Preemption and the Efficiency of Entry*

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Abstract

This paper studies an entry timing game. Firms differ in their efficiency. In a game with two firms, the more efficient firm always enters first in equilibrium. In a game with three firms instead, the equilibrium order of entry does not necessarily reflect the efficiency ranking. This result is also illustrated with an example in which post-entry profits arise from Cournot competition and differences in efficiency are due to differences in marginal costs.

Keywords: Timing Games, Preemption, Dynamic Entry

JEL Classification: C73, L13, O3.

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1 Introduction

This paper examines whether the order of entry in a market always reflects the efficiency of the entering firms. A more efficient firm earns higher per-period profits upon entry, hence one may conclude that it has a stronger incentive to enter than a less efficient firm. Therefore, it should enter earlier, even if this comes at a higher cost. This intuition is correct for a dynamic entry game with entry cost declining over time played by two firms but, as we show in this paper, can fail when there are three firms.

In particular, we consider a game where three firms have to decide when to enter a new market. Post-entry profits are declining in the number of rival entrants, and the cost of entry declines exogenously over time. This is a preemption game: Firms want to delay entry in order to reduce the entry cost, but they also want to enter earlier than the rivals, in order to earn higher flow profits before the following entries.

In our model, there are one efficient firm (“type A” firm) and two inefficient firms (“type B” firms). We show that with a general payoff structure, the unique equilibrium outcome may be such that the order of entry is $B - A - B$. That is, one of the inefficient firms enters first, the efficient firm follows strictly later, to be followed by the remaining inefficient firm. We also provide an example in which post-entry profits are derived from Cournot competition, where the relative efficiency of firm A stems from a lower marginal cost.

The intuition behind our result is as follows. When a firm considers entering first, it takes into account for how long it will earn monopoly profits. Thus the incentive to enter first, to preempt its rivals, depends on the timing of next entry in the ensuing two-player subgame. If firm $A$ enters first, the resulting subgame among the two type $B$ firms can involve a relatively intense preemption race, and thus second entry will occur relatively soon. If however a type $B$ firm enters first, the resulting subgame among firm $A$ and the remaining type $B$ firm involves relatively weak preemption incentives: The second entrant is firm $A$, and it can afford to wait long and enter at a relatively low cost, because $B$ is a weak competitor. As we show, it can hold that the first entrant enjoys monopoly profits
for a shorter period if it is of type A than if it is of type B. This shorter monopoly period can outweigh the fact that the monopoly profit flow earned would be higher for the more efficient firm. As a result, in equilibrium one of the inefficient type B firms will enter first. The exact timing of first entry will be determined by a preemption race among the two inefficient firms. Rents among the two inefficient type B firms are equalized.

Our model builds on the classic literature on two-player preemption games. We rely on Fudenberg and Tirole (1985) to derive the outcome of the two-player symmetric subgame played by two inefficient firms. From Riordan’s (1992) analysis, we obtain the outcome of the two-player asymmetric subgame. It follows from Riordan (1992) that in a two-player asymmetric game, the more efficient firm always enters first. We contribute to this literature by showing that this order of entry may be reversed when adding a third firm.

Our result can also be relevant to the recent empirical literature on static entry games with asymmetric potential entrants. Following Berry (1992), it is often assumed that entry occurs in the order of profitability to solve the inherent multiplicity problem. Indeed, Quint and Einav (2005) show that this assumption can be rationalized by the outcome of a war of attrition where entry cost are sunk gradually: The entrants in the unique subgame perfect equilibrium in such a game are the most efficient firms. Our game shows that this result may be reversed if the underlying game is a preemption game: At any given point in time, the firms observed in the market are not necessarily the most efficient ones.

Similarly, our result may be important when analyzing the role of entry on mitigating anticompetitive effects of a merger. Section 3.4 of the Merger Guidelines\(^1\) requires entry to be “sufficient” in the sense that entry drives down prices. If our game is played among three potential entrants after a merger in the industry, the constraint from entry on pricing will not come from the most efficient potential entrant.

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2 Model

We model entry in a new market as an infinite horizon dynamic game in continuous time. Our assumptions correspond to those made by Fudenberg and Tirole (1985) and Riordan (1992), when specialized to the case of a new market, and with a third firm added to the model. In particular, we consider a model with one efficient firm, firm A, and two identical “type B” firms. Each firm has to decide whether and when to enter a new market. Before entry, it receives no profits. Upon entry, firm $i$ (for $i = A, B$) earns flow profits $\pi_i(m, -i)$, where $m$ is the total number of firms that have entered, hence $m \in \{1, 2, 3\}$, and $-i$ stands for the identity of rival firms that have entered. For example, $\pi_B(2, A)$ stands for the profits of a type B firm in duopoly if its rival is firm A. The following payoffs are relevant in our model:

<table>
<thead>
<tr>
<th>Firm</th>
<th>Type B firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopoly</td>
<td>$\pi_A(1)$</td>
</tr>
<tr>
<td>Duopoly</td>
<td>$\pi_A(2)$</td>
</tr>
<tr>
<td>Triopoly</td>
<td>$\pi_A(3)$</td>
</tr>
</tbody>
</table>

In monopoly there are no rival firms and in triopoly the identity of rivals is uniquely identified by the identity of firm $i$. Similarly, in a duopoly firm A will always oppose a type B firm. Hence, to economize on notation, we leave out the $-i$ term in the monopoly and triopoly cases as well as for firm A’s duopoly profits.

All the above profits are positive. For a given firm, profits decline in the number of competitors. Moreover, firm A’s higher efficiency is reflected in payoffs. Firm A always earns higher profits than a type B firm, for a given number of competitors. Also, a type B firm earns lower profits if its opponent is firm A than if its opponent is the other type B firm, i.e. profits decline in the efficiency of rival firms. Formally:

**Assumption 1**

(\(i\)) $\pi_i(m, -i) > 0 \quad \forall (m, -i)$
\[(ii)\]
\[
\begin{align*}
\pi_A (1) & > \pi_B (1) \\
\pi_A (2) & > \pi_B (2, B) > \pi_B (2, A) \\
\pi_A (3) & > \pi_B (3) \\
\pi_A (1) & > \pi_A (2) > \pi_A (3) \\
\pi_B (1) & > \pi_B (2, B) > \pi_B (2, A) > \pi_B (3)
\end{align*}
\]

In section 4 we consider an example of post-entry competition that gives rise to a payoff structure that satisfies Assumption 1: Firms compete à la Cournot with constant marginal costs, and firm A’s marginal cost \( c_A \) is strictly less than the type B firms’ marginal cost \( c_B \).

The present value at time zero of entering the market at time \( t \) is \( c (t) \). Following the literature,\(^2\) we assume that it declines over time, at a decreasing rate:

**Assumption 2**

\[(i)\] \( (c(t)e^{rt})' < 0 \ \forall t \)

\[(ii)\] \( (c(t)e^{rt})'' > 0 \ \forall t \)

The payoff function for firm \( i \), conditional upon a given entry order in which \( i \) is the \( j \)-th entrant, as a function of its own entry time \( t_j \) and the competitors’ entry times \( t_{-j} \) is:

\[
f_i(t_j, t_{-j}) \equiv \sum_{m=1}^{3} I [j \leq m] \cdot \int_{t_m}^{t_{m+1}} \pi_i(m, -i)e^{-rs}ds - c(t_j) \quad (*)
\]

where \( I [\cdot] \) is the indicator function and \( t_4 \equiv +\infty \). Before \( t_j \), firm \( i \) receives zero profits. Then, it receives flow profits \( \pi_i(m, -i) \) depending on the number and identity of the competitors present in the market. Finally, \( c(t_j) \) denotes entry cost.

We denote with \( T_i^*(m, -i) \) the time that maximizes \( f_i(\cdot) \) with respect to \( t_j \), i.e. the optimal time for firm \( i \) to enter the market, conditional upon being the \( j \)-th entrant.\(^3\) Given assumptions 1 and 2, the above payoff function is strictly quasi-concave in \( t_j \). The next two assumptions guarantee that it admits a strictly positive finite maximum.

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\(^3\)For the case of first entry, \( T^*_1(m, -i) \) corresponds to firm \( i \)'s stand-alone entry time (see Katz and Shapiro (1987)).
Assumption 3 normalizes time as to guarantee that entry at time 0 is not profitable.

**Assumption 3**

(i) \( \pi_A(1) - c(0) < 0 \)

(ii) \( -c'(0) > \pi_A(1) \)

Part (i) implies that even if a firm could preempt all its rivals and earn monopoly profits forever, entry at time zero would be too costly. Part (ii) guarantees that \( f_i(t_j) \) is increasing at time zero.

Assumption 4 instead guarantees that the entry cost eventually becomes so low, and slows down so much, that even entry with the lowest possible profits dominates staying out.

