealed-bid, second-price auction with no reserve price. Buyers do not observe how many other buyers are matched with the same seller. The bids are not revealed ex post, so bidders learn only whether they have won with their submitted bid. Trading at price p yields payoffs $v - p$ and $p - c$ for a buyer or a seller, respectively (where c is the common cost of selling).

4. A winning buyer pays the second highest bid, obtains the good and leaves the market. A losing buyer stays in the market with probability $\delta \in [0, 1)$ and is matched with another seller in the next period.[19] Similarly for sellers. Those traders who do not stay, exit the market permanently. A trader who exits the market without trading has zero payoffs.

5. Upon losing, buyers update their beliefs, based on the information gained from losing with their bids. The remaining traders who neither trade nor exit, stay in the market.

Given a stationary bidding strategy, the ratio and the distribution of beliefs define a distribution of the highest competing bid. The distribution of the highest competing bid is also constant over time and depends only on the state of nature. We denote its distribution

[19] The exit probability is equivalent to a discount rate δ from the perspective of the individual buyers. On the aggregate level, it ensures existence of a steady state, see Satterthwaite and Shneyerov (2008).

by F_w.[20] Given a bid b, the probability of winning is $F_w(b)$. If a bidder loses, he proceeds to the next auction in which he faces the same distribution of competing bids. Losing bidders learn nothing beyond the mere fact of having lost. Hence, a bidder's history consists only of his own losing bids. The bidding problem of an individual buyer is therefore simply to choose a sequence of bids $\{b_t\}_{t\in\mathbb{N}}$. The expected payoff of a bidding strategy in state w is denoted $EU(\{b_t\}|w)$, and it satisfies

$$EU(\{b_t\}|w) = \int_0^{b_1} (v-\tau)\, dF_w(\tau) + \delta(1-F_w(b_1))\, EU(\{b_t\}_{t=2}^{\infty}|w). \tag{9}$$

The bidding problem of an individual bidder in the dynamic auction can be analyzed using a two-stage game, equivalent to the one described in Section 2: The first stage consists of a second-price auction. The number of bidders is Poisson distributed with parameter μ_w and unobservable.[21] Each bidder receives a signal. Signals induce a distribution of posteriors, Φ_w. The winning bidder receives the good at the second highest bid. A bidder who loses proceeds to the second stage. The second state consists of a single-person decision problem. The action set in the decision problem is given by the set of all sequences $\{b_t\}$. The payoffs from taken an action $\{b_t\}$ in state w are given by $EU(\{b_t\}|w)$, implicitly defined by (9). The optimal bid in the first period depends on the bidders' expectation of the continuation payoffs, just as in Section 3.1. Information that a losing bidders receives changes his expectations of his continuation payoffs, and, hence, his bid.

Given the equivalence result above, we can use the analysis of the two-stage model to provide insights into an individual seller's auction design problem in this dynamic setting. Suppose a single seller has the chance to choose a different information policy, taking as given the continuation payoffs of the bidders provided by the dynamic market described before. The continuation payoffs derived from the infinite horizon economy have the same properties as continuation payoffs in our two-stage reduced form framework. In particular, continuation payoffs are convex in beliefs. Therefore, a single seller in the dynamic auction market faces the same design problem as the auctioneer in the two-stage model.[22] For example, our results from Section 5.1 imply that the auctioneer does not want to reveal the winning bid to the losers. An interesting avenue for future research is to study dynamic settings where *each*

[20] For brevity, we do not provide a complete analysis of steady-state equilibrium. A steady-state equilibrium consists of a bidding strategy, an updating rule, and a characterization of the endogeneous stock of buyers and sellers. If buyers follow a symmetric, strictly increasing bidding strategy $b:[0,1]\to[0,v]$, with $b(\rho)$ being the bid given belief ρ, then F_w is given by $F_w(\beta) = e^{-\mu_w(1-\Phi_w(b^{-1}(\beta)))}$.

[21] The assumption that the number of bidders is a random variable that depends on the state is what allows us to introduce aggregate uncertainty about the market condition in this section. We chose not to make the number of bidders random in Section 2 in order to keep the model simple.

