Joining forces to attract consumers: location choice in a consumer search model.*

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March 18, 2009

Abstract

This paper studies the location choice of competing shops in a model with costly consumer search. Shops either are isolated or locate in a mall where one or more competitors are present. The paper shows that an increase in the number of competing shops in a mall leads to lower expected prices, but also makes more consumers willing to search and buy. Moreover, mall shops attract a larger share of consumers than isolated shops. The equilibrium mall size is computed for several parameter values, showing that the mall size can take values from one to all shops in the market.

Keywords: location choice, consumer search, pricing

JEL codes: D43, D83, L11, L13

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*I am indebted to Maarten Janssen, Marco Haan and Jose Luis Moraga-Gonzalez for their useful comments. The paper has also benefitted from presentations at Erasmus University Rotterdam, Tinbergen Institute Rotterdam, University of Groningen and the EARIE 2008 meeting (Toulouse). Financial support from Marie Curie Excellence Grant MEXTCT-2006-042471 is gratefully acknowledged

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1 Introduction

Shopping centers come in different forms and sizes, ranging from small in-town mini-malls or strip malls to regional and super-regional malls. A mini-mall offers a limited array of shops, while regional and super-regional malls can contain more than 500 different shops. Given this difference in size, it is no surprise that regional and super-regional malls typically contain several grocery stores, hair and nail salons, clothing shops, restaurants, and so on, while strip malls typically offer only one shop per category. This means that the shops in a small mall typically do not have direct competitors, while shops in regional malls have to compete with other shops offering similar products. In this paper I will use the term isolated shop for a shop in a small mall that has no direct competitors, while a shop that does have competitors will be called a mall shop. Both types of shops coexist, and one might wonder how this is possible. At first sight, it seems unattractive to locate next to some direct competitors in a regional mall. On the other hand, once a regional mall with strong competition and low prices exists, how can isolated shops survive?

Previous research typically only answers the question why large malls exist, see e.g. Stahl (1982a, 1982b), Gehrig (1998) and Konishi (2005). None of these papers finds the existence of isolated shops. The intuition that these papers give revolves around the heterogeneity of goods. When goods are heterogeneous, consumers prefer to visit a mall with a large variety to increase the probability of finding a good match. This increases the volume of sales in a large mall and makes it profitable to locate together. This effect of heterogeneous goods is however not the complete story. Even when goods are heterogeneous and malls are more attractive for consumers, isolated shops could survive by lowering their price. This does not occur in equilibrium because of a simplifying assumption that the papers mentioned before make: consumers can only visit one mall, independent of whether it is a large or small mall. When a consumer would decide to visit an isolated shop, he or she is stuck there and the isolated shop has an incentive to ask a monopoly price. Consumers anticipate this and prefer to go to the large mall where prices are lower and the choice is greater. Consequently, isolated shops attract no consumers and in equilibrium do not exist.

The current paper assumes that consumers can visit different shops in different malls. There are costs to travel between market places, but if an isolated shop sets a price that is much higher than in a mall, consumers who initially visited the isolated shop will not buy there, but instead will

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1 Some of the papers mentioned above assume a spatial structure where consumers and shops are spread along a line or plane. In this case there are some consumers who prefer the isolated shop because it is much closer to their home than the mall. But because of the high price and limited choice in the isolated shop, the number of consumers they attract is very small and generally not enough to make an isolated shop profitable.
continue to search in other shops. Isolated shops are thus forced to adjust their price towards the mall prices and because of this they attract a share of consumers. The current paper shows that in equilibrium isolated shops can exist, even when there also is mall with several competitors located next to each other.

In order to focus on the role of this pricing mechanism, the current paper assumes homogenous goods. Even though the variety argument does not hold anymore and the (price) competition between mall shops is tough, there still can be a mall with several competing shops. The reason for this is twofold. First, the search costs of consumers are defined in such a way that once in a mall shop a consumer incurs some small cost to search another shop in the same mall. This softens the competition between mall shops enough to make above-zero pricing in the mall possible, which is a necessary condition for a mall to exist. Second, as will be discussed later, mall shops attract more consumers than isolated shops, especially when the mall is small. This increased sales volume makes locating in a mall attractive. Thus, even in a setting where variety does not play a role a mall can exist. Heterogeneity could be introduced in the current model, but as long as the heterogeneity is moderate it most likely will not change the results. When heterogeneity is introduced, the variety argument will be added to the pricing mechanism. This will somewhat increase the attractiveness of a mall and decrease the attractiveness of an isolated shop. But isolated shops will only cease to exist when the variety argument gets too pronounced.

There are some other papers on the location choice of shops that should be mentioned here. First, as far as I know, Dudey (1990, 1993) are the only papers also assuming homogenous goods. In these models, consumers can only visit one market place, but once in a mall a consumer can visit all shops in that mall for free. To make sure mall prices are above zero, shops compete in quantities. Because consumers can visit only one market place, and because the largest mall has the lowest prices, all consumers visit the largest mall and isolated shops do not exist. Wolinsky (1983) is one of the few papers that assume consumers can visit more than one market place. Wolinsky however only analyzes conditions under which all shops locate in the same mall, and does not give any attention to the possibility of isolated shops. The paper that comes closest to the current paper is Fischer and Harrington (1996). In their model products are heterogeneous, consumers can visit more than one market place and once in the mall all shops in the mall can be visited for free. To keep their model tractable, Fischer and Harrington however need to assume that consumers expect an infinite number of isolated shops, even while in equilibrium there is a finite number of isolated shops. In the current paper, consumers know the (finite) number of isolated shops beforehand, and act accordingly. Moreover, the current paper shows that product heterogeneity is not necessary to find an equilibrium with both mall and isolated shops.
Apart from the joint existence of mall and isolated shops, the current paper adds some other interesting insights to the literature. First of all, the pricing behavior of shops has some special features. The model assumes that there are some consumers, called shoppers, who incur no costs when searching different shops. They therefore know all prices and buy at the cheapest store. All other consumers, called non-shoppers, incur some cost to search a shop. Mall shops compete for the shoppers, but have some power over the non-shoppers. The shops balance these two effects by randomizing over prices. Isolated shops also randomize over prices, but choose a different support than the mall shops. An isolated shop either sets a price that is above the maximum supported mall price or set a price that is relatively low. This is caused by the costs that consumers incur when traveling between different market places. When a non-shopper is currently in an isolated shop he has to incur costs to travel to another shop. When a non-shopper is in a mall shop, continuing search within the mall comes at lower costs. Therefore, when in an isolated shop non-shoppers are willing to pay a somewhat higher price than when in a mall shop. On the other hand, an isolated shop has to make sure that consumers still want to visit it. It does so by sometimes offering a low price relative to the mall prices, such that the expected isolated price equals the expected mall price. This could be interpreted as isolated shops usually being more expensive than mall shops, but sometimes offering a large discount. An isolated shop will attract non-shoppers who hope to be lucky enough to find a discount. But even if the non-shopper does not find a discount he will stay at the isolated shop since to search further the non-shopper has to incur travel costs. Note also that setting a low price increases the probability of attracting the shoppers. This ensures that low and high prices are equally profitable.

Another interesting result is that mall shops attract more non-shoppers than isolated shops. The intuition behind this is fairly straightforward. If isolated shops would attract many non-shoppers, they would make more profits on the high prices than on the low prices, which cannot be an equilibrium situation. Put differently, isolated shops are only willing to randomize over prices when the shoppers are relatively important for them. For mall shops, the difference between high and low prices is smaller and therefore they are willing to randomize over prices even when they attract many non-shoppers. A simple consequence of the uneven distribution of non-shoppers over mall and isolated shops is that mall shops make more profits than isolated shops. Still, this does not imply that all isolated shops want to join the mall shops. If an isolated shop relocates and joins a mall, the mall size increases, which increases competition in the mall and decreases expected prices. The remaining isolated shops have to adjust their prices as well to attract some consumers, and as a result the profits of both mall and isolated shops decrease when an isolated shop relocates to a mall. Whether
it is profitable to join a mall depends on the number of consumers that are gained and on the size of the decrease in prices. As the paper will show, in some cases it is profitable to join a mall and capture additional non-shoppers, while in other cases the decrease in prices is too strong to make joining a mall profitable.

When the costs of visiting a shop are high, a third effect plays an important role in the location choice of shops. As in Janssen et al. (2005), when the costs to visit a shop are high, some non-shoppers stay at home and do not buy at all. When this happens and when more shops locate in the same mall, prices tend to decrease and the participation of non-shoppers increases. Thus, when an isolated shop joins a mall it will capture a larger share of non-shoppers and the total amount of non-shoppers will increase. The joint effect on the sales of the isolated shop that relocates is so strong that it is always profitable to join a mall. So for high enough search costs all isolated shops will want to join a mall and the only possible equilibrium has no isolated shops.

In this paper a three stage game is considered. In the first stage shops choose a location that will maximize their individual profits, in the second stage shops jointly set prices and in the last stage consumers decide whether and where to search and buy. The next section presents the model that will be used. Section 3 analyzes the second and third stage for two extreme cases. In one case all shops are isolated and in the other case all shops are located in the same mall. In both cases consumers perceive no difference between shops and spread evenly over shops. This implies that only pricing and the total amount of active non-shoppers play a role. The two extreme cases are thus very suitable to isolate the effect of the total amount of active non-shoppers. Moreover, Section 3 will build some intuition before heading on to the more complicated case with both mall and isolated shops. Section 4 will analyze the second and third stage of the model for this more complicated case. Section 5 discusses some of the results found in Section 4. Finally, Section 6 analyzes the location choice of shops and Section 7 concludes. The proofs are in the Appendix.

2 The model

The model has $n > 2$ shops in the market that sell a homogeneous good. Production costs are linear and without loss of generality they are taken to be zero. As mentioned in the Introduction, the model has three stages. In the first stage shops choose a location that will maximize their individual future profits. I assume that only one mall can be formed. One can think of a town that has one regional mall with ample space for new shops and several much smaller in-town mini malls that have no space to expand and
accommodate new shops. A shop thus has to choose between locating in the regional mall, next to some competitors, or locating in a mini mall without any direct competitors. In the remainder of this paper I will refer to the regional mall with several competitors as the 'mall'. A mall with $k^*$ shops is an equilibrium when none of the mall shops can increase its profits by leaving the mall and none of the isolated shops can increase its profits by joining the mall.

In the second stage, the shops choose a price. I explicitly allow for a mixed strategy and therefore the strategy of a shop $i$ can be denoted by a price distribution $F_i(p)$, where $F_i(p)$ is the cdf, the maximum price is denoted by $P_i$ and the minimum price by $p_i$. Note that if shop $i$ chooses a pure price strategy with price $p_i$ the price distribution is given by $F_i(p) = 0$ for $p < p_i$ and $F_i(p) = 1$ for $p \geq p_i$. In the next sections it will however become clear that there is no symmetric pure strategy equilibrium.

In the third stage of the model consumers decide on whether and where to search and buy. The model has a unit mass of consumers, all having unit demand and a valuation $\theta$ for the product. The consumers are aware of all the locations of the shops, but they do not know the prices in the shops. They however form rational price expectations and base their decisions on these expectations.

There are two different types of consumers. A fraction $\gamma$ of consumers consists of shoppers who have zero search costs. As a consequence shoppers know all the prices and buy at the cheapest shop. A fraction $1 - \gamma$ of consumers consists of consumers that incur strictly positive search costs. These consumers are referred to as non-shoppers. Non-shoppers incur costs $c_e$ when entering a shop. These costs are incurred whenever a not previously visited shop is entered and do not depend on whether a shop is in a mall with several shops or is an isolated shop. The entering costs are equivalent to the continuation costs in a standard consumer search model. These costs reflect the time spent in the shop, finding the product on the shelf, finding the price of the product, waiting for a shop assistant to help you, etc. Note that positive entering costs are essential in the model. Without entering costs non-shoppers could without additional costs search all the shops in the mall. This would drive the mall prices to zero, and no shop would ever locate in a mall. In addition to the entering costs, non-shoppers incur travel costs $c_t$ whenever they travel from their house to a market place or travel between market places, where a market place can be either a shopping mall

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2One could think of shoppers as consumers who obtain a strictly positive utility from the shopping experience, even if travel expenses are taken into account. For the results it is not strictly necessary that there are consumers with zero search costs who know all prices. The less restrictive assumption that some fraction $\gamma$ of consumers gets to know the prices of two or more random shops without incurring search costs would be sufficient to obtain the results in this paper. For simplicity I however assume the presence of a fraction of consumers with zero search costs.
with several shops selling the product or an isolated shop. The travel costs are incurred every time a non-shopper travels between market places, and therefore are also incurred when returning to a previously visited shop that is in a different market place than the market place where the non-shopper currently is. The travel costs can be interpreted as the costs of, say, a bus ticket or petrol costs. The travel costs ensure that searching \( h \) shops in the same mall comes at less costs than searching \( h \) shops spread over different clusters. Finally, the analysis is restricted to values of \( c_e \) and \( c_t \) for which \( c_t + c_e \leq \theta \).

Non-shoppers search sequentially. This means that non-shoppers first decide on whether to stay at home, visit a mall shop or visit an isolated shop. Let \( \mu \) denote the fraction of non-shoppers who decide to visit a shop and let \( 1 - \mu \) denote the fraction of non-shoppers who decide to stay at home. The fraction \( \mu \) of non-shoppers will be referred to as active non-shoppers. Based on the price found in the first shop, an active non-shopper decides on whether to search a second shop and whether this second search will be in the same market place as the first search (if possible) or in another market place. Then, based on the outcome of the second search, active non-shoppers decide on whether or not to search a third time and where the third search will be, etc.

In the analysis below I will derive a subgame perfect equilibrium of the three stage game described in this section. I will focus on symmetric equilibria in the sense that all shops in the same market place choose identical price distributions and market places of the same size have identical price distributions as well. Note that identical price distributions does not necessarily imply identical prices since realized prices could differ from each other. Since price distributions are symmetric where possible and since I only consider situations where there is at most one shopping mall I drop the shop index \( i \) in \( F_i(p) \) and in \( p_i \) and \( p^m \) but instead use an index \( k \) denoting the number of firms in the shopping mall. Where necessary I add an index \( m \) for mall or \( i \) for an isolated shop. So \( F_i(p) \) is the price distribution used by all shops when they are all isolated shops and \( F_n(p) \) is the price distribution when all shops are in the same shopping mall. \( F_i^k(p) \) and \( F_i^m(p) \) are the price distributions of an isolated shop while there also is a cluster of \( k \) shops and of a shop located in a mall with \( k \) shops, respectively. The index \( k \) will also be added to \( \mu \), such that \( \mu_k \) denotes the fraction of active non-shoppers when

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3These return costs are necessary to prevent arbitrage. Imagine a situation of one mall with shops 1 and 2 and two isolated shops, 3 and 4. Now suppose a non-shopper’s first search was in shop 1 and the second search was in shop 3. If there are no return costs and the non-shopper would like to visit shop 2 in his third search he could go there immediately at cost \( c_t + c_e \), but he could also return to shop 1 at no costs and then visit shop 2 at cost \( c_e \). To prevent such a situation return costs of at least \( c_t \) are necessary when returning to a shop in a different market place.
there is a mall with \( k \) shops.

Because shops choose symmetric pricing strategies, non-shoppers a priori have no preferences over shops that are located in the same mall. Moreover, non-shoppers a priori have no preferences over the isolated shops. Once a non-shopper has chosen to visit the mall he will therefore choose a random shop from this mall. In the same vein, once a non-shopper has decided to visit an isolated shop he will choose such a shop at random.

3 Two opposite cases: only isolated shops and only mall shops

3.1 Only isolated shops

In this subsection the consumer behavior and pricing behavior of shops are analyzed when all shops are isolated. In this case each visit to a shop comes at cost \( c_t + c_e \), and each return visit to a previously visited shop comes at cost \( c_t \). This model is equivalent to the model in Janssen, Moraga-Gonzalez and Wildenbeest (2005) (henceforth JMW) except for the return costs, which are absent in the JMW model. It is relatively easy to show that the equilibrium derived in JMW also holds in a model with return costs and in this section I will focus on this equilibrium.\(^4\) The discussion here will necessarily be short. A formal proof of all assertions in this subsection can be found in the Appendix. JMW also provides more details.