**Assumption 4**

(i) \( \exists \tau \text{ such that } c(\tau) e^{r\tau} < \frac{\pi_B(3)}{r} \)

(ii) \( \lim_{t \to \infty} c'(t)e^{rt} \epsilon \in (\pi_B(3), 0] \)

Given Assumptions 1 to 4, \( T^*_i(m, -i) \) solves the first order condition:

\[-\pi(m, -i)e^{-rt} - c'(t) = 0.\]

The condition is easily interpreted: a marginal delay of entry implies foregone profits \( \pi_i(m, -i)e^{-rt} \) and cost savings \( c'(t) \). Given the quasiconcavity of the payoff function (*) in \( t_j \), it follows that \( T^*_i(m, -i) \) is decreasing in \( \pi_i(m, -i) \). Hence, the following inequalities follow from assumption 1(ii):

\[
T_A^*(1) < T_B^*(1) \quad T_A^*(2) < T_B^*(2, B) < T_B^*(2, A) \\
T_A^*(3) < T_B^*(3) \quad T_A^*(1) < T^*_A(2) < T^*_A(3) \\
T_B^*(1) < T_A^*(2) < T^*_B(2, B) < T_B^*(2, A) < T_B^*(3)
\]

6
It is clear that firm A’s optimal entry time, for a given rank in the entry order, is always earlier than that of a less efficient firm: By delaying entry, A would forego a higher profit than a type B firm would.

In order to model entry as a preemption game of complete information in continuous time, we follow Hoppe and Lehmann-Grube (2005), who illustrate how to adopt the framework introduced by Simon and Stinchcombe (1989) for this class of games. In particular, we restrict play to pure strategies and interpret continuous time as “discrete time, but with a grid that is infinitely fine.”

Moreover, we need to address the issue of nonexistence of an equilibrium in pure strategies in preemption games, which is related to the possibility of coordination failures. Since we adopt the Simon and Stinchcombe (1989) framework, we need to explicitly rule out the possibility of coordination failures, and we do so using a randomization device as in Katz and Shapiro (1987), Dutta, Lach and Rustichini (1995), and Hoppe and Lehmann-Grube (2005):

**Assumption 5**

*If n firms invest at the same instant t (with n ∈ [2, N]), then only one firm, each with probability \( \frac{1}{n} \), succeeds.*

Assumption 5 rules out the possibility of coordination failures and thus ensures existence of an equilibrium in pure strategies.

### 3 Inefficient Entry

In this section, it is shown that while in a preemption race with only two asymmetric players the first firm to enter is always the most efficient one, this result can be reversed if the game is played by more than two players.

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4 For an illustration of this methodology, see Hoppe and Lehmann-Grube (2005).
5 See the example in Simon and Stinchcombe (1989, p. 1178-1179).
6 For a discussion, see Dutta, Lach and Rustichini (1995) and Argenziano and Schmidt-Dengler (2008).
3.1 The Game with Two Asymmetric Firms.

Let us first consider a preemption game played by two asymmetric firms, that is one efficient firm of type A and one inefficient firm of type B. In such a game, it follows from the analysis in Riordan (1992) that the first entrant is A. Below, we illustrate the key mechanism behind Riordan’s (1992) result in the context of our model.

Consider the payoff for each firm in case it is the “leader” in this game, i.e. the first entrant, or the “follower”, i.e. the second entrant. If firm A enters first at time \( t \), firm B follows at its optimal time for second-entry, \( T_B^*(2, A) \). Hence, firm A receives payoff

\[
\bar{L}_A(t) = \pi_A(1) \int_t^{T_B^*(2, A)} e^{-rs} ds + \pi_A(2) \int_{T_B^*(2, A)}^\infty e^{-rs} ds - c(t)
\]

and firm B receives payoff

\[
\bar{F}_B(t) = \pi_B(2) \int_{T_B^*(2, A)}^\infty e^{-rs} ds - c(T_B^*(2, A)).
\]

If instead firm B enters first at time \( t \), firm A follows at its own optimal time for second entry, \( T_A^*(2) \), and the leader’s and follower’s payoff are

\[
\bar{L}_B(t) = \pi_B(1) \int_t^{T_A^*(2)} e^{-rs} ds + \pi_B(2, A) \int_{T_A^*(2)}^\infty e^{-rs} ds - c(t)
\]

and

\[
\bar{F}_A(t) = \pi_A(2) \int_{T_A^*(2)}^\infty e^{-rs} ds - c(T_A^*(2)).
\]

respectively. Notice that, as we argue in section 2, the fact that \( \pi_A(2) > \pi_B(2, A) \) implies that firm A’s optimal entry time as a follower is earlier than firm B’s: \( T_A^*(2) < T_B^*(2, A) \).

Now consider the incentive for each firm to preempt the competitor and be the leader,
rather than the follower. Firm A prefers to enter as a leader at time $t$ whenever

$$\tilde{D}_A(t) = \tilde{L}_A(t) - \tilde{F}_A(t)$$

$$= \pi_A(1) \int_t^{T_B(2,A)} e^{-rs} ds - \pi_A(2) \int_{T_A(2)}^{T_B(2,A)} e^{-rs} ds +$$

$$-[c(t) - c(T_A^*(2))]$$

is positive. Similarly, firm B prefers to enter as leader at $t$ if

$$\tilde{D}_B(t) = \tilde{L}_B(t) - \tilde{F}_B(t)$$

$$= \pi_B(1) \int_t^{T_B(2)} e^{-rs} ds + \pi_B(2, A) \int_{T_A(2)}^{T_B(2,A)} e^{-rs} ds$$

$$-[c(t) - c(T_B^*(2,A))]$$

is positive.

It follows immediately from Riordan (1992) that firm A is the leader in equilibrium. This result is based on the comparison between the two $\tilde{D}_i(t)$ functions. More precisely, on the fact that for any time $t$ earlier than $T_A^*(2)$, the incentive to preempt is always stronger for A than for B, that is $\tilde{D}_A(t) > \tilde{D}_B(t)$. The crucial observation to prove this inequality is that $T_A^*(2) < T_B^*(2,A)$. Consider the first term in $\tilde{D}_A(t)$ and $\tilde{D}_B(t)$. Monopoly profits are not only higher for A, but also earned for a longer period, that is until $T_B^*(2,A)$ rather than until $T_A^*(2)$. Consider then the third term in $\tilde{D}_A(t)$ and $\tilde{D}_B(t)$. Anticipating entry from $T_A^*(2)$ to $t$ is cheaper than anticipating entry from $T_B^*(2,A)$ to $t$. To complete the argument, one only needs to show that the increase in duopoly profits for B and the decrease in duopoly profits for A do not offset the previous two effects. The intuition is as follows.7 By preempting B, firm A delays the date from which it earns duopoly profits from $T_A^*(2)$ to $T_B^*(2,A)$. In the interval $[T_A^*(2), T_B^*(2,A)]$ duopoly profits are replaced by monopoly profits, so the total effect is still positive. B instead, by preempting A, anticipates the date from which it earns duopoly profits, from $T_B^*(2,A)$ to $T_A^*(2)$. By definition of $T_B^*(2,A)$, anticipating entry as a duopolist to the left of this point is detrimental: extra duopoly profits are more than offset.

7For a formal proof of a similar argument, see the proof of Lemma 4 in the Appendix.
by the increase in entry cost.

Consider now the remaining information we have about $\tilde{D}_A(t)$ and $\tilde{D}_B(t)$. First, they are both negative for $t = 0$: by Assumption 2 preemption is too costly at time zero. Moreover, they are both strictly quasi-concave, because of the convexity of the cost function, and have a maximum in $T^*_i(1)$ for $i = A, B$ respectively. Finally, in $t = T^*_A(2)$, the function $\tilde{D}_A(t)$ is strictly positive.

Following the argument in Riordan (1992), the equilibrium has the following features. First entry cannot take place for $t$ very close to zero, because $\tilde{D}_A(t)$ and $\tilde{D}_B(t)$ are both negative. From some $\tilde{T}^*_A < T^*_A(2)$ onwards, $\tilde{D}_A(t)$ becomes positive: firm $A$ would rather be leader than follower, and ideally it would like to delay first entry until $T^*_A(1)$. If $\tilde{D}_B(t)$ is negative in the interval $[\tilde{T}^*_A, T^*_A(1)]$, firm $B$ has no incentive to enter before $T^*_A(1)$, and firm $A$ can therefore not only be the first to enter, but also enter at its preferred time. If instead $\tilde{D}_B(t)$ is positive from some point $\tilde{T}^*_B \in (\tilde{T}^*_A, T^*_A(1))$ onwards, then $A$ is forced to anticipate first entry to $t = \tilde{T}^*_B$ by the threat of preemption. Following the terminology in Riordan (1992), we refer to firm $A$ as a “strong leader” if $T^*_A(1) < T^*_B$ and as a “weak leader” otherwise.

In any case, $B$ cannot enter first in equilibrium. In a candidate equilibrium with first entry by $B$ at some time $t$, it has to hold that in $t$ firm $B$ strictly prefers the leader’s payoff to the follower’s payoff. But then $\tilde{D}_A(t)$ is also positive, hence $A$ can profitable deviate preemption $B$ and entering at $(t - \varepsilon)$.

