Disclosure Policy in Contest with Stochastic Abilities

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Abstract

This paper investigates whether the contest organizer should disclose his private information about the abilities of the contestants in a perfectly discriminating contest. The private abilities of the contestants are stochastic and they are observed by the contest organizer who decides whether to disclose this information publicly. The organizer may care about total effort or rent dissipation. We find that concealing the abilities of the contestants elicits higher expected total effort, regardless of the distribution of the abilities. For rent dissipation, we find the rent dissipation rate does not depend on the disclosure policy. This finding is robust in settings with multiple prizes as long as effort cost function is linear. We then explore the robustness of our results while allowing endogenous ability distribution and endogenous entry of contestants. We find that our findings are robust to these generalized settings. However, when the cost function is nonlinear, the finding from the benchmark model is not robust, the organizer may prefer disclosure.

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Keywords: All Pay Auction; Disclosure; Concealment; Stochastic Abilities

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1 Introduction

It has been widely recognized that contestants' incentive to make effort and the resultant rent dissipation crucially depend on the rules of the contest. Most of the received rent-seeking literature deals with how a forward-looking sponsor implements an optimal structure to achieve a given objective. Early contributions assume that a player's ability, measured by his or her cost of expending effort, is fixed and common knowledge. However, contestants usually do not know the actual abilities of their rival at the time they make their decision. For example, advertising firms that are vying for a commercial project are not fully aware of each other's advertising ideas and thus unable to fully assess their relative competitiveness. Consider another example whereby a university is actively sourcing for new research professionals to join its teaching faculty. Each prospective candidate is unlikely to be fully aware of other candidates' research background and capabilities, thus seriously challenging the assumptions of common knowledge.

This paper analyzes contests where contestants have private information about their abilities, and are observed by the contest organizer. Following Konrad and Kovenock (2009), a player's ability measured by his or her cost of expending effort is determined as the outcome of a stochastic process. Players with lower cost can be thought of as stronger (more able) players. This assumption is reasonable since in reality, many aspects of a contestant's actual effectiveness or ability have transitory ups and downs, the value of unit cost of effort is not known to the rival contestants, but are easily observed by a sponsor. Similar to research tournaments, the sponsor can form tentative judgements regarding a firm's ability based on its research proposal. In addition, in the job market, a candidate's curriculum vitae usually pre-signals his ability to the prospective employer. One important consideration when designing a tournament with commitment power concerns the control of information. The organizer should strategically plan whether the information about agents' abilities should be revealed back to them. In other words, we want to compare between two policies: revealing or concealing the abilities of contestants.

On the other hand, most literature on contests design has focused on the case where the contestants compete in order to win a unique prize. More work needs to be done because the more prevalent form of contests in the real world involves multiple rather than single prize. For instance, in public recruitment, departments normally offer several identical positions at all ranks to the candidates. We extend the all pay contest models to allow for multiple (homogenous) prizes. Our extension can be used at the theoretical level to examine where established properties of the single prize all pay contests carry over to the more general case.

In the meanwhile, contestants often make costly investment to improve competency prior to the formal competition. For instance, an R&D company may purchase laboratory equipment, which improves the efficiency of subsequent research activities. We endogenize the distribution of abilities by allowing a contestant to reduce his marginal cost by making technological investment prior to the contest. Moreover, taking the time and trouble to enter the contest is a major concern for the contestants. We further assume that potential contestant have to incur a positive entry cost to participate in the contest. Potential bidders simultaneously make symmetric pure-strategy entry decisions so that their expected profits are exactly zero. In this way, we endogenize the entry probabilities. The comparison may differ.

The focus of the paper is to study how disclosure policy would affect the contestants' effort supply and rent dissipation. The private abilities of the contestants are observed by the contest organizer. The organizer may care about total effort or rent dissipation. She decides whether to disclose this information publicly.

Within the auction and contest literature, under disclosure policy, these kinds of all-pay auction with complete information has been carefully studied by Hillman and Riley (1989), Baye, Kovenock and de Vries (1996), Konrad and Kovenock (2009), Clark and Riis (1998) and Siegel (2009). They characterize the unique Nash equilibrium and calculate the expected effort of each agent. Especially, Clark and Riis (1998) extend the complete information single-prize all-pay auction for multiple (homogeneous) prizes. Moreover, Siegel (2009) provide a closed-form formula for players' equilibrium payoffs, and analyze player participation in all-pay contests. Last but not least, Hillman and Riley (1989) provides for the equilibrium under an all-pay auction with incomplete information and concealment policy.

Following the methodology provided by this literature, we find that concealing the abilities of the contestants elicits higher expected total effort, regardless of the distribution of the abilities. For rent dissipation, we find that the rent dissipation rate does not depend on the disclosure policy. To check the robustness of our main results and to deepen our basic analysis, we extend our model to allow any number of prizes. In order to have a less technical exposition, we focus on the organizer's problem in the case where she can award two identical prizes, we find that our findings are robust in this two-prize contest structure. We then study the robustness of our results while allowing endogenous ability distribution and endogenous entry of contestants. We find that our findings remain robust to these generalized settings. Further generalization are taken by exploring nonlinear cost function. We find that our findings are not robust when cost function is strictly concave or convex.

This paper is connected to a few strands of economic literature on contests and tournaments. Firstly, it is inspired by Lim and Matros (2009), Aoyagi (2010), Fu, Jiao and Lu (2010), Kovenock, Morath and Münster (2010) and Denter and Sisak (2011). All of these papers study the information revelation on contest design. Lim and Matros (2009) and Fu, Jiao and Lu (2010) investigate the impact of disclosure policy on expected effort in contests with a stochastic number of contestants. Aoyagi (2010) studies optimal feedback policy about agents' performance in a multi-stage tournament. Kovenock, Morath and Münster (2010) consider the information sharing in a two player all pay contest where firms have independent values and common values of winning the contest. Denter and Sisak (2011) focus on information transmission between lobbying groups and the consequences for disclosure policy in general rent seeking contests. Our analyses consider how disclosure policy would affect the contestants' effort supply and rent dissipation when players' abilities are stochastic.

This paper is also related to the literature on contests with asymmetric information. Hurley and Shogen (1998) explore how one-side asymmetric information over values affects effort levels in a Cournot Nash contest. Wärneryd (2003) studies a model of a common value contest under different assumptions about the information held by the players. In addition, our paper is linked to Fu and Lu (2009, 2010), who study contest with pre-investment and optimal endogenous entry in an imperfectly discriminating contest. Our analysis departs from these papers in that we focus on perfectly discriminating contests (All pay contests).

The paper is organized as follows. Section 2 sets up the general all pay contest model with n contestants, the unique equilibrium is characterized, the total expected effort and rent dissipation rate is calculated under both policies, and the optimal disclosure policy is explored. In section 3, the robustness of optimal disclosure policy is checked in a general all pay contest model with m prizes. In sections 4 and 5, we check the robustness of two endogenous cases. Section 6 further explores nonlinear cost cases. Some concluding remarks are presented in section 7.

2 A Model with unique prize

We study a contest with n players. A prize normalized to unity is awarded to the winner. The competition for this prize is organized as a perfectly discriminating contest (all-pay auction), in which each player simultaneously expend effort $x_i \geq 0$. The costs of effort are equal to $c_i x_i$. Here, c_i is the bid effort cost of player i. Assume that unit cost c_i is an independent random variable that is absolutely continuous with finite support $[\underline{c}, \overline{c}]$. The cumulative distribution functions of c_i is $F(c_i)$ with corresponding densities $f(c_i)$. The contest organizer knows all the information about players' cost.

We assume further that the contest organizer is allowed to commit to her disclosure policy - either to disclose the actual ability of participants, or to conceal this information - and announces this policy choice publicly. We denote the former policy by D, and the latter by C. Nature then determines c_i, the actual value of abilities. The organizer observes this information, and discloses it if and only if she has committed to a disclosure. The participants then submit their effort entry simultaneously $\mathbf{x} = (x_i)$ to compete for the prize.

2.1 Disclosure

We assume that when the efforts are chosen, the contest organizer will disclose each player's unit cost to the public; hence, this problem is a perfectly discriminating contest with complete information, the payoff to player i is given by

$$\pi_i(x_1, \ldots, x_n) = \begin{cases} -c_i x_i & \text{if } \exists j \text{ such that } x_j > x_i, \\ \frac{1}{m} - c_i x_i & \text{if } i \text{ ties for the high bid with } m-1 \text{ others,} \\ 1 - c_i x_i & \text{if } x_i > x_j \ \forall j \neq i. \end{cases}$$

This game has been carefully analyzed by Hillman and Riley (1989) and Baye, Kovenock, and de Vries (1996). They demonstrated that the equilibrium of the perfectly discriminating contest for given values of $c_1, c_2, \ldots c_n$ is unique and this is described as follows.

Proposition 1 *(Baye et al. 1996) The unique equilibrium of n players all pay contest with complete information is a set of mixed strategies. Assume $c_1 = \min\{c_1, c_2, \ldots, c_n\}$, $c_2 = \min\{c_2, c_3, \ldots, c_n\}$. Then the unique equilibrium is for the two players with the lowest cost to compete as if they were the only two players. All other players remain passive. And bids are described by the following cumulative distribution functions:*

$$G_1(x_1) = \begin{cases} c_2 x_1 & \text{for } x_1 \in [0, \frac{1}{c_2}), \\ 1 & \text{for } x_1 \geq \frac{1}{c_2}; \end{cases}$$

$$G_2(x_2) = \begin{cases} 1 - \frac{c_1}{c_2} + c_1 x_2 & \text{for } x_2 \in [0, \frac{1}{c_2}), \\ 1 & \text{for } x_2 \geq \frac{1}{c_2}. \end{cases}$$

With homogeneous abilities ($c_1 = c_2 = c_3 \ldots = c_n$), there exists a unique symmetric equilibrium and a continuum of asymmetric equilibria. All of the equilibria imply the same expected payoff (zero) for each player, and yield the organizer the same expected revenue.

This result is easy to obtain because in an all-pay contest, one can interpret differences in the c_i's as arising from differences in valuations or differences in abilities of players to convert an entry into a prize. Dividing contestant i's expected payoff by c_i we obtain an affine transformation of the expected payoff given by Hillman and Riley (1989) and Baye et al. (1996). Indeed, in our case, c_i plays the same role as the value v_i in the current literature as the adjusted-value $\frac{1}{c_i}$ is equal to v_i.