One key component of the equilibrium will be the so-called reservation price \( r_1 \), implicitly defined by

\[
\int_{p_1}^{r_1} (r_1 - p) dF_1(p) = c_t + c_e.
\]

I will concentrate on an equilibrium with \( p_1 \leq r_1 \). The derivation of this equilibrium shows that this is the only equilibrium with \( p_1 \leq r_1 \), but because of the return costs there is a possibility that equilibria with \( p_1 > r_1 \) exist as well. First consider the behavior of non-shoppers. Suppose a non-shopper has visited at least one shop, and that he found a price \( p^* \) in the shop he last visited. Also denote by \( p^{\min} \) the lowest price he found in previously visited shops, with \( p^{\min} \) infinitely large when no shops were previously visited. If the non-shopper stops searching, he can buy at a price \( q \equiv \min(p^{\min} + c_t, p^*) \).

When he searches one more time he expects to gain \( \int_{p_1}^{p} (q - p) dF_1(p) \), but incurs costs \( c_t + c_e \). Thus, a non-shopper will continue to search when

\(^4\)Return costs complicate a full analysis considerably and could potentially lead to multiple equilibria. See Janssen and Parakhonyak (2008) for an analysis of consumer behavior under the assumption of return costs.
\( q > r_1 \) and will stop searching when \( q \leq r_1 \). Note that because \( \overline{p}_1 \leq r_1 \), non-shoppers expect to stop searching after their first search.

Now suppose a non-shopper has not yet searched any shops. Searching a shop will give expected gains \( \theta - E_{p_1} - c_t - c_e \), with \( E_{p_1} \) the expected price. Because \( \overline{p}_1 \leq r_1 \), \( E_{p_1} = \int_{p_1}^{r_1} p \, dF_1(p) \) and the definition of \( r_1 \) gives \( r_1 - E_{p_1} = c_t + c_e \). Using this, the expected gains of search can be written as \( \theta - r_1 \). It is clear that all non-shoppers will search when \( r_1 < \theta \). In this case \( \mu_1 = 1 \) and this situation will be referred to as 'full search'. In the Appendix it is shown that in equilibrium \( r_1 < \theta \) holds if and only if \( c_t + c_e \) is below some threshold \( C^* \). When \( r_1 = \theta \), non-shoppers are indifferent between searching and not searching. In this case, only a fraction of non-shoppers will be active (\( 0 < \mu_1 < 1 \)) and this situation will be called 'partial search'. At first sight, it might seem that \( r_1 = \theta \) only holds for very specific values of \( c_t + c_e \). This is however not true. The reservation value \( r_1 \) depends on \( F_1(p) \) and in turn depends on \( \mu_1 \). For \( c_t + c_e \) above \( C^* \) it is always possible to find a value of \( \mu_1 \) that satisfies \( 0 < \mu_1 < 1 \) and that defines \( F_1(p) \) in such a way that \( \int_{p_1}^{r_1} (r_1 - p) \, dF_1(p) = c_t + c_e \). The case \( \theta < r_1 \) cannot occur in equilibrium. If \( \theta < r_1 \), non-shoppers would not search at all. Then the shoppers would drive the prices down to zero and consequently the non-shoppers expect a utility \( \theta - c_t - c_e \). Since the model assumes \( c_t + c_e \leq \theta \) the non-shoppers expect a non-negative utility from searching, contradicting \( \theta < r_1 \).

Specifying the optimal pricing behavior of shops now is a fairly straightforward exercise. When all shops apart from one deviating shop set a price at or below \( r_1 \), the deviating shop will never sell to the shoppers. Moreover, the non-shoppers who visit the deviating shop will continue to search and will buy at a cheaper shop. The deviating shop thus will not sell anything, showing that deviating from \( \overline{p}_1 \leq r_1 \) is not profitable. The profit function for \( p \leq r_1 \) is given by

\[
\pi_1(p) = p\gamma(1 - F_1(p))^{n-1} + p(1 - \gamma)\mu_k \frac{1}{n}
\]

with \( \mu_k = 1 \) in a full search equilibrium and \( 0 < \mu_k < 1 \) in a partial search equilibrium. This profit function is the same as in JMW, and the analysis is identical. See their paper for more details. The next Propositions summarize.

**Proposition 3.1** (Full search equilibrium)
If \( c_t + c_e < \theta(1 - \int_0^1 \frac{1}{1+\gamma ny^n-dy}) \) then all non-shoppers are active and non-shoppers will stop searching as soon as \( \min(p^*, p^{\text{min}} + c_t) \leq r_1 \), with \( r_1 \) defined as

\[
r_1 = \frac{c_t + c_e}{1 - \int_0^1 \frac{1}{1+\gamma ny^n-dy}} .
\]
Shops randomize over prices according to the price distribution

\[ F_1(p) = 1 - \left( \frac{1 - \gamma}{\gamma n} r_1 - p \right) \frac{1}{n-1}. \]

The maximum price asked by a shop is \( r_1 \), the minimum price equals \( \frac{1 - \gamma}{1 + \gamma (n-1)} r_1 \) and expected profits are given by \( \pi_1 = r_1 \frac{1 - \gamma}{n} \).

**Proposition 3.2.** (Partial search equilibrium)

If \( c_t + c_e > \theta (1 - \int_0^1 \frac{1}{1 + (1 - \gamma) \mu_1} y^{\mu_1-1} dy) \) a fraction \( 0 < \mu_1 < 1 \) of the non-shoppers is active, while the remaining fraction \( 1 - \mu_1 \) of non-shoppers stays at home and does not buy at all. The fraction \( \mu_1 \) is implicitly defined by

\[ h(\mu_1) = \int_0^1 \frac{1}{1 + (1 - \gamma) \mu_1} y^{\mu_1-1} dy = \frac{\theta - c_t - c_e}{\theta}. \]

Active non-shoppers stop searching as soon as \( \min(p^*, p^{\min} + c_t) \leq \theta \). Shops randomize over prices according to the price distribution

\[ F_1(p) = 1 - \left( \frac{(1 - \gamma) \mu_1}{\gamma n} \theta - p \right) \frac{1}{n-1}. \]

The maximum price asked by a shop is \( \theta \), the minimum price equals \( \frac{(1 - \gamma) \mu_1}{\gamma n + (1 - \gamma) \mu_1} \theta \) and expected profits are given by \( \pi_1 = \theta \mu_1 \frac{1 - \gamma}{n} \).

Figure 1 shows the expected profits as a function of the search costs \( c_t + c_e \). In this figure, the number of firms \( n \) equals 10, \( \gamma = 0.1 \) and \( \theta = 1 \). With these parameter values the full search equilibrium holds for \( c_t + c_e < 0.073 \) and the partial search equilibrium holds for \( 0.073 < c_t + c_e < 1 \). At first sight the full search equilibrium seems to hold only for very small search costs but note that a search cost value of 0.073 implies that the search costs are still 7.3% of the valuation of the product. The expected profits are plotted for \( c_t + c_e < 0.45 \); for higher values of \( c_t + c_e \) the profits are decreasing and when \( c_t + c_e \) approaches 1 the profits approach 0. In the full search equilibrium the maximum price \( r_1 \) increases linearly with \( c_t + c_e \) and the profits are linearly increasing in \( c_t + c_e \) as well. As soon as the search costs \( c_t + c_e \) are above 0.073 profits however decrease in search costs. In this case the partial search equilibrium obtains, the maximum price equals \( \theta \) and the profits depend on the fraction of consumers who search, \( \mu_1 \). This fraction decreases in \( c_t + c_e \), leading to decreasing profits.

### 3.2 Only mall shops

When all the shops are in the same mall non-shoppers incur costs \( c_e + c_t \) for the first search, they incur costs \( c_e \) for every next search and have no return
Figure 1: Expected profits as a function of the search costs when all shops are isolated. This figure is based on 10 shops, 10% shoppers and a valuation of the product of 1.

costs. In such a setup it is possible to derive a unique equilibrium, again with a parameter region where all non-shoppers search and a parameter region where only a fraction of non-shoppers searches.

The analysis of this case follows the same lines as the analysis of the model where all shops are isolated. More specifically, define a reservation price \( r_n \) by

\[
\int_{\mathcal{P}_n} (r_n - p) dF_n(p) = c_e.
\]

Suppose a non-shopper has visited at least one shop, and suppose that \( p_{\text{min}} \) is the lowest price he found in all shops he visited, including the shop where he currently is. Then he will continue to search when \( p_{\text{min}} > r_n \) and he will stop searching when \( p_{\text{min}} \leq r_n \). Note that return costs are absent in the definition of \( p_{\text{min}} \). Moreover, note that the definition of the reservation price \( r_n \) uses only \( c_e \) instead of \( c_e + c_t \). This is because in the model where all shops are in the same mall continuing search comes at cost \( c_e \), while in section 3.1 continuing search comes at cost \( c_e + c_t \).

Non-shoppers who have to decide whether to search or stay at home also make the same kind of consideration as before. They will all search when
\( \theta - Ep_n - c_t - c_e > 0 \). The definition of \( r_n \) gives \( r_n - Ep_n = c_e \) and so all non-shoppers will be active when \( r_n < \theta - c_t \). A partial search equilibrium occurs when \( r_n = \theta - c_t \). Note that the travel costs explicitly occur in this expression. This is because non-shoppers incur travel costs when they search for the first time, while the expected prices \( (r_n - c_e) \) are based on their behavior once they are in the mall, where travel costs do not play a role anymore.

This has implications for the maximum prices that can be asked in equilibrium. In a full search equilibrium the maximum price is, as before, the reservation price. In a partial search equilibrium the maximum price however is \( \theta - c_t \). If shops would set a higher maximum price, none of the non-shoppers would be active anymore. The logic is as follows. Non-shoppers expect that in equilibrium the maximum possible price equals the expected price plus \( c_e \). Higher maximum prices would make non-shoppers willing to continue search, while at a lower maximum price shops could profitably deviate to \( Ep_n + c_e \). When the maximum price is above \( \theta - c_t \), the non-shoppers thus expect an average price above \( \theta - c_t - c_e \). When the costs of the first search are \( c_t + c_e \), the expected gains from searching are negative and no consumer is willing to search.

The full search equilibrium has the following form.

**Proposition 3.3** *(Full search equilibrium)*  
If \( c_e < \left( 1 - \int_0^1 \frac{1}{1+y^{n-1}} \frac{1}{n\gamma} \, dy \right) (\theta - c_t) \) all non-shoppers are active and non-shoppers will stop searching as soon as they find a price at or below \( r_n \), with \( r_n \) defined as

\[
r_n = \frac{c_e}{1 - \int_0^1 \frac{1}{1+y^{n-1}} \frac{1}{n\gamma} \, dy}.
\]

Shops randomize over prices according to the price distribution

\[
F_n(p) = 1 - \left( \frac{(r_n - p)(1 - \gamma)}{n\gamma p} \right)^{\frac{1}{n-1}}.
\]

The maximum price asked by a shop is \( r_n \), the minimum price equals \( \frac{1 - \gamma}{(1 - \gamma) + \gamma n} r_n \) and expected profits are given by \( \pi_n = r_n \frac{1 - \gamma}{n} \).

The partial search equilibrium is as follows.

**Proposition 3.4** *(Partial search equilibrium)*  
If \( c_e > \left( 1 - \int_0^1 \frac{1}{1+y^{n-1}} \frac{1}{n\gamma} \, dy \right) (\theta - c_t) \) a fraction \( 0 < \mu_n < 1 \) of non-shoppers is active and a fraction \( 1 - \mu_n \) of non-shoppers does not search at all, where \( \mu_n \) is defined by

\[
h(\mu_n) \equiv \int_0^1 \frac{1}{1+ \frac{\mu_n}{\gamma} y^{n-1}} \, dy = \frac{\theta - c_t - c_e}{\theta - c_t}.
\]
Active non-shoppers will stop searching as soon as they find a price at or below $\theta - c_t$. Shops randomize over prices according to price distribution $F_n(p) = 1 - \left(\frac{(\theta - c_t - p)(1 - \gamma)\mu_n}{n\gamma p}\right)^{\frac{1}{n-1}}$.

The maximum price asked by a shop is $\theta - c_t$, the minimum price equals $(\theta - c_t - p)\frac{\mu_n(1-\gamma)}{\gamma n + \mu_n(1-\gamma)}$ and expected profits are given by $\pi_n = (\theta - c_t)\frac{\mu_n(1-\gamma)}{n}$.

Figure 2 shows the expected profits as a function of the search costs $c_t + c_e$. Recall that the reservation value $r_n$ depends only on the continuation costs of search, $c_e$. Moreover, the decision whether or not to search depends on $c_t$. Therefore, in contrast to the model where all shops are isolated, the expected profits do not depend on total costs $c_e + c_t$, but on $c_t$ and $c_e$ in isolation. To be able to make a plot of the expected profits as a function of the total costs $c_e + c_t$ I assume that $c_e$ and $c_t$ are related to each other in a fixed proportion, that is, $c_t = \beta(c_t + c_e)$ and $c_e = (1 - \beta)(c_t + c_e)$, or consequently $c_t = \frac{\beta}{1 - \beta}c_e$. In the figure, $\beta = 0.8$. As before, the number of firms $n$ equals 10, $\gamma = 0.1$ and $\theta = 1$. The expected profits are plotted for $c_t + c_e < 0.8$. For higher values of $c_t + c_e$ the expected profits decrease to 0. The figure shows the same pattern as in the case where all the shops are isolated. When the search costs are low enough (for the current parameter values $c_t + c_e$ should be below 0.28) the full search equilibrium holds. In this case the expected profits increase in the search costs since the maximum price that can be asked, $r_n$, increases in the search costs. When the search costs are high enough ($c_t + c_e$ above 0.28) the partial search equilibrium holds and the expected profits decrease in the search costs. As before, the fraction of searching consumers, $\mu_n$, decreases in the search costs and moreover the maximum price $\theta - c_t$ decreases in the search costs as well.

3.3 Comparing the two opposite cases

Figure 3 combines figures 1 and 2 by showing the expected profits as a function of the search costs $c_t + c_e$ in the case where all shops are isolated and in the case where all shops are located in the same shopping mall. Again, the number of firms $n$ equals 10, $\gamma = 0.1$ and $\theta = 1$. In the figure $c_t = 0.8(c_t + c_e)$ and $c_e = 0.2(c_t + c_e)$. The expected profits are plotted for $c_t + c_e < 0.5$.

The figure can be split in different parts. First, when the search costs $c_t + c_e$ are small enough (for the current parameter values $c_t + c_e < 0.073$) the full search equilibrium holds in both cases and $\pi_1 > \pi_n$. The intuition for this result is straightforward. By locating together in a single shopping mall shops decrease the costs to continue search from $c_e + c_t$ to $c_e$, leading to stronger competition and lower prices and profits. Second, when the search costs $c_t + c_e$ have an intermediate value (for the current parameter values
Figure 2: Expected profits as a function of the search costs when all shops are located in the same shopping mall. This figure is based on 10 shops, 10% shoppers and a valuation of the product of 1. The travel costs $c_t$ are set at 80% of the total search costs $c_t + c_e$.

Figure 3: Expected profits as a function of the search costs when all shops are isolated and when all shops are located in the same shopping mall. This figure is based on 10 shops, 10% shoppers and a valuation of the product of 1. The travel costs $c_t$ are set at 80% of the total search costs $c_t + c_e$. 
0.073 < c_t + c_e < 0.28) the full search equilibrium holds in the case where all shops are located together and the partial search equilibrium holds in the case where all shops are isolated. The intuition for this is as before: when all shops are located together consumers expect lower prices and therefore consumers are more willing to search. This implies that when all shops are located together all non-shoppers are active and the expected profits increase in the search costs c_t + c_e. When all shops are isolated however only a fraction of the non-shoppers is active and expected profits decrease in the search costs c_t + c_e. When the search costs c_t + c_e are high enough the expected profits when locating together are higher than the expected profits when all shops are isolated. Finally, when the search costs are high enough (for the current parameter values c_t + c_e > 0.28) the partial search equilibrium holds in both cases. The fraction of active consumers is however higher when all firms are located together and this leads to higher expected profits when all firms are located together.

The pattern shown in Figure 3 does not depend on the specific parameter values chosen. Name the value of c_t + c_e where the full search equilibrium changes into a partial search equilibrium the inflection value. A close look at Propositions 3.1 and 3.3 shows that the inflection value is always higher when all shops are located together. It is also easy to see that when in both cases a full search equilibrium holds, that is, when the search costs c_t + c_e are below the inflection value for the case when all shops are isolated, π_1 > π_n. With somewhat more effort it can be shown that for c_t + c_e at or above the inflection value when all shops are located together π_n > π_1. This gives Proposition 3.5.