[22] The fact that the number of buyers is random is not consequential for the analysis.

seller chooses any information policy he wishes.[23]

Finally, note that the large market assumption in the dynamic auction model is crucial. If a buyer's bid today influences his belief about the future distribution of bids, that is, if the distribution of bids depends not only on the state but also on the history of bids, then we cannot reduce the dynamic bidding problem to a static auction followed by a simple decision problem. Therefore, our analysis does not apply to small markets in which losing bidders from one auction are likely to interact again in the future.

7 Conclusion

We study auctions with endogenous outside options determined through actions taken after the auction. We find that auctions with less information revelation may yield higher revenues: Opaque auctions decrease the information available to losing bidders, which leads to worse decisions in the aftermarket. This leads to worse outside options, which results in more aggressive bidding in the original auction. As an application, we show that the auctioneer may or may not reveal an informative signal about the aftermarket. We also ask whether the seller has an incentive to facilitate information exchange between the privately informed bidders. We show that a less transparent auction format, the second-price auction, can yield higher revenues than an English auction, as the English auction fosters learning and provides higher continuation values for the bidders. Comparative statics analysis shows that non-transparent auctions arise under a small payoff difference between the two states, a large value of information in the continuation problem, and imprecise signals of the bidders.

Our model provides a novel explanation for the prevalence of opaque trading mechanisms: Sellers may decrease the information flow to losing bidders to reduce the value of their outside options. Our result that opaque auctions may be prevalent has further implications for market design and public policy. For example, when buyers have imprecise information to begin with, the sellers may not want to reveal information. In this case, a public policy that promotes public information among buyers is doubly effective, as it also encourages the sellers to reveal more information themselves.

[23] The literature on "information percolation" initiated by Duffie and Manso (2007), and Duffie et al. (2009), studies information policy choices in dynamic matching models where agents learn about market conditions. In their settings, agents have no incentives to hide their information. Identifying general characteristics of market interaction under which agents are willing to share information is left for future research.

8 Appendix

In this Appendix, we prove Proposition 1. The proofs of Propositions 5, and 6 are in the online Appendix.

Let $\alpha(\rho)$ be the set of optimal actions when the belief is ρ. Define $U_H(\rho) = \max_{x\in\alpha(\rho)} u_H(x)$ and $U_L(\rho) = \min_{x\in\alpha(\rho)} u_L(x)$.[24] First, we show some useful results in regards to the continuation problem:

Lemma 1 *(i) Function U_H is monotone increasing in ρ, while U_L is monotone decreasing in ρ.*

(ii) For all $\rho \in [0,1]$, the value function satisfies $V(\rho) = \rho U_H(\rho) + (1-\rho)U_L(\rho) \geq \rho U_H(\rho') + (1-\rho)U_L(\rho')$.

(iii) For almost all $\rho \in [0,1]$,

$$V'(\rho) = U_H(\rho) - U_L(\rho). \tag{10}$$

(iv) The value function V is strictly decreasing if and only if $U_H(\rho) < U_L(\rho)$ for all $\rho < 1$.

Proof We start the proof by establishing that

$$\min_{x\in\alpha(\rho')} (u_H(x) - u_L(x)) \geq \frac{V(\rho') - V(\rho)}{\rho' - \rho} \geq \max_{x\in\alpha(\rho)} (u_H(x) - u_L(x)). \tag{11}$$

By construction, for all $z' \in \alpha(\rho')$, $z \in \alpha(\rho)$

$$V(\rho') = \rho' u_H(z') + (1-\rho')u_L(z') \geq \rho' u_H(z) + (1-\rho')u_L(z),$$

and

$$V(\rho) = \rho u_H(z) + (1-\rho)u_L(z) \geq \rho u_H(z') + (1-\rho)u_L(z').$$

Combining the last two formulas yields (11).

