Vertical or Horizontal: Endogenous Merger Waves in Vertically Related Industries

Zhiyong Yao and Wen Zhou*

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Abstract

Endogenous merger waves in vertically related industries are studied assuming firms can engage in both vertical and horizontal mergers. The analysis explains why and how merger waves occur endogenously so that the market structure transforms from one stable configuration in which no firm is merged, to another in which many firms are merged. Merger waves are shown to be possible with or without any fundamental change in the underlying economic conditions. Whether a merger wave is vertical or horizontal depends on the balance between vertical and horizontal externalities, and the balance between upstream and downstream competition.

Keywords: merger waves, endogenous mergers, vertical mergers, horizontal mergers, stable market structure

JEL Code: L13, L42, D43

1 Introduction

It has been well documented that mergers occur in waves, clustering in time and by industry (e.g., Andrade et al. 2001; Mitchell and Mulherin, 1996). Some merger waves are horizontal, consisting mainly of mergers between competing firms in the same industry, while some others are vertical, made up mainly of mergers between suppliers and customers. For example, the first great merger wave in the U.S. (1890s–1904) was horizontal. Consolidation between competitors generated corporate giants in the steel, telephone, oil, mining, railroad, and other major manufacturing and service industries. The second great merger wave (1910s–1929), by contrast, was mainly vertical. Ford and General Motors emerged as the major automobile manufacturers through vertical integration, acquiring every business along the supply chain from iron and coal mining

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to railroads and water transportation, steel mills, all the way to finished vehicles (Lipton, 2006; Martynova and Renneboog, 2008). In the U.S., the cement and ready-mixed concrete industries underwent at least two waves of vertical integration in the 1960s and 1980s (Hortacsu and Syverson, 2007). More recently, in February 2011 Nokia and Microsoft announced a comprehensive plan to ally vertically in the smartphone area in response to the success of Apple's iPhone. Several months later, Google acquired Motorola Mobility in a similar move.

Horizontal mergers may cluster not only in a single industry, but also across several industries that are vertically related. For example, a series of mergers in the US pharmaceutical industry occurred in 2007 to counteract the increased bargaining power in the downstream health care industry, where consolidation had been taking place.[1] Furthermore, a wave of integration along one dimension may follow a wave of disintegration along another. In other words, the market may switch between vertical and horizontal integrations. Semiconductor companies used to be vertically integrated. With rising costs and a changing cost structure, economies of scale became more important, and firms started to disintegrate vertically and merge horizontally.[2]

These examples highlight the need to study horizontal and vertical mergers together, as a merger will alter the incentives for other mergers, and the impacts are likely to be different depending on whether the mergers are vertical or horizontal. Given the observed regularity of horizontal and vertical merger waves, the following questions immediately emerge: Why do mergers cluster? What causes a merger wave to occur in the first place? And what determines whether the wave is vertical or horizontal? There have been some economic studies on horizontal merger waves (in a single industry) and a few on vertical merger waves, but virtually none in which the two are considered together. This research studied mergers in vertically related industries and is therefore well positioned to address the last question. In doing so, the analysis has shed new light on the answers to the first two questions.

Consider two vertically related industries, each consisting of two firms that may engage in both horizontal and vertical mergers.[3] Although merger decisions are made simultaneously, the solution concept is such that each firm considers possible "responses" from other firms when contemplating a merger. A merger wave is an equilibrium phenomenon which satisfies two conditions. First, it is stable for multiple firms to merge, meaning that the firms' merger incentives must be analyzed together. Second, merging is endogenous, meaning that the original market structure, i.e., all firms remaining

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[1] "Meandering giants," *The Economist*, March 24, 2007.

[2] "Under new management," *The Economist,* April 2, 2009.

[3] Since each industry has only two firms, a horizontal merger "wave" in this context would be horizontal mergers in both industries, which is slightly different from the usual meaning of multiple mergers in a single industry.

independent, must also be stable. Therefore, a merger wave is a change in the market structure from one stable configuration to another.

The analysis will show that any stable market structure results in either no merger or multiple mergers, so mergers always occur in waves. Depending on whether the two structures are stable under different or identical conditions, a merger wave may be triggered in two ways. Market structure may change because the underlying conditions have changed, or the wave may be a switch between equilibria without any change in the fundamentals. The first case corresponds, for example, to an economic shock that changes the cost, demand, or regulatory situation, so the trigger is tangible. In the second case, the trigger may be intangible, corresponding to something trivial or totally unrelated to the underlying economic conditions, such as rumors or changes in mood or expectations.

The main contribution of this study was to link the two triggers to demand and cost properties, and to identity the conditions determining whether a merger wave is horizontal or vertical. It was found that these conditions affect stable market structure through the relative intensity between vertical and horizontal externalities, and the relative intensity of competition in the upstream and downstream industries. Vertical mergers eliminate vertical externalities (double markup) but intensify horizontal externalities (competition in the downstream industry). Horizontal mergers lessen horizontal externalities in both industries but exacerbate vertical externalities. If the two externalities are balanced, one will mitigate the other as long as all four firms remain independent. This explains why firms may refrain from merging, a necessary component for the endogenization of merger waves. If the balance is disrupted, i.e., if the externalities become more severe along either dimension, firms will start to merge. For example, when the demand for the final product becomes more convex, double markup is more damaging, and firms have an inventive to merge vertically.

While the externalities mainly affect the four firms' total profits, the relative intensity of competition in the two industries affects how the total profit is distributed among them. When competition in the two industries is balanced, firms tend to remain independent or merge vertically. When the balance is disrupted, firms will merge horizontally. For example, when the final products become closer substitutes, horizontal competition is intensified and firms merge horizontally in both industries. So competition in one industry may induce mergers in a vertically related industry. Not merging and vertical mergers tend to share the same conditions while horizontal mergers correspond to different conditions, so vertical merger waves are more likely to be associated with intangible triggers, while a horizontal merger wave tends to be associated with a tangible trigger.

Research has long established that mergers tend to occur in waves, and scholars have usually explained the waves in terms of economic shocks at the industry or econ-

omy level (Andrade et al. 2001; Andrade and Stafford, 2004; Harford, 2005; Jensen, 1993; Jovanovic and Rousseau, 2002; Mitchell and Mulherin, 1996; Shleifer and Vishny, 2003). In the industrial organization literature, there are only a few theoretical papers on merger waves, all in the same industry. Nilssen and Sorgard (1998) analyzed sequential horizontal mergers to demonstrate their temporal interdependence. Fauli-Oller (2000) studied sequential mergers among firms with asymmetric costs, and found that a merger's profitability is affected by both the demand level and mergers among other firms. Fridolfsson and Stennek (2005) showed that a merger may induce further mergers due to the negative impact a merger exerts on other firms. Qiu and Zhou (2007) and Toxvaerd (2008) endogenized multiple mergers and attributed merger waves to economic shocks. This paper is mainly interested in a setting where both horizontal and vertical mergers are possible, and we have found equilibrium conditions that are indeed unique to such settings: the balance between vertical and horizontal externalities and between upstream and downstream competition.