**Proposition 3.5** Let c_t = β(c_t + c_e) with 0 < β < 1. Then there exists a number c with

\[
\theta(1 - \int_0^1 \frac{1}{1 + \frac{1}{1-y} ny^{n-1}} dy) < c < \frac{1 - \int_0^1 \frac{1}{1 + \frac{1}{1-y} ny^{n-1}} dy}{1 - \beta \int_0^1 \frac{1}{1 + \frac{1}{1-y} ny^{n-1}} dy}
\]

such that for c_t + c_e < c π_1 > π_n and for c_t + c_e > c π_1 < π_n.

4 The intermediate case

The previous section has analyzed the two extreme cases of no mall shops and no isolated shops. In this section I will investigate the situation where 2 ≤ k ≤ n − 1 shops are located together in a shopping mall and the remaining n − k shops are located outside the shopping mall and separately from each other.

Recall that F^m_k(p) is the price distribution used by the shops that are in a shopping mall with k shops. Denote by π^m_k the expected profits of such a shop and define r^m_k as
The same can be done for the isolated shops: \( F_k^i(p) \) is the price distribution used by them, \( \pi_k^i \) denotes the expected profits and \( r_k^i \) is defined as

\[
\int_{p_k^i}^{r_k^i} (r_k^i - p) dF_k^i(p) = c_e. 
\]

Note that the definition of \( r_k^m \) uses \( c_e \) while the definition of \( r_k^i \) uses \( c_t + c_e \). The reason for this is that a non-shopper who is in an isolated shop and wants to continue search has to incur a search cost \( c_e + c_t \), while a non-shopper who is in a mall can continue searching in the mall at cost \( c_e \).

As before, the reservation prices determine whether a consumer wants to continue search and moreover determine whether a full search or a partial search equilibrium holds. As in the previous section, I will concentrate on equilibria where \( \overline{p}_k^m \leq r_k^m \) and \( \overline{p}_k^i \leq r_k^i \).

The optimal consumer behavior is quite complex because of the wealth of options for consumers. After one or more searches they can decide to buy at the current shop, possibly return to a previously visited shop (incurring return costs), continue search in the mall or continue search in an isolated shop. The complete specification of optimal consumer behavior is only used in the formal proofs of the propositions in this section and to save space the complete specification of consumer behavior is therefore placed in the appendix.

The first result that can be derived is on the relation between \( r_k^i \) and \( r_k^m \). When \( \overline{p}_k^m \leq r_k^m \) and \( \overline{p}_k^i \leq r_k^i \) (4) and (4) can be rewritten as \( r_k^m = E\overline{p}_k^m + c_e \) and \( r_k^i = E\overline{p}_k^i + c_t + c_e \), where \( E\overline{p}_k^m \) is the expected mall price, and \( E\overline{p}_k^i \) is the expected price in an isolated shop. If \( E\overline{p}_k^i < E\overline{p}_k^m \) all active non-shoppers prefer to search in an isolated shop and mall shops only attract shoppers. Since the number of mall shops, \( k \), is at or above 2, this drives the prices in the mall shops down to zero and \( E\overline{p}_k^m < E\overline{p}_k^i \) cannot hold. When there are at least two isolated shops (\( k \leq n - 2 \)), the reverse argument holds for \( E\overline{p}_k^m < E\overline{p}_k^i \), showing that in equilibrium \( E\overline{p}_k^m = E\overline{p}_k^i \). This also holds when there is only one isolated shop (\( k = n - 1 \)), but the argument for that is less intuitive. The appendix provides more details. Using (4) and (4) \( E\overline{p}_k^m = E\overline{p}_k^i \) implies that \( r_k^i = r_k^m + c_t \).

**Proposition 4.1** In any equilibrium with \( \overline{p}_k^m \leq r_k^m \) and \( \overline{p}_k^i \leq r_k^i \), \( r_k^i = r_k^m + c_t \).

A consequence of Proposition 4.1 is that non-shoppers who have to choose where to search first are indifferent between searching in the mall and in
isolated shops. In equilibrium they will therefore spread randomly over mall shops and isolated shops. But I will show later that they do not spread evenly: mall shops attract a larger share of non-shoppers than isolated shops. Applying Proposition 4.1 to the optimal consumer behavior as defined in the Appendix shows that when \( p_{m}^{i} \leq r_{m}^{i} \) and \( p_{k}^{i} \leq r_{k}^{i} \) non-shoppers will stop searching after their first search. When a mall shop deviates to a price above \( r_{m}^{i} \) every non-shopper who finds this price will continue to search in the mall. Similarly, when an isolated shop deviates to a price above \( r_{k}^{i} \) every non-shopper who finds this price will continue to search in some other isolated shop. Using this optimal consumer behavior it is easy to see that indeed an equilibrium with \( p_{m}^{i} \leq r_{m}^{i} \) and \( p_{k}^{i} \leq r_{k}^{i} \) can exist. If a shop would deviate to a higher price it would not sell anything and profits would be zero. Non-deviating shops obtain strictly positive profits since they at least sell to some non-shoppers. This shows that deviating is not profitable.

A natural equilibrium would be an equilibrium where \( F_{k}(p) = F_{m}(p) = F_{k}(p) \). This is however not possible. Note that \( p_{k} = p_{m}^{i} \leq r_{m}^{i} = r_{k}^{i} - c_{t} \) and that because of the shoppers \( F_{k}(p) \) should be atomless. An isolated shop setting price \( p_{k} \) would sell only to those non-shoppers who visit the isolated shop on their first search. But raising price to \( r_{k}^{i} \) would not deter any of these non-shoppers from buying and profits would be higher. Note that non-shoppers are willing to buy at price \( r_{k}^{i} \) since to continue search they not only incur continuation costs \( c_{e} \) but also travel costs \( c_{t} \). In equilibrium therefore \( F_{m}^{i}(p) \neq F_{k}(p) \).

Let \( x_{k} \) be the fraction of active non-shoppers who decide to first visit a shop in the shopping mall and let \( 1 - x_{k} \) be the fraction of active non-shoppers who first visit an isolated shop, with \( 0 < x_{k} < 1 \). Note that \( x_{k} = \frac{k}{n} \) means an equal division of non-shoppers over all the shops.

The specification of optimal consumer behavior shows that all non-shoppers will be active (\( \mu_{k} = 1 \)) when \( r_{k}^{i} < \theta \) and that only a fraction of non-shoppers will be active (\( \mu_{k} < 1 \)) when \( r_{k}^{i} = \theta \). Using the fact that non-shoppers stop searching after their first search the profit functions are as follows. For \( p \leq r_{m}^{i} \),

\[
\pi_{m}^{i}(p) = \gamma p(1 - F_{m}^{i}(p))^{k-1}(1 - F_{k}^{i}(p))^{n-k} + (1 - \gamma)\mu_{k} x_{k} p.
\]

For \( p \leq r_{k}^{i} \)

\[
\pi_{k}(p) = \gamma p(1 - F_{k}^{m}(p))^{k}(1 - F_{k}^{i}(p))^{n-k-1} + (1 - \gamma)\mu_{k} \frac{1 - x_{k}}{n - k} p.
\]

Assume for the moment that \( k < n - 1 \). A standard undercutting argument shows that atoms in \( F_{m}^{i}(p) \) are only possible for those prices \( p^{*} \) at which
\(F^i_k(p^*) = 1\). Similarly, atoms in \(F^i_k(p)\) are only possible for those prices \(p^*\) at which \(F^m_k(p^*) = 1\). The profit functions also show that in equilibrium \(\overline{F^i_k} = r^m_k\), since for a lower maximum price it would be profitable to deviate to \(r^m_k\). Similarly, in equilibrium \(\overline{F^i_k} = r^i_k\). Exact results on atoms and maximum prices hold for \(k = n - 1\), but the proof is less intuitive and is placed in the appendix. Equilibrium expected profits are 
\[
\pi^m_k = r^m_k \frac{1 - x_k}{n - k} \mu_k (1 - \gamma) \quad \text{and} \quad \pi^i_k = r^i_k \frac{1 - x_k}{n - k} \mu_k (1 - \gamma).
\]

Note that for \(p \geq r^m_k\), \(\pi^i_k(p) = (1 - \gamma) \mu_k \frac{1 - x_k}{n - k} p\).

This shows that isolated shops will never set a price between \(r^m_k\) and \(r^i_k\) and that there will be an atom at \(r^i_k\). \(F^i_k(p)\) should also have some probability mass below \(r^m_k\) since else the definition of \(r^i_k\) as given by (4) cannot hold. This probability mass is atomless, as well as \(F^m_k(p)\). Proposition 4.2 summarizes.

**Proposition 4.2** In any equilibrium, \(0 < F^i_k(r^m_k) < 1\) and for \(p < r^m_k\) \(F^i_k(p)\) is atomless. \(F^i_k(p)\) is constant for \(r^m_k \leq p < r^i_k\) and has an atom at \(p = r^i_k\). Moreover, in equilibrium \(F^m_k(p)\) is atomless and \(\overline{F^m_k} = r^m_k\).

The intuition behind \(F^m_k(p)\) is fairly standard: shops in a mall randomize over prices to balance the effects of the shoppers and the non-shoppers. \(F^i_k(p)\) has a non-standard shape, with an atom at \(\overline{F^i_k}\) and a gap below \(\overline{F^i_k}\). Moreover, \(\overline{F^i_k} > \overline{F^m_k}\) and the difference between the two maximum prices is exactly \(c_t\). The maximum prices differ because of the search costs. Once a non-shopper is in the mall he can search at a relatively low cost \(c_e\) while if a non-shopper is in an isolated shop, continuing search will cost \(c_e + c_t\). Consequently, an isolated shop has more power over the non-shoppers, which leads to a maximum price that is higher by exactly the difference in search costs. Despite the higher maximum price, non-shoppers are willing to search in isolated shops. This is because isolated shops randomize over the high maximum price and much lower prices. In this way isolated shops balance the effects of shoppers and non-shoppers. Note that the difference in maximum price and the other prices is at least \(c_t\). Balancing the effects of shoppers and non-shoppers therefore only is possible when the fraction of non-shoppers who decide to search in an isolated shop, \(1 - x_k\), is relatively low, making the fraction of shoppers more important for the shop. The next Proposition addresses this.

**Proposition 4.3** In any equilibrium, \(\pi^i_k < \pi^m_k\) and \(\frac{1 - x_k}{n - k} < \frac{x_k}{k}\) or, equivalently, \(x_k > \frac{k}{n}\).

The main intuition behind this result is as argued above. When the isolated shops have expected profits at or above the expected profits of shops in the
mall then the fraction of non-shoppers that isolated shops attract, \(1 - x_k\), necessarily is relatively high\(^5\). The proof of Proposition 4.3 shows that when \(1 - x_k\) is that high an isolated shop makes more profit from setting a price \(r^i_k\) than from setting a lower price. A situation where isolated shops ask a price \(r^i_k\) for sure however cannot be an equilibrium situation, since in that case consumers prefer to search in the mall and \(x_k\) would be 1.

The same line of thought can be used when thinking about the minimum prices. When an isolated shop sets a price \(p^i_k\) it competes with the mall for the shoppers. To make sure that setting this minimum price is as profitable as setting price \(r^i_k\), \(1 - x_k\) should be relatively small, but also the probability of attracting the shoppers when setting price \(p^i_k\) should be high. To give an extreme example, if \(p^i_k = p^m_k\), the probability of selling to the shoppers would be zero and profits from a price \(p^i_k\) are below the profits from a price \(r^i_k\). To make sure that the probability of attracting the shoppers is high enough, \(p^i_k\) cannot be much higher than \(p^m_k\). In fact, it can be proven that in equilibrium \(p^i_k \leq p^m_k\).

**Proposition 4.4** In any equilibrium, \(p^i_k \leq p^m_k\).

It is now possible to derive some statements about the supports of \(F^m_k(p)\) and \(F^i_k(p)\). The support of \(F^m_k(p)\) is defined by all prices for which \(f^m_k(p) > 0\), with \(f^m_k(p)\) the probability density function corresponding to \(F^m_k(p)\). The support of \(F^i_k(p)\) is defined similarly. Proposition 4.2 already defined \(F^m_k(p)\) and \(F^i_k(p)\) for \(p > r^m_k\), so only the supports for \(p \leq r^m_k\) are of interest. When \(k < n - 1\), there are three possible types of supports, while for \(k = n - 1\) only one type of support can hold.

**Proposition 4.5** When \(k < n - 1\), the supports of \(F^m_k(p)\) and \(F^i_k(p)\) belong to one of the following three types.

1. \(f^i_k(p) > 0\) for \(p^i_k \leq p \leq a\), \(f^i_k(p) = 0\) for \(a < p \leq r^m_k\), \(f^m_k(p) = 0\) for \(p^i_k \leq p \leq a\) and \(f^m_k(p) > 0\) for \(a < p \leq r^m_k\).

2. \(f^i_k(p) > 0\) for \(p^i_k \leq p \leq b\), \(f^i_k(p) = 0\) for \(b < p \leq r^m_k\), \(f^m_k(p) = 0\) for \(a < p < b\) and \(f^m_k(p) > 0\) for \(p^i_k \leq p \leq a\) and \(b \leq p \leq r^m_k\) (with \(a < b\)).

3. \(f^i_k(p) > 0\) for \(p^i_k \leq p \leq a\), \(f^i_k(p) = 0\) for \(a < p \leq r^m_k\) and \(f^m_k(p) > 0\) for \(p^i_k \leq p \leq r^m_k\).

When \(k = n - 1\) only type 3 can hold.

\(^5\)Note that it need not be the case that \(\frac{1 - x_k}{n - k} \geq \frac{r^i_k}{k}\), but \(x_k\) should be such that \(r^i_k \frac{n - k}{k} \geq r^m_k\).
Note that the support of $F_k^i(p)$ always covers some region of low prices, $[p_k^*, p^*]$, with $a < r_k^m$ and combines that with an atom at $r_k^i$. The support of $F_k^m(p)$ always covers the region $[p^*, r_k^m]$ that is not covered by $F_k^i(p)$. Next to that, the support of $F_k^m(p)$ can also include lower prices, that are covered by $F_k^i(p)$ as well.

Each equilibrium type has a full search variant with $\mu_k = 1$ and $r_k^i < \theta$ and a partial search variant with $0 < \mu_k < 1$ and $r_k^i = \theta$. For $k < n - 1$ this gives a total of six equilibria and for $k = n - 1$ this gives a total of two equilibria. The derivation of these equilibria is a standard exercise, the details are in the appendix. Unfortunately, none of the equilibria has closed form expressions for the integrals in 4 and 4. For equilibria with full search this implies that numerical methods are necessary to calculate expressions for $r_k^m$ and $r_k^i$. Moreover, $x_k$ depends on $r_k^m$ and $r_k^i$, so the value of $x_k$ can also only be evaluated numerically. For equilibria with partial search the integrals in 4 and 4 determine $\mu_k$ and $x_k$, so again numerical methods are needed to evaluate $\mu_k$ and $x_k$. In the next section some results of a numerical analysis are provided.

5 Numerical analysis

Recall that in section 3 $\beta$ has been defined as a constant such that $c_t = \beta(c_t + c_e)$ and $c_e = (1 - \beta)(c_t + c_e)$. It can be shown that in all full search equilibria for $2 \leq k \leq n - 1$ the parameter $x_k$ only depends on $\beta$, $\gamma$, $n$ and $k$, and not on $c_t$ and $c_e$. Moreover, all reservation prices and profits can be written as $c_t + c_e$ times some function of $\beta$, $\gamma$, $n$ and $k$. The partial search equilibria for $2 \leq k \leq n - 1$ are more complicated, in the sense that $x_k$ depends not only on $\beta$, $\gamma$, $n$ and $k$, but also on $c_t + c_e$. Moreover, the reservation prices and profits are nonlinear in $c_t + c_e$. This implies that when $\beta$, $\gamma$, $n$ and $k$ are fixed, the full search equilibria can be numerically calculated. To calculate the partial search equilibria, $c_t + c_e$ also needs to be specified. In this section I will therefore concentrate on the full search equilibria; partial search equilibria will be discussed in the next section.