Formula (11) implies that for all $\rho' > \rho$,

$$\min_{x\in\alpha(\rho')} (u_H(x) - u_L(x)) \geq \max_{x\in\alpha(\rho)} (u_H(x) - u_L(x)). \tag{12}$$

Given this, $z' \in \alpha(\rho')$ and $z \in \alpha(\rho)$ imply $u_H(z') \geq u_H(z)$ and $u_L(z') \geq u_L(z)$,[25] which concludes the proof of result *(i)*.

[24] This formalization amounts to a particular selection of optimal actions $a^*(\rho) \in \alpha(\rho)$ for all ρ.

[25] Otherwise, suppose that $u_H(z') < u_H(z)$. Then $u_L(z') < u_L(z)$ follows from (12), which implies that z' is worse than z for any beliefs, and thus $z' \in \alpha(\rho')$ could not hold.

Take $x_1,\ x_2 \in \alpha(\rho)$. By construction, $V(\rho) = \rho u_H(x_1) + (1-\rho)u_L(x_1) = \rho u_H(x_2) + (1-\rho)u_L(x_2)$. Therefore, $u_H(x_1) \geq u_H(x_2) \Leftrightarrow u_L(x_1) \leq u_L(x_2)$, and thus $\underset{x\in\alpha(\rho)}{\arg\max} u_H(x) = \underset{x\in\alpha(\rho)}{\arg\min} u_L(x)$. Picking any action $a \in \underset{x\in\alpha(\rho)}{\arg\max} u_H(x) = \underset{x\in\alpha(\rho)}{\arg\min} u_L(x)$, we obtain $V(\rho) = \rho u_H(a) + (1-\rho)u_L(a)$. Since $a \in \underset{x\in\alpha(\rho)}{\arg\max} u_H(x)$, and $a \in \underset{x\in\alpha(\rho)}{\arg\min} u_L(x)$, therefore $U_H(\rho) = u_H(a)$, and $U_L(\rho) = u_L(a)$. Combining the last observations with $V(\rho) = \rho u_H(a) + (1-\rho)u_L(a)$ yields the equality result in *(ii)*. Picking any other belief ρ', and choosing $a' \in \underset{x\in\alpha(\rho')}{\arg\max} u_H(x) = \underset{x\in\alpha(\rho')}{\arg\min} u_L(x)$, we obtain that $U_w(\rho') = u_w(a')$ for $w = L, H$. By the incentive condition, $V(\rho) \geq \rho u_H(a') + (1-\rho)u_L(a') = \rho U_H(\rho') + (1-\rho)U_L(\rho')$, which establishes the inequality result in *(ii)*.

Noting, that V is convex, and is thus almost everywhere differentiable, we can use (11) to conclude that for almost all ρ, it holds that $V'(\rho) = \min_{x\in\alpha(\rho)} (u_H(x) - u_L(x)) = \max_{x\in\alpha(\rho)} (u_H(x) - u_L(x))$. Thus for almost all ρ, $\min_{x\in\alpha(\rho)} (u_H(x) - u_L(x)) = \max_{x\in\alpha(\rho)} (u_H(x) - u_L(x))$. Therefore, for almost all ρ it holds for all $a, b \in \alpha(\rho)$ that $u_H(a) - u_L(a) = u_H(b) - u_L(b)$. This implies that for almost all ρ it holds for all $a, b \in \alpha(\rho)$, that $u_H(a) = u_H(b)$, and $u_L(a) = u_L(b)$, because neither a nor b can be better in both states than the other optimal action. Consequently, for almost all ρ and all $a \in \alpha(\rho)$ we have that $U_w(\rho) = u_w(a)$ for $w \in \{L, H\}$. Combining this observation, with $V'(\rho) = \min_{x\in\alpha(\rho)} (u_H(x) - u_L(x))$ we obtain for almost all ρ that $a \in \alpha(\rho) \Rightarrow V'(\rho) = u_H(a) - u_L(a) = U_H(\rho) - U_L(\rho)$, concluding the proof of result *(iii)*. Result *(iv)* follows directly from results *(i)* and *(iii)*. ∎

We are ready to characterize the equilibrium bidding function. To relate our work to static interdependent-value auctions, we chose an indirect approach that relies on constructing an appropriate interdependent-value auction first. Let us recall some notation from the main text. First, y is the highest type of bidders other than i. Second, the following conditional beliefs were defined: $\rho(s, x) = \Pr(H \mid s, y = x)$, and $\rho_{lose}(s, x) = \Pr(s, y \geq x)$ denote the probability of the high state conditional on losing with a signal s, and the highest other signal being exactly x, and at least x, respectively.