Toxvaerd (2008) has shown that merging and not merging can be unique or multiple equilibria under identical or different conditions. Although not discussed explicitly, Toxvaerd's result implies the possibility of tangible and intangible triggers. Nevertheless, his reason for why no merger and multiple merger co-exist is very different from ours. We have further related the two triggers to the types of merger waves and the specific demand and cost conditions. Colangelo (1995) studied vertical and horizontal mergers together, but his merger game is very different from the one considered here. Colangelo was mainly interested in exclusive, single mergers that are assumed to be substitutes, while this study considers multiple mergers that turn out to be complements. His merger decisions are sequential with an exogenous target and ordered moves, while in this study everything including the merger process is endogenous (i.e., the pre-merger and post-merger market structures must both be at equilibrium).

Greenhut and Ohta (1979) have demonstrated that successive duopolists would both merge vertically, but Bonanno and Vickers (1988) and Lin (1988) showed that firms may choose vertical disintegration in order to dampen downstream competition. Our model synthesizes these contradictory results, as we show that both market structures can be stable even under the same conditions. Like Greenhut and Ohta (1979), we emphasize the role of double markup in driving vertical mergers. Like Bonanno and Vickers (1988) and Lin (1988), we show that firms may refrain from profitable vertical mergers because vertical disintegration generates a vertical externality that mitigates the horizontal externality associated with horizontal competition. Unlike their setting in which the only alternative to not merging is vertical integration, firms in our model have the additional option of horizontal merger.

In applying the equilibrium concept for one-to-one matching games with externalities

(Sasaki and Toda, 1996; Hafalir, 2008),[4] our simultaneous merger game provides a useful framework for merger studies. The equilibrium concept is able to justify both no merger and multiple mergers as stable configurations and thereby identify an intangible trigger of endogenous merger waves. It also captures the essential interdependence of firms' merger incentives without having to specify details such as the order of moves, which so often will greatly affect the equilibrium outcome.

The plan of the paper is as follows. Section 2 presents the major findings in a baseline model with specific assumptions about demand, cost, and the nature of competition. The next section relaxes and generalizes these assumptions and demonstrates that the findings are robust. More importantly, variations in the setting put the findings in perspective and help us relate merger waves to specific demand and cost conditions. Finally Section 4 concludes.

2　Model

2.1　Setup

Consider two vertically related industries. The upstream industry consists of two identical firms, A and B, and the downstream industry also consists of two identical firms, 1 and 2. A homogeneous input is produced by the upstream firms at constant marginal cost, c, and sold through arm's-length transaction to the downstream firms, which then transform it into a homogeneous final product at zero extra cost on a one-for-one basis. Firms compete à la Cournot in both industries. Each downstream firm regards the input price, denoted t, as given and chooses its output based on the demand for the final product, $p = \alpha - Q$, where Q is the total output produced by the downstream firms. The downstream Cournot equilibrium will give rise to Q as a function of t, which is then inverted to generate the inverse demand for the input, i.e., t as a function of Q. Facing this derived demand, the upstream firms engage in their Cournot competition.[5]

The firms play a two-stage merger game. In stage one, each firm chooses simultaneously a merger partner. A firm can commit to independence by choosing itself as the partner. A merger can be either horizontal (between two downstream firms or two upstream firms) or vertical (between a downstream firm and an upstream firm), and it takes place if and only if two firms choose each other as partners. A merged entity is not allowed to participate in any further mergers.[6] The merger decisions result in a market

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[4]Work on coalition formation with externalities (Bloch 1996; Ray and Vohra, 1999) has borrowed and modified various equilibrium concepts from cooperative game theory, but those concepts lead to either a very large set of equilibria or an empty set, therefore are not suitable for the questions studied here.

[5]The successive Cournot oligopoly setting has been widely used. See Greenhut and Ohta (1976), and Salinger (1988) for examples.

[6]This is to avoid trial monopolies.

configuration, which is publicly announced.

The game then proceeds to stage two. Given the market configuration from the first stage, all the resulting firms compete à la Cournot, choosing their quantities in order to maximize their payoffs. Within a merged entity, the two partners share the merger surplus equally,[7] where the surplus is calculated assuming a fixed configuration among the other firms. For example, a merger between firms A and 1 in $N \equiv \{A, B; 1, 2\}$ leads to $S \equiv \{B; A1, 2\}$, so $\pi_A^S = \pi_A^N + \frac{1}{2}\left(\pi_{A1}^S - \pi_A^N - \pi_1^N\right)$ and $\pi_1^S = \pi_1^N + \frac{1}{2}\left(\pi_{A1}^S - \pi_A^N - \pi_1^N\right)$, where $A1$ denotes the entity resulting from a merger between firms A and 1, and π_i^X is firm i's payoff in configuration X with $i \in X$.

A configuration is stable if no deviation by any individual firm or pair of firms is profitable. A deviation is profitable if all deviators are better off (strictly for at least one) for any possible configuration among the other firms. Once stability is characterized, a transition from one stable configuration in which no firm is merged to another in which all firms are merged will be an endogenous merger wave.

This is essentially a one-to-one matching game, so both unilateral deviations by individual firms and collective deviations by a pair of firms are allowed (e.g., Roth and Sotomayor, 1990). Since a merger involves two firms, collective deviation must also be considered, as deviating into a new merger requires two firms to change their strategies simultaneously. Furthermore, because a merged entity's profits depend on other firms' merger structures, this merger game is essentially a one-to-one matching with externalities. Following Sasaki and Toda (1996), our equilibrium concept requires a deviation to be profitable under all possible configurations among the remaining firms. Such a requirement is obviously sufficient to justify the deviation. Sasaki and Toda (1996) have proved the existence of such an equilibrium in the context of one-to-one matching with externalities, and it works well for this merger game, as the equilibrium set thus generated is small but not empty.

2.2 Analysis

The game is solved by backward induction. Suppose that at the beginning of stage two the market configuration contains v vertically integrated firms, u independent upstream firms, and d independent downstream firms. Because marginal costs are constant and identical within an industry, a horizontal merger is equivalent to eliminating one of the merging firms. A vertically integrated firm participates only in the downstream competition,[8] and it differs from an independent downstream firm in that its input is

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[7]This assumption can be justified by implementing alternating-offer bargaining à la Rubinstein (1982).

[8]Salinger (1988) has pointed out that a vertically integrated entity will withdraw from the upstream competition—it neither buys the input from other upstream firms nor sells it to other downstream firms. Some researchers have raised concerns about commitment power in Salinger's setting, but the issue is not serious in this analysis because firms compete on quantity rather than price (Hart and Tirole, 1990).

procured at cost c rather than the market price t. In the downstream competition, therefore, a vertically integrated firm chooses q_v to maximize $\pi_v \equiv (\alpha - Q - c)q_v$, which leads to the first-order condition $\alpha - q_v - Q = c$. Since $Q = vq_v + dq_d$ (using symmetry), the first-order condition can be rewritten as:

$$\alpha - (v+1)q_v - dq_d = c.$$

Likewise, the first-order condition for an independent downstream firm is $\alpha - q_d - Q = t$, or

$$\alpha - (d+1)q_d - vq_v = t.$$

These two equations lead to

$$q_v = \frac{\alpha - (d+1)c + dt}{d+v+1} \quad \text{and} \quad q_d = \frac{\alpha - (v+1)t + vc}{d+v+1}.$$

