An analysis of the dependence among financial markets by spatial contagion*

Fabrizio Durante† Enrico Foscolo‡

Abstract

Spatial contagion between two financial markets $X$ and $Y$ appears when there is more dependence between $X$ and $Y$ when they doing badly than when they exhibit typical performance. In this paper, we introduce an index to measure the contagion effects. This tool is based on the use of suitable copulas associated with the markets and on the calculation of the related conditional Spearman’s correlation coefficients. As an empirical application, the proposed index is exploited to create a clustering of European stock market indices in order to assess their behaviour in the recent years. The whole procedure is expected to be useful for portfolio diversification in crisis periods.

Keywords: Cluster analysis, Contagion, Copula, Financial asset returns, Fuzzy Cluster, Spearman’s conditional correlation.

1 Introduction

Financial contagion is usually referred to as the process that describes the spread of financial difficulties from one economy to others in the same region and beyond. Numerous theoretical papers have described the various channels by which contagion could occur, but empirical works often disagree on a common working definition of contagion. For instance, Pericoli and Sbracia [31] have specified five different notions of contagion stressing the various facets of this concept. For a recent overview about contagion, see [23] and the references therein.

In this paper, we consider contagion as “significant increase in comovements of prices and quantities across markets, conditional on a crisis occurring in one market or group of markets”. In other words, financial contagion is related to a change in the positive association among markets, when a group of markets is affected by a financial shock. Specifically, following the approach in [4] (see, also, [3, 2]), we say that there is contagion between market $X$ and market $Y$ if there

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is more dependence between $X$ and $Y$ when they doing badly than when they exhibit typical performance. Thus, contagion is related to the strength of the dependence between $X$ and $Y$ at different regions of the domain of the joint distribution of $(X, Y)$. For such a reason, we usually refer to such a specification as spatial contagion.

Starting with these ideas, Durante and Jaworski [15] (see, also, [14]) have proposed a spatial contagion test that is grounded on the concept of copula, which is a function that is able to capture the rank-invariant dependency among random variables. Such an approach is based on the determination of a suitable threshold $\alpha \in (0, 0.5)$ that draws a distinction between normal comovements, due to simple interdependence among markets, and excessive comovements.

In this work, we underpin the approach in [15] by deriving a related index of contagion, which is based on the conditional Spearman’s correlation coefficient.

Notice that the use of copulas (or copula-based coefficients) for checking contagion between financial markets is not new. [32] an analysis of financial contagion in Latin American and Asian countries through Markov switching copula model is given; an approach that has been also employed in [22]. An index of contagion based on copula densities has been introduced in [10]. Other approaches based also on volatility have been presented in [1, 30]. However, contrarily to the these cases, our methodology is essentially non-parametric and does not require the specification of a suitable copula family to model the dependence between the time series.

The paper is organized as follows. In Section 2 we review the basic ideas about contagion described in [15]. Then, we present the main ideas related to the new introduced index (Section 3) and the algorithm to calculate it for a group of different markets (Section 4). The procedure is hence used in order to check the presence of contagion among European markets in the last years (Section 5). In particular, the index is used to derive a dissimilarity matrix that could be used for implementing suitable cluster procedures in a crisp as well as fuzzy setting. Section 6 concludes.

2 Spatial contagion via copulas

As discussed in [15] a notion of spatial contagion can be introduced by using suitable copulas associated with the random variables $X$ and $Y$ representing the returns of two financial markets. We recall here some basic aspects that will be useful in the following.

Let $(X, Y)$ be a pair of continuous random variables (=r.v.’s) on the probability space $(\Omega, \mathcal{F}, P)$. It is well known that the dependence of $(X, Y)$ can be conveniently described by means of the copula associated with it. For basic definitions and properties about copulas, we refer the reader to [20, 11, 26]. Actually, this idea also extends to conditional distribution functions (=d.f.’s). In fact, if $B$ is a Borel set in $\mathbb{R}^2$ such that $P((X, Y) \in B) > 0$, we may consider the copula of the conditional d.f. of $(X, Y)$ given $(X, Y) \in B$. Such a copula is usually called threshold copula (see [28, 13]).