Let us describe the utility of the bidders in our auction from placing different bids. Let $h_s(y)$ denote the density of the highest type among the other bidders if one has signal s.[26] Suppose that a bidder with signal s bids $b(x)$, and all other bidders use the symmetric, strictly increasing bidding function b. Then the bidder's expected utility is

$$\pi(s,x) = \int_{\underline{s}}^{x} (v - b(y))h_s(y)dy + \int_{x}^{\overline{s}} (\rho(s,y)U_H(\rho_{lose}(s,x)) + (1-\rho(s,y))U_L(\rho_{lose}(s,x))h_s(y)dy.$$

[26]The distribution function of the first order statics from $n-1$ i.i.d. draws is $H_s(y) = \rho(s)G_H^{n-1}(y) + (1-\rho(s))G_L^{n-1}(y)$, with $h_s(y) = \partial H_s/\partial y$.

To explain this formula, imagine that the highest type among the other bidders is $y < x$. Then the bidder wins and obtains a utility of $v - b(y)$. If $y > x$, then the bidder loses. In this case the high state has probability $\rho(s, y)$, but the bidder knows only that he lost, that is he only knows that the highest other type is greater than x. Therefore, the bidder's belief is $\rho_{lose}(s, x)$, and takes his action accordingly. Result *(iii)* in Lemma 1 states that

$$\rho U_H(\rho) + (1 - \rho)U_L(\rho) \geq \rho U_H(\rho') + (1 - \rho)U_L(\rho'). \tag{13}$$

Take the belief of a bidder who received signal s and lost with bid $b(x)$, that is set $\rho = \rho_{lose}(s, x) = \int_x^1 \rho(s, t)h_s(t)dt$. Note that for any ρ'

$$\int_x^{\overline{s}} \left(\rho(s, t)U_H(\rho') + (1 - \rho(s, t))U_L(\rho')\right) h_s(t)dt =$$

$$= \rho_{lose}(s, x)U_H(\rho') + (1 - \rho_{lose}(s, x))U_L(\rho'). \tag{14}$$

By (13) and (14), $\rho_{lose}(s, x) \in \arg\max_{\rho'} \int_x^{\overline{s}}(\rho(s, t)U_H(\rho') + (1 - \rho(s, t))U_L(\rho'))h_s(t)dt$, and thus

$$\frac{\partial}{\partial \rho'} |_{\rho' = \rho_{lose}(s,x)} \left[\int_x^{\overline{s}} (\rho(s, t)U_H(\rho') + (1 - \rho(s, t))U_L(\rho'))h_s(t)dt\right] = 0. \tag{15}$$

Note that (15) is a consequence of the envelope theorem applied to the learning problem in the post auction decision problem. Using (15), the partial derivative of π with respect to the second variable becomes

$$\pi^{(2)}(s, x) = h_s(x)[v - (\rho(s, x)U_H(\rho_{lose}(s, x)) + (1 - \rho(s, x))U_L(\rho_{lose}(s, x)) - b(x)].$$

We establish that a common-value auction with an appropriately defined value function and our second-price auction provide the same bidding incentives. In particular, let $v(s, y)$ denote the value of winning when one's signal is s, and the highest other signal is y. Take an interdependent-value (second-price) auction with value function

$$v(s, y) = v - [\rho(s, y)U_H(\rho_{lose}(s, y)) + (1 - \rho(s, y))U_L(\rho_{lose}(s, y))]. \tag{16}$$

The profit from bidding $b(x)$ in such an auction is $\pi_{static} = \int_{\underline{s}}^x (v(s, t) - b(t))h_s(t)dt$, which gives rise to $\pi^{(2)}_{static}(s, x) = \pi^{(2)}(s, x)$. This implies that there exists a function D such that for a given s it holds that

$$\forall x, \; \pi(s, x) = \pi_{static}(s, x) + D(s).$$

Consequently, the bidding incentives in the two auctions are identical, and thus they give rise to the same (monotone and symmetric) equilibrium bid functions.