The demand for the input for independent upstream firms is therefore

$$Q_i \equiv dq_d = \frac{d[\alpha + vc - (v+1)t]}{d+v+1},$$

or

$$t = \frac{\alpha + vc}{v+1} - \frac{d+v+1}{d(v+1)}Q_i,$$

where $Q_i \equiv dq_d \equiv uq_u$ is the total quantities produced by the independent upstream or downstream firms. Facing this derived demand, an upstream firm will choose q_u to maximize $\pi_u \equiv \left[\frac{\alpha+vc}{v+1} - \frac{d+v+1}{d(v+1)}Q_i - c\right]q_u$. The Cournot equilibrium is then given by

$$q_u = \frac{d(\alpha - c)}{(u+1)(d+v+1)}.$$

Consequently,

$$q_d = \frac{u(\alpha - c)}{(u+1)(d+v+1)}, \quad q_v = \frac{[d + (u+1)(v+1)](\alpha - c)}{(u+1)(v+1)(d+v+1)},$$

$$\text{and } Q = \frac{[(u+1)(v+1)(d+v) - d](\alpha - c)}{(u+1)(v+1)(d+v+1)}.$$

It is clear that all profits are proportional to $(\alpha - c)^2$, so we can normalize $\alpha - c \equiv 1$ without losing any generality. The profits of the three types of the firm are then:

$$\pi_v(v, u, d) = \frac{[d + (u+1)(v+1)]^2}{(u+1)^2(v+1)^2(d+v+1)^2},$$

$$\pi_u(v, u, d) = \frac{d}{(u+1)^2(v+1)(d+v+1)}, \quad \text{and}$$

$$\pi_d(v, u, d) = \frac{u^2}{(u+1)^2(d+v+1)^2}.$$

Schrader and Martin (1998) provide further arguments in support of Salinger's conclusion.

Now move back to stage one. There are six possible market configurations as listed in Table 1. Firms' payoffs can be calculated using the formulas derived above. For example, in $S_1 \equiv \{A, B; 1, 2\}$, $v = 0$, $u = 2$, $d = 2$. As specified earlier, each merged entity divides the merger surplus/deficit equally between the two partners.

Table 1: Payoffs in the six configurations [9]

Configuration	π_A	π_B	π_1	π_2	Total profits
$S_1 \equiv \{A, B; 1, 2\}$	74	74	49.4	49.4	247
$S_2 \equiv \{AB; 12\}$	62.5	62.5	31	31	188
$S_3 \equiv \{A1, B2\}$	62.5	62.5	48.6	48.6	222
$S_4 \equiv \{B; A1, 2\}$	99	42	74.5	28	243
$S_5 \equiv \{AB; 1, 2\}$	83	83	28	28	222
$S_6 \equiv \{A, B; 12\}$	55.5	55.5	55.5	55.5	222

Proposition 1. *Only S_1 and S_3 are stable.*

Proof: S_1 is stable. There are three possible deviations, but none of them are profitable: $A + B$ is unprofitable when 1 and 2 merge; $1 + 2$ is unprofitable when A and B merge; and $A + 1$ is unprofitable when B and 2 merge.

S_2 is not stable because $A + 1$ is a profitable deviation: If B and 2 merge, $A + 1$ will leave A indifferent but 1 strictly better off; if B and 2 remain independent, both A and 1 are strictly better off.

S_3 is stable. There are three possible deviations (excluding symmetric ones), but none is profitable: Breaking up $B2$ is unprofitable when A and 1 remain merged; $A + B$ is unprofitable when 1 and 2 merge; $1 + 2$ is unprofitable when A and B merge.

S_4 is not stable because $B + 2$ is a profitable deviation (whether A and 1 separate or remain merged).

S_5 is not stable because $1 + 2$ is a profitable deviation (whether A and B separate or remain merged).

S_6 is not stable because $A + B$ is a profitable deviation (whether 1 and 2 separate or remain merged). **Q.E.D.**

Proposition 1 says that the four firms either remain independent or carry out two vertical mergers. To understand this conclusion, it is useful to summarize the payoffs in Table 1 into the following three rankings.

R1: An exogenous merger, one fixing the configuration among the remaining two firms, is always profitable. This is because a merger internalizes either the horizontal externality (due to horizontal competition) or the vertical externality (due to double markup), which benefits the merging firms.

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[9]For ease of comparison, the fractions of the payoffs are multiplied by $1,000$ and turned into numerical values.

R2: A merger always hurts other firms and consequently a breakup always benefits other firms. By eliminating double markup, a vertical merger makes the merged entity more aggressive in the downstream competition, which hurts other firms. A horizontal merger between duopolists will hurt the other industry by reducing quantities supplied or demanded.

R3: S_1 Pareto dominates S_3, which in turn dominates S_2. Firms face horizontal externality in S_3 and therefore tend to produce too much from the viewpoint of their joint profits. They face vertical externality in S_2 and tend to produce too little. Both externalities are present in S_1, but their opposite effects mitigate each other, so S_1 dominates both S_2 and S_3. The horizontal externality in S_3 is moderate because the downstream competition (Cournot rather than Bertrand) is mild, while the vertical externality in S_2 is severe because it leads to successive monopoly, so S_3 dominates S_2.

These three profit ranks help us understand the intuition. First, it cannot be stable to have only one merger because the remaining two independent firms will be better off merging. They gain if the originally merged firm remains merged (*R1*), and will gain even more if the merged firm breaks up (*R2*). Second, having two horizontal mergers is unstable, as an upstream firm and a downstream firm would rather merge vertically. Such a deviation is profitable if the other two firms also merge vertically (*R3*), and is even more profitable if the other two firms do not merge (*R2*).

That leaves two configurations: all firms remain independent (S_1) or carry out two vertical mergers (S_3). Both are stable. Consider first the no-merger case. An exogenous merger would have been profitable (*R1*). However, if the other two firms also merge, the participants in the first merger would be worse off (*R3*). Therefore, no deviation (in the form of a merger) will be carried out in S_1. Now consider two vertical mergers. It is unprofitable to either break up (if the other merged firm remains merged—*R1*) or switch to a horizontal merger (if the other two firms also merge horizontally—*R3*).

2.3   Discussion

Four conclusions can be drawn from Proposition 1. First, mergers occur in waves. Proposition 1 shows that it is stable to have two vertical merged firms (S_3), and it is unstable to have a single merger (S_4, S_5, and S_6). Therefore, whenever a merger occurs, it must be accompanied by another. A single merger hurts the other two firms, which will then find it profitable to merge regardless of what the first pair does (dissolves or remains merged). So the driving force for mergers to cluster is the negative impact of a merger on other firms.[10]

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[10] The negative impact has also been emphasized by Nilssen and Sorgard (1998) and Fridolfsson and Stennek (2005). This result contrasts with Qiu and Zhou's (2007) discovery that a positive impact may also account for merger waves.

Second, it is also stable to have no merger (S_1): all four firms may remain independent even though any individual merger would have been profitable. When a given merger is said to be profitable, the configuration among the remaining two firms is implicitly assumed to be fixed, i.e., they remain independent. If those two firms also merge, however, the first pair will be hurt. Concerns about a second merger's negative impact therefore prevent the first one from taking place.