Tables 5, 5 and 5 give simulation results for the full search equilibria. Table 5 uses a small value of $\gamma$; $\gamma = 0.05$. Table 5 has an intermediate value of $\gamma$ ($\gamma = 0.1$) and table ?? has a large value of $\gamma$ ($\gamma = 0.25$). Each table gives results for different values of $k$ and $\beta$, while in all tables $n = 10$. Each table gives a panel with results on $r_k^i$ and a panel with results on the reservation price. For $2 \leq n - 1$, $r_k^i$ is reported. Note that a full search equilibrium only holds when $r_k^i < \theta$, which translates to $c_t + c_e$ being small enough. For $k = 1$, the tables give the equivalence of $r_k^i$, $r_1$, and the full search equilibrium holds when $r_1 < \theta$. For $k = n$, the tables give $r_n + c_t$. This is
the equivalence of \( r^i_k \) as a full search equilibrium holds when \( r_n + c_t < \theta \). Proposition 4.5 states that for \( 2 \leq k \leq n - 2 \) three equilibrium types are possible. The simulations suggest that these equilibria do not overlap and together fill the complete parameter space. In the tables lines denote when each equilibrium type holds. The equilibria in the upper right corner, for high \( \beta \) and low \( k \), are of type 1. The equilibria in the lower left corner, for low \( \beta \) and high \( k \), are of type 3. Note that also all equilibria with \( k = n - 1 \) are of type 3. The intermediate equilibria are of type 2.

When \( k \) is small, equilibrium type 1 occurs more often than when \( k \) is large. To explain this, take a closer look at the pricing behavior of mall and isolated shops in equilibrium type 1. Using the notation from proposition 4.5, mall shops set a price between \( a \) and \( r^m_k \), where \( a \) is defined as the lowest mall price such that \( \pi^m_k(a) = \pi^m_k(r^m_k) \). Note that when a mall shop sets a price \( a \) it sells to its captive non-shoppers, but also sells to all shoppers when all isolated shops set a price \( r^i_k \). When \( k \) is low, and \( n - k \) is high, the probability that all isolated shops set a price \( r^i_k \) is low. Consequently, the difference between \( r^m_k \) and \( a \) is relatively small.

Isolated shops either set a price \( r^i_k \) or a price at or below \( a \). As in the case of mall shops, when setting a price \( a \) an isolated shop sells to its captive consumers and moreover sells to all shoppers when all other isolated shops set a price \( r^i_k \). The probability that this happens is small for large \( n - k \) and since the difference between \( r^i_k \) and \( a \) is larger than the difference between \( r^m_k \) and \( a \), isolated shops find a price \( a \) less attractive than mall shops. To make sure that isolated shops still want to ask a price \( a \) they should attach much weight to capturing the shoppers, which occurs when the fraction of non-shoppers, \( \frac{1 - \bar{x}_k}{n - k} \), is small, or \( \frac{\bar{x}_k}{k} \) is high. Tables ??, ?? and ?? indeed show that for low \( k \) \( \frac{\bar{x}_k}{k} \) is high. This high value of \( \frac{\bar{x}_k}{k} \) in turn implies that mall shops attract many non-shoppers and do not want to attract shoppers by deviating to \( p^i_k \), thus making equilibrium type 1 possible. Another implication of the high value of \( \frac{\bar{x}_k}{k} \) is that \( r^m_k \) (and consequently \( r^i_k \)) is high. When the mall shops attract mainly non-shoppers, they will focus on these consumers, thereby driving prices up.

Equilibrium type 1 also is more likely to occur when \( \beta \) is large. Recall that the difference between the isolated and mall reservation prices, \( r^i_k - r^m_k \), is \( c_t \), and thus increases in \( \beta \). In equilibrium, the price distributions should be such that the expected mall price equals the expected isolated price. When \( r^i_k - r^m_k \) is large, the expected isolated price tends to be larger than the expected mall price, unless equilibrium type 1, with \( p^i_k < p^m_k \) holds.

Note from the tables that an increase in \( \beta \) seems to lead to an increase in \( \frac{\bar{x}_k}{k} \) and a decrease in \( r^i_k \), except for \( k = 1 \). The increasing difference

\[^6\text{For } k = 1 \text{ the equilibrium does not depend on } \beta. \text{ For every visit to a shop consumers...} \]
(a) Values of $\frac{x}{k}$ for $n = 10$ and $\gamma = 0.05$.

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<td>0.1018</td>
<td>0.1028</td>
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(b) Reservation prices for $n = 10$ and $\gamma = 0.05$. The first row gives $r_1$, the last row gives $r_n + c_t$ and the intermediate rows give $r_k^i$ for $2 \leq k \leq 9$. For ease of notation, $c_t + c_e$ is denoted by $c$.

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Table 1: Values of $\frac{x}{k}$ and reservation prices for $n = 10$ and $\gamma = 0.05$. 
(a) Values of $x_k^*$ for $n = 10$ and $\gamma = 0.1$.

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(b) Reservation prices for $n = 10$ and $\gamma = 0.1$. The first row gives $r_1$, the last row gives $r_n + c_t$ and the intermediate rows give $r_k^*$ for $2 \leq k \leq 9$. For ease of notation, $c_t + c_e$ is denoted by $c$.

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Table 2: Values of $x_k^*$ and reservation prices for $n = 10$ and $\gamma = 0.1$. 
(a) Values of $\frac{x_k}{k}$ for $n = 10$ and $\gamma = 0.25$.

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(b) Reservation prices for $n = 10$ and $\gamma = 0.25$. The first row gives $r_1$, the last row gives $r_n + c$, and the intermediate rows give $r_k^s$ for $2 \leq k \leq 9$. For ease of notation, $c_1 + c$ is denoted by $c$.

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Table 3: Values of $\frac{x_k}{k}$ and reservation prices for $n = 10$ and $\gamma = 0.25$. 
between \( r^i_k \) and \( r^m_k \) makes the high price more attractive for isolated shops. In equilibrium, isolated shops should be indifferent between setting a price \( r^i_k \) and a low price. To counter the increasing attractiveness of \( r^i_k \) the fraction of captive consumers an isolated shop attracts, \( \frac{1-x_k}{n-k} \), should decrease, and consequently \( \frac{\mu_k}{k} \) should increase.

For \( k \geq 2 \) the reservation prices in the mall are determined by \( c_e \). An increase in \( \beta \) implies a decrease in \( c_e \) and consequently \( r^m_k \) (\( r_n \) for \( k = n \)) decreases. The difference between \( r^i_k \) and \( r^m_k \) increases in \( \beta \), but this increase is not enough to reverse the decrease in \( r^m_k \): tables 5, 5 and 5 show that \( r^i_k \) decreases in \( \beta \).

When \( \gamma \) increases, the simulation results in tables 5, 5 and 5 suggest that \( r^i_k \) decreases, while \( \frac{\mu_k}{k} \) can both increase and decrease. When \( \gamma \) increases there is a stronger competition for the shoppers. As a result, the isolated shops are less tempted to ask price \( r^i_k \) and the mall shops also prefer lower prices. The tables show that indeed equilibrium type 1 occurs less often when \( \gamma \) is larger: the mall shops are more tempted to set \( p^m_k = p^i_k \). Since both types of shops prefer lower prices, it is no surprise that \( r^i_k \) decreases. The effect on \( \frac{\mu_k}{k} \) is twofold. Isolated shops need to be indifferent between asking the high price \( r^i_k \) and lower prices. When the fraction of shoppers increases, isolated shops are less tempted to ask a high price, but at the same time the behavior of the mall shops leads to more competition in the lower price range, making a high price more attractive. When the first effect dominates, \( \frac{1-x_k}{n-k} \) needs to increase and consequently \( \frac{\mu_k}{k} \) needs to decrease to make sure that isolated shops still want to ask \( r^i_k \). When the second effect dominates, \( \frac{1-x_k}{n-k} \) needs to decrease and \( \frac{\mu_k}{k} \) consequently increases to ensure that isolated shops still want to set a low price. Note that in equilibrium type 1 the second effect is absent and that indeed \( \frac{\mu_k}{k} \) decreases in \( \gamma \).

Even though a numerical analysis is needed to evaluate the equilibria, it is possible to analytically derive some limiting results.

**Proposition 5.1**

- When \( \beta \to 0 \) \( p^i_k = p^m_k \), \( r^i_k - r^m_k \to 0 \), \( F^i_k(p) - F^m_k(p) \to 0 \) and \( \frac{\mu_k}{k} \to \frac{1}{n} \).

- When \( \beta \to 1 \), \( r^m_k \to 0 \), \( F^i_k(r^m_k) \to 1 \) and \( \frac{1-x_k}{n-k} \to 0 \).

- When \( \gamma \to 0 \) \( \mu_k \to 0 \).

- When \( \gamma \to 1 \) \( p^m_k \to 0 \), \( \frac{\mu_k}{k} \to 0 \), \( F^m_k(p) \to 1 \) and \( F^i_k(p) \to 1 \).

These results are in line with previous consumer search models, see e.g. Janssen et al. (2005). Recall that \( \beta \) determines the relative sizes of \( ct \) and have to incur a cost \( ct + ce \), making the equilibrium independent of \( ce \) and \( ct \) in isolation.
When $\beta \to 0$, $c_t \to 0$ and in the limit consumers only incur entering costs. In that case, the difference between mall shops and isolated shops vanishes, which leads to equal price distributions and an equal distribution of non-shoppers over the firms. When $\beta \to 1$, $c_e \to 0$. In this case, once a consumer is in the mall, he can visit all mall shops almost for free. This leads to large competition between mall shops and consequently to prices of almost zero in the mall. To make mall and isolated shops equally attractive, isolated shops should set prices to almost zero as well. To make this possible, isolated shops should attract almost no non-shoppers. If they would attract too many non-shoppers, an isolated shop could set a price $c_t$ and make a profit on the non-shoppers who visited the isolated shop in the first place. When the fraction of shoppers, $\gamma$, vanishes, firms tend to focus completely on the captive consumers. This raises prices to monopoly levels, and consequently many non-shoppers drop out of the market. When the fraction of non-shoppers vanishes, firms compete strongly for the shoppers, leading to very low prices.

### 6 Location choice

In this section I will consider the equilibrium location choice of shops. A mall with $k^*$ shops is considered an equilibrium when none of the mall shops has an incentive to leave the mall and when none of the isolated shops has an incentive to join the mall. Thus, a mall with $k^*$ shops is an equilibrium when $\pi^m_{k^*} \geq \pi^i_{k^*-1}$ and $\pi^i_{k^*} \geq \pi^m_{k^*+1}$. Note that if a mall shop would deviate and leave the mall, the mall size would decrease by one. Also, if an isolated shop would deviate and join the mall, the mall size would increase by one. I will first consider location choice when $c_t + c_e$ is small, such that a full search equilibrium holds. As mentioned section 5, in a full search equilibrium profits can be written as $c_t + c_e$ times some function of $\beta$, $\gamma$, $n$ and $k$. Table 4 gives the profits in a full search equilibrium when $n = 10$ and $\gamma = 0.1$. Three different values of $\beta$ are considered and profits are given for every possible mall size $k$.

When comparing $\pi_1$ with $\pi_2^i$ and $\pi_2^m$ note that $\pi_1$ can both be below and above $\pi_2^m$, while $\pi_1$ is always above $\pi_2^i$. When two shops decide to form a mall the reservation prices (maximum prices a shop will ask) decrease. At the same time, the fraction of non-shoppers going to a mall shop, $\frac{x_k}{k}$, is clearly above $\frac{1}{n}$, the fraction of non-shoppers that a shop attracts when there is no mall. The two mall shops thus set lower maximum prices but sell more, and the total effect is ambiguous. The isolated shops also set lower maximum

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7 Rental costs that differ between locations and relocation costs could easily be introduced, but will only change the equilibrium conditions with a constant.

8 Note that $\frac{x_k}{k}$ is larger when $\beta$ is large and thus the positive effect on mall profits is
\[ \beta = 0.1 \] 
\[ \beta = 0.4 \] 
\[ \beta = 0.7 \]

<table>
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</table>

Table 4: Profits for different values of \( k \) and \( \beta \) when a full search equilibrium holds. The number of firms, \( n \), is fixed to 10 and \( \gamma = 0.1 \). For ease of notation \( c_t + c_e \) is denoted by \( c \). Bold profits indicate an equilibrium.

prices, but on top of that lose customers. As a consequence, \( \pi^i_k < \pi^i_1 \). Once a mall exists \( (k \geq 2) \), both mall and isolated profits seem to decrease in mall size. The reason for this again is that the reservation prices decrease in mall size. For mall shops it is also important that the fraction of captive consumers, \( \hat{\pi}^m_k \), decreases in mall size. This decreases the mall profits even more. Isolated shops attract more non-shoppers when the mall size increases, but this increase in non-shoppers does not offset the lower reservation prices. It does show however in the table that mall profits decrease faster in mall size than isolated profits.

Once \( \gamma, \beta \) and \( n \) are fixed, a table with profits for different mall sizes is sufficient to find the equilibrium mall size. Take for example the case \( n = 10, \gamma = 0.1 \) and \( \beta = 0.1 \), which is the left panel in table 4. When no mall exists, profits are 1.2303\((c_t + c_e)\). When a shop decides to deviate and join another shop, it will make profits \( \pi^m_2 = 1.2123(c_t + c_e) \). These profits are below 1.2303\((c_t + c_e)\) and thus \( k = 1 \) is an equilibrium. This is indicated in the table by bold profits. For \( 2 \leq k \leq 8 \), it is profitable for a mall shop to leave the mall: \( \pi^m_k < \pi^i_{k-1} \). When \( k = 9 \), a mall shop has profits 1.1203\((c_t + c_e)\). Leaving the mall would give smaller profits (1.1194\((c_t + c_e)\)), so a mall shop has no incentive to deviate. An isolated shop has profits 1.1123\((c_t + c_e)\) and joining the mall would give profits of 1.1073\((c_t + c_e)\). An isolated shop therefore has no incentive to join the mall and consequently \( k = 9 \) is another equilibrium (denoted in bold). For \( k = 10 \), a mall shop would find it profitable to leave the mall, so \( k = 10 \) is not an equilibrium. In the middle and right panel of table 4 the same analysis gives the equilibria that are denoted more pronounced for large values of \( \beta \).
Table 5: Equilibrium mall sizes for several values of $\beta$ and $\gamma$. The number of firms, $n$, is fixed to 10.

<table>
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Table 5 gives equilibrium mall sizes for different values of $\beta$ and $\gamma$, keeping $n$ fixed to 10. To understand the intuition behind the results in this table, first look at the incentives of mall and isolated shops to deviate. A mall shop that leaves the mall loses some of its captive consumers, but at the same time it can set a higher maximum price. On its own, this is not sufficient to leave the mall: $\pi_m^k > \pi_i^k$. But when a shop leaves the mall, the mall size decreases. This lowers $1 - x/n$ and increases the reservation prices. This magnifies the positive and negative effects mentioned before, such that $\pi_m^k < \pi_i^{k-1}$ is possible. For an isolated shop that joins the mall the effects on profits are reversed. After joining the mall, the shop attracts more captive consumers than before, but the maximum price it can ask is lower. These two effects on their own would be sufficient to join the mall ($\pi_i^k < \pi_m^k$), but once again joining the mall changes the size of the mall. This will decrease $x/n$ and decrease the reservation prices. Both of these effects negatively affect the profits of a deviating isolated shop, such that $\pi_i^k > \pi_m^{k+1}$ is possible.

When $\beta$ is high and $k$ is small, $x/n$ is very high. Intuitively, when the travel costs $c_t$ are large, many consumers prefer the mall. If joining the mall would not affect the mall size, isolated shops would have a large incentive to join the mall and capture this large share of non-shoppers, instead of the tiny share of non-shoppers they capture as isolated shop. As a counteracting effect, joining the mall increases the mall size and therefore decreases $x/n$ and the reservation prices. But because $x/n$ is very large from the outset and $1 - x/n$ is very low, an isolated shop can still gain from joining the mall. This will only stop when the mall size has grown large and $x/n$ is relatively small. Thus, for high $\beta$ the equilibrium mall size is fairly large.

When the fraction of shoppers, $\gamma$, is large, the only reason for an isolated shop to join the mall is not so important. An isolated shop will join the
mall when it leads to a large increase in captive consumers, $\frac{c}{k}$. When $\gamma$ is large, many consumers are shopping for the best deal and an isolated shop cannot gain many non-shoppers by joining the mall. As a consequence, the equilibrium mall size is smaller when $\gamma$ is larger.