Lemma 1 *In an all pay contest with complete information, the total expected effort for contest sponsor is*

$$R^D = n(n-1) \int_{\underline{c}}^{\overline{c}} \int_{c_1}^{\overline{c}} \left[\frac{1}{2c_2} + \frac{c_1}{2c_2^2} \right] [1 - F(c_2)]^{n-2} \, dF(c_2) \, dF(c_1). \quad (1)$$

Proof. See Appendix. ■

We now examine the expected payoff for each player. Take player i with unit cost c_i as a representative contestant and let $\widetilde{c} = \min\{c_{-i}\}$. Then the cumulative distribution function of $\widetilde{c}$ is $1 - [1 - F(\widetilde{c})]^{n-1}$. Player i can get a positive payoff if and only if $c_i < \widetilde{c}$. Apply the payoff characterization in Siegel (2009), in any equilibrium of a generic contest, only the contestant with minimum cost can get a positive payoff. i.e., $\pi_i(x_i; c_i, c_{-i}) = 1 - \frac{c_i}{\widetilde{c}}$ if and only if $c_i < \widetilde{c}$. [1]

Therefore, the expected payoff of player i is

$$E\pi_i^D = \int_{\underline{c}}^{\overline{c}} \left\{ \int_{c_i}^{\overline{c}} \left[1 - \frac{c_i}{\widetilde{c}}\right] d\left[1 - [1 - F(\widetilde{c})]^{n-1}\right] \right\} dF(c_i). \tag{2}$$

Assume $\frac{[1-F(\widetilde{c})]^{n-2}}{\widetilde{c}}$ is integrable and define

$$A(\widetilde{c}) = -(n-1) \int \frac{[1 - F(\widetilde{c})]^{n-2}}{\widetilde{c}} dF(\widetilde{c}), \tag{*}$$

which will be used repeatedly throughout the analysis.

2.2 Concealment

In this subsection, we assume when the efforts are chosen, the contest organizer will conceal his information about every player's unit cost, causing each player to know only his own unit cost; hence, at this stage, the problem describes a perfectly discriminating contest with incomplete information.

Take player i with unit cost c_i as a representative contestant, let $\widetilde{c} = \min\{c_{-i}\}$. Then the cumulative distribution functions of $\widetilde{c}$ is $1 - [1 - F(\widetilde{c})]^{n-1}$. Player i will get the prize if and only if $c_i < \widetilde{c}$.

Assuming all bidder other than i adopt a bidding strategy $x_{-i}(\cdot)$. The payoff of player i given as

$$\begin{aligned} \pi_i(\tilde{x}_i, x_{-i}(\cdot); c_i) &= \Pr(\tilde{x}_i > x_j(c_j), \forall j \neq i)\, 1 - c_i \tilde{x}_i \\ &= \Pr\left(c_j > x_j^{-1}(\tilde{x}_i), \forall j \neq i\right) 1 - c_i \tilde{x}_i \\ &= \left[1 - F\left(x_{-i}^{-1}(\tilde{x}_i)\right)\right]^{n-1} - c_i \tilde{x}_i. \end{aligned} \tag{3}$$

Player i will choose $x_i(c_i)$ to maximize his expected payoff.

Lemma 2 *In an pay contest with incomplete information, the equilibrium bid of each player is*

$$x_i(c_i) = (n-1) \int_{c_i}^{\overline{c}} \frac{[1 - F(\widetilde{c})]^{n-2}}{\widetilde{c}} dF(\widetilde{c}),$$

[1] Following definitions of Siegel (2009), with a unique prize $m = 1$, and initial score $a_i = 0$. The payoff is given as $v_i(x_i) = V_i - c_i(x_i) = 1 - c_i x_i$. The contestant with marginal cost $\widetilde{c}$ is the *marginal player*, his *reach* $\widetilde{r} = \max\{x_i | v_i(x_i) = 0\} = \frac{1}{\widetilde{c}}$. And player i's *power* $w_i = v_i(\max\{0, \widetilde{r}\}) = \max\{0, v_i(\widetilde{r})\} = \begin{cases} 1 - \frac{c_i}{\widetilde{c}} > 0 & \text{if } c_i < \widetilde{c} \\ 0 & \text{if } c_i \geq \widetilde{c} \end{cases}$. The expected payoff of every player equals the maximum of his power and 0.

and the total expected effort for contest sponsor is

$$R^C = n(n-1) \int_{\underline{c}}^{\overline{c}} \frac{F(c_2)[1-F(c_2)]^{n-2}}{c_2} dF(c_2). \quad (4)$$

Proof. See Appendix. ■

Given c_i, the expected payoff of each player is given by

$$\begin{aligned} \pi_i^C(x_i, \mathbf{x}_{-i}; c_i) &= \Pr(c_i < \widetilde{c})\, 1 - c_i x_i(c_i) \\ &= [1 - F(c_i)]^{n-1} - c_i x_i(c_i). \end{aligned}$$

The expected payoff of each player under concealment policy is given by

$$E\pi_i^C = \int_{\underline{c}}^{\overline{c}} \left\{[1-F(c_i)]^{n-1} - c_i x_i(c_i)\right\} dF(c_i). \quad (5)$$

2.3 Optimal Disclosure Policy

For a contest organizer who is interested in maximizing the total expected effort, the following equilibrium analysis allows us to investigate the structure of the optimal disclosure policy. Lemma 1 and 2 imply the following.

Theorem 1 *In an N players all pay contests, concealing the abilities of contestants elicits higher expected total revenue to contest sponsor.*

Proof. See Appendix. ■

It follows that expected payoff (2) and (5) take exactly the same form, the following result can be obtained immediately.

Theorem 2 *Given the number of participant n, both disclosure and concealment policies give each contestant same expected payoff .*

Proof. See Appendix. ■

Theorems 1 and 2 indicate that contestants' expected payoffs are identical in the two cases. However, readers should note that the contest sponsor would nonetheless like to conceal contestant's ability to induce higher expected effort. Note that the total expected effort and player's expected payoff are both *ex ante*. Before disclosure policy is implemented, each player expects to get a positive payoff if and only if he is the most able player, which produces the same *ex ante* expected payoff, regardless subsequent disclosure policy .

Under concealment policy, each contestant has incomplete knowledge on his competitiveness or lack of comparative advantage over other contestants, thus leading him to have a positive *ex post* expected payoff and do their best in the bidding process. The player with highest ability wins the unique prize with probability one. However, when players' abilities are common knowledge, competition is less fierce as the *ex post* expected payoff of contestants with lower abilities is zero, but they also have positive probability of winning since the equilibrium is in mixed strategies.

We now turn to analyzing the property of contestant's expected payoff, recall

$$E\pi_i(n) = \int_{\underline{c}}^{\overline{c}} \left\{ \int_{c_i}^{\overline{c}} \left[1 - \frac{c_i}{\widetilde{c}}\right] d\left[1 - [1 - F(\widetilde{c})]^{n-1}\right] \right\} dF(c_i),$$

where c_i is the unit cost of a representative contestant i when there are $n-1$ other bidders, and $\widetilde{c} = \min\{c_{-i}\}$.

Corollary 1 *$E\pi_i(n) \geq 0$ and monotonically decreases with $n \leq N$.*

Proof. The conditional payoff of a bidder given c_i is

$$\begin{aligned} & \int_{c_i}^{\overline{c}} \left[1 - \frac{c_i}{\widetilde{c}}\right] d\left[1 - [1 - F(\widetilde{c})]^{n-1}\right] \\ = & \left[1 - \frac{c_i}{\widetilde{c}}\right] \left[1 - [1 - F(\widetilde{c})]^{n-1}\right] |_{c_i}^{\overline{c}} - \int_{c_i}^{\overline{c}} \left[1 - [1 - F(\widetilde{c})]^{n-1}\right] d\left[1 - \frac{c_i}{\widetilde{c}}\right] \\ = & (1 - c_i) - 0 - \int_{c_i}^{\overline{c}} \left[1 - [1 - F(\widetilde{c})]^{n-1}\right] \frac{c_i}{\widetilde{c}^2} d\widetilde{c} \text{ since } F(\overline{c}) = 1. \end{aligned}$$

since $[1 - F(\widetilde{c})]^{n-1}$ decreases with n, then the term $-\int_{c_i}^{\overline{c}} \left[1 - [1 - F(\widetilde{c})]^{n-1}\right] \frac{c_i}{\widetilde{c}^2} d\widetilde{c}$ decreases with n. Therefore $\int_{c_i}^{\overline{c}} \left[1 - \frac{c_i}{\widetilde{c}}\right] d\left[1 - [1 - F(\widetilde{c})]^{n-1}\right]$ decreases with n as well.

$E\pi_i(n) \geq 0$ is apparent since both $1 - \frac{c_i}{\widetilde{c}}$ and $1 - [1 - F(\widetilde{c})]^{n-1}$ are nonnegative. ■

This result parallels the finding in a standard all pay contest that the expected payoff decreases in the number of contestants as the competition intensifies.

Following part of the paper further explores the issue of information disclosure from three additional dimensions. First, we generalize the disclosure policy in the basic setting by allowing n participants competing for more than one prizes. Second, we endogenize the distribution of abilities by allowing a contestant to reduce his marginal cost through making technological investment prior to the contest, and endogenize the entry probabilities by assuming that entry incurs a fixed cost to each contestant. Third, an extension that generalizes cost function to nonlinear form is explored.

3 Multi-prize Contests

Consider an all pay auction in which there are m identical prizes to be won. There are n players who are ranked according to their abilities. To simplify the model assume $c_1 < c_2 < \ldots < c_n$ and each prize is normalized to unity $V_1 = V_2 = \ldots V_m = 1$. The players with m highest efforts win these prizes and everyone can only win one prize. Abilities are draw independently of each other from an interval $[\underline{c}, \overline{c}]$ according to the absolutely continuous distribution function $F(c_i)$ which is common knowledge.

3.1 Disclosure

We assume at the point in time when the efforts are chosen, the contest organizer will disclose each player's unit cost to public; In this case, $c_1 < c_2 < \ldots < c_n$ are common knowledge. Hence, this problem is a perfectly discriminating contest with complete information, the payoff to player i is given by

$$P_i(x_1, x_2, \ldots, x_n) 1 - c_i x_i$$

Let $G_i(x)$ represent the cumulative density function of player i's equilibrium mixed strategy. This game has been carefully analyzed by Clark and Riis (1998). They have shown that the equilibrium of the perfectly discriminating contest for given values of $c_1, c_2, \ldots c_n$ is necessarily in mixed strategies and described as follows.