Now, let $X$ and $Y$ be the r.v.’s representing the returns of two financial markets. In the study of contagion, we are often interested (see, for instance, [24, 7]) in the conditional d.f. of $[X, Y \mid (X, Y) \in B]$, where the conditioning set $B \subset \mathbb{R}^2$ is of the following type:
• $T_{\alpha_1, \alpha_2} = [-\infty, q_X(\alpha_1)] \times [-\infty, q_Y(\alpha_2)]$

• $M_{\beta_1, \beta_2} = [q_X(\beta_1), q_X(1 - \beta_1)] \times [q_Y(\beta_2), q_Y(1 - \beta_2)],$

for some $\alpha_1, \alpha_2 \in \left]0, \frac{1}{2}\right] \text{ and } \beta_1, \beta_2 \in \left[0, \frac{1}{2}\right]$. Here $q_X$ and $q_Y$ are the quantile functions associated with $X$ and $Y$, respectively. Specifically, we may interpret such sets in the following way:

• $T_{\alpha_1, \alpha_2}$, which is called tail set, represents the “risky scenario”, since it includes the observations related to $X$ (respectively, $Y$) that are less than a given threshold.

• $M_{\beta_1, \beta_2}$, which is called central set (or mediocre set), represents “untroubled scenario”, and is related to the observations that are not judged to be extremal.

As showed in [15], spatial contagion can be introduced in terms of a suitable comparison between threshold copulas associated with a tail and a central set, as stated in the following definition. Here we recall that, given two copulas $C_1$ and $C_2$, we write $C_1 \prec C_2$ when $C_1(u, v) \leq C_2(u, v)$ for all $(u, v) \in [0, 1]^2$, but $C_1(u, v) \neq C_2(u, v)$ for at least one $(u, v) \in [0, 1]^2$.

**Definition 2.1.** Let $X$ and $Y$ be the r.v.’s representing the returns of two financial markets. There is symmetric contagion between the markets $X$ and $Y$ at the level $\alpha \in \left]0, \frac{1}{2}\right]$ if

$$C_{M_\alpha} \prec C_{T_\alpha},$$

where $C_{M_\alpha}$ is the copula of the conditional d.f. of $(X, Y)$ given $M_\alpha = [q_X(\alpha), q_X(1 - \alpha)] \times [q_Y(\alpha), q_Y(1 - \alpha)]$, while $C_{T_\alpha}$ is the copula of the conditional d.f. of $(X, Y)$ given $T_\alpha = [-\infty, q_X(\alpha)] \times [-\infty, q_Y(\alpha)].$

Intuitively speaking, there is contagion when the dependence among the markets increases in the case both the markets are doing bad. Thus, spatial contagion means that an additional amount of positive dependence is present in the tail of the joint distribution of $(X, Y)$ compared to the center of the distribution.

The above definition of contagion has the following interesting features:

• it is a distribution-based notion, since it is grounded on the comparison between the dependence in a tail region (the copula related to the tail set) and in a central region (the copula of the central set) of the joint d.f. of $(X, Y)$;

• being based on copulas, it is more informative than other methods based on Pearson’s correlation coefficient or tail dependence coefficients;

• it is strictly dependent on a suitable definition of tail (and central) set, according to the threshold $\alpha$;

• it appears when there is a strict order between the copulas; for instance, two markets that are perfectly comonotone (i.e., one market is an increasing function of the other) exhibit no contagion, since their dependence does not change at any time (in fact, these markets are simply interdependent, as clarified by [17]).
Checking directly spatial contagion by means of the comparison of copulas as in Definition 2.1 could be difficult to implement. In fact, in general, the calculation of threshold copulas is not easily manageable. In order to avoid such troubles, a non-parametric procedure as been suggested by [15]. The idea is based on the following fact. Given two copulas \( C \) and \( D \), if \( C \prec D \), then \( \rho(C) \leq \rho(D) \), where \( \rho \) is the Spearman’s rank correlation coefficient, given, for any pair of continuous r.v.’s \( X \) and \( Y \) with copula \( C \), by

\[
\rho(C) = 12 \int_{[0,1]^2} C(u, v) \, du \, dv - 3
\]