When both $\beta$ and $\gamma$ are small, there are two possible equilibria: $k^* = 1$ and $k^* = 9$. As explained before, when $\beta$ is low, $\frac{c}{k}$ is relatively low and isolated shops do not have much to gain from joining the mall. This explains why $k^* = 1$ is an equilibrium. To see why $k^* = 9$ also can be an equilibrium, recall the consideration shops make. If leaving or joining the mall would not change the mall size, it is clear that isolated shops would always want to join the mall and mall shops would never want to leave the mall ($\pi^m_k > \pi^i_k$).

The change in mall size can distort this behavior. Now when $\beta$ and $\gamma$ are small and the initial mall size is large, a change in mall size will only lead to a very small change in $\frac{c}{k}$ and the reservation prices. For small $\beta$ and high $k$, $\frac{c}{k}$ is relatively low, and changes in $\frac{c}{k}$ are small by definition. When $\gamma$ is low and $c_e$ is high ($\beta$ is low), the competition between mall shops is mild, leading to only small changes in reservation prices. As a result, the effect of a change in mall size is negligible and an equilibrium with a large mall can occur.\(^9\)

An equilibrium with $k^* > 1$ in general gives lower profits for both firm types than a situation where there is no mall at all. To understand the intuition behind this, take a better look at the most right panel of table 4. Starting from a situation with no mall it is profitable to join two shops. These shops will attract much more non-shoppers than when they were isolated. But the remaining isolated shops suffer from this. Because they lose non-shoppers their profits are much lower than in the case no mall existed. Consequently, an isolated shop finds it profitable to join the mall. This drives down the mall profits to below the level when there were no mall at all, but the resulting mall profits are still higher than the isolated profits in the case $k = 2$. Intuitively, by joining two shops, the remaining isolated shops are hurt so badly that too many of those isolated shops want to join the mall, even when mall profits are below $\pi_1$.

Thus far, I have looked at full search equilibria. In the remainder of this section I will also look at partial search equilibria. This equilibrium type is more complicated to analyze, since the profits depend on $c_l$ and $c_e$ in a nonlinear way. Therefore, instead of tables, I will provide several plots of expected profits as a function of $c_l + c_e$. Simulations show that the plots of the profits as function of $c_l + c_e$ when both full and partial search equilibria

\(^9\)For the values of $n$, $\beta$ and $\gamma$ listed in table 5 the initial mall size should be 8 for the argument to hold. For smaller mall sizes, the changes in $\frac{c}{k}$ and reservation prices are too large. Moreover, there is a discontinuity when from an initial mall size of 9 the last isolated shop joins the mall. In that case, the change in reservation prices is considerably larger than when the mall grows from 8 to 9.
Figure 4: Expected profits as a function of the search costs when all shops are located separately and when there is a mall of two shops. This figure is based on 10 shops, 10% shoppers and a valuation of the product of 1. The travel costs $c_t$ are set at 70% of the total search costs $c_t + c_e$.

are considered have the same pattern as in the extreme cases analyzed in Section 3. Again, there is a value of $c_t + c_e$, called the inflection value, such that for $c_t + c_e$ below this value the full search equilibrium holds and above this value the partial search equilibrium holds. The fraction of active non-shoppers, $\mu_k$, is decreasing in $c_t + c_e$ and as a consequence the profits in the partial search equilibrium decrease in $c_t + c_e$.

Figures 4 and 5 show the expected profits for several values of $k$. In these figures $\gamma$ is set at 0.1, $n = 10$, $\beta = 0.7$ and $\theta = 1$. Figure 4 depicts $\pi_1$, $\pi_2^m$ and $\pi_2^i$ and figure 5 depicts $\pi_2^m$, $\pi_2^i$, $\pi_3^m$ and $\pi_3^i$. A first observation is that the inflection point shifts to the right when $k$ increases. Note that this can also be inferred from table 5 since the inflection point simply is defined as the value of $c_t + c_e$ for which $r_k^i = \theta$. Intuitively, competition will be stronger when more shops are located in the mall. Therefore, more non-shoppers will be tempted to search, shifting the inflection point to the right. A consequence of this is that for $k \geq 2$ $\mu_k \geq \mu_{k-1}$. Moreover, recall that in a partial search equilibrium $r_k^i = \theta$ and $r_k^m = \theta - c_t$. This implies that a change in mall size does not affect the maximum prices the shops can ask. For an isolated shop, joining the mall therefore gives more captive consumers ($\frac{x_{k+1}}{k+1} > \frac{1}{n}$ instead of $\frac{1-x_k}{n-k} < \frac{1}{n}$) and increases the fraction of
active consumers $\mu_k$, while not affecting maximum prices. Therefore, for large enough values of $c_t + c_e$, $\pi_{k+1}^m \geq \pi_k^i$. Figures 4 and 5 indeed show this. To save space, figures of the profits for $k > 3$ are not included in the paper, but they show the same pattern. Thus, for large enough values of $c_t + c_e$, isolated shops have an incentive to join the mall and the only possible equilibrium is one that has no isolated shops at all ($k = n$). Note that also the remaining isolated shops profit when one of the isolated shops joins the mall ($\pi_k^i > \pi_{k-1}^i$), since they benefit as well from the increase in active non-shoppers.

7 Conclusion

This paper analyzed the incentives of a shop to locate together with similar shops in a shopping mall. As in a standard sequential consumer search model, the consumers incur costs when entering a shop, independent of where the shop is located. Consumers also incur travel costs when traveling between shops that are not in the same mall, a novel feature in a sequential search setting. The addition of travel costs implies that searching in a shopping mall is more attractive than searching isolated shops. This has several implications for the profitability of shops. First, the lower search
costs in a shopping mall reduce the prices compared to a situation without a shopping mall. Second, the shopping mall attracts more consumers than the isolated shops. And, third, because the existence of a shopping mall leads to more competition, the fraction of active consumers is increasing in the size of the shopping mall.

Dependent on the relative sizes of these three effects, several equilibria are possible. When the search costs are low, the third effect is absent because all consumers are active, independent of the size of the mall. It is shown that the equilibrium mall size can range from no mall at all to a mall with \( n - 1 \) shops, dependent on the relative importance of the first and second effect. When the search costs are high enough, the third effect plays an important role, while the first effect is absent. The second and third effect both are positive, so for large enough search costs the only possible equilibrium is one where all the shops are in the mall. Interestingly, this paper is the first in the literature to find equilibrium mall sizes that are strictly between no mall and all shops in a mall.

An interesting feature of the equilibria in this model is that for a fixed mall size the profits of mall shops are always above the profits of isolated shops. This has two causes. First, as mentioned before, mall shops attract more consumers than isolated shops. Second, even though isolated shops can set prices above the mall prices, the increase in prices is limited. If isolated shops would set a price that is too high all consumers would go to the mall. Consumers in an isolated shop are willing to buy at a slightly higher price since to continue search they not only incur the entering costs from the standard model but also the travel costs. When the price difference between mall shops and isolated shops is however above the travel costs consumers in an isolated shop continue their search in the mall.

An interesting simulation result is that for high enough search costs isolated shops not necessarily lose profits when a mall is formed. Even though isolated shops lose customers who go to the mall, the fraction of active consumers increases and the isolated shops get some share of these consumers. This increase in active consumers is high enough to offset the decrease in sales caused by consumers visiting the mall instead of an isolated shop.
References


A Proofs of Section 3

Proof of Propositions 3.1 and 3.2
To prove Propositions 3.1 and 3.2 first note that in an equilibrium where some non-shoppers search a shop will never ask a price above $\theta$. If a shop would ask a price above $\theta$ it would not make any sales and profits would be 0. Asking a price $c_t + c_e$ however prevents non-shoppers from searching further and guarantees a strictly positive profit. This implies that if a non-shopper is in a shop he can always obtain a non-negative utility by buying from this shop.

To prove the optimality of the consumer behavior stated in the Propositions an induction argument will be used. Consider a non-shopper who expects the shops to price according to some price distribution $F_1(p)$ with $\overline{p} \leq \min(\theta, r_1)$, where $r_1$ is defined by

$$\int_{\overline{p}}^{r_1} (r_1 - p) dF_1(p) = c_e + c_t.$$

Denote by $p^*$ the price the non-shopper found in his last search and denote by $p^{\text{min}}$ the minimum price he found in previous searches, with $p^{\text{min}}$ infinite when there are no previous searches. Let $q$ denote $\min(p^* + c_t, p^{\text{min}} + c_t)$. If the non-shopper has already searched $n - 1$ shops the utility from buying is $\theta - \min(p^*, p^{\text{min}} + c_t)$. If the non-shopper decides to search the $n$th shop as well and he finds a price below $q$ he will buy in the $n$th shop. Else he will return to a previously visited shop. The expected utility from searching is given by

$$U(\text{search}) = -c_t - c_e + \int_q^{\overline{p}} (\theta - p) dF_1(p) + (1 - F_1(q))(\theta - q).$$

Note that the utility above holds even when $q > \theta$. For $p > \theta$ $F_1(p) = 1$ and therefore when $q > \theta$ the utility above reduces to $-c_t - c_e + \int_{\overline{p}}^{\theta} (\theta - p) dF_1(p)$, which is exactly the expected utility of search in case $q > \theta$. The utility from searching can be rewritten as

$$U(\text{search}) = -c_t - c_e + \theta - q + \int_q^{\overline{p}} (q - p) dF_1(p).$$

If $\min(p^*, p^{\text{min}} + c_t) > r_1$ it must be that $q > r_1$. Using that $\overline{p} \leq r_1$, $\int_q^{\overline{p}} (q - p) dF_1(p) = \int_{\overline{p}}^{r_1} (q - p) dF_1(p) = q - r_1 + \int_{\overline{p}}^{r_1} (r_1 - p) dF_1(p) = q - r_1 + c_t + c_e$. This gives that the utility of searching equals $\theta - r_1$ and since the utility of buying immediately is $\theta - \min(p^*, p^{\text{min}} + c_t)$ searching is profitable for $\min(p^*, p^{\text{min}} + c_t) > r_1$. 

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If \( \min(p^*, p^{\min} + c_t) \leq r_1 \) both \( q > r_1 \) and \( q \leq r_1 \) are possible. For \( q > r_1 \) the utility of search equals \( \theta - r_1 \) (see previous paragraph) and \( U(buy) \geq \theta - r_1 \). Search therefore is not profitable. For \( q \leq r_1 \), \( \int_{p^*}^{\theta} (q - p)dF_1(p) < c_t + c_e \) and \( U(search) < \theta - q \). Since \( U(buy) \geq \theta - r_1 \geq \theta - q \) search is not profitable. So for \( \min(p^*, p^{\min} + c_t) \leq r_1 \) the non-shopper will stop searching while for \( \min(p^*, p^{\min} + c_t) > r_1 \) he will continue to search.

This shows that the consumer behavior stated in the Propositions is indeed optimal when a consumer has searched \( n - 1 \) shops. Now suppose that he has searched \( h \geq 1 \) shops and that the stated consumer behavior holds whenever he has searched \( h + 1 \) or more shops. Since the consumer expects \( p \) to be at or below \( r_1 \) the optimal consumer behavior tells him to stop searching after searching the \( h + 1 \)th shop. Therefore, after searching the \( h \)th shop, the consumer expects to search only one more shop and the utilities of continuing search and of stopping search are the same as before. After searching the \( h \)th shop the non-shopper will therefore continue his search if and only if \( \min(p^*, p^{\min} + c_t) > r_1 \) and the stated consumer behavior also holds in the case \( h \geq 1 \) shops have been searched.

This leaves the case where no shops have been searched yet. Again, given the optimal consumer behavior for \( h \geq 1 \), the non-shopper expects to search only once and the utility of search equals

\[
U(search) = -c_t - c_e + \int_{p^*}^{\min(\theta, r_1)} (\theta - p)dF_1(p).
\]

When \( r_1 < \theta \) this reduces to

\[
-c_t - c_e + \int_{p^*}^{r_1} (\theta - p)dF_1(p) = -c_t - c_e + \theta - r_1 + \int_{p^*}^{\theta} (r_1 - p)dF_1(p) = \theta - r_1 > 0,
\]

so for \( r_1 < \theta \) all non-shoppers will search. When \( r_1 = \theta \) the expression above can be rewritten as

\[
-c_t - c_e + \int_{p^*}^{r_1} (r_1 - p)dF_1(p) = 0
\]

and so non-shoppers are indifferent between searching and staying home.

Before deriving an explicit expression for \( r_1 \), consider the pricing behavior of shops. First look at the full search case with \( r_1 < \theta \). A standard undercutting argument shows that the price distribution has no atoms. For \( p \leq r_1 \) profits are given by

\[
\pi_1(p) = p\gamma(1 - F_1(p))^{n-1} + p(1 - \gamma)\frac{1}{n}
\]

Under the assumption \( \overline{p} \leq r_1 \) it must be that \( \overline{p} = r_1 \). If \( \overline{p} < r_1 \) deviation to a price \( r_1 \) would be profitable. This gives that in equilibrium profits equal \( \pi_1(r_1) = r_1(1 - \gamma)\frac{1}{n} \) and equating this with \( \pi_1(\overline{p}) \) gives
Finally, the minimum price is the price $p$ such that $F_1(p) = 0$. This gives $p = r_1 \frac{1 - \gamma}{\gamma n + (1 - \gamma) \mu_1}$. Note that deviation to a price below $p$ is not profitable and that deviation to a price above $r_1$ gives zero profits and therefore is not profitable as well.

Given $F_1(p)$ the reservation price $r_1$ can be derived. Rewriting the definition of $r_1$ gives $r_1 - \int_p^{r_1} p dF_1(p) = c_t + c_e$. Rewriting $F_1(p)$ gives

$$p = \frac{r_1}{1 + \frac{\gamma n}{1 - \gamma} (1 - F_1(p))^{n-1}}$$

and therefore

$$\int_p^{r_1} p dF_1(p) = \int_0^1 \frac{r_1}{1 + \frac{\gamma n}{1 - \gamma} (1 - y)^{n-1}} dy.$$ 

This can be rewritten as

$$\int_p^{r_1} p dF_1(p) = \int_0^1 \frac{r_1}{1 + \frac{\gamma n}{1 - \gamma} y^{n-1}} dy.$$ 

The definition of $r_1$ then finally gives

$$r_1 = \frac{c_t + c_e}{1 - \int_0^1 \frac{1}{1 + \frac{\gamma n}{1 - \gamma} y^{n-1}} dy}.$$ 

Now look at the partial search case with $r_1 = \theta$. A standard undercutting argument shows that the price distribution has no atoms. For $p \leq r_1$ profits are given by

$$\pi_1(p) = p \gamma (1 - F_1(p))^{n-1} + p(1 - \gamma) \frac{\mu_1}{n}$$

It must be that $\bar{p} = r_1 = \theta$. If $\bar{p} < \theta$ deviation to a price $\theta$ would be profitable. This gives that in equilibrium profits equal $\pi_1(\theta) = \theta (1 - \gamma) \frac{\mu_1}{n}$ and equating this with $\pi_1(p)$ gives

$$F_1(p) = 1 - \frac{(1 - \gamma) \mu_1 r_1 - p}{\gamma n} \frac{1}{p^{n-1}}.$$ 

Finally, the minimum price is the price $p$ such that $F_1(p) = 0$. This gives $p = r_1 \frac{1 - \gamma \mu_1}{\gamma n + (1 - \gamma) \mu_1}$. Note that deviation to a price below $p$ is not profitable.