Proposition 2 *(Clark and Riis 1998) There exists a unique mixed strategy equilibrium of the game in which the* $m+1$ *highest ranked of players bid* $x_i, i = 1, 2, \ldots, m+1$*, from probability distribution functions* $G_i(x)$ *over* $\left[\frac{1}{c_i^l}, \frac{1}{c_{m+1}}\right]$*, with common upper support* $\frac{1}{c_i^u} = \frac{1}{c^u} = \frac{1}{c_{m+1}}$ *and lower supports given by*

$$\frac{1}{c_{m+1}^l} = 0,$$

$$\frac{1}{c_i^l} = \left[1 - \Pi_{j=i}^m \left(\frac{c_i}{c_j}\right)\right] \frac{1}{c_{m+1}} \quad i = 1, 2, \ldots, m,$$

and where

$$G_i(x) = 1 - \frac{1}{c_i} \Pi_{j=k}^m c_j^{1/(m+1-k)} (1 - c_{m+1} x)^{1/(m+1-k)} \quad i = 1, 2, \ldots, m,$$

where

$$\begin{aligned} k &= 1 \quad \textit{if } \frac{1}{c_1^l} \leq x \leq \frac{1}{c_{m+1}}, \\ k &= s \quad \textit{if } \frac{1}{c_s^l} \leq x < \frac{1}{c_{s-1}^l}, \\ s &= 2, 3, \ldots, m. \end{aligned}$$

Player $m+1$ *bids* $x_{m+1} > 0$ *with probability* c_m / c_{m+1}*. The conditional distribution function of this player is*

$$G_{m+1}(x | x > 0) = 1 - \frac{1}{c_m} \Pi_{j=k}^m c_j^{1/(m+1-k)} (1 - c_{m+1} x)^{1/(m+1-k)}.$$

When there are two prizes $m = 2$, the unique equilibrium is for the three players with the lowest cost to compete as if they were the only three players. All other players remain passive. And bids are described by the following cumulative distribution functions:

$$G_1(x_1) = 1 - \left(\frac{c_2}{c_1}\right)^{\frac{1}{2}} (1 - c_3 x_1)^{\frac{1}{2}} \quad \text{for } x_1 \in [\left(1 - \frac{c_1}{c_2}\right) \frac{1}{c_3}, \frac{1}{c_3});$$

$$G_2(x_2)=\begin{cases} c_3x_2 & \text{for } x_2\in[0,\left(1-\frac{c_1}{c_2}\right)\frac{1}{c_3}),\\ 1-\left(\frac{c_1}{c_2}\right)^{\frac{1}{2}}(1-c_3x_2)^{\frac{1}{2}} & \text{for } x_2\in[\left(1-\frac{c_1}{c_2}\right)\frac{1}{c_3},\frac{1}{c_3});\end{cases}$$

$$G_3(x_3)=\begin{cases} 1-\frac{c_2}{c_3}+c_2x_3 & \text{for } x_3\in[0,\left(1-\frac{c_1}{c_2}\right)\frac{1}{c_3}),\\ 1-\left(\frac{c_1c_2}{c_3}\right)^{\frac{1}{2}}\left(\frac{1}{c_3}-x_3\right)^{\frac{1}{2}} & \text{for } x_3\in[\left(1-\frac{c_1}{c_2}\right)\frac{1}{c_3},\frac{1}{c_3}).\end{cases}$$

Corollary 2 *The unique equilibrium of n players all pay contest with complete information is in mixed strategies. The unique equilibrium is for the three players with the lowest cost to compete as if they were the only three players. All other players remain passive. the total expected effort for contest sponsor is*

$$\begin{aligned} R^D &= n(n-1)(n-2)\\ &\int_{\underline{c}}^{\overline{c}}\int_{c_1}^{\overline{c}}\int_{c_2}^{\overline{c}}\left[\left(\frac{3}{2}-\frac{c_1}{3c_2}+\frac{c_1^2}{6c_2^2}\right)\frac{1}{c_3}+\left(c_2+\frac{c_1^2}{3c_2}\right)\frac{1}{2c_3^2}\right]\\ &[1-F(c_3)]^{n-3}dF(c_3)\,dF(c_2)\,dF(c_1). \end{aligned} \quad (6)$$

Proof. See Appendix. ■

3.2 Concealment

In this case, the ability of contestant i is private information to i. Player i's maximization problem is

$$\max_{x(c)}\sum_{j=0}^{m-1}C_{n-1}^{j}\left[F\left(x^{-1}(c)\right)\right]^{j}\left[1-F\left(x^{-1}(c)\right)\right]^{n-1-j}-c\cdot x(c).$$

Fix agent i, and let $F_s(c)$, $1\leq s\leq m$, denote the probability that agent i with type $c\in[\underline{c},\overline{c}]$ meets $n-1$ competitors such that $s-1$ of them have lower types, while $n-s$ have higher types. Hence, F_s is the probability of winning the s'th prize, where $s=1,2,\ldots,m$. We now have

$$F_s(c)=\frac{(n-1)!}{(s-1)!\,(n-s)!}\times[1-F(c)]^{n-s}[F(c)]^{s-1}. \quad (7)$$

The corresponding derivatives are given by

$$F_1^{'}(c)=-(n-1)(1-F(c))^{n-2}F^{'}(c) \quad (8)$$

when $s=1$,

$$F_s^{'}(c)=\frac{(n-1)!}{(s-1)!\,(n-s)!}\times[1-F(c)]^{n-s-1}[F(c)]^{s-2}F^{'}(c)\times[(1-n)F(c)+(s-1)] \quad (9)$$

when $2\leq s\leq m$.

Moldovanu and Sela (2001) states the symmetric equilibrium as follows.

Proposition 3 *(Moldovanu and Sela 2001) The equilibrium bid under concealment policy for any number of prizes $m \geq 2$ and $n \geq m$ contestants is given by*

$$x_i(c_i) = \sum_{s=1}^{m} \int_{c_i}^{\overline{c}} -\frac{1}{c} F_s^{'}(c)\, dc,$$

where $F_s^{'}(c)$ is given by (8) and (9).

In the special case where there are two prizes $m = 2$, the contestants with the highest and second-highest effort win the two prizes.

The symmetric equilibrium of the perfectly discriminating contest with incomplete information is described as follows

Corollary 3 *Assume that there are two prizes, $V_1 = V_2 = 1$, and $n \geq 3$ contestants. In a symmetric equilibrium, the bid function of each contestant is given by $x_i(c) = P(c) V_1 + Q(c) V_2 = P(c) + Q(c)$, where*

$$P(c) = (n-1) \int_{c}^{\overline{c}} \frac{1}{c_i} [1 - F(c_i)]^{n-2} dF(c_i),$$

$$Q(c) = (n-1) \int_{c}^{\overline{c}} \frac{1}{c_i} [1 - F(c_i)]^{n-3} [(n-1) F(c_i) - 1] dF(c_i),$$

the total expected effort for contest sponsor is

$$R^C = n \int_{\underline{c}}^{\overline{c}} x_i(c)\, dF(c). \tag{10}$$

3.3 Optimal Disclosure Policy

The goal of the contest designer is to maximize the total expected effort (i.e., the expected sum of the bids) at the contest. In order to keep the analysis as simple and tractable as possible, and in order to compare our results directly, we focus, however, on the total expected effort comparison with *two* identical prizes.[2] Given the characterization of the total expected effort under disclosure and concealment policy, we can now compare (6) and (10) to address the issue of optimal contest design.

Theorem 3 *In an N players all pay contest with two identical prizes, concealing the abilities of contestants elicits higher total expected revenue to contest sponsor.*

Proof. See Appendix. ■

Theorem 3 shows that our findings in Theorem 1 are robust in multi-prizes contests.

3.4 Payoff Equivalent

In this section, we further explore the rent dissipation rate comparison under both policies.

[2] It will become clear that none of our qualitative results change if we allow for more than two prizes. We leave the analysis of more general environments to future works.

3.4.1 Payoff under Disclosure

We first get the expected payoff of each player under disclosure. Assume there are m identical prizes $V_1 = V_2 = \ldots V_m = 1$. As $c_1 < c_2 < \ldots < c_n$ are common knowledge, the c.d.f of the mth lowest among $n-1$ bidders' costs is

$$H(c) = \sum_{j=m}^{n-1} C_{n-1}^{j} F(c)^j [1 - F(c)]^{n-1-j}.$$

Following the result provided by Siegel (2009), only the contestants with marginal cost $c_i < c$ can get a positive payoff.[3]

Given c_i,the conditional expected payoff of each player is given by

$$\pi_i^D(c_i) = \int_{c_i}^{\bar{c}} \left[1 - \frac{c_i}{c}\right] dH(c). \tag{11}$$

The expected payoff of player i is

$$\begin{aligned} E\pi_i^D &= \int_{\underline{c}}^{\bar{c}} \pi_i^D(c_i)\, dF(c_i) \\ &= \int_{\underline{c}}^{\bar{c}} \left\{ \int_{c_i}^{\bar{c}} \left[1 - \frac{c_i}{c}\right] dH(c) \right\} dF(c_i). \end{aligned}$$

note that $F(c = \bar{c}) = 1$ and $H(c = \bar{c}) = 1$ since $c \in [\underline{c}, \bar{c}]$.

3.4.2 Payoff under Concealment

In the following part, we will get the equilibrium bid and expected payoff of each player under concealment. Proposition 3 has stated the equilibrium bid. Since $c_1 < c_2 < \ldots < c_m < c_{m+1} < \ldots < c_n$, player i will get a prize if and only if $c_i < c$. Here c is the mth lowest among $n-1$ bidders' costs, and the c.d.f of c is

$$H(c) = \sum_{j=m}^{n-1} C_{n-1}^{j} F(c)^j [1 - F(c)]^{n-1-j}.$$

Given c_i,the conditional expected payoff of each player is given by

$$\begin{aligned} \pi_i^C(c_i) &= \Pr(c_i < c)\, 1 - c_i x_i(c_i) \\ &= 1 - H(c_i) - c_i x_i(c_i). \end{aligned} \tag{12}$$

And the expected payoff of player i is

$$\begin{aligned} E\pi_i^C &= \int_{0}^{\bar{c}} \pi_i^C(c_i)\, dF(c_i) \\ &= \int_{\underline{c}}^{\bar{c}} \{1 - H(c_i) - c_i x_i(c_i)\}\, dF(c_i). \end{aligned}$$

[3] Applying the payoff characterization in Siegel (2009), the payoff of player i is given as $v_i(x_i) = 1 - c_i x_i$, and his *reach* is $r_i = \frac{1}{c_i}$, his *power* is $1 - \frac{c_i}{c}$. Since $\frac{1}{c_1} > \frac{1}{c_2} > \ldots > \frac{1}{c_m}$, in this m prizes model, player $m+1$ is the *marginal* player.

Lemma 3 *Given c_i, both policies give each player same conditional expected payoff.*

Proof. See Appendix. ■

Since the conditional expected payoffs under both policies are identical, each player will get equal expected payoff finally, i.e., $E\pi_i^D = E\pi_i^C$. The following result can be established immediately.

Theorem 4 *In multi-prize perfectly discriminating contests, each player will get identical expected payoff under both policies, and rent dissipation rates are identical as well.*

Note that rent dissipation rate $= \frac{\text{total effort cost}}{\text{prize value}} = \frac{1- \text{total expected payoff}}{\text{prize value}}$. Both policies will induce the same rent dissipation rate as long as each contestant gets the same expected payoff. Theorem 4 shows that our findings in Theorem 2 are robust in multi-prizes contests.

Theorem 3 and 4 and their intuition are similar to the corresponding findings in the previous section. In all pay auction with multiple prizes, a policy maker interested in maximizing his total rent-seeking revenues always prefers concealment to disclosure, while disclosure policy does not affect contestant's *ex ante* expected payoff.

The results discussed in the previous sections offer a sharp characterization of the optimal contest design. But to what extent can one generalize our result in a model when the distribution of player's ability and contestant's entry can be endogenized? Also, does the main result continue to hold if a more general class of effort costs becomes feasible? In subsequent analysis, we discuss the robustness of our main results to each of these issues.