(for more details, see [33]). Then, we might check the absence of contagion by comparing not the copulas, but the values of the associated Spearman’s rank-correlation. For instance, the following hypothesis test can be performed:

\[
H_0: \ \rho(C_{T_n}) \leq \rho(C_{M_n}) \quad \text{ (no contagion)}
\]

against

\[
H_1: \ \rho(C_{T_n}) > \rho(C_{M_n})
\]

where \( C_{T_n} \) and \( C_{M_n} \) are the copulas associated with the conditional d.f. of the markets \( X \) and \( Y \) with respect to tail and central set, respectively.

A formal test of contagion may have an inconvenience that could limit its practical use. In fact, it cannot quantify the size of the “jump” in the comovement, an information that could be instead useful in order to classify different markets according to their “extreme” linkages.

In order to cover also this aspect, an empirical measure of contagion is proposed in the following.

### 3 The Spearman’s contagion index

In order to provide an empirical measure of contagion, we propose here the so-called Spearman’s Contagion Index (briefly, SCI), which is based on the idea previously described.

**Definition 3.1.** Let \( X \) and \( Y \) be the r.v.’s representing the returns of two financial markets. The SCI at level \( \alpha \in (0, 0.5) \) is defined via the formula

\[
SCI = \frac{\rho(C_{T_n}) - \rho(C_{M_n})}{2}
\]

As noted in section 2 (see also [15]), fixed a level \( \alpha \), a contagion test is based on the behaviour of

\[
\Delta \rho(\alpha) = \rho(C_{T_n}) - \rho(C_{M_n})
\]

a statistics that quantifies the difference between Spearman’s conditional correlation in a tail and a central set. The SCI is exactly based on such a difference up to a normalizing constant.

The advantage of SCI is that it is well know that its empirical estimation \( \Delta \hat{\rho}(\alpha) \) could be easily calculated from some available data by using the empirical version of Spearman’s correlation.
Moreover, under some technical assumptions on the involved threshold copulas, as the length of the time series \( n \to +\infty \),

\[
\sqrt{n} \left( \Delta \hat{\rho}(\alpha) - \Delta \rho(\alpha) \right) \xrightarrow{d} \mathcal{N}(0, \sigma^2_{T_n,M_n}).
\]  

(3.2)

For more details, see [15].

The value of SCI may be interpreted as follows. If \( SCI > 0 \), then \( \rho(C_{T_n}) > \rho(C_{M_n}) \), which could indicate the presence of contagion. Analogously, if \( SCI < 0 \), then \( \rho(C_{T_n}) < \rho(C_{M_n}) \) and contagion does not occur. If there is no change in the conditional correlation between tail and central set, then \( SCI = 0 \), meaning that no shift of the dependence structure is realized.

**Remark 3.1.** It has been recognized by [14] that the occurrence of contagion strongly depends on the choice of the threshold, a fact that could represent a limitation when one wants to draw conclusions from the empirical analysis. To provide an assistance to the decision maker, here it is suggested to choose a threshold \( \alpha = 0.10 \) or \( \alpha = 0.25 \).

### 4 Calculation of the Spearman’s contagion index

Here we describe the algorithm for calculating the SCI between a set of different markets. Before proceeding, a preliminary comment is necessary.

SCI is based on the calculation of conditional Spearman’s correlation coefficients in specific regions of the domain of the joint distribution. However, as pointed out, for instance, by [17], the estimation of these coefficients may be problematic, since the choice of conditioning event and the heteroscedasticity in returns of financial markets may lead to spurious conclusions. In other words, one might mistakenly believe that a change of structure of the dependence of two markets has taken place when in fact this is merely an artefact of the heteroscedasticity of the individual return time series and/or choice of conditioning event. In order to avoid such problems, before calculating SCI, we remove the heteroscedastic effects from the univariate time series, as will be clear from the procedure we are going to illustrate.