### 3.2 The Game with Three Firms.

Here we consider the game with one $A$ firm and two $B$ firms. We show that the efficient-entry result in the two-firm game can be reversed when there are three firms. More precisely, we show that the unique equilibrium outcome can be that the entry order is $B - A - B$.

With a construction similar to the one presented in subsection 3.1, we first derive the payoff from being the leader, or one of the followers, for each firm. Then, we compute the preemption incentives, that is the difference between the leader’s and the follower’s payoff.

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8The fact that $D_A(t) > D_B(t)$ guarantees that $T_B(1) < T_A(1)$. 10
for each firm. Finally, we show under which circumstances the entry order in equilibrium will be $B - A - B$. We will show that the main difference with the two-firm case is that with three asymmetric firms it is not always the case that the most efficient firm has a strictly stronger preemption incentive.

If firm $A$ enters first in this game, the ensuing subgame will be played by two firms of type $B$. We call this subgame the “$BB$-subgame.” If on the other hand a firm of type $B$ enters first, the ensuing subgame will be played by firm $A$ and the remaining type $B$ firm. We call this the “$AB$-subgame.” The unique equilibrium outcomes of these two-player subgames are readily known from work by Fudenberg and Tirole (1985) for the $BB$-subgame and by Riordan (1992) for the $AB$-subgame. It follows from their analysis that the first entry times of both subgames are uniquely determined. We will denote them as $t_2(BB)$ and $t_2(AB)$ respectively. They further show that in either subgame last entry is by a $B$ firm, and occurs at $T_B(3)$, a type $B$’s optimal entry time as a last entrant.

Consider the case in which the type $A$ firm preempts its rivals and enters first at time $t$. The $B$ firms will follow at $t_2(BB)$ and $T_B(3)$ respectively, hence firm $A$ will earn a leader payoff:

$$L_A(t) = \pi_A(1) \int_t^{t_2(BB)} e^{-rs} ds + \pi_A(2) \int_{t_2(BB)}^{T_B(3)} e^{-rs} ds + \pi_A(3) \int_{T_B(3)}^{\infty} e^{-rs} ds - c(t)$$

and each of the $B$ firms will earn a follower payoff:

$$F_B(t) = \pi_B(2,A) \int_{t_2(BB)}^{T_B(3)} e^{-rs} ds + \pi_B(3) \int_{T_B(3)}^{\infty} e^{-rs} ds - c(t_2(BB))$$

$$= \pi_B(3) \int_{T_B(3)}^{\infty} e^{-rs} ds - c(T_B(3))$$

where the l.h.s. of the last equality represents the payoff of the early entrant in the subgame, the r.h.s. represents the payoff of the late entrant, and the equality describes the “rent equalization” result in Fudenberg-Tirole (1985).

If instead a type $B$ firm enters first, a two-player game with asymmetric firms ensues in which the type $A$ firm enters at $t_2(AB)$ and the remaining $B$ firm at time $T_B(3)$. The early
B firm obtains the leader payoff:

\[ L_B(t) = \pi_B(1) \int_t^{t_2(AB)} e^{-rs} ds + \pi_B(2, A) \int_{t_2(AB)}^{T_B(3)} e^{-rs} ds + \pi_B(3) \int_{T_B(3)}^\infty e^{-rs} ds - c(t). \]

Firm A is preempted and thus earns the follower payoff:

\[ F_A(t) = \pi_A(2) \int_{t_2(AB)}^{T_B(3)} e^{-rs} ds + \pi_A(3) \int_{T_B(3)}^\infty e^{-rs} ds - c(t_2(AB)) \]

and the late B firm obtains follower’s payoff \( F_B(t) \).

Next, we compute the incentive to be the first entrant in the game for an efficient and an inefficient firm, respectively.

Firm A would like to preempt its rivals whenever

\[
D_A(t) = L_A(t) - F_A(t) = \pi_A(1) \int_t^{t_2(BB)} e^{-rs} ds + \pi_A(2) \int_{t_2(BB)}^{t_2(AB)} e^{-rs} ds - c(t) + c(t_2(AB))
\]

is positive. By preempting the rivals, firm A gains monopoly profits from time \( t \) until \( t_2(BB) \), achieves duopoly profits starting from \( t_2(BB) \) rather than \( t_2(AB) \), and finally sustains a higher entry cost because it enters earlier.

Similarly, a type B firm prefers to be the leader rather than the follower if

\[
D_B(t) = L_B(t) - F_B(t) = \pi_B(1) \int_t^{t_2(AB)} e^{-rs} ds + \pi_B(2, A) \int_{t_2(AB)}^{t_2(BB)} e^{-rs} ds - c(t) + c(t_2(BB))
\]

is positive. By preempting the rivals, a B firm gains monopoly profits from \( t \) until \( t_2(AB) \), achieves duopoly profits starting from \( t_2(AB) \) rather than \( t_2(BB) \), and finally sustains a higher entry cost because it enters earlier.

We now present our main result: if the preemption incentive as described by the functions \( D_i(t) \) for \( i = A, B \) is “stronger” for \( A \) than for the type-\( B \) firms in a sense that

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9 Observe that a B firm’s follower payoff is independent of whether the subgame following first entry is between two type B firms or one type A and one type B firm.
we will specify later, the equilibrium entry order is $B - A - B$. In the next subsection, we illustrate when it is the case that the necessary condition holds.

Consider the $D_i(t)$ functions, for $i = A, B$. In $t = 0$, they are both negative, because by Assumption 3 preemption is too costly at time zero. Moreover, they are both strictly quasi-concave because of the convexity of the cost function, and have a maximum in $T_i^*(1)$ for $i = A, B$ respectively.

Our main result is the following. Suppose that the earliest point in time in which a $B$ firm weakly prefers to be a leader rather than a follower is even earlier than the first point in time in which A weakly prefers to be the leader. That is, the smallest point in which $D_B(t)$ intersects zero, is earlier than the earliest point in which $D_A(t)$ intersects zero. Then, the equilibrium outcome will be that one type-$B$ firm enters first, exactly at the earliest time in which $D_B(t)$ intersects zero, and an AB subgame ensues, with firm A and the remaining type-$B$ firm entering at $t_2(AB)$ and $T_B^*(3)$ respectively.

Formally, let $T^1_B$ and $T^1_A$ be defined as:

\[
T^1_B = \begin{cases} 
\min \{ \tau \text{ such that } D_B(\tau) = 0 \} & \text{if } D_B(t) \text{ admits at least one zero} \\
+\infty & \text{otherwise}
\end{cases}
\]

\[
T^1_A = \begin{cases} 
\min \{ \tau \text{ such that } D_A(\tau) = 0 \} & \text{if } D_A(t) \text{ admits at least one zero} \\
+\infty & \text{otherwise}
\end{cases}
\]

The following Proposition holds:

**Proposition 1** If $T^1_B < T^1_A$, the game admits a unique SPNE outcome, in which the entry order is $B - A - B$, and entry times are $t_1 = T^1_B$, $t_2 = t_2(AB)$, $t_3 = T^*_B(3)$. In equilibrium, both inefficient firms achieve the same equilibrium payoff, and the efficient firm achieves a higher payoff.

In equilibrium, first entry takes place exactly at $T^1_B$. Clearly, no firm has an incentive to enter earlier, because for $t < T^1_B$, each firm prefers to be the follower rather than a very early leader. Moreover, first entry cannot take place later than $T^1_B$ because in a right-neighbourhood of $T^1_B$, $D_B(t)$ is positive. If first entry took place at $t > T^1_B$, one of the
two $B$ firms would enter third at $T_B^1(3)$ and receive $F_B(t)$. That firm would rather deviate, preempt the rivals, and be a leader at $T_B^1 + \varepsilon$. Following a logic that is analogous to that in Fudenberg-Tirole (1985), the preemption race between the two $B$ firms guarantees that first entry takes place exactly at $T_B^1$ so that there is rent equalization for the two $B$ firms and neither has an incentive to preempt further. Moreover, $T_B^1 < T_A^1$ guarantees that in $T_B(1)$ firm $A$ strictly prefers to be follower rather than leader, hence has no incentive to deviate either.

3.3 Discussion

In this subsection, we provide an intuition for what guarantees that $T_B^1$ is indeed smaller than $T_A^1$. As shown in the proof of Proposition 1, a necessary condition for $T_B^1 < T_A^1$ is that $t_2(AB) > t_2(BB)$.

We first consider the case where this condition is violated. Then, the comparison between the preemption incentives for the efficient and the inefficient firms is similar to the discussion presented in subsection 3.1: In both cases, if the leader is an $A$ type firm, second entry takes place later than if the leader is a type-$B$ firm. From this observation, it follows once again that in the relevant range the preemption incentive $D_i(t)$ is always stronger for $A$ than for $B$, hence first entry cannot be by a $B$ firm.

