On the Duration of Technology Licensing

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Abstract

We model an innovator’s choice of payment scheme and duration as a joint decision in a multi-period licensing game with potential sequential innovations and some irreversibility of technology transfer. We find that it may be optimal to license the innovation for less than the full length of the patent and that royalty contracts can be more profitable than fixed-fee licensing even in the absence of information asymmetry and risk aversion. Moreover, licensing contracts based on royalty have a longer duration than fixed-fee licenses and are more likely to be used in industries where sequential innovations are frequent and intellectual property protection is weak. Our paper also highlights an important link between the study of technology licensing and the theory of durable goods. (JEL D86, L13, L24)

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Technology transfer through licensing is a common method to utilize a patent. A large literature on technology licensing has studied the optimal payment scheme of selling a cost-reducing innovation (Arrow 1962, Kamien and Tauman 1984, 1986, 2002, Katz and Shapiro 1986, Kamien, Oren and Tauman 1992; see Kamien 1992 for a survey). It has been shown that licensing by means of a royalty is inferior to that of a fixed-fee or an auction for an outside innovator, regardless of the industry size or the magnitude of the innovation.

Subsequent studies have tried to explain the wide prevalence of royalties in practice by examining the many variants of the standard model. These studies include models with asymmetric information (Gallini and Wright 1990; Macho-Stadler and Pérez-Castrillo, 1991; Beggs, 1992; Poddar and Sinha, 2002; Sen, 2005), variation in the quality of innovation (Rockett, 1990), product differentiation (Muto, 1993; Wang and Yang, 1999; Poddar and Sinha, 2004; Stamatopoulos and Tauman, 2003), moral hazard (Macho-Stadler, Martinez-Giralt and Perez-Castrillo, 1996; Choi, 2001; Jensen and Thursby, 2001), risk aversion (Bousquet, Cremer, Ivaldi and Wolkowicz, 1998), incumbent innovator (Shapiro, 1985; Wang, 1998, 2002; Kamien and Tauman, 2002; Sen, 2002; Sen and Tauman, 2003), Stackelberg leadership (Filippini, 2001; Kabiraj, 2004, 2005) or strategic delegation (Saracho, 2002).

However, surprisingly few studies have examined the duration of technology licensing, even though it is an important dimension of every contract. More concretely, should the innovator license the innovation for the entire length of the patent, or should a series of short-term contracts be used? While existing theoretical models implicitly assume that a license remains in effect for the duration of the patent, most actual contract agreements terminate before the underlying patents expire. Anand and Khanna (2000) study the structure of licensing contracts that involved at least one US participant and were signed during the period 1990-93. They find that no contract agreement lasts more than 10 years, even though the length of patent protection ranges from 14 to 20 years in the US.

A more interesting fact is the variation in the duration of licensing contracts. Macho-Stadler, Martinez-Giralt and Perez-Castrillo (1996) study a sample of 241 contracts between
Spanish and foreign firms and find that contracts based on royalties tend to have a longer duration than fixed-fee contracts. Of the contracts containing fixed payments, 24.5% are one-year contracts, while this proportion falls to 6.2% in the set of contracts containing royalty payments. At the other extreme, 58% of the 174 contracts with royalty payments are long-term contracts (at least five years), while only 15% of the contracts with fixed payments had a duration of at least five years. Using the same dataset, Mendi (2005) studies the impact of contract duration in determining scheduled payments in technology transfer. He finds a positive relationship between contract duration and the probability of the parties including royalties in the first period of the agreement.\footnote{The author also provides a theoretical model to explain the finding, but it differs from ours in many aspects, among which the most crucial is that the duration of contract is exogenous in his but not in ours.}

In this paper, we introduce a model of technology licensing that analyzes the duration of contracts as well as the optimal payment scheme. We focus on an outsider innovator’s optimal licensing policy in a setting with potential sequential innovations and some irreversibility of technology transfers. We find that it can be optimal to issue a license for less than the length of the patent; even in the absence of information asymmetry and risk aversion, royalty can be more profitable than fixed-fee licensing. Moreover, licensing contracts based on royalty tend to have a longer duration and are more likely to be used in industries where sequential innovations are frequent and intellectual property protection is weak. These findings are broadly consistent with the stylized empirical facts.\footnote{The empirical support for the finding that relates the use of royalty to sequential innovations and intellectual property protection is discussed in Section IV.D.}

Our model builds on two observations. First, technology advances are destructive. A new innovation often renders past ones obsolete. This means that an innovator who engages in a series of innovations potentially faces a time-consistency problem in technology licensing: once a license is sold, the innovator may have an excessive incentive to invest in new technologies. This decreases the value of the initial license. At the same time, it may be too costly to write a complete long-term contract in which license fees are contingent upon...
the outcome of risky investments for future improvements (Williamson 1975). Therefore, a long-term fixed-fee license may be sub-optimal.

Second, the transfer of knowledge is irreversible. Once transferred, it is difficult for the innovator to retract the knowledge from a licensee (Caves, Crookell and Killing 1983; Brousseau, Coeurderoy and Chaserant 2007). This means that a licensee may be able to utilize an innovation even after the license has expired. We call this "technology leakage" and model it as the licensee retaining a fraction of the cost savings of the initial innovation without renewing the license.\(^3\) Conceptually, we can think of a technology as embodying both tangible assets and intangible know-how. While the termination of a license may stop the use of tangible assets by past licensees, it is difficult, if possible at all, to prevent them from utilizing the technology know-how. The existence of technology leakage creates a potential downside for short-term contracts.

The innovator’s problem in our model is reminiscent of the classic time-consistency problem of a durable goods monopolist (Coase 1972, Waldman 1996, Rey and Tirole 2007). Indeed, one can view the choice of short-term contracts by the innovator as analogous to the rental solution for the durable goods monopolist.\(^4\) However, there is a fundamental difference between the transfer of intellectual property and the rental of a durable good: whereas for the latter the use of the property is immediately terminated upon its physical removal at the conclusion of the contract, it is not so for the former. To quote Richard Posner (2005), "Because intellectual property is readily appropriable simply by being copied (in contrast to a rental car, for example, which can be appropriated only by being stolen), preserving one’s property rights when one licenses intellectual property is often difficult." This means that a series of short-term contracts are sufficient in resolving a durable goods monopolist’s commitment problem, but not an innovator’s. The main novelty of our paper is to capture this

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\(^3\)Our use of the term "technology leakage" should be distinguished from the occasional uses in newspaper articles (e.g., "Expulsions Tied To Fear Of Technology Leaks", Philip Taubman, New York Times, April 24, 1983) that refer to the more blatant theft of technologies. In our model, technology leakage is not illegal and is present only because intellectual property protection is imperfect.

\(^4\)To push the analogy further, royalty corresponds to "metering" used by a durable goods monopolist to approximate the rental outcome (Burstein 1960, Bulow 1982).
observation by formally modelling technology leakage and studying its role in the design of licensing contracts.

In our model, there are two periods. An innovator sells licenses, which can last a single period or two periods, by either fixed fees or royalties. Whereas long-term fixed-fee contracts potentially prevent technology leakage, they distort the innovator’s incentive to invest in subsequent innovations. Short-term fixed-fee contracts or long-term fixed-fee contracts with opt-out clauses do better, but each is unable to entirely resolve the time-consistency problem because of technology leakage. Long-term royalty contracts do not have a time-consistency problem, but royalty does not maximize the value of the initial innovation. Based on these tradeoffs, we derive conditions under which it is optimal for the innovator to license the technology for less than the length of the patent and conditions under which the uses of royalty contracts are optimal.

To our knowledge, Gandal and Rockett (1995) and Antelo (2009) are the only theoretical papers that have examined the optimal duration of licensing contracts.\(^5\) The first paper focuses on the licensing of a sequence of exogenous innovations by fixed fees. They derive conditions under which the innovator licenses the initial technology bundled with all future improvements and conditions under which licenses to each innovation are sold period-by-period. The other paper focuses on royalties in a model of asymmetric information, in which a licensee’s output in a short-term contract signals her cost. Neither paper compares different payment schemes, nor are they concerned with the innovator’s time-consistency and technology leakage problems identified in this paper.\(^6\)

\(^5\)Farrell and Shapiro (2008) consider a variable royalty rate, contingent upon the outcome of a court challenge of the validity of the patent. In their model, the innovator offers licenses to all downstream firms by assumption, therefore a fixed-fee license is offered only if the downstream firm has no competition. In an extension of their model, they consider short-term licenses, which are contracts that do not survive a finding of validity.

\(^6\)A number of papers study the optimal patent policy in markets with sequential innovation (Green and Scotchmer 1995, Scotchmer 1996, O’Donoghue, 1998, Besen and Maskin, 2000, Denicolo 2002). In these models, a sequence of innovations is undertaken by different firms rather than being concentrated in one firm and their focus is on the length and breadth of patents. Oster (1996) is the only other paper that considers the optimal licensing scheme under sequential innovation. By way of an example, she explores the strategic opportunities created by exclusive licensing in a research-intensive market with sequential innovations, but contracts are short-term by assumption in her model.
Our conceptualization of technology leakage is also related to papers by Macho-Stadler et al. (1996) and Choi (2001), who have developed incomplete contract models of a licensing relationship that is susceptible to moral hazard. They assume that the transfer of technology know-how is costly and cannot be contracted directly. A royalty-based contract is optimal because it reduces the innovator’s temptation of not actually transferring all the know-how. While these papers and ours share the prediction that the use of royalty is positively correlated with the amount of know-how involved in technology transfer, there are subtle differences. They implicitly assume that a technology can be transferred without transferring all necessary know-how; our paper complements theirs by assuming that technology know-how, once transferred, cannot be withdrawn even after the contractual relationship ends.

The remainder of the paper is organized as follows: Section I presents the environment and assumptions of our model of innovation and licensing. In Section II, we consider a simple example to illustrate the basic intuition. In Section III, we solve the innovator’s period 2 problem. In Section IV, we find the optimal licensing scheme in period 1 and report comparative statics results. In Section V, we discuss the robustness of our results. Section VI concludes. Any formal proofs omitted from the main text are contained in the appendix.

I. The Model

We consider an industry consisting of \( n \geq 2 \) identical firms all producing the same good with a linear cost function, \( C(q) = c_0q \), where \( q \) is the quantity produced and \( c_0 > 0 \) is the constant marginal cost of production. In addition to the \( n \) firms, there is an innovator that engages in a series of innovations. She seeks to license the innovations to all or some of the \( n \) firms so as to maximize her profit.

The game lasts two periods. At the beginning of period 1, the innovator owns a patent on a cost-reducing innovation, which reduces the marginal cost of production from \( c_0 \) to \( c_1 \). The patent is valid for both periods. At the beginning of period 2, the innovator can make
Figure 1: The Timeline

a further investment in R&D. If the new R&D effort is successful, then it will generate a second innovation that reduces the cost of production further to $c_2$; hence $c_2 < c_1 < c_0$.

