An Analysis of Price-based Tests of Antitrust Market Delineation*

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Abstract

There are well known concerns regarding the use of price correlation tests in determining antitrust markets. However, this has not deterred the use of these tests or the development of using Granger causality, stationarity and cointegration tests for the determination of antitrust markets. In this paper we explore the empirical performance of these various tests. In particular we want to know whether these tests are capable of generating the correct inference both when products are in the same relevant market and when they are not. Our results suggest that while simple price correlations are capable of determining anti-trust markets in the absence of common shocks other commonly employed price based tests provide no economically meaningful information to the antitrust practitioners.

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1 Introduction

Market definition is the foundation of antitrust analysis. In order to understand the competitive effects of merger and non-merger cases, antitrust agencies must first determine the extent of the relevant antitrust market. Determining the extent of the antitrust market allows identification of the degree of monopoly power. To determine the scope of an antitrust market, antitrust agencies rely on the “hypothetical monopolist test”, which determines a group of products and geographic areas in which a sole supplier (hypothetical monopolist) would be able to exert significant market power.¹ A properly defined antitrust market will determine the set of products such that market shares are reflective of market power. If the market is defined too broadly, then the degree of market power will be underestimated. Likewise, if market definition is too narrow, then the degree of market power will be overestimated.

While this approach is theoretically appealing, it can be difficult to implement empirically. Scheffman and Spiller (1987) developed an econometric analysis of residual demand that is consistent with the determination of an antitrust market. The shortcoming of this methodology is that it requires a stable structure for demand and a stable structure of oligopoly.² It is also data intensive, requiring data on prices, quantities and costs. The problem is that the appropriate data may not be available in the initial stages of an inquiry, and in some cases, may not be available at all.

In order to work around this problem, antitrust practitioners have suggested the use of price-based tests to determine the scope of the antitrust market. The four most common price-based tests are (i) price correlations; (ii) Granger causality tests; (iii) stationarity (unit root) tests; and (iv) cointegration tests. The concern with using these tests is that while they may be useful in identifying economic markets (areas in which prices are linked together through arbitrage), they may not identify relevant antitrust markets.³

¹See Commissioner of Competition, Merger Enforcement Guidelines, September 2004: Part 3, and U.S. Department of Justice and Federal Trade Commission, Horizontal Merger Guidelines, April 1997:§1.0. In Canada, significant market power is considered to be the ability to impose and sustain a significant (five percent) and non-transitory (one year) price increase.
²See Scheffman and Spiller (1996), page 166. However, they note that the demand and oligopoly structure can be adequately captured with the use of appropriate exogenous data.
The critique that price correlations are subject to spurious correlation — the prices of two products may be correlated due to having common costs, even though the products are not substitutable — is well understood.\textsuperscript{4} On the other hand, the potential limitations of the other main price tests have not received as much attention. The counterfactuals provided in the critiques of these tests are based on actual data. For example, Werden and Froeb (1993) look at high fructose corn syrup and sugar and note fairly high price correlations even though the government and court of appeals concluded that these goods were not in the same antitrust market.\textsuperscript{5} However using statistics derived from actual data to evaluate empirical methods can be problematic. For example, for at least part of Werden and Froeb’s sample many consumers were switching from sugar to high fructose corn syrup and so high price correlations are not surprising, see Sherwin (1993).

The purpose of this paper is to explore the empirical performance of the four main price-based tests in determining relevant antitrust markets. In doing so, we use synthetic data generated by a differentiated product model in which the parameters are set such that two products are in, and one product is out, of the relevant antitrust market as defined by the hypothetical monopolist test. Thus, we can determine whether these tests are capable of generating the correct inference both when products are in the same relevant antitrust market and when they are not. We prefer synthetic data to actual data for this exercise as it allows us complete control over the data generating process and therefore we can avoid the sort of ambiguity over the interpretation of results as illustrated by the sugar and high fructose corn syrup example. Furthermore, by using synthetic data we can examine the empirical performance of the price-based tests with respect to the degree of substitutability between the products, the structure of cost shocks, serial correlation and sample size.

We find that in the absence of common cost shocks, price correlations do a good job of determining relevant antitrust markets. However, in the presence of common cost shocks, price correlations do not perform as well, and not surprisingly tend to be over-inclusive (the relevant antitrust market is defined too broadly). However, price correlations perform much better than

\textsuperscript{5}See also Hosken and Taylor (2004).
the other three tests regardless of the type of cost shock. The Granger causality, stationarity, and cointegration tests are unable to distinguish between the case where the goods are in the same antitrust market, and the case where they are not. In fact, our simulation results suggest that these tests provide absolutely no meaningful information to antitrust practitioners. We find that they are equally likely to generate statistics consistent with two goods being in the same antitrust market regardless of whether or not the two goods actually are in the same antitrust market.

In the next section we provide a brief discussion of the four main price-based tests. Section three presents the product differentiation model used to generate the synthetic data. In section four, we calibrate that model, generate synthetic data and examine the empirical performance of the price-based tests in defining relevant antitrust markets. Section five concludes.

2 Price-based Tests

The idea behind price-based tests is that products should be grouped together into a single market if their prices move together. The reason being that price differentials greater than transportation costs provide the opportunity for profitable arbitrage. As Church and Ware (2000) point out, “if two markets are linked by arbitrage, then a disturbance — either a cost or demand shock — that changes the price in one will also change the price in the other, implying price correlation.”6 The various price-based tests provide different methodologies for determining this price relationship. The four most common price-based tests are: (i) price correlations; (ii) Granger causality tests; (iii) stationarity tests; and (iv) cointegration tests.7

We now discuss each of these approaches in turn.

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6 See Church and Ware (2000), page 613.
2.1 Price Correlations

Stigler and Sherwin (1985) argue that if two goods can be considered to be part of the same market then their price movements should be correlated. For example, if a supply shock were to raise the price of good $i$, then consumers would substitute away from this good into good $j$. This increase in demand pushes up the price of good $j$, and so we see a positive correlation between the changes in the prices of the two goods.

The strongest (and most common), argument against using price correlations to determine antitrust markets is the problem of spurious correlation. In this case, the price correlation is the result of common influences (such as a common cost shock), rather than product substitutability. This is not the only concern with using price correlations. Werden and Froeb (1993) argue that price correlations can provide misleading results due to individual cost variation, individual demand variation and variation in the price of the other product. They go on to conclude, that “more generally, the various economic forces that affect both price correlations and monopoly markups do not affect the two in quite the same way, so correlations can be misleading.” However, Baker (1987, pp. 26-27) argues that price correlations can be informative for determining antitrust markets if the price correlation is due to cost shocks that only occur for firms in the antitrust market. It should be noted that Baker is quick to point out that if there is enough information to determine that the cost shock only affects the firms within the relevant antitrust market, then there is probably enough information to perform a residual demand analysis.

2.2 Granger Causality Tests

In addition to problems associated with common factors price correlations can be uninformative if the adjustment across goods takes place with a delay. Suppose the supply shock that raises the price of a good only has an effect on the price of a substitute good in the following period. In this case the contemporaneous correlation between the price changes will be equal to zero.

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8 The correlation coefficient between goods $i$ and $j$ is given by $\rho_{ij} = \frac{\text{cov}(p_i, p_j)}{\sigma_i \sigma_j}$, where $\text{cov}(p_i, p_j)$ is the covariance between the prices of goods $i$ and $j$, and $\sigma_i$ and $\sigma_j$ are the standard deviations of the prices of goods $i$ and $j$ respectively.

even if the goods are very close substitutes. Slade (1986) makes this point and advocates the use of Granger causality tests to explore the possibility that the prices of two goods may be related, but that the effects are not instantaneous. Granger causality tests ask whether one variable contains marginal predictive content for another variable above and beyond what is contained in that variable’s own lags. More specifically, variable 1 is said to Granger cause variable 2 if lagged values of variable 1 are statistically significant in a regression for variable 2 which contains lagged values of variable 2.