More precisely: The preempting firm achieves monopoly profits for some time. For firm $A$, these profits would be higher than for firm $B$, and achieved for a longer time (until $t_2(BB)$, rather than until $t_2(AB)$). Moreover, by entering at $t$ and being a leader rather than a follower, each firm sustains a higher entry cost. The cost increases less for firm $A$, who would otherwise enter at $t_2(AB)$, than for firm $B$, who would otherwise enter at $t_2(BB)$. Finally, by being leader rather than follower, a firm changes the time from which it starts earning duopoly profits. For an $A$-type leader, this date is delayed from $t_2(AB)$ to $t_2(BB)$. In that interval, duopoly profits are replaced by monopoly profits, so the total effect is positive. For $B$ instead, this date is anticipated from $t_2(BB)$ to $t_2(AB)$. Nonetheless, the extra duopoly profits earned in this period are more than offset by the increase in entry.
cost, so the total effect is negative.\textsuperscript{10} Hence, the total preemption incentive at $t$ is stronger for firm $A$ than for a type-$B$ firm.

Consider now the opposite case, in which $t_2(BB) < t_2(AB)$. In this case, the comparison between the preemption incentives of an efficient and an inefficient firm is substantially different, and it is possible that the incentive is stronger for a $B$ type firm than for $A$.

The cost increase due to earlier entry is higher for an $A$-type leader, who would otherwise enter at $t_2(AB)$, than for a $B$-type leader, who would instead enter at $t_2(BB)$. Monopoly profits are higher for $A$ than for $B$, but a $B$-type leader achieves them for a longer time (until $t_2(AB)$ rather than just until $t_2(BB)$). Finally, the date from which duopoly profits are achieved is delayed for a $B$-type leader, and this is beneficial because in the interval from $t_2(AB)$ to $t_2(BB)$ duopoly profits are now replaced by monopoly profits. For an $A$-type leader instead, the date from which duopoly profits are earned is now anticipated from $t_2(AB)$ to $t_2(BB)$. This is detrimental, because in that interval the entry cost varies at a rate higher than $\pi_B(2, A)$. The sum of these different effects can give rise to a situation in which $D_B(t) > D_A(t)$ in some range, and in particular that $T^1_B < T^1_A$.

We now illustrate under which conditions first entry in an $AB$-subgame occurs later than in a $BB$-subgame, i.e. $t_2(AB) > t_2(BB)$.

Let us first consider an $AB$-subgame. In such a subgame, the strategic interaction is analogous to that described in section 3.1 for the case of an asymmetric preemption race between one $A$ firm and one $B$ firm. Hence, it follows from the analysis of Riordan (1992) that $A$ enters first in the subgame, at the earliest of $T^*_A(2)$, which is $A$’s preferred entry date, and $T^{2,B}_B$, which is defined as the earliest time at which $B$ can credibly threaten to preempt $A$, that is the earliest date where the difference between the “leader” payoff $L^{2,B}_B(t)$ and the “follower” payoff $F^{2,B}_B$ for a $B$ firm:

\textsuperscript{10}For a formal argument, see the Proof of Lemma 5.
\[ D^2_B(t) = \mathcal{I}^2_B(t) - F^2_B \]
\[ = \left[ \pi_B(2, B) \int_t^{T_A^*(3)} e^{-rs} ds + \pi_B(3) \int_{T_A^*(3)}^{\infty} e^{-rs} ds - c(t) \right] - \left[ \pi_B(3) \int_{T_B^*(3)}^{\infty} e^{-rs} ds - c(T_B^*(3)) \right] \]
\[ = \pi_B(2, B) \int_t^{T_A^*(3)} e^{-rs} ds + \pi_B(3) \int_{T_A^*(3)}^{T_B^*(3)} e^{-rs} ds - [c(t) - c(T_B^*(3))] \]

is weakly positive.

Next, consider a BB-subgame. This is the classic Fudenberg and Tirole (1985) symmetric two-player preemption game. Here, by the principle of rent equalization, first entry takes place at the earliest point in time when either of the players has a weak incentive to preempt the rival, namely at \( T^2_B \), in which the difference between the leader and follower payoff:

\[ D^2_A(t) = \mathcal{I}^2_A(t) - F^2_A \]
\[ = \left[ \pi_B(2, A) \int_t^{T_B^*(3)} e^{-rs} ds + \pi_B(3) \int_{T_B^*(3)}^{\infty} e^{-rs} ds - c(t) \right] - \left[ \pi_B(3) \int_{T_B^*(3)}^{\infty} e^{-rs} ds - c(T_B^*(3)) \right] \]
\[ = \pi_B(2, A) \int_t^{T_B^*(3)} e^{-rs} ds - [c(t) - c(T_B^*(3))] \]

is weakly positive.

So, one possible case in which \( t_2(BB) < t_2(AB) \), is when \( t_2(AB) = T^2_B \) and \( T^2_A < T^2_B \). In other words, when \( A \) is a “weak leader” in the AB subgame, and the preemption incentive for a \( B \) firm is stronger in a BB subgame than in an AB subgame. Let us compare \( D^2_A(t) \) to \( D^2_B(t) \). The former involves a long period of relatively low duopoly profits \( \pi_B(2, A) \) until \( T_A^*(3) \). The latter involves relatively high duopoly profits \( \pi_B(2, B) \) only until \( T_A^*(3) \), followed by triopoly profits until \( T_B^*(3) \). For a region of the parameter space, the “duration” effect dominates the “level” effect: low duopoly profits for a long period give a higher total payoff than high duopoly profits for a short period. If this is indeed the case, \( D^2_A(t) \) is positive at \( t = T^2_B \), where \( D^2_B(t) \) is zero. That is, where a type \( B \) firm is just indifferent between preemption or not preempting a type \( A \) firm, it can
credibly threaten to preempt another type $B$ firm. Thus $T_{B}^{2,A}$ will be to the left of $T_{B}^{2,B}$.

The last result in this section is the characterization of one limit case for the parameter values in which the necessary condition for the entry order to be $B - A - B$, namely that $T_{B}^{1} < T_{A}^{1}$, is satisfied. In the next section, we show that this example is not special.

**Claim 1** If $\pi_A(1) = \pi_B(1)$, $\pi_B(2, A) = \pi_B(2, B) = \pi_A(2)$ and $\pi_A(3) > \pi_B(3)$, then $T_{B}^{1} < T_{A}^{1}$.

Claim 1 states that a limit case in which the conditions of Proposition 1 are satisfied is one in which all the firms in the game receive the same profits if they are monopolist, or duopolists, but different profits if all of them compete in the market.

In the next section, we show that the equilibrium with entry order $B - A - B$ is robust to more general assumptions on the parameters.

## 4 Example

We conclude the analysis providing an example in which post-entry profit flows are derived from a fairly standard asymmetric oligopoly model, and the condition for a $B - A - B$ type of equilibrium is satisfied. Consider a model where flow profits arise from Cournot competition and firms face a constant elasticity demand function. That is, demand is given by

$$P(Q) = Q^{-\eta}$$

where $Q$ is total output in the industry, $\eta$ is the inverse demand elasticity. Firms’ cost functions are given by $C_i(q_i) = c_i q_i$. In particular, we assume that the marginal costs satisfy:

$$0 < c_A < c_B.$$ 

It is easy to verify that the resulting payoff structure satisfies Assumption 1. Moreover, unlike the limit case characterized in Claim 1, the profit difference between efficient and inefficient firms is always positive. This difference is amplified as the number of competi-
tors in the market increases. For a small level of efficiency difference, this is sufficient to guarantee that the condition in Proposition 1 is satisfied.

We further assume that the present value cost of entry declines exponentially\(^{11}\) so that

\[
c(t) = \bar{c}e^{-(r+\alpha)t}.
\]

Now consider a specific numerical example with \(\eta = .5, c_A = 1, c_B = 1.05, r = .03, \bar{c} = 10,\) and \(\alpha = .08.\) This yields a payoff matrix of

\[
\begin{array}{cc}
\pi_A (1) & \pi_B (1) \\
\pi_A (2) & \pi_B (2, B) & \pi_B (2, A) \\
\pi_A (3) & \pi_B (3)
\end{array}
\begin{array}{cc}
.250 & .238 \\
.105 & .089 & .079 \\
.060 & .038
\end{array}
\]

Figure 1 illustrates the “Leader-Follower” curves \(D_i (\cdot)\) for this example. It is useful to first consider the two-player subgames. In the \(BB\)-subgame, the first entry time is determined by the dotted curve \(D_{2,B}^A (t),\) describing the advantage of being leader over being follower. First entry occurs at the time where \(D_{2,B}^A (t)\) is equal to zero, at \(t_2(BB) = T_{2,B}^A = 26.60,\) and second entry at the time \(T_B^B(3) = t_3 = 42.11.\)

Next consider the \(AB\)-subgame. Firm \(B\)'s incentive to preempt firm \(A\) is described by the dash-dot \(D_{2,B}^B (t).\) In this example, \(D_{2,B}^B (t)\) lies below \(D_{2,A}^A (t)\) and firm \(B\) is only willing to preempt firm \(A\) from \(T_{2,B}^B = 29.21,\) three time units later, than it would have been willing to preempt another type \(B\) firm. Firm \(A\)'s ideal entry time as a second is \(T_A^2(2) = 29.32 > 29.21 = T_{2,B}^B.\) Hence firm \(A\) is a “weak leader” and first entry in an \(AB\)-subgame takes place at \(t_2(AB) = T_{2,B}^A = 29.21.\) Therefore, in this example first entry would occur earlier in the \(BB\)-subgame than in the \(AB\)-subgame, i.e. \(t_2(BB) < t_2(AB).\)

Consider now preemption incentives at the beginning of the game, with all firms still active. The solid and dashed curves represent \(D_A(t)\) and \(D_B(t),\) respectively. \(D_B(t)\) is negative at zero, intersects zero at \(T_B^1 = 11.93\) and is strictly positive until \(t_2(BB).\) The reason why it is positive in \(t_2(BB)\) is very simple: by entering first at \(t_2(BB),\) firm \(B\) would

\(^{11}\)This choice of cost function is motivated by the example given in Fudenberg and Tirole (1985).
pay the current cost, earn monopoly profits until \( t_2(AB) \), duopoly profits from then until \( T^*_B(3) \), and later triopoly profits. If preempted instead, B would still enter (as a second) at \( t_2(BB) \), pay the current cost, but only receive duopoly profits until \( T^*_B(3) \), and triopoly profits later on\(^{12} \). Hence, in \( t_2(BB) \), each B firm strictly prefers to be a leader rather than a follower.