Following Waldman (1996), we assume that the probability of a successful second innovation is $Pr$ and it increases with the amount of investment $I$. For ease of exposition, we assign a particular functional form to $Pr(I)$ such that it equals $2\sqrt{\rho I}$, where $\rho \leq \left[2\pi^M(c_2)\right]^{-1}$. The innovator stops all R&D activities after two periods and the game ends. Figure 1 shows the timeline of the game.

In each period, the inverse demand function of the industry is given by $p = \max\{0, a - Q\}$, where $a > c_0$ and $Q$ is the total production level. Denote by $p^M(c)$ the monopoly price in the downstream market when the marginal cost is $c$, we assume that the initial innovation is drastic, i.e., $p^M(c_1) < c_0$, but the second innovation can be drastic or non-drastic, i.e., $p^M(c_2)$ can be below or above $c_1$.

In order to model technology leakage, we assume that if a downstream firm that licenses the technology $c_1$ in period 1 does not license any innovation in period 2, its marginal cost of production in period 2 is $c' \in [c_1, c_0)$. According to this assumption, a licensee can retain

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7This guarantees that the optimal amount of investment will be an interior solution. Our qualitative results will not change if a more general form of the function is adopted.

8Only some of our results rely on the assumption of a linear demand, which is the most often used demand function in the technology licensing literature. They will be clearly indicated where applicable.

The assumption of a constant market demand is for ease of exposition, but our model can be easily extended to allow shifts in market demand across periods, as shown in Section V.C.

9In the case of a drastic innovation, the granting of an exclusive license offers such a large cost advantage that the licensee can effectively monopolize the industry (Arrow, 1962). The case of non-drastic initial innovations is discussed in Section V.
some fraction of the cost saving from the initial technology transfer, even if he does not license that technology in period 2.

Our main interest is in the innovator’s choice of period 1 licensing contracts. We assume that the amount of investment is not observable to outside parties; hence it cannot be contracted upon. While it is possible to write a contract that is contingent upon the outcome of the period 2 innovation, it costs \( \varphi \) to write such a contract.\(^{10}\) Since we do not explicitly model the transaction cost \( \varphi \) and its impact on the choice of contracts is rather obvious, we assume that \( \varphi \) is so large that a contingent contract is never optimal.\(^{11}\) Therefore, we only consider licenses that specify the payment scheme, the number of licensees and the duration of the contract.

In both periods, the innovator licenses her innovations to \( k \leq n \) firms either by a fixed-fee or by a royalty.\(^{12}\) The duration of a license issued in period 1 can be either one period (short-term) or two periods (long-term). This means that there are four possible types of licensing contracts: short-term fixed fee (\( SF \)), long-term fixed-fee (\( LF \)), short-term royalty (\( SR \)) and long-term royalty (\( LR \)). If the license is a long-term fixed-fee contract, it specifies a payment plan \( (f_1, f_2) \), where \( f_1 \) and \( f_2 \) are fees due at the beginning of period 1 and 2, respectively.\(^{13}\) If the license is a royalty contract, then it specifies the royalty rate \( r \) for each unit that a licensee sells. All individuals maximize their expected total profits, with a common discount factor of \( \delta \). Our solution concept is the subgame perfect equilibrium.

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\(^{10}\)There are a variety of reasons why conditional contracts may be even more costly. For example, there may be search costs associated with thinking through the contracts’ implications (Klein 2002) or simply ink costs associated with writing lengthy contracts (Dye 1985). The costs become even more pronounced in an auction setting, which necessarily involves multiple parties.

\(^{11}\)To be more specific, it suffices for \( \varphi \) to be greater than \( \delta \rho (\tau_1 - \tau_2)^2 \), where \( \tau_1 \) and \( \tau_2 \) are defined in Section III.B.a.

\(^{12}\)Since contracts based on an auction are typically associated with a fixed-fee payment, for conciseness, we lump contracts based on a fixed fee or an auction together and call them fixed-fee contracts. In fact, since contracts based on a fixed fee are dominated by contracts based on an auction when buyers are symmetric (Kamien and Tauman 1986, 2002, Katz and Shapiro 1986), it suffices for us to consider only the latter type of contracts and this is the approach that we have taken, except where noted, in this paper. When buyers are asymmetric, a fixed-fee policy can be more profitable than an auction policy (Stamatopoulos and Tauman 2008, Miao 2009).

\(^{13}\)By offering a payment plan, the innovator gives a licensee the right to terminate the contract in period 2 without paying the second installment \( f_2 \).
Here is a collection of notations that will be used throughout the paper. Some additional notations on the cost of technology leakage are introduced in Section 3.

\( p^M(c) \): Single-period monopoly price in the downstream market when the marginal cost is \( c \).

\( q^M(c) \): Single-period monopoly output in the downstream market when the marginal cost is \( c \).

\( \pi^M(c) \): Single-period monopoly gross profit for the licensee who has a marginal cost of \( c \).

\( \Gamma^{LS(k)} \): Gross licensing revenue from a single-innovation game for licensing scheme \( LS \in \{R, FF\} \), where \( R \) is royalty, \( FF \) is fixed-fee, and \( k \in Z^+ \) denotes the number of licensees.

\( \Pi_t^{LS(k)} \): The innovator’s gross licensing revenue at time \( t \) for licensing scheme \( LS \in \{SF, LF, SR, LR\} \) and \( k \in Z^+ \) denotes the number of licensees. For notational simplicity, we drop the superscript when doing so is unambiguous; particularly in period 2, since there are two innovation outcomes, for each licensing scheme we use \( \Pi_2 \) to denote the period 2 gross licensing revenue if the new innovation is unsuccessful and \( \Pi'_2 \) the revenue if it is successful.

The innovator’s period 2 R&D incentive largely depends on the difference between \( \Pi_2 \) and \( \Pi'_2 \) and we denote it by \( \Delta \).

\( \Pi^{LS} \): The innovator’s maximal profit (licensing revenue net of the investment) for licensing scheme \( LS \in \{SF, LF, SR, LR\} \). We also denote by \( \Pi^B \) the total profit for a vertically integrated innovator.

## II. An Example: The Period 2 Innovation is Drastic

In the standard one-innovation setting, a fixed-fee license for the duration of the patent is optimal, but this result does not extend to a model with sequential innovations. The reason is that the innovator has an incentive to over-invest when presented with the opportunity for new innovations; moreover, this time-consistency problem cannot be solved by a series of
short-term contracts due to technology leakage. The intuition is best illustrated by a simple example, in which the period 2 innovation is also drastic, i.e., \( p^M(c_2) < c_1 \). Since \( c' \geq c_1 \), we must have \( p^M(c_2) < c' \). Hence a firm who licenses the new innovation will become an effective monopoly in period 2. This means that the innovator can sell an exclusive license on the new innovation for a fee equal to the period 2 monopoly profit.\(^{14}\) To put it differently, although technology leakage may weaken the innovator’s bargaining position in period 2, it has no such impact should she succeed in the new innovation. It is this feature that makes the example particularly tractable and illustrative.

The innovator’s incentive to invest in the period 2 innovation is driven by the payoff difference from the two outcomes of the innovation. The optimal level of investment is obtained when the innovator is either vertically integrated with a downstream firm or able to commit to an investment level in period 2 at the time when period 1 licenses are issued. Under either of these circumstances the incentive to invest is driven by the payoff difference \( \pi^M(c_2) - \pi^M(c_1) \).

However, if the innovator is neither vertically integrated nor able to commit, then the outcome of a successful innovation may become more attractive. Suppose that the initial license is a standard long-term exclusive contract with an upfront fee, then the innovator receives no income in period 2 unless the new innovation is successful. This means that her incentive to innovate will be driven by \( \pi^M(c_2) \). Therefore, the innovator has an incentive to over-invest in period 2, relative to the investment level she would choose if she were able to commit in period 1.

Now suppose that the initial license is a short-term fixed-fee exclusive contract. The original licensee is not willing to pay the entire monopoly profit from renewing the license in period 2 because she will enjoy some of the cost savings from the innovation even if she does not renew the license. Further, her possession of the leaked technology means that no

\(^{14}\)We view exclusivity as the exclusive use of a technology, but not the exclusive position in the market, although an exclusive license for a drastic innovation does lead to a market monopoly. This means that the original licensee may still enjoy the exclusive use of the initial innovation, but does not necessarily maintain its monopoly unless it gains the exclusive license to the new innovation.
other firm will be willing to pay the entire monopoly profit. Thus, the innovator is unable to receive the entire monopoly profit as licensing revenue in period 2. Let the revenue loss from leakage be $\tau_1$, the innovator’s incentive to innovate will be driven by $\pi^M(c_2) - [\pi^M(c_1) - \tau_1]$. Therefore, the innovator still has an incentive to over-invest in period 2, but the degree of over-investment is smaller.

This example gives us the basic intuition why a short-term fixed-fee contract may be preferred to a long-term fixed fee contract and why neither contract can achieve the first-best outcome. In the standard long-term fixed-fee contract with upfront payments, the innovator faces a classic time-consistency problem: Once a license is sold, the innovator is then tempted to invest in new technologies that render the initial license obsolete; expecting this, firms will pay less for the license. At the same time, a short-term contract entails technology leakage; so the innovator has an incentive to choose an investment level to minimize the negative impact of technology leakage, but this investment level generally deviates from the optimal.

Of course, the above analysis is far from complete. Clearly, the innovator may want to structure a contract that deals with the time-consistency problem. Since the initial license will be worthless once the period 2 innovation succeeds, a possible solution is to use an installment payment plan, in which the second installment is paid only if a licensee wishes to continue the contract. It is easy to see that the second installment has to be as high as $\pi^M(c_1)$ in order for the innovator to overcome her excessive incentive to invest, but for a payment this high the original licensee will terminate the contract even if the period 2 innovation fails. In other words, a long-term contract that stipulates an opt-out clause with a high continuation fee effectively becomes a short-term contract, which may alleviate the time-consistency problem but not eliminate it. This and other points will be discussed in more detail when we solve the complete model.
III. Investment and Licensing in Period 2

We solve the game via backward induction. In this section, we consider the innovator’s period 2 problem. We first find the optimal licensing scheme under a cost asymmetry. It allows us to more precisely define the cost of technology leakage. We then derive the optimal investment level at the beginning of period 2.

A. Licensing Under Cost Asymmetry

In period 2, downstream firms are no longer identical in their pre-licensing costs. Licensees of the initial innovation will have lower marginal costs than non-licensees, either because the former has signed long-term contracts or because of technology leakage. Here we focus on a scenario that is relevant to our model, in which an exclusive license is granted in period 1 so that the period 2 cost asymmetry is between the original licensee and all others. We show that the optimal licensing scheme under such a cost asymmetry is to once again issue a fixed-fee exclusive license to the original licensee.