It is well known that in a second-price common-value auction the symmetric equilibrium bid function is $b(s) = v(s,s)$, that is, the bid is equal to the expected value conditional on being tied at the top. This equilibrium bid can be rewritten as $b(s) = v - [\rho(s,s)U_H(\rho_{lose}(s,s)) + (1-\rho(s,s))U_L(\rho_{lose}(s,s))]$. We summarize our findings in the following Lemma:

Lemma 2 *If $v(s,y)$ in formula (16) is strictly monotone in s and y, then a monotone equilibrium exists in the (static) interdependent-value auction characterized by the value function $v(s,y)$. In the monotone equilibrium of the static auction for almost all s each bidder bids according to*

$$b(s) = v - [\rho(s,s)U_H(\rho_{lose}(s,s)) + (1-\rho(s,s))U_L(\rho_{lose}(s,s))]. \tag{17}$$

If $v(s,y)$ is strictly monotone in s and y, then (17) also forms a (monotone) equilibrium in our second-price auction.

Proof The first two statements follow from the analysis of Milgrom and Weber (1982). The last is an immediate consequence of our discussion above. ∎

Proof of Proposition 1:

Proof To prove result *(i)* in the Proposition, it is sufficient to establish that function $v(s,y)$ is strictly monotone, if V is decreasing. Let $s' > s$, and $\rho' = \rho(s',y) > \rho = \rho(s,y)$, and $\rho'_{lose} = \rho_{lose}(s',y) > \rho_{lose} = \rho_{lose}(s,y)$. We now prove the following chain of inequalities:

$$\rho U_H(\rho_{lose}) + (1-\rho)U_L(\rho_{lose}) > \rho U_H(\rho'_{lose}) + (1-\rho)U_L(\rho'_{lose}) > \rho' U_H(\rho'_{lose}) + (1-\rho')U_L(\rho'_{lose}). \tag{18}$$

The second inequality is immediate from $\rho' > \rho$ and $U_H(\rho'_{lose}) < U_L(\rho'_{lose})$, which follows from result *(iv)* in Lemma 1, and the assumption that V is decreasing. For the first inequality, consider the formula that follows from result *(iii)* in Lemma 1:

$$\rho_{lose}U_H(\rho_{lose}) + (1-\rho_{lose})U_L(\rho_{lose}) \geq \rho_{lose}U_H(\rho'_{lose}) + (1-\rho_{lose})U_L(\rho'_{lose}),$$

which implies

$$\frac{U_H(\rho'_{lose}) - U_H(\rho_{lose})}{U_L(\rho'_{lose}) - U_L(\rho_{lose})} \leq \frac{1-\rho_{lose}}{\rho_{lose}}.$$

The fact that $\rho_{lose} > \rho$ implies that $\frac{1-\rho_{lose}}{\rho_{lose}} < \frac{1-\rho}{\rho}$ and thus $\frac{U_H(\rho'_{lose})-U_H(\rho_{lose})}{U_L(\rho'_{lose})-U_L(\rho_{lose})} < \frac{1-\rho}{\rho}$. This inequality yields the first inequality in (18), which then yields that $v(s,y)$ is monotone in s.