As for \( D_A(t) \), it also starts negative at zero and it is negative at \( t_2(BB) \). In \( t_2(BB) \), if A enters first, a B firm follows instantly, so A receives duopoly profits until \( T^*_B(3) \) and triopoly profits later on. But taking for given the entry of a B firm at \( t_2(BB) \), A would rather be a follower and wait until \( t_2(AB) \), as we know from the analysis of the AB-subgames.

In this specific example, \( D_A(t) \) intersects zero twice before \( t_2(BB) \)\(^{13} \) and the earliest intersection is in \( T^1_A = 12.81 \). First entry cannot take place after \( T^1_A \) because the second and third entrant would have a strict incentive to preempt the first one. If first entry took place at \( T^1_A \), each B firm would have an incentive to preempt both rivals and enter at \( T^1_A - \varepsilon \). Similarly, if first entry took place in the interval \( (T^1_B, T^1_A) \) each B firm would have an incentive to preempt, and be first rather than third entrant. Therefore, a preemption race between the two B firms pushes back the first entry time to \( T^1_B \), where both firms are indifferent between being the follower (in this case the third entrant) and being the leader (the first entrant.) Hence, the equilibrium outcome is given by one type B firm entering at \( t_1 = T^1_B = 11.93 \), firm A entering second at \( t_2(AB) = T^2_B = 29.21 \), and the remaining type B firm entering at \( t_3 = T^*_B(3) = 42.11 \).

5 Conclusions

We presented a preemption game of entry into a new market with asymmetric players. It is well known from the literature that in a two-player game the equilibrium entry order reflects the efficiency ranking. We show that this result can be reversed if the game is played by more than two firms.

\(^{12}\) Alternatively, B would enter at \( T^*_B(3) \) and receive the same payoff by rent equalization.

\(^{13}\) It is also possible that the function stays negative in the whole interval.
Figure 1: Preemption Incentives in the Three Player Game
6 Appendix

Proof of Proposition 1. We prove the Proposition through a series of Lemmata. First, we analyze the equilibrium of subgames with one active firm.

Lemma 1 In any subgame starting at time \( \tau \), with only one active firm \( i \), firm \( i \) enters at \( \max\{\tau, T_i^*(3)\} \).

Proof. For \( t \geq \tau \), the function

\[
f_i(t_1, t_2, t) \equiv \int_t^{+\infty} \pi_i(3) e^{-rs} ds - c(t)
\]

represents firm \( i \)'s payoff from entering last at time \( t \). As discussed in section 2, it admits a unique maximum at \( T_i^*(3) \). By assumptions 4(i) and 2(i) it is strictly positive for every \( t \) larger than some finite \( t' \). Hence its maximum value is strictly positive. Therefore, if \( \tau < T_i^*(3) \) firm \( i \) will wait until \( T_i^*(3) \) and then enter, while if \( \tau \geq T_i^*(3) \) then it will enter immediately.

An immediate consequence of Lemma 1 is the following:

Lemma 2 In any subgame starting at time \( \tau \geq T_B^*(3) \), with any number of active firms, all firms enter immediately.

Proof. The proof is analogous to the proof of Lemma 2 in Argenziano and Schmidt-Dengler (2008).

The next Lemma analyzes subgames with two active firms of type B, starting at \( \tau < T_B^*(3) \). It follows immediately from our assumptions and the analysis in Fudenberg and Tirole (1985) that there exists a point \( T_{B}^{2,A} \in (0, T_B^*(2, A)) \) such that

\[
\pi_B(2, A) \int_{T_{B}^{2,A}}^{T_B^*(3)} e^{-rs} ds - \left[ c(T_{B}^{2,A}) - c(T_B^*(3)) \right] = 0
\]

and that the following Lemma holds:

Lemma 3 In any SPNE, in any BB subgame starting at time \( \tau \) there is a unique equilibrium outcome, such that:

(i) entries take place at \( t_2 = \max\{\tau, T_{B}^{2,A}\} \) and \( t_3 = T_B^*(3) \).

(iii) If \( \tau < (T_{B}^{2,A}) \), both B firms achieve payoff \( \left[ \pi_B(3) e^{-rT_B^*(3)} - c(T_B^*(3)) \right] \), while if \( \tau > (T_{B}^{2,A}) \) payoffs for the early and late entrant are \( L_{B}^{2,A}(\tau) \) and \( F_{B}^{2,A}(\tau) < L_{B}^{2,A}(\tau) \) respectively.\(^{14}\)

\(^{14}\)Both functions were defined in section 3.2.

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The next Lemma analyzes \( AB \) subgames starting at \( \tau < T_B^*(3) \).

Consider the function \( D^{2,B}_B(t) \) defined in section 3.2. It is strictly quasiconcave and admits a unique global maximum in \( t = T_B^*(2, B) \in (T_A^*(2), T_B^*(3)) \). It takes negative value at zero by assumptions 1 and 3(i), and in \( t = T_A^*(3) \) by definition of \( T_B^*(3) \). Hence, in the interval \( t \in [0, T_A^*(3)] \) the following cases are possible:

Case (1) The function is negative everywhere.

Case (2) The function has two (possibly coinciding) intersections with zero, \( \{ T_B^{2,B}, T_A^{2,B} \} \) such that \( T_B^{2,B} \leq T_B^{2,B} \), and \( T_A^{2,B} < T_A^{2,B} \).

Case (3) The function has two (possibly coinciding) intersections with zero, \( \{ T_B^{2,B}, T_B^{2,B} \} \) such that \( T_B^{2,B} \leq T_B^{2,B} \), and \( T_B^{2,B} < T_A^{2,B} \).

Letting \( T_B^{2,B} \) be defined as follows:

\[
T_B^{2,B} = \begin{cases} 
  +\infty & \text{in case 1} \\
  T_B^{2,B} & \text{in cases 2 and 3}
\end{cases}
\]

the following Lemma holds:

**Lemma 4** In any SPNE, in any \( AB \) subgame starting at time \( \tau < T_B^*(3) \), it holds that:

(i) If \( \tau \leq \min \{ T_B^{2,B}, T_A^{2,B} \} \), firm A enters first in the subgame, at \( t_2 = \min \{ T_B^{2,B}, T_A^{2,B} \} \) and firm B enters later, at \( t_3 = T_B^*(3) \).

(ii) If \( \tau > \min \{ T_B^{2,B}, T_A^{2,B} \} \):

- In case (1), firm A enters first in the subgame, at \( t_2 = \tau \) and firm B enters later, at \( t_3 = T_B^*(3) \).
- In cases (2) and (3), for \( \tau < \min \{ T_B^{2,B}, T_B^{2,B} \} \), firm A enters first in the subgame, at \( t_2 = \tau \) and firm B enters later, at \( t_3 = T_B^*(3) \), while for \( \tau \in \{ T_B^{2,B}, T_B^{2,B} \} \) either firm A enters first in the subgame, at \( t_2 = \tau \) and firm B enters later, at \( t_3 = T_B^*(3) \), or firm B enters first in the subgame, at \( t_2 = \tau \) and firm A enters later, at \( t_3 = T_A^*(3) \).

**Proof.** Given our assumptions, part (i) of the result follows immediately from Theorem 1 in Riordan (1992). Similarly, part (ii) follows from the analysis in the proof of Riordan (1992)’s Theorem 1. In particular, this is true because in our model it always holds that \( T_A^2 < T_B^2 \), where \( T_A^2 \) is defined as the smallest value of \( t \) such that the following function is null:

\[
D^{2}(t) = \pi_A(2) \int_{t}^{T_A^*(3)} e^{-rs}ds + [\pi_A(2) - \pi_A(3)] \int_{T_A^*(3)}^{T_B^*(3)} e^{-rs}ds - [c(t) - c(T_A^*(3))].
\]

This condition is equivalent to the condition \( \Sigma_j (y_j, z_j, z_i) < \Sigma_i (y_i, z_i, z_j) \) in Riordan’s Theorem 1, with the interpretation that \( i = A \) and \( j = B \).
The function $D_A^2(t)$ is strictly quasi-concave in $t$, strictly negative for $t = 0$, it has strictly positive value for $t = T_A^*(3)$, and admits a unique global maximum in $t = T_A^*(2) < T_A^*(3)$. Hence, $T_A^2$ is well defined and belongs to the interval $(0, T_A^*(2))$.