Lemma 1 Suppose that firm 1 has a cost of $c_a$ and the other $n - 1$ firms have a cost of $c$, where $c_a \leq c$. If an innovation allows a firm to produce at a cost of $c_b$, where $p^M(c_b) < c$, then it is optimal to issue an exclusive license to firm 1 for a fixed fee via a Right of First Offer.

Proof. Suppose that an optimal licensing scheme $S$ exists, in which firm 1’s net profit (profit minus the payment for a license) is $\pi_0$. Since the industry profits are no larger than $\pi^M(c_b)$, the innovator’s licensing revenue cannot be greater than $\pi^M(c_b) - \pi_0$ under scheme $S$. Now consider an alternative scheme, in which an exclusive contract is offered for a fixed-fee of $\pi^M(c_b) - \pi_0$ and firm 1 is given the Right of First Offer: if firm 1 accepts, then the game ends; if firm 1 rejects, then the innovator uses scheme $S$ to sell the innovation. Since $p^M(c_b) < c$, firm 1 will be able to earn the monopoly profit if it gets the exclusive license. Therefore, in the subgame perfect equilibrium, firm 1 accepts the offer and the innovator.
receives $\pi^M(c_b) - \pi_0$ as her revenue. This means that a fixed-fee exclusive contract is at least as profitable as scheme $S$ and is therefore optimal.

The intuition behind Lemma 1 is straightforward. Offering an exclusive license to firm 1 ensures that the market continues to be monopolized so that the industry profits are shared just between the innovator and firm 1. After leaving firm 1 a surplus that it could have earned otherwise, the innovator keeps all the gain in the industry profits. In such a case, any licensing scheme that maximizes the industry profits also maximizes the innovator’s payoff. But Lemma 1 does not always hold if $p^M(c_b) > c$.\textsuperscript{15} It is our assumption that the initial innovation is drastic and thus any improvement upon the initial innovation is also drastic against the old technology that allows us to dwell on this case, which greatly simplifies our task.

B. The Cost of Technology Leakage

A prominent feature of our model is technology leakage in short-term contracts. Because of technology leakage, the innovator may obtain a smaller licensing revenue in period 2 than she would if the technology transfer were reversible. This loss in period 2 licensing revenue is what we call the cost of technology leakage. Notice that this is not the same as the decrease in total licensing revenue across both periods that results from technology leakage. Potential licensees, expecting technology leakage in period 2, are willing to pay a higher price for the initial license.

\textbf{Definition 1} The cost of technology leakage for an innovator is the difference between the licensing revenue she earns in case past licensees retain some of the cost savings, and the licensing revenue in case they do not.

In order to find the cost of technology leakage, one compares the innovator’s period 2 licensing revenues with and without leakage, which, in general, is not an easy task. However,\textsuperscript{15} For example, in a duopoly, if the high cost firm obtaining a new technology leads to a higher industry profit than the low cost one obtaining the same technology, then it is optimal to license the technology to the former (Stamatopoulos and Tauman 2008).
the comparison in this model is made simpler by the assumption that the initial innovation is drastic. Due to this assumption and the fact that the period 2 innovation is necessarily an improvement over the initial one, without technology leakage the innovator can always sell an exclusive license in period 2 for a fee equal to the monopoly profits. In other words, the period 2 licensing revenue without leakage is \( \pi^M(c_2) \) if the period 2 innovation succeeds or \( \pi^M(c_1) \) if it fails. Therefore, the cost of technology leakage is either \( \pi^M(c_2) - \Pi'_2 \) or \( \pi^M(c_1) - \Pi_2 \).

a. Technology Leakage From an Exclusive License

Moreover, if an exclusive license was issued in period 1, then the cost of technology leakage is exactly equal to the profit that the original licensee can earn in period 2. This is because, according to Lemma 1, it is optimal to offer a second exclusive license to the original licensee so that the innovator and the original licensee split the monopoly profits in period 2. This means that any gain in the bargaining power of the original licensee directly translates into the innovator’s loss of revenue. It is this linkage that allows us to further quantify the cost of technology leakage based on the latter’s profit in this special, but important case, of our model.

The cost of the technology leakage not only depends on the amount of cost saving, but also the technologies available to firms that it competes with. Suppose that firm 1 has a cost of \( c_a \) and the other \( n - 1 \) firms have a cost of \( c \geq p^M(c_b) \) before a new technology that lowers the cost to \( c_b \) is introduced by the innovator. Denote by \( \tau(c_a, c_b) \) firm 1’s net profits (profits minus the licensing fee) from licensing the new technology \( c_b \). The cost of technology leakage if the period 2 innovation fails can thus be written as \( \tau_1 = \tau(c'_1, c_1) \). Similarly, the cost of technology leakage if the period 2 innovation succeeds is \( \tau_2 = \tau(c'_1, c_2) \). In addition, we find the value of owning an exclusive license to the initial innovation in period 2 in the event of a successful period 2 innovation to be another important variable. Using the notation just introduced, we can write it as \( \tau_l = \tau(c_1, c_2) \).
Throughout the paper, we make the following assumptions on the costs of technology leakage:

**Assumption 1**  If $c_a < \pi^M(c_b)$, then $\tau(c_a, c_b) > 0$; if $c_a \geq \pi^M(c_b)$, then $\tau(c_a, c_b) = 0$.

**Assumption 2**  $\tau_1 < \pi^M(c_1)$.

**Assumption 3**  $\tau_1 - \tau_2 < \pi^M(c_1) - \tau_1$.

Assumption 1 states that the cost of technology leakage is zero if and only if $c_b$ is drastic against $c_a$, i.e., the availability of the new technology renders the leaked technology obsolete. Assumption 2 states that the value of having an exclusive access to a production technology of $c_1$ cannot exceed the monopoly profit earned with that technology. Assumption 3 further narrows down the range of the costs of technology leakage. In Appendix B, we verify that these assumptions are met in homogeneous good, conjectural variation oligopoly models. In Appendix C, we consider more general models in which some of these assumption are not met.

One may wonder whether $\tau_2$ is always smaller than $\tau_1$, since the leakage appears to be less of a concern should the period 2 innovation succeed. The answer is no, due to the integer constraint in the number of licenses that the innovator can sell in period 2. As shown in Appendix B, if the original licensee refuses the offer of an exclusive license, then the innovator will auction either 1 or 2 licenses in stage 2 of the period 2 licensing game. For a fixed $c_a$, when $c_b$ is close to $c_a$, 2 is the optimal number of licenses to sell in stage 2; when $c_b$ decreases, the optimal number of licenses to sell in stage 2 will also decrease and at some point that number will "jump" from 2 to 1, diminishing the threat that can be imposed on the original licensee. It is this discontinuity in the number of licenses that causes the non-monotonicity in the cost of technology leakage, because of which we cannot rule out the possibility that $\tau_1 < \tau_2$. 

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C. The Investment Decision and the Value of Investment

Now we solve the innovator’s problem at the investment stage in period 2. Let \( \Delta = \Pi'_2 - \Pi_2 \), we have

**Lemma 2** The optimal amount of investment is \( \rho \Delta^2 \), the probability of a successful innovation is \( 2 \rho \Delta \), and the innovator’s expected profit in period 2 is \( \Pi_2 + \rho \Delta^2 \).

**Proof.** The innovator’s investment decision is \( \max_I \Pr (I) \Delta - I = 2\sqrt{\rho I} \Delta - I \). Denote by \( I^* \) the optimal choice of \( I \), we must have \( \sqrt{\rho / I^*} \Delta = 1 \), i.e., \( I^* = \rho \Delta^2 \). So \( \Pr (I^*) = 2 \rho \Delta \) and the expected profit in period 2 is \( \Pi_2 + \Pr (I^*) \Delta - I = \Pi_2 + \rho \Delta^2 \).

Lemma 2 shows that the innovator’s incentive to invest in period 2 is entirely determined by \( \Delta \), the difference in period 2 licensing revenues from the two outcomes. Hence, \( \Delta \) can serve as a convenient indicator of a licensing scheme’s optimality, which we will use repeatedly in this paper. Another result that allows us to easily compare licensing schemes is the following:

**Lemma 3** The innovator’s expected profit in period 2 increases in both \( \Pi_2 \) and \( \Pi'_2 \).

**Proof.** The innovator’s expected profit in period 2 is \( \max_I \Pi_2 + \Pr (I) (\Pi'_2 - \Pi_2) - I \). Denote it by \( \Pi^*_2 \). By the envelope theorem, \( d\Pi^*_2/d\Pi_2 = \frac{\partial}{\partial \Pi_2} (\Pr (I) (\Pi'_2 - \Pi_2) - I) \bigg|_{I=I^*} = 1 - \Pr (I^*) \geq 0 \) and \( d\Pi^*_2/d\Pi'_2 = \Pr (I) \geq 0 \).

IV. Licensing in Period 1

In this section, we find the optimal licensing scheme in period 1, which is the central concern of this paper. We start with the first-best scenario for the innovator, whose solution is then used as our benchmark. Then we solve for the payoffs associated with each of the possible licensing schemes. We compare them with the benchmark and discuss each scheme’s advantages/disadvantages. Finally, we carry out some comparative statics exercises by varying the rate of innovation parameter and the cost of technology leakage.
A. The Benchmark

In a first-best scenario, the innovator is vertically integrated with a downstream firm and sells the final output by herself. There is neither a commitment problem nor technology leakage. Since \( p^M(c_2) < p^M(c_1) < c_0 \), the innovator can monopolize the industry in both periods.\(^\text{16}\) Therefore, her incentive to innovate in period 2 is perfectly aligned with the gain in industry profits, which is \( \pi^M(c_2) - \pi^M(c_1) \).

**Proposition 1** If the innovator markets the final output by herself, then \( \Delta = \pi^M(c_2) - \pi^M(c_1) \) and her total profit is

\[
\Pi^B = (1 + \delta) \pi^M(c_1) + \delta p[\pi^M(c_2) - \pi^M(c_1)]^2.
\]

Proposition 1 gives us the upper bound of licensing revenue that the innovator can obtain, which serves as a useful benchmark in comparing different licensing schemes. It also provides a necessary condition for any licensing scheme to generate the benchmark profits: \( \Delta \) (i.e., \( \Pi'_2 - \Pi_2 \)) must be equal to \( \pi^M(c_2) - \pi^M(c_1) \). This is true because the optimal level of investment is proportional to \( \Delta^2 \), so the period 2 investment must be inefficient if \( \Delta \) deviates from \( \pi^M(c_2) - \pi^M(c_1) \).