4 Endogenous Distribution of Abilities

We have thus far assumed that the cumulative distribution functions of unit effort cost is taken as given. We now consider endogenous distribution of abilities where contestants can independently make investment to improve their distributions of types. Specifically, the distribution of marginal cost c_i of contestant i is determined by her pre-contest investment α_i, with investment cost $I(\alpha_i)$, where $I'(\alpha_i) < 0$ and $I''(\alpha_i) > 0$.

In an N players all pay auction, player i with investment $\alpha_i \in [0,1]$ will have a corresponding cumulative distribution functions $F_i(c_i; \alpha_i)$ on support $[0,1]$. In addition, the investment cost is $I(\alpha_i)$ with $I(1) = 0, I(0) = \infty$. It is clear that contestants' pre-investment affect the distribution of their abilities. In our setting, ability is interpreted as the value of per-unit-of-bid effort cost.

Consider the following game:

Stage 1: The contest organizer commits to and announces publicly her disclosure policy, D or C;

Stage 2: Upon observing the disclosure rule, each player simultaneously engage in technological investment α_i with investment cost $I(\alpha_i)$ in order to lower their marginal costs, so that effort cost with c.d.f $F_i(c_i; \alpha_i)$, and the investments of contestants are private information;

Stage 3: Nature draw abilities c_i, and each player privately learns his own ability, the value of marginal cost is revealed if and only if the organizer committed to policy D;

Stage 4: Players bids x_i simultaneously in the all pay auction.

The subgame perfect equilibrium in stage 4 is the same as in Section 2, except that players have endogenized distribution functions of unit cost.

4.1 Disclosure

Without loss of generality, take player i with unit cost c_i as a representative contestant, assume $\widetilde{c} = \min\{c_{-i}\}$. We concentrate on characterizing a symmetric pure-strategy equilibrium. Assume all bidders other than i adopt an optimal investment level at stage 1, that is, $\alpha_k = \alpha_j = \alpha_D^*$ for any $k \neq j \neq i$. Then the cumulative distribution functions of $\widetilde{c}$ is $1 - [1 - F(\widetilde{c}; \alpha_D^*)]^{n-1}$. Player i will get a positive payoff if and only if $c_i < \widetilde{c}$.

We first look for a subgame equilibrium at stage 4. Recall the results in section 2.1, the expected payoff of player i is

$$E\pi_i = \int_{\underline{c}}^{\overline{c}} \left\{ \int_{c_i}^{\overline{c}} \left[1 - \frac{c_i}{\widetilde{c}}\right] d\left[1 - [1 - F(\widetilde{c}; \alpha_D^*)]^{n-1}\right] \right\} dF_i(c_i; \alpha_i).$$

Then, given c_i, the expected payoff of player i is

$$\begin{aligned} \pi_i^D(c_i; \alpha_D^*) &= \int_{c_i}^{\overline{c}} \left[1 - \frac{c_i}{\widetilde{c}}\right] d\left[1 - [1 - F(\widetilde{c}; \alpha_D^*)]^{n-1}\right] \\ &= [1 - F(c_i; \alpha_D^*)]^{n-1} - c_i A(c_i), \end{aligned} \tag{13}$$

where $A(c_i)$ is defined by (*).

Note that for any realization of abilities c_i, $\pi_i^D(c_i; \alpha_D^*)$ is irrelevant with their pre-contest investment α_i.

The expected payoff of player i at stage 4 is

$$E\pi_i^D(c_i; \alpha_D^*, \alpha_i) = \int_{\underline{c}}^{\overline{c}} \pi_i^D(c_i; \alpha_D^*) \, dF_i(c_i; \alpha_i).$$

At stage 1, player i will make a pre-contest investment α_i to maximize $\int_{\underline{c}}^{\overline{c}} \pi_i^D(c_i; \alpha_D^*) \, dF_i(c_i; \alpha_i) - I(\alpha_i)$. Therefore, if α_D^* is a symmetric equilibrium solution, player i's best reply is $\alpha_D^* = \arg\max_{\alpha_i} \left\{ \int_{\underline{c}}^{\overline{c}} \pi_i^D(c_i; \alpha_D^*) \, dF_i(c_i; \alpha_i) - I(\alpha_i) \right\}$.

4.2 Concealment

In this perfectly discriminating contest with incomplete information, take player i with unit cost c_i as a representative contestant, let $\widetilde{c} = \min\{c_{-i}\}$. Then the cumulative distribution functions of $\widetilde{c}$ is $1 - \prod_{j=1, j\neq i}^{n} [1 - F_j(\widetilde{c})]$. Player i will get the prize if and only if $c_i < \widetilde{c}$.

We still concentrate on characterizing a symmetric pure-strategy equilibrium. Assume all bidders adopt an optimal investment level α_C^* at stage 1. Therefore the cumulative distribution functions of $\widetilde{c}$ is $1 - [1 - F(\widetilde{c}; \alpha_C^*)]^{n-1}$ since $\alpha_k = \alpha_j = \alpha_C^*$ for any $k \neq j \neq i$.

Then the subgame equilibrium at stage 4 is the same as section 2. Recall the results in section 2.2, a representative contestant's optimal bidding strategy is given as

$$x^*(c_i) = A(c_i) = -(n-1)\int \frac{[1 - F(c_i; \alpha_C^*)]^{n-2}}{c_i} dF(c_i; \alpha_C^*).$$

Then, given c_i, the expected payoff of player i at stage 4 is given by

$$\begin{aligned} \pi_i^C(c_i; \alpha_C^*) &= \Pr(c_i < \tilde{c}) \cdot 1 - c_i x^*(c_i) \\ &= [1 - F(c_i; \alpha_C^*)]^{n-1} - c_i x^*(c_i). \end{aligned} \tag{14}$$

Note that for any realization of ability c_i, $x^*(c_i)$ is player i's optimal bidding strategy which maximize his payoff, i.e., $\frac{\partial \pi_i}{\partial \tilde{x}_i}|_{\tilde{x}_i = x^*(c_i)} = 0$. While $x^*(c_i)$ and $\pi_i^C(c_i; \alpha_C^*)$ is irrelevant with his pre-contest investment α_i.

The expected payoff of player i at stage 4 is

$$E\pi_i^C(c_i; \alpha_C^*, \alpha_i) = \int_{\underline{c}}^{\overline{c}} \pi_i^C(c_i; \alpha_C^*) \, dF_i(c_i; \alpha_i).$$

At stage 1, player i will make a pre-contest investment α_i to maximize $\int_{\underline{c}}^{\overline{c}} \pi_i^C(c_i; \alpha_C^*) \, dF_i(c_i; \alpha_i) - I(\alpha_i)$. Therefore, if α_C^* is a symmetric equilibrium solution, player i's best response is $\alpha_C^* = \arg\max_{\alpha_i} \left\{ \int_{\underline{c}}^{\overline{c}} \pi_i^C(c_i; \alpha_C^*) \, dF_i(c_i; \alpha_i) - I(\alpha_i) \right\}$.

4.3 Comparison

At stage 1, a representative contestant i will make investment to maximize his pre-contest expected payoff $\int_{\underline{c}}^{\overline{c}} \pi_i(c_i; \alpha^*) \, dF_i(c_i; \alpha_i) - I(\alpha_i)$. Recall (13) and (14), for any realization of abilities, his expected payoffs at stage 4 are identical under different policies $\pi_i^D(c_i; \alpha^*) = \pi_i^C(c_i; \alpha^*)$. Therefore, the optimal symmetric investment levels are same under both policies. i.e., $\alpha_D^* = \alpha_C^* = \alpha^*$.

The existence of symmetric equilibrium implies that players will make the same level of investment at stage 1, the endogenized distribution of abilities are parallel under different policies. Therefore both policies give players equivalent pre-contest expected payoff at stage 1. Given the same distribution of abilities, however, hiding the information leads to higher effort according to the exogenous entry result. Therefore, contest sponsor still prefers to conceal the actual value of contestants' abilities to elicit higher total expected effort. The following result can be established immediately.

Theorem 5 *Consider the symmetric equilibria of endogenous distribution of abilities, both policies implement the same level of investment and the same contestant's expected payoff, while concealing the abilities of contestants still elicits higher expected total revenue to contest sponsor.*

In addition, one should note that when $\alpha^* = 1$, no one makes pre-contest investment. $F(c_i; \alpha^* = 1) = F(c_i)$, and the distribution of abilities is taken as given, similar to the case in section 2.

4.4 An Example

In a two players all pay auction, player i with investment $\alpha_i \in [0,1]$ will have a corresponding cumulative distribution functions $F_i(c_i;\alpha_i) = c_i^{\alpha_i}$ on support $[0,1]$. And the investment cost is $I(\alpha_i) = \frac{5}{36}\left(\frac{1}{\alpha_i} - 1\right)$ which satisfy $I(1) = 0, I(0) = \infty, I'(\alpha_i) < 0$ and $I''(\alpha_i) > 0$.

Under disclosure policy, apply the results in Konrad and Kovenock (2009), given values $c_1 < c_2$, bidder 1 gets payoff $\pi_1 = 1 - \frac{c_1}{c_2}$ and bidder 2 gets 0; given values $c_2 < c_1$, bidder 2 gets payoff $\pi_2 = 1 - \frac{c_2}{c_1}$ and bidder 1 gets 0.

When $\alpha_2 \in [0,1)$, the expected payoff of player 1 is given by

$$\begin{aligned} E\pi_1^D(\alpha_1,\alpha_2) &= \int_0^1 \left[\int_{c_1}^1 \left(1 - \frac{c_1}{c_2}\right) dF(c_2)\right] dF(c_1) - I(\alpha_1) \\ &= \frac{\alpha_2}{(1+\alpha_1)(\alpha_1+\alpha_2)} - \frac{5}{36}\left(\frac{1}{\alpha_1} - 1\right); \end{aligned}$$

When $\alpha_2 = 1$, the expected payoff of player 1 is given by

$$\begin{aligned} E\pi_1(\alpha_1,\alpha_2) &= \int_0^1 \left[\int_{c_1}^1 \left(1 - \frac{c_1}{c_2}\right) dc_2\right] dF(c_1) - I(\alpha_1) \\ &= 1 - \left[1 - \frac{1}{\alpha_1+1}\right]^2 - \frac{5}{36}\left(\frac{1}{\alpha_1} - 1\right). \end{aligned}$$

Under concealment policy, with only two players, the payoff of the player 1 given as

$$\begin{aligned} \pi_1(x_1,x_2;c_1) &= \Pr(x_1 > x_2^*(c_2))\, 1 - c_1 x_1\left(c_1'\right) - I(\alpha_1) \\ &= \Pr\left(c_2^* > c_1'\right) 1 - c_1 x_1\left(c_1'\right) - I(\alpha_1) \\ &= \left[1 - F_2\left(c_1'\right)\right] - c_1 x_1\left(c_1'\right) - I(\alpha_1). \end{aligned}$$

Note that $x_i(c_i)$ is decreasing with c_i, the higher cost, the lower effort. Take first order condition with respect to c_1,

$$\frac{\partial \pi_1}{\partial c_1'} = -f_2\left(c_1'\right) - c_1 \frac{dx_1\left(c_1'\right)}{dc_1'}.$$

When $c_1' = c_1^*$, $\frac{\partial \pi_i}{\partial c_i'} = 0$, hence $\frac{dx_1(c_1)}{dc_1} = \frac{-f_2(c_1;\alpha_2)}{c_1}$.