The calculation of the SCI between two time series \( X_1 \) and \( X_2 \) representing two financial markets goes as follows.

**(Load the data)** For \( i \in \{1, 2\} \) let \( P^i_t \) be a time series from a stock market index. Then, calculate \( L^i_t \) be the time series of the log-returns defined as \( \log(P^i_t / P^i_{t-1}) \).

**(Fit a joint model)** Following a copula approach, the joint model of \( (L^1_t, L^2_t) \) may be determined in two steps (see, for instance, [9, 29]): first, we select a model for the marginal times series; then, we choose a suitable copula between them. We suppose that, for \( i \in \{1, 2\} \), \( L^i_t \) is modelled by an ARMA–GJR–GARCH process [18] with innovation distribution being Student distributed; while the innovations are assumed to be independent and identically distributed with a common copula \( C \). Such a model takes into account possible differences of market behaviour in bearish and bullish periods.
Symbol | Stock Market
--- | ---
AEX | Netherlands
ATHEX20 | Greece
ATX | Austria
BEL20 | Belgium
CAC40 | French
DAX30 | Germany
FTSE-MIB | Italy
IBEX35 | Spain
ISEQ | Ireland
LUXXGEN | Luxembourg
OMXH | Finland
PSI20 | Portugal

Table 1: Stock market index and related symbols.

(Find the residuals) After applying the ARMA–GJR–GARCH filter to each univariate time series, the standardized residuals \((R_1^t, R_2^t)\) can be recovered. These will be rescaled to \([0, 1]^2\) via the respective empirical cumulative distribution functions by obtaining the sample \((S_1^t, S_2^t)\). Such a rescaling has no effect in the calculation of Spearman’s \(\rho\) that is invariant under rank transformation.

(Calculate the SCI) For fixed \(\alpha \in (0, 0.5)\), calculate:

- the empirical version \(\hat{\rho}(C_{T, \alpha})\) by computing the Spearman’s correlation of the observations \((S_1^t, S_2^t)\) such that \(S_1^t \leq \alpha\) and \(S_2^t \leq \alpha\);
- the empirical version \(\hat{\rho}(C_{M, \alpha})\) by computing the Spearman’s correlation of the observations \((S_1^t, S_2^t)\) such that \(\alpha \leq S_1^t \leq 1 - \alpha\) and \(\alpha \leq S_2^t \leq 1 - \alpha\);
- Return the empirical value of SCI given by

\[SCI_{12} = \frac{(\rho(C_{T, \alpha}) - \rho(C_{T, \alpha}))}{2}.\]

5 An empirical analysis for European stock market indices

The data we are considering are related to the stock market indices listed in Table 1 in the last 10 years. Specifically, the indices refers to all the countries that in the whole period August 2002 – July 2012 have used the Euro as national currency. To alleviate non synchronous trading hours in the past we stick to a weekly frequency (see, e.g., [8]). The data have been downloaded from Datastream.

Following the procedure described in Section 4, each univariate time series of log–returns has been fitted with an ARMA(1,1)–GJR–GARCH(1,1) model with Student-t distributed errors to account for heavy tails. Parameter estimates are available upon request from the authors.
Table 2: ARMA(1,1)–GJR–GARCH(1,1) residual diagnostic test for the univariate time series.

<table>
<thead>
<tr>
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<th>BP (1)</th>
<th>BP (4)</th>
<th>LB (1)</th>
<th>LB (4)</th>
<th>ARCH (1)</th>
<th>ARCH (4)</th>
</tr>
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<td>0.5784</td>
<td>0.8211</td>
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</tr>
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<td>0.5314</td>
<td>0.8555</td>
<td>0.2497</td>
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<td>0.9476</td>
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</table>

For all time series we then perform Box-Pierce and Ljung-Box to check for residual autocorrelation, and ARCH test for residual volatility at lag 1 and 4. As shown in Table 2, our main assumptions are globally satisfied at the standard significance level of 5%.