Vertical integration is not the only way for the innovator to obtain the benchmark profit. If the transaction cost, \( \varphi \), is zero, then the benchmark outcome can also be achieved via a fixed-fee license whose payments are contingent upon the innovation outcome. Denote by \( f_1 \) the period 1 license fee, \( f_2 (f'_2) \) the period 2 license fee if the period 2 innovation fails (succeeds).

**Proposition 2** A long-term fixed-fee exclusive contract with \( f_1 = \pi^M(c_1), f_2 = \pi^M(c_1) \) and \( f'_2 = \pi^M(c_2) \) replicates the benchmark outcome if and only if \( \varphi = 0 \).

\(^\text{16}\)This also means that an innovator who is an incumbent in the industry will not license either innovation in our model.
In reality, however, both vertical integration and writing complete contracts may be impractical: a research university may want to keep arms’ length from the product market in order to avoid conflicts of interest; certain “transaction costs” may prevent future contingencies from being contracted ex ante. Therefore, we must also examine the optimal licensing scheme when the above two options are unavailable.

B. Fixed-fee Licenses

We first consider fixed-fee licenses. Since we assume that the initial innovation is drastic, a fixed-fee exclusive license is optimal in an one-innovation model (Katz and Shapiro 1986, Kamien and Tauman 1986), but we show in this section that it is generally not true when there are sequential innovations and technology leakage. In so doing, we also solve for the optimal fixed-fee contracts. To streamline our exposition, we restrict our attention to exclusive contracts in period 1. We will verify in Section V. that this restriction is inconsequential.

a. Short-term Fixed-Fee Exclusive License

In a short-term fixed-fee exclusive contract, a licensee has the right to use the cost-reducing technology of \( c_1 \) for just one period, during which he earns the monopoly profit \( \pi^M(c_1) \). After the contract expires at the end of period 1, the original licensee enjoys a cost of \( c' < c_0 \) because of technology leakage, while the other \( n - 1 \) firms only have the old technology of \( c_0 \). In period 2, new licensing takes place regardless of whether the innovator is successful in her R&D efforts.

Lemma 4 If a short-term fixed-fee exclusive license is offered in period 1, then \( \Pi^{SF} = \Pi^B - \delta \rho (\tau_1 - \tau_2)^2 \); if \( c' > p^M(c_1) \), then \( \tau_1 = \tau_2 = 0 \) and a short-term fixed-fee exclusive license replicates the benchmark outcome.
Proof. According to Lemma 1, the period 2 license will be granted to the original licensee, who will gain a monopoly, regardless of the innovation outcome. Thus $\Pi_2 = \pi^M(c_1) - \tau_1$ and $\Pi'_2 = \pi^M(c_2) - \tau_2$. Hence, $I = \rho[\pi^M(c_2) - \pi^M(c_1) + \tau_1 - \tau_2]^2$, $\Pi_{2SF}^* = \pi^M(c_1) - \tau_1 + \rho(\pi^M(c_2) - \pi^M(c_1) + \tau_1 - \tau_2)^2$ and $\Pi_{1SF}^* = \pi^M(c_1) + \delta(1 - Pr)\tau_1 + \delta Pr\tau_2$, where $Pr = 2\rho(\pi^M(c_2) - \pi^M(c_1) + \tau_1 - \tau_2)$. Therefore,

\[
\Pi_{SF}^* = (1 + \delta)\pi^M(c_1) - 2(\tau_1 - \tau_2)\delta\rho(\pi^M(c_2) - \pi^M(c_1) + \tau_1 - \tau_2) + \delta\rho(\pi^M(c_2) - \pi^M(c_1) + \tau_1 - \tau_2)^2
\]

\[
= \Pi^B - \delta\rho(\tau_1 - \tau_2)^2.
\]

If $c' > \rho^M(c_1)$, then $\tau_1 = \tau_2 = 0$ by Assumption 1 hence $\Pi_{SF}^* = \Pi^B$. It is also easy to verify that the innovator will choose the optimal investment level $\rho[\pi^M(c_2) - \pi^M(c_1)]^2$ if she has the ability to commit in period 1.

If the game lasts only one period, then the standard model predicts that a fixed-fee license is optimal (Kamien and Tauman 1986). Upon first glance of our model, extending the game into two periods adds little new: the innovator and potential licensees can contract period by period and this reduces a two-period game into two one-period standard games. However, Lemma 4 tells us that the benchmark outcome can be replicated by a series of short-term contracts only if there is no technology leakage, otherwise technology leakage will cost the innovator $\delta\rho(\tau_1 - \tau_2)^2$, where $\tau_1 - \tau_2$ represents the difference in the costs of technology leakage between the two outcomes of the period 2 innovation.

It should be noted that the innovator’s period 2 revenue loss from technology leakage does not directly translate into a loss in total licensing revenue: after all, expecting a leakage, firms will pay more for the initial license. Rather, it is the innovator’s attempt to minimize the revenue loss from leakage that causes a distortion in her incentive to invest in sequential innovations and this lowers a licensee’s willingness to pay for the initial innovation. It can
be seen most clearly by examining $\Delta = \Pi_2 - \Pi_2$, which equals $\pi^M(c_2) - \pi^M(c_1) + (c_1 - c_2)$ under a short-term contract. Thus, as long as the costs of technology leakage are not identical under different innovation outcomes, the innovator’s incentive to invest will deviate from the optimal level. It is this deviation that results in the innovator’s loss in total revenue. In other words, the presence of both technology leakage and sequential innovation are essential for short-term fixed-fee contracts not to be able to replicate the benchmark outcome.

b. Long-term Fixed-fee Exclusive License

Now we examine in detail long-term fixed-fee contracts and their optimality. In our simple example, only contracts with an upfront payment are considered and we find that long-term fixed-fee contracts entail a time-consistency problem. To deal with the problem, the innovator may choose to add an opt-out clause, which allows a licensee to terminate a long-term contract after the innovation outcome is realized in period 2. More specifically, the period 1 contract specifies the fees to be paid in each of the two periods and we denote them by $f_1$ and $f_2$; if a licensee opts out the contract in period 2, then the contract terminates and $f_2$ will not be paid.$^{17}$ Figure 2 illustrates the period 2 subgame. For the purpose of comparison, we also include the subgame tree for short-term fixed-fee contracts in Figure 2.

Clearly, a long-term fixed-fee contract $(f_1, f_2)$ without the opt-out clause is equivalent to a long-term fixed-fee contract $(f_1 + \delta f_2, 0)$ with the opt-out clause. This means that any long-term fixed-fee contracts without the opt-out clause are just special cases of a long-term fixed-fee contract with an opt-out clause. Therefore, it suffices to find the optimal long-term fixed-fee contract with an opt-out clause.

$^{17}$Here we implicitly assume a zero termination fee, but this assumption is without loss of generality, since only the difference in the payments affects a licensee’s decision whether to continue or to terminate the contract and the innovator’s incentive to invest in a new innovation. If the contract instead specifies a non-zero termination fee of $f_0^2$, then such a contract is equivalent to $(f_1 + \delta f_2, f_2 - f_0^2)$.

$^{18}$A contract with an opt-out clause can be implemented via an auction, in which the innovator first announces the period 2 payment $f_2$ and then invites bids such that the winning bid becomes the period 1 payment $f_1$. 
Lemma 5  In a long-term fixed-fee exclusive contract \((f_1, f_2)\) with an opt-out clause, if \(\tau_1 - \tau_2 > 0\) and \(f_2 \leq \pi^M(c_1) - \tau_1\), then \(\Pi^{LF}\) increases with \(f_2\).

Proof.  In period 2, there are two states of nature: (i) innovation is not successful; (ii) innovation is successful. In case (i), since \(\pi^M(c_1) - f_2 > \tau_1\), the original licensee will continue the contract and get \(\pi^M(c_1) - f_2\). Hence \(\Pi_2 = f_2\).

In case (ii), we separate \(f_2\) further into two regions: a) \(f_2 \leq \tau_1 - \tau_2\) and b) \(\tau_1 - \tau_2 < f_2 < \pi^M(c_1) - \tau_1\).

\((ii.a)\) \(f_2 \leq \tau_1 - \tau_2\). If the original licensee continues the initial contract and produces at a cost of \(c_1\), then he gets \(\tau_1 - f_2\); if he opts out, then he gets \(\tau_2\). Since \(\tau_1 - f_2 \geq \tau_2\), it is optimal for the original licensee to continue the original license. This means that \(\Pi'_2 = \pi^M(c_2) - \tau_1 + f_2\) and \(\Delta = \pi^M(c_2) - \tau_1\). So the innovator’s incentive to invest is independent of \(f_2\). Therefore, her licensing revenue is a constant if \(f_2 \leq \tau_1 - \tau_2\).
(ii.b) $f_2 > \tau_1 - \tau_2$. If the original licensee continues the initial contract and produces at a cost of $c_1$, then he gets $\tau_1 - f_2$; if he opts out, then he gets $\tau_2$. Since $\tau_1 - f_2 < \tau_2$, the original licensee’s right to use the old innovation has no value and he will opt out the initial contract. This means that $\Pi' = \pi^M(c_2) - \tau_2 - f_2$. In period 1, a firm is willing to pay $\Pi_1^{LF} = \pi^M(c_1) + \delta(1 - 2\rho(\pi^M(c_2) - \tau_2 - f_2)) \pi^M(c_1) - f_2 + 2\delta\rho(\pi^M(c_2) - \tau_2 - f_2)\tau_2$ for an exclusive license. At the same time, $\Pi_2^{LF} = f_2 + \rho(\pi^M(c_2) - \tau_2 - f_2)^2$. Hence, the total licensing revenue is $\Pi^{LF} = \pi^M(c_1) + \delta(1 - 2\rho(\pi^M(c_2) - \tau_2 - f_2)) \pi^M(c_1) - f_2 + 2\delta\rho(\pi^M(c_2) - \tau_2 - f_2)\tau_2 + \delta(f_2 + \rho(\pi^M(c_2) - \tau_2 - f_2)^2)$, so $\frac{\partial}{\partial f_2} \Pi^{LF} = 2\rho\delta(\pi^M(c_1) - \tau_2 - f_2) \geq 2\rho\delta(\tau_1 - \tau_2) > 0$. Last, it is also easy to verify that $\Pi^{LF}$ is continuous at $f_2 = \tau_1 - \tau_2$.

**Lemma 6** If $\tau_1 - \tau_2 > 0$, then any equilibrium long-term fixed-fee exclusive contract $(f_1, f_2)$ with an opt-out clause and $f_2 \geq \pi^M(c_1) - \tau_1$ is equivalent to a short-term exclusive contract with a fixed fee of $f_1$.