A convenient way to conduct these tests is in the context of a vector autoregression (VAR). Let $\Delta p_t$ be an $N \times 1$ vector containing the first differences of the natural logarithm of the prices of $N$ goods at time $t$. A VAR can be used to describe the dynamic behavior of these prices and relates the vector $\Delta p_t$ to its own lags, that is

$$\Delta p_t = \sum_{k=1}^{K} \psi_k \Delta p_{t-k} + e_t$$

(1)

where $\psi_k$ are $N \times N$ matrices that contain parameters that describe the dynamic relationships between prices. The lag order of the VAR, given by $K$, is chosen to ensure that the residuals $e_t$ are serially uncorrelated. A test for Granger (non-)causality from $\Delta p_i$ to $\Delta p_j$ requires testing the null hypothesis:

$$\psi_{ij,1} = \psi_{ij,2} = \ldots = \psi_{ij,K} = 0$$

(2)

If this null hypothesis is rejected in favor of the alternative that at least one of these parameters is non-zero then $\Delta p_i$ is said to Granger cause $\Delta p_j$.

### 2.3 Unit Root Tests

Simple arbitrage predicts that if two goods belong in the same market their is a limit as to how far their prices can diverge from one another. Forni (2004) exploits this to suggest the use of unit root tests in determining whether two goods are in the same market. If two goods are in the same market then a shock which raises the price of one good relative to the other can only have transitory effects. As consumers begin to substitute away from the good which
has become more expensive, the price of that good falls, and the price of the other good rises, such that the relative price returns to some long-run equilibrium value. If this is the case then the relative price is said to be stationary. On the other hand if two goods are not in the same market then shocks can have a permanent effect on the relative price, which is then said to be non-stationary.

Suppose the log of the relative price of goods $i$ and $j$ can be written as a 1st order autoregressive process, that is:

$$p_{i,t} - p_{j,t} = \rho(p_{i,t-1} - p_{j,t-1}) + e_t$$

where $e_t$ is a serially uncorrelated, mean-zero random variable and $\rho$ measures the persistence of innovations to the relative price series.\(^\text{10}\) If we repeatedly lag equation (3) and substitute the result back into (3) we can write the relative price as an initial condition plus a moving average of past and current $e_t$, that is:

$$p_{i,t} - p_{j,t} = \rho^T(p_{i,0} - p_{j,0}) + \sum_{k=0}^{T-1} \rho^k e_{t-k}.$$  

(4)

The partial derivative of (4) with respect to $e_{t-k}$ measures the effect of a shock to the relative price at time $t - k$ at time $t$. Notice that when $-1 < \rho < 1$ this partial derivative tends to zero as $k$ tends to infinity. In other words shocks have purely transitory effects. However, when $\rho = 1$ the partial derivative is equal to one for all $k$ and so shocks have permanent effects.

The most common unit root test is the augmented Dickey-Fuller (ADF) test. This involves testing the null hypothesis that $\phi = 0$, where $\phi = \rho - 1$, against the alternative that $\phi < 0$ using the regression:

$$\Delta(p_{i,t} - p_{j,t}) = \phi(p_{i,t-1} - p_{j,t-1}) + \sum_k \varphi_k \Delta(p_{i,t-k} - p_{j,t-k}) + e_t$$

(5)

where $k$ is chosen to ensure $e_t$ is serially uncorrelated.\(^\text{11}\) A failure to reject the null hypothesis

\(^{10}\)More generally this equation can include an intercept to represent a non-zero equilibrium relative price as well as additional lags of the dependent variable.

\(^{11}\)In practice the ADF regression typically contains an intercept allowing for a non-zero equilibrium price differential. In subsequent drafts we will discuss alternate tests.
implies that shocks to the relative price are permanent and therefore that the two goods are in separate markets. On the other hand if the two goods are in the same market then the null hypothesis should be rejected in favor of the alternative that the relative price is stationary.

An important caveat applies here. If both $p_{i,t}$ and $p_{j,t}$ are stationary then the ratio $p_{i,t} - p_{j,t}$ is also stationary regardless of whether the two goods are in the same market. Therefore a rejection of the null hypothesis does not necessarily imply the two goods are in the same market. In our simulation exercise we are able to sidestep this issue by constructing our price series such that they are individually non-stationary. Therefore, in our synthetic data a finding that the relative price is stationary implies the two goods belong to the same market.

2.4 Cointegration Tests

Closely related to unit root tests are cointegration tests. Two non-stationary time series are said to be cointegrated if a linear combination of those two series is stationary. If the two price series are cointegrated then there exists a stable long-run relationship between the two series described by $p_{i,t} - bp_{j,t} = 0$.\footnote{Again, more generally this long-run relationship can contain a constant term.} A test for cointegration is then a test that $p_{i,t} - bp_{j,t}$ is stationary.\footnote{Note that when $b = 1$ this is equivalent to a unit root test on the relative price.} If the null hypothesis that the two series are not cointegrated is not rejected then the implication is that there is no long-run relationship between the two price series and the two goods are in separate markets. On the other hand if the null hypothesis is rejected in favor of the alternative that the two prices are cointegrated then this is consistent with the two goods being in the same market.\footnote{Note that as with the unit root test on the relative price, if both $p_{i,t}$ and $p_{j,t}$ are stationary then $p_{i,t} - bp_{j,t}$ will also be stationary, even if the goods are in separate markets.}

3 Model of Product Differentiation

In this section we outline a model of product differentiation that we use as a data generating process. In the next section we discuss parameter values and use this model to generate synthetic data.
3.1 Demand

Consider a representative consumer with a quadratic utility function defined over three differentiated goods\(^{15}\)

\[ U(Q) = \alpha Q - (1/2)Q'\Gamma Q \]  

(6)

where

\[
\alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}, \quad Q = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} \quad \text{and} \quad \Gamma = \begin{bmatrix} \beta_1 & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \beta_2 & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \beta_3 \end{bmatrix} \]  

(7)

For analytical ease we will assume symmetric two-way substitutability between goods.\(^{16}\) In other words, consumers view the degree of substitutability of good 1 for good 2 the same as good 2 for good 1 and so we assume \(\gamma_{12} = \gamma_{21}, \gamma_{23} = \gamma_{32}\) and \(\gamma_{31} = \gamma_{13}\). The inverse demands follow from the first-order conditions of the consumer’s utility maximization problem and are given by

\[ P_i = \frac{\partial U_i}{\partial Q_i} \]  

(8)

for \(Q_i > 0\) and \(i = 1, 2, 3\). Therefore, the inverse demand system is given by

\[ P(Q) = \alpha - \Gamma Q. \]  

(9)

The direct demand system is determined by inverting the inverse demand system (9), which yields

\[ D(P) = \Gamma^{-1}(\alpha - P), \]  

(10)

where

\[
\Gamma^{-1} = \begin{bmatrix} \frac{\beta_2\beta_3 - \gamma_{23}^2}{\Delta} & \frac{\gamma_{23}\gamma_{12} - \beta_3\gamma_{12}}{\Delta} & \frac{\gamma_{23}\gamma_{13} - \beta_3\gamma_{13}}{\Delta} \\ \frac{\gamma_{23}\gamma_{12} - \beta_3\gamma_{12}}{\Delta} & \frac{\beta_1\beta_3 - \gamma_{13}^2}{\Delta} & \frac{\gamma_{23}\gamma_{13} - \beta_3\gamma_{13}}{\Delta} \\ \frac{\gamma_{23}\gamma_{13} - \beta_3\gamma_{13}}{\Delta} & \frac{\gamma_{23}\gamma_{13} - \beta_3\gamma_{13}}{\Delta} & \frac{\beta_1\beta_2 - \gamma_{12}^2}{\Delta} \end{bmatrix} \]  

(11)

\(^{15}\)For a detailed discussion of the representative consumer model, see chapter 6 of Vives (2001).