The condition $r_1 = \theta$ defines $\mu_1$ and the condition $0 < \mu_1 < 1$ defines the parameter region for which the equilibrium holds. The definition of $r_1$ and
\( r_1 = \theta \) gives \( \theta - \int_0^\theta p dF_1(p) = c_t + c_e \). Using the same method as before, this can be rewritten as

\[
\theta (1 - \int_0^1 \frac{1}{1 + \gamma \mu_1 y^{\gamma - 1}} dy) = c_t + c_e
\]

or,

\[
h(\mu_1) = \int_0^1 \frac{1}{1 + \gamma \mu_1 y^{\gamma - 1}} dy = \frac{\theta - c_t - c_e}{\theta}
\]

defining \( \mu_1 \). Note that \( h(\mu_1) \) is increasing in \( \mu_1 \), with \( h(0) = 0 \) and \( h(1) = \int_0^1 \frac{1}{1 + \gamma y^{\gamma - 1}} dy \). The condition \( 0 < \mu_1 < 1 \) therefore gives

\[
0 < \frac{\theta - c_t - c_e}{\theta} < \int_0^1 \frac{1}{1 + \gamma y^{\gamma - 1}} dy.
\]

Recall that by assumption \( \theta - c_t - c_e > 0 \) and so the only relevant part is

\[
\frac{\theta - c_t - c_e}{\theta} < \int_0^1 \frac{1}{1 + \gamma y^{\gamma - 1}} dy, \quad \text{or} \quad c_t + c_e > \theta (1 - \int_0^1 \frac{1}{1 + \gamma y^{\gamma - 1}} dy).
\]

**Proof of Propositions 3.3 and 3.4**

Once a non-shopper has searched one shop he is in the situation described by Stahl (1989) with search costs \( c_e \) and so he will stop searching as soon as he finds a price at or below \( r_n \), with \( r_n \) defined by

\[
\int_{\mathcal{P}} (r_n - p) dF_n(p) = c_e.
\]

Stahl (1989) also shows that the maximum price is at or below \( r_n \). This implies that non-shoppers search at most once. The expected utility of the first search therefore is

\[
-c_t - c_e + \int_{\mathcal{P}} (\theta - p) dF_n(p).
\]

For \( r_n \leq \theta \) this can be rewritten as

\[
-c_t - c_e + \theta - r_n + \int_{\mathcal{P}} (r_n - p) dF_n(p),
\]

which equals \( \theta - r_n - c_t \). Therefore, for \( r_n < \theta - c_t \) all non-shoppers will search and for \( r_n = \theta - c_t \) non-shoppers are indifferent between searching and staying home. For \( \theta - c_t < r_n \leq \theta \) searching clearly is not profitable and for \( r_n > \theta \) \( \int_{\mathcal{P}} (\theta - p) dF_n(p) < c_e \) and so the utility of searching is strictly negative as well.
In a full search equilibrium \( r_n < \theta - c_t \) and the profits for \( p \leq r_n \) are given by

\[
\pi_n(p) = p \frac{1 - \gamma}{n} + p\gamma(1 - F_n(p))^{n-1}.
\]

This expression shows that \( \bar{p} = r_n \) since else deviation to \( r_n \) would be profitable. Equilibrium profits are therefore \( \pi_n(r_n) = r_n \frac{1 - \gamma}{n} \) and equating \( \pi_n(p) \) and \( \pi_n(r_n) \) gives

\[
F_n(p) = 1 - \left( \frac{r_n - p}{n\gamma p} \right)^{\frac{1}{n\gamma - 1}}
\]

with \( p_n = r_n \frac{1 - \gamma}{\gamma n + (1 - \gamma)} \). It is clear that deviation to a price below \( p_n \) is not profitable. The same argument as in the proof of Propositions 3.1 and 3.2 finally shows that

\[
r_n = \frac{c_e}{1 - \int_0^1 \frac{1}{1 + \frac{\gamma n}{n \gamma - 1} y^{n-1}} dy}
\]

In a partial search equilibrium \( r_n = \theta - c_t \) and a fraction \( \mu_n \) of the non-shoppers searches. For \( p \leq r_n \) the profits are

\[
\pi_n(p) = p \frac{\mu_n(1 - \gamma)}{n} + p\gamma(1 - F_n(p))^{n-1}.
\]

This expression shows that \( \bar{p}_n = r_n \) and equilibrium profits are \( \pi_n(r_n) = r_n \frac{\mu_n(1 - \gamma)}{n} \). Equating \( \pi_n(p) \) with \( \pi_n(r_n) \) gives

\[
F_n(p) = 1 - \left( \frac{(r_n - p)(1 - \gamma)\mu_n}{n\gamma p} \right)^{\frac{1}{n\gamma - 1}}
\]

with \( p_n = r_n \frac{\mu_n(1 - \gamma)}{\gamma_n + \mu_n(1 - \gamma)} \). It is clear that deviating to a price below \( p_n \) is not profitable. The fraction of searching non-shoppers, \( \mu_n \), is defined by the condition \( r_n = \theta - c_t \). The same procedure as in the proof of Propositions 3.1 and 3.2 gives

\[
h(\mu_n) \equiv \int_0^1 \frac{1}{1 + \frac{\gamma n}{(1 - \gamma)\mu_n} y^{n-1}} dy = \frac{\theta - c_t - c_e}{\theta - c_t}.
\]

Finally, because \( h(\mu_n) \) is increasing in \( \mu_n \) the condition \( 0 < \mu_n < 1 \) gives

\[
0 < \frac{\theta - c_t - c_e}{\theta - c_t} < \int_0^1 \frac{1}{1 + \frac{\gamma n}{(1 - \gamma)\mu_n} y^{n-1}} dy
\]

where the first part, \( \frac{\theta - c_t - c_e}{\theta - c_t} > 0 \), is automatically satisfied because of the assumption \( \theta - c_t - c_e > 0 \).
Proof of Proposition 3.5

For ease of notation, let $q$ denote $\int_0^1 \frac{1}{1 + \frac{\gamma}{1 - \gamma} y^n} dy$.

For $c_t + c_e < \theta(1 - q)$ the full search equilibrium holds both when all shops are located together and when all shops are isolated. Therefore $\pi_t = \frac{1 - q}{n} > \frac{1 - q}{n} = \pi_n$.

Next, I show that for $c_t + c_e > \theta\frac{1 - q}{(1 - \beta)q}$ $\pi_1 < \pi_n$. In this case the partial search equilibrium holds both when all shops are located together and when all shops are isolated. Define the function $g(\mu) = \int_0^1 \frac{1}{1 + \frac{1}{\theta - c_t} \mu} y^{\beta - 1} dy$ and note that $\mu_1$ is defined by $g(\mu_1) = \frac{\theta - c_t - c_e}{\theta - c_t}$ and that $\mu_n$ is defined by $g(\mu_n) = \frac{\theta - c_t - c_e}{\theta - c_t}$. Expected profits are given by $\pi_1 = \theta \mu_1 \frac{1}{n}$ and $\pi_n = (\theta - c_t) \mu_n \frac{1}{n}$ and therefore $\pi_1 < \pi_n$ holds if and only if $\mu_n > \frac{\theta}{\theta - c_t} \mu_1$. Using that $g(\mu)$ is strictly increasing in $\mu$ this can be rewritten as $g(\mu_n) > g(\frac{\theta}{\theta - c_t} \mu_1)$ or

$$\frac{\theta - c_t - c_e}{\theta - c_t} > \int_0^1 \frac{1}{1 + \frac{1}{\theta - c_t} \mu} y^{\beta - 1} dy.$$

Since $\int_0^1 \frac{1}{1 + \frac{1}{\theta - c_t} \mu} y^{\beta - 1} dy = \frac{\theta}{\theta - c_t} \int_0^1 \frac{1}{y^{\beta - 1} + \frac{1}{\theta - c_t} \mu} y^{\beta - 1} dy$, $\pi_1 < \pi_n$ if and only if

$$\int_0^1 \frac{\theta}{\theta - c_t} + \frac{1}{\gamma \mu_1} y^{\beta - 1} dy < \frac{\theta - c_t - c_e}{\theta}.$$

The definition of $\mu_1$ gives $\frac{\theta - c_t - c_e}{\theta} = \int_0^1 \frac{1}{1 + \frac{1}{\theta - c_t} \mu} y^{\beta - 1} dy$ and so $\pi_1 < \pi_n$ if and only if

$$\int_0^1 \frac{\theta}{\theta - c_t} + \frac{1}{1 - \gamma \mu_1} y^{\beta - 1} dy < \int_0^1 \frac{1}{1 + \frac{1}{\theta - c_t} \mu} y^{\beta - 1} dy$$

and this holds always since $\frac{\theta}{\theta - c_t} > 1$.

For $\theta(1 - q) < c_t + c_e < \theta\frac{1 - q}{(1 - \beta)q}$ a full search equilibrium holds when all shops are located together. This implies that $\pi_n = \frac{(1 - \beta)(c_t + c_e)}{1 - q}$ is linearly increasing in $c_t + c_e$. When all shops are located separately a partial search equilibrium holds with $\pi_1 = \theta \mu_1 \frac{1}{n}$ and $g(\mu_1) = \frac{\theta - c_t - c_e}{\theta}$. Since $g(\mu)$ is strictly increasing in $\mu$ and $\frac{\theta - c_t - c_e}{\theta}$ decreases in $c_t + c_e$, $\mu_1$ decreases in $c_t + c_e$ and therefore $\pi_1$ decreases in $c_t + c_e$. Since for $c_t + c_e < \theta(1 - q)$ $\pi_1 > \pi_n$ and for $c_t + c_e > \theta\frac{1 - q}{(1 - \beta)q}$ $\pi_1 < \pi_n$ this implies that there exists a unique value $\theta$ with $\theta(1 - q) < c < \theta\frac{1 - q}{(1 - \beta)q}$ where $\pi_1 = \pi_n$, with $\pi_1 > \pi_n$ for $c_t + c_e < c$ and $\pi_1 < \pi_n$ for $c_t + c_e > c$.
B Optimal consumer behavior

A first useful result is the following.

**Proposition B.1** In equilibrium $\pi^m_k > 0$ and $\pi^i_k > 0$. Consequently, $\pi^m_k \leq \theta$ and $\pi^i_k \leq \theta$.

**Proof**

Suppose to the contrary that $\pi^m_k = 0$ and $\pi^i_k > 0$. This implies that $p^i_k > 0$ since for $p^i_k = 0$ $\pi^i_k(p^i_k) = 0$. If some of the non-shoppers visit the shopping mall in their first search then for a shop in the mall setting a price $c_e$ will prevent the non-shoppers from continuing search, leading to positive profits, a contradiction. If none of the non-shoppers search in the shopping mall shops in the mall compete for the shoppers, leading to a maximum price of 0. But then non-shoppers would prefer to search in the shopping mall, a contradiction.

The case $\pi^m_k > 0$ and $\pi^i_k = 0$ is the same as above, reversing the roles of the shops inside and outside the mall.

This leaves the case $\pi^m_k = 0$ and $\pi^i_k = 0$. If non-shoppers would search then the shops attracting some non-shoppers could set a price $c_e$ and make a strictly positive profit. If non-shoppers do not search the firms compete for the shoppers and all set a price 0. But in that case non-shoppers would find it optimal to search, a contradiction.

Since a price above $\theta$ would not lead to any sales, the profits of setting such a price are 0, which contradicts the fact that in equilibrium profits are strictly positive.

□

Using Proposition B.1 and assuming that $\pi^m_k \leq r^m_k$ and $\pi^i_k \leq r^i_k$, the behavior of non-shoppers can be derived. This behavior depends on the prices found in previous searches and on whether the consumer currently is in a shop inside the shopping mall or whether he currently is in a shop outside the shopping mall. First consider the case where the consumer currently is in the shopping mall and let $\tilde{p}^m_k$ denote the lowest price found on current and previous searches in the shopping mall. Let $\tilde{p}^i_k$ denote the lowest price found on previous searches outside the shopping mall, with $\tilde{p}^i_k = \infty$ when on previous searches no shops outside the mall have been visited. Note that if the non-shopper decides to stop searching and $\tilde{p}^m_k \leq \tilde{p}^i_k + c_t$ then he will buy from the cheapest shop outside the shopping mall, with $\tilde{p}^i_k = \infty$ when $\tilde{p}^m_k \leq \theta$ and therefore buying at $\tilde{p}^m_k$ is a better strategy than not buying at all. If the non-shopper decides to stop searching and $\tilde{p}^m_k > \tilde{p}^i_k + c_t$ then he will buy from the cheapest shop outside the shopping mall, incurring return
costs $c_t$ and buying at price $\tilde{p}_k^i$. In the proposition that follows I will use the term 'buy from the cheapest option' to denote this behavior.

**Proposition B.2** Consider a non-shopper who expects $\overline{\pi}_k^m \leq r_k^m$ and $\overline{\pi}_k^r \leq r_k^i$ and who currently is in a shop in the shopping mall.

When $r_k^m \leq r_k^i$ ($r_k^m > r_k^i$) his optimal behavior is as follows. When $\min(\overline{\pi}_k^m, \tilde{p}_k^i + c_t) \leq r_k^m$ ($\min(\overline{\pi}_k^m, \tilde{p}_k^i + c_t) \leq r_k^i$) stop search and buy from the cheapest option. When $\min(\overline{\pi}_k^m, \tilde{p}_k^i + c_t) > r_k^m$ ($\min(\overline{\pi}_k^m, \tilde{p}_k^i + c_t) > r_k^i$) search further in (outside) the shopping mall, if possible. If there are no shops left to search in (outside) the shopping mall and $\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) \leq r_k^m$ ($\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) \leq r_k^i$) buy from the cheapest option. If there are no shops left to search in (outside) the shopping mall and $\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) > r_k^m$ ($\min(\tilde{p}_k^m, \tilde{p}_k^i + c_t) > r_k^i$) search further outside (in) the shopping mall, if possible. If there also are no shops left to search outside (in) the shopping mall buy from the cheapest option.

Now consider the case where the consumer currently is outside the shopping mall and let $\tilde{p}_k^i$ denote the price found in the current shop. Let $\overline{\pi}_k^m$ denote the lowest price found on previous searches outside the shopping mall, with $\overline{\pi}_k^m = \infty$ when on previous searches no shops outside the mall have been visited. Let $\overline{\pi}_k^r$ denote the lowest price found on previous searches inside the shopping mall, with $\overline{\pi}_k^r = \infty$ when on previous searches no shops inside the mall have been visited. Note that if the non-shopper decides to stop searching and to buy and $\min(\overline{\pi}_k^m, \tilde{p}_k^i + c_t, \overline{\pi}_k^r + c_t) = \tilde{p}_k^i$ then he will buy from the shop he currently is, at price $\tilde{p}_k^i$. If $\min(\overline{\pi}_k^m, \tilde{p}_k^i + c_t, \overline{\pi}_k^r + c_t) = \overline{\pi}_k^m + c_t$ then he will buy from the cheapest shop outside the shopping mall visited before, incurring return costs $c_t$ and buying at price $\overline{\pi}_k^m$. If $\min(\overline{\pi}_k^m, \tilde{p}_k^i + c_t, \overline{\pi}_k^r + c_t) = \tilde{p}_k^r + c_t$ then he will buy from the cheapest shop inside the shopping mall, incurring return costs $c_t$ and buying at price $\tilde{p}_k^r$. In the proposition that follows I will use the term 'buy from the cheapest option' to denote this behavior.

**Proposition B.3** Consider a non-shopper who expects $\overline{\pi}_k^m \leq r_k^m$ and $\overline{\pi}_k^r \leq r_k^i$ and who currently is in a shop outside the shopping mall.

When $r_k^i \leq r_k^m + c_t$ ($r_k^i > r_k^m + c_t$) his optimal behavior is as follows. When $\min(\overline{\pi}_k^i, \tilde{p}_k^i + c_t, \overline{\pi}_k^m + c_t) \leq r_k^i$ ($\min(\overline{\pi}_k^i, \tilde{p}_k^i + c_t, \overline{\pi}_k^m + c_t) \leq r_k^m + c_t$) stop search and buy from the cheapest option. When $\min(\overline{\pi}_k^i, \tilde{p}_k^i + c_t, \overline{\pi}_k^m + c_t) > r_k^i$ ($\min(\overline{\pi}_k^i, \tilde{p}_k^i + c_t, \overline{\pi}_k^m + c_t) > r_k^m + c_t$) search further outside (in) the shopping mall, if possible. If there are no shops left to search outside (in) the shopping mall and $\min(\overline{\pi}_k^i, \tilde{p}_k^i + c_t, \overline{\pi}_k^m + c_t) \leq r_k^i$ ($\min(\overline{\pi}_k^i, \tilde{p}_k^i + c_t, \overline{\pi}_k^m + c_t) \leq r_k^m + c_t$) buy from the cheapest option. If there are no shops left to search outside (in) the shopping mall and $\min(\overline{\pi}_k^i, \tilde{p}_k^i + c_t, \overline{\pi}_k^m + c_t) > r_k^i$ ($\min(\overline{\pi}_k^i, \tilde{p}_k^i + c_t, \overline{\pi}_k^m + c_t) > r_k^m + c_t$) search further in (outside) the shopping mall, if possible. If there also are no shops left to search in (outside) the shopping mall buy from the cheapest option.
Proof

A complete proof of Propositions B.2 and B.3 is available on request. Here I only give a short sketch of the proof.