Third, a merger wave may take place without any fundamental change in the underlying conditions. Proposition 1 predicts two stable configurations: The firms may remain in S_1 (or 4I, meaning four independent firms, for a more informative notation) or they may carry out two vertical mergers, ending up with S_3 (or 2V). A change of the market structure from 4I to 2V duly endogenizes the vertical merger wave. Because the two structures are stable under the same conditions, shifting from 4I to 2V does not require any change in the economic fundamentals such as demand or technology. The trigger may be something trivial or totally unrelated: mood, expectation, rumor, etc.

Although 4I and 2V are both stable, they are not on an equal footing. Switching from 4I to 2V is possible or even likely because if, for whatever reason, a pair of firms expect the other pair to merge, they will prepare to follow suit. The reverse process of jumping from 2V to 4I, a divestiture wave, is more difficult. If for some reason a pair breaks up, it is in the best interest of the second pair not to follow. This asymmetry between the two stable configurations may explain why in real life merger waves are much more common than divestiture waves.

Fourth, a merger wave may be vertical or horizontal depending on the tradeoff between the vertical and horizontal externalities. In this 2×2 setting a merger wave may consist of two vertical mergers (2V) or two horizontal mergers (S_2 or 2H), referred to respectively as vertical and horizontal merger waves. A firm chooses a vertical or horizontal merger taking into account possible reactions by the remaining firms, so ultimately it is a choice between 2V and 2H. As mentioned earlier, 2V eliminates double markup but intensifies horizontal competition; 2H does the opposite. In the present setting, the damage of double markup is greater than that of horizontal competition, so the firms end up making two vertical mergers. In other settings, however, the comparison may be reversed and horizontal mergers may emerge in equilibrium. If, for example, the competition is à la Bertrand rather than Cournot, a merger wave could be horizontal.

3 Generalization

The conclusions discussed so far are derived from a highly stylized model with symmetric 2×2 firms, constant marginal costs, homogeneous products, linear demand and Cournot competition. To what extent do they hold under more general assumptions? In this sec-

tion we will investigate five variations of the baseline model by relaxing its assumptions, one at a time, so that products are differentiated, the competition is in price rather than quantity, the marginal cost is increasing, or the demand is non-linear.[11] Since the baseline model is a special case of some of the variation models, we are able to put the conclusions in perspective and understand better their reason, conditions and implications. In particular, we will discuss the conditions for a merger wave to be vertical or horizontal, and identify a second trigger of merger waves.

3.1   Product differentiation

Suppose that the final products are differentiated (though the inputs are still homogeneous) so that the demand for firm i's product is $p_i = \alpha - q_i - \beta q_j$, where $i, j \in \{1, 2\}$ with $i \neq j$, and $\beta \in [0, 1]$ represents the degree of product differentiation. Further, assume that each industry may compete on either quantity or price. This gives rise to the following three combinations: Cournot competition in both industries (Cournot-Cournot); upstream Cournot competition and downstream Bertrand competition (Cournot-Bertrand); and Bertrand competition in both industries (Bertrand-Bertrand). All other aspects of the model remain the same as in the baseline model. For each of the three cases, a payoff table similar to Table 1 can be constructed (an example is shown in the appendix), where each payoff is a function of β only.[12]

Proposition 2.

(1) In the Cournot-Cournot case, S_1 is stable for $\beta > 0.29$ and S_3 is stable for any β.

(2) In the Cournot-Bertrand case, S_1 is stable for $0.22 < \beta < 0.8$, S_2 is stable for $\beta > 0.66$, and S_3 is stable for $\beta < 0.66$.

(3) In the Bertrand-Bertrand case, S_3 is stable for $\beta < 0.66$ and S_2 is stable for any β.

Proof: See the appendix.

As Proposition 2 suggests, a single merger (S_4, S_5 and S_6) can never be stable. The reason has been discussed in the baseline model and will be further elaborated later. The remaining three configurations, no-merger (S_1) and horizontal and vertical merger waves (S_2 and S_3), can all be stable under certain conditions. A general pattern is that as competition intensifies (moving from Cournot to Bertrand), horizontal mergers become more likely, while vertical mergers and no-merger become less likely.

The equilibrium results shown in Table 2 exemplify the implications of Proposition 3. The parameter range indicates the condition for a particular configuration to be stable,

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[11] Furthermore, a 3×3 case is analyzed in the appendix.

[12] Again, $\alpha - c$ can be normalized without losing any generality.

and the parenthesis below indicates the profitable deviation that makes the configuration unstable when the condition is violated. For example, in the Cournot-Cournot case, S_1 is stable for $\beta > 0.29$. If $\beta < 0.29$, S_1 becomes unstable because $A + B$ is a profitable deviation.

Table 2: Results of Propositions 2 and 3

Cases	S_1 (*4I*)	S_2 (*2H*)	S_3 (*2V*)
Cournot-Cournot	$\beta > 0.29$ $(A + B)$	*Never* $(A + 1)$	*Always*
Cournot-Bertrand	$0.22 < \beta < 0.80$ $(A + B) \qquad (1 + 2)$	$\beta > 0.66$ $(A + 1)$	$\beta < 0.66$ $(A + B)$
Bertrand-Bertrand	*Never* $(A + B)$	*Always*	$\beta < 0.66$ $(A + B)$
Increasing mc	$\gamma < 2$ $(1 + 2)$	$\gamma > 2$ $(A + 1)$	$\gamma < 2$ $(1 + 2)$
General demand	*Always*	$\sigma > 1$ $(A + 1)$	$\sigma \leq 1$ $(A + B)$

■ Cournot-Cournot

The setting is the same as the baseline model except that the final products are differentiated, so the baseline model is a special case with $\beta = 1$. Greater differentiation between the final products weakens the downstream competition. If products are sufficiently differentiated (β is small), the downstream horizontal externality is greatly weakened. 4I is no longer stable because there is no need to use vertical externality to mitigate the horizontal externality. In the baseline model, 2V dominates 2H because downstream horizontal competition is less damaging than double markup. Product differentiation further reduces the damage of horizontal competition, so 2V is always stable while 2H is never stable for any β.

■ Cournot-Bertrand

When downstream firms compete on price rather than quantity, downstream competition is intensified. 4I is stable only if the products are modestly differentiated (intermediate β). When products are highly differentiated the downstream competition is too weak; when products are close substitutes the downstream competition is too strong. 4I is unstable in either case. As explained earlier, product differentiation favors 2V over 2H, so 2V continues to be stable when products are sufficiently differentiated (small β). Conversely, when products are close substitutes (large β), fierce competition in the downstream industry raises both the benefits of horizontal mergers and the drawbacks of vertical mergers, so 2H becomes stable.