\(^{16}\)However, two-way substitutability is not necessary for two goods to be in the same antitrust market. Two goods can be in the same antitrust market with one-way substitutability.
and $\Delta \equiv \beta_1\beta_2\beta_3 + 2\gamma_{12}\gamma_{23}\gamma_{13} - \beta_1\gamma_{23}^2 - \beta_2\gamma_{13}^2 - \beta_3\gamma_{12}^2 > 0$. Furthermore, we define $B$ to be the diagonal matrix consisting of the absolute value of the slopes of the direct demands $(\partial D_i(P)/\partial P_i)$, which is the diagonal of $-\Gamma^{-1}$

$$B = \begin{bmatrix}
\frac{\beta_2\beta_3 - \gamma_{23}^2}{\Delta} & 0 & 0 \\
0 & \frac{\beta_1\beta_3 - \gamma_{13}^2}{\Delta} & 0 \\
0 & 0 & \frac{\beta_1\beta_2 - \gamma_{12}^2}{\Delta}
\end{bmatrix}. \quad (12)$$

Thus, the diagonal of $B$ is simply the own-price elasticities of demand.

### 3.2 Pricing

There exist three firms that each produce one of the goods, with marginal cost $M_i < \alpha_i, i = 1, 2, 3$. We assume Bertrand competition with firm profits given by

$$\pi_i = P_iD_i(P) - M_iD_i(P). \quad (13)$$

Each firm maximizes profit by choosing the price of the good it produces. This yields the following first-order conditions for each firm $i$

$$(P_i - M_i)\frac{\partial D_i(P)}{\partial P_i} + D_i(P) = 0. \quad (14)$$

Using (10) and (12), the system of first-order conditions can be rewritten as

$$-B(P - M) + \Gamma^{-1}(\alpha - P) = 0, \quad (15)$$

where

$$P = \begin{bmatrix}
P_1 \\
P_2 \\
P_3
\end{bmatrix} \quad \text{and} \quad M = \begin{bmatrix}
M_1 \\
M_2 \\
M_3
\end{bmatrix}. \quad (16)$$

Note that $\Delta > 0$ and $\beta_1\beta_2 - \gamma_{12}^2 > 0$ are necessary conditions for $U(Q)$ to be strictly concave.
Solving (15) for $P$, yields the following Bertrand-Nash equilibrium prices and quantities

\[ P^* = (I + \Gamma B)^{-1}(\alpha - M) + M, \quad (17) \]

\[ Q^* = (\Gamma + B^{-1})^{-1}(\alpha - M). \quad (18) \]

### 3.3 Hypothetical Monopolist Test

In Canada, the Competition Bureau uses the hypothetical monopolist test to define the relevant antitrust market. The *Merger Enforcement Guidelines* state:\(^{18}\)

The market definition analysis begins by postulating a candidate market for each product of the merging parties. For each candidate market, the analysis proceeds by determining whether a hypothetical monopolist controlling the group of products in that candidate market would be able to impose a five per cent increase assuming the terms of sale of all other products remained constant. If the price increase would likely cause buyers to switch their purchases to other products in sufficient quantity to render the price increase unprofitable, the postulated candidate market is not the relevant market, and the next-best substitute is added to the candidate market. ...

The smallest set of products in which the price increase can be sustained is defined as the relevant product market.

Provided that the next-best substitute can be identified unambiguously, the *Merger Enforcement Guidelines* define a unique relevant antitrust market.\(^{19}\) In the simulation exercises that follow we choose parameter values that satisfy the hypothetical monopolist test such that goods 1 and 2 are in the same antitrust market and good 3 is not.

### 4 Calibration and Simulation

As mentioned above, we calibrate the model of the previous section so that consumers regard goods one and two substitutable, but not good three. We then use this model to generate

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\(^{18}\) Commissioner of Competition, *Merger Enforcement Guidelines*, September 2004, at ¶3.5, footnotes removed. Note that this hypothetical monopolist test is identical to the one used by the U.S. Department of Justice and Federal Trade Commission.

\(^{19}\) Note that this procedure is also used in non-merger cases. The only difference is that the reference price for determining the magnitude of the price increase is not the prevailing market price. In non-merger cases, the competitive price is used in order to avoid the problems related to the *Cellophane Fallacy*. We follow the merger application and use the prevailing market price as the reference price.
synthetic price data for all three goods. Using this synthetic data, for each price pair, we calculate correlation coefficients, estimate a VAR and perform Granger causality tests, test for a unit root in the (log) relative price and test for cointegration between the two (log) prices. We then examine the extent to which the results of these tests differ across the three price pairs and therefore the extent to which these tests provide useful information to the antitrust practitioner.

4.1 Calibration

As discussed earlier when two price series are stationary then a linear combination of those two series will also be stationary. Therefore in order to interpret the unit root and cointegration tests as tests of market definition it is important that the two price series are individually non-stationary. That is shocks to these price series must be permanent. We can introduce shocks into the model of the previous section either through the cost side or the demand side. On the cost side this can be achieved by allowing shocks to marginal cost processes, $m_i$, and on the demand side by allowing shocks to $\alpha_i$. In this draft we focus purely on cost shocks. In later drafts we plan to explore the effects of introducing demand shocks. In general we calibrate the model to match the the spaghetti sauce data used by Capps et al (2003). In particular we choose parameter values such that the prices of goods one and two have similar properties to the prices of Classico and Newman’s in the Capps et al dataset and good three has similar properties to the price of Hunt’s.

We begin by discussing the demand parameters. We set $\beta_i = 0.4$ for all three goods. In our baseline calibrations we set $\gamma_{12} = 0.1$ and $\gamma_{13} = \gamma_{23} = 0$. These parameter values imply own price elasticities of $-2.67$ for goods one and two and $-2.50$ for good three, as well as a cross-price elasticity of 0.67 between goods one and two. The own-price elasticities are within the ranges estimated by Capps et al, while the cross-price elasticity is higher than their estimates. We also experiment with values of $\gamma_{12} = 0.2$ and $\gamma_{12} = 0.3$ which imply cross-price elasticities of 1.67 and 4.29 respectively. We set $\alpha_1 = \alpha_2 = 5$ and $\alpha_3 = 2$. These figures in combination with our initial conditions for the marginal cost processes imply initial prices similar to the means reported for Classico, Newman’s and Hunt’s in table 1 of Capps et al.
Given the marginal cost processes we describe below and in the absence of any cost shocks, when $\gamma_{12} = 0.1$ the equilibrium prices are $p_1^* = p_2^* = 2.20$ and $p_3^* = 1.03$. If a hypothetical monopolist produces goods one and two, conditional on $p_3^* = 1.03$, then the equilibrium prices for goods one and two increase to 2.55, a 14% increase. If a hypothetical monopolist produces all three goods then the equilibrium prices $p_1^* = p_2^* = 2.55$ and $p_3^* = 1.03$. Thus, goods one and two are in the same antitrust market and good three is not.