Note that $x_1(c_1 = 1) = 0$, then

$$x_1(c_1^*) = \int_{c_1}^1 \frac{f_2(c_1;\alpha_2)}{c_1} dc_1 = \int_{c_1}^1 \frac{\alpha_2 c_1^{\alpha_2 - 1}}{c_1} dc_1 = \frac{\alpha_2}{\alpha_2 - 1}\left(1 - c_1^{\alpha_2 - 1}\right).$$

Then given c_1,the payoff of player 1 is

$$\begin{aligned} \pi_1(x_1,x_2;c_1) &= [1 - F_2(c_1)] - c_1 x_1(c_1) - I(\alpha_1) \\ &= 1 - c_1^{\alpha_2} - c_1 \frac{\alpha_2}{\alpha_2 - 1}\left(1 - c_1^{\alpha_2 - 1}\right) - \frac{5}{36}\left(\frac{1}{\alpha_1} - 1\right) \\ &= 1 - \frac{\alpha_2}{\alpha_2 - 1} c_1 + \frac{1}{\alpha_2 - 1} c_1^{\alpha_2} - \frac{5}{36}\left(\frac{1}{\alpha_1} - 1\right), \end{aligned}$$

and the expected payoff of player 1 is given by

$$\begin{aligned} E\pi_1^C(\alpha_1,\alpha_2) &= \int_0^1 [\pi_1(x_1,x_2;c_1)]\,dF(c_1) - I(\alpha_1) \\ &= \int_0^1 \left[1-\frac{\alpha_2}{\alpha_2-1}c_1+\frac{1}{\alpha_2-1}c_1^{\alpha_2}\right] d\alpha_1 c_1^{\alpha_1-1}dc_1 - \frac{5}{36}\left(\frac{1}{\alpha_1}-1\right) \\ &= \frac{\alpha_2}{(1+\alpha_1)(\alpha_1+\alpha_2)} - \frac{5}{36}\left(\frac{1}{\alpha_1}-1\right). \end{aligned}$$

The expected payoff of each player is the same regardless of the disclosure policy. And by taking first order condition with respect to α_1, we get

$$\frac{\partial E\pi_1(\alpha_1,\alpha_2)}{\partial\alpha_1} = \frac{\alpha_2}{1-\alpha_2}\left[\frac{1}{(1+\alpha_1)^2}-\frac{1}{(\alpha_2+\alpha_1)^2}\right]+\frac{5}{36\alpha_1^2}.$$

The symmetric equilibrium investment level is $\alpha_1=\alpha_2=\frac{1}{2}$ with expected payoff $\frac{7}{36}$. Figure 1 provides the shape of $E\pi_1\left(\alpha_1;\alpha_2=\frac{1}{2}\right)=\frac{1}{(1+\alpha_1)(2\alpha_1+1)}-\frac{5}{36}\left(\frac{1}{\alpha_1}-1\right)$.

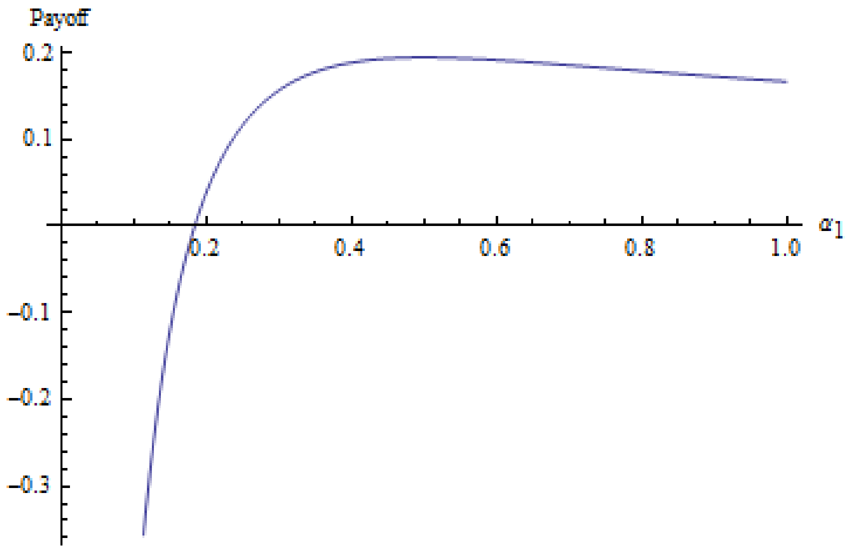

Figure 1: Expected payoff $E\pi_1\left(\alpha_1;\alpha_2=\frac{1}{2}\right)$

The symmetric equilibrium pre-contest investment level does exist in this two players all pay contests example.

5 Endogenous Entry

In this subsection, we further explore the role of disclosure policy in rent dissipation. We now consider strategic contestants. Instead of entering the contest with a fixed probability, each contestant makes his entry decision. We assume that entry incurs a fixed positive sunk cost Δ to each contestant, which is irreversible once entry decision has been made. One enters if and only if his expected payoff in the subsequent contest at least offsets the fixed cost.

The game then proceeds as follows. The contest organizer commits to and announces publicly her disclosure policy, D or C. Upon observing the disclosure rule, contestants simultaneously choose their entry strategies, and each participant sinks a fixed cost $\Delta > 0$ upon entry. The value of marginal cost is revealed if and only if the organizer committed to policy D. Participating contestants then simultaneously submit their effort entries.

5.1 Disclosure

Given the number of participants k, the equilibrium expected payoff of each contestant under disclosure policy is solved in section 2.1, where

$$E\pi_i^D(k) = \frac{1}{k} - \int_{\underline{c}}^{\overline{c}} c_i A(c_i, k)\, dF(c_i). \tag{15}$$

At the equilibrium entry probability $p \in [0, 1)$, every contestant is indifferent between participation and nonparticipation.

$$\pi(p) = \sum_{k=1}^{n} C_{n-1}^{k-1} p^{k-1} (1-p)^{n-k} E\pi_i^D(k) = \Delta. \tag{16}$$

5.2 Concealment

Following the methodology in section 2.2, for each participant i with marginal cost c_i, the probability of k contestants showing up is $C_{n-1}^{k-1} p^{k-1} (1-p)^{n-k}$, the payoff of player i is given as

$$\pi_i(x_i; k, c_i) = \sum_{k=1}^{n} C_{n-1}^{k-1} p^{k-1} (1-p)^{n-k} \left[1 - F\left(x_{-i}^{-1}(x_i)\right)\right]^{n-1} - c_i x_i. \tag{17}$$

At a symmetric equilibrium, $x_i(\cdot) = x_{-i}(\cdot) = x(\cdot)$. We thus have

$$\frac{dx(c_i)}{dc_i} = -\sum_{k=1}^{n} (k-1) C_{n-1}^{k-1} p^{k-1} (1-p)^{n-k} f(c_i) \left[1 - F(c_i)\right]^{n-2} / c_i.$$

Apply (*)[4]

$$A(c_i; k) = -(k-1) \int \frac{\left[1 - F(c_i)\right]^{k-2}}{c_i} dF(c_i).$$

Therefore, the equilibrium bid of each player is

$$\begin{aligned} x_i(c_i) &= \sum_{k=1}^{n} C_{n-1}^{k-1} p^{k-1} (1-p)^{n-k} A(c_i) \\ &= \sum_{k=1}^{n} (k-1) C_{n-1}^{k-1} p^{k-1} (1-p)^{n-k} \int_{c_i}^{\overline{c}} \frac{\left[1 - F(\widetilde{c})\right]^{n-2}}{\widetilde{c}} dF(\widetilde{c}). \end{aligned}$$

[4]Note that $A(\overline{c}) = 0$ since $F(\overline{c}) = 1$. And $x(\overline{c}) = 0$.

Given c_i, the expected payoff of each player is given by

$$\begin{aligned}\pi_i^C(x_i, \mathbf{x}_{-i}; c_i) &= \sum_{k=1}^{n} C_{n-1}^{k-1} p^{k-1}(1-p)^{n-k} \Pr(c_i < \widetilde{c})\, 1 - c_i x_i(c_i) \\ &= \sum_{k=1}^{n} C_{n-1}^{k-1} p^{k-1}(1-p)^{n-k} \left\{ [1 - F(c_i)]^{k-1} - c_i A(c_i; k) \right\}.\end{aligned}$$

Next, the expected payoff of each player under concealment policy is given by

$$E\pi_i^C = \sum_{k=1}^{n} C_{n-1}^{k-1} p^{k-1}(1-p)^{n-k} \left\{ \frac{1}{k} - \int_{\underline{c}}^{\overline{c}} c_i A(c_i; k)\, dF(c_i) \right\}. \tag{18}$$

At the equilibrium entry probability $p \in [0, 1)$, every contestant's expected payoff offsets his entry cost.

$$E\pi_i^C = \Delta. \tag{19}$$

Moreover, each potential contestant has 0 expected payoff at the equilibrium[5], and the expected total cost of effort can be written as

$$TEC(p) = [1 - (1-p)^n] - np\Delta. \tag{20}$$

where $n \geq 2$ is the number of potential contestants.

5.3 Comparison

Note that (16) and (19) take identical forms, then we get the equivalent equilibrium entry probability and from (20) we get the same rent dissipation rate. Given the same entry, however, hiding the information leads to higher effort according to the exogenous entry result. Therefore, contest sponsor still prefers to conceal the actual value of contestants' abilities to elicit higher total expected effort. Then we have the following result,

Theorem 6 *When entry is endogenized, both policies induce equal equilibrium entry probability $p(k)$ and the same dissipation rate, while concealing the abilities of contestants still elicits higher expected total revenue to contest sponsor.*

Theorem 5 and 6 strengthen the argument of Theorem 1 and 2. The results of Theorem 1 and 2 are robust even when an endogenous ability distribution or endogenous entry is allowed in the game. It further verifies that the expected payoff for each contestant and equilibrium level of total effort cost in the contest do not depend on whether the contest organizer disclose information, while concealing the abilities of the contestants elicits higher expected total effort, despite the endogeneity of ability distribution and entry.

[5] Please refer to Fu and Lu (2010) for detailed interpretation and proof.

6 Contests with Nonlinear Cost

In this section, we will check whether our result about disclosure policy is robust when cost function is nonlinear. A bid x_i costs a contestant $c(x_i)$, with $c'(\cdot) > 0$ and $c''(\cdot) \geq 0$. For the sake of tractability, we assume that player i's bidding cost function takes the form $c(x_i) = c_i x_i^{\beta}$. We just focus on the basic model with two contestants and a unique prize.