**Proof.** If the period 2 innovation is not successful, then the period 2 surplus that the original licensee can obtain is $\pi^M(c_1) - f_2$ by continuing the contract and $\tau_1$ by opting out. Since $\tau_1 \geq \pi^M(c_1) - f_2$, the contract will be terminated after period 1.

If the period 2 innovation is successful, then the period 2 surplus that the original licensee can obtain is $\tau_1 - f_2$ by continuing the initial contract and $\tau_2$ by opting out. Since $\tau_1 - f_2 < \tau_1 - \tau_2$, the initial contract will also be terminated after period 1.

**Lemma 7** If $\tau_1 - \tau_2 < 0$, then there exists a long-term fixed-fee exclusive contract that replicates the benchmark outcome.

**Proof.** Consider a long-term fixed-fee contract $(f_1, f_2)$ with $f_1 = \pi^M(c_1) + \delta\tau_2$ and $f_2 = \pi^M(c_1) - \tau_2$.

If the period 2 innovation is not successful, then the period 2 surplus that the original licensee can obtain is $\pi^M(c_1) - f_2 = \tau_2$ by continuing the contract and $\tau_1$ by opting out. Since $\tau_1 < \tau_2$, the contract will be continued after period 1 and thus the original licensee is willing to pay $f_1 = \pi^M(c_1) + \delta\tau_2$ in period 1. Also, we obtain that $\Pi_2 = f_2 = \pi^M(c_1) - \tau_2$. 

22
If the period 2 innovation is successful, then the period 2 surplus that the original licensee can obtain is $\tau_1 - f_2$ by continuing the initial contract and $\tau_2$ by opting out. Since $\tau_1 - \pi^M(c_1) + \tau_2 < \tau_2$, the initial contract will be terminated after period 1 and the original licensee’s period 2 surplus is $\tau_2$. This again means that the original licensee is willing to pay $\pi^M(c_1) + \delta \tau_2$ in period 1, so we obtain that $\Pi'_2 = \pi^M(c_2) - \tau_2$.

Since $\Delta = \Pi'_2 - \Pi_2 = \pi^M(c_2) - \pi^M(c_1)$, the innovator’s incentive to invest in period 2 is optimal. Therefore, the given contract replicates the benchmark outcome.

Using the above lemmas, we obtain the following result for fixed-fee contracts.

**Proposition 3** For homogeneous good, conjectural variation oligopoly models, (i) If $\tau_1 - \tau_2 \leq 0$, then there exists a long-term fixed-fee contract that replicates the benchmark outcome; (ii) if $\tau_1 - \tau_2 > 0$, then a short-term contract is optimal among fixed-fee licenses.

The intuition for the above result is easy to understand. As shown in the simple example, in a long-term fixed-fee contract, the innovator has an incentive to over-invest in order to make the initial license obsolete. To mitigate this incentive, the innovator can increase $f_2$, the continuation fee on the initial license. But too high a continuation fee will lead the original licensee to terminate the initial contract regardless of the innovation outcome, replicating a short-term contract. Hence $f_2$ can not exceed $\pi^M(c_1) - \tau_1$. On the other hand, the continuation fee that allows the innovator to replicate the benchmark outcome is $\pi^M(c_1) - \tau_2$. The two conditions can both be met only if $\pi^M(c_1) - \tau_2 < \pi^M(c_1) - \tau_1$, i.e., $\tau_1 < \tau_2$.\(^{19}\) otherwise a long-term fixed-fee contract is at best as profitable as a short-term one.

As shown in the proof, the optimal fixed-fee contract depends on comparing the costs of technology leakage, especially $\tau_1 - \tau_2$ and $\pi^M(c_1) - \tau_1$. In homogenous good, conjectural variation models, we have Assumption 2 and 3, which significantly reduce the number of

\(^{19}\)It should also be noted that the continuation fee does not have to be positive. In fact, if $\pi^M(c_1) < \tau_2$, then the optimal continuation fee will be negative; or to put it differently, the continuation fee will be greater than the termination fee.
cases to consider. For more general models, the results are analogous, but the proof are somewhat tedious, so we leave them in the appendix.

C. Royalty Licenses

Next we consider the optimality of short-term and long-term royalty licensing schemes. We will use a result attributed to Kamien and Tauman (1986): in an one-innovation licensing game, under Cournot competition with a linear demand, the licensing revenue from royalty $\Gamma^{R(k)}(r)$ on a drastic innovation that reduces the production cost from $c_0$ to $c_1$ is maximized at $r^* = (a - c_1)/2$ and $k^* = n$ for a maximum of $\Gamma^{R}(r^*) = \frac{n}{n+1}M(c_1)$; under Bertrand competition, $r^* = (a - c_1)/2$, $k^* = n$ and $\Gamma^{R}(r^*) = M(c_1)$.

a. Short-term Royalty

Like a short-term fixed-fee license, short-term royalty contracts last only one period, but they generally admit more licensees in period 1. Hence, in period 2, more than one firms may have access to the part of cost saving that is irreversible. This makes an explicit solution to the period 2 licensing game difficult to obtain. Therefore, we simply compare the two licensing schemes and rule out short-term royalty as a possible optimal scheme.

Lemma 8 Short-term royalty is less profitable than short-term fixed-fee exclusive licensing.

Proof. Recall that the innovator’s total licensing revenue net of investment is $\Pi = \Pi_1 + \Pi_2 + \delta \rho (\Pi'_2 - \Pi_2)^2$. Our plan of the proof is to show that all three terms, $\Pi_1, \Pi_2$ and $\Pi'_2$, are lower under short-term royalty ($SR$) than under short-term fixed-fee exclusive licensing ($SF$) and therefore the same must be true for $\Pi$ according to Lemma 3.

First, it is easy to see that $\Pi_1^{SR} \leq \pi^M(c_1) \leq \Pi_1^{SF}$. Next we consider period 2 licensing if the innovation fails so that the best technology available remains $c_1$. Because of technology leakage, $k_1 \geq 1$ firms have a cost of $c'$ at the beginning of period 2 under $SR$ but only 1 firm has $c'$ under $SF$. All other firms have a cost of $c_0 > c'$.
Let the period 2 optimal licensing scheme under $SR$ be $O$. We want to show that under $SF$ a scheme based on $O$ can give the innovator a period 2 licensing revenue at least as much as $\Pi_2^{SR}$. Consider scheme $O^+$, under which scheme $O$ is used along with a royalty contract offered to $k_1 - 1$ firms that allows them to use the cost-reducing technology $c_1$ for a rate of $c' - c_1$. For the $k_1 - 1$ firms offered a royalty, their cost effectively becomes $c_1 + (c' - c_1) = c'$. Thus, in total, $k_1$ of firms will have a cost of $c'$ when they participate in scheme $O$. This means that scheme $O^+$ allows period 2 licensing under $SF$ to replicate the licensing game played under scheme $O$ and therefore $\Gamma^{O^+} \geq \Pi_2^{SR}$, where the inequality holds if the royalty offer is taken by a positive number of firms. Since $O^+$ is not necessarily the optimal scheme under $SF$, we must have $\Pi_2^{SF} \geq \Gamma^{O^+}$ and therefore $\Pi_2^{SF} \geq \Pi_2^{SR}$. The same argument, except that the royalty rate should be set at $c' - c_2$, can be applied to the case of a successful period 2 innovation to show that $\Pi_2^+$ is lower under $SR$ than under $SF$.

The intuition behind the proof goes as follows: when compared with short-term fixed-fee licensing, short-term royalty generates a smaller period 1 licensing revenue and leads to a greater degree of technology leakage, which lowers the licensing revenue in period 2 regardless of the innovation outcome. In particular, under a fixed-fee contract the innovator is able to capture a licensee's gain from technology leakage, whereas under a royalty the innovator is unable to. This, however, suggests that a two-part tariff can potentially improve upon pure royalty, an observation that we will return to in Section V.

b. Long-term Royalty

Under a long-term royalty scheme, a licensee is entitled to use the period 1 innovation of $c_1$ for both periods and pay $r_1$ (reps. $r_2$) for every unit of output produced in period 1 (reps. 2). Here we allow the royalty rate to change because the optimal royalty rate varies with market demand, which may be different across periods. However, in the constant demand case we consider in this model the two royalty rates coincide. More importantly, they are shown to be the same as the royalty rate that maximizes revenue in a one-innovation licensing game.
Lemma 9 Under Cournot competition with linear demand, the optimal royalty rate in each period is \( r^* = (a - c_1)/2 \) in a long-term royalty contract and the licensing revenue is \( \Pi^{LR} = (1 + \delta) \Gamma^R(r^*) + \delta \rho \left( \pi^M(c_2) - \Gamma^R(r^*) \right)^2 \); under Bertrand competition, the optimal royalty rate in each period is \( r^* = (a - c_1)/2 \) in a long-term royalty contract and the licensing revenue is \( \Pi^{LR} = \Pi^B \).

Proof. If the period 2 innovation is not successful, then \( \Pi_2 = \Gamma^R(r_2) \). Hence \( \Pi^{LR} = (1 + \delta) \Gamma^R(r_2) + \delta \rho \left( \Pi'_2 - \Gamma^R(r_2) \right)^2 \). In order to find the optimal royalty rate in period 2, we separate the possible choice of \( r_2 \) into two regions: (i) \( r_2 \geq r^* \) or (ii) \( r_2 < r^* \), where \( r^* = (a - c_1)/2 \) is the optimal royalty rate in a one-innovation licensing game.

(i) If \( r_2 \geq r^* \), then the period 2 innovation \( c_2 \) is drastic against an original licensee’s total cost \( c_1 + r_2 \), since \( p^M(c_2) < p^M(c_1) \leq c_1 + r_2 \). Therefore, if the period 2 innovation is successful, then the licensing revenue from it is maximized via a fixed-fee exclusive license. This means that \( \Pi_2 = \pi^M(c_2) \) and \( \Pi^{LR} = \Gamma^R(r_1) + \delta \Gamma^R(r_2) + \delta \rho \left( \pi^M(c_2) - \Gamma^R(r_2) \right)^2 \). We can see that the innovator’s total profits \( \Pi^{LR} \) increases with \( \Gamma^R(r_2) \), since

\[
\frac{\partial}{\partial \Gamma^R(r_2)} \Pi^{LR} = \delta [1 - 2 \rho (\pi^M(c_2) - \Gamma^R(r_2))] \geq 0.
\]

Therefore, \( r_2 \) should be chosen so as to maximize \( \Gamma^R(r_2) \), i.e., \( r_2 = r^* \). Thus,

\[
\Pi^{LR} = (1 + \delta) \Gamma^R(r^*) + \delta \rho \left( \pi^M(c_2) - \Gamma^R(r^*) \right)^2.
\]

(ii) In order for \( r_2 < r^* \) to be a profitable deviation from \( r^* \), we must have \( \Gamma^R(r_2) + \rho \left( \Pi'_2 - \Gamma^R(r_2) \right)^2 > \Gamma^R(r^*) + \rho \left( \pi^M(c_2) - \Gamma^R(r^*) \right)^2 \) and \( \Pi'_2 > \Gamma^R(r_2) \). But \( \Gamma^R(r^*) + \rho \left( \pi^M(c_2) - \Gamma^R(r^*) \right)^2 \geq \Gamma^R(r_2) + \rho \left( \pi^M(c_2) - \Gamma^R(r_2) \right)^2 \), a contradiction. The first inequality is due to Eq. (4) and the second is due to \( \Pi'_2 \leq \pi^M(c_2) \).