For each firm we use an AR(2) process for the log of the marginal cost process ($m_{it}$):

$$m_{i,t} = \lambda_1 m_{i,t-1} + \lambda_2 m_{i,t-2} + \varepsilon_{it} \quad (19)$$

where $\varepsilon_{it}$ is the shock to the marginal cost process. We assume that this shock can be decomposed into two orthogonal components, a common shock $U_t$ which affects all three firms and a firm-specific shock $u_{i,t}$ which only affects firm $i$. Both of these shocks are drawn from zero-mean normal distributions with standard deviations given by $\sigma_U$ and $\sigma_{u_i} = \sigma_u$ for all $i$.

In our first set of experiments we focus on firm specific shocks only and so set $\sigma_U = 0$. In the second set of experiments we allow for both a common shock and a firm-specific shock. In this case we set $\sigma_U = \sigma_u$ so that the common and firm specific shock contribute equally to price volatility. In subsequent drafts we plan to relax this assumption. We set $\sigma_u$ in order that the standard deviation of $\Delta p_i$ in our synthetic data matches that of Classico and Newman’s in the Capps et al data.\(^{20}\)

Modeling marginal costs as an AR(2) process allows us to generate synthetic data for which the first difference of the (log) price series exhibit first order serial correlation. We consider three sets of values for $\lambda_1$ and $\lambda_2$. It is important to note that in each case we choose $\lambda_1$ and $\lambda_2$ such that the shocks to marginal cost are permanent and so the individual (log) price series contain a unit root. In the first set of simulations we set $\lambda_1 = 1$ and $\lambda_2 = 0$. This corresponds to the standard random walk case in which the first difference of the marginal costs and therefore the first difference of the price series are serially uncorrelated. In the second set of simulations we $\lambda_1$ and $\lambda_2$ such that the first difference of the price series exhibit negative

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\(^{20}\)The values of $\sigma_u$ that do this vary with the other parameters of the model and so we report these in the footnotes to each table.
first order autocorrelation in the range of $-0.3$ which matches that in the Capps et al spaghetti sauce data. Finally in our third set of simulations the price series exhibit positive first order autocorrelation in the range of $0.3$. This just leaves initial conditions for the marginal cost processes which we set at $m_1 = m_2 = 0.1$ and $m_3 = 0.05$ in order that the initial observation in the synthetic price data matches the sample means of the Capps et al data.

4.2 Simulation

Using these parameter values we generate 10000 samples of length $T + 200$ and then discard the first two hundred observations in order to minimize the influence of starting values. Given that we have calibrated our model to the Capps et al dataset which is weekly, we consider values of $T = 26, 52, 104$ and $260$ which implies datasets of length 6 months, 1 year, 2 years and 5 years. Our impression is that most empirical studies using price data have sample sizes in the range of 1 - 2 years, however shorter samples are occasionally used. The figure $T = 260$ represents what we imagine to be an upper bound on the amount of data currently available in price-based studies. For each sample of synthetic data we calculate the correlation coefficient for each pair of $\Delta p$. We then estimate a VAR using all three price series (again in first difference of logs) and perform pairwise Granger causality tests. Finally, we also test for a unit root in the (log of) each relative price and for cointegration between each pair of log prices.

If these tests are to provide useful information to practitioners then they should be able to distinguish between the the three different price pairs. Goods one and two are substitutes and so we expect to see a high correlation coefficient for $\Delta p_1$ and $\Delta p_2$, but not for $\Delta p_1$ and $\Delta p_3$ or $\Delta p_2$ and $\Delta p_3$. Similarly, if Granger causality tests are to provide useful information we expect to see a rejection of the null of no Granger causality between $\Delta p_1$ and $\Delta p_2$ but not for the other two pairs. We also expect to see rejection of a unit root in relative price $p_2 - p_1$, but not in the relative prices $p_3 - p_1$ and $p_3 - p_2$ and a rejection of the null hypothesis of no cointegration between $p_1$ and $p_2$, but not the other two pairs.
4.2.1 Firm specific cost shocks only

Table one shows results for which the data generating process contains only firm specific cost shocks and the marginal cost process is constructed such that the first difference of the log price does not exhibit serial correlation, that is \( \lambda_1 = 1 \) and \( \lambda_2 = 0 \). The first three rows of this table show the mean values of \( \rho_{12}, \rho_{13} \) and \( \rho_{23} \) across the 10000 samples of synthetic data. The remaining rows show rejection rates across the 10000 samples for various hypotheses based on a nominal size of 0.05.

The first thing to note from table 1 is that in the absence of common shocks the price correlations do a very good job of differentiating between goods that are in the same market and those which are not. For example consider our baseline case in which \( \gamma_{12} = 0.1 \) and a sample size of \( T = 104 \). Here the average correlation coefficient between \( \Delta p_1 \) and \( \Delta p_2 \), \( \rho_{12} \) is 0.73 while the averages for \( \rho_{13} \) and \( \rho_{23} \) are essentially zero. This marked difference in mean correlation coefficients is true across all sample size and values of \( \gamma_{12} \). As one might expect, the mean of \( \rho_{12} \) increases with \( \gamma_{12} \). The ability of the price correlation approach to differentiate between the different cases is well illustrated by figure 1 which shows the densities of \( \rho_{12} \) and \( \rho_{13} \) constructed using estimates from the 10000 samples of synthetic data (again \( \gamma_{12} = 0 \) and \( T = 104 \)).

However, probably the most striking thing to take from table 1 is how poorly the other price-based tests of market delineation perform in our simulations. For each of these tests the null hypothesis is consistent with the two goods being in separate markets, while the alternative is consistent with the goods being substitutes and in the same antitrust market. The Granger causality tests tend to over-reject the null hypothesis when it is true. Note that good three is not in the same antitrust market as goods one and two, however for \( \Delta p_1 \) and \( \Delta p_3 \), and \( \Delta p_2 \) and \( \Delta p_3 \) the rejection rates, based on a nominal size of 0.05, are typically in the range of 0.15 to 0.20. On the other hand, the low rejection rates for the null hypothesis of non-causality between \( \Delta p_1 \) and \( \Delta p_2 \) indicate that this is also a test of low power against the alternative implied by a value of \( \gamma_{12} = 0.1 \). Indeed the rejection rates are only marginally higher than those involving \( \Delta p_3 \). For example, when \( T = 104 \) the rejection rate for the null hypothesis that \( \Delta p_2 \) does not Granger cause \( \Delta p_1 \) is 0.1857, while the rejection rate for \( \Delta p_3 \)
not Granger causing $\Delta p_1$ is 0.1610.

This inability of the Granger causality tests to distinguish between the different cases is illustrated in figure 2 which shows the densities of the $F-$statistics for the test that $\Delta p_2$ does not Granger cause $\Delta p_1$, and $\Delta p_3$ does not Granger cause $\Delta p_1$. In sharp contrast to the densities in figure 1, these two densities are almost identical. This figure and the statistics in table 1 suggest that Granger causality tests provide very little information that is useful to antitrust practitioners looking to distinguish between the two cases described by the model of the previous section. Table 1 also shows that raising the sample size to $T = 260$ or $\gamma_{12}$ to either 0.2 or 0.3 has little effect on this result.

The next three rows of table 1 show rejection rates for the null hypotheses that the relative price $p_{2,t} - p_{1,t}$, $p_{3,t} - p_{1,t}$ and $p_{3,t} - p_{2,t}$ contain a unit root against the alternatives that they are stationary. The test employed here is the standard augmented Dickey-Fuller test. These figures show a very similar story to the Granger causality tests, with the rejection rates showing a tendency to over-reject the true null hypothesis and very little difference between rejection rates across relative prices. Here the rejection rates are between ten and twenty percent for the $p_{3,t} - p_{1,t}$ and $p_{3,t} - p_{2,t}$, and typically only about one or two percentage points higher for $p_{2,t} - p_{1,t}$. As with the Granger causality tests there is very little improvement in the performance of this test as we raise either $T$ or $\gamma_{12}$. Figure 3 shows the densities for the ADF statistics for the null hypotheses that $p_{2,t} - p_{1,t}$ and $p_{3,t} - p_{1,t}$ contain unit roots. Again these densities are almost indistinguishable from one another. The implication; this test provides little economically meaningful information to the practitioner.