For $T_A^2 < T_B^{2,B}$ to hold, it is sufficient that $D_A^2(t) - D_B^{2,B}(t) > 0$ for every $t < T_A^*(3)$. To see that this condition holds, notice that $D_A^2(t)$ can be rewritten as

$$[\pi_A(2) - \pi_B(2,B)]\int_t^{T_A^*(3)} e^{-rs} ds + [\pi_A(2) - \pi_A(3)]\int_{T_A^*(3)}^{T_B^*(3)} e^{-rs} ds$$

$$-\pi_B(2,B)\int_{T_A^*(3)}^{T_B^*(3)} e^{-rs} ds + c(T_A^*(3)) - c(T_B^*(3)).$$

The first two terms are positive by assumption 1(ii) and the last one by definition of $T_B^*(3)$.

Next, we assume that $T_A^1 < T_A^2$, as in the Proposition, and show that it implies that first entry in a $BB$ subgame has to be earlier than first entry in an $AB$ subgame.

Consider subgames with three active firms starting at $\tau < \min\{T_B^{2,A}, T_B^{2,B}\}$. At time $\tau$, if all three firms are active and $A$ enters first, it follows from Lemma 3 that the two $B$ firms follow at $T_B^{2,A}$ and $T_B^*(3)$ respectively. Payoffs are $L_A(\tau)$ for $A$ and $F_B(\tau)$ (as defined in section 3.2) for both $B$ firms. If instead one of the $B$ firms enters at $\tau$, $A$ follows at $\min\{T_A^*(2), T_B^{2,B}\}$ and the other $B$ firm enters last at $T_B^*(3)$. In this case, payoffs for the first, second and third entrant are: $L_B(\tau)$, $F_A(\tau)$ and $F_B(\tau)$ respectively (as defined in section 3.2).

Consider the functions $D_A(\cdot)$ and $D_B(\cdot)$ as defined in section 3.2. Both functions are negative at zero by assumptions 1 and 3(i), they are strictly quasiconcave, and maximized at $T_A^*(1)$ and $T_B^*(1) > T_A^*(1)$ respectively. The following Lemma holds:

**Lemma 5** If, $T_A^1 < T_A^2$, then it has to be the case that $T_B^{2,A} < \min\{T_A^*(2), T_B^{2,B}\}$.

**Proof.** We prove the result by contradiction. Suppose $\min\{T_A^*(2), T_B^{2,B}\} \leq T_B^{2,A}$. Then for any $t \leq \min\{T_A^*(2), T_B^{2,B}\}$, $D_A(t)$ and $D_B(t)$ can be written as

$$D_A(t) = \pi_A(1)\int_t^{\min\{T_A^*(2), T_B^{2,B}\}} e^{-rs} ds + [\pi_A(1) - \pi_A(2)]\int_{\min\{T_A^*(2), T_B^{2,B}\}}^{T_B^{2,A}} e^{-rs} ds - c(t) + c\left(\min\{T_A^*(2), T_B^{2,B}\}\right)$$

and

$$D_B(t) = \pi_B(1)\int_t^{\min\{T_A^*(2), T_B^{2,B}\}} e^{-rs} ds + \pi_B(2,A)\int_{\min\{T_A^*(2), T_B^{2,B}\}}^{T_B^{2,A}} e^{-rs} ds - c(t) + c\left(T_B^{2,A}\right).$$
Both functions are strictly quasiconcave and negative at zero. $D_A \left( \min \left\{ T_A^*(2), T_B^2 \right\} \right)$ is strictly positive, so it has to be the case that $T_A(1) < \min \left\{ T_A^*(2), T_B^2 \right\}$. Moreover, $D_B \left( \min \left\{ T_A^*(2), T_B^2 \right\} \right)$ is strictly negative. This follows from the fact that the function

$$
\pi_B(2, A) \int_{\min \left\{ T_A^*(2), T_B^2 \right\}}^{+\infty} e^{-rs} ds - c(t)
$$

is strictly quasiconcave and maximized at $T_B^*(2, A) > T_B^2$, hence it is strictly increasing in $\left[ \min \left\{ T_A^*(2), T_B^2 \right\}, T_B^2 \right]$. It follows that either $T_B(1) > \min \left\{ T_A^*(2), T_B^2 \right\}$ or $T_A(1)$, in which case the statement follows, or $T_B(1) > \min \left\{ T_A^*(2), T_B^2 \right\}$. For the latter case, we show that $D_A(t) > D_B(t)$ for any $t \in \left[ 0, \min \left\{ T_A^*(2), T_B^2 \right\} \right]$ which in turn implies the result.

First, notice that by Assumption (1) the following inequalities hold:

$$
\pi_A(1) \int_{t}^{\min \left\{ T_A^*(2), T_B^2 \right\}} e^{-rs} ds > \pi_B(1) \int_{t}^{\min \left\{ T_A^*(2), T_B^2 \right\}} e^{-rs} ds
$$

$$
\left[ \pi_A(1) - \pi_A(2) \right] \int_{\min \left\{ T_A^*(2), T_B^2 \right\}}^{T_B^2} e^{-rs} ds > 0
$$

Moreover,

$$
-\pi_B(2, A) \int_{\min \left\{ T_A^*(2), T_B^2 \right\}}^{T_B^2} e^{-rs} ds + c \left( \min \left\{ T_A^*(2), T_B^2 \right\} \right) - c \left( T_B^2 \right) > 0
$$

by definition of $T_B^*(2, A)$. We can therefore conclude that even if $T_B(1) > \min \left\{ T_A^*(2), T_B^2 \right\}$, $D_A(t) > D_B(t)$ for any $t \in \left[ 0, \min \left\{ T_A^*(2), T_B^2 \right\} \right]$, which in turn implies it cannot be the case that $T_B^1 < T_A^1$.

Next, we assume that the condition $T_B^1 < T_A^1$ in the statement of the Proposition holds, and consider subgames starting at $\tau \in \left[ T_B^2, T_B^3 \right]$ with all the three players still active. The analysis will rely on the implication derived in the previous Lemma, namely that $T_B^2 < \min \left\{ T_A^*(2), T_B^2 \right\}$.

At time $\tau$, if all three firms are active and $A$ enters first, it follows from Lemma 3 that the two $B$ firms follow at $\tau$ and $T_B^3$ respectively. Payoffs are

$$
L_A(\tau) = L_A^1(\tau) = \pi_A(2) \int_{\tau}^{T_B^3} e^{-rs} ds + \pi_A(3) \int_{T_B^3}^{\infty} e^{-rs} ds - c(\tau)
$$

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for $A$ and a lottery between $L^2_A(\tau)$ and $F^2_A(\tau)$ for both $B$ firms, with $L^2_A(\tau) > F^2_A(\tau)$.

If instead one of the $B$ firms enters at $\tau$, it follows from Lemma 4 that if $\tau \in [T^2_B, \min\{T^2_B, T^*_A(2)\}]$, then the entry order is $B-A-B$, entry times are $\left(\tau, \min\{T^2_B, T^*_A(2)\}, T^*_B(3)\right)$ and payoffs are $L_B(\tau)$, $L^*_A(\min\{T^2_B, T^*_A(2)\})$ and $F^2_B(\tau)$ for the first, second and third entrant, respectively.

If instead $\tau \in \left[\min\{T^2_B, T^*_A(2)\}, T^*_B(3)\right]$, then:

- in case (1), entry order is $B-A-B$, entry times are $\left(\tau, \tau, T^*_B(3)\right)$ and payoffs are $L^2_B(\tau)$, $L^*_A(\tau)$ and $F^2_B(\tau)$ for the first, second and third entrant, respectively.

- in cases (2) and (3), for $\tau \not\in [T^2_B, T^*_B]$ entry order, entry times and payoffs are those described for case (1), while for $\tau \in [T^2_B, T^*_B]$ either entry order, entry times and payoffs are those described for case (1), or entry order is $B-B-A$, entry times are $\left(\tau, \tau, T^*_A(3)\right)$ and payoffs are $L^2_B(\tau)$ for the first two entrant and

$$F^2_A(\tau) \equiv \pi_A(3) \int_{T^*_A(3)}^{+\infty} e^{-rs} ds - c(T^*_A(3)).$$

for firm $A$.