6.1 Disclosure

We assume at the point in time when the efforts are chosen, the contest organizer will disclose each player's unit cost to public; hence, this problem is a perfectly discriminating contest with complete information, the payoff to player i is given by

$$\pi(x_1, x_2; c_1, c_2) = P_i(x_1, x_2)\, 1 - c_i x_i^{\beta}.$$

Following the method outlined by Hillman and Riley (1999), the equilibrium of the perfectly discriminating contest for given values of c_1 and c_2 is unique and described as follows,

Proposition 4 *The unique equilibrium of the perfectly discriminating contest for given values of c_1 and c_2 $(c_1 < c_2)$ is in mixed strategies and bids are described by the following cumulative distribution functions:*

$$G_1(x_1) = \begin{cases} c_2 x_1^{\beta} & \text{for } x_1 \in [0, \left(\frac{1}{c_2}\right)^{\frac{1}{\beta}}), \\ 1 & \text{for } x_1 \geq \left(\frac{1}{c_2}\right)^{\frac{1}{\beta}}; \end{cases}$$

$$G_2(x_2) = \begin{cases} 1 - \frac{c_1}{c_2} + c_1 x_2^{\beta} & \text{for } x_2 \in [0, \left(\frac{1}{c_2}\right)^{\frac{1}{\beta}}), \\ 1 & \text{for } x_2 \geq \left(\frac{1}{c_2}\right)^{\frac{1}{\beta}}. \end{cases}$$

The expected value of bids for bidder 1 and bidder 2 are

$$Ex_1 = \int_0^{\left(\frac{1}{c_2}\right)^{\frac{1}{\beta}}} \beta c_2 x_1^{\beta} dx_1 = \frac{\beta}{\beta+1}\left(\frac{1}{c_2}\right)^{\frac{1}{\beta}},$$

$$Ex_2 = 0 \times \Pr(x_2 = 0) + \int_{0^+}^{\left(\frac{1}{c_2}\right)^{\frac{1}{\beta}}} \beta c_1 x_2^{\beta-1} x_2 dx_2 = \frac{\beta c_1}{\beta+1}\left(\frac{1}{c_2}\right)^{\frac{\beta+1}{\beta}}.$$

Therefore, with general cumulative distribution function $F(c_i)$ and $c_i \in [\underline{c}, \overline{c}]$, the total expected effort for contest sponsor is

$$\begin{aligned}
R^D &= 2\int\int_{c_1<c_2} \left[Ex_1 + Ex_2\right] dF\left(c_1\right) dF\left(c_2\right) \\
&= 2\int_{\underline{c}}^{\overline{c}}\int_{\underline{c}}^{c_2} \left[\frac{\beta}{\beta+1}\left(\frac{1}{c_2}\right)^{\frac{1}{\beta}} + \frac{\beta c_1}{\beta+1}\left(\frac{1}{c_2}\right)^{\frac{\beta+1}{\beta}}\right] dF\left(c_1\right) dF\left(c_2\right) \\
&= \frac{4\beta}{\beta+1}\int_{\underline{c}}^{\overline{c}}\left(\frac{1}{c_2}\right)^{\frac{1}{\beta}} F\left(c_2\right) dF\left(c_2\right) - \frac{2\beta}{\beta+1}\int_{\underline{c}}^{\overline{c}}\left[\left(\frac{1}{c_2}\right)^{\frac{\beta+1}{\beta}}\left(\int_{\underline{c}}^{c_2} F\left(c_1\right) dc_1\right)\right] dF\left(c_2\right).
\end{aligned} \tag{21}$$

6.2 Concealment

In this subsection, we assume that at the point in time when the efforts are chosen, the contest organizer will conceal information regarding player's unit cost, and each player does not know the rival player's unit cost; hence, at this stage, the problem describes a perfectly discriminating contest with incomplete information.

The payoffs of the player is given as

$$\begin{aligned}
\pi_i\left(x_1, x_2, c_1, c_2\right) &= p_i\left(x_i^{'} > x_j\right) 1 - c_i x_i^{\beta}\left(c_i^{'}\right) \\
&= p_i\left(c_i^{'} < c_j\right) 1 - c_i x_i^{\beta}\left(c_i^{'}\right) \\
&= \left[1 - F\left(c_i^{'}\right)\right] - c_i x_i^{\beta}\left(c_i^{'}\right).
\end{aligned} \tag{22}$$

Note that $x_i\left(c_i\right)$ is decreasing with c_i,thus the higher the cost, the lower the effort. Take first order condition with respect to c_i we obtain

$$\frac{\partial \pi_i}{\partial c_i^{'}} = -f\left(c_i^{'}\right) - c_i \frac{dx_i^{\beta}\left(c_i^{'}\right)}{dc_i^{'}},$$

when $c_i^{'} = c_i$, $\frac{\partial \pi_i}{\partial c_i^{'}} = 0$. Hence

$$\frac{dx_i\left(c_i\right)}{dc_i} = \frac{-f\left(c_i\right)}{\beta c_i x_i^{\beta-1}}.$$

Note that $x_i\left(\overline{c}\right) = 0$, therefore, the individual equilibrium effort is given by

$$x_i\left(c_i\right) = \left[\int_{c_i}^{\overline{c}} \frac{f\left(c_i\right)}{c_i} dc_i\right]^{\frac{1}{\beta}}.$$

The total expected effort for contest sponsor is

$$\begin{aligned}
R^C &= 2\int_{\underline{c}}^{\overline{c}} x_i\left(c_i\right) dF\left(c_i\right) \\
&= 2\int_{\underline{c}}^{\overline{c}}\left(\int_{c_1}^{\overline{c}} \frac{1}{c_2} dF\left(c_2\right)\right)^{\frac{1}{\beta}} dF\left(c_1\right).
\end{aligned} \tag{23}$$

6.3 Optimal Disclosure Policy

We now compare (21) and (23) to investigate the effort maximizing disclosure policy.

Compare R^D and R^C,

$$\begin{aligned} R^D &= \frac{4\beta}{\beta+1}\int_{\underline{c}}^{\overline{c}} \left(\frac{1}{c_2}\right)^{\frac{1}{\beta}} F(c_2)\,dF(c_2) \\ &\quad -\frac{2\beta}{\beta+1}\int_{\underline{c}}^{\overline{c}} \left[\left(\frac{1}{c_2}\right)^{\frac{\beta+1}{\beta}} \left(\int_{\underline{c}}^{c_2} F(c_1)\,dc_1\right)\right] dF(c_2) \\ &= \frac{2\beta}{\beta+1}\int_{\underline{c}}^{\overline{c}} \left\{2\left(\frac{1}{c_2}\right)^{\frac{1}{\beta}} F(c_2) - \left(\frac{1}{c_2}\right)^{\frac{\beta+1}{\beta}} \left(\int_{\underline{c}}^{c_2} F(c_1)\,dc_1\right)\right\} dF(c_2), \end{aligned} \tag{21}$$

$$\begin{aligned} R^C &= 2\int_{\underline{c}}^{\overline{c}} \left(\int_{c_1}^{\overline{c}} \frac{1}{c_2} dF(c_2)\right)^{\frac{1}{\beta}} dF(c_1) \\ &= 2\int_{\underline{c}}^{\overline{c}} \left(\underset{c_1<c_2}{E}\frac{1}{c_2}\right)^{\frac{1}{\beta}} dF(c_1). \end{aligned} \tag{23}$$

With linear cost $c(x_i) = c_i x_i$, we have shown that $R^C > R^D$ in section 2.

However, it is possible that at some level where $\beta \neq 1$, $R^C < R^D$.

Example 1 *In a two player perfectly discriminating contest, when $F(c) = c$, and $c \in [1, 2]$, with either convex cost $c(x_i) = c_i x_i^2$ or concave cost $c(x_i) = c_i x_i^{0.5}$, disclosing the abilities of contestants elicits higher total expected revenue to contest sponsor.*

Proof. When $F(c) = c, c \in [1, 2]$,

(1) With convex cost $\beta = 2$,

$$\begin{aligned} R^D &= \frac{4}{3}\int_1^2 \left\{2\left(\frac{1}{c_2}\right)^{\frac{1}{2}} c_2 - \left(\frac{1}{c_2}\right)^{\frac{3}{2}} \left(\int_1^{c_2} c_1 dc_1\right)\right\} dc_2 \\ &= \frac{4}{3}\left(2^{\frac{3}{2}} - 2^{-\frac{1}{2}}\right) = 2.8284, \end{aligned}$$

$$R^C = 2\int_1^2 \left(\int_{c_1}^2 \frac{1}{c_2} dc_2\right)^{\frac{1}{2}} dc_1 = 1.0325.$$

(2) With concave cost $\beta = \frac{1}{2}$,

$$\begin{aligned} R^D &= \frac{2}{3}\int_1^2 \left\{2\left(\frac{1}{c_2}\right)^{2} c_2 - \left(\frac{1}{c_2}\right)^{3} \left(\int_1^{c_2} c_1 dc_1\right)\right\} dc_2 \\ &= \ln 2 + \frac{1}{8} = 0.8182, \end{aligned}$$

$$\begin{aligned} R^C &= 2\int_1^2 \left(\int_{c_1}^2 \frac{1}{c_2} dc_2\right)^2 dc_1 \\ &= 0.2665. \end{aligned}$$

Then $R^D > R^C$ in both cases. ■

The following theorem regarding nonlinear cost is therefore obvious,

Theorem 7 *The optimal disclosure policy depends on convexity of cost functions, when cost function is nonlinear, disclosing the abilities of contestants may elicit higher total expected revenue.*

Theorem 7 reveals that the nonlinearity of effort cost function does affect the optimal disclosure policy of contest organizer. In the above analysis, we adopted a power form effort cost function. The analysis shows that the form of the cost function plays a pivotal role in determining the optimal disclosure policy of the contest organizer. With a linear effort cost function, a concealment policy leads to the best outcome in terms of expected aggregate effort, although this need not to be true when effort cost is nonlinear. Hence the effect of optimal disclosure policy is ambiguous and no general results can be obtained to guide contest organizer.

7 Conclusion

This paper investigates the optimal disclosure policy of the contest organizer in a perfectly discriminating contest. The private abilities of the contestants are stochastic and they are observed by the contest organizer who decides whether to disclose this information publicly. The organizer may care about total effort or rent dissipation. We find that in a benchmark model with unique prize and linear effort cost, concealing the abilities of the contestants elicits higher expected total effort, regardless of the distribution of the abilities. For rent dissipation, we find that the rent dissipation rate does not depend on the disclosure policy. We then develop these results in the context of a tractable two-prize model. While we believe that our main insights are robust in settings with multiple prizes as long as effort cost function is linear, we leave the analysis of more generalized environments to future work.

We further study the robustness of our results while allowing endogenous ability distribution and endogenous entry of contestants. We find that our findings are robust to these generalized settings. Another natural extension of our model would be to allow the cost function to be nonlinear. However, the analysis of this extension would depend critically on the cost function form. The finding from the benchmark model is not robust, and the organizer may prefer disclosure instead.