Thus, we may calculate the SCI for all the considered stock indices. The resulting SCI matrix is reported in Table 3 for $\alpha = 0.10$ and in Table 4 for $\alpha = 0.25$, respectively. As can be immediately noticed, the matrix present quite different values. In fact, for $\alpha$ becoming smaller, contagion effects seem to disappear, as already noticed in [14]. In most of the pairs, we do not see clear evidence of contagion effects. These findings are in line with the general idea that usual contagion tests are biased and inaccurate due to heteroskedasticity. Nevertheless, when heteroscedasticity is removed, the occurrence of contagion remains questionable (compare with [17]).

In order to show the contagion effects, it could be convenient to transform the SCI matrix into a dissimilarity matrix in order to obtain suitable clusters that could split the original group of markets into different subsets according to their contagion effects. This procedure is quite common in the financial literature: in [27] a Pearson correlation matrix is used as a starting point; clustering methods based on tail dependence measures of association are used in [12, 16]; methods based on symbolic time series analysis are given, for instance, in [6, 5].

To this aim, the SCI matrix containing the contagion effects among the markets is transformed into a dissimilarity matrix, i.e. a symmetric matrix with non-zero entries apart from the main diagonal (when the entries are 0’s). This could be done, for instance, by considering the matrix $\Delta = (d_{ij})$, where $d_{ij} = (1 - SCI_{ij})/2$ and $SCI_{ij}$ denotes the SCI between market $i$ and market $j$. In fact, if $d_{ij} = 1$, then markets $i$ and $j$ have SCI equal to −1, thus they are very dissimilar in the extreme behaviour and should possibly belong to different clusters. Instead, if $d_{ij} = 0$, then markets $i$ and $j$ have SCI equal to 1, thus they are very similar in the extreme behaviour.

The matrix $\Delta$ could be used as a starting point for considering various clustering procedures.
Table 3: SCI index (multiplied by 100) for the considered stock exchange indices. $\alpha = 0.10$. 

<table>
<thead>
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<th>ATX</th>
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<td>-18.69</td>
<td>5.40</td>
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</table>
Table 4: SCI index (multiplied by 100) for the considered stock exchange indices. $\alpha = 0.25$. 

<table>
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<tr>
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<th>IBEX35</th>
<th>ISEQ</th>
<th>LUXGEN</th>
<th>OMXH</th>
<th>PSI20</th>
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<td>10.26</td>
<td>13.65</td>
<td>7.88</td>
<td>17.81</td>
<td>20.33</td>
<td>11.96</td>
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<tr>
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<td>15.92</td>
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<td>13.40</td>
</tr>
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<td>12.40</td>
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<tr>
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<td>9.77</td>
<td>10.69</td>
<td>0.00</td>
<td>15.57</td>
</tr>
</tbody>
</table>
in the given dataset. One may consider, for instance, the complete linkage method, which has proved to be useful in a variety of situations (see, for instance, [19]), as well as single linkage and average linkage. In this work, we find quite useful the use of a fuzzy clustering procedure, as described by [21] (“fanny”). The calculation have been performed by using the R package “cluster” [25].

The fuzzy clusterings for $\alpha = 0.10$ and $\alpha = 0.25$ are displayed in Figure 1 and Figure 2, respectively. As it is apparent, in the extreme scenario ($\alpha = 0.10$) the clusters seems to be more evident than in the other case.

The graphs provided below may give a graphical tool for pointing different subgroups in a set of markets. In particular, for diversification purposes, one may be interested in selecting assets from different subgroups in order to mitigate the effects of simultaneous sharp downturns.

6 Conclusions

Spatial contagion between two financial markets $X$ and $Y$ appears when there is more dependence between $X$ and $Y$ when they doing badly than when they exhibit typical performance. In this paper, an index to measure the contagion effects between a group of markets is provided. It is based on the use of threshold copulas associated with $(X, Y)$ and on the calculation of the related Spearman’s correlation coefficients.

As an empirical application, the new procedure is applied in order to create a fuzzy clustering
of European stock market indices in the last years.

We observe that detecting the contagion effects are crucial ingredients for successful portfolio management. In fact, an increase in correlation during turbulent market conditions usually results in a reduction of benefits from portfolio diversification.

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**References**