Let $h$ denote the number of shops that have not yet been searched, let $h^m$ denote the number of shops in the mall that have not yet been searched and let $h^i$ denote the number of isolated shops that have not yet been searched, with $h = h^m + h^i$. The proof uses several induction arguments. First, it is easy to see that both propositions hold when $h = 0$. A second step is to prove that both propositions hold when $h^m = 1$ and $h^i = 0$. Using a standard induction argument it can then be shown that both propositions also hold for $h^i = 0$ and $h^m > 1$. A third step is to prove that both propositions also hold when $h^m = 0$ and $h^i = 1$. Again using a standard induction argument it can then be shown that both propositions also hold for $h^m = 0$ and $h^i > 1$.

These three steps together prove Propositions B.2 and B.3 for some corner cases. These cases together form the basis of one final induction step. This final step shows the following. If the propositions hold for $h = x - 2$ and for $h = x - 1$ then the propositions also hold for $h = x$, with $h^m \geq 1$ and $h^i \geq 1$. Since the propositions hold for $h = 0$ and $h = 1$, they will also hold for $h > 1$, $h^m \geq 1$ and $h^i \geq 1$. Note that steps two and three have already shown that the propositions hold for $h^m = 0$, $h^i > 1$ and for $h^i = 0$, $h^m > 1$.

Propositions B.2 and B.3 specify the optimal behavior of non-shoppers when they have searched at least one shop under the conditions $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k \leq r_k^i$. Proposition B.4 specifies the optimal behavior of non-shoppers when they have not yet searched any shop.

**Proposition B.4** Let shops price according to $\overline{p}_k^m \leq r_k^m$ and $\overline{p}_k \leq r_k^i$. If $r_k^i < r_k^m + c_t$ (or $r_k^m + c_t$) non-shoppers prefer to first search an isolated shop (shop in the mall) above first searching a shop in the mall (an isolated shop). If $r_k^i < \theta$ (or $r_k^m + c_t < \theta$) all non-shoppers will search, if $r_k^i = \theta$ (or $r_k^m + c_t = \theta$) non-shoppers are indifferent between staying at home and searching an isolated shop (shop in the mall) and if $r_k^i > \theta$ (or $r_k^m + c_t > \theta$) all non-shoppers prefer to stay at home.

If $r_k^i = r_k^m + c_t$ non-shoppers are indifferent between searching in an isolated shop or in a shop in the mall. When $r_k^i < \theta$ all non-shoppers will search, when $r_k^i = \theta$ non-shoppers are indifferent between searching and staying at home and when $r_k^i > \theta$ all non-shoppers prefer to stay at home.

Proof

First look at the case $r_k^i = r_k^m + c_t$. If a non-shopper would start his search in a shop in the mall he expects to find a price at or below $r_k^m$ and as
Proposition B.2 shows the non-shopper thus expects to stop searching after the first search. Expected utility of searching in the mall is

\[ U(\text{mall}) = -c_t - c_e + \int_{\mathbb{P}_k^m} (\theta - p) dF_k^m(p), \]

which can be rewritten as

\[ U(\text{mall}) = -c_t - c_e + \theta - r_k^m + \int_{\mathbb{P}_k^m} (r_k^m - p) dF_k^m(p) = \theta - r_k^m - c_t. \]

A non-shopper who starts his search in an isolated shop expects to find a price at or below \( r_i^k \) and as Proposition B.3 shows he expects to stop searching after the first search. Expected utility is

\[ U(\text{isolated}) = -c_t - c_e + \int_{\mathbb{P}_k^i} (\theta - p) dF_k^i(p), \]

which can be rewritten as

\[ U(\text{isolated}) = -c_t - c_e + \theta - r_k^i + \int_{\mathbb{P}_k^i} (r_k^i - p) dF_k^i(p) = \theta - r_k^i. \]

Since \( r_k^i = r_k^m + c_t \), \( U(\text{mall}) = U(\text{isolated}) \) and non-shoppers are indifferent between searching in an isolated shop or in a shop in the mall. When \( r_k^i < \theta \) \( U(\text{isolated}) > 0 \) and all non-shoppers will search. When \( r_k^i = \theta \) \( U(\text{isolated}) = 0 \) and non-shoppers are indifferent between searching and staying at home. When \( r_k^i > \theta \) \( U(\text{isolated}) < 0 \) and all non-shoppers prefer to stay at home.

The proof for the cases \( r_k^i > r_k^m + c_t \) and \( r_k^i < r_k^m + c_t \) follows the same arguments, but is mathematically slightly more complicated since non-shoppers sometimes expect to search twice instead of once. Details are available on request.

\[ \square \]

## C  Proofs of Section 4

### Proof of Proposition 4.1

Recall that in the model \( c_t + c_e < \theta \) and therefore at least some non-shoppers will search. First assume that \( r_k^i < r_k^m + c_t \). Proposition B.4 shows that under this assumption all searching non-shoppers will first search in an isolated shop. Using Proposition B.3 and using that \( \overline{p}_k \leq r_k^i \) it is easy to see
that the searching non-shoppers will stop searching after their first search and will buy from the isolated shop they visited. Consequently, shops in the mall will compete for the shoppers and mall prices will be zero. The definition of \( r_k^m \) in that case gives \( r_k^m = c_e \), a contradiction of the initial assumption that \( r_k^i < r_k^m + c_t \).

For \( r_k^i > r_k^m + c_t \) all searching non-shoppers will first search in the mall, and because \( \bar{p}_k^m \leq r_k^m \) they will stop searching after their first search and buy from the mall shop they visited. In case \( k < n-1 \) there are 2 or more isolated shops and these isolated shops would compete for the shoppers. Isolated prices would be zero and \( r_k^i = c_t + c_e \), a contradiction of \( r_k^i > r_k^m + c_t \).

To show that \( r_k^i > r_k^m + c_t \) cannot hold when \( k = n-1 \), a lengthy argument is required. Here only a brief outline is given; full details are available on request. First, it can be argued that for \( r_k^i > r_k^m + c_t \) and \( k = n-1 \) \( \bar{p}_k^i \geq \bar{p}_k^m \). The next step then is to derive the equilibrium price distributions \( F_k^m(p) \) and \( F_k^i(p) \). It can be shown that \( F_k^i(p) \) is strictly increasing for \( 0 < p < r_k^i \). If \( (p)_k^m < \bar{p}_k^i \) it should hold that \( F_k^i(\bar{p}_k^m) < 0 \). But then \( \pi_k^i(\bar{p}_k^m) > \pi_k^i(\bar{p}_k^i) \). This shows that an equilibrium with \( r_k^i > r_k^m + c_t \) and \( k = n-1 \) has \( \bar{p}_k^i = \bar{p}_k^m \). Using this, a full equilibrium can be derived, including analytical expressions for the two reservation prices \( r_k^m \) and \( r_k^i \). An analysis then shows that \( r_k^i \leq r_k^m + c_t \) for all relevant parameter values, contradicting the initial assumption that \( r_k^i > r_k^m + c_t \).

**Proof of Proposition 4.3**

First note that

\[
\pi_k^i = \pi_k^i(\bar{p}_k^i) = \gamma \bar{p}_k^i (1 - F_k^m(\bar{p}_k^i))^k + (1 - \gamma) \frac{1 - x_k}{n - k} \bar{p}_k^i \mu_k
\]

and

\[
\pi_k^m \geq \pi_k^m(\bar{p}_k^i) = \gamma \bar{p}_k^i (1 - F_k^m(\bar{p}_k^i))^{k-1} + (1 - \gamma) \frac{x_k}{n-k} \bar{p}_k^i \mu_k.
\]

Suppose contrary to the proposition that \( \pi_k^m = \pi_k^i \), implying that \( \bar{r}_k^m \frac{1-x_k}{n-k} = \frac{1-x_k}{n-k} \frac{1-x_k}{n-k} \). Since, using Proposition 4.1, \( \frac{x_k}{n-k} > \frac{1-x_k}{n-k} \). This gives \( \pi_k^i = \pi_k^i(\bar{p}_k^i) < \pi_k^m(\bar{p}_k^i) \leq \pi_k^m \), a contradiction to the assumption \( \pi_k^m = \pi_k^i \).

Now suppose contrary to the proposition that \( \pi_k^m < \pi_k^i \). Note that \( \pi_k^m < \pi_k^i \) implies \( \pi_k^m(\bar{p}_k^i) = \pi_k^m(\bar{p}_k^i) \) and therefore \( \frac{1-x_k}{n-k} > \bar{p}_k^i \). Moreover, \( \pi_k^m(\bar{p}_k^i) \geq \pi_k^m(\bar{p}_k^i)(1 - \gamma) \bar{p}_k^i \mu_k \geq \gamma \bar{p}_k^i (1 - F_k^m(\bar{p}_k^i))^{k-1} \). Combining these two inequalities gives
\[ 
\pi_k^i(p_k^i) \leq \gamma p_k^i (1 - F_k^m(p_k^i))^{k-1} + (1 - \gamma) \frac{1 - x_k}{n - k} p_k^i \mu_k \\
\leq (r_k^m - p_k^i)(1 - \gamma) \frac{x_k}{k} \mu_k + (1 - \gamma) \frac{1 - x_k}{n - k} p_k^i \mu_k \\
< r_k^m (1 - \gamma) \frac{1 - x_k}{n - k} \mu_k \\
< r_k^i (1 - \gamma) \frac{1 - x_k}{n - k} \mu_k = \pi_k^i(r_k^i),
\]
a contradiction.

Since both \( \pi_k^m = \pi_k^i \) and \( \pi_k^m < \pi_k^i \) are not feasible, it should be that \( \pi_k^m > \pi_k^i \), or \( r_k^m x_k \frac{1}{k} > r_k^i \frac{1}{n - k} \). Proposition 4.1 then gives that \( \frac{x_k}{k} > \frac{1 - x_k}{n - k} \).

**Proof of Proposition 4.4**

Suppose to the contrary that \( p_k^m < p_k^i \). I will show that in that case isolated shops have an incentive to deviate to a price \( p_k^m \).

For \( p \leq p_k^i \)
\[ 
\pi_k^m(p) = \gamma p(1 - F_k^m(p))^{k-1} + (1 - \gamma) \frac{x_k}{k} p.
\]

Proposition 4.2 shows that for \( p \leq p_k^i \) there will not be any atoms in \( F_k^m(p) \). Moreover, for \( p \leq p_k^i \) a gap in \( F_k^m(p) \) is not possible. To prove this, suppose \( F_k^m(p) \) would be constant for all prices in \((p_k^i, p_k^m)\), with \( p_k^m \leq p_k^i \). Then \( \pi_k^m(p_k^i) < \pi_k^m(p_k^m) \), a contradiction. If \( F_k^m(p) \) would be constant for all prices in \((p_k^i, p_k^m)\), with \( p_k^m > p_k^i \), then \( \pi_k^m(p_k^m) < \pi_k^m(p_k^i) \), again a contradiction.

Given that there are no atoms or gaps for \( p \leq p_k^i \), in equilibrium \( \pi_k^m(p_k^m) = \pi_k^m(p_k^i) \). This gives
\[ 
1 - F_k^m(p_k^i) = \left[ \frac{\gamma p_k^m + (1 - \gamma) \frac{x_k}{k} (p_k^m - p_k^i)}{\gamma p_k^i} \right]^{\frac{1}{k-1}}.
\]

Note that \( \pi_k^i(p_k^i) = \gamma p_k^i (1 - F_k^m(p_k^i))^{k-1} + (1 - \gamma) \frac{1 - x_k}{n - k} p_k^i \). Plugging in the expression given above gives
\[ 
\pi_k^i(p_k^i) = (1 - F_k^m(p_k^i)) (\gamma p_k^m + (1 - \gamma) \frac{x_k}{k} (p_k^m - p_k^i)) + (1 - \gamma) \frac{1 - x_k}{n - k} p_k^i.
\]

Note that \( \pi_k^i(p_k^m) = \gamma p_k^m + (1 - \gamma) \frac{1 - x_k}{n - k} p_k^m \) and that deviation to \( p_k^m \) is profitable when \( \pi_k^i(p_k^m) > \pi_k^i(p_k^i) \). This can be rewritten as
\[ \gamma p^m_k + (1 - \gamma) \frac{1 - x_k}{n-k} (p^m_k - p^i_k) > (1 - F^m_k(p^i_k))(\gamma p^m_k + (1 - \gamma) \frac{x_k}{k} (p^m_k - p^i_k)). \]

I will show that this inequality always holds. First note that \(1 - F^m_k(p^i_k) \geq 0\) implies that \(\gamma p^m_k + (1 - \gamma) \frac{x_k}{n-k} (p^m_k - p^i_k) \geq 0\).

Now if \(0 < 1 - F^m_k(p^i_k) \leq 1\), \((1 - F^m_k(p^i_k))(\gamma p^m_k + (1 - \gamma) \frac{x_k}{n-k} (p^m_k - p^i_k)) \leq \gamma p^m_k + (1 - \gamma) \frac{x_k}{n-k} (p^m_k - p^i_k)\). Since \(\frac{x_k}{n-k} > \frac{1 - x_k}{n-k}\) (Proposition 4.3) and \(p^m_k - p^i_k < 0\), \(\gamma p^m_k + (1 - \gamma) \frac{x_k}{n-k} (p^m_k - p^i_k) < \gamma p^m_k + (1 - \gamma) \frac{1 - x_k}{n-k} (p^m_k - p^i_k)\). Combining gives \((1 - F^m_k(p^i_k))(\gamma p^m_k + (1 - \gamma) \frac{x_k}{n-k} (p^m_k - p^i_k)) < \gamma p^m_k + (1 - \gamma) \frac{1 - x_k}{n-k} (p^m_k - p^i_k)\).

If \(1 - F^m_k(p^i_k) = 0\), the inequality reduces to \(\gamma p^m_k + (1 - \gamma) \frac{x_k}{n-k} (p^m_k - p^i_k) > 0\). Since \(\gamma p^m_k + (1 - \gamma) \frac{x_k}{n-k} (p^m_k - p^i_k) \geq 0\) this inequality always holds.

**Proof of Proposition 5.1**

- Recall that \(c_t = \beta(c_t + c_e)\), so \(\beta \to 0\) implies \(c_t \to 0\). Thus \(r^m_k - r^n_k = c_t \to 0\). In equilibrium type 1, \(\pi^i_k(a) = \pi^i_k(r^i_k)\) gives

\[
\gamma p^m_k \frac{x_k}{k} (\gamma z^{n-k-1} + (1 - \gamma) \frac{1 - x_k}{n-k}) - r^i_k \frac{1 - x_k}{n-k} (\gamma z^{n-k} + (1 - \gamma) \frac{x_k}{k}) = 0,
\]

where \(z = 1 - F^i_k(a)\). Note that \(r^m_k \frac{x_k}{k} (\gamma z^{n-k-1} + (1 - \gamma) \frac{1 - x_k}{n-k}) - r^i_k \frac{1 - x_k}{n-k} (\gamma z^{n-k} + (1 - \gamma) \frac{x_k}{k}) \to r^m_k \gamma z^{n-k-1} (\frac{x_k}{k} - \frac{1 - x_k}{n-k})\) and therefore the equilibrium can only hold when either \(z \to 0\) or \(z \to 1\) and \(\frac{x_k}{k} \to \frac{1}{n}\).

When \(z \to 0\), \(Ep^i_k < Ep^m_k\), which cannot be an equilibrium, but when \(z \to 1\), \(Ep^i_k > Ep^m_k\), which also cannot be an equilibrium. Thus, equilibrium type 1 cannot hold when \(\beta \to 0\). Note that equilibrium types 2 and 3 have \(p^i_k = p^m_k\). For those equilibria, \(r^m_k = \frac{1 - x_k}{n-k} \gamma + (1 - \gamma) \frac{1 - x_k}{n-k} c_t\).

Since by definition \(r^m_k \geq c_e\) it must be that \(\frac{x_k}{k} - \frac{1 - x_k}{n-k} \to 0\), or \(\frac{x_k}{k} \to \frac{1}{n}\).