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Appendix

Proof of Lemma 1

Proof. According to the bidding strategy, the expected value of bids for bidder 1 and bidder 2 are

$$Ex_1 = \int_0^{\frac{1}{c_2}} c_2 x_1 dx_1 = \frac{1}{2c_2},$$

$$Ex_2 = 0 \times \Pr(x_2 = 0) + \int_{0^+}^{\frac{1}{c_2}} c_1 x_2 dx_2 = \frac{c_1}{2c_2^2}.$$

And for all the other remaining $n-2$ players, their marginal costs are above c_2, they will remain passive and exert zero effort.

In a model with n players, there are $n(n-1)$ cases that two of their marginal costs are ranked as the lowest and second lowest. Therefore, with general cumulative distribution function $F(c_i)$ with $c_i \in [\underline{c}, \overline{c}]$, the total expected effort for contest sponsor is given as

$$\begin{aligned}
R^D &= n(n-1)\int\int\cdots\int [Ex_1 + Ex_2]\, dF(c_n)\cdots dF(c_3)\, dF(c_2)\, dF(c_1) \\
&= n(n-1)\int_{\underline{c}}^{\overline{c}}\int_{c_1}^{\overline{c}}\left\{\int_{c_2}^{\overline{c}}\cdots\int_{c_2}^{\overline{c}}\left[\frac{1}{2c_2}+\frac{c_1}{2c_2^2}\right] dF(c_n)\cdots dF(c_3)\right\} \\
&\quad dF(c_2)\, dF(c_1) \\
&= n(n-1)\int_{\underline{c}}^{\overline{c}}\int_{c_1}^{\overline{c}}\left[\frac{1}{2c_2}+\frac{c_1}{2c_2^2}\right][1-F(c_2)]^{n-2} dF(c_2)\, dF(c_1).
\end{aligned}$$

■

Proof of Lemma 2

Proof. Note that $x_i(c_i)$ is decreasing with c_i, the higher cost, the lower effort. Take first order condition with respect to c_i,

$$\begin{aligned}
\frac{\partial \pi_i}{\partial \tilde{x}_i} &= -(n-1)\left[1-F\left(x_{-i}^{-1}(\tilde{x}_i)\right)\right]^{n-2} f\left(x_{-i}^{-1}(x_i')\right)\frac{dx_{-i}^{-1}(\tilde{x}_i)}{d\tilde{x}_i} - c_i \\
&= -(n-1)\left[1-F\left(x_{-i}^{-1}(\tilde{x}_i)\right)\right]^{n-2} f\left(x_{-i}^{-1}(x_i')\right)[x'_{-i}(x_{-i}^{-1}(\tilde{x}_i))]^{-1} - c_i.
\end{aligned}$$

Given $x_{-i}(\cdot)$, $x_i(\cdot)$ is the optimal strategy of i, we must have

$$\begin{aligned}
&\frac{\partial \pi_i}{\partial \tilde{x}_i}|_{\tilde{x}_i = x_i(c_i)} \\
&= -(n-1)\left[1-F\left(x_{-i}^{-1}(x_i(c_i))\right)\right]^{n-2} f\left(x_{-i}^{-1}(x_i(c_i))\right)[x'_{-i}(x_{-i}^{-1}(x_i(c_i)))]^{-1} - c_i \\
&= 0.
\end{aligned}$$

At a symmetric equilibrium, $x_i(\cdot) = x_{-i}(\cdot) = x(\cdot)$. We thus have

$$\frac{dx(c_i)}{dc_i} = \frac{-(n-1)f(c_i)[1-F(c_i)]^{n-2}}{c_i}.$$

Therefore, the equilibrium bid of each player is

$$x_i(c_i) = (n-1)\int_{c_i}^{\overline{c}} \frac{[1-F(\widetilde{c})]^{n-2}}{\widetilde{c}} dF(\widetilde{c}).$$

Then the total expected effort for contest sponsor is

$$\begin{aligned}
R^C &= n\int_{\underline{c}}^{\overline{c}} x_i(c_i)\, dF(c_i) \\
&= n\int_{\underline{c}}^{\overline{c}} (n-1)\left\{\int_{c_1}^{\overline{c}} \frac{[1-F(c_2)]^{n-2}}{c_2} dF(c_2)\right\} dF(c_1) \\
&= n(n-1)\int_{\underline{c}}^{\overline{c}} \left\{\left[\int_{\underline{c}}^{c_2} dF(c_1)\right] \frac{[1-F(c_2)]^{n-2}}{c_2}\right\} dF(c_2) \\
&= n(n-1)\int_{\underline{c}}^{\overline{c}} \frac{F(c_2)[1-F(c_2)]^{n-2}}{c_2} dF(c_2).
\end{aligned}$$

■

Proof of Theorem 1

Proof. Recall (1) and (4), compare R^D and R^C,

$$\begin{aligned}
R^D &= n(n-1)\int_{\underline{c}}^{\overline{c}} \int_{c_1}^{\overline{c}} \left[\frac{1}{2c_2} + \frac{c_1}{2c_2^2}\right][1-F(c_2)]^{n-2}\, dF(c_2)\, dF(c_1) \\
&= n(n-1)\int_{\underline{c}}^{\overline{c}} \frac{[1-F(c_2)]^{n-2}}{c_2} F(c_2)\, dF(c_2) \\
&\quad -\frac{n(n-1)}{2}\int_{\underline{c}}^{\overline{c}} \left[\frac{1}{c_2^2}\int_{\underline{c}}^{c_2} F(c_1)\, dc_1\right][1-F(c_2)]^{n-2}\, dF(c_2) \\
&= \frac{n(n-1)}{2}\left\{\begin{array}{c} \int_{\underline{c}}^{\overline{c}} \frac{[1-F(c_2)]^{n-2}}{c_2} F(c_2)\, dF(c_2) \\ +\int_{\underline{c}}^{\overline{c}} \left[\frac{1}{c_2^2}\int_{\underline{c}}^{c_2} c_1 dF(c_1)\right][1-F(c_2)]^{n-2}\, dF(c_2) \end{array}\right\},
\end{aligned}$$

$$R^C = n(n-1)\int_{\underline{c}}^{\overline{c}} \frac{F(c_2)[1-F(c_2)]^{n-2}}{c_2} dF(c_2).$$

Since

$$
\begin{aligned}
& \frac{2}{n\left(n-1\right)}\left(R^{D}-R^{C}\right) \\
= & \int_{\underline{c}}^{\overline{c}}\left[\frac{1}{c_2^2}\int_0^{c_2} c_1 dF\left(c_1\right)\right]\left[1-F\left(c_2\right)\right]^{n-2} dF\left(c_2\right) \\
& -\int_{\underline{c}}^{\overline{c}}\frac{F\left(c_2\right)\left[1-F\left(c_2\right)\right]^{n-2}}{c_2} dF\left(c_2\right) \\
= & \int_{\underline{c}}^{\overline{c}}\frac{1}{c_2}\left[\frac{1}{c_2}\int_0^{c_2} c_1 dF\left(c_1\right)-F\left(c_2\right)\right]\left[1-F\left(c_2\right)\right]^{n-2} dF\left(c_2\right) \\
= & \int_{\underline{c}}^{\overline{c}}\frac{1}{c_2}\left[-\frac{1}{c_2}\int_{\underline{c}}^{c_2} F\left(c_1\right) dc_1\right]\left[1-F\left(c_2\right)\right]^{n-2} dF\left(c_2\right) \\
= & -\int_{\underline{c}}^{\overline{c}}\frac{1}{c_2^2}\left[\int_0^{c_2} F\left(c_1\right) dc_1\right]\left[1-F\left(c_2\right)\right]^{n-2} dF\left(c_2\right)<0,
\end{aligned}
$$

then $R^D < R^C$. ■

Proof of Theorem 2

Proof. Recall (2), the expected payoff of player i under disclosure policy is

$$
E\pi_i^D=\int_{\underline{c}}^{\overline{c}}\left\{\int_{c_i}^{\overline{c}}\left[1-\frac{c_i}{\widetilde{c}}\right] d\left[1-\left[1-F\left(\widetilde{c}\right)\right]^{n-1}\right]\right\} dF\left(c_i\right).
$$

Then given c_i, the expected payoff of each player is

$$
\begin{aligned}
& \pi_i^D\left(x_i;c_i\right) \\
= & \int_{c_i}^{\overline{c}}\left[1-\frac{c_i}{\widetilde{c}}\right] d\left[1-\left[1-F\left(\widetilde{c}\right)\right]^{n-1}\right] \\
= & \left(n-1\right)\int_{c_i}^{\overline{c}}\left[1-\frac{c_i}{\widetilde{c}}\right]\left[1-F\left(\widetilde{c}\right)\right]^{n-2} dF\left(\widetilde{c}\right) \\
= & \left(n-1\right)\int_{c_i}^{\overline{c}}\left[1-F\left(\widetilde{c}\right)\right]^{n-2} dF\left(\widetilde{c}\right)-\left(n-1\right)c_i\int_{c_i}^{\overline{c}}\frac{\left[1-F\left(\widetilde{c}\right)\right]^{n-2}}{\widetilde{c}} dF\left(\widetilde{c}\right) \\
= & \left[1-F\left(c_i\right)\right]^{n-1}+\left[A\left(\overline{c}\right)-A\left(c_i\right)\right]c_i \\
= & \left[1-F\left(c_i\right)\right]^{n-1}-c_iA\left(c_i\right) \quad \text{since } A\left(\overline{c}\right)=0, F\left(\overline{c}\right)=1 \text{ by definition (*).}
\end{aligned}
$$

Therefore the equilibrium expected payoff of each player under disclosure policy is given by

$$
\begin{aligned}
E\pi_i^D &= \int_{\underline{c}}^{\overline{c}}\left[\left[1-F\left(c_i\right)\right]^{n-1}-c_iA\left(c_i\right)\right] dF\left(c_i\right) \\
&= \int_{\underline{c}}^{\overline{c}}\left[1-F\left(c_i\right)\right]^{n-1} dF\left(c_i\right)-\int_{\underline{c}}^{\overline{c}} c_iA\left(c_i\right) dF\left(c_i\right) \\
&= \frac{1}{n}-\int_{\underline{c}}^{\overline{c}} c_iA\left(c_i\right) dF\left(c_i\right).
\end{aligned}
$$

Recall (5), the expected payoff under concealment policy is given by

$$
\begin{aligned}
E\pi_i^C &= \int_{\underline{c}}^{\overline{c}} \left\{ [1-F(c_i)]^{n-1} - c_i x_i(c_i) \right\} dF(c_i) \\
&= \frac{1}{n} - \int_{\underline{c}}^{\overline{c}} c_i x(c_i)\, dF(c_i).
\end{aligned}
$$

Note that

$$
\begin{aligned}
x_i(c_i) &= (n-1) \int_{c_i}^{\overline{c}} \frac{[1-F(\widetilde{c})]^{n-2}}{\widetilde{c}} dF(\widetilde{c}) \\
&= -[A(\overline{c}) - A(c_i)] = A(c_i),
\end{aligned}
$$

therefore,

$$
E\pi_i^D = E\pi_i^C.
$$

■

Proof of Corollary 2

Proof. The expected value of bids for player 1, 2 and 3 are

$$
\begin{aligned}
Ex_1 &= \frac{1}{c_3}\left(1 - \frac{c_1}{3c_2}\right), \\
Ex_2 &= \frac{1}{2c_3}\left(1 + \frac{c_1^2}{3c_2^2}\right), \\
Ex_3 &= \frac{1}{c_3^2}\left(\frac{c_1^2}{6c_2} + \frac{c_2}{2}\right).
\end{aligned}
$$

And for all the other remaining $n-3$ players, their marginal costs are above c_2, they will remain passive and exert zero effort.