■ Bertrand-Bertrand

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The upstream firms also compete on price now, so upstream competition is intensified. In fact it is intensified to the greatest extent as inputs are homogeneous. 4I is never stable because upstream firms earn zero profits in the 4I situation and they will always attempt to merge. As in the previous case, 2V is stable when the products are sufficiently differentiated. 2H is stable for any β because the upstream horizontal competition is too strong.[13]

3.2 Cost and demand

Let us now move back to the setting with a homogeneous final product and Cournot competition in both industries. In the baseline model, the cost of input production and the demand for the final product are both linear: $C(q) = cq$ and $p = \alpha - Q$. On the cost side, the model can be generalized to allow increasing marginal cost (mc): $C(q) = \gamma q^2$ with $\gamma \geq 0$. Note that the baseline model is a special case of increasing mc with $\gamma = 0$.[14] On the demand side, it can also become more general in the form of $p = \alpha - Q^\sigma$ with $\alpha > 0$ and $\sigma > 0$. Again the baseline model is a special case with $\sigma = 1$. Note a peculiar property of this general demand function: Its concavity, defined as $\frac{p''(Q)Q}{p'(Q)}$, is constant.[15]

Proposition 3.

  (1) In the increasing marginal cost case with $C(q) = \gamma q^2$, S_1 and S_3 are stable for $\gamma < 2$, and S_2 is stable for $\gamma > 2$.

  (2) In the general demand case with $p = \alpha - Q^\sigma$, S_1 is stable for any σ, S_2 is stable for $\sigma > 1$, and S_3 is stable when $\sigma \leq 1$.

Proof: See the appendix.

The results of Proposition 3 have already been summarized in Table 2. Again, a single merger (S_4, S_5 and S_6) is never stable.

  ■ Increasing marginal cost

Producing the inputs at increasing marginal cost will reduce the upstream competition.[16] When the mc curve is steep (γ is large), 4I is unstable because an upstream merger will not raise the input price by much and therefore will do little damage to the

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[13]Starting from 2H, if an upstream and a downstream firm deviate to form a vertical merger, one scenario they may face is that the other two firms do not merge. The upstream firm's profit share in the merger will then be very low because it is calculated based on the situation where the merger does not take place, in which case its profit will be zero due to upstream price competition.

[14]In the baseline model the exact value of c is inconsequential, as all profits are proportional to $(\alpha - c)^2$ and consequently $\alpha - c$ has been normalized without any loss of generality. We may as well think that $c = 0$.

[15]In fact, $\frac{p''(Q)Q}{p'(Q)} = \sigma - 1$. This function has been used by Greenhut and Ohta (1976) and other scholars of merger incentives and/or vertically related industries.

[16]In terms of oligopoly behavior, product homogeneity with increasing marginal costs is mathematically equivalent to product differentiation with constant marginal cost (Vives, 1999). In a sense, the increasing marginal cost considered here introduces differentiation to the inputs.

downstream firms. As a result, the downstream firms will find it profitable to merge. An increasing marginal cost favors 2H over 2V. The benefit of a vertical merger comes from output expansion due to the elimination of double markup. When the marginal cost of the input increases with quantity, expansion is increasingly costly, so the benefits of vertical mergers decrease. When the mc curve is flat (γ is small), such a decrease is small, so 2V continues to dominate 2H, meaning that 2V is stable while 2H is not.

■ General demand

An increase in the concavity of demand lessens the severity of the double markup problem.[17] As will be argued below, 4I is always threatened by horizontal mergers rather than vertical ones. But demand concavity affects mainly double markup, which is present in both the 4I and 2H configurations, so 2H is never a real threat and consequently 4I is stable for any value of the demand's concavity. When the demand is convex ($\sigma < 1$) the double markup problem is very severe, so firms merge vertically to avoid the damage, leading to 2V. Conversely, when the demand is concave ($\sigma > 1$) double markup is less of a problem. Avoiding horizontal competition becomes the major concern, so 2H is stable.

3.3   Discussion

As was seen with the baseline model, the stability of a configuration depends crucially on the relative intensity of double markup and horizontal competition in the two industries. Each of the five variation models introduces a parameter that affects one of the three intensities. Compared to the baseline model, differentiation of the final products (Cournot-Cournot) weakens the downstream competition; Bertrand competition downstream (Cournot-Bertrand) intensifies the downstream competition; Bertrand competition upstream (Bertrand-Bertrand) intensifies the upstream competition. Increasing marginal cost for input production weakens the upstream competition, whereas greater demand concavity lessens the severity of double markup. The equilibrium outcomes of the five variation models are presented graphically in Figure 1, where the bullet point represents the baseline model's result as a special case.

3.3.1   Merging in waves

In all cases with any parameter values, there is always a stable configuration involving two mergers, either horizontal or vertical, while a single merger (S_4, S_5 and S_6) is never stable. So mergers always come in waves. The driving force is still the two profitability

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[17]A single-level monopolist facing constant marginal cost c and demand $p = \alpha - Q^\sigma$ will choose quantity $Q_1 = \left(\frac{\alpha-c}{1+\sigma}\right)^{\frac{1}{\sigma}}$. In successive monopoly with the same cost and demand, the quantiy chosen will be $Q_2 = \left[\frac{\alpha-c}{(1+\sigma)^2}\right]^{\frac{1}{\sigma}} < Q_1$. As a measure of the severity of double markup, $\frac{Q_1}{Q_2} \equiv (1+\sigma)^{\frac{1}{\sigma}}$ declines with σ, indicating that double markup is less of a problem when σ increases. Basically, when σ is larger, the demand for input is closer to the demand for the final product.
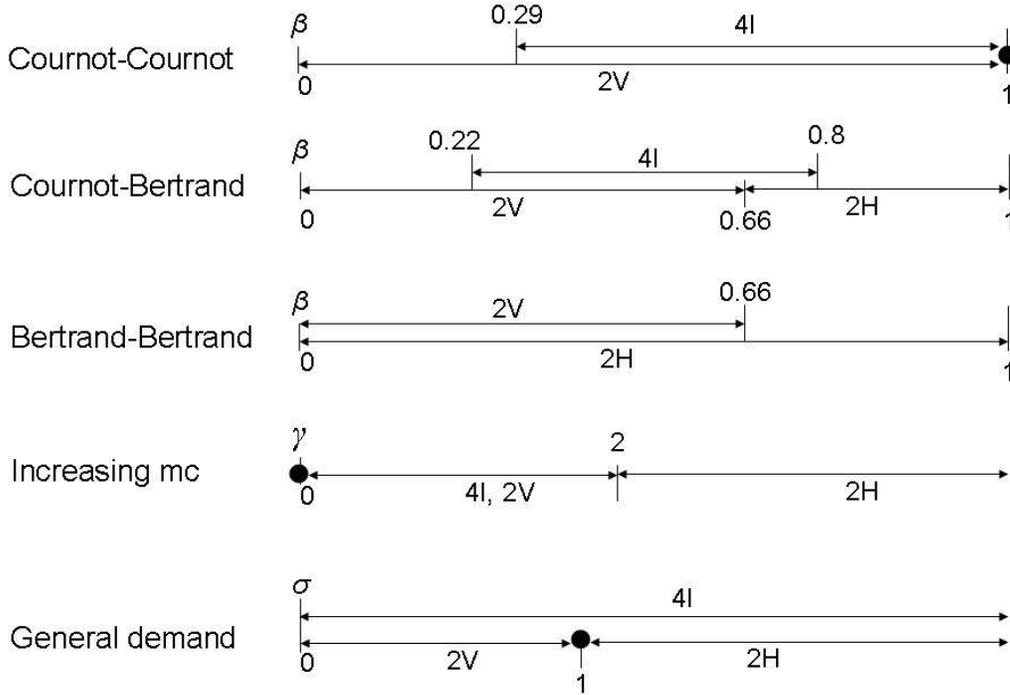
Figure 1: Stable configurations in the five variation models

rankings seen in the baseline model. An exogenous merger between any two firms is always profitable (*R1*), and a merger always hurts other firms (*R2*). Below we explain why they still hold in general. Once they are established, our game rule immediately implies that configurations with only single merger can never be stable.