The last six rows of table 1 show that the cointegration tests do slightly better than the Granger causality and unit root tests. However, they still yield very little useful information. We consider two cointegration tests, the Engle-Granger (1987) two-step and Johansen’s (1991) trace statistic.\textsuperscript{21} As with the Granger causality tests both these tests have a tendency to over-reject the null hypothesis when it is true. The rejection rates for the null of no cointegration between $p_{1,t}$ and $p_{2,t}$ are higher than those under the null hypothesis. However, as in the

\textsuperscript{21}We obtain very similar results when using Johansen’s maximum eigenvalue statistic and so omit these in the interest of space. They are available from us on request.
previous cases the differences do not inspire confidence in the ability of these tests to provide reliable inference. For example when $\gamma_{12} = 0.1$ and $T = 104$ the rejection rate for the Engle-Granger using $p_{1,t}$ and $p_{3,t}$ is 0.0843 while for $p_{1,t}$ and $p_{2,t}$ the figure is only slightly higher at 0.1487. Figure 4 shows the densities of the Engle-Granger tests statistics for the null hypothesis that $p_{1,t}$ and $p_{2,t}$ are not cointegrated and the null hypothesis that $p_{1,t}$ and $p_{3,t}$ are not cointegrated. Finally, the Engle-Granger test does reject the false null more frequently as $T$ increases, however even when we use 5 years of weekly observations ($T = 260$) the rejection rate is still only 25%.

The rejection rates for the Johansen trace statistics imply a level of performance similar to that of the Engle-Granger test. That is, we see a tendency to over-reject the null hypothesis when it is true and only slightly higher rejection rates when the null hypothesis is false. Across sample sizes and combinations of prices we see higher rejection rates for the trace statistics than the Engle-Granger statistic and as with the Engle-Granger test the rejection rates increase with $T$. However, as illustrated in figure 5 for the case where $\gamma_{12} = 0.1$ and $T = 104$ our results suggest that this test provides very little useful information to policymakers.

One might argue that the inability of these tests to distinguish between the two cases is driven by the fact that the degree of substitutability between goods one and two implied by $\gamma_{12} = 0.1$ is relatively low. Table 1 shows that in fact this is not the case. While raising $\gamma_{12}$ to 0.2 or 0.3 has the effect of increasing the mean of $\rho_{12}$ it has little effect on the rejection rates for the Granger causality, ADF, Engle-Granger and Trace statistics. Regardless of our choice of $\gamma_{12}$ these tests seem incapable of distinguishing between two goods that are in the same antitrust market and two goods that are not. Finally tables 1a and 1b present the results of simulations for which the marginal cost process is constructed so that the first difference of the log price series contain negative or positive serial correlation. Again these results are very similar to those in table 1 and the simple correlation coefficient seems to be the only statistic capable of providing reliable inference.
4.2.2 Firm specific and common shocks

The most common criticism of the price-based approach to market definition is that the presence of common shocks can lead to an incorrect conclusion that two goods are in the same anti-trust market. In table two we explore this possibility by allowing the marginal cost process of each firm to be subject to a common shock and a firm specific shock. The introduction of common shocks raises the mean correlations between goods 1 and 3 and goods 2 and 3 from approximately zero to numbers in the range 0.40 to 0.50. While these figures are still below the mean value of $\rho_{12}$, which is greater than 0.80, they do reflect the fact that the presence of common shocks makes the interpretation of correlation coefficients as tests of market definition problematic. The performance of the Granger causality, unit root and cointegration tests is essentially unaffected by the presence of common shocks, that is, it remains very poor.

5 Conclusion

Price-based tests of market delineation remain popular for preliminary work in antitrust cases despite existing criticisms. In this paper we explore the extent to which these tests can provide antitrust practitioners with useful information. We use a three good product differentiation model in which two goods are in the same market but a third is not to generate synthetic data. We then apply an number of price-based tests using this synthetic data. We find that even in the absence of common shocks only simple price correlations are capable of generating correct inference regarding which goods are in the same antitrust market. Granger causality, unit root and cointegration test statistics are essentially identical regardless of whether or not the two goods being studied are in the same market. Therefore, our results suggest that the application of such tests is not a fruitful avenue of research in antitrust analysis.

22In this draft we assume that the standard deviations of these shocks are equal.
References


<table>
<thead>
<tr>
<th>Table 1: Simulation Results - Firm Specific Cost Shocks Only. $\Delta \rho$, serially uncorrelated.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_1 = 0.1$</td>
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<td>$T - 26$</td>
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<tr>
<td>$\rho_{12}$</td>
</tr>
<tr>
<td>$\rho_{13}$</td>
</tr>
<tr>
<td>$\rho_{23}$</td>
</tr>
</tbody>
</table>

Means

Rejection Rates (nominal size 0.05)

$GC_{01}$ | 0.1811 | 0.1738 | 0.1857 | 0.2174 | 0.1823 | 0.1705 | 0.1766 | 0.2116 | 0.1838 | 0.1701 | 0.1710 | 0.1941 |
$GC_{31}$ | 0.1736 | 0.1671 | 0.1610 | 0.1615 | 0.1718 | 0.1705 | 0.1616 | 0.1631 | 0.1717 | 0.1711 | 0.1614 | 0.1644 |
$GC_{32}$ | 0.1808 | 0.1777 | 0.1869 | 0.2270 | 0.1793 | 0.1739 | 0.1781 | 0.2164 | 0.1823 | 0.1713 | 0.1714 | 0.1977 |
$GC_{32}$ | 0.1742 | 0.1704 | 0.1615 | 0.1557 | 0.1729 | 0.1735 | 0.1609 | 0.1595 | 0.1731 | 0.1747 | 0.1605 | 0.1609 |
$GC_{13}$ | 0.1764 | 0.1634 | 0.1620 | 0.1597 | 0.1752 | 0.1654 | 0.1642 | 0.1604 | 0.1738 | 0.1696 | 0.1672 | 0.1592 |
$GC_{23}$ | 0.1805 | 0.1737 | 0.1613 | 0.1606 | 0.1796 | 0.1725 | 0.1605 | 0.1602 | 0.1788 | 0.1711 | 0.1609 | 0.1612 |
$ADF_{15}$ | 0.1911 | 0.1593 | 0.1496 | 0.1777 | 0.1839 | 0.1489 | 0.1333 | 0.1463 | 0.1775 | 0.1393 | 0.1235 | 0.1271 |
$ADF_{15}$ | 0.1823 | 0.1521 | 0.1359 | 0.1426 | 0.1726 | 0.1423 | 0.1245 | 0.1201 | 0.1716 | 0.1344 | 0.1180 | 0.1063 |
$ADF_{25}$ | 0.1835 | 0.1526 | 0.1340 | 0.1446 | 0.1754 | 0.1417 | 0.1237 | 0.1235 | 0.1725 | 0.1352 | 0.1139 | 0.1075 |
$EG_{19}$ | 0.0894 | 0.1075 | 0.1487 | 0.2512 | 0.0894 | 0.0909 | 0.1224 | 0.2018 | 0.0815 | 0.0808 | 0.1017 | 0.1640 |
$EG_{15}$ | 0.0836 | 0.0715 | 0.0843 | 0.1128 | 0.0796 | 0.0629 | 0.0734 | 0.0920 | 0.0765 | 0.0584 | 0.0638 | 0.0800 |
$EG_{25}$ | 0.0802 | 0.0727 | 0.0866 | 0.1181 | 0.0752 | 0.0654 | 0.0747 | 0.0975 | 0.0741 | 0.0606 | 0.0671 | 0.0826 |
$TR_{15}$ | 0.2515 | 0.2385 | 0.2867 | 0.4057 | 0.2357 | 0.2158 | 0.2456 | 0.3413 | 0.2243 | 0.1988 | 0.2176 | 0.2888 |
$TR_{15}$ | 0.2359 | 0.2024 | 0.2325 | 0.3077 | 0.2211 | 0.1843 | 0.2057 | 0.2656 | 0.2114 | 0.1715 | 0.1852 | 0.2348 |
$TR_{25}$ | 0.2368 | 0.2074 | 0.2340 | 0.3094 | 0.2258 | 0.1917 | 0.2087 | 0.2653 | 0.2165 | 0.1768 | 0.1876 | 0.2325 |