In a model with n players, there are $n(n-1)(n-2)$ cases that three of them are the three players with the lowest cost. Therefore, the total expected effort for contest sponsor is given as

$$
\begin{aligned}
R^D &= n(n-1)(n-2) \int\int\cdots\int [Ex_1 + Ex_2 + Ex_3]\, dF(c_n)\cdots dF(c_3) \\
&\quad dF(c_2)\, dF(c_1) \\
&= n(n-1)(n-2) \int_{\underline{c}}^{\overline{c}} \int_{c_1}^{\overline{c}} \int_{c_2}^{\overline{c}} \\
&\quad \{ \int_{c_3}^{\overline{c}} \cdots \int_{c_3}^{\overline{c}} \left[\frac{1}{c_3}\left(1 - \frac{c_1}{3c_2}\right) + \frac{1}{2c_3}\left(1 + \frac{c_1^2}{3c_2^2}\right) + \frac{1}{c_3^2}\left(\frac{c_1^2}{6c_2} + \frac{c_2}{2}\right) \right] \\
&\quad dF(c_n)\cdots dF(c_4) \} dF(c_3)\, dF(c_2)\, dF(c_1) \\
&= n(n-1)(n-2) \\
&\quad \int_{\underline{c}}^{\overline{c}} \int_{c_1}^{\overline{c}} \int_{c_2}^{\overline{c}} \left[\left(\frac{3}{2} - \frac{c_1}{3c_2} + \frac{c_1^2}{6c_2^2}\right)\frac{1}{c_3} + \left(c_2 + \frac{c_1^2}{3c_2}\right)\frac{1}{2c_3^2} \right] \\
&\quad [1-F(c_3)]^{n-3}\, dF(c_3)\, dF(c_2)\, dF(c_1).
\end{aligned}
$$

■

Proof of Theorem 3

Proof. Recall (6)

$$\begin{aligned} R^{D} &= n(n-1)(n-2) \\ &\int_{\underline{c}}^{\overline{c}}\int_{c_1}^{\overline{c}}\int_{c_2}^{\overline{c}}\left[\left(\frac{3}{2}-\frac{c_1}{3c_2}+\frac{c_1^2}{6c_2^2}\right)\frac{1}{c_3}+\left(c_2+\frac{c_1^2}{3c_2}\right)\frac{1}{2c_3^2}\right] \\ &[1-F(c_3)]^{n-3}\,dF(c_3)\,dF(c_2)\,dF(c_1). \end{aligned}$$

Under concealment, the equilibrium bid

$$\begin{aligned} x_i(c) &= A(c)+B(c) \\ &= (n-1)\left\{\begin{array}{c} \int_c^{\overline{c}}\frac{1}{c_i}[1-F(c_i)]^{n-2}\,dF(c_i) \\ +\int_c^{\overline{c}}\frac{1}{c_i}[1-F(c_i)]^{n-3}[(n-1)F(c_i)-1]\,dF(c_i) \end{array}\right\} \\ &= (n-1)(n-2)\int_c^{\overline{c}}\frac{F(c_i)}{c_i}[1-F(c_i)]^{n-3}\,dF(c_i). \end{aligned}$$

Then the total expected effort for contest sponsor is

$$\begin{aligned} R^{C} &= n\int_{\underline{c}}^{\overline{c}}x_i(c)\,dF(c) \\ &= n(n-1)(n-2)\int_{\underline{c}}^{\overline{c}}\left\{\int_c^{\overline{c}}\frac{F(c_i)}{c_i}[1-F(c_i)]^{n-3}\,dF(c_i)\right\}dF(c). \end{aligned}$$

It is sufficient to forget the coefficient before integration. By swapping integrations,

$$\begin{aligned} R^{\prime C} &= \int_{\underline{c}}^{\overline{c}}\left\{\int_c^{\overline{c}}\frac{F(c_i)}{c_i}[1-F(c_i)]^{n-3}\,dF(c_i)\right\}dF(c) \\ &= \int_{\underline{c}}^{\overline{c}}\left[\int_{\underline{c}}^{c_i}dF(c)\right]\frac{F(c_i)}{c_i}[1-F(c_i)]^{n-3}\,dF(c_i) \\ &= \int_{\underline{c}}^{\overline{c}}\frac{F^2(c_i)}{c_i}[1-F(c_i)]^{n-3}\,dF(c_i). \end{aligned}$$

In addition

$$\begin{aligned} R^{\prime D} &= \int_{\underline{c}}^{\overline{c}}\int_{c_1}^{\overline{c}}\int_{c_2}^{\overline{c}}\left[\left(\frac{3}{2}-\frac{c_1}{3c_2}+\frac{c_1^2}{6c_2^2}\right)\frac{1}{c_3}+\left(c_2+\frac{c_1^2}{3c_2}\right)\frac{1}{2c_3^2}\right] \\ &\quad [1-F(c_3)]^{n-3}\,dF(c_3)\,dF(c_2)\,dF(c_1) \\ &= \int_{\underline{c}}^{\overline{c}}\frac{[1-F(c_3)]^{n-3}}{c_3}dF(c_3)\int_{\underline{c}}^{c_3}\int_{\underline{c}}^{c_2}\left(\frac{3}{2}-\frac{1}{3}\frac{c_1}{c_2}+\frac{1}{6}\left(\frac{c_1}{c_2}\right)^2\right)dF(c_1)\,dF(c_2) \\ &\quad +\int_{\underline{c}}^{\overline{c}}\frac{[1-F(c_3)]^{n-3}}{c_3}dF(c_3)\int_{\underline{c}}^{c_3}\int_{\underline{c}}^{c_2}\left(c_2+\frac{c_1^2}{3c_2}\right)\frac{1}{2c_3}dF(c_1)\,dF(c_2) \\ &< \int_{\underline{c}}^{\overline{c}}\frac{[1-F(c_3)]^{n-3}}{c_3}dF(c_3)\int_{\underline{c}}^{c_3}\int_{\underline{c}}^{c_2}\left(\frac{3}{2}-\frac{1}{3}+\frac{1}{6}\right)dF(c_1)\,dF(c_2) \\ &\quad +\int_{\underline{c}}^{\overline{c}}\frac{[1-F(c_3)]^{n-3}}{c_3}dF(c_3)\int_{\underline{c}}^{c_3}\int_{\underline{c}}^{c_2}\frac{2}{3}dF(c_1)\,dF(c_2) \\ &= \frac{1}{2}\left(\frac{4}{3}+\frac{2}{3}\right)\int_{\underline{c}}^{\overline{c}}\frac{F^2(c_3)}{c_3}[1-F(c_3)]^{n-3}\,dF(c_3)=R^{\prime C}, \end{aligned}$$

where the first inequality is derived as follows: by assumption $\underline{c} \leq c_1 < c_2 < c_3 \leq \overline{c}$, this easily implies that

$$\frac{3}{2} - \frac{1}{3}\frac{c_1}{c_2} + \frac{1}{6}\left(\frac{c_1}{c_2}\right)^2 < \frac{3}{2} - \frac{1}{3} + \frac{1}{6} = \frac{4}{3},$$

$$\left(c_2 + \frac{c_1^2}{3c_2}\right)\frac{1}{2c_3} < \frac{c_2}{2c_3} + \frac{c_1^2}{6c_2c_3} < \frac{1}{2} + \frac{1}{6} = \frac{2}{3},$$

and the last equality is derived by

$$\int_{\underline{c}}^{c_3}\int_{\underline{c}}^{c_2} dF(c_1)\,dF(c_2) = \int_{\underline{c}}^{c_3} F(c_2)\,dF(c_2) = \frac{1}{2}F^2(c_3)$$

Therefore $R'^D < R'^C$, and we can conclude $R^D < R^C$. ■

Proof of Lemma 3

Proof. Recall (11), given c_i,the conditional expected payoff of each player under disclosure policy is given by

$$\begin{aligned}
\pi_i^D(c_i) &= \int_{c_i}^{\overline{c}}\left[1 - \frac{c_i}{c}\right] dH(c) \\
&= \int_{c_i}^{\overline{c}} dH(c) - c_i\int_{c_i}^{\overline{c}}\frac{1}{c}dH(c) \\
&= H(\overline{c}) - H(c_i) - c_i\int_{c_i}^{\overline{c}}\frac{1}{c}dH(c) \\
&= 1 - H(c_i) - c_i\int_{c_i}^{\overline{c}}\frac{1}{c}dH(c).
\end{aligned}$$

Recall (12), given c_i,the conditional expected payoff of each player under concealment policy is given by

$$\begin{aligned}
\pi_i^C(c_i) &= 1 - H(c_i) - c_i x_i(c_i) \\
&= 1 - H(c_i) - c_i\sum_{s=1}^{m}\int_{c_i}^{\overline{c}} -\frac{1}{c}F_s^{'}(c)\,dc \\
&= 1 - H(c_i) - c_i\int_{c_i}^{\overline{c}}\frac{1}{c}d\left[-\sum_{s=1}^{m}F_s(c)\right].
\end{aligned}$$

Since

$$\begin{aligned}
\sum_{s=1}^{m}F_s(c) &= \sum_{s=1}^{m}\frac{(n-1)!}{(s-1)!\,(n-s)!}\times[1-F(c)]^{n-s}[F(c)]^{s-1} \text{ let } s = j+1 \\
&= \sum_{j=0}^{m-1}\frac{(n-1)!}{j!\,(n-1-j)!}F(c)^j[1-F(c)]^{n-1-j} \\
&= \sum_{j=0}^{m-1}C_{n-1}^{j}F(c)^j[1-F(c)]^{n-1-j}.
\end{aligned}$$

Note that

$$H(c)=\sum_{j=m}^{n-1}C_{n-1}^{j}F(c)^{j}\left[1-F(c)\right]^{n-1-j}\ ,$$

and

$$\begin{aligned}&\sum_{j=0}^{n-1}C_{n-1}^{j}F(c)^{j}\left[1-F(c)\right]^{n-1-j}\\ =&\sum_{j=0}^{m-1}C_{n-1}^{j}F(c)^{j}\left[1-F(c)\right]^{n-1-j}+\sum_{j=m}^{n-1}C_{n-1}^{j}F(c)^{j}\left[1-F(c)\right]^{n-1-j}\\ =&\ 1,\end{aligned}$$

then

$$\sum_{s=1}^{m}F_{s}(c)+H(c)=1,$$

therefore

$$dH(c)=d\left[1-\sum_{s=1}^{m}F_{s}(c)\right]=d\left[-\sum_{s=1}^{m}F_{s}(c)\right].$$

We can conclude $\pi_i^D(c_i)=\pi_i^C(c_i)$. ■