Consider first the configuration with only a vertical merger (S_4). The merged firm becomes more competitive in the downstream competition, so the merger hurts the other two firms. Fixing the first merger, a merger between the remaining two firms is profitable because it internalizes the vertical externality generated by double markup.

Next consider the configuration with only an upstream horizontal merger (S_5). This merger raises the input price and thereby hurts downstream firms. Fixing the upstream merger, the downstream firms can merge from duopoly to monopoly, which should always be profitable. But the complication is the presence of the upstream industry. In particular, upstream firms may try to share the benefit of the downstream merger by raising the input price, which will reduce the profitability of the downstream merger. However, it can be verified that in all the models we have considered so far, a downstream merger never changes the equilibrium input price.[18] As a result, a downstream

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[18]In the baseline model, for example, the equilibrium input price is $t = c + \frac{\alpha - c}{(u+1)(v+1)}$, which does not

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merger continues to be profitable despite the upstream industry's response.

Finally consider the configuration with only a downstream horizontal merger (S_6). This merger reduces the quantity of inputs demanded without changing the input price, so it hurts upstream firms. Fixing the downstream merger, the upstream firms may merge from duopoly into a monopoly, which again should always be profitable. And again the complication is the presence of a vertically related industry downstream. However, the demand that upstream firms face is derived from the downstream competition. As long as the downstream configuration is fixed, that demand does not change. An upstream merger continues then to be profitable despite the downstream industry's response.

3.3.2 The stability of not merging

In all cases except Bertrand-Bertrand competition, it is stable under certain conditions for all firms to remain independent (4I). Firms in 4I may deviate into either a horizontal or a vertical merger, but what really matters is how the deviators expect to do in a situation where the other two firms also merge. That is, the alternative to 4I is essentially a merger wave resulting in either 2H or 2V.

The stability of 4I depends on the four firms' total profits and their distribution among them. In terms of total profits, 4I tends to outperform both 2H and 2V. In all cases except Bertrand-Bertrand competition, 2V eliminates double markup but retains horizontal competition, so the firms over-produce from the viewpoint of their joint profits, a horizontal externality. By contrast, 2H removes horizontal competition but retains double markups, so the firms under-produce, which can be viewed as a vertical externality. 4I has both externalities, but since the two externalities move in opposite directions, one mitigates the other and the firms' total profits tend to increase.

The distribution of the total profit also matters for the stability of 4I. For a deviating vertical merger in 4I to be profitable, the merging pair will have to earn more. Because the two vertical pairs are symmetrical, the four firms' total profits must therefore be higher in 2V than in 4I, which is unlikely. By contrast, 4I can easily be disrupted by a horizontal merger; such a deviation need only benefit one horizontal pair, not both. So the stability of 4I is always threatened by a horizontal merger, not a vertical one.

Whether or not the threat will materialize depends on the relative intensity of horizontal competition in the upstream and downstream industries. If the two competitions

depend on d. So if there is a downstream merger (i.e., d decreases by 1), the equilibrium input price will not change. A downstream merger rotates the demand for inputs clockwise around the intercept. This rotation has two opposite effects on t: For any given t, the quantity demanded is smaller, so t tends to drop. At the same time, the demand becomes less elastic, so t tends to rise. In the case of linear demand for the final product, the two forces exactly cancel each other so the equilibrium input price will be independent of the number of downstream firms. For non-linear demand, Greenhut and Ohta (1976) and Ziss (2005) have shown the same invariance as long as the demand has constant concavity, which is satisfied by the general demand adopted here.

are more or less balanced, 4I can be stable. For example, in the Bertrand-Bertrand situation, the upstream competition is very strong, so 4I is never stable at any β. In the Cournot-Bertrand situation, the competition is balanced only for moderate differentiation in the final products.

3.3.3   Vertical and horizontal merger waves

In all cases and with any parameter values there is always a stable configuration involving two mergers, either horizontal (2H) or vertical (2V). Furthermore, except with Bertrand-Bertrand competition, the two structures are mutually exclusive: When 2H is stable, 2V is unstable, and vice versa. For 2H, the threat to stability comes mainly from a vertical merger. Given the assumptions, the deviating firms are mainly concerned about the situation in which the other two firms also merge vertically, so the alternative market structure to 2H is basically 2V. Similarly, the alternative to 2V is 2H. Therefore, 2H and 2V will be essentially the only market structures that the firms consider when they ponder a merger or deviation. At any given set of parameter values, either 2H dominates 2V (for a horizontal pair) or 2V dominates 2H (for both vertical pairs), so one and only one of the two configurations is stable.

Which is stable depends crucially on how the total profits distribute among the four firms, which in turn depends on the relative intensity of upstream and downstream competition. Because firms within an industry are assumed symmetric, the two vertical pairs are symmetric while the two horizontal pairs are not. For 2H to be stable, 2H needs to dominate 2V for only one horizontal pair, either upstream or downstream. By contrast, for 2V to be stable, 2V has to dominate 2H for all four firms. When competition in both industries is balanced, 2V tends to be stable. When an industry's competition is greatly intensified or weakened, 2H tends to be stable. For example, price (rather than quantity) competition or closer substitutability between final products will lead to a horizontal merger wave. Greatly weakened competition in the upstream industry, due to increasing marginal costs of input production, will have a similar effect, because downstream competition is now relatively intensified and firms there would like to merge horizontally. These results demonstrate that changes of competition in one industry may induce mergers in a vertically related industry.

To summarize, both 4I and 2V are disrupted by 2H, while 2H is disrupted by 2V. 4I tends to outperform both 2H and 2V in terms of the four firms' total profits. Between 2H and 2V, the comparison of total profits depends on the tradeoff between double markup and downstream horizontal competition. For example, when the demand for the final products is convex, double markup is a severe problem, so 2V prevails. When the downstream competition is Bertrand and the final products are close substitutes, the downstream competition is intense, so firms merge horizontally. These predictions

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are straightforward. More interesting and somewhat surprising is the role of the relative intensity of competition in the two industries. When the intensities are balanced, both 4I and 2V tend to be stable. When the competition in one industry is greatly intensified or weakened, 2H tends to be stable.

3.3.4   Causes of merger waves

A model of endogenous merger waves must explain not only why a merger wave takes place, but also why it did not take place earlier. In other words, the structure before and after the wave must both be stable. As Figure 1 indicates, in all five cases at any parameter value, there are at most two stable configurations and at least one. For some ranges of the parameters there is a unique stable configuration, which invariably contains two mergers, either 2V or 2H. If the parameter falls into that range, the mergers should take place immediately, so the only explanation for why they did not take place earlier is that the parameter was outside the range, where it was stable for all firms to remain independent. Such a change of parameter value can be interpreted as an economic shock. For example, in the Cournot-Cournot situation, 2V is the unique equilibrium for $\beta < 0.29$, and 4I is an equilibrium (though not unique) for $\beta > 0.29$. If β drops from a value greater than 0.29 to a value below it, the industry will undergo a wave of vertical mergers. So an increase in the differentiation of the final products (due to, say, more investment in R&D or advertising) may trigger a vertical merger wave. Likewise, a more convex demand curve may also trigger a vertical merger wave. Similarly, a horizontal merger wave may be triggered by greater substitutability between the final products (Cournot-Bertrand), steeper marginal cost in input production, or greater concavity of demand.