These results are based on 10000 replications of the simulation exercise described in the text. The data is constructed such that the marginal cost process for each firm contains a unit root and the first difference of each of the price series is serially uncorrelated. This is achieved by setting $\rho_{12} = 1$ and $\rho_{13} = 1$ for all $t$. The standard deviations of the cost shocks are set such that the standard deviation of the first difference of the log of the synthetic price data matches that of Classic and Neumann's in the spaghetti salad data of Capps et al. (2003). This means that the calibrated value of $\sigma_4$ differs with $\gamma_1$. In particular, when $\gamma_1 = 0.1$ we set $\sigma_4 = 0.18$ when $\gamma_1 = 0.2$ we set $\sigma_4 = 0.15$ and when $\gamma_1 = 0.3$ we set $\sigma_4 = 0.13$. Our calibrations are based on weekly data and sum of values of $T$ correspond to samples of 6 months, 1 year, 2 years and 5 years. $\rho_{12}$ is the correlation coefficient between the first differences of the log of prices of goods $i$ and $j$. $ADF_i$ is the augmented Dickey-Fuller test for a unit root in the log of the price of good $i$ minus the log of the price of good $j$ and $EG_{15}$ and $TR_{15}$ are the Engle-Granger and Trace statistics for cointegration between the logs of the prices of goods $i$ and $j$. Finally $GC_{14}$ is the test statistic for the null hypothesis that $\Delta \rho_i$ does not Granger cause $\Delta \rho_j$ based on the estimation of an unrestricted VAR for $p_1$, $p_2$, $p_3$. 

20
### Table 1a: Simulation Results - Firm Specific Cost Shocks Only, Δπ, Exhibits Negative Autocorrelation

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<th></th>
<th>γ12 = 0.1</th>
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<th>γ12 = 0.2</th>
<th></th>
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<td>1' - 52</td>
<td>1' - 104</td>
<td>1' - 26C</td>
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<td></td>
<td></td>
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Rejection Rates (nominal size 0.05):

<p>| | | | | | | | | | | | | |</p>
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See notes to Table 1a. The only change in the data generating process for this table is that the data is constructed such that the marginal cost process for each firm contains a unit root and the first difference of each of the price series exhibits negative 1st order autocorrelation with an autocorrelation coefficient of approximately −0.3. This is achieved by setting λ11 = 0.7 and λ12 = 0.3015 for all i.
Table 1B: Simulation Results - Firm Specific Cost Shocks Only $\Delta p_t$ exhibits positive autocorrelation

<table>
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<th>$\gamma_{12}$</th>
<th>$\Delta p_t$ exhibits positive autocorrelation</th>
<th>$\gamma_{12}$</th>
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<td>0.0043</td>
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</table>

Means

Rejection Rates 'nominal size 0.05'

| $GC_{10}$ | 0.1896                                      | 0.1737        | 0.1966                                        | 0.2327        | 0.1836                                        | 0.1790 |
| $GC_{31}$ | 0.1845                                      | 0.1748        | 0.1636                                        | 0.1608        | 0.1851                                        | 0.1760 |
| $GC_{19}$ | 0.1851                                      | 0.1752        | 0.1952                                        | 0.2390        | 0.1835                                        | 0.1774 |
| $GC_{29}$ | 0.1851                                      | 0.1779        | 0.1641                                        | 0.1542        | 0.1833                                        | 0.1763 |
| $GC_{18}$ | 0.1805                                      | 0.1845        | 0.1633                                        | 0.1582        | 0.1843                                        | 0.1737 |
| $GC_{28}$ | 0.1875                                      | 0.1813        | 0.1641                                        | 0.1592        | 0.1860                                        | 0.1808 |
| $ADF_{15}$ | 0.0617                                      | 0.0431        | 0.0371                                        | 0.0473        | 0.0554                                        | 0.0348 |
| $ADF_{13}$ | 0.0525                                      | 0.0342        | 0.0276                                        | 0.0251        | 0.0492                                        | 0.0261 |
| $ADF_{25}$ | 0.0532                                      | 0.0334        | 0.0268                                        | 0.0250        | 0.0498                                        | 0.0273 |
| $EG_{19}$ | 0.1044                                      | 0.1233        | 0.1714                                        | 0.2795        | 0.1013                                        | 0.1071 |
| $EG_{13}$ | 0.0845                                      | 0.0783        | 0.0916                                        | 0.1240        | 0.0795                                        | 0.0676 |
| $EG_{29}$ | 0.0851                                      | 0.0786        | 0.0947                                        | 0.1292        | 0.0823                                        | 0.0714 |
| $TR_{15}$ | 0.2905                                      | 0.2777        | 0.3136                                        | 0.4394        | 0.2806                                        | 0.2504 |
| $TR_{13}$ | 0.2657                                      | 0.2359        | 0.2551                                        | 0.3316        | 0.2515                                        | 0.2095 |
| $TR_{25}$ | 0.2715                                      | 0.2334        | 0.2567                                        | 0.3348        | 0.2602                                        | 0.2154 |

See notes to table 1a. Here, the data is constructed such that the marginal cost process for each firm contains a unit root and the first difference of each of the price series exhibits positive 1st order autocorrelation with an autocorrelation coefficient of approximately 0.3. This is achieved by setting $\lambda_{11} = 1.3$ and $\lambda_{12} = -0.2$ for all $t$. The standard deviations of the cost shocks are set such that the standard deviation of the 1st difference of the synthetic price data for goods 1 and 2 matches that of Classen and Newman in the spaghetti sauce data of Capps et al. (2003). This means that the calibrated value of $\sigma_i$ differs with $\gamma_{12}$. In particular when $\gamma_{12} < 0.1$ we set $\sigma_i = 0.14$ when $\gamma_{12} = 0.2$ we set $\sigma_i = 0.12$ and when $\gamma_{12} = 1.3$ we set $\sigma_i = 0.11$. 
### Table 2: Simulation Results - Firm Specific and Common Cost Shocks, $\Delta \phi_t$, Serially Uncorrelated