This gives \(\frac{\frac{x_k}{k}(r^m_k - p)}{\frac{1}{n-k}(r^i_k - p)} \to 1\) and thus \(F^i_k(p) - F^m_k(p) \to 0\) for \(p \leq a\). In equilibrium type 2, \(\pi^i_k(b) = \pi^m_k(r^m_k)\) can be rewritten as

\[
\frac{1 - x_k}{n-k} (r^i_k - b)(\frac{a(r^i_k - b)}{b(r^i_k - a)})^{\frac{1}{n-k-1}} = (r^m_k - b) \frac{x_k}{k}.
\]

Since \(\frac{x_k}{k} \to \frac{1}{n}\) and \(r^i_k \to r^m_k\), this can only hold when \(a \to b\). Therefore, in the limit for both equilibrium type 2 and equilibrium type 3 we have

\[
\int_p^{r^m_k} F^m_k(p)dp = \int_p^{a} F^m_k(p)dp + \int_a^{r^m_k} F^m_k(p)dp = c_e
\]
When equilibrium type 1 holds and 

\[ \int_p^m F_k^i(p)dp = \int_p^a F_k^i(p)dp + (r_k^m - a)F_k^i(a)dp = c_e. \]

Since for \( p \leq a \) \( F_k^i(p) - F_k^m(p) \to 0 \) the expressions above can only hold when \( a \to r_k^m \).

- Note that \( \beta \to 1 \) implies \( c_e \to 0 \). In any equilibrium this gives
  \[ \int_p^m F_k^m(p)dp = c_e \to 0, \text{ thus } p_k^m \to r_k^m. \]
  In equilibrium type 1
  \[ p_k^m = r_k^m \frac{\mu_k(1-\gamma)}{\gamma z^{n-k} + \mu_k(1-\gamma) k}, \]
  with \( z = 1 - F_k^i(r_k^m) \). To ensure that
  \( p_k^m \to r_k^m \), it should hold that \( x_0 = 0 \) and thus \( F_k^i(r_k^m) \to 1 \). In equilibrium,
  \[ \int_p^m F_k^i(p)dp = c_e + c_t. \]
  \[ F_k^i(r_k^m) \to 1 \] gives
  \[ \int_p^m F_k^i(p)dp \to \int_p^m F_k^i(p)dp + c_t \text{ and thus } c_e \to 0 \]
  gives \( \int_p^m F_k^i(p)dp \to 0 \), or \( p_k^i \to r_k^m \).
  \[ p_k^i = r_k^i \frac{\mu_k(1-\gamma)}{\gamma + \mu_k(1-\gamma) k}, \]
  so if \( \mu_k \to 0 \), \( p_k^i \to 0 \) and consequently \( r_k^m \to 0 \), contradicting \( \mu_k \to 0 \) \((\mu_k < 1 \text{ implies } r_k^m = \theta - c_t)\). Thus, \( \mu_k \to 0 \).

Using \( z \to 0 \) the equilibrium condition

\[ r_k^m \frac{z^{n-k-1} \mu_k(1-\gamma)^{1-x_k}}{\gamma z^{n-k} + \mu_k(1-\gamma) k} = \mu_k(1-\gamma) \frac{1-x_k}{n-k} (r_k^i - r_k^m \frac{\mu_k(1-\gamma)^{1-x_k}}{\gamma z^{n-k} + \mu_k(1-\gamma) k}) \]

provides \( \mu_k(1-\gamma)c_k \frac{1-x_k}{n-k} \to 0 \), or \( \frac{1-x_k}{n-k} \to 0 \). Consequently, \( p_k^i \to 0 \) and \( r_k^m \to 0 \).

In equilibrium types 2 and 3 \( r_k^m - p_k^m \to 0 \) gives

\[ r_k^m \frac{(1-\gamma)^{1-x_k}}{\gamma + (1-\gamma) \mu_k k} \to 0, \]

thus \( r_k^m \to 0 \). \( r_k^m = \frac{(1-\gamma)^{1-x_k}}{\gamma + (1-\gamma) \mu_k k} c_t \) so \( r_k^m \to 0 \) if and only if

\[ \frac{1-x_k}{n-k} \to 0. \]

Finally, \( \int_p^m F_k^i(p)dp = c_t + c_e \) with \( p = p_k^i = p_k^m \to r_k^m \)
gives \( F_k^i(r_k^m) \to 1 \).

- When equilibrium type 1 holds and \( \mu_k = 1 \), \( \pi_k^i(a) = \pi_k^i(r_k^i) \) gives

\[ r_k^m \frac{x_k(\gamma z^{n-k-1} + (1-\gamma) \frac{1-x_k}{n-k} - r_k^i \frac{1-x_k}{n-k} (\gamma z^{n-k} + (1-\gamma) \frac{x_k}{k}) = 0, \]

where \( z = 1 - F_k^i(a) \). Taking the limit when \( \gamma \to 0 \) this gives

\[ r_k^m \frac{x_k}{n-k} - r_k^i \frac{1-x_k x_k}{n-k} \frac{k}{k}. \]

This is clearly above 0, thus in equilibrium type 1 \( \mu_k < 1 \).

One of the equilibrium conditions then is
\[(\theta - c_t) \int_0^1 1 - \frac{1}{1 + \frac{\gamma}{\mu_k} \frac{1 - x_k}{x_k} \frac{y}{y - 1}} dy = c_e.\]

When \(\frac{\gamma}{\mu_k} \rightarrow 0\) the integral goes to 0 and the condition cannot hold.

Thus, in equilibrium, \(\mu_k \rightarrow 0\).

In equilibrium types 2 and 3, \(p_k = r_k^m(1 - \frac{\gamma}{\gamma + (1-\gamma)\mu_k x_k})\), with \(r_k^m = \theta - c_t\) when \(\mu_k < 1\). When \(\frac{\mu_k}{\gamma} \rightarrow \infty\), \(p_k = r_k^m\), such that \(\int p_k^m F_k^m(p)dp \rightarrow 0\).

This cannot hold, so in equilibrium \(\mu_k \rightarrow 0\).

- In equilibrium type 1, \(p_k^i = r_k^i(1-\gamma)\mu_k \frac{1-x_k}{x_k} p_k^i\). When \(\gamma \rightarrow 1\), \(p_k^i \rightarrow 0\).

Moreover, \(F_k^i(p) = 1 - \frac{r_k^i}{\gamma \mu_k} - \frac{1-x_k}{x_k} p_k^i\) \(\nrightarrow \frac{1}{x_k}\) \(\rightarrow 1\). Together, this gives \(r_k^i \rightarrow c_t + c_e\) and consequently \(r_k^m \rightarrow c_e\). But \(r_k^m \rightarrow c_e\) can only happen when \(p_k^m \rightarrow 0\) and \(F_k^m(p) \rightarrow 1\).

In equilibrium types 2 and 3, \(p_k^i = p_k^m = r_k^m(1 - \frac{\gamma}{\gamma + (1-\gamma)\mu_k x_k}) \rightarrow 0\).

Moreover,

\[(1 - F_k^m(p))^{n-1} = \frac{(1-\gamma)\mu_k \frac{1-x_k}{x_k} (r_k^i - p)}{\gamma p} \left( \frac{1-x_k}{x_k} (r_k^i - p) \right)^{n-1} \rightarrow 0,
\]

so \(F_k^m(p) \rightarrow 1\). Consequently, \(r_k^m \rightarrow c_e\) and \(r_k^i \rightarrow c_e + c_t\). This finally gives \(F_k^i(p) \rightarrow 1\).

Details on the derivation of the equilibrium of Section ??

In the full search equilibrium \(\pi_k^m = r_k^m(1 - \gamma) \frac{x_k}{x_k} \) and equating \(\pi_k^m(p)\) with \(\pi_k^m\) gives

\[F_k^m(p) = 1 - \frac{\left( r_k^m - p \right) \left( 1 - \gamma \right) \frac{x_k}{x_k} }{\gamma p (z_k)^{n-1}} \pi_k^m\]

with

\[p_k^m = r_k^m \frac{1 - \gamma}{\gamma (z_k)^{n-k} + (1-\gamma) \frac{x_k}{x_k}}.\]

Isolated shops expect profits equal to \(\pi_k^i = r_k^i(1 - \gamma) \frac{1-x_k}{x_k} \) and equating this with \(\pi_k^i(p)\) gives that for \(p \leq p_k^m\)

\[F_k^i(p) = 1 - \frac{\left( r_k^i - p \right) \left( 1 - \gamma \right) \frac{1-x_k}{n-k} }{\gamma p} \pi_k^i(1-\gamma) \frac{1-x_k}{x_k} \]

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with

\[ \hat{\ell}_k^i = r_k^i \frac{(1 - \gamma)^{1 - x_k} n - k}{\gamma + (1 - \gamma)^{1 - x_k} n - k}. \]

The maximum prices \( r_k^m \) and \( r_k^i \) are defined by

\[ \int_{\hat{\ell}_k^m}^{r_k^m} (r_k^m - p) dF_k^m(p) = c_e \]

and

\[ \int_{\hat{\ell}_k^i}^{r_k^i} (r_k^i - p) dF_k^i(p) = c_e + c_t. \]

Plugging in the expressions for \( F_k^m(p) \) and \( F_k^i(p) \) and rewriting (details are in the appendix) gives

\[ r_k^m = \frac{c_e}{\int_{0}^{1} \frac{1}{1 + \gamma (z_k)^{n-k} x_k} dy} \]

and

\[ r_k^i = \frac{c_e + c_t}{\int_{z_k}^{1} \frac{1}{1 + \gamma + (1 - \gamma)^{x_k} y^{n-k}} dy}. \]

The probability that an isolated shop sets a price equal to \( r_k^i, z_k \), is implicitly defined by \( z_k = 1 - F_k^i(\hat{\ell}_k^m) \). Moreover, as Proposition 4.1 shows, \( r_k^i = r_k^m + c_t \). These two equalities together define \( x_k \) and \( z_k \). Unfortunately, it is impossible to solve explicitly for \( x_k \) and \( z_k \), and in the next section I will use computer simulations to obtain numerical values.

The full search equilibrium can only hold when \( r_k^i < \theta \) (see the optimal consumer behavior) and no shop has an incentive to deviate. Using the expressions for \( F_k^m(p) \) and \( F_k^i(p) \) given above it can be shown that an isolated shop never has an incentive to deviate from \( F_k^i(p) \). For a shop in the mall it is clear that deviation to a price above \( r_k^m \) is never profitable. Deviating to a price below \( p_k^i \) and \( p_k^m \) could be profitable. In that case deviating gives profits \( \pi_k^m(p) = \gamma p (1 - F_k^i(p)) n - k + (1 - \gamma) \frac{r_k^i}{n - k} p \). Plugging in \( F_k^i(p) \) and twice differentiating shows that the second derivative is positive. This implies that the maximum value of \( \pi_k^m(p) \) is obtained either at \( p = p_k^i \) or at \( p = p_k^m \). Deviating is not profitable if and only if \( \pi_k^m(\hat{\ell}_k^i) \geq \pi_k^m(\hat{\ell}_k^m) \), or \( r_k^m x_k (\gamma + (1 - \gamma) \frac{1 - x_k}{n - k}) \geq r_k^i \frac{1 - x_k}{n - k} (\gamma + (1 - \gamma) \frac{1}{n - k}). \)
Using the same method as before one can derive that in a partial search equilibrium for $p \leq r^m_k$

$$F_k^m(p) = 1 - \left[ \frac{(r^m_k - p)(1 - \gamma)\mu_k \frac{x_k}{n-k}}{\gamma p (z_k)^{n-k}} \right]^{\frac{1}{k-1}}$$

with

$$\bar{r}_k^m = r^m_k \frac{(1 - \gamma) \frac{x_k}{n-k} \mu_k}{\gamma (z_k)^{n-k} + (1 - \gamma) \frac{x_k}{n-k} \mu_k}$$

and that for $p \leq \bar{r}_k^m$

$$F_i^i(p) = 1 - \left[ \frac{(r^i_i - p)(1 - \gamma) \frac{1-x_k}{n-k} \mu_k}{\gamma p} \right]^{\frac{1}{n-k-1}}$$

with

$$\bar{r}_k^i = r^i_i \frac{(1 - \gamma) \frac{1-x_k}{n-k} \mu_k}{\gamma + (1 - \gamma) \frac{1-x_k}{n-k} \mu_k}.$$ 

Since this is a partial search equilibrium, $r^i_i = \theta$ and $r^m_k = \theta - c_t$. The three parameters $x_k$, $z_k$ and $\mu_k$ are jointly defined by $z_k = 1 - F^i_k(\bar{r}_k^i)$, $r^m_k = \int_{\bar{r}_k^m}^{\theta - c_t} p dF^m_k(p) + c_e = \theta - c_t$ and $r^i_i = \int_{\bar{r}_k^i}^{\theta} p dF^i_k(p) + c_t + c_e = \theta$.

Again, solving explicitly for $x_k$, $z_k$ and $\mu_k$ is not possible and I will resort to simulations.

An analysis similar to the one for the full search equilibrium shows that deviating is never profitable for isolated shops and that mall shops will not deviate if and only if $r^m_k \frac{x_k}{k} (\gamma + (1 - \gamma) \frac{1-x_k}{n-k} \mu_k) \geq r^i_i \frac{1-x_k}{n-k} (\gamma + (1 - \gamma) \frac{x_k}{n-k} \mu_k)$.

**More details**

In the full search equilibrium $r^m_k$ is defined by

$$\int_{\bar{r}_k^m}^{r^m_k} (r^m_k - p) dF^m_k(p) = c_e.$$ 

Rewriting gives

$$r^m_k - \int_{\bar{r}_k^m}^{r^m_k} p dF^m_k(p) = c_e.$$ 

Note that

$$p = \frac{r^m_k}{1 + \frac{z_k}{1-\gamma} (z_k)^{n-k} \frac{r^m_k}{z_k} (1 - F^m_k(p))^{k-1}}.$$ 

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Therefore,
\[
\int_{p_{ik}^m}^{r_{ik}^m} p dF_{ik}^m(p) = \int_0^1 \frac{r_{ik}^m}{1 + \frac{\gamma}{1-\gamma}(z_k)^{n-k} x_k} dy
\]
and so the definition of \( r_{ik}^m \) gives
\[
r_{ik}^m - \int_0^1 \frac{r_{ik}^m}{1 + \frac{\gamma}{1-\gamma}(z_k)^{n-k} x_k} dy = c_e,
\]
or
\[
r_{ik}^m(\int_0^1 1 - \frac{1}{1 + \frac{\gamma}{1-\gamma}(z_k)^{n-k} x_k} dy) = c_e.
\]
In the full search equilibrium \( r_{ik}^i \) is defined by
\[
\int_{p_{ik}^i}^{r_{ik}^i} (r_{ik}^i - p) dF_{ik}^i(p) = c_e + c_t.
\]
Since \( F_{ik}^i(p) \) is constant for \( r_{ik}^m \leq p < r_{ik}^i \) and since for \( p = r_{ik}^i r_{ik}^i - p = 0 \) the definition can be rewritten as
\[
\int_{p_{ik}^i}^{r_{ik}^i} (r_{ik}^i - p) dF_{ik}^i(p) = c_e + c_t.
\]
Rewriting gives
\[
r_{ik}^i F_{ik}^{r_{ik}^m}(r_{ik}^m) - \int_{p_{ik}^i}^{r_{ik}^m} pdF_{ik}^i(p) = c_e + c_t,
\]
or, using that \( F_{ik}^{r_{ik}^m}(r_{ik}^m) = 1 - z_k \) by definition of \( z_k \),
\[
r_{ik}^i (1 - z_k) - \int_{p_{ik}^i}^{r_{ik}^m} pdF_{ik}^i(p) = c_e + c_t.
\]
Note that
\[
p = \frac{r_{ik}^i}{1 + \frac{\gamma}{1-\gamma} \frac{n-k}{1-x_k} (1 - F_{ik}^i(p))^{n-1}}.
\]
Therefore,
\[
\int_{p_{ik}^i}^{r_{ik}^m} pdF_{ik}^i(p) = \int_0^{1-z_k} \frac{r_{ik}^i}{1 + \frac{\gamma}{1-\gamma} \frac{n-k}{1-x_k} (1 - y)^{n-1}} dy.
\]
A change of variables gives
\[ \int_{P_k}^{p_m} p dF_k(p) = \int_{z_k}^{1} \frac{r^i_k}{1 + \frac{\gamma_{n-k} y^{n-k-1}}{1-x_k y^{n-k-1}}} dy. \]

The definition of \( r^m_k \) now gives

\[
\begin{aligned}
& r^i_k (1 - z_k) - \int_{z_k}^{1} \frac{r^i_k}{1 + \frac{\gamma_{n-k} y^{n-k-1}}{1-x_k y^{n-k-1}}} dy = c_e + c_t, \\
\text{or} \\
& r^i_k \left( \int_{z_k}^{1} 1 - \frac{1}{1 + \frac{\gamma_{n-k} y^{n-k-1}}{1-x_k y^{n-k-1}}} dy \right) = c_e.
\end{aligned}
\]