For some other ranges of the parameters there are exactly two stable structures, and in most cases one is 4I and the other is either 2V or 2H. When that is the case, it is possible for a merger wave to take place without any change in the underlying economic conditions, as the baseline model shows. For example, a vertical merger wave is possible when the products are close substitutes (Cournot-Cournot), when marginal cost rises slowly, or when the demand is convex. A horizontal merger wave can occur when product differentiation is moderate (Cournot-Bertrand) or the demand is concave.

So both tangible and intangible causes may trigger a merger wave.[19] A tangible trigger is an economic shock that changes business conditions. An intangible trigger might be a rumor or a sudden change in expectations or mood which shifts the market structure between two stable configurations without any change in the underlying economic

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[19]In a study of horizontal mergers, Toxvaerd (2008) found no merger as the unique equilibrium at one end of the fundamentals spectrum, multiple mergers at the other end, and both structures as multiple equilibria for intermediate values of the fundamentals. The two types of triggers are therefore implied by Toxvaerd's result.

conditions.

The equilibrium may also switch between 2H and 2V. In a sense this is also a merger wave, although it involves more restructuring because firms have to disintegrate and then re-integrate. Both tangible and intangible triggers also can be found for such merger waves. For example, with Bertrand-Bertrand competition the economy may switch from 2V to 2H, the pattern of structural changes observed in the semiconductor industry. Such a wave of vertical disintegration followed by horizontal integration can have an intangible trigger when the products are poor substitutes, or a tangible trigger when the final products become closer substitutes.

4   Conclusions

We have studied endogenous merger waves in vertically related industries where firms can engage in both vertical and horizontal mergers. Whether, why and how a merger wave takes place has been shown to depend on the balance between vertical and horizontal externalities and the balance between upstream and downstream competition. A merger wave may happen with or without any fundamental change in the underlying economic conditions. This research has generated many empirically testable predictions relating merger waves to conditions such as product differentiation, cost structure, and demand concavity.

The simplest possible setting with two upstream and two downstream firms was studied. In this simple setting an exogenous merger always benefits the merging firms and hurts non-merging firms. These effects conform to common understanding of what a merger means, particularly for vertically related industries, and they hold regardless of whether firms compete on price or quantity. The drawback of this simple setting is that horizontal and vertical merger waves cannot co-exist.[20] In fact they are mutually exclusive and always generate a tradeoff. Merging from duopoly into a monopoly is also special, so it is important to go beyond the 2×2 setting. Presented in the appendix, an analysis of the 3×3 setting shows that there are two equilibria, one in which all firms remain independent and the other in which three vertical mergers take place. Similar patterns have been found in the 3×4, 4×3 and 4×4 settings (not shown).

Merger decisions in this model are made simultaneously. A sequential game would

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[20]Interesting industry dynamics involving both horizontal and vertical mergers can be seen in the vertically related iron ore mining and steelmaking industries. In November 2007, the world's two largest iron ore producers, BHP Billiton and Rio Tinto, announced a plan for a horizontal merger. Concerned that the concentration of the iron ore market would raise iron ore prices and jeopardize the downstream steel industry, one of Rio Tinto's biggest customers, Chinese state-owned Aluminum Corporation of China (Chinalco), launched a preemptive move in February 2009 to purchase 14.9% of Rio Tinto's shares. Such a (partial) vertical merger was apparently meant to preempt a horizontal merger in the upstream industry.

have the advantage of generating strategic incentives for mergers and may therefore yield other explanations for merger waves (Qiu and Zhou, 2007). But the disadvantage is that the equilibrium will be sensitive to the order of moves and other details of the game,[21] which a researcher has no compelling reason or sufficient information to specify exogenously. So this study has adopted a simultaneous game and an equilibrium concept borrowed from one-to-one matching games with externalities. This has enabled us to capture the essential interdependence of merger incentives without having to specify the details. It may therefore prove useful in studying other merger-related issues.

The merger surplus was calculated assuming a fixed configuration among remaining firms, which seems to contradict the idea that firms consider all possible configurations when contemplating a merger. However, if configurations are allowed to vary in calculating profit sharing, the profits will be ill-defined because there are multiple configurations. The assumption of arm's length transaction may seem problematic when an industry has only one firm, but what is really needed is double markup and the property that vertical transactions depend on the market structure in both industries. Such assumptions are relevant in many situations. An alternative to arm's length transactions may be two-part tariff. The Bertrand-Bertrand assumption removes double markup and may therefore have already captured what will happen when a two-part tariff is used between upstream and downstream firms.

This has been a first step in studying endogenous merger waves in vertically related industries. Many interesting extensions remain to be explored, such as asymmetric firms, dynamic game rules, and more sophisticated settings and sharing rules.

5   Appendix

Procedures for proving propositions 2 and 3

The calculation and proofs are tedious. Here we provide only the procedures. In each case, we construct a payoff table similar to Table 1. Each payoff will be a function containing a single parameter (it turns out that $\alpha - c$ can be normalized to 1 without any loss of generality). For example, in the Cournot-Cournot case the following payoff table results. Other tables are available from the corresponding author on request.

Table 3: Payoffs in the Cournot-Cournot case

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[21]For example, Colangelo's (1995) sequential game leads to very different predictions about the equilibrium market structure depending on the identity of the first acquisition target, which is exogenously given.

Configuration	π_A	π_B	π_1	π_2	Total profits
$S_1 = \{A, B; 1, 2\}$	$\frac{2}{9(2+\beta)}$	$\frac{2}{9(2+\beta)}$	$\frac{4}{9(2+\beta)^2}$	$\frac{4}{9(2+\beta)^2}$	$\frac{4(4+\beta)}{9(2+\beta)^2}$
$S_2 = \{AB; 12\}$	$\frac{1}{8(1+\beta)}$	$\frac{1}{8(1+\beta)}$	$\frac{1}{16(1+\beta)}$	$\frac{1}{16(1+\beta)}$	$\frac{3}{8(1+\beta)}$
$S_3 = \{A1, B2\}$	$\frac{10-\beta^2}{16(2+\beta)^2}$	$\frac{10-\beta^2}{16(2+\beta)^2}$	$\frac{6+\beta^2}{16(2+\beta)^2}$	$\frac{6+\beta^2}{16(2+\beta)^2}$	$\frac{2}{(2+\beta)^2}$
$S_4 = \{A1, B, 2\}$	$\frac{144+104\beta+9\beta^2}{288(2+\beta)^2}$	$\frac{2-\beta}{8(2+\beta)}$	$\frac{144+40\beta+9\beta^2}{288(2+\beta)^2}$	$\frac{1}{4(2+\beta)^2}$	$\frac{28+8\beta-\beta^2}{16(2+\beta)^2}$
$S_5 = \{AB; 1, 2\}$	$\frac{1}{4(2+\beta)}$	$\frac{1}{4(2+\beta)}$	$\frac{1}{4(2+\beta)^2}$	$\frac{1}{4(2+\beta)^2}$	$\frac{3+\beta}{2(2+\beta)^2}$
$S_6 = \{A, B; 12\}$	$\frac{1}{9(1+\beta)}$	$\frac{1}{9(1+\beta)}$	$\frac{1}{9(1+\beta)}$	$\frac{1}{9(1+\beta)}$	$\frac{4}{9(1+\beta)}$

Given the payoffs, the stability of each configuration is easily examined. For a configuration to be stable, every possible deviation must be checked to show that it is unprofitable for at least one firm. For a configuration to be unstable, we need only find one deviation that is profitable for both deviators (when it is a joint deviation).