<table>
<thead>
<tr>
<th></th>
<th>$\gamma_{12} = 0.1$</th>
<th></th>
<th></th>
<th>$\gamma_{12} = 0.2$</th>
<th></th>
<th></th>
<th>$\gamma_{12} = 0.3$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$I = 26$</td>
<td>$I = 52$</td>
<td>$I = 104$</td>
<td>$I = 260$</td>
<td>$I = 26$</td>
<td>$I = 52$</td>
<td>$I = 104$</td>
<td>$I = 260$</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{12}$</td>
<td>0.8745</td>
<td>0.8345</td>
<td>0.8316</td>
<td>0.8273</td>
<td>0.8975</td>
<td>0.8980</td>
<td>0.8972</td>
<td>0.8964</td>
<td>0.9357</td>
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<tr>
<td>$\sigma_{13}$</td>
<td>0.4946</td>
<td>0.4792</td>
<td>0.4435</td>
<td>0.3925</td>
<td>0.5094</td>
<td>0.4933</td>
<td>0.4693</td>
<td>0.4227</td>
<td>0.5206</td>
</tr>
<tr>
<td>$\rho_{23}$</td>
<td>0.4933</td>
<td>0.4729</td>
<td>0.4432</td>
<td>0.3926</td>
<td>0.5086</td>
<td>0.4940</td>
<td>0.4692</td>
<td>0.4227</td>
<td>0.5199</td>
</tr>
</tbody>
</table>

Means

<table>
<thead>
<tr>
<th></th>
<th>$\rho_{12}$</th>
<th>$\rho_{13}$</th>
<th>$\rho_{23}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$GC_{11}$</td>
<td>0.2076</td>
<td>0.2236</td>
<td>0.2566</td>
</tr>
<tr>
<td>$GC_{12}$</td>
<td>0.2003</td>
<td>0.2146</td>
<td>0.2351</td>
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<tr>
<td>$GC_{13}$</td>
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<td>0.2276</td>
<td>0.2551</td>
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<tr>
<td>$GC_{14}$</td>
<td>0.1966</td>
<td>0.2154</td>
<td>0.2328</td>
</tr>
<tr>
<td>$GC_{15}$</td>
<td>0.2012</td>
<td>0.2083</td>
<td>0.2362</td>
</tr>
<tr>
<td>$GC_{16}$</td>
<td>0.2006</td>
<td>0.2164</td>
<td>0.2325</td>
</tr>
<tr>
<td>$ADF_{11}$</td>
<td>0.2036</td>
<td>0.1843</td>
<td>0.1907</td>
</tr>
<tr>
<td>$ADF_{12}$</td>
<td>0.1997</td>
<td>0.1763</td>
<td>0.1763</td>
</tr>
<tr>
<td>$ADF_{13}$</td>
<td>0.1980</td>
<td>0.1758</td>
<td>0.1794</td>
</tr>
<tr>
<td>$EG_{10}$</td>
<td>0.0976</td>
<td>0.1106</td>
<td>0.1737</td>
</tr>
<tr>
<td>$EG_{11}$</td>
<td>0.0877</td>
<td>0.0873</td>
<td>0.1204</td>
</tr>
<tr>
<td>$EG_{12}$</td>
<td>0.0882</td>
<td>0.0886</td>
<td>0.1248</td>
</tr>
<tr>
<td>$TR_{15}$</td>
<td>0.2701</td>
<td>0.2668</td>
<td>0.3468</td>
</tr>
<tr>
<td>$TR_{16}$</td>
<td>0.2528</td>
<td>0.2503</td>
<td>0.3127</td>
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<tr>
<td>$TR_{25}$</td>
<td>0.2484</td>
<td>0.2492</td>
<td>0.3146</td>
</tr>
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</table>

**Rejection Rates (nominal size 0.05)**

See notes to Table 1a. In these experiments the standard deviations of the firm specific shocks and the common shock are equal. The standard deviations of these cost shocks are set such that the standard deviation of the 1st difference of the log of the synthetic price data matches that of Classicc and Newman's in the spaghetti sauce data of Cappe et al (2003). This means that the calibrated values of $\sigma_{\nu}$ and $\sigma_{\psi}$ differ with $\gamma_{12}$. In particular, when $\gamma_{12} = 0.1$ we set $\sigma_{\psi} = \sigma_{\nu} = 0.15$ when $\gamma_{12} = 0.5$ we set $\sigma_{\psi} = \sigma_{\nu} = 0.17$ and when $\gamma_{12} = 0.3$ we set $\sigma_{\psi} = \sigma_{\nu} = 0.10$.

23
<table>
<thead>
<tr>
<th>$\gamma_{12} = 0.1$</th>
<th>$\gamma_{12} = 0.2$</th>
<th>$\gamma_{12} = 0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I - 26$</td>
<td>$I - 52$</td>
<td>$I - 104$</td>
</tr>
<tr>
<td>$\rho_{12}$</td>
<td>0.8214</td>
<td>0.8231</td>
</tr>
<tr>
<td>$\rho_{13}$</td>
<td>0.5038</td>
<td>0.4886</td>
</tr>
<tr>
<td>$\rho_{23}$</td>
<td>0.5025</td>
<td>0.4886</td>
</tr>
</tbody>
</table>

**Rejection Rates** (nominal size 0.05):

| $GC_{11}$ | 0.1965 | 0.2145 | 0.2395 | 0.2866 | 0.1941 | 0.2089 | 0.2230 | 0.2655 | 0.1916 | 0.1999 | 0.2050 | 0.2431 |
| $GC_{31}$ | 0.1924 | 0.2062 | 0.2263 | 0.2543 | 0.1846 | 0.2004 | 0.2147 | 0.2488 | 0.1818 | 0.1896 | 0.2033 | 0.2316 |
| $GC_{19}$ | 0.2003 | 0.2178 | 0.2344 | 0.2815 | 0.1986 | 0.2044 | 0.2218 | 0.2630 | 0.1940 | 0.1967 | 0.2073 | 0.2396 |
| $GC_{32}$ | 0.1881 | 0.2083 | 0.2252 | 0.2558 | 0.1826 | 0.1992 | 0.2105 | 0.2455 | 0.1760 | 0.1897 | 0.2014 | 0.2309 |
| $GC_{13}$ | 0.1947 | 0.2091 | 0.2275 | 0.2512 | 0.1890 | 0.2046 | 0.2144 | 0.2475 | 0.1878 | 0.1968 | 0.2033 | 0.2341 |
| $GC_{23}$ | 0.1974 | 0.2087 | 0.2247 | 0.2554 | 0.1903 | 0.2010 | 0.2187 | 0.2448 | 0.1863 | 0.1942 | 0.2032 | 0.2309 |
| $ADF_{11}$ | 0.5417 | 0.5991 | 0.6692 | 0.7272 | 0.5284 | 0.5822 | 0.6403 | 0.6923 | 0.5239 | 0.5706 | 0.6177 | 0.6569 |
| $ADF_{15}$ | 0.5292 | 0.5926 | 0.6478 | 0.6910 | 0.5234 | 0.5772 | 0.6165 | 0.6676 | 0.5184 | 0.5610 | 0.5975 | 0.6303 |
| $ADF_{28}$ | 0.5361 | 0.5968 | 0.6446 | 0.6952 | 0.5260 | 0.5809 | 0.6196 | 0.6657 | 0.5196 | 0.5680 | 0.5963 | 0.6326 |
| $EG_{19}$ | 0.0834 | 0.0921 | 0.1395 | 0.2457 | 0.0770 | 0.0749 | 0.1086 | 0.1841 | 0.0735 | 0.0657 | 0.0895 | 0.1456 |
| $EG_{13}$ | 0.0769 | 0.0767 | 0.0957 | 0.1529 | 0.0725 | 0.0614 | 0.0806 | 0.1205 | 0.0689 | 0.0582 | 0.0709 | 0.0984 |
| $EG_{28}$ | 0.0777 | 0.0755 | 0.1018 | 0.1528 | 0.0716 | 0.0687 | 0.0844 | 0.1217 | 0.0695 | 0.0622 | 0.0710 | 0.0992 |
| $TR_{11}$ | 0.2278 | 0.2267 | 0.2921 | 0.4166 | 0.2084 | 0.1971 | 0.2473 | 0.3427 | 0.1973 | 0.1756 | 0.2145 | 0.2869 |
| $TR_{15}$ | 0.2154 | 0.2126 | 0.2604 | 0.3544 | 0.2002 | 0.1900 | 0.2263 | 0.3006 | 0.1931 | 0.1750 | 0.2010 | 0.2595 |
| $TR_{28}$ | 0.2147 | 0.2124 | 0.2658 | 0.3535 | 0.2004 | 0.1920 | 0.2283 | 0.2981 | 0.1928 | 0.1768 | 0.2026 | 0.2578 |