Under Bertrand competition, \( \Gamma^R(r^*) = \pi^M(c_1) \) hence \( \Pi^{LR} = \Pi^B \).

Royalty contracts generally cannot replicate the benchmark outcome under Cournot competition. In standard one-innovation models, they are inferior to contracts based on a fixed
fee or an auction. But in a model with sequential innovations, a long-term royalty contract can avoid both the technology leakage problem in a short-term contract and the time-consistency problem in a fixed-fee contract. Intuitively, the use of royalty to collect payments on an ongoing basis eliminates the innovator’s commitment problem. It is this advantage that makes royalty a potentially optimal licensing scheme.

It should be noted that the licensing revenue obtained in Eq. (5) is likely to be the lower bound for a long-term royalty contract. If renegotiations are allowed, the innovator can potentially increase her revenue. For example, in the above discussion, we have implicitly assumed that the innovator cannot change the licensing scheme for the period 1 innovation in period 2 if the new innovation is not successful. Now suppose that the innovator can modify licensing contracts with individual licensees, then it is optimal to move from the royalty scheme to a fixed-fee exclusive licensing in period 2. This change will increase the period 2 licensing revenue without affecting the period 1 royalty rate and thus may increase the innovator’s total profits. Under Cournot competition, it can be shown that the innovator’s total revenue will then become $\Gamma^R(r^*) + \delta \left(1 - \frac{1}{(n+1)^2}\right) \pi^M(c_1) + \delta \rho \left(\pi^M(c_2) - \left(1 - \frac{1}{(n+1)^2}\right) \pi^M(c_1)\right)^2$, which is greater than $\Pi^{LR}$ obtained in Eq. (5), since $\Gamma^R(r^*) = \frac{n}{n+1} \pi^M(c_1) < \left(1 - \frac{1}{(n+1)^2}\right) \pi^M(c_1)$.

D. Summary of results and Comparative Statics

Now we summarize the comparison of different licensing schemes and discuss how the choice of licensing schemes varies with the model parameters. In so doing, we also provide some potential testable hypotheses, which can serve as guidance for future empirical work.

**Proposition 4** Under Cournot competition with linear demand, (i) If $\tau_1 - \tau_2 < 0$, then the optimal licensing contract is a long-term fixed-fee contract with an opt-out clause, where $\Pi^{LF} = \Pi^B$; (ii) if $\tau_1 - \tau_2 > 0$, then the optimal licensing contract is either SF or LR.

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20This is due to the fact that licensees, on top of the royalty fee charged by the innovator, will charge an additional mark-up. Only as the number of licensees gets large, so that each licensee has no market power, is the innovator able to extract the entire monopoly profit.
where $\Pi^{SF} = \Pi^B - \rho \delta (\tau_1 - \tau_2)^2$ and $\Pi^{LR} = (1 + \delta) \Gamma^R(r^*) + \delta \rho \left( \pi^M(c_2) - \Gamma^R(r^*) \right)^2$, which approaches $\Pi^B$ when $n \to \infty$. Under Bertrand competition, the optimal licensing contract is a long-term royalty with $\Pi^{LR} = \Pi^B$.

For any fixed-fee contract, short-term or long-term, the payoff of the innovator stays the same for any size of oligopoly and is bounded away from the benchmark profit, but the payoff from a long-term royalty always increases in $n$, the size of the oligopoly, and can be made arbitrarily close to the benchmark profit by increasing $n$. Therefore, even in the absence of information asymmetry and risk aversion, royalty licensing can be more profitable than fixed-fee contracts when the size of the oligopoly is sufficiently large.\footnote{Sen (2005) uses the same bounds approach to show that royalty licensing can be superior to both fixed fee and auction in a one-innovation setting due to the integer constraint on the number of licensees.}

**Corollary 1** Denote by $\tilde{n}$ the size of the oligopoly such that $\Pi^{LR} = \Pi^{SF}$ and $A = \pi^M(c_2)/\pi^M(c_1)$. Under Cournot competition with linear demand, $\partial \tilde{n}/\partial \rho < 0$.

**Proof.** From Proposition 4, we get

$$\frac{\partial}{\partial \rho} \tilde{n} = -\delta (\tau_1 - \tau_2)^2 (n + 1) + (\tau_1 - \tau_2)^2 (n + 1)^2) / ((n + 1) (1 + \delta - 2A\rho \delta + 2\delta \rho n) < 0.$$ 

When we vary the parameter governing the probability of innovation, we find that non-exclusive royalty contracts are optimal for higher levels of innovation. The reason is that increasing the rate of innovation magnifies the innovator’s incentive to engage in R&D, thereby exacerbating the over-investment problem under fixed-fee contracts. This suggests that industries where sequential innovations are common are more likely to use non-exclusive royalty contracts. Although there is no direct evidence to support this prediction, Anand and Khanna (2000) do find that the incidence of exclusivity varies considerably across industries.\footnote{They have not been able to gather reliable information on the form of payment (royalties versus fixed fees) agreed to in the licensing contracts.} In Computers (18%) and Electronics (16%), two industries that are well known
for sequential innovations (Bessen 2004, Bessen and Maskin 2000), exclusive transfers are much less common than in the other industries (38%).\textsuperscript{23}

Our model also suggests that exclusive contracts may be more likely used in industries that have strong intellectual property protection so that technology leakage is of less concern. This hypothesis is compatible with the pattern of licensing observed by Anand and Khanna (2000): the chemical industries, which have high invent-around costs and patents that deliver strong appropriability (Levin et al. 1987, Cohen et al. 2000), also have higher incidences of exclusive licensing than other industries.\textsuperscript{24}

V. Discussion

In this section, we explore the effects of relaxing some of our assumptions made in the basic model. Our results appear robust to these extensions.

A. Multiple Fixed-Fee Licenses

For ease of exposition, we only allowed the innovator to sell an exclusive fixed-fee license on the initial innovation in the main results. Here we consider the possibility that the innovator sells multiple fixed-fee licenses in period 1. First, if these licenses are short-term, then their optimality will not change our results. Second, if these licenses are long-term, then the industry profits in period 2 will be lower than the monopoly profit if the new innovation is not successful; in itself, it lowers the innovator’s potential licensing revenue; at the same time, it also creates incentive for the innovator to over-invest. In the appendix, we prove that issuing multiple long-term fixed-fee licenses is never optimal in our model.

\textsuperscript{23}Anand and Khanna (2000) offer an alternative explanation based on Katz and Shapiro (1986): network externalities are important in the computer and electronics industries and thus may have led firms to license technologies non-exclusively, with the aim of ‘setting the standard’ early on. Note that the two explanation are not mutually exclusive.

\textsuperscript{24}It should be noted, however, that the two factors, sequential innovation and technology leakage, may be confounding each other in explaining the above pattern of licensing, since Anand and Khanna (2000)’s study concentrates on only three industries: Chemicals, Computers and Electronics. Further research is needed to identify which of the two factors is more responsible for the observed pattern.
B. Non-drastic Innovation

Our current analysis has been limited to drastic innovations in period 1. If the innovation in period 1 is not drastic, then it may no longer be optimal to offer an exclusive license in period 1.$^{25}$ Instead, a fixed-fee license may be offered to multiple firms, either as a long-term contract or a short-term contract. A detailed analysis will be complicated and is beyond the scope of this paper.$^{26}$ However, the basic trade-off between the value of the initial innovation and the incentive to engage in future innovations remains the same. More importantly, we find that short-term royalty is never optimal even if the period 1 innovation is non-drastic, as shown in the appendix, while short-term fixed-fee licensing can be optimal when there is no technology leakage. Therefore, we expect our result that royalties on average have a longer duration than fixed-fee contracts to continue to hold.

C. Shifts in Market Demand

Another possibility that we want to consider is the change in the size of the market across two periods. As long as the period 2 demand is common knowledge, our results do not change qualitatively. Under both short-term and long-term fixed-fee schemes, the innovator is able to extract the entire period 1 monopoly profit as her licensing revenue. Therefore, the size of the period 1 monopoly profit does not affect the comparison between short-term and long-term fixed-fee contracts. Royalty contracts are slightly more complicated, since they involve a trade-off between resolving the time-consistency problem in period 2 and lowering licensing revenue in period 1, but it is not difficult to see that royalties are more likely to be used when the market demand is larger in period 2.

One trivial exception is the possibility of delayed licensing, i.e. the innovator can choose not to offer any license in period 1 and only offer licenses after the outcome of the period

$^{25}$In Kamien and Tauman (1986), it has been shown that a fixed-fee license sold to multiple firms is optimal in a one-period game if the innovation is not drastic.

$^{26}$The major complication involves solving the period 2 competition outcome with four different types of firms, whose marginal costs can be any of $c_0, c_1, c_2$ and $c'$.
2 innovation is realized. Such a delay allows the innovator to get the same expected net profit in period 2 as in the benchmark case, though she loses the monopoly profits in period 1. Clearly, if the market demand in period 2 is sufficiently large relative to the period 1 demand, then delayed licensing can be optimal. In the case of constant market demand delayed licensing is not optimal, since the revenue loss from delayed licensing is $\pi^M(c_1)$ and we have $\pi^M(c_1) > \delta[\pi^M(c_1)]^2/[2\pi^M(c_2)] \geq \delta \rho (\pi^M(c_1))^2 \geq \delta \rho (\pi^M(c_1) - \tau l)^2$, where the last term is the revenue loss from a long-term fixed-fee contract with an upfront payment.