For $S_1 = \{A, B; 1, 2\}$ to be stable: (i) $A + B$ is unprofitable: $\pi_A^{S_1} > \min\{\pi_A^{S_2}, \pi_A^{S_5}\}$. (ii) $1 + 2$ is unprofitable: $\pi_1^{S_1} > \min\{\pi_1^{S_2}, \pi_1^{S_6}\}$. (iii) $A + 1$ is unprofitable: $\pi_i^{S_1} > \min\{\pi_i^{S_3}, \pi_i^{S_4}\}$ for $i = A$ or 1.

For $S_2 = \{AB; 12\}$ to be stable: (i) Breaking AB is unprofitable: $\pi_A^{S_2} > \min\{\pi_A^{S_1}, \pi_A^{S_6}\}$. (ii) Breaking 12 is unprofitable: $\pi_1^{S_2} > \min\{\pi_1^{S_1}, \pi_1^{S_5}\}$. (iii) $A + 1$ is unprofitable: $\pi_i^{S_2} > \min\{\pi_i^{S_3}, \pi_i^{S_4}\}$ for $i = A$ or 1.

For $S_3 = \{A1, B2\}$ to be stable: (i) Breaking $B2$ is unprofitable: $\pi_i^{S_3} > \min\{\pi_i^{S_1}, \pi_i^{S_4}\}$ for $i = B$ and 2. (ii) $A+B$ is unprofitable: $\pi_A^{S_3} > \min\{\pi_A^{S_2}, \pi_A^{S_5}\}$. (iii) $1+2$ is unprofitable: $\pi_1^{S_3} > \min\{\pi_1^{S_2}, \pi_1^{S_6}\}$.

For $S_4 = \{B; A1, 2\}$ to be unstable, we only need one of following cases to hold: (i) $B+2$ is profitable: $\pi_i^{S_4} < \min\{\pi_i^{S_3}, \pi_j^{S_4}\}$ for both $\{i = B, j = A\}$ and $\{i = 2, j = 1\}$. (ii) $A + B$ is profitable: $\pi_A^{S_4} < \min\{\pi_A^{S_2}, \pi_A^{S_5}\}$. (iii) $1 + 2$ is profitable: $\pi_1^{S_4} < \min\{\pi_1^{S_2}, \pi_1^{S_6}\}$. (iv) Breaking $A1$ is profitable: $\pi_i^{S_4} < \min\{\pi_i^{S_1}, \pi_j^{S_4}\}$ for either $\{i = A, j = B\}$ or $\{i = 1, j = 2\}$. (v) $A + 2$ is profitable: $\pi_i^{S_4} < \min\{\pi_i^{S_3}, \pi_j^{S_4}\}$ for both $i = j = A$ and $\{i = 2, j = 1\}$. (vi) $B + 1$ is profitable: $\pi_i^{S_4} < \min\{\pi_i^{S_3}, \pi_j^{S_4}\}$ for both $\{i = B, j = A\}$ and $i = j = 1$.

For $S_5 = \{AB; 1, 2\}$ to be unstable: $1 + 2$ is profitable since $\pi_1^{S_5} < \min\{\pi_1^{S_2}, \pi_1^{S_6}\}$.

For $S_6 = \{A, B; 12\}$ to be unstable: $A + B$ is profitable since $\pi_A^{S_6} < \min\{\pi_A^{S_2}, \pi_A^{S_5}\}$.

Q.E.D.

The 3×3 case

Suppose there are three identical upstream firms (A, B, C) and three identical downstream firms $(1, 2, 3)$. The other assumptions remain the same as in the baseline model. There are then 10 possible market configurations as shown in the following payoff table. As the next proposition reveals, the firms will either all remain independent or carry out three vertical mergers, which is consistent with the result of the baseline model.

Table 4: Payoffs in the 3×3 case

Configurations	π_A	π_B	π_C	π_1	π_2	π_3	Total profits
$S_1 = \{A, B, C; 1, 2, 3\}$	47	47	47	35	35	35	246
$S_2 = \{A1, B2, C3\}$	34	34	34	28.5	28.5	28.5	187.5
$S_3 = \{C; A1, B2, 3\}$	42.5	42.5	21	42.5	42.5	15.5	206.5
$S_4 = \{B, C; A1, 2, 3\}$	61.5	28	28	49.5	28	28	222
$S_5 = \{BC; A1, 23\}$	99	21	21	74.5	14	14	243
$S_6 = \{BC; A1, 2, 3\}$	98	31	31	42.5	15.5	15.5	234.5
$S_7 = \{B, C; A1, 23\}$	65	18.5	18.5	86	24.5	24.5	237.5
$S_8 = \{A, BC; 1, 23\}$	74	37	37	49.5	25	25	247
$S_9 = \{A, BC; 1, 2, 3\}$	83.5	41.5	41.5	28	28	28	250
$S_{10} = \{A, B, C; 1, 23\}$	41.5	41.5	41.5	62.5	31	31	250

Proposition 4. *In the three by three case, only S_1 and S_2 are stable.*

Proof: S_1 is stable because there is no profitable deviation. There are three possible collective deviations excluding symmetric ones: $B + C$, $2 + 3$, and $A + 1$. $B + C$ is not profitable since both B and C are worse off in S_5. $2 + 3$ is not profitable since both firms are worse off in S_5. $A + 1$ is not profitable because both A and 1 are worse off in S_2.

S_2 is stable. First, there is no profitable unilateral deviation, as no firm wants to break up with its merger partner. For example, if $C3$ breaks up, both C and 3 are worse off in S_3 as compared to S_2. Second, there is no profitable collective deviation. There are two possible collective deviations: $B + C$ and $2 + 3$. $B + C$ is not profitable since both of them are worse off in S_6 as compared to S_2. $2 + 3$ is not profitable either since both of them are worse off in S_7 as compared to S_2.

S_3 is not stable since $C + 3$ is a profitable deviation. S_4, S_5, S_6 and S_7 are not stable since $B + 2$ is a profitable deviation in any of these configurations. S_8, S_9 and S_{10} would have been stable using our stability criterion as specified in the baseline model, but they are unreasonable equilibrium market structures. BC prefers to break up in both S_8 and S_9, while 23 prefers to break up in S_{10}. Those firms won't merge horizontally due to the merger paradox argument (Salant et al. 1983). **Q.E.D.**

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