See notes to Table 2a. The only change in the data generating process for this table is that the data is constructed such that the marginal cost process for each firm contains a unit root and the first difference of each of the price series exhibits negative 1st order autocorrelation with an autocorrelation coefficient of approximately $-0.3$. This is achieved by setting $\lambda_{11} = 0.7$ and $\lambda_{12} = 0.3015$ for all $i$. 
Table 2b: Simulation Results - Firm Specific and Common Cost Shocks. $\Delta P_i$ exhibits positive autocorrelation.

<table>
<thead>
<tr>
<th></th>
<th>$\gamma_{12} = -0.1$</th>
<th></th>
<th>$\gamma_{12} = -0.2$</th>
<th></th>
<th>$\gamma_{12} = -0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$t - 26$</td>
<td>$t - 52$</td>
<td>$t - 104$</td>
<td>$t - 260$</td>
<td>$t - 26$</td>
</tr>
<tr>
<td>$\omega_{12}$</td>
<td>0.8415</td>
<td>0.8401</td>
<td>0.8355</td>
<td>0.8324</td>
<td>0.9097</td>
</tr>
<tr>
<td>$\omega_{13}$</td>
<td>0.4924</td>
<td>0.4994</td>
<td>0.4395</td>
<td>0.3888</td>
<td>0.5006</td>
</tr>
<tr>
<td>$\omega_{23}$</td>
<td>0.4912</td>
<td>0.4698</td>
<td>0.4397</td>
<td>0.3891</td>
<td>0.4998</td>
</tr>
</tbody>
</table>

Rejection Rates (nominal size 0.05):

$GC_{21}$ | 0.2156 | 0.2256 | 0.2681 | 0.2975 | 0.2115 | 0.2274 | 0.2576 | 0.2885 | 0.2050 | 0.2056 | 0.2336 | 0.2640 |
$GC_{31}$ | 0.2087 | 0.2202 | 0.2387 | 0.2646 | 0.2056 | 0.2168 | 0.2302 | 0.2611 | 0.1980 | 0.2066 | 0.2183 | 0.2559 |
$GC_{12}$ | 0.2196 | 0.2325 | 0.2595 | 0.3038 | 0.2176 | 0.2333 | 0.2520 | 0.2924 | 0.2070 | 0.2024 | 0.2270 | 0.2667 |
$GC_{29}$ | 0.2096 | 0.2257 | 0.2337 | 0.2660 | 0.2037 | 0.2191 | 0.2282 | 0.2624 | 0.1970 | 0.2067 | 0.2156 | 0.2537 |
$GC_{13}$ | 0.2133 | 0.2185 | 0.2395 | 0.2616 | 0.2083 | 0.2071 | 0.2375 | 0.2582 | 0.1988 | 0.2021 | 0.2206 | 0.2448 |
$GC_{53}$ | 0.2134 | 0.2192 | 0.2396 | 0.2615 | 0.2086 | 0.2117 | 0.2316 | 0.2565 | 0.2019 | 0.2043 | 0.2187 | 0.2468 |

$ADF_{15}$ | 0.0677 | 0.0498 | 0.0498 | 0.0591 | 0.0637 | 0.0456 | 0.0443 | 0.0557 | 0.0531 | 0.0380 | 0.0337 | 0.0442 |
$ADF_{15}$ | 0.0636 | 0.0433 | 0.0425 | 0.0440 | 0.0601 | 0.0389 | 0.0371 | 0.0405 | 0.0554 | 0.0306 | 0.0290 | 0.0331 |
$ADF_{55}$ | 0.0645 | 0.0439 | 0.0413 | 0.0425 | 0.0608 | 0.0370 | 0.0374 | 0.0387 | 0.0548 | 0.0331 | 0.0303 | 0.0344 |

$EG_{12}$ | 0.1031 | 0.1133 | 0.1763 | 0.3056 | 0.0985 | 0.1038 | 0.1588 | 0.2796 | 0.0868 | 0.0844 | 0.1225 | 0.2031 |
$EG_{13}$ | 0.0938 | 0.0907 | 0.1216 | 0.1950 | 0.0884 | 0.0854 | 0.1109 | 0.1702 | 0.0834 | 0.0737 | 0.0896 | 0.1301 |
$EG_{39}$ | 0.0945 | 0.0936 | 0.1255 | 0.1945 | 0.0912 | 0.0874 | 0.1142 | 0.1696 | 0.0852 | 0.0795 | 0.0916 | 0.1325 |

$TR_{15}$ | 0.3014 | 0.2846 | 0.3565 | 0.5056 | 0.2944 | 0.2702 | 0.3316 | 0.4576 | 0.2747 | 0.2374 | 0.2723 | 0.3674 |
$TR_{15}$ | 0.2843 | 0.2677 | 0.3263 | 0.4391 | 0.2785 | 0.2489 | 0.3021 | 0.4017 | 0.2637 | 0.2258 | 0.2605 | 0.3355 |
$TR_{25}$ | 0.2857 | 0.2672 | 0.3271 | 0.4367 | 0.2769 | 0.2527 | 0.3067 | 0.3991 | 0.2599 | 0.2294 | 0.2594 | 0.3356 |

See notes to Table 2a. Here, the data is constructed such that the marginal cost process for each firm contains a unit root and the first difference of each of the price series exhibits positive 1st order autocorrelation with an autocorrelation coefficient of approximately 1.3. This is achieved by setting $\lambda_{11} = 1.3$ and $\lambda_{12} = -0.5$ for all $i$. The standard deviations of the two cost shocks are set such that the standard deviation of the 1st difference of the synthetic price data for goods 1 and 2 matches that of Classic and Newmans in the spaghetti-saus data of Capps et al. (2003). This means that the calibrated values of $\sigma_{\gamma V}$ and $\sigma_{\gamma}$ differ with $\gamma_{12}$. In particular, when $\gamma_{12} = 1.1$ we set $\sigma_{\gamma V} = \sigma_{\gamma} = 3.11$, when $\gamma_{12} = 0.2$ we set $\sigma_{\gamma} = 0.10$ and when $\gamma_{12} = 0.5$ we set $\sigma_{\gamma V} = \sigma_{\gamma} = 3.08$. 
Figure 1: Densities for $\rho_{12}$ and $\rho_{13}$; $\gamma_{12}$, 0.1 and $T$.
Figure 2: Densities for $GC_{21}$ and $GC_{31}$: $\gamma_{12} = 0.1$ and $T$
Figure 3: Densities for $DF_{12}$ and $DF_{13}$: $\gamma_{12} = 0.1$ and $T = 104$.
Figure 4: Densities for $EG_{12}$ and $EG_{12}^{-1}$, $\gamma_{12} = 0.1$ and $T' = 104$.
Figure 5: Densities for $TR_{12}$ and $TR_{13}$; $\gamma_{12} = 0.1$ and $T = 104$