D. Two-part Tariff

In this model, the licensing policies are confined to either pure fixed fees or pure royalty. Here we discuss what happens if the innovator can use two-part tariffs, i.e., a combination of fixed fees and royalties, as a possible payment scheme.\footnote{A two-part tariff can be implemented via an auction plus royalty policy where the innovator first announces the level of royalty and then auctions off one or more licenses so that the upfront fee that a licensee pays is its winning bid (Sen and Tauman 2007).} Since both fixed fee and royalty are special cases of the more general licensing scheme, the innovator cannot do worse by having the ability to use the combination of the two. The question is therefore about which existing licensing scheme can be improved by its combination with the other. First, it is clear that any pure fixed-fee contracts cannot be improved by adding a positive royalty, for otherwise the fee currently set could not have been optimal; second, royalty contracts can potentially become more profitable since the innovator can use the fixed-fee part of a two-part tariff to extract licensees’ profits. Thus, allowing two-part tariffs increases the circumstances under which contracts with positive royalty rates are used. In fact, Vishwasrao (2007) finds that contracts of longer duration are generally associated with a combination of fees and royalties rather than royalties alone. Our model provides a useful starting point to understand her empirical finding.
VI. Conclusion

In this paper, we have extended the literature on technology licensing by adding to the literature on the duration of contracts, sequential innovations and a model of technology leakage. We show in this framework that it may be optimal for the innovator to limit the length of fixed-fee licenses to less than that of the underlying patent. We find that long-term, but not short-term, royalty contracts can be optimal, even under complete information and risk neutrality, because they allow the innovator to resolve a time-consistency problem caused by sequential innovation and technology leakage. This implies that royalty contracts are on average of longer duration than fixed-fee contracts, a result generally consistent with empirical findings.

It has long been recognized that the market of technology licensing is imperfect (e.g., Caves, Crookell and Killing 1983). While other papers in the literature of technology licensing have dwelt on incomplete information, moral hazard, risk and uncertainty, our paper focuses on the irreversibility of technology transfer and the incentive to engage in sequential innovations. In particular, we introduce the notion of technology leakage, which is shown to be an important determinant in an innovator's choice of licensing contracts. Nonetheless, it remains an under-explored topic, which we believe will lead to fruitful researches.

The model presented here has made strong assumptions that can potentially relaxed. First, one may examine whether our results extend beyond the artificial two-period model. Second, in our model, technology leakage is studied in detail only when one firm has a cost advantage over other firms hence an exclusive license is the best form of contract. It can be challenging yet worthwhile to quantify technology leakage in more general cases. On a related point, the optimal contract when there is cost asymmetry among potential licensees deserves more attention in the literature.\(^{28}\) Third, one can extend the analysis by allowing

\(^{28}\text{Jehiel and Moldovanu (1996, 2000) and Jehiel and Moldovanu and Stacchetti (1996) include models of licensing to asymmetric firms, but there is only one license which may be sold to a single buyer. Hoppe, Jehiel and Moldovanu (2006) considers a game in which multiple licenses are auctioned, but they impose restrictions on parameter values that are not always satisfied in our model. Bagchi (2007) analyzes the}
more general licensing schemes, including two-part tariffs. Last, we have contented ourselves with a positive analysis, but new and interesting questions will arise in a normative analysis. These questions are left for future research.
A Proofs

Proposition 5 If the period 1 innovation is drastic, then any long-term fixed-fee contract with \( k \geq 2 \) licensees in period 1 cannot be optimal.

Proof. Under Cournot competition with linear demand, if \( k \geq 2 \) long-term licenses are sold in period 1 for a fixed fee, then the period 2 industry profits will be at most \( 8\pi^M(c_1)/9 \) if the new innovation fails, since \( \pi^D(c_1) = 4\pi^M(c_1)/9 \). This means that the total licensing revenue will be at most \( \pi^M(c_1) + \delta [A + \rho(\pi^M(c_2) - A)^2] \) where \( A = 8\pi^M(c_1)/9 \). It is less than the benchmark profit \( \Pi^B \) by \( \delta \rho(\pi^M(c_1) - A) \left[ \rho(\pi^M(c_1) + A) + (1 - 2\rho\pi^M(c_2)) \right] \geq \delta \rho(\pi^M(c_1) - A) (\pi^M(c_1) + A) = 17\delta \rho[\pi^M(c_1)]^2/81 \). Now consider two cases. First, if \( \tau_1 \leq \tau_2 \) then according to Lemma 7, a long-term fixed-fee exclusive contract can replicate the benchmark outcome. Therefore, issuing multiple fixed-fee licenses cannot be optimal. Second, if \( \tau_1 > \tau_2 \), then \( \delta \rho(\tau_1 - \tau_2)^2 \leq \delta \rho \tau_1^2 \leq (1/16) \delta \rho[\pi^M(c_1)]^2 \), where \( \tau_1 \leq \pi^M(c_1)/4 \) (see the proof of Assumption 2 in Appendix B). Since \( \Pi^B - \Pi^{SF} = \delta \rho(\tau_1 - \tau_2)^2 \), we must have \( \Pi^{LF}(k \geq 2) - \Pi^{SF} \leq (1/16) \delta \rho[\pi^M(c_1)]^2 - (17/81) \delta \rho[\pi^M(c_1)]^2 < 0 \).

Under Bertrand competition, if \( k \geq 2 \) long-term licenses are sold in period 1, then the industry profits will be zero in period 2 if the new innovation fails. This means that the total licensing revenue will be at least \( \delta \rho[\pi^M(c_1)]^2 \) less than the benchmark profit. According to Assumption 3 and its proof in Appendix B, \( 0 < \tau_1 - \tau_2 < \pi^M(c_1) - \tau_1 \), we have \( \delta \rho(\tau_1 - \tau_2)^2 < \delta \rho(\pi^M(c_1) - \tau_1)^2 \leq \delta \rho[\pi^M(c_1)]^2 \). Hence \( \Pi^{LF}(k \geq 2) < \Pi^{SF} \).

Proposition 6 If the period 1 innovation is non-drastic, then short-term royalties cannot be optimal.

Proof. Under Cournot competition with linear demand, we prove by showing that, for any short-term royalty, there exists a fixed-fee licensing scheme that issues a smaller or equal number of short-term licenses and generates a greater period 1 licensing revenue. By the
same argument used in the proof of Lemma 8, we can then conclude that the short-term fixed-fee licensing scheme must be more profitable than the short-term royalty.

Denote by \( k^* \) the optimal number of licensees in a one-innovation game when fixed-fee licensing is used. We have \( \Gamma^{R(n)} < \Gamma^{FF(k^*)} \) according to Kamien and Tauman (1986). Consider a short-term royalty scheme \( SR \) that issues \( k \) royalty licenses in period 1. If \( k > k^* \), then a short-term fixed-fee contract with \( k^* \) licensees generates a greater period 1 licensing revenue and has smaller technology leakage. Now if \( k < k^* \), then we can consider a short-term fixed-fee contract with \( k \) licensees. The period 1 licensing revenue is \( \Pi_1^{SF} > \Gamma^{FF(k)} = k[q(p - c_1) - q'(p - c_0)] \) where \( p \) is the equilibrium price and \( q \) (resp. \( q' \)) is the quantity produced by a licensee (resp. non-licensee), while \( \Pi_1^{SR} = \Gamma^{R(k)} = kq(n)(c_0)(c_0 - c_1) \), where \( q(n)(c_0) \) is Cournot quantity with \( n \) identical firms whose costs are \( c_0 \). Since \( k < k^* \), we have \( p > c_0 \) (Kamien and Tauman 1986). In addition, \( q > \max[q', q^{(n)}(c_0)] \). Hence \( \Pi_1^{SF(k)} > k[q(p - c_1) - q'(p - c_0)] = k[(q - q') (p - c_0) + q(c_0 - c_1)] > kq(c_0 - c_1) > \Pi_1^{SR(k)} \).

Under Bertrand competition, any fixed-fee license must be exclusive regardless of whether the innovation is drastic, so the proof used in the drastic innovation case also applies here.

B The cost of technology leakage in Homogenous Good, Conjectural Variation Oligopoly Models

In the main text, we make assumptions on the costs of technology leakage. Here we verify that these assumptions are met in homogenous good, conjectural variation oligopoly models including Cournot competition with linear demand and Bertrand competition.

**Proof of Assumption 1.** Suppose that a licensing scheme \( S \) exists such that the innovator’s licensing revenue equals \( \pi^M(c_b) \). Since post-licensing competition takes place among firms who sell homogenous good at a uniform price, we must have \( p = p^M(c_b) > c_a \) and \( \pi_i = 0 \) for all \( i = 1, 2, ..., n \). Now if firm 1 chooses not to license the innovation, then \( \pi_1 = q_1 \left( p^M(c_b) \right) \left[ p^M(c_b) - c_a \right] > 0 \). This means that \( S \) cannot be an equilibrium. Contradiction.
Proof of Assumption 2 and 3 Under Cournot Competition with Linear Demand.

Following Lemma 1, we consider a licensing scheme that involves the Right of First Offer to the firm who has a cost of $c_b$ (firm 1) with the threat of an auction of fixed-fee licenses: in stage 1, the innovator offers firm 1 a fixed-fee exclusive license; if firm 1 accepts, then the game ends; but if firm 1 rejects, then the innovator sells $k$ licenses in stage 2 via a sealed-bid first-price auction. The cost of technology leakage is firm 1’s net profit in stage 2, which depends on the number of licenses that will be issued and whether firm 1 places a winning bid.

Divide the range of possible values for $c_a$ into three regions: (i) $c_a \in [c_b, \frac{a+2c_b}{3})$, (ii) $c_a \in [\frac{a+2c_b}{3}, \frac{a+c_b}{2})$ and (iii) $c_a \geq \frac{a+c_b}{2}$. In region (i), a firm with cost $c_a$ will earn positive profits in competition with two firms that have cost $c_b$. In region (ii), a firm with cost $c_a$ will earn positive profits in competition with one firm that has cost $c_b$, but zero profit with two or more firms. In region (iii), a firm with cost $c_a$ will be driven out of the market if any other firm has a cost of $c_b$.

i) Within this region we consider two possibilities, either $k \geq 2$ or $k = 1$. We will show that the licensing revenue from auctioning $k \geq 2$ licenses has a lower bound and that it is greater than the licensing revenue from $k = 1$ licenses. Therefore, the cost of technology leakage is firm 1’s net profit when $k \geq 2$ licenses are auctioned, which has an upper bound.

If $k = 1$, then firm 1 will outbid others, since $\pi^M(c_b) > \pi^D(c_a, c_b) + \pi^D(c_b, c_a)$. In this case, the winning bid will be the profits that a licensee would earn in competition with firm 1, $\pi^D(c_b, c_a) = (a + c_a - 2c_b)^2/9$.