The exact same argument implies that $v(s,y)$ is monotone in y. Uniqueness of b as in (1) follows from the above argument as well, since upon tieing, indifference has to hold in an ex post equilibrium. This yields exactly (1) after taking into account that the equilibrium is symmetric and monotone. The only caveat is that the bid function is not determined at the (at most countably many) discontinuity points of U_H, U_L. At such a belief ρ, the optimal action in the continuation problem is not unique, which introduces multiple optimal bids when the belief is ρ. However, there are at most countably many such jump points, so this multiplicity arises only for a small set of types, and for all other beliefs, the equilibrium bid is pinned down by formula (1).[27]

Finally, to establish result *(ii)* in Proposition 1, suppose that V is not (strictly) monotone. We show that the unique candidate equilibrium bid function (1) is not strictly increasing, which proves part *(ii)* of the Proposition. Let us assess whether b is increasing at $s = \overline{s}$. To simplify exposition assume that b is differentiable at all s, but the argument can be modified in a straightforward manner to cover other cases. We sometimes use the shorthand notation $\rho_{tie} = \rho(s,s)$ below. We obtain

$$b'(s) = -\rho'_{tie}(s)(U_H(\rho_{lose}) - U_L(\rho_{lose})) - \rho'_{lose}(s)U'_H(\rho_{lose})\rho(s,s) - \rho'_{lose}(s)U'_L(\rho_{lose})(1-\rho(s,s)).$$

Since signal $\overline{s}$ is perfectly informative, it follows that $\lim\limits_{s\to\overline{s}}\rho_{lose}(\overline{s}) = \lim\limits_{s\to\overline{s}}\rho_{tie}(\overline{s}) = 1$. From the incentive condition it follows that $\lim\limits_{s\to\overline{s}}U'_H(\rho_{lose}(\overline{s})) = 0$. Inspect $\frac{b'(s)}{\rho'_{tie}(s)} = -(U_H(\rho_{lose}(s)) - U_L(\rho_{lose}(s))) - \frac{\rho'_{lose}(s)}{\rho'_{tie}(s)}U'_H(\rho_{lose}(s))\rho(s,s) - \rho'_{lose}(s)U'_L(\rho_{lose}(s))\frac{1-\rho(s,s)}{\rho'_{tie}(s)}$. We show next that $\lim\limits_{s\to\overline{s}}\frac{\rho'_{lose}(s)}{\rho'_{tie}(s)}U'_H(\rho_{lose}(s)) = 0$ and that $\lim\limits_{s\to\overline{s}}\frac{1-\rho(s,s)}{\rho'_{tie}(s)} = 0$, which will then imply that $\lim\limits_{s\to\overline{s}}\frac{b'(\overline{s})}{\rho'_{tie}(s)} < 0$, since $U_H(1) - U_L(1) < 0$.[28] Since ρ_{tie} is strictly increasing, therefore if s is large enough, then $b'(s) < 0$ follows contradicting monotonicity of b.

To prove those two claims, consider the following, which is by l'Hospital's rule: $\lim\limits_{s\to\overline{s}}\frac{1-\rho(s,s)}{\rho'_{tie}(s)} = \lim\limits_{s\to\overline{s}}\frac{-\rho'_{tie}(s)}{\rho''_{tie}(s)} = 0$ as $\rho'_{tie}(s) = 0$, $\rho''_{tie}(s) < 0$. Finally, $\lim\limits_{s\to\overline{s}}U'_H(\rho_{lose}(s)) = 0$ as $\lim\limits_{s\to\overline{s}}\rho_{lose}(s) = 1$. Also, $\rho_{lose}(s) \geq \rho_{tie}(s)$ for all s, and $\rho_{lose}(\overline{s}) = \rho_{tie}(\overline{s}) = 1$, implies that $\lim\limits_{s\to\overline{s}}\frac{\rho'_{lose}(s)}{\rho'_{tie}(s)} \leq 1$, implying that $\lim\limits_{s\to\overline{s}}\frac{\rho'_{lose}(s)}{\rho'_{tie}(s)}U'_H(\rho_{lose}(s)) = 0$. ∎

[27] If the value function is smooth, then such discontinuity of U_H, U_L cannot occur and the equilibrium bid is unique for all x. Moreover, the function b is continuous in this case.

[28] If V is not monotone decreasing, then convexity requires that V must be increasing at 1, which implies (using Lemma 1) that $U_H(1) > U_L(1)$.

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