The following Lemma holds:

**Lemma 6** In any SPNE of the game, the outcome of subgames with three active firms starting at $\tau \in (T^2_B, T^*_B(3))$ is as follows:

(i) If $\tau \in \left[T^2_B, \min\{T^2_B, T^*_A(2)\}\right]$, one of the $B$ firms enters at $t_1 = \tau$, the $A$ firm enters at $t_2 = \min\{T^2_B, T^*_A(2)\}$ and the remaining $B$ firm enters at $t_3 = T^*_B(3)$;

(ii) If $\tau \in \left[\min\{T^2_B, T^*_A(2)\}, T^*_B(3)\right]$:

(iia) for any $\tau$ in the interval in case (1), and for any $\tau$ in the interval such that $\tau \not\in [T^2_B, T^*_B]$ in cases (2) and (3), the unique outcome is that firm $A$ and one of the $B$ firms enter at $t_1 = t_2 = \tau$ and the remaining $B$ firm enters at $t_3 = T^*_B(3)$;

(iib) moreover, in cases (2) and (3), for $\tau \in \left[T^2_B, T^*_B\right]$ the outcome is either that firm $A$ and one of the $B$ firms enter at $t_1 = t_2 = \tau$ and the remaining $B$ firm enters at $t_3 = T^*_B(3)$, or that both $B$ firms enter at $t_1 = t_2 = \tau$ and the $A$ firm enters at $t_3 = T^*_A(3)$.

**Proof.** For simplicity, we develop the proof of this Lemma under the following assumption: Suppose that at any time $t$, if a firm is indifferent between being the $m$-th investor at $t$ and the $(m+1)$-th investor, then it invests at $t$. It is immediate to verify that even without this assumption the result still holds.
First, consider subgames with three active firms starting at $\tau \in [T_A^*(2), T_B^*(3))$ for case (1), or $\tau \in [T_B^{2,B}, T_B^*(3))$ for case (2). In equilibrium, at $\tau$, it has to be the case that both $B$ firms play Enter and $A$ plays either Enter or Wait. Assumption 5 and Lemmas 3 and 4 then guarantee the result.

If firms play either of these action profiles, payoffs are $L_A^2(\tau)$ for the $A$ firm, and a lottery between $L_B^{2,A}(\tau)$ and $F_B^{2,A}(\tau)$ for the $B$ firms. By Lemma 3, in this interval $L_B^{2,A}(\tau) > F_B^{2,A}(\tau)$ so no $B$ firm has an incentive to deviate and receive $F_B^{2,A}(\tau)$ with probability one. By the same argument, there cannot be an equilibrium in which at $\tau$ only one of the type-$B$ firms plays Enter, regardless of $A$’s action, because the other one would rather deviate and play Enter. Consider now firm $A$. Given that both $B$ firms play Enter, $A$’s action does not affect its payoff, so $A$ has no profitable deviation from either profile described above.

Next, we prove that there are no other action profiles at $\tau$ compatible with equilibrium. There cannot be an equilibrium in which only firm $A$ plays Enter at $\tau$, because each of the $B$ firms would receive a lottery between $L_B^{2,A}(\tau)$ and $F_B^{2,A}(\tau)$ and would rather deviate and play Enter at $\tau$ as well, thus receiving a similar lottery but with higher probability to obtain $L_B^2(\tau)$.

Finally there cannot be an equilibrium in which all three firms play Wait at $\tau$. In such an equilibrium, the first entry would happen at some time $t$ later than $\tau$. By Lemma 2 first entry would happen at some later $t \in (\tau, T_B^*(3)]$. From the arguments presented so far in the proof of part (ii) of this Lemma, it could only be the case that the $A$ firm and one of the $B$ firms enter simultaneously at $t$ and the remaining $B$ firm follows at $T_B^*(3)$. Since the function $L_A^3(\tau)$ is strictly quasiconcave and maximized at $T_A^*(2) \leq \tau$, $A$ would then have an incentive to deviate and preempt the rivals playing Enter at $(t - \varepsilon)$. So, in case (1), for $\tau \in [T_A^*(2), T_B^*(3))$ there cannot be an equilibrium in which all three firms play Wait at $\tau$. Hence, we can conclude that for any $\tau$ in this interval the unique equilibrium outcome is the one described in part (iii) of the Lemma.

Next, consider subgames with three active firms starting at $\tau \in [T_B^{2,B}, T_B^*]$. We prove that in equilibrium, all firms play Enter at $\tau$. Assumption 5(i), together with Lemmas 3 and 4 then guarantee the result.

If firms play the above profile, $A$ receives $L_A^2(\tau)$ with probability $\frac{2}{3}$ and $F_A^3(\tau)$ with probability $\frac{1}{3}$. By the proof of Lemma 4, $D_A^3(\tau) = L_A^2(\tau) - F_A^3(\tau)$ is weakly positive in $[T_A^*(2), T_A^*(3)]$, hence in the interval we are considering. It follows that $A$ has no incentive to deviate because it would then receive $L_A^2(\tau)$ or $F_A^3(\tau)$ with probabilities $\frac{1}{2}, \frac{1}{2}$. (By an analogous argument, there cannot be an equilibrium in which at $\tau$ $A$ plays Wait and either one or both $B$ firms play Enter). As for the $B$ firms, if the above profile is played, each $B$ firm receives $L_B^{2,B}(\tau)$, $L_B^{2,A}(\tau)$ and $F_B^{2,A}(\tau)$ with probabilities $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ while by deviating it
would receive a similar lottery with probabilities $(\frac{1}{4}, \frac{1}{4}, \frac{1}{2})$. Since in this interval $D_B^{2, B}(t) = L_B^{2, B}(\tau) - F_B^{2, B}(\tau) > 0$ and $L_B^{2, A}(\tau) > F_B^{2, A}(\tau)$, the deviation is not profitable. (By an analogous argument, there cannot be an equilibrium in which at $\tau$ $A$ plays Enter, and one or both of the $B$ firms play Wait).

Finally, there cannot be an equilibrium in which all three firms play Wait at $\tau$. In such an equilibrium, by the argument presented above, the first entry would take place at some later time $t \leq T_B^{2, B}$. If in $t$ only one or two firms plays Enter, any firm who plays Wait has an incentive to deviate and play Enter at $(t - \varepsilon)$. Similarly, if in $t$ all three firms play Enter, each of them has an incentive to deviate and play Enter at $(t - \varepsilon)$. Hence, we can conclude that for any $\tau$ in this interval the unique equilibrium outcome is the one described in part (iib) of the Lemma.

Next, for case (2), consider subgames with three active firms starting at $\tau \in \left[T_B^{2, A}(2), T_B^{2, B}\right]$. Given part (iib) of this Lemma, the equilibrium outcome of any such subgame must be that first entry happens weakly before $T_B^{2, B}$. Then, the same arguments presented in the first part of this proof guarantee that in equilibrium, at $\tau$ both $B$ firms play Enter and $A$ plays either Enter or Wait, which in turn guarantees that for any $\tau$ in this interval the unique equilibrium outcome is the one described in part (iiia) of the Lemma.

Finally, consider subgames with three active firms starting at $\tau \in \left[T_B^{2, A}, \min\left\{T_B^{2, B}, T_A^{2, A}(2)\right\}\right]$. In equilibrium, at $\tau$, it has to be the case that the $B$ firms play Enter and the $A$ firm plays Wait. Then, Assumption 5(i), together with Lemma 4, guarantees the result. If firms play the prescribed actions, $A$ receives $F_A(\tau)$, and the $B$ firms a lottery between $L_B(\tau)$ and $F_B^{2, A}(\tau) = F_B^{2, B}(\tau)$ with probabilities $(\frac{1}{2}, \frac{1}{2})$. By Assumption 1 $L_B(\tau) > L_B^{2, A}(\tau)$ which is turn larger that $F_B^{2, A}(\tau)$. Therefore, no $B$ firm has an incentive to deviate and receive $F_B^{2, A}(\tau)$ with probability 1. By the same argument, there cannot be an equilibrium in which at $\tau$ only one $B$ firm plays Enter.

As for firm $A$, by deviating it would receive a lottery between $F_A(\tau)$ and $L_A(\tau)$. It is easy to verify that this deviation is not profitable, using the fact that the function

$$\pi_A(2) \int_t^{T_B(3)} e^{-rs} ds + \pi_A(3) \int_{T_B(3)}^{+\infty} e^{-rs} ds - c(t)$$

is strictly quasiconcave and maximized at $T_A^{2}(2) > \tau$. By an identical argument, a strategy profile in which the $A$ firm and one or two $B$ firms play Enter at $\tau$, cannot be an equilibrium, since $A$ would want to deviate and play Wait.

Finally, we prove that a profile in which all three firms play Wait at $\tau$ cannot be part of an equilibrium. By part (ii), first entry would then take place at some later time $t \leq$
min \{T_B^{2,B}, T_A(2)\}. In \( t \), it holds that \( L_B(t) > T_B^{2,A}(t) > F_B^{2,A}(t) \).