If $k \geq 2$, then the winning bid is at least $(a + c_a - 2c_b)^2/(k + 2)^2$, the profit that a firm with cost $c_a$ can earn as a licensee if firm 1 is not among the licensees. Thus, the licensing revenue must be at least $\max_k k(a + c_a - 2c_b)^2/(k + 2)^2 = (a + c_a - 2c_b)^2/8 > (a + c_a - 2c_b)^2/9$, the licensing revenue when $k = 1$. Next, if firm 1 wins a license, then its bid must be at least $(a + c_a - 2c_b)^2/(k + 2)^2$. Therefore, $\tau(c_a, c_b) \leq \pi^k(c_b) - (a + c_a - 2c_b)^2/(k + 2)^2 \leq [(a - c_b)^2/(k + 1)^2 - (a - c_b)^2/(k + 2)^2]_{k=2} < \pi^M(c_b)/4$. If firm 1 does not win a license, then
its profit as well as \( \tau(c_a, c_b) \) will be 
\[
(a - c_a - kc_a + kc_b)^2 / (k + 2)^2 \leq (a - c_b)^2 / 16 = \pi^M(c_b)/4.
\]
Further, since \( c_a < \frac{2 + 2c_b}{3} \), we get 
\[
\tau(c_a, c_b) \leq \pi^M(c_b)/4 < \pi^M(3c_b-a)/4 = 9\pi^M(c_a)/16.
\]

\( ii) \) Within this region, if \( k \geq 2 \), then firm 1 can compete in the market only if it wins a license. This means that the innovator can earn at least \( 2\pi^D(c_b) = 8\pi^M(c_b)/9 \) by auctioning 2 licenses. Therefore, 
\[
\tau(c_a, c_b) \leq \pi^M(c_b)/9 < \pi^M(2c_a - a)/9 = 4\pi^M(c_a)/9 < 9\pi^M(c_a)/16.
\]

\( iii) \) \( \tau(c_a, c_b) = 0. \)

In sum, \( \tau(c_a, c_b) < \pi^M(c_b)/4 < 9\pi^M(c_a)/16 \). Applying the definitions of \( \tau_1, \tau_2 \) and \( \tau_3 \), we obtain that \( \tau_1 < \pi^M(c_1)/4 \) and \( \tau_3 < 9\pi^M(c_1)/16 \), therefore \( \tau_1 - \tau_2 + \tau_3 < \pi^M(c_1) \) and \( \tau_3 < \pi^M(c_1) \).

**Proof of Assumption 2 and 3 Under Bertrand Competition.** Under Bertrand competition, licenses can only be profitably sold to a single firm in each period. This is because Bertrand competition yields zero profit to each firm, unless there is only one firm with a superior technology. Thus, we can obtain an explicit solution for technology leakage:

\[
(6) \quad \tau(c_a, c_b) = \begin{cases} 
\pi^M(c_b) - (c_a - c_b)D(c_a) & \text{if } c_a < p^M(c_b); \\
0 & \text{otherwise.}
\end{cases}
\]

First, suppose that \( c_2 \) is not drastic relative to \( c' \), then 
\( \tau_1 = \pi^M(c_1) - (c' - c_1)D(c') \) and 
\( \tau_2 = \pi^M(c_2) - (c' - c_2)D(c') \). Because \( q^M(c_1) \) maximizes profits when cost is \( c_1 \), we know 
\( \tau_1 - \tau_2 = \pi^M(c_1) - \pi^M(c_2) + (c_1 - c_2)D(c') > (p^M(c_2) - c_1)q^M(c_2) - \pi^M(c_2) + (c_1 - c_2)D(c'). \)

Note that 
\( \pi^M(c_2) = (p^M(c_2) - c_1)q^M(c_2) + (c_1 - c_2)q^M(c_2). \) Therefore, 
\( \tau_1 - \tau_2 > (p^M(c_2) - c_1)q^M(c_2) - (p^M(c_2) - c_1)q^M(c_2) - (c_1 - c_2)q^M(c_2) + (c_1 - c_2)D(c') = (c_1 - c_2)(D(c') - q^M(c_2)) = (c_1 - c_2)(D(c') - D(p^M(c_2))). \) Since \( c_2 \) is not drastic relative to \( c' \) this must be positive.

Second, suppose that \( c_2 \) is drastic relative to \( c' \). In this case 
\( \tau_1 = \pi^M(c_1) - (c' - c_1)D(c') \) and \( \tau_2 = 0 \), thus \( \tau_1 - \tau_2 > 0. \)

Last, 
\( \tau_1 - \tau_2 + \tau_3 = \pi^M(c_1) - (c_1 - c_2)[D(c_1) - D(c')] < \pi^M(c_1). \)
More General Classes of Downstream Competition

In the main text, we have focused on the case of homogenous good, conjectural variation oligopoly models, which allows us to impose restrictions on the costs of technology leakage. For completeness, in this appendix, we solve for the optimal fixed-fee licensing contracts without restricting the nature of downstream competition. In addition to generalizing our main result, the following results further illustrate the important role played by technology leakage in our model.

**Lemma 10** In a long-term fixed-fee contract \((f_1, f_2)\) with the opt-out clause, if either (i) \(0 < \tau_1 - \tau_2 < \pi^M(c_1) - \tau_1\) and \(f_2 \leq \pi^M(c_1) - \tau_1\), or (ii) \(\pi^M(c_1) - \tau_1 < \tau_1 - \tau_2 < 0\) and \(f_2 \leq \tau_1 - \tau_2\), then \(\Pi^{LF}\) increases with \(f_2\).

**Proof.** Case (i) is proved in the proof of Lemma 5. An analogous proof can be constructed for case (ii).

**Lemma 11** In a long-term fixed-fee contract \((f_1, f_2)\) with the opt-out clause, if either (i) \(\tau_1 - \tau_2 < \pi^M(c_1) - \tau_1 < 0\) and \(f_2 \leq \pi^M(c_1) - \tau_1\), or (ii) \(\tau_1 - \tau_2 > \pi^M(c_1) - \tau_1 > 0\) and \(f_2 \leq \tau_1 - \tau_2\), then \(\Pi^{LF}\) decreases with \(f_2\).

**Proof.** The following proof applies to case (i). An analogous proof can be constructed for case (ii).

In period 2, there are two states of nature: 1. innovation is not successful; 2. innovation is successful. In case 1, the original licensee will continue the contract and get \(\pi^M(c_1) - f_2\), since \(\pi^M(c_1) - f_2 > \tau_1\). Hence \(\Pi_2 = f_2\).

In case 2, we separate \(f_2\) further into two regions: a) \(f_2 \leq \tau_1 - \tau_2\) and b) \(\tau_1 - \tau_2 < f_2 < \pi^M(c_1) - \tau_1\).

(2.a) \(f_2 \leq \tau_1 - \tau_2\). If the original licensee continues the initial contract and produces at a cost of \(c_1\), he gets \(\tau_1 - f_2\); if he opts out, he gets \(\tau_2\). Since \(\tau_1 - f_2 \geq \tau_2\), it is optimal for
the original licensee to continue the original license. This means that $\Pi'_2 = \pi^M(c_2) - \tau_1 + f_2$ and $\Delta = \pi^M(c_2) - \tau_1$. So the innovator’s incentive to invest is independent of $f_2$. Therefore, her licensing revenue is a constant if $f_2 \leq \tau_1 - \tau_2$.

(2.b) $f_2 > \tau_1 - \tau_2$. If the original licensee continues the initial contract and produces at a cost of $c_1$, then he gets $\tau_1 - f_2$; if he opts out, then he gets $\tau_2$. Since $\tau_1 - f_2 < \tau_2$, the original licensee’s right to use the old innovation has no value and he will opt out the initial contract. This means that $\Pi'_2 = \pi^M(c_2) - \tau_2$ and $\Delta = \pi^M(c_2) - \tau_2 - f_2$. In period 1, a firm is willing to pay $\Pi^{LF}_1 = \pi^M(c_1) + \delta (1 - 2\rho (\pi^M(c_2) - \tau_2 - f_2)) (\pi^M(c_1) - f_2) + 2\delta \rho (\pi^M(c_2) - \tau_2 - f_2) \tau_2$ for an exclusive license. At the same time, $\Pi^{LF}_2 = f_2 + \rho (\pi^M(c_2) - \tau_2 - f_2)^2$. Hence, the total licensing revenue is $\Pi^{LF} = \pi^M(c_1) + \delta (1 - 2\rho (\pi^M(c_2) - \tau_2 - f_2)) (\pi^M(c_1) - f_2) + 2\delta \rho (\pi^M(c_2) - \tau_2 - f_2) \tau_2 + \delta \left( f_2 + \rho (\pi^M(c_2) - \tau_2 - f_2)^2 \right)$, so $\frac{\partial}{\partial f_2} \Pi^{LF} = 2\rho \delta (\pi^M(c_1) - \tau_2 - f_2) < 2\rho \delta (\pi^M(c_1) - \tau_1) < 0$. Last, it is easy to verify that $\Pi^{LF}$ is continuous at $f_2 = \tau_1 - \tau_2$.

**Lemma 12** If either (i) $0 < \tau_1 - \tau_2 < \pi^M(c_1) - \tau_1$ and $f_2 \geq \pi^M(c_1) - \tau_1$, or (ii) $\pi^M(c_1) - \tau_1 < \tau_1 - \tau_2 < 0$ and $f_2 \geq \tau_1 - \tau_2$, or (iii) $\tau_1 - \tau_2 < \pi^M(c_1) - \tau_1 < 0$ and $f_2 \geq \pi^M(c_1) - \tau_1$, or (iv) if $\tau_1 - \tau_2 > \pi^M(c_1) - \tau_1 > 0$ and $f_2 \geq \tau_1 - \tau_2$, then any equilibrium long-term fixed-fee contract $(f_1, f_2)$ with the opt-out clause is equivalent to a short-term fixed-fee contract with $f = f_1$.

**Proof.** Case (i) is proved in the proof of Lemma 6. Analogous proofs can be constructed for the other cases.

**Lemma 13** If either (i) $\tau_1 - \tau_2 < 0$ and $\pi^M(c_1) - \tau_1 > 0$, or (ii) $\pi^M(c_1) - \tau_1 < 0$ and $\tau_1 - \tau_2 > 0$, then there exists a long-term fixed-fee contract that replicates the benchmark outcome.

**Proof.** Case (i) is proved in the proof of Lemma 7. An analogous proof can be constructed for case (ii).

From the above lemmas, we can conclude the following for fixed-fee contracts.
Proposition 7 (i) If $\tau_1 - \tau_2$ and $\pi^M(c_1) - \tau_l$ have different signs, then there exists a long-term fixed-fee contract that replicates the benchmark outcome; (ii) if they have the same sign, then the optimal fixed-fee license depends on the comparison of their absolute values: if $|\tau_1 - \tau_2| > |\pi^M(c_1) - \tau_l|$, then a long-term contract with an upfront payment is optimal and generates a licensing revenue that is $\delta \rho \left[ \pi^M(c_1) - \tau_l \right]^2$ below the benchmark profit, otherwise a short-term contract is optimal and generates a licensing revenue that is $\delta \rho (\tau_1 - \tau_2)^2$ below the benchmark profit.
References