Suppose at time \( t \) only the two \( B \) firms play Enter. By continuity, regardless of what \( A \) plays at \( t \), each of the \( B \) firms has a strict incentive to preempt the rival and enter at time \( (t - \varepsilon) \). Similarly, if at time \( t \) only one of the \( B \) firm plays Enter, then the other \( B \) firm has an incentive to preempt and enter at time \( (t - \varepsilon) \). Finally, if at time \( t \) only the \( A \) firm plays Enter, then each \( B \) firm has an incentive to preempt and enter at time \( (t - \varepsilon) \). Therefore, there cannot be an equilibrium of the subgame starting at \( t \) in which the first entry happens later than \( \tau \).

Next, we analyze subgames with three active firms starting at \( \tau \in [0,T_B^{2,A}] \). Consider again the functions \( D_A(\tau) \) and \( D_B(\tau) \). Evaluated at \( T_B^{2,A} \), \( D_B(\tau) \) is positive, because

\[
D_B(T_B^{2,A}) = [\pi_B(1) - \pi_B(2,A)] \int_{T_B^{2,A}}^{\min(T_A^*(2),T_B^{2,B})} e^{-rs} ds > 0
\]

by Assumption (1). It follows that there exists one and only one point \( T_B(1) \in (0,T_B^{2,A}) \) such that \( D_B(T_B^1) = 0 \). As for \( D_A(\tau) \) instead,

\[
D_A(T_B^{2,A}) = \pi_A(2) \int_{T_B^{2,A}}^{\min(T_A^*(2),T_B^{2,B})} e^{-rs} ds - c(T_B^{2,A}) + c\left( \min\{T_A^*(2),T_B^{2,B}\} \right) < 0
\]

because the function

\[
\pi_A(2) \int_{t}^{\min(T_A^*(2),T_B^{2,B})} e^{-rs} ds - c(t)
\]

is strictly quasiconcave, maximized at \( T_A^*(2) \), hence strictly increasing for \( t \in [T_B^{2,A}, \min\{T_A^*(2),T_B^{2,B}\}] \).

It follows that two cases are possible:

(a) \( D_A(\tau) < 0 \forall \tau \in [0,T_B^{2,A}] \), and \( T_A^1 = +\infty \); 

(b) there exist two points, \( T_A(1) \) and \( T_A(1) \), with \( 0 < T_A(1) \leq T_A(1) < T_B^{2,B} \), in which \( D_A(\tau) \) is null, and \( T_A^1 = T_A(1) \).

Given the assumption \( T_B(1) \leq T_A^1 \), the following Lemma holds:

**Lemma 7** In any SPNE of the game, the outcome of subgames with three active firms starting at \( \tau \in [0,T_B^{2,A}] \) is as follows:

(i) If \( \tau \leq T_B(1) \) one of the \( B \) firms enters at \( t_1 = T_B(1) \), the \( A \) firm enters at \( t_2 = \min\{T_B^{2,B},T_A^*(2)\} \) and the remaining \( B \) firm enters at \( t_3 = T_B^3(3) \);

(ii) If \( \tau \in (T_B(1),T_B^{2,A}) \):

(iia) for any \( \tau \) in the interval in case (a), and for any \( \tau \) in the interval such that \( \tau \notin [T_A^*(1),T_A(1)] \) in case (b), the unique outcome is that one of the \( B \) firms enters at \( t_1 = \tau \),
the A firm enters at \( t_2 = \min \left\{ T^2_B, T^A_A(2) \right\} \) and the remaining B firm enters at \( t_3 = T^B_B(3) \) (iib) moreover, in case (b), for \( \tau \in [T_A(1), T_A(1)] \) the outcome is either as in case (iia), or that firm A enters at \( t_1 = \tau \) and the B firms enter at \( t_2 = T^2_B \) and \( t_3 = T^B_B(3) \) respectively.

**Proof.** First, consider subgames with three active firms starting at \( \tau \in [T_B(1), T^2_B] \) for case (a), or \( \tau \in (T_A(1), T^2_B] \) for case (b). In equilibrium, at \( \tau \), it has to be the case that A plays Wait, and the B firms play Enter. If firms play the above profile, A receives \( F_A(\tau) \) and each B firm a lottery between \( L_B(\tau) \) and \( F_B(\tau) \). By deviating, A would receive \( L_A(\tau) \) with positive probability and a B firm would receive \( F_B(\tau) \). Then, the fact that \( D_A(\tau) < 0 \) and \( D_B(\tau) > 0 \) guarantees that no firm has an incentive to deviate. There cannot be an equilibrium in which both firms play Enter at \( \tau \), because the A firm and at least one of the B firms play Enter, because the A firm would then receive a lottery between \( L_A(\tau) \) and \( F_A(\tau) \) and would rather deviate and receive \( F_A(\tau) \). There cannot be an equilibrium in which both firms play Enter at \( \tau \), because both B firms would receive \( F_B(\tau) \) and would rather deviate and receive a lottery between \( L_B(\tau) \) and \( F_B(\tau) \). Finally, there cannot be an equilibrium in which all three firms play Enter at \( \tau \). In such an equilibrium, by Lemma 4 first entry would take place at some later time \( t \leq T^2_B \). But this cannot be part of an equilibrium, because at \( t \) one of the following action profiles would have to be played:

- A plays Enter and either one or both B firms play Enter: then the A firm would rather deviate and play Wait.
- A plays Enter and both B firms play Enter: then each B firm would rather deviate and play Enter
- A plays Wait and either one or both B firms play Enter: then each B firm would rather deviate and play Enter at \( (t - \varepsilon) \).

Hence, we can conclude that for any \( \tau \) in this interval the unique equilibrium outcome is the one described in part (iaa) of the Lemma.

Next, for case (b), consider any subgame with three active firms starting at \( \tau \in [T_A(1), T_A(1)] \). In equilibrium, all firms play Enter at \( \tau \). If firms play the above profile, each firm i receives a lottery between \( L_i(\tau) \) and \( F_i(\tau) \), and the fact that in this interval \( D_A(\tau) > 0 \) and \( D_B(\tau) > 0 \) guarantees that there are no profitable deviations. Similarly, this fact guarantees that there cannot be an equilibrium in which either one or two firms only play Enter at \( \tau \), because in that case there is at least one firm which plays Wait and has an incentive to deviate and play Enter. Finally, there cannot be an equilibrium in which all
three firms play Wait at \( \tau \). In such an equilibrium, by the argument presented above first entry would happen at some later time \( t \leq T_A(1) \). If at \( t \) only one or two firms plays Enter, any firm who plays Wait has an incentive to deviate and play Enter at \( (t - \varepsilon) \). Similarly, if in \( t \) all three firms play Enter, each of them has an incentive to deviate and play Enter at \( (t - \varepsilon) \). Hence, we can conclude that for any \( \tau \) in this interval the unique equilibrium outcome is the one described in part (iiib) of the Lemma.

Next, for case (b), consider any subgame with three active firms starting at \( \tau \in [T_B(1), T_A(1)) \). Given part (iib), the equilibrium outcome of any such subgame must be that first entry happens weakly before \( T_A(1) \), then the same arguments presented in the first part of this proof guarantee that in equilibrium, at \( \tau \), the A firm plays Wait and the B firms play Enter, which in turn guarantees that for any \( \tau \) in this interval the unique equilibrium outcome is the one described in part (iiiia) of the Lemma.

Finally, consider subgames with three active firms starting at \( \tau \leq T_B(1) \). In equilibrium, all firms play Wait for any \( t \in [\tau, T_B(1)) \). If they do so, the outcome is the one described in part (i) of the Lemma and firm \( i \) receives payoff \( F_i(\tau) \). (Notice that each B firm receives a lottery between \( L_B(T_B(1)) = F_B(T_B(1)) \) by definition of \( T_B(1) \), and \( F_B(\tau) = F_B(T_B(1)) \)). The fact that in this interval \( D_A(\tau) < 0 \) and \( D_B(\tau) < 0 \) guarantees that there are no profitable deviations. By the same argument, there cannot be an equilibrium in which any number of firms plays Enter at \( \tau \), because then there would be at least one firm receiving \( L_i(\tau) \) with positive probability, and this firm would rather deviate and receive \( F_i(\tau) \) with probability one. Hence, we can conclude that for any \( \tau \) in this interval the unique equilibrium outcome is the one described in part (i) of the Lemma.

The statement in Proposition 1 follows immediately from the above Lemmas.

**Proof of Claim 1.** Given the assumptions, \( D_{B}^{2,A}(t) > D_{B}^{2,B}(t) \), hence \( T_{B}^{2,A} < T_{B}^{2,B} \). Moreover, it follows from Fudenberg and Tirole (1985) that \( T_{B}^{2,A} < T_{B}^{2,B}(2, A) \). Since \( \pi_B(2) = \pi_A(2, A) \) implies \( T_{A}^{*}(2) = T_{B}^{*}(2, A) \), it follows that \( T_{B}^{2,A} < T_{A}^{*}(2) \). Therefore, it holds that \( T_{B}^{2,A} < \min \{ T_{A}^{*}(2), T_{B}^{2,B} \} \). Given this result, and the fact that \( \pi_A(1) = \pi_B(1) \), it follows that \( D_B(t) > D_A(t) \). Hence \( T_{B}^{1} < T_{A}^{1} \